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TRANSPARENCY, PRICES, AND WELFARE WITH IMPERFECT SUBSTITUTES

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Transparency, prices, and welfare with imperfect substitutes

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Abstract

Markets that are not completely transparent feature complex comparative statics with respect to the e®ect of number of ¯rms, elasticity of substitution between goods and degree of transparency on equilibrium prices. The main result is that the following 'common wisdom' is incorrect: more transparent markets always feature lower prices, higher consumer welfare and lower price disperion.

1 Introduction

It is often suggested that transparency will increase competition, reduce prices and price dispersion, and increase consumer surplus. "The internet is a nearly perfect market because information is instantaneous and buyers can compare the o®erings of sellers worldwide. The result is ¯erce price competition", (BusinessWeek, 1998). "The Internet cuts costs, increases competition and improves the functioning of the price mechanism. It thus moves the economy closer to the textbook model of perfect competition.(...) By improving the °ow of information between buyers and sellers, it makes markets more e±cient (...) By increasing price transparency, the Internet will give more power to the consumers " (The Economist, 2000). It is easy to ¯nd many similar assertions in both academic and more popular accounts.

In this paper we present a caveat against this argument.[1] If goods are

[1] Empirical results are quite mixed. One of the ¯rst studies is Bailey (1998) who ¯nds that prices for books, CDs, and software are higher on the internet than for traditional retailers. Lee (1999) ¯nds a similar result for the market for used cars. Clay et al. (2001) ¯nd no tendency of prices of online bookstores to decrease over time. Brynjolfsson and Smith (1999), on the other hand, found that internet retailers had lower prices for books and CDs than conventional outlets.

imperfect substitutes and consumers value variety, a rise in transparency increases consumer awareness about di®erent products. This increases total demand, and may lead to higher prices. Note that in this case prices and welfare may move in the same direction, but the opposite can happen as well! Of course, this e®ect disappears if goods are perfect substitutes. Then there is no reason for consumers to buy more products if transparency increases. In this case we are left with the e®ect of transparency on competition: if consumers become better informed about prices this makes demand more elastic and decreases prices.[2] Finally, if some ¯rms target the market of less informed consumers, there is a range of values of transparency where a small increase in transparency increases price dispersion.

We illustrate this argument with a model in which there are two types of buyers. One type is perfectly informed about all prices o®ered by di®erent sellers; the other type is only informed about the product and price of one seller and is restricted to buy from this particular seller. The fraction of buyers that is perfectly informed is an exogenous parameter which we take as a measure for the level of transparency in the market. Thus we take a short cut on modeling the search process by which buyers become informed explicitly.[3] Doing so, however, allows us to study a richer demand side. In particular, we accommodate for decreasing (individual) demand, whereas search models typically focus on the case of unit demand. More importantly, it allows us to study the impact of the level of product di®erentiation. This feature of our model is important since, as will be seen, the impact of transparency on prices and welfare hinges crucially on the degree to which products are substitutes.

In the next section we present our market model. Section 3 analyzes the Cournot-Nash equilibrium of the model Section 4 concludes.

2 Model

Consider a market where n producers are active, $1; 2; :::; n$ and consumers are modelled on the unit interval $[0; 1]$. Consumers' utility functions are of the

[2]Another reason why transparancy may increase prices is that it facilitates collusion. Studies which focus on this possibility are M¿llgaard and Overgaard (2000) and Nilsson (1999).

[3]This transparancy parameter in a sense captures in a reduced form the e®ect the exogenous search cost parameters in models with endogenous search (e.g., Bakos, 1997, Janssen and Moraga, 2000, Stahl, 1989, Varian, 1980).

form

$$u(x_1, \ldots, x_n; M) = \sum_{i=1} \left(x_i - x_i^2 - 2\sigma \sum_{j>i} x_i x_j \right) + M \qquad (1)$$

where x_i is the amount of good i $(= 1, \ldots, n)$, M is a composite good of all the other products in the economy, and σ indicates the degree to which goods from different producers are substitutes. We use the composite good M as numeraire (hence the price of good M equals 1) Let a fraction τ of consumers be aware of all the n products in this market. Hence, they maximize this utility function above subject to a budget constraint. It is routine to verify that the inverse demand functions of these informed customers are of the form

$$p_k(x_k; x_{-k}) = 1 - 2x_k - 2\sigma \sum_{l \neq k} x_l$$

Further, of the $(1 - \tau)$ other consumers, a fraction $\frac{1}{n}$ knows only of product i and is not aware of any other product. Hence their utility function is $u(x_i; M) = x_i - x_i^2 + M$. The inverse demand function of an uninformed consumer equals

$$p_i(x_i) = 1 - 2x_i$$

We say that transparency increases in this market as the fraction of people τ who are aware of all products increases.

