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STRATEGIC AND TACTICAL ALLOCATION TO COMMODITIES FOR RETIREMENT SAVINGS SCHEMES

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Strategic and Tactical Allocation to Commodities for Retirement Savings Schemes[*]

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Abstract

We examine whether the variance risk of investment portfolios of pension schemes investing in traditional asset classes can be reduced by extending the set of traditional investment opportunities with commodities We investigate the economic and statistical significance of shifts in the strategic (three year), myopic (uarterly), and tactical (quarterly rebalancing) mean-variance frontier for pension schemes with a fixed liability portfolio. We find substantial differences in optimal strategic allocations for pension schemes with nominal and inflation-indexed pensions. While our results suggest that commodities reduce the risk on the funding ratio from an inflation-indexed scheme more than 30 percent, the optimal expected return and risk trade-off is unaffected for pension schemes with nominal claims. Similar results are obtained for the unconditional myopic investor with a quarterly investment horizon. When conditioning information about the macro economic situation is used, a pension scheme with nominal claims can during certain periods also improve its efficient risk-return trade-off by investing in commodities. Moreover, we investigate the use of quarterly timing strategies switching between commodities and stocks, in addition to the buy-and-hold investments in the traditional assets and commodities. Both for nominal and real pension schemes, timing strategies can be useful in addition to the strategic allocation. The liability hedging property of commodities is likely to reduce the probability of underfunding.

Keywords: Asset Liability Management, Commodities, Optimal portfolio choice, Pension funds, Strategic asset allocation, Tactical asset allocation

JEL classification: G11, G18, G23

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1 Introduction

Institutions such as insurance companies and pension funds are investigating the benefits of investing part of their wealth in alternative asset classes. Recently, investments in commodities, inflation-indexed bonds, and hedge funds have become increasingly popular. Especially the trade-off between risk and return in these relatively unknown asset classes and the portfolio implications of investing part of their wealth in these asset classes are not yet fully explored. This paper aims to shed further light on the benefits of investing in commodities for investors with a liability structure sensitive to the nominal or real interest rate and inflation.

The interest in commodity investments for institutions dates at least back to Bodie (1980), who points out the potential benefits of commodities for pension funds. In the following years, several papers have confirmed the risk reducing characteristics of commodities. Froot (1995) suggests that commodities are better diversifiers than, for example, real estate and stocks of commodity-related companies. Chow et al. (1999) indicate that commodities can be particularly valuable in adverse economic circumstances, when other alternatives tend to correlate more with traditional assets. These studies focus on nominal asset returns only and find that the benefits of investing in (derivatives on) commodities are most pronounced for investors with high risk aversion. For many investors the optimal portfolio is determined by a trade-off between expected return and volatility of the surplus of assets minus liabilities rather than by the distribution of asset returns only. Individuals or institutions that anage a defined contribution pension scheme will often be primarily concerned with the surplus of the asset value over the discounted value of expected future payments in real terms. They will therefore take interest and inflation risks in the liabilities into account when selecting their asset portfolio. This is a fortiori true for defined benefit schemes. Individuals or institutions with nominal future liabilities will take the interest risk involved into account. Commodities might be particularly suitable for pension savings because of their positive relation with inflation.

The first contribution of this paper is our examination of the benefits of investing in commodities for investors with financial liabilities. We treat these financial liabilities as fixed, because there is in general no liquid market to trade these pension claims. The terminology 'fixed' indicates that we assume that desired annual future cash flow is predetermined, not that the value of the liabilities does not change over time. This is an extension of the asset-only approach, which is used in the existing literature. Incorporating correlations between the returns on assets and liabilities may lead to substantially different portfolio weights in an optimal asset allocation than in the traditional asset-only approach. We consider two types of pension schemes, one with liabilities in nominal terms, and one with liabilities in real terms (i.e. protected against inflation).

Our second contribution is that in addition to the existing evidence on the *economic* significance of adding commodities to an existing portfolio of stocks and bonds, we also provide evidence on the *statistical* significance of the outward shift of the mean-variance frontier. Although Bodie (1980) and Froot (1995) provide empirical evidence on the benefits of commodities as alternative investments, a statistical analysis on the importance of the shift of the mean-variance frontier is not included. These tests for significant improvements are based on regression analysis; see, e.g., Huberman & Kandel (1987).

Finally, we contribute by investigating multiple investment horizons. We distinguish asset allocation for a long-term strategic (three-year), and short-term myopic and tactical (three-month) horizon.[1] For the short-term cases we investigate whether commodities expand the three-month buy-and-hold frontier, but also whether these short-term deviations expand the strategic buy-and-hold frontier. Since there is some evidence that expected returns and covariances change over time, we examine both unconditional and conditional spanning of commodities by the traditional asset classes. Whereas tests for unconditional spanning indicate whether commodities expand the mean-variance frontier without using information about the current economic state, conditional spanning tests answer whether the frontier shifts given the economic state we are currently in. We also aim to answer whether quarterly tactical timing strategies between commodities and stocks further reduce the variance risk of efficient strategic portfolios.

The current discussion among regulators in the European Community concentrates on

[1] We are aware that strategic might refer to many, possibly longer, investment horizons than used in the paper. However, longer horizons are rarely used for buy-and-hold strategies. Note that we consider a one-period model, and hence the three-year horizon is in some sense also myopic.

introducing/relaxing restrictions on the asset side, as well as finding a “fair value” for pension claims on the liability side. A lively debate was triggered by the Spanish government proposing restrictions on investments in alternative asset classes and derivatives by pension funds. The decision has been taken that pension funds that operate in across country borders in the European Community are severely restricted in their use of derivatives and investments in alternatives.[2] In this light, our paper aims at answering whether alternative asset classes such as commodities might reduce overall risk, and hence we investigate whether introducing restrictions for these asset classes are more likely to harm then protect the fund’s participants.

Our results indicate that there is a substantial difference in the optimal asset allocation for a pension scheme with nominal liabilities compared to a scheme that compensates its beneficiaries with inflation. Within the framework analyzed here, a pension scheme with nominal liabilities that already optimally invests in long-term government bonds, domestic stocks, and foreign stocks, cannot significantly improve the trade-off between expected return and volatility of the portfolio by investing in commodities. In contrast to pension schemes with nominal liabilities, pension schemes with real liabilities can significantly (both economically and statistically) improve the strategic risk-return trade-off by investing part of their wealth in commodities. This is due to the inflation-hedge provided by commodity investments.

In the quarterly myopic setting, when information about the economic situation is ignored, the spanning hypothesis is not rejected for the nominal scheme, but is rejected for the inflation-indexed scheme. This result was also obtained from the strategic spanning tests. However, when we use conditional information about the bond yield, term spread, default spread, and inflation, it turns out that commodities can shift the mean-variance frontier significantly outward for pension schemes with nominal liabilities. Thus, even when from a strategic perspective commodity returns are spanned by the traditional assets, at a quarterly horizon they might provide additional diversification benefits using dynamic strategies. Tactical timing strategies between commodities and stocks may shift the strategic frontier even further outward, suggesting that active short-term investment strategies can reduce the long-term portfolio risk even further.

[2]See for example ‘Spain to put the clock back’ (*Investments and Pensions Europe*, March 2002, p. 2) and Legge (2002) for comments on the recent proposal of pension reforms in the European community.

The remainder of this paper is organized as follows. In Section 2, we investigate the strategic mean-variance efficient (MVE) frontier for a pension scheme with fixed liabilities (in real or nominal terms). In particular, we examine whether a strategic allocation to commodities expands the MVE frontier. In Section 3, we examine the optimal asset allocation for a myopic investor with a quarterly investment horizon. We analyze unconditional spanning and moreover allow the use of macro economic information to determine the optimal expected risk-return trade-off. In this section we also aim to answer whether tactical timing strategies between commodities and stocks expand the MVE frontier of strategic buy-and-hold portfolios even further. In Section 4 we perform some robustness analyses. Finally, Section 5 concludes.

2 Strategic asset allocation

The portfolio problem of investors in pension schemes is defined in return on assets *relative* to return on liabilities rather than the usual asset-only approach. For a defined benefit pension scheme the volatility of the *return on assets relative to liabilities* is of concern rather than the asset returns themselves. Thus, high volatility in asset returns is not necessarily perceived as risky by these investors, because the correlation of assets with liability returns determines the volatility of the net position of the scheme.