Now we analyze the equilibrium in this market. In particular, we are looking for the prices that clear the market in this context. We distinguish two sets of firms. First, there are the firms that produce for the whole market. That is, they sell to both the $\frac{1-\tau}{n}$ consumers that only know their product and to the τ consumers that know all n products. Second, there are firms that only produce for their $\frac{1-\tau}{n}$ captive consumers that are not aware of any other product.

Starting with the group of the producers that produce for all consumers that are aware of their products. These producers we denote by $\{1, 2, \ldots, m\}$. Let x_j denote total output of firm $j \in \{1, 2, \ldots, m\}$. Further, let x_j^U (x_j^I) denote firm j's output per head of uninformed (informed) customers. Since there are τ informed customers and $\frac{1-\tau}{n}$ uninformed customers who (only) know firm j's product, the following relationship holds between these output levels: $\tau x_j^I + \frac{1-\tau}{n} x_j^U = x_j$. Next, we assume that sellers cannot distinguish an informed from an uninformed customer and cannot price discriminate, implying that $p_j(x_1^I, \ldots, x_n^I; M) = p_j(x_j^U; M)$.

The second (and possibly empty) set of firms is denoted $\{m+1, \ldots, n\}$. These firms produce only for their captive consumers. Hence it must be the case that informed consumers are not willing to buy from such a firm.

Summarizing, we obtain the following definition of a market equilibrium.

Definition 1 Given any vector of total outputs $(x_1, \ldots, x_n)$ with $x_1 \geq x_2 \geq \ldots \geq x_n$, a market equilibrium is determined by the set of firms $\{1, 2, \ldots, m\}$ (with $m \leq n$) that produce for both the informed and uninformed markets, the set of firms $\{m+1, \ldots, n\}$ that produce only for their uninformed market and vector of prices $(p_1, \ldots, p_n)$ satisfying:

$$p_j\left(x_1^I, \ldots, x_m^I; M\right) = p_j\left(x_j^U; M\right) = p_j \tag{2}$$

$$\tau x_j^I + \frac{1-\tau}{n} x_j^U = x_j \tag{3}$$

for $j = 1, \ldots, m$, with m the highest value for which we have

$$1 - \sigma \sum_{j=1}^{m-1} x_j^I > \lambda p_m \tag{4}$$

where λ $(=1)$ is the marginal utility of income and

$$p_k\left(x_k^U; M\right) = p_k \tag{5}$$

$$x_k^U = \frac{n}{1-\tau} x_k \tag{6}$$

for $k = m+1, \ldots, n$.

Equation (2) says that prices are equal on both market segments, and we will refer to it as a no-arbitrage condition. Equation (3) is an accounting identity: total output x_j has to be allocated over the informed and uninformed market segments. The inequality (4) ensures that informed consumers are willing to buy from firm m but not from firm $m+1$, because the marginal value of buying good m evaluated at $x_m^I = 0$ $(1 - \sigma \sum_{j=1}^{m-1} x_j^I)$ is greater than the price of m multiplied by λ $(=1)$, the marginal value of income. If the informed consumers find the price for product $m+1$ too high, they also find the price for products $m' > m+1$ too high because of the assumption that output levels are ordered $x_1 \geq x_2 \geq \ldots \geq x_n$. Finally, equations (5) and (6) define the price and demand for those firms that only sell to their own captive consumers.

The following lemma gives an equivalent characterization of a market equilibrium.

Lemma 2 Given any vector of total outputs $(x_1, \ldots, x_n)$ with $x_1 \geq x_2 \geq \ldots \geq x_n$, the distribution of outputs over the informed and uninformed consumers in a market equilibrium is determined by the following equations

$$x_j^I + \sigma \sum_{\substack{l=1 \\ l \neq j}}^{n} x_l^I = x_j^U \tag{7}$$

$$\tau x_j^I + \frac{1-\tau}{n} x_j^U = x_j \tag{8}$$

for $j = 1, \ldots, m$, with m the highest value for which we have

$$x_m > \frac{1}{m - 1 + \frac{1-\sigma}{\sigma} + \frac{n\tau}{\sigma(1-\tau)}} \sum_{j=1}^{m-1} x_j \tag{9}$$

and

$$\begin{aligned} x_k^I &= 0 \\ x_k^U &= \frac{n}{1-\tau} x_k \end{aligned}$$

for $k = m+1, \ldots, n$.

Lemma 3 The following result about m will be used below and follows immediately from equation (9).

Corollary 4 If $x_1 = x_2 = \ldots = x_n$ then $m = n$.

So if each firm produces the same amount of output, then the unique market equilibrium is the one where all firms produce for both the informed and the uninformed market.

Lemma 5 Given the total output vector $(x_1, \ldots, x_n)$, the market equilibrium price for product $j \in \{1, \ldots, m\}$ equals

$$p_j(x_1, \ldots, x_n) = 1 - \beta x_j - \gamma \sum_{i \neq j; i=1}^{n} x_i$$

where

$$\gamma = \gamma(\sigma, \tau, n, m) = \frac{2n^2 \sigma \tau}{[(1-\tau)(1-\sigma) + n\tau][n\tau + (1-\tau)(1-\sigma+m\sigma)]} \tag{10}$$

$$\beta = \beta(\sigma, \tau, n, m) = \gamma + \frac{2n(1-\sigma)}{(1-\tau)(1-\sigma) + n\tau} \tag{11}$$

and for product $k \in \{m+1, \ldots, n\}$

$$p_k(x_k) = 1 - \frac{2n}{1-\tau} x_k$$

Note that for $\tau = 1$ we are back again in the usual case where $\beta = 2$ and $\gamma = 2\sigma$. That is, if the market is perfectly transparent we get the demand function which corresponds to the case where every consumer has utility function (1). In particular, in that case β and γ do not depend on the number of firms n and β does not depend on σ.

Also note that for the case where $\tau < 1$ and goods are perfect substitutes ($\sigma = 1$) and where all firms produce for both informed and uninformed market segments ($m = n$), we get $\beta = \gamma = 2$. So in that case β and γ do not depend on τ nor on n. The intuition for this is important as it explains many of the results that follow. The reason why β and γ (for the case where $\sigma < 1$ and $m = n$) depend on τ and n is that the firm faces two markets: one where decreasing marginal utility is strong (the uninformed market where consumers only buy the firm's own product) and one where it is weaker (because with $\sigma < 1$ the informed consumers like variety and hence are willing to consume more)[4]. The parameters τ and n determine the size of the uninformed market, which equals $\frac{1-\tau}{n}$. If $\sigma = 1$ and $m = n$, goods are perfect substitutes and marginal utility decreases at the same speed in both markets. Hence the relative sizes of these market are irrelevant. If $\sigma = 1$ and $m < n$, however, we see that β $(= \gamma)$ decreases with τ and the informed price rises with τ for given output levels $x_1, \ldots, x_n$. The intuition is as follows. Some people buy from firms that only sell on the uninformed market segment. In other words, these consumers buy from firms that charge high prices. As τ increases, some of these consumers switch to sellers that sell on both segments. For given output levels, this will cause prices to rise on the informed market.

To see how the values of σ and τ affect the demand curve a firm faces, the following result derives some comparative statics properties.

Lemma 6 Changes in τ and σ have the following effects on the demand curve if $\tau < 1$, $\sigma < 1$ and $m \leq n$

[4] The observation that marginal utility goes down faster on the uninformed market suggests that the firm would prefer to have informed consumers only. At first sight, this seems to contradict the idea that the firm is a monopolist on the uninformed market which makes this uninformed market more profitable than the informed one. The point is that we are taking output levels $x_1, \ldots, x_n$ fixed at this moment. The next section introduces the idea that output levels are determined in Nash equilibrium. This introduces the competition effect of transparancy τ.

(a) $\frac{\partial \beta}{\partial \tau} < 0; \frac{\partial \beta}{\partial \sigma} < 0;$

(b) $$\frac{\partial \gamma}{\partial \tau} \begin{cases} > 0 & \text{if} \quad 0 < \tau < \sqrt{\dfrac{\sigma + \frac{1}{n-1} - \frac{n-m}{n-1}\sigma}{(\sigma + n - 1)\left(1 + \frac{\sigma}{1-\sigma}\frac{n-m}{n-1}\right)}} \\ < 0 & \text{if} \quad \sqrt{\dfrac{\sigma + \frac{1}{n-1} - \frac{n-m}{n-1}\sigma}{(\sigma + n - 1)\left(1 + \frac{\sigma}{1-\sigma}\frac{n-m}{n-1}\right)}} < \tau < 1 \end{cases} ; \frac{\partial \gamma}{\partial \sigma} > 0;.$$

(c) $\frac{\partial(\beta + (m-1)\gamma)}{\partial \tau} < 0; \frac{\partial(\beta + (m-1)\gamma)}{\partial \sigma} > 0.$

First, consider the effect of τ on β. As τ increases the price of firm j decreases more slowly with a rise in j's total output level x_j. When τ is low, a relatively big part of j's output is sold on the uninformed market which features strong decreasing marginal utility. Hence a small increase in total output x_j causes a big fall in p_j. In other words, a big fall in the price is needed to convince the uninformed agents to buy the additional output. As τ increases, more output is sold to informed agents who (with $\sigma < 1$) have a taste for variety and hence are willing to absorb the increase of output x_j at a smaller price discount. That is, the price p_j falls by less in response to a rise in x_j, the higher τ is.

As goods become closer substitutes (σ goes up) an increase in x_j leads to a smaller fall in p_j ($\frac{\partial \beta}{\partial \sigma} < 0$). When goods are closer substitutes, it is easier for the informed consumers to buy more per head from j's good and substitute away from the competing goods. As a consequence, the decrease in p_j needed to absorb the increase in x_j is moderated.