We assume that the investor has a mean-variance utility function in the return on the funding ratio, i.e.,

$$U(R_t^{FR}) = \mathrm{E}\{R_t^{FR}\} - \gamma \mathrm{Var}\{R_t^{FR}\}, \tag{1}$$

where U denotes the utility function, γ the investor's risk aversion, and R_t^{FR} the return on the funding ratio. The funding ratio is defined as the value of assets devided by the discounted value of liabilities. The return on the funding ratio is defined as the difference between the return on the assets and liabilities,

$$R_t^{FR} = R_t^A - R_t^L, \tag{2}$$

where R_t^A, and R_t^L are the return on the assets and liabilities.[3] When the investor has no liabilities, the problem reduces to the usual mean-variance optimization problem. This approach also fits with the full surplus maximization for a pension fund with funding ratio equal to one, as indicated by Sharpe & Tint (1990).[4] Since we analyze a pension scheme with utility derived from the mean and variance on the funding ratio instead of the surplus, our approach does not depend on the initial funding ratio; see, e.g., Leibowitz, Kogelman & Bader (1994). The optimal portfolio weights can straightforwardly be derived to equal

$$w^{opt} = \gamma^{-1}\Sigma_{RR}^{-1}(\mu_R - \eta\iota) - \Sigma_{RR}{}^{-1}\Sigma_{RL}\mu_L, \tag{3}$$

where μ_R and μ_L are the expected return on the assets and liabilities, Σ_{RR}, is the variance matrix of asset returns, and Σ_{RL} the covariance between assets and liabilities. Note that η is known as the the zero-beta rate, and is a function of γ.[5] The first term of this expression is the asset-only optimal portfolio. The second term in equation (3) accounts for the covariance between the returns on the assets and liabilities. Obviously, if these returns are uncorrelated, the optimal portfolio does not change. A positive correlation between the asset and the liabilities leads to an increase of the weight of this asset in the optimal portfolio, since it decreases the volatility of the funding ratio.

We examine two types of pension schemes in the analyses below. First, we consider a pension scheme that pays nominal pensions to its beneficiaries. The second scheme is committed to pay real (or inflation-indexed) pensions. The value of pension liabilities is not always easy to determine, since there is frequently no liquid market in which these claims are traded. This holds especially for price- or wage-inflation indexed liabilities; see, e.g., Head et al. (2000). For simplicity, this fact is ignored in this paper, and funding ratios are

[3] This is due to log-approximation of the definition of the return on the funding ratio,

$$\ln\{1 + R_1^{FR}\} = \ln\{FR_1\} - \ln\{FR_0\} = R_1^A - R_1^L$$

[4] In their setup, the surplus is defined as $S_t(k) = A_t - k \cdot L_t$, with k the 'importance' of liabilities, with $k = 1$ for a full surplus optimization. Dividing the surplus at the end of next period by the current value of the assets yields shows that suplus maximization is equal to maximization of $R_t^A - k \cdot FR_0^{-1} \cdot R_t^L$. Our funding ratio return is the same as surplus maximization when $k = FR_0$.

[5] There is a one-to-one relation between the zero-beta rate η and the risk aversion parameter γ, $\gamma = \mu'\Sigma^{-1}\iota - \iota'\Sigma^{-1}\iota \cdot \eta$. In this notation, μ is the expected return, and Σ the variance matrix of the returns. The elements of vector ι are all equal to one.

determined using market based valuation of liabilities as if they are fully liquid. The valuation concepts are described in more detail below.

In our first stylized pension scheme, the claims are in *nominal* terms. The return on the (marked-to-market nominal) liabilities is primarily driven by the changes in the yield of bonds. The liabilities can be viewed as a portfolio of nominal bonds and hence appropriate valuation techniques from this line of literature can be used to obtain a "fair" or market value.[6]

The duration of the portfolio of nominal bonds depends on the characteristics (such as age) of the beneficiaries of the pension scheme. We analyze schemes as if the there is one claim that has to be paid 10 years from now.[7] The value of the liabilities for pension schemes with a high duration is sensitive to changes in the nominal interest rate. The return on the nominal liabilities is[8]

$$RNOM_t^L = y_t^{(DUR-1)} - DUR \cdot \left(y_t^{(DUR-1)} - y_{t-1}^{(DUR)} \right), \tag{4}$$

where DUR is fixed over time (in our example equal to 10 years), and y_t the nominal yield at the end of period t.

The second example considers a pension scheme with future claims in *real* terms rather than nominal terms as in the previous example. This is more rational from an economic perspective and in line with the actual practice in Europe in the last decades, although the indexation of pension claims is much debated recently since the substantial world-wide drop in stock prices in the beginning of this decade. The payment of real benefits complicates the computation of fair values for the liabilities, since this means that future claims should be discounted with the *real* yield instead of the *nominal* yield. Since the US started issuing index-linked bonds only in 1997, historical data about the real yield is not directly available for our empirical analysis. However, Bridgewater has modeled the historical expected inflation and hence have come up with estimated yields of index-linked bonds as if they have been traded from 1970.[9] The value of the liabilities can be regarded as an index-linked bond, and

[6] In this paper we use government bond yields to value the liabilities of pension schemes.
[7] This maturity can also be interpreted as the average duration of the total portfolio of claims.
[8] See, for example, Campbell, Lo & MacKinlay (1997), p. 398.
[9] See also `http://www.bwater.com/research_ibonds.htm`.

the valuation methods for these assets can be used to determine the value of the liabilities. We again assume that the value of the liabilities is only affected by the real yield. Note that we assume that the inflation risk premium is zero. The return on the liabilities is now

$$\widetilde{R}_t^L = \widetilde{y}_t^{(DUR-1)} - \widetilde{DUR} \cdot \left(\widetilde{y}_t^{(DUR-1)} - \widetilde{y}_{t-1}^{(DUR)} \right), \tag{5}$$

where $\widetilde{\cdot}$ refers to real variables rather than nominal ones. For the duration of the portfolio of liabilities we again use 10 years. The nominal return on the real liabilities is the real return plus the realized inflation. Thus,

$$RREAL_t^L = \widetilde{R}_t^L + \pi_t, \tag{6}$$

where π_t is the annual inflation rate. Thus, assets that reduce the risk of the portfolio should either be positively correlated with inflation, the real interest rate, or changes in the real interest rate. For portfolios with long durations, the latter component is the most relevant when real yields are changing substantially.

In the remainder of this section we analyze whether it is optimal for the two stylized pension schemes to add commodities to its strategic asset allocation consisting of domestic government bonds, domestic stocks, and international stocks. The strategic investment horizon we investigate is three years. During this period we do not rebalance the portfolio to maintain the initial weights, but analyze a buy-and-hold portfolio instead. This assumption is relaxed when investigating tactical timing strategies in Section 4.[10]

Our sample period is from January 1970 to December 2001. In order to gain efficiency, we do not use 10 tri-annual observations, but make use of monthly overlapping tri-annual samples. Thus, the first observation is from Jan-1970 until Dec-1972, the second from Feb-1970 to Jan-1973, etcetera. This way, the number of observations is increased substantially. Since the error terms in a regression model with overlapping samples are by definition autocorrelated, we use the Newey & West (1987) method to account for heteroskedasticity and autocorrelation of general form. The interpretation of this correction method is that we take

[10]See Campbell & Viceira (2002) for long-term investors who quarterly rebalance their strategic portfolios.

into account that we are using the same information several times.

We take as traditional assets the Ibbotson long-term government bond index, the MSCI USA (domestic stocks), and the MSCI EAFE (foreign stocks).[11] The alternative asset offered to the investor is the Goldman Sachs Commodity Total Return Index (GSCI). This is a fully cash-collateralized index of commodity futures; see Ankrim & Hensel (1993) for a description. This index reflects the return of an investor that is restricted to take full cash collateralization. In practice other assets than cash may serve as collateral for the futures which implies that the use of the GSCI yields a lower bound for the potential for commodity strategies. In Section 4.3 we analyze the impact of the assumption about the fully cash-collateralized commodity futures position. The use of the GSCI total return index is common in this line of literature, see Appendix A for a short summary. For reasons of brevity, we refer to this investment object simply as *commodities* in the remainder of this paper.

The descriptive statistics of the log returns on the assets are displayed in Table 1. Panel A and B contain the tri-annual, and Panel C and D the quarterly statistics. The highest average returns are obtained by investing in domestic stocks, which returned on average 12.2 percent over a three year horizon. Bonds have the lowest average return, 8.70 percent. Bond returns also have the lowest volatility, and commodity returns are the most volatile. The correlation matrix suggests a potential diversification benefit from investing in commodities, since correlations between commodities and traditional assets are negative at both horizons.