Now consider the cross-price effects. The effect of τ on γ is the following. If $\tau = 0$, every firm serves its own uninformed market. Hence there is no interaction between firms and $\gamma = 0$. As τ increases there is interaction between firms and $\gamma > 0$. That is, with positive (but small) τ, a rise in the output level of firm i reduces the equilibrium price for firm j. With two firms it is, in fact, always the case that $\frac{\partial \gamma}{\partial \tau} > 0$. The more markets overlap, the more the price of firm j is negatively effected by an increase in the output of firm i. With more than two firms the effect is reversed for relatively high value of τ. To absorb an an increase in x_i the informed consumers will consume less per head of the products of all the other firms. If τ is large and there are many firms, this has a mitigating effect on the price decline that firm j will experience from an increase in x_i ($\frac{\partial \gamma}{\partial \tau} < 0$ for large n and large τ).

As goods are closer substitutes the price of firm j is more strongly affected by an increase in the output of firm i: $\frac{\partial \gamma}{\partial \sigma} > 0$. To absorb an increase in x_i the informed consumers will buy less from firm j. This effect is stronger the easier it is for the informed consumers to substitute x_j^I for x_i^I. If x_j^I decreases more strongly then, by (3), x_j^U must increase more strongly. It is this increase on the market for uninformed consumers that causes a bigger effect on the price of firm j.

Now turn to the last line (c) of the lemma. This considers the case where all firms raise their output simultaneously by the amount dx. What happens to firm j's price? Clearly it is the case that

$$\frac{dp_j}{dx} = -(\beta + (n-1)\gamma) < 0$$

Line (c) shows that the fall in p_j is bigger the smaller is τ, and the bigger is σ. The intuition for the first effect is that as τ decreases, more is sold on the uninformed market where marginal utility decreases faster than on the informed market. Hence a given rise dx leads to a bigger fall in prices. If σ is bigger, goods are closer substitutes. This implies two things. First, the informed consumers have less taste for variety and hence prices fall faster on the informed market. Second, the interaction between firms is bigger. Both work in the direction of a bigger fall in prices in response to dx if σ is higher.

From (c) we see immediately that for given output levels it is possible that a rise in transparency increases all prices p_j. If firms choose symmetric output levels, $x = x_1 = ... = x_n$ then we have $p_j = 1 - (\beta + (n-1)\gamma)x$. For given x, a rise in transparency τ increases prices p_j. The intuition for this effect is that if consumers become better informed about the availability of goods which are imperfect substitutes then total demand will increase. But if total supply is taken as given, the market equilibrium price must increase in order to restore the equality of demand and supply. This we call the demand effect of transparency, because supply is fixed in this analysis.

From (c) we also see that for given symmetric output levels, market prices will decrease if goods become closer substitutes (σ increases) or if the number of firms (n) increases.

Now we turn to the comparative static results with respect to n. Here we consider two cases. First, the case where the new firms that enter produce only for the uninformed markets. In other words, n goes up but m does not change as the new firms enter. This implies that the new firms are rather small (produce low output levels). The second case derives the effect of n when all firms produce for both the informed and uninformed market segment ($m \equiv n$).

Lemma 7 Changes in n have the following effects on the demand curve if $\tau < 1$ and $\sigma < 1$

(a) $\frac{\partial \beta(\sigma,\tau;n;m)}{\partial n} > 0; \frac{\partial \beta(\sigma,\tau;n;n)}{\partial n} > 0;$

(b) $\frac{\partial \gamma(\sigma,\tau;n;m)}{\partial n} > 0; \frac{\partial \gamma(\sigma,\tau;n;n)}{\partial n} > 0;.$

(c) $\frac{\partial(\beta(\sigma,\tau;n;m)+(m-1)\gamma(\sigma,\tau;n;m))}{\partial n} > 0; \frac{\partial(\beta(\sigma,\tau;n;n)+(n-1)\gamma(\sigma,\tau;n;n))}{\partial n} > 0.$

To understand the effect of n on β, note that as n increases the number of uninformed consumers $(1-\tau)$ is spread more thinly over the firms. A given rise in output by firm i leads to a bigger rise in output per uninformed head. Since the uninformed market has the faster decreasing marginal utility, such an increase in output leads to a bigger fall in p_i if n is higher.

The main effect of an increase in n is that it decreases $\frac{1-\tau}{n}$. The number of uninformed consumers per firm decreases. As we have just seen, an increase in x_i implies a decrease in x_j^I. Because $\frac{1-\tau}{n}$ is smaller as n is larger, a given reduction of x_j^I leads to a bigger increase in the consumption per head of the uninformed, x_j^U. To accommodate this increase, a bigger fall in p_j is needed. Therefore, we have $\frac{\partial \gamma}{\partial n} > 0$.