The descriptive statistics of the returns on the liabilities with a duration of 10 years can also be found in Table 1.[12] The correlations between the traditional assets and the liabilities are presented in Panels B and D. Note that because variables such as inflation rates and yields are highly autocorrelated, the covariance structure at the larger horizon differs substantially form the one at the quarterly horizon. From the correlation structure it becomes clear that future benefits defined in nominal or real returns matters for the correlation structure between assets and liabilities. Whereas over our sample period long-term government

[11]We also included the North American Real Estate Investment Trust (NAREIT) index, but this does not materially alter our results.

[12]The tri-annual returns on nominal liabilities are calculated as follows: $R_t = \left[(1+Y_t^{(7)})^3 - 1\right] - 10 \cdot \left(Y_t^{(7)} - Y_{t-36}^{(10)}\right)$, where the yields Y are expressed in annual terms, and t in months. The tri-annual returns on real liabilities are the aggregated monthly log returns of the Bridgewater index-linked bond return series.

bonds correlate for 0.96 with nominal liabilities, the correlation with real liabilities is negative. This observation is crucial for our results. The opposite holds for the alternative asset class commodities, which correlates negatively with the nominal, but positive with the real liabilities. In fact, commodities are the *only* asset from our set with a positive correlation with real liabilities, indicating that inclusion of these assets can lead to risk reduction for pension schemes with real liabilities.

In Figure 1 the mean-variance frontiers for the nominal pension scheme are plotted. One frontier is constructed using only the three traditional assets, while the other frontier also allows investments in commodities. There is little difference in both frontiers, suggesting that the expected return of the funding ratio can hardly be increased with keeping the volatility equal. This is due to the close relation between the investments in government bonds and the nominal liability structure. Adding commodities leaves the efficient risk-return trade-off virtually unchanged. The expected return on the minimum variance portfolio is negative, indicating that investing in the lowest-risk strategy results in an expected deterioration of the solvency of the pension scheme.

The mean-variance frontier of the stylized pension scheme with real benefits is depicted in Figure 2. This figure is different from the previous one in several respects. First, the expected return on the minimum-variance portfolio is positive, which means that for the portfolio with lowest funding ratio volatility the solvency of the scheme increases rather than decreases. Second, the volatility is much larger than for the nominal scheme, i.e. 14 percent versus 5 percent on a three year horizon. This is due to the mismatch between the value of the investable assets and the inflation-indexed pensions. Lastly, visual inspection suggests that while for nominal pension obligations the addition of commodities does not increase the investment opportunity set, for real pension schemes they provide additional diversification. The volatility of the funding ratio reduces in some cases even more than 30 percent.

At several points of the frontiers in Figure 1 and 2 the strategic weights to commodities in the new optimal portfolio are displayed. For the nominal pension scheme the weights are almost zero near the minimum-variance portfolio, and 20 percent at the expected return of 4 percent per annum. Even when the optimal weight is 20 percent, the volatility of the funding ratio seems to be reduced only marginally. The strategic weights in the inflation-index pension scheme are substantially higher on the efficient side of the frontier, amplifying the diversification benefits of commodities for all risk-averse investors.

We are not aware of papers investigating the properties of commodities in strategic asset allocation. Our qualitative results, however, are consistent with conclusions based on a short term investment horizon from Bodie (1980) and Froot (1995). These papers deal, however, with short term (annual or quarterly) asset-only investors and ignore possible correlation with the liability structure of the pension scheme.

The aforementioned papers lack to test for statistical significance of the shifts in the mean-variance frontier. Huberman & Kandel (1987) show how regression analysis can be used to perform these statistical tests. Such tests are equivalent with testing whether the optimal portfolio weight of the additional asset is significantly positive. The null hypothesis for *intersection* is that for a mean-variance investor with a given risk-aversion the optimal portfolio weight of the new asset is zero. The null hypothesis for *spanning* is that this new optimal weight is zero for all risk-aversions, and it can be shown that this hypothesis equals.

$$H_0^{span} : \mu_{com} - \beta' \mu_R = 0 \quad \text{and} \quad \beta' \iota = 1, \tag{7}$$

where μ_{com} and μ_R are the expected returns on commodities and the basic assets, while β is the vector with covariances between the commodities and the traditional assets.

The hypothesis in (7) can be tested using the following linear regression equation,

$$R_t^{com} = \alpha + \beta' R_t^{basic} + \varepsilon_t, \tag{8}$$

where R_t^{com} is the return on commodities and R_t^{basic} is the vector of returns on the set of basic or traditional assets. Note that $\alpha = \mu_{com} - \beta' \mu_R$. Substitution of α in the null hypotheses for intersection and spanning gives the test in terms of the parameters of the regression equation. It is shown in, e.g. De Roon & Nijman (2001), that a modified version of this test can be used when the investor faces fixed liabilities. The only necessary alteration is that the liability return is subtracted from both the commodity and traditional returns.

We start by investigating whether commodity returns are spanned by the returns on the traditional assets by using regression equation (8). The results of this regression analysis are reported in Table 3. The p-value of the test statistic is 0.52 for the nominal scheme, confirming the intuition obtained from Figure 1 that the shift is insignificant. For the inflation-indexed scheme, the p-value of the test statistic is below 0.001, amplifying the economical significance of the difference in frontiers in Figure 2. Thus, our formal tests indicate that for real pension schemes commodities significantly improve the three year risk-return trade-off on the funding ratio, while this is not the case for pension schemes with nominal liabilities. The test statistic on spanning can be rewritten as the sum of two intersection hypotheses. The first is for an extreme risk-averse investor, and the second for a risk-neutral investor. We report both intersection tests also in Table 3, in order to examine which type of investor is driving the spanning rejection, and hence which type of investor can benefit most from investing in commodities. The last line of Table 3 suggests that, when spanning is rejected, this is mostly because the extremely risk averse investor can improve his risk-return trade-off significantly.

The above illustrates that if regulation does not allow pension funds to invest in alternative asset classes such as commodities, this might imply that the probability of underfunding is increased instead of decreased. Recalling that the asset labeled commodities in this paper actually is a dynamic (with an a priori fixed trading rule) derivatives trading strategy, this analysis suggests that the solvency of efficient pension funds can be harmed when regulators introduce restrictions on derivatives trading.

3 Short-term myopic and tactical asset allocation

In this section we relax the assumption of a three-year buy-and-hold asset allocation and investigate the short-term benefits of investing in commodities. We start this section by an unconditional test for spanning on a three-month investment horizon. Next, we analyze whether information about the state of the economy might improve the conditional frontier, by allowing expected asset returns and covariances to change depending on these variables. We end this section by examining the potential benefits of short-term timing strategies between stocks and bonds on the long-term frontier. This answers the question whether such tactical timing strategies are useful in addition to a three-year buy-and-hold strategy in bonds, domestic stocks, foreign stocks, and commodities.

Though pension schemes have in principle a long term objective, their performance is mostly evaluated at shorter horizons too. The short horizon perspective is particularly important for asset managers, whose incentives are usually based on short term (relative) performance. In addition, regulatory bodies require that the probability of underfunding in the short run should be at reasonable levels. We examine whether the efficient short term (quarterly) mean-variance frontier is spanned by the traditional asset classes when the liability structure is also taken into account. These results from spanning on the short and long term (buy-and-hold) investments may be different due to changing covariance structures at various horizons. The short term persistence in inflation in particular causes this horizon effect in covariance structure.[13] The quarterly horizon analysis is performed unconditionally as well as by a model that allows expected returns and covariances to vary depending on conditional macro economic information.

The descriptive statistics of the quarterly returns can be found in Table 1, Panel C and D.[14] The most noteworthy is the correlation structure in Panel D, which is different from the tri-annual returns for the pension scheme with real liabilities in Panel B. The correlation with long-term government bonds has become positive, while the correlations with the other assets is close to zero. The correlation between the nominal and real liability returns is also positive, compared to a negative for the three-year horizon. This change in correlation structure, due to the fact that the return on real liabilities is far from uncorrelated, may lead to different optimal asset allocations for the myopic short term investor than for a strategic investor with a three year horizon.

In Figure 3 and 4 we depict the mean-variance frontier of the nominal and real pension scheme, respectively. Inclusion of commodities as an asset class shifts the mean-variance frontier only marginally to the left for the pension scheme with nominal liabilities, while it shifts substantially to the left for the real liability scheme. Again, we conduct a statistical test to investigate whether the shifts in the frontier are statistically significant. The conclusions

[13]A regression of the three-month index-linked bond return on its three-month lag (and a constant) results in an insignificant estimate of -0.02 (t-value: -0.24), while a regression of the three-year index-linked bond return on its three-year lag (and a constant) yields a significant estimate of 0.43 (t-value: 3.70).

[14]For the return on the nominal liabilities we have now taken both the 10-year yield and not the 9.75-year yield as is required. In the previous example we used the 7-year yield when needed. We expect the yields for this short duration difference to be so close that the results are not influenced by this approximation.

from the visual inspection of the graphs are statistically confirmed, with a p-value of 0.13 for the nominal case, and below 0.001 for the real case. Thus, even when the correlation between the real liabilities and government bonds has increased substantially, investing in commodities has a significantly positive impact on the risk-return trade-off for the real pension scheme. This is consistent with the findings of for example Froot (1995), who investigates the diversification properties of commodities to an *asset-only* portfolio of stocks and bonds on a quarterly horizon. He finds that commodities are better diversifiers than real estate and equity of commodity-related firms.

Thus far, we have investigated the unconditional mean-variance frontier, which implies that only investment strategies that can be fixed a priori and do not use the most recent information about the state of the economy are considered.[15] There is a large body of literature claiming that asset returns are predictable up to a certain degree. Moreover, the covariances of asset returns might depend on specific economic circumstances (see, e.g., Campbell (2000) for an overview on asset return predictability). We allow expected asset returns and covariances to vary depending on the economic situation; see, e.g., Shanken (1990), and test whether efficient investment strategies can exploit time variation in these quantities.

The conditioning information we use in order to characterize the economic situation is the yield on a 10-year government bond, the term spread, the default spread, and the inflation rate. Similar economic information is also used in, e.g., Ferson & Schadt (1996). The term spread is defined as the yield of a 10-year government bond minus the yield of a 1-year government bond. The default spread is defined as the Moody's seasoned Baa corporate bond yield minus the Moody's seasoned Aaa corporate bond yield. These yield data are obtained from the Federal Reserve Bank of St Louis. We construct annual inflation rates on a monthly basis using the same methodology as the Bureau of Labor Statistics. Hence, our results for December correspond to the annual inflation rate as is published each year. We incorporate a reporting lag of one month in our studies, since the inflation rate is typically published with a delay. The descriptive statistics on our conditioning variables can be found

[15] Cochrane (2001) refers to this frontier the unconditional *fixed-weight* mean-variance frontier, since it does not contain managed portfolios.

in Table 3. The difference between the average nominal yield and the average inflation is three percent per annum. During some period of our sample, an inverse term structure could be observed, since the minimum term spread is negative.

Conditional spanning can also be tested for by using regression analysis. The regression equation suitable for this type of conditional spanning is

$$R_t^{comm} = \alpha_0 + \alpha_1' Z_{t-1} + \beta_0' R_t^{basic} + \beta_1' \left(Z_{t-1} \otimes R_t^{basic} \right) + \varepsilon_t, \tag{9}$$

where Z_{t-1} is a vector with macro economic variables known at the end of period $t-1$, and β_1 a 15-dimensional vector with coefficients of the cross-products between the basic assets and the macro-economic variables. This regression allows to test for the validity of the restrictions implied by conditional spanning, which are

$$\text{conditional spanning: } H_0 : \alpha_0 + \alpha_1' Z_{t-1} = 0, \quad \beta_0' \iota + \beta_1' \left(Z_{t-1} \otimes \iota \right) = 1. \tag{10}$$

Testing for spanning for all economic situations means means that

$$\text{conditional spanning: } H_0 : \alpha_0 = 0, \quad \alpha_1 = 0, \quad \beta_0' \iota = 1, \quad \beta_1 = 0. \tag{11}$$

Tests for conditional spanning indicate whether it is efficient to include commodities in the portfolio given the current economic situation. We estimate the regression model of (9) and test the hypotheses in (10) and (11) in order to investigate whether allowing for conditional expected return and covariances changes the benefits from investing in commodities. For the ease of presentation, we report the results for the economic situation at the end of each five-year period of our sample. Table 4 contains the parameter estimates as well as spanning tests. These results indicate that the mean-variance frontier shifts significantly outward for the inflation-index pension scheme for each of the economic circumstances that we analyze.

For the nominal scheme, we observe some differences by introducing conditioning information. Whereas the unconditional mean-variance frontier of the traditional asset spans the set including commodities, this does not hold for the conditional mean-variance frontier at each of the periods under investigation. In 1984 and 1989, the spanning test is rejected at the five percent significance level. In other words, the investment opportunity set created by the three basis assets can be expanded by introducing the alternative assets commodities. For example, in the end of 1984, the high term spread and low inflation cause commodities to have a low conditional expected return. This low conditional expected return, in combination with the estimated covariance structure, causes the mean-variance spanning hypothesis to be rejected. From this model to incorporate conditional information, it follows that in certain economic environments short term asset allocation may contain commodities, while at a strategic horizon commodities should not be included. We observe this for the pension scheme with nominal claims. For a pension scheme with real claims investing in commodities seems to be beneficial both on the long and short run, both unconditionally and conditionally.

In the remainder of this section, we examine whether active short-run tactical allocation between stocks and commodities increases the strategic buy-and-hold mean-variance frontier including commodities. This provides an answer to the question whether there are possible longer-horizon gains to engage in tactical allocation based on macro-economic news. These dynamic trading strategies (or managed portfolios) can be interpreted as new asset classes, on which the pension scheme can decide to invest in or not.

We use the same four macro variables as above to investigate the effects of timing between stocks and commodities.[16] We normalize the variables for ease of interpretation by subtracting the mean and dividing by the standard deviation. This normalized variable is the signal for the timing strategy.[17] A signal with value 1 means that a long position in commodities is taken, which is financed by a short position in the stock market. We simultaneously evaluate all possible combinations of linear strategies in the bond yield, term spread, default spread, and inflation rate. This can be done by checking whether the four timing strategies based on a single macro variable are simultaneously spanned by the traditional buy-and-hold portfolio, which now also contains a fixed strategic position in commodities. Similar active trading

[16] In addition, we also performed a test on the timing strategy proposed by Johnson & Jensen (2001), using conditioning information about restrictive or expansive policies by the Federal Reserve. The spanning hypothesis is rejected for both the real and nominal pension schemes when bonds, domestic stocks, and foreign stocks are the basic assets. However, when domestic real estate is added the spanning hypothesis is no longer rejected. Results of these additional tests are available from the authors upon request.

[17] The investment strategy is $\frac{z_{i,t}-\overline{z}_i}{\sigma_i(z)} \cdot R^e_{t+1}$, where $z_{i,t}$ is the value of the macro economic variable i at time t, and R^e_{t+1} is the excess return of commodities over the stock market.

strategies for currency futures have been investigated by De Roon, Nijman & Werker (2003).

The spanning hypothesis for adding quarterly timing strategies to a strategic portfolio on a three year horizon based on the inflation rate, default spread, term spread, and bond yield, is rejected with a p-value below 0.001 for both the nominal and real pension scheme. This means that quarterly timing strategies between commodities and stocks based on these macro economic variables enhance the efficient risk-return trade-off for pension schemes significantly. The excess returns of the strategies based on the single variables are in Table 7. The p-values of the spanning tests for each of the variables separately is also tabulated, and indicates that timing on the basis of the term spread and default spread is expanding the efficient set for both the nominal and real pension scheme, and the inflation rate and yield are only significant for one type of the liabilities.

4 Robustness and extensions

In this section, we address some issues of robustness and some possible extensions for further research. In the first subsection the analyses are repeated for the post 1984 subperiod, which is characterized by more modest inflation than the first 15 years of our sample. We also use an alternative commodity index, computed by the Commodity Research Bureau (CRB), to examine whether our results are dependent on the choice of commodity index. In the second subsection we briefly address the issue of cash as the futures collateral. The third subsection addresses how to test for spanning when the utility function of the investor is not the usual mean-variance utility function.

4.1 Subperiod analysis 1984- 001

Since Paul Volcker became the chairman of the Board of Governors of the Federal Reserve system in 1979, the central bank's target has become to keep inflation under control. Since this policy is likely to continue in the future, the first part of our sample period might not be representative. We decide to investigate the subsample ranging from January 1984 – December 2001 in order to examine the influence of this period characterized by high inflation. First, we investigate whether there is spanning on the strategic three year horizon. Next, we check whether there is spanning for the myopic investor with an investment horizon of three months. Finally, we investigate the benefits of timing strategies in addition to strategic allocations.

The descriptive statistics over the subperiod are displayed in Table 5. The regression results used to test for spanning can be found in Table 6. The conclusions for the strategic three year horizon are similar to the entire sample period. Spanning cannot be rejected for the pension scheme with nominal interest rates, but spanning is rejected for the inflation-indexed pension scheme.

For the myopic short term investor with real liabilities the results are qualitatively the same as before. However, when nominal liabilities are concerned, the spanning hypothesis is rejected at the 10 percent level for this subperiod, indicating that for myopic pension schemes with nominal liabilities commodities are also expanding the MVE frontier.