Finally, as n increases a given rise in output dx per firm, leads to a bigger rise in total output. Also, a given rise in i's output level and a given rise in j's output level have a bigger effect on i's price because as n increases a firm's uninformed market becomes smaller. All these effects cause a bigger fall in p_j in response to dx as n is bigger.

Up till now we have discussed the demand effects of parameter changes of the market equilibrium prices, given the supply levels of output. But, of course, output levels themselves will also be affected by these parameters. In the next section, we consider the effects of the parameters when firms choose their output levels in Cournot Nash equilibrium. The additional effects that we get in this case, we call competition effects.

3 Cournot Nash equilibrium

This section analyzes the equilibrium levels of output and price under the assumption that firms compete in quantities. As will be seen, an increase in transparency, will increase firms' output levels in the Cournot-Nash equilibrium. In this sense transparency makes the market more competitive. This effect is called the competition effect of transparency. It does not follow in general though that prices will fall as a consequence. This is due to the demand effect of transparency that we have seen in the previous section.

Let firms have identical costs functions denoted by $c(x_j)$, with $c' \geq 0$ and (unless explicitly stated otherwise) we assume $c'' \geq 0$: Then profits functions are

$$\pi_j(x_j; x_{-j}) = p_j(x_j; x_{-j})x_j - c(x_j) = (1 - \beta x_j - \gamma \sum_{i \neq j} x_i)x_j - c(x_j)$$

The first order conditions for the unique symmetric Cournot-Nash equilibrium ($x^N = x_1^N = ... = x_n^N$ and hence m = n by corollary 4) are

$$1 - (2\beta + (n-1)\gamma)x^N - c'(x^N) = 0 \qquad (12)$$

Note that the second order condition is

$$-2\beta - c''(x^N) < 0 \tag{13}$$

Using the implicit function theorem we find

$$\frac{dx^N}{d\tau} = -\frac{x^N}{2\beta + \gamma(n-1) + c''(x^N)} \frac{d(2\beta + (n-1)\gamma)}{d\tau} \tag{14}$$

As derived in Lemma 6 (with $m = n$), we have $\frac{d(\beta+(n-1)\gamma)}{d\tau} < 0$ and $\frac{d\beta}{d\tau} < 0$, for $\sigma < 1$ and $\tau < 1$. Hence, we have

Proposition 8 If firms have identical cost functions, there is a symmetric Cournot Nash equilibrium where each firm produces output level x^N. Furthermore, when $\sigma < 1$ we have

$$\frac{dx^N}{d\tau} > 0$$

Firms increase their output levels if more consumers become informed about available products and prices. This effect and its intuition are in line with the conventional wisdom regarding competition on transparent (Internet) markets. The competition between firms becomes fiercer if the group of informed consumers for which they compete becomes larger.

The competition effect of transparency can be dominated by the demand effect of transparency, as the next result shows. Further, we consider the competition effects of σ and n. In this result and other results below, we condition on the value of $c''(x^N)$ which we think about in the following way. First, in terms of a Taylor expansion around $x_i = x^N$, we can write $c(x)$ as

$$c(x) = c(x^N) + c'(x^N)\left(x - x^N\right) + \frac{1}{2}c''(\xi)\left(x - x^N\right)^2$$

for some ξ between x and x^N. Now if we change $c''(\xi)$, keeping $c'(x^N)$ constant, we stay in the same Nash equilibrium. However, changes in parameters will have different effects depending on the value of $c''(\xi)$. In this sense, we can do comparative static analysis with the second derivative of the cost function $c(\cdot)$.

Figure 1 makes the same point graphically. The downward sloping line is the marginal revenue curve for the symmetric Nash equilibrium, that is $1 - (2\beta + (n-1)\gamma)x$ as a function of x. The solid upward sloping curve is the marginal cost curve $c'(x)$. The point where these two lines intersect determines the Cournot Nash outcome x^N. The dotted upward sloping curve is the cost curve with the same value for $c'(x^N)$, but different value for the second derivative around x^N. Again, a small change in, say, τ shifts the marginal revenue curve and the change in x^N is then determined by the value of $c''(x)$ evaluated at $x = x^N$.