4.2 Alternative commodity index

For the shorter sample period 1984-2001, we can also use the Commodity Research Bureau (CRB) total return index to check for robustness on the choice of commodity index. The descriptive statistics of this alternative commodity index can be found in Table 5. The results from this commodity index amplify the results from our analysis. In Panels A and B of Table 6 the results can be found. The spanning hypothesis is again rejected on both the quarterly and three year horizon, indicating that the empirical evidence for the addition of commodities to the investor's portfolio is not due to the choice of the particular commodity index.

4.3 Futures positions wit out cash-collateral

In accordance with the existing literature, we use the fully cash-collateralized total return index as the commodity asset in this paper. In practical situations, however, other assets in the portfolio may also serve as collateral for the commodity futures positions. An alternative and more extreme view is to assume that no cash-collateral whatsoever is needed, i.e., the assets in the existing portfolio may always serve as collateral for the commodity future positions. This changes the analysis as presented above, because a futures return is not a true return, since no initial investment is required. In addition to the true returns on the asset side and the fixed liabilities, the return on the funding ration now also consists of the excess return from the futures position.

The regression-based spanning tests used in the previous sections can be straightforwardly generalized for portfolios containing futures positions; see De Roon et al. (2003), amongst others. The results from this robustness analysis confirm our previous findings.[18] The spanning hypothesis on the three-year horizon cannot be rejected for the nominal pension scheme (with p-value 0.11). For the inflation-indexed pension scheme, spanning is rejected (with p-value < 0.001), implying that the shift in the mean-variance frontier by adding commodity futures is not due to the cash-collateralization assumed in the previous sections.

4.4 Non-mean variance utility functions

In the analyses of this paper, we have assumed that the pension scheme has a mean-variance utility function in the funding ratio. If the returns on the assets and liabilities are not normally distributed (which is an approximation at best) the mean-variance approach is restrictive and the optimal asset allocation will depend on the utility function of the investor. A power utility function with a large risk aversion parameter might well characterize the preferences of an investor that puts a lot of weight on assuring a funding ratio that is at least equal to one.[19]

Spanning for more general utility functions than the standard mean-variance case can be dealt with in a regression framework as well. For *generalized spanning*, the regression equation (8) changes to

$$r_t = \alpha + \beta' R_{t+1} + \sum_i \gamma_i U_i'(\phi_i^{*\prime} R_{t+1}) + \varepsilon_{t+1}, \tag{12}$$

where $U_i'()$ denotes the derivative of the i^{th} utility function. The scaled optimal portfolio weights for investors with non-mean-variance utility are denoted by ϕ_i^*. Generalized spanning restrictions imply, next to the usual $\alpha = 0$ and $\beta'\iota = 1$ for each asset, that $\gamma_i = 0$ for all i. For a more detailed derivation of the hypotheses and tests from this section see, e.g., the

[18] For reasons of brevety, we do not report the full table with results from this analysis. These tables are available from the authors upon request.

[19] A power utility function has the form $u(W) = \frac{W^{1-\rho}-1}{1-\rho}$, with W the future wealth, defined as $1 + R \cdot w$, where R are the expected returns on the assets, and w the corresponding portfolio weights.

survey article by De Roon & Nijman (2001).

In addition to the mean-variance utiltiy function, we use power utility functions with various degrees of weights on negative returns on the funding ratio to investigate the benefits of commodities in the institutional portfolio. The results of the spanning tests are displayed in Table 8. When a utility function with high aversion to negative returns on the funding ratio ($\rho = 15$) is investigated, spanning at the quarterly horizon is rejected for both the nominal and inflation-indexed pension scheme. Thus, pension schemes that are more averse to negative returns on their funding ratio have even more reason to add commodities to increase efficiency of their portfolios. So, extending our set of utility functions with power utility functions yields that investing in commodities can be also efficient for a nominal pension scheme with high aversion to negative returns.

5 Conclusions

In this paper we analyze the benefits for pension schemes to invest part of their wealth in commodities. We leave the traditional asset-only framework and incorporate market-based returns for both nominal and inflation-indexed liabilities. Our results indicate that for nominal pension schemes the use of commodities is limited, while for real pensions they reduce the volatility on the funding ratio more than 30 percent. In addition to the significant economic magnitude of this risk reduction, we contribute to the existing literature on commodity investments by providing statistical significance as well.

Our analysis aims in the first place at a strategic three year buy-and-hold investment horizon. Since the returns on the real liabilities are correlated over time, this might lead to different allocations than for an investor with a short, say quarterly, investment horizon. While our unconditional results remain unchanged for the short term, there is no conditional spanning for both the nominal and real pension scheme. Thus, in certain economic situations, also for nominal pension schemes with a short horizon commodities improve the efficient risk-return trade-off.

Finally, we investigate whether timing strategies between commodities and stocks are improving the strategic mean-variance frontier even further. We find that the timing strategies based on macro economic information may expand the three-year horizon frontier significantly.

Our results are robust for the choice of sample period, as our results over the 1984-2000 period suggest. Our results do not seem to depend on our choice of commodity index either. The empirical results against spanning are even stronger for an alternative commodity index which is available over the 1984-2000 period. As a final robustness check, we have extended the usual set of mean-variance utility functions with power utility functions, to capture the non-normalities in the returns on assets and liabilities. Addition of these functions do not affect our previous results.

The liability hedge potential of alternative asset classes such as commodities should also be taken into account by regulators. The call for more strict regulation on the use of alternative assets by institutional investors has become stronger, especially in Europe. Our results suggest that the presence of alternative assets (which are frequently constructed by combinations of derivative products) could protect the solvency position of the fund, and hence benefit the participant in the fund. The merit of alternative assets in strategic asset allocation should be confronted with the other assets in the portfolio, and the liability structure of the scheme.

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Table 1: Descriptive statistics of assets and liability returns. In Panel A, the tri-annual log returns (in US dollars) over the sample 1972:12 - 2001:12 can be found. The assets are long-term US government bonds, domestic stocks, foreign stocks, commodities, and the liabilities are from the pension scheme with nominal obligations and inflation adjusted (real) obligations. The column labeled average contains the average three year return, and the following column contains the annualized average. The same is done for the standard deviation. The columns with minimum and maximum contain the lowest returns on a consecutive three-year period. Panel B contains the correlation matrix ($\times 100$) of the assets and liabilities on a three-year basis. Panels C and D contain the same information, but then at a three month instead of three-year horizon, ranging from 1970:3 - 2001:12.

Panel A: three years	average	annual	stdev	annual	minimum	maximum
domestic gov-bonds	26.1	8.70	16.9	9.7	-18.7	68.0
domestic stocks	36.6	12.19	24.5	14.2	-41.5	87.1
foreign stocks	35.4	11.81	34.8	20.1	-27.2	137.9
commodities	32.7	10.91	36.4	21.0	-38.2	137.5
nominal liabilities	28.5	9.50	19.5	11.3	-17.6	79.8
real liabilities	26.9	8.96	7.7	4.4	6.8	43.9

Panel B: three years correlation matrix	domestic gov-bonds	domestic stocks	foreign stocks	commodities	nominal liabilities	real liabilities
domestic gov-bonds	100	51	34	-38	96	-39
domestic stocks	51	100	37	-51	47	-65
foreign stocks	34	37	100	-5	38	-10
commodities	-38	-51	-5	100	-30	52
nominal liabilities	96	47	38	-30	100	-32
real liabilities	-39	-65	-10	52	-32	100

Table 1: (continued)

Panel C: quarterly	average	annual	stdev	annual	mininum	maximum
domestic gov-bonds	2.21	8.84	5.3	10.6	-15.7	21.8
domestic stocks	2.76	11.04	7.7	15.5	-34.8	23.5
foreign stocks	2.62	10.48	8.7	17.4	-23.7	30.1
commodities	2.63	10.52	9.7	19.5	-36.6	43.9
nominal liabilities	2.18	8.72	6.7	13.5	-20.6	33.0
real liabilities	2.26	9.04	1.9	3.8	-3.4	10.5

Panel D: quarterly correlation matrix	domestic gov-bonds	domestic stocks	foreign stocks	commodities	nominal liabilities	real liabilities
domestic gov-bonds	100	29	13	-22	87	68
domestic stocks	29	100	59	-19	31	1
foreign stocks	13	59	100	-9	19	-8
commodities	-22	-19	-9	100	-18	0
nominal liabilities	87	31	19	-18	100	60
real liabilities	68	1	-8	0	60	100

Table 2: Mean-variance spanning of commodities for pension schemes. The added asset is commodities, while the basic assets in the top panel are domestic bonds (bnd), domestic stocks (dom), foreign stocks (for), and in the bottom panel domestic real estate (nareit) is added as a basic asset. The parameter estimates are obtained by OLS on $R_t^{com} - R_t^{liab} = \alpha + \beta'(R_t^{basic} - R_t^{liab}) + \varepsilon_t$, where R_t^{com} is the commodity return, R_t^{liab} the return on the liabilities, and R_t^{basic} the vector with returns on the basic assets. The Newey & West (1987) standard errors (with lag 35) are displayed in the column behind the parameter estimate, and are used to calculate the spanning and intersection test statistics. With $\gamma \to \infty$ we test for intersection for an investor with extremely high risk aversion. In Panel A the results for the three-year, and in Panel B the three-month investment horizon are displayed. Sample periods are 1972:12–2001:12 and 1970:3–2001:12, respectively.