Proposition 9 With $\sigma < 1$ we find the following effects on the symmetric Nash price $p^N = 1 - (\beta + (n-1)\gamma)x^N$:

$$\frac{dp^N}{d\tau} \begin{cases} > 0 & \text{if } c''(x^N) \geq (n-1)\gamma \text{ and } \tau \in \left\langle \sqrt{\frac{\sigma + \frac{1}{n-1}}{\sigma + n - 1}}; 1 \right\rangle \\ < 0 & \text{if } c''(x^N) \leq (n-1)\gamma \text{ and } \tau \in \left\langle 0; \sqrt{\frac{\sigma + \frac{1}{n-1}}{\sigma + n - 1}} \right\rangle \end{cases}$$

$$\frac{dp^N}{d\sigma} < 0 \text{ if } c''(x^N) \leq (n-1)\gamma$$

$$\frac{dp^N}{dn} < 0 \text{ if } c''(x^N) \geq (n-1)\gamma$$

This result gives sufficient conditions under which a rise in competition through a rise in $\tau; \sigma$ or n reduces the Nash price. However, it also derives sufficient conditions under which the demand effect of transparency outweighs the competition effect. In that case, a rise in transparency raises the Nash equilibrium price. To get this result, firms' output levels should be relatively unresponsive to the rise in transparency; that is, c'' should be relatively big. Further, if $\tau > \sqrt{\frac{\sigma + \frac{1}{n-1}}{\sigma + n - 1}}$, a rise in transparency causes a fall in γ which, ceteris paribus, leads to a rise in the price level as well.

3.1 Transparency and consumer surplus

It is often assumed that consumers will benefit from a rise in transparency. In this section we show that under the Cournot Nash assumption, consumer welfare can be decreasing in transparency.

Since the equilibrium level of quantities x^N is increasing in τ (see proposition 8), social welfare (defined as the sum of consumer and producer surplus) is also increasing in τ. This can be seen as follows. Social welfare is here defined as consumer surplus minus production costs as a function of output levels. The first derivative of welfare with respect to output of product i then equals the marginal utility of good i minus the marginal cost of good i. In this case, consumers' marginal utility of good i equals the price of good i. So the sign of the derivative of welfare with respect to the output level of good i, equals the sign of $p_i - c'(x_i^N)$ which is positive under Cournot Nash.

Furthermore, in those cases in which prices are decreasing in τ, profits are decreasing and consumer surplus is increasing in τ. However, it is not necessarily the case that consumers benefit from a rise in transparency. Again, if supply is rather inelastic, consumers may loose, despite the fact that total output increases.

Let u^I denote the utility level of an informed agent, that is

$$u^I \equiv \max_{x_1,\ldots,x_n,M} \left\{ \sum_{i=1} \left(x_i - x_i^2 - \sigma \sum_{j>i} x_i x_j \right) + M \left| \sum_{i=1} p_i x_i + M \le y \right. \right\}$$

where we assume that each agent (whether informed or uninformed) has the same income level y. Let u_j^U denote the utility level of an uninformed agent who only knows of good j (besides good M), that is

$$u_j^U \equiv \max_{x_j,M} \left\{ x_j - x_j^2 + M \left| p_j x_j + M \le y \right. \right\}$$

Then we have the following intuitive result.

Lemma 10 If $x_i^I > 0$ for some good i, then $u^I > u_k^U$ for some $k \neq i$.

The lemma implies that the informed agent is always better off than the uninformed agent who only consumes good k, if there is another good $i \neq k$ that the informed agent consumes in a positive amount. The proof of this result is straightforward. Because $x_i^I > 0$, a small increase in the price of good i leads (by the envelope theorem) to a decrease in the utility of the informed agent. Since the uninformed agent (who only knows good k) can be viewed as facing a price of good i equal to $+\infty$, his utility level is strictly lower than the informed agent's utility level.

Consumers welfare is defined as

$$W = \tau g(u^I) + \frac{1-\tau}{n} \sum_{j=1} g(u_j^U)$$

with $g'(\cdot) > 0$. We consider three cases for $g(\cdot)$ which differ in the weight assigned to the informed (high utility) and the uninformed (low utility) consumers: (i) $g(\cdot)$ is linear (welfare is the sum of utilities), (ii) $g(\cdot)$ is concave (a marginal change in utility of the uninformed has a bigger impact on welfare than a change in the utility of the informed), and (iii) $g(\cdot)$ is convex (utility of the informed has a bigger impact on welfare than utility of the uninformed). As the next Proposition shows, in all three cases it is possible that welfare falls with an increase in transparency.

Proposition 11 Assume each firm j has the same cost function $c(x_j)$ with $c'(.) > 0$ and $c''(.) \geq 0$. If $c''(x^N)$ is "big"
(i) and if $g(.)$ is concave, then there exists a value $\tau^* > 0$ such that

$$\frac{dW}{d\tau} < 0$$

for each $\tau < \tau^*$;
(ii) and if $g(.)$ is convex, then there exists a value $\tau^{**} < 1$ such that

$$\frac{dW}{d\tau} < 0$$

for each $\tau > \tau^{**}$;
(iii) and if $g(.)$ is linear, then it is the case that

$$\frac{dW}{d\tau} < 0$$

for each value of τ.

The lemma shows that, irrespective of the shape of $g(.)$, there always exist situations in which a rise in τ leads to a fall in consumer surplus. The intuition for this result is as follows. A rise in τ has two positive effects. First, it raises total output as established in proposition 8. Second, it increases the number of informed agents who have higher utility than the uninformed agents, as established in lemma 10. The first positive effect can be reduced to zero by increasing $c''(.)$ at the Cournot Nash equilibrium. The second positive effect is outweighed by the loss in utility of both informed and uninformed agents. These agents lose utility, because the increase in the number of informed agents raises demand for all goods and hence raises prices. If $c(.)$ is very convex, the competition effect of transparency is limited and welfare can fall as τ rises.

3.2 Price dispersion

Above we have focussed on the unique symmetric pure strategy equilibrium. For some parameter values there also exist asymmetric pure strategy equilibria and mixed strategy equilibria. These equilibria involve price dispersion. We will not give a complete characterization of the equilibria but proceed by means of example. We will illustrate that it is not generally the case that an increase in transparency will lead to a reduction in price dispersion.

In an asymmetric pure strategy equilibrium there are m firms ($j = 1; \ldots; m$) who sell to the informed consumers as well as to their own captive consumers and $n - m$ firms ($k = m + 1; \ldots; n$) who sell only to their own captive consumers. For simplicity assume that all firms face the cost function $c(x_i) = 0$. According to Lemma 5 a firm in the first group faces demand function $p_j = 1 - \beta x_j - \gamma \sum_{i \neq j; i=1}^{m} x_i$. The first order condition for firm j is $1 - 2\beta x_j - \gamma \sum_{i \neq j; i=1}^{m} x_i = 0$. Symmetry among the first m firms then leads to $x_j = x^m \equiv \frac{1}{2\beta + (m-1)\gamma}$ with corresponding price

$$p^m(\sigma, \tau, n) = \frac{\beta}{2\beta + (m-1)\gamma}$$

The firms producing for their own uninformed consumers face demand function $p_k = 1 - \frac{2n}{1-\tau} x_k$ They maximize profits by setting $x_k = x^U \equiv \frac{1-\tau}{4n}$. The corresponding price is

$$p^U(\sigma, \tau, n) \equiv \frac{3}{4}$$

From Lemma 6 it follows that p^m is decreasing in both τ and σ (see also Proposition 9). At the same time we have $p^U > p^m$ and p^U independent of both τ and σ. Hence, as τ or σ increase the gap between p^U and p^m widens and price dispersion increases.

For this to be an equilibrium, two conditions must be satisfied. Firstly, the m firms producing for the informed should not want to switch to servicing only their own uninformed consumers. Secondly, the $n - m$ firms producing for their own uninformed consumers should not want to make the opposite switch. To derive these two conditions for the general case is straightforward but tedious. Therefore, as said, we proceed by means of example.

Consider the case $n = 3, m = 2$ and $\sigma = 1$.[5] From $\beta(1, \tau, 3 | m = 2) = \gamma(1, \tau, 3 | m = 2) = \frac{6}{2+\tau}$, it follows that $x_1 = x_2 = x^m(1, \tau, 3, 2) = \frac{1}{18}(2 + \tau)$ with corresponding profits $\pi^m = \frac{1}{54}(2 + \tau)$. Furthermore, we have $x_3 = \frac{1}{12}(1 - \tau)$ and $\pi^U = \frac{1}{24}(1 - \tau)$. For this to be an equilibrium, the first condition is that $\pi^M \geq \pi^U$, which requires $\tau \geq \frac{1}{13}$. The second condition is that firm 3 does not want to switch to producing for the informed consumers. In that case we would have $\gamma(1, \tau, 3 | m = 3) = 2$ and firm 3 would have demand function $p_3 = 1 - 2(x_1 + x_2 + x_3)$. Given $x_1 = x_2 = x^M$ the optimal deviation for firm 3 would be to produce $x_3 = x^D = \frac{1}{36}(5 - 2\tau)$ and earn $\pi^D(1, \tau, 3, 2) = \frac{1}{648}(5 - 2\tau)^2$. This profit does not exceed π^U if $\tau \leq \frac{1}{4}$. Hence we have an asymmetric equilibrium if $\frac{1}{13} \leq \tau \leq \frac{1}{4}$. For the prices we find: $p_1 = p_2 = 1 - \beta(x_1 + x_2) = \frac{1}{3}$ and $p_3 = 1 - \frac{6}{1-\tau} x_3 = \frac{3}{4}$. Hence, within the range $\frac{1}{13} \leq \tau \leq \frac{1}{4}$, prices do not depend on τ. However, this equilibrium

[5] It is straightforward to show that in any asymmetric pure strategy equilibrium we must have $m \geq 2$.

can be slightly perturbed to generate an asymmetric equilibrium in which price dispersion is increasing in τ: For $\sigma = 1 - \epsilon$, (products are almost but not perfect substitutes), we have an asymmetric equilibrium for a similar range of values of τ. For this equilibrium we have $p_1 = p_2 = \frac{\beta}{2\beta+\gamma}$ which is decreasing in τ if $\sigma < 1$. At the same time, we have $p_1 = p_2 < p_3 = \frac{3}{4}$, with p_3 independent of τ. Hence, the gap between $p_1 = p_2$ and p_3 widens as τ increases locally. At some point, a further increase in τ will destroy the asymmetric equilibrium. At that point, the fraction of informed consumers becomes so large that firm 3 is better off serving these consumers as well, rather than focussing on its own captive consumers (condition $\pi^D \leq \pi^U$ will be violated). We then enter the region in which the unique pure strategy equilibrium is the symmetric equilibrium.