Panel A:

commodity spanning	nominal liabilities		inflation-indexed	
three year horizon	estimate	hac.se	estimate	hac.se
intercept	0.109	0.033	0.080	0.080
domestic gov-bonds	2.085	1.212	-0.338	0.246
domestic stocks	-0.495	0.548	-0.369	0.251
foreign stocks	0.340	0.277	0.135	0.138
number of obs.	349		349	
spanning test [p-val]	1.31	[0.52]	73.19	[0.00]
inters. $\gamma \to \infty$ [p-val]	1.07	[0.30]	63.39	[0.00]

Panel B:

commodity spanning	nominal liabilities		inflation-indexed	
three month horizon	estimate	hac.se	estimate	hac.se
intercept	0.003	0.009	0.004	0.007
domestic gov-bonds	1.068	0.222	-0.771	0.145
domestic stocks	-0.015	0.126	-0.126	0.088
foreign stocks	0.301	0.111	0.076	0.090
number of obs.	382		382	
spanning test [p-val]	4.09	[0.13]	93.14	[0.00]
inters. $\gamma \to \infty$ [p-val]	3.95	[0.05]	88.05	[0.00]

Table 3: Descriptive statistics of conditioning variables, 1970:1 - 2001:12. This table contains the monthly average, standard deviation, minimum, and maximum of the macro economic variables. The bond yield is the level of the 10-year government bond yield. The term spread is defined as the yield of a 10-year government bond minus the yield of a 1-year government bond. The default spread is defined as the Moody's seasoned Baa corporate bond yield minus the Moody's seasoned Aaa corporate bond yield. Monthly inflation is calculated in similar fashion to the Bureau of Labor Statistics calculates its annual inflation, so that the December values correspond to the officially announced annual inflation rate.

monthly	average	stdev	mininum	maximum
bond yield	8.08	2.31	4.53	15.32
term spread	0.88	1.11	-3.07	3.29
default spread	1.09	0.44	0.55	2.69
inflation	4.97	2.78	1.54	12.75

Table 4: Conditional spanning with time-varying expected returns and covariances. The estimated regression equation is $r_t = \gamma_0 + \gamma' Z_{t-1} + \beta_0' R_t + \beta' (Z_{t-1} \otimes R_t) + \varepsilon_t$, where r_t is the commodity return, the conditioning information Z_{t-1} consists of the long yield, term spread, default spread, and inflation, and R_t are the returns on long bonds, domestic stocks, and foreign stocks. Since we use monthly overlapping quarterly returns, the standard errors are corrected using the Newey & West (1987) method. The sample period is May-1970 – Dec-2001. The following rows contain the values of the conditioning variables at the end of five-year periods, for which the p-value for a spanning test under these conditions is presented in the final two columns. The null hypothesis for spanning implies that $\gamma_0 + \gamma' Z_{t-1} = 0$ and $\beta_0' R_t + \beta' (Z_{t-1} \otimes \iota) = 1$. The null hypothesis to check for spanning for all economic situation is $\gamma_0 = \gamma = \beta = 0$, and $\beta_0' \iota = 1$.

Spanning	yield	term	default	inflation	p-value nom	p-value real
Dec-1974	7.43	0.12	1.74	10.21	0.49	0.00
Dec-1979	10.39	-1.59	1.32	10.35	0.16	0.00
Dec-1984	11.50	2.17	1.27	4.16	0.00	0.00
Dec-1989	7.84	0.12	0.96	4.67	0.04	0.00
Dec-1994	7.81	0.67	0.64	2.58	0.04	0.00
Dec-1999	6.28	0.44	0.64	2.08	0.08	0.00
	Under all economic situations				0.00	0.00

Table 5: Descriptive statistics subperiod analysis. In Panel A, the tri-annual log returns (in US dollars) over the sample 1987:1 - 2001:12 can be found. Panels C and D contain the same information, but then at a three month instead of three-year horizon, ranging from 1984:3 - 2001:12. In addition, descriptive statistics of the alternative commodity index CRB are provided as well.

Panel A: three years	average	annual	stdev	annual	minimum	maximum
domestic gov-bonds	30.73	10.24	11.3	6.5	9.39	66.30
domestic stocks	46.90	15.63	20.6	11.9	-6.02	87.10
foreign stocks	34.90	11.63	39.1	22.6	-27.23	137.91
commodities	27.08	9.03	31.1	18.0	-38.24	95.80
commodities CRB	11.27	3.76	17.8	10.3	-24.17	42.42
nominal liabilities	32.19	10.73	14.8	8.6	7.88	79.82
real liabilities	22.10	7.37	5.6	3.2	6.81	31.30

Panel B: 3Y correlation	bonds	domstock	forstock	GSCI	CRB	nomliab	realliab
domestic gov-bonds	100	36	47	-7	-10	95	45
domestic stocks	36	100	29	-30	-10	25	-45
foreign stocks	47	29	100	-2	24	49	9
commodities	-7	-30	-2	100	65	1	63
commodities CRB	-10	-10	24	65	100	-12	37
nominal liabilities	95	25	49	1	-12	100	51
real liabilities	45	-45	9	63	37	51	100

Panel C: Quarterly	average	annual	stdev	annual	minimum	maximum
domestic gov-bonds	2.74	10.96	5.0	9.9	-10.15	18.01
domestic stocks	3.41	13.64	7.6	15.2	-34.84	20.43
foreign stocks	2.67	10.67	8.9	17.8	-23.74	30.08
commodities	1.86	7.45	8.9	17.9	-22.30	43.93
commodities CRB	0.59	2.34	4.4	8.7	-12.38	11.68
nominal liabilities	2.78	11.13	5.8	11.5	-12.50	20.68
real liabilities	1.92	7.67	1.7	3.3	-3.45	6.06

Panel D: Q correlation	bonds	domstock	forstock	GSCI	CRB	nomliab	realliab
domestic gov-bonds	100	21	17	-23	-29	88	77
domestic stocks	21	100	58	-15	-8	24	2
foreign stocks	17	58	100	1	19	24	-2
commodities	-23	-15	1	100	67	-25	2
commodities CRB	-29	-8	19	67	100	-32	-15
nominal liabilities	88	24	24	-25	-32	100	70
real liabilities	77	2	-2	2	-15	70	100

Table 6: Robustness of results for subsample 1984–2001 and CRB index. This table is the equivalent of Table 2, but now for the sample period 1984–2001. Panel A contains the results for the three-year horizon, and Panel B for the three-month horizon. This is done for both the GSCI and CRB total return index.

Panel A:

	GSCI strategic				CRB strategic			
Sample: 1984-2001	nominal liabilities		real liabilities		nominal liabilities		real liabilities	
Horizon: Three year	estimate	hac.se	estimate	hac.se	estimate	hac.se	estimate	hac.se
intercept	0.030	0.165	0.181	0.124	-0.150	0.063	-0.083	0.056
domestic gov-bonds	1.849	1.512	-1.304	0.396	2.884	0.584	-1.168	0.265
domestic stocks	-0.363	0.517	-0.131	0.263	-0.149	0.238	0.199	0.155
foreign stocks	-0.026	0.159	0.107	0.143	0.166	0.077	0.202	0.042
spanning test	0.16	[0.92]	53.00	[0.00]	35.86	[0.00]	542.39	[0.00]
inters. $\gamma \rightarrow \infty$ [p-val]	0.14	[0.70]	36.30	[0.00]	12.91	[0.00]	51.46	[0.00]

Panel B:

	GSCI myopic				CRB myopic			
Sample: 1984-2001	nominal liabilities		real liabilities		nominal liabilities		real liabilities	
Horizon: Quarterly	estimate	hac.se	estimate	hac.se	estimate	hac.se	estimate	hac.se
intercept	-0.009	0.011	0.009	0.009	-0.022	0.007	-0.008	0.004
domestic gov-bonds	1.222	0.276	-0.974	0.179	1.083	0.181	-0.634	0.106
domestic stocks	0.046	0.145	-0.202	0.123	0.095	0.120	-0.094	0.059
foreign stocks	0.319	0.130	0.194	0.088	0.317	0.099	0.220	0.045
spanning test	4.99	[0.08]	67.17	[0.00]	20.73	[0.00]	222.13	[0.00]
inters. $\gamma \rightarrow \infty$ [p-val]	4.65	[0.03]	57.43	[0.00]	8.53	[0.00]	166.15	[0.00]

Table 7: Quarterly timing strategies 1970-2001. P-values for quarterly timing strategies between stocks and bonds. These p-values correspond to the hypothesis that a quarterly timing strategy between stocks and commodities does not shift the three-year mean-variance frontier to the left.

timing strategy	nominal	real
bond yield	0.00	0.97
inflation	0.11	0.03
term spread	0.00	0.00
default spread	0.00	0.00

Table 8: Non-mean-variance spanning. This table contains the p-values of mean-variance spanning tests, as well as tests for power utility spanning in addition to mean-variance spanning. The power utility function is given by $u(W) = \frac{W^{1-\rho}-1}{1-\rho}$, where future wealth W is defined as $1 + R \cdot w$, where R are the asset returns, and w the corresponding weights in the portfolio. Regression equation $R_t^{com} - R_t^{liab} = \alpha + \beta'(R_t^{basic} - R_t^{liab}) + \gamma \cdot u'\left(\left(R_t^{basic} - R_t^{liab}\right) \cdot \widehat{\varphi}\right) + \varepsilon_t$ is estimated, and the hypothesis for generalized spanning is $\alpha = \gamma = 0$ and $\beta'\iota = 1$. In the regression equation, $\widehat{\varphi}$ is the adjusted optimal weights vector. The covariance matrix is corrected for overlapping samples and heteroskedasticity using the Newey & West (1987) method..

Horizon	Three month		Three year	
p-values	nominal	real	nominal	real
mv-span	0.14	0.00	0.55	0.00
$\rho = 1$	0.23	0.00	0.01	0.00
$\rho = 3$	0.25	0.00	0.00	0.00
$\rho = 6$	0.03	0.00	0.36	0.00
$\rho = 15$	0.00	0.00	0.07	0.00

A The Goldman Sachs Commodity Index

The Goldman Sachs Commodity Index (GSCI) is a composite index of five commodity sectors.[20] The returns are unleveraged, fully cash-collateralized long-only investments in commodity futures with full reinvestment. The individual components are determined on the basis of liquidity and are weighted by their respective world production quantities. A table of the weights in the index in December 2001 can be found in the figure below. The GSCI has been used in many recent papers studying commodity investments. A summary of this research can be found in the table below. We display for each of the papers the commodity index used and the frequency of the analysis.

Paper	Commodity Index	Frequency
Bodie (1980)	raw futures data	annual
Ankrim & Hensel (1993)	GSCI, ICI	monthly
Lummer & Siegel (1993)	GSCI	annual
Froot (1995)	GSCI, CRB	quarterly
Becker & Finnerty (1997)	GSCI, CRB	monthly, quarterly
Anson (1999)	GSCI, CPCI, ICI, JPMCI	quarterly
Johnson & Jensen (2001)	GSCI, JPMCI	monthly
Georgiev (2001)	GSCI	monthly

[20] More information can be found at the website of Goldman Sachs, `www.gs.com/gsci`.

Figure 1: Tri-annual mean variance frontiers of a pension scheme with nominal pension payments of 10-year duration. The vertical axis is the expected annual return on the funding ratio on a three-year strategic investment horizon. The horizontal axis is the volatility of the return on the funding ratio on an annual basis. The basis assets are long-term government bonds, domestic stocks, and foreign stocks. The sample period is Dec-1972 to Dec-2001. We use monthly overlapping tri-annual returns. At several points on the frontier the optimal portfolio weight of commodities is shown.

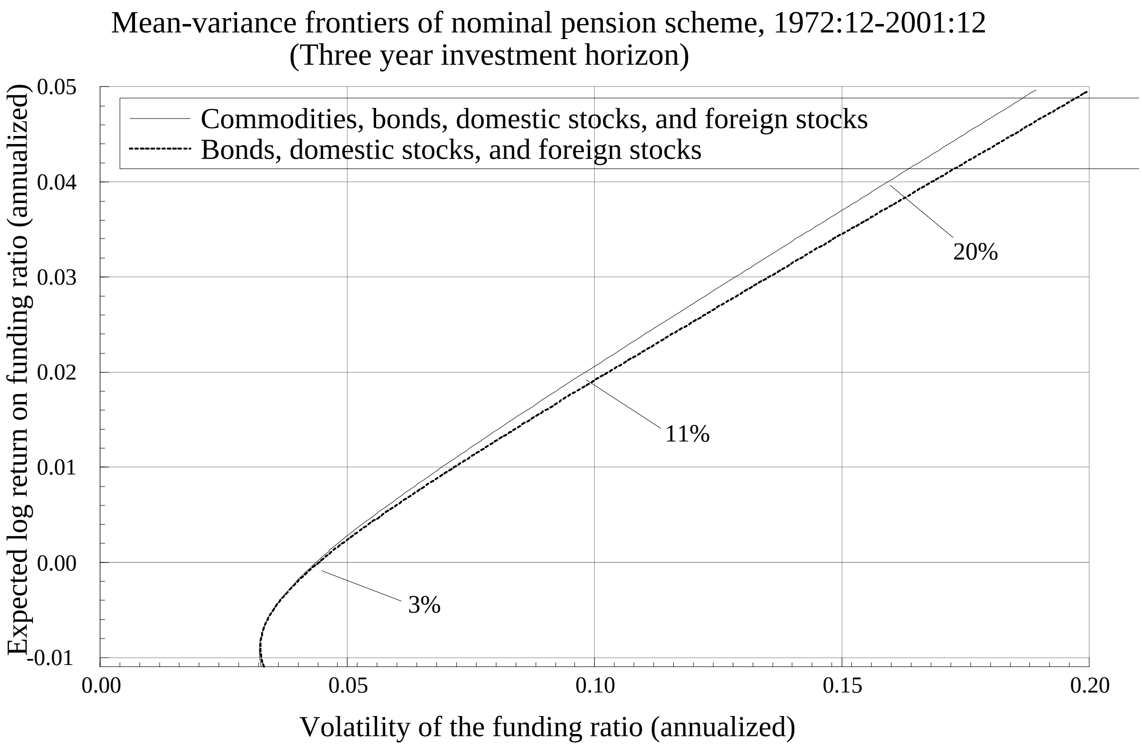

Figure 2: Tri-annual mean variance frontiers of a pension scheme with inflation-indexed pension payments of 10-year duration. The vertical axis is the expected annual return on the funding ratio on a three-year strategic investment horizon. The horizontal axis is the volatility of the return on the funding ratio on an annual basis. The basis assets are long-term government bonds, domestic stocks, and foreign stocks. The sample period is Dec-1972 to Dec-2001. We use monthly overlapping tri-annual returns. At several points on the frontier the optimal portfolio weight of commodities is shown.

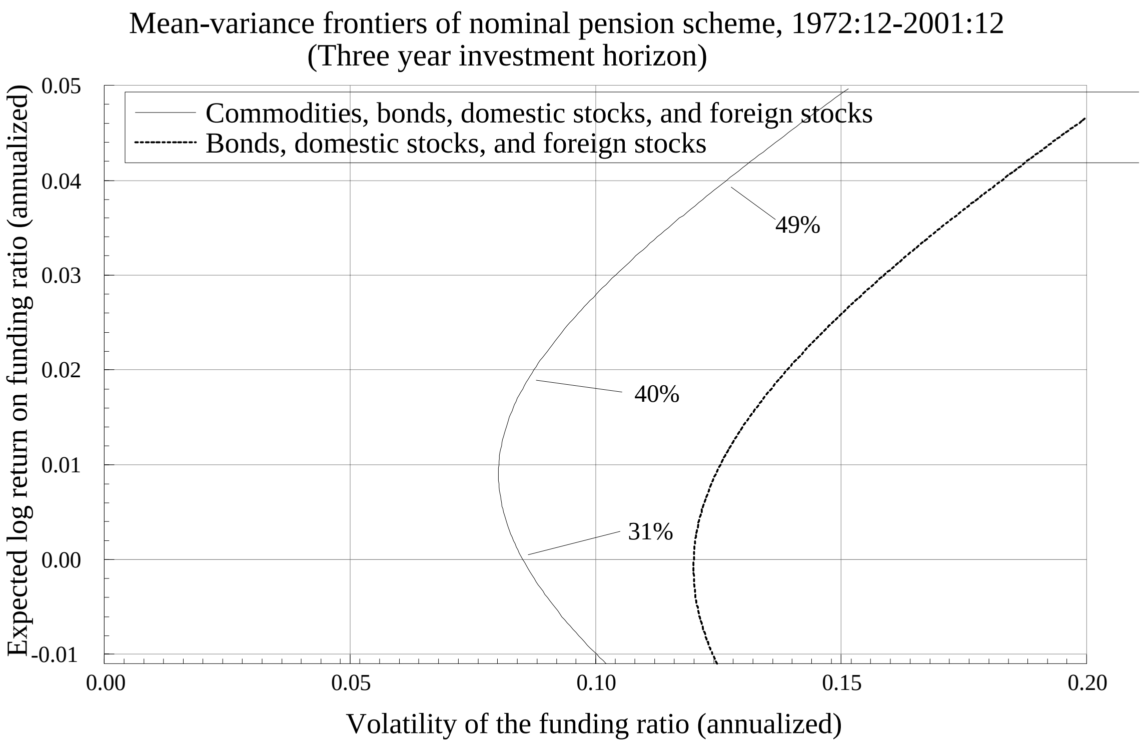

Figure 3: Quarterly mean variance frontiers of a pension scheme with nominal pension payments of 10-year duration. The vertical axis is the expected annualized return on the funding ratio on a quarterly strategic investment horizon. The horizontal axis is the volatility of the return on the funding ratio on an annual basis. The basis assets are long-term government bonds, domestic stocks, and foreign stocks. The sample period is Mar-1970 to Dec-2001. We use monthly overlapping quarterly returns. At several points on the frontier the optimal portfolio weight of commodities is shown.

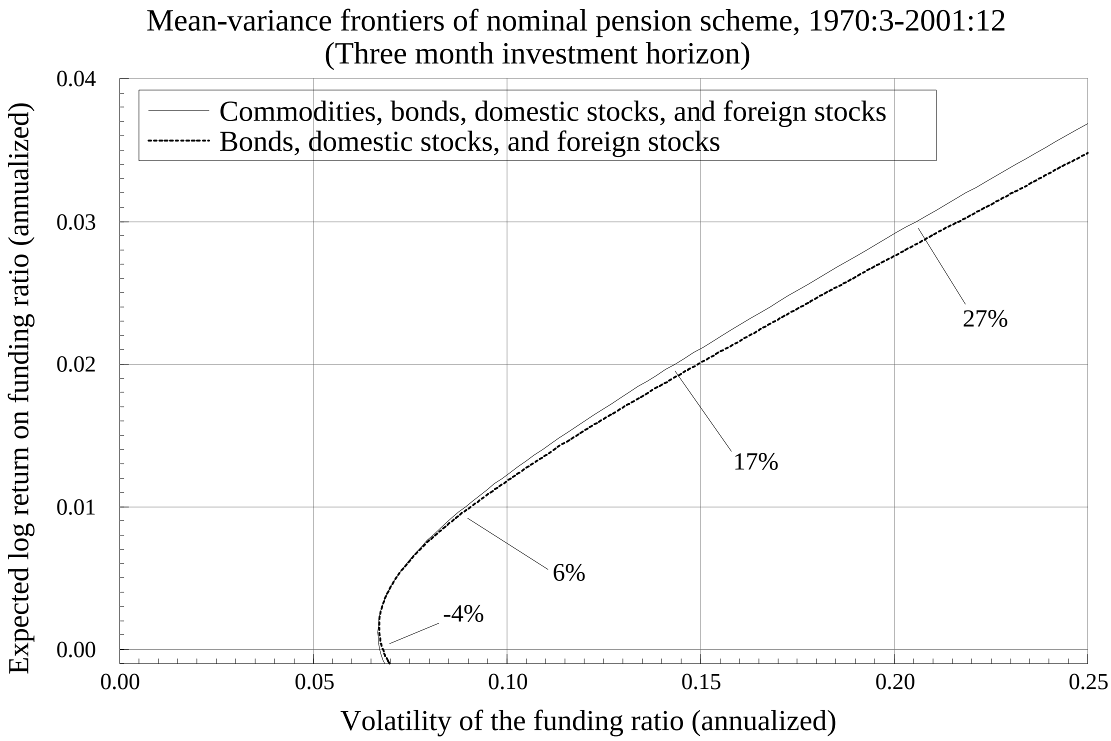

Figure 4: Quarterly mean variance frontiers of a pension scheme with real pension payments of 10-year duration. The vertical axis is the annualized expected return on the funding ratio on a quarterly strategic investment horizon. The horizontal axis is the volatility of the return on the funding ratio on an annual basis. The basis assets are long-term government bonds, domestic stocks, and foreign stocks. The sample period is Mar-1970 to Dec-2001. We use monthly overlapping quarterly returns. At several points on the frontier the optimal portfolio weight of commodities is shown.

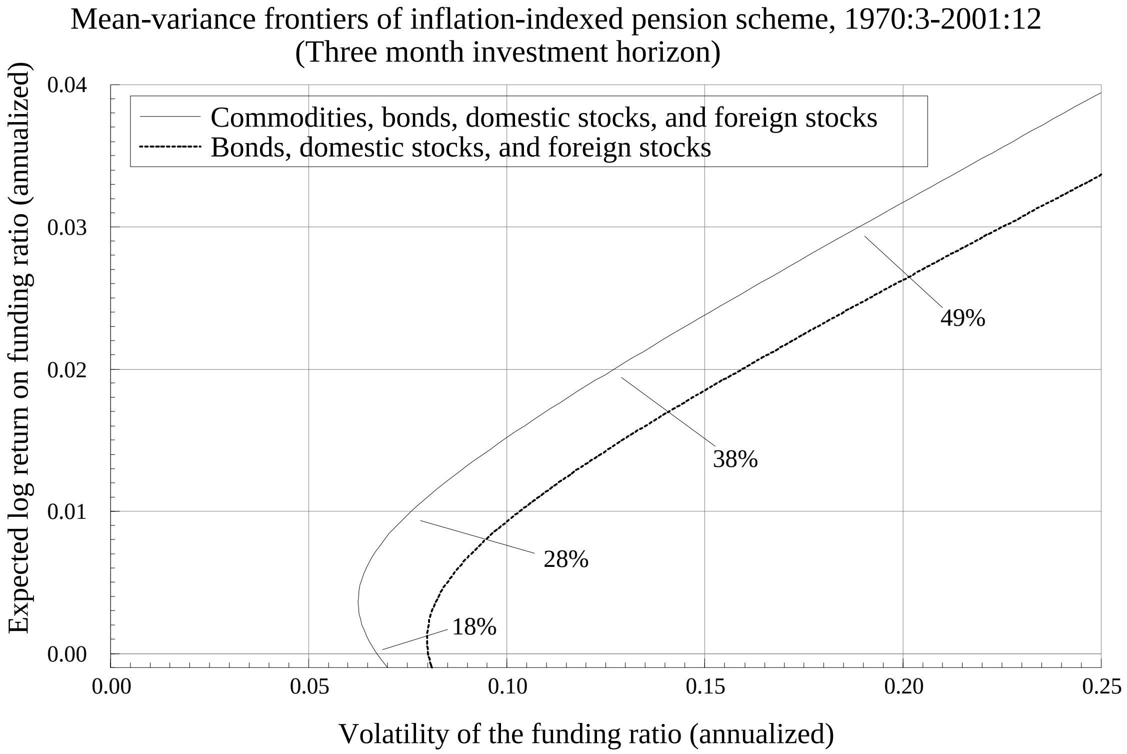

Figure 5: Composition of the Goldman Sachs Commodity Index at 7 December 2001

Table 1: GSCI Components and Dollar Weights (%)(December 7, 2001)

Energy	57.75	Industrial Metals	7.90	Precious Metals	2.83	Agriculture	20.55	Livestock	10.97
Crude Oil	24.72	Aluminium	4.18	Gold	2.35	Wheat	5.10	Live Cattle	7.67
Brent Crude Oil	11.34	Copper	2.10	Platinum	0.24	Red Wheat	1.67	Lean Hogs	3.30
Unleaded Gas	4.44	Lead	0.31	Silver	0.24	Corn	5.50		
Heating Oil	6.17	Nickel	0.54			Soybeans	2.49		
GasOil	2.95	Tin	0.11			Cotton	1.82		
Natural Gas	8.13	Zinc	0.67			Sugar	2.37		
						Coffee	0.70		
						Cocoa	0.41		
						Orange Juice	0.50		