4 Conclusion

The impact of transparency on prices and consumer surplus is more subtle than conventional wisdom seems to suggest. In our model an increase in transparency under some circumstances leads to an increase in prices, an increase in price dispersion and a decrease in consumer surplus. These effects are due to, what we have called, the demand effect of transparency. If goods are imperfect substitutes and consumers have a taste for variety, more widespread information about the availability of goods wets consumers appetite and shifts demand outward. This demand effect may counterbalance the downward pressure on prices due the competition effect of transparency.

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6 Appendix: proofs

Proof of lemma 2

From $p_j\left(x_1^{\omega};...;x_m^{\omega};M\right) = p_j(x_j^U;M)$ it follows that $1 - 2x_j^{\omega} - 2\sigma\sum_{l\neq k} x_l^{\omega} = 1 - 2x_j^U$ which gives us (7). The only part of the Lemma that is not self-evident is inequality (9). Substitution of $p_m = 1 - \frac{2n}{1-\tau}x_m$ into (4) gives

$$x_m > \frac{(1-\tau)\sigma}{n}\sum_{j=1}^{m-1} x_j^{\omega} \tag{15}$$

Substituting (7) into (8) gives (remember that we evaluate at $x_m^{\omega} = 0$)

$$(\tau + \frac{1-\tau}{n})x_j^{\omega} + \frac{(1-\tau)\sigma}{n}\sum_{\substack{l=1\\ l\neq j}}^{m-1} x_l^{\omega} = x_j$$

that is,

$$(\tau + \frac{1-\tau}{n})x_j^{\omega} + \frac{(1-\tau)\sigma}{n}\sum_{l=1}^{m-1} x_l^{\omega} - \frac{(1-\tau)\sigma}{n}x_j^{\omega} = x_j$$

Rearranging and taking the sum over j yields

$$[\tau + \frac{(1-\tau)}{n}(1-\sigma) + (m-1)\frac{(1-\tau)\sigma}{n}]\sum_{j=1}^{m-1} x_j^{\omega} = \sum_{j=1}^{m-1} x_j$$

Substitution in (15) gives us inequality (9).

Q.E.D.

Proof of lemma 5

Let $\{1, 2, \ldots, m\}$ denote the set of firms that produce on both the informed and uninformed market segments and $\{m+1, \ldots, n\}$ the set of firms that only produce on the uninformed market segment. We start with the former group of firms.

Substituting equation (2) into (3) and rewriting yields

$$x_k^I = \frac{n}{n\tau + (1-\tau)(1-\sigma)} x_k - \frac{\sigma(1-\tau)}{n\tau + (1-\tau)(1-\sigma)} \sum_{j=1}^{m} x_j^I \qquad (16)$$

Substituting into the expression for the price yields

$$\begin{aligned} p_k &= 1 - 2(1-\sigma)x_k^I - 2\sigma \sum_{j=1}^{m} x_j^I \\ &= 1 - \frac{2(1-\sigma)n}{n\tau + (1-\sigma)(1-\tau)} x_k - \left[2\sigma - \frac{2(1-\sigma)\sigma(1-\tau)}{n\tau + (1-\sigma)(1-\tau)}\right] \sum_{j=1}^{m} x_j^I \\ &= 1 - \frac{2(1-\sigma)n}{n\tau + (1-\sigma)(1-\tau)} x_k - \left[\frac{2n\sigma\tau}{n\tau + (1-\sigma)(1-\tau)}\right] \sum_{j=1}^{m} x_j^I \end{aligned}$$

Summation of (16) over k and rewriting yields

$$\sum_{k=1}^{m} x_k^I = \frac{n}{n\tau + (1-\tau)(1-\sigma+\sigma m)} \sum_{k=1}^{m} x_k$$

Inserting in the expression for p_k gives

$$\begin{aligned} p_k = {} & 1 - \frac{2(1-\sigma)n}{n\tau + (1-\sigma)(1-\tau)} x_k - \\ & \frac{2n^2\sigma\tau}{[n\tau + (1-\sigma)(1-\tau)][n\tau + (1-\tau)(1-\sigma+\sigma m)]} \sum_{j=1}^{m} x_j \end{aligned}$$

which corresponds to the expression in the Lemma.

Turning to the firms $\{m+1, \ldots, n\}$ we substitute $x_k^U = \frac{n}{1-\tau} x_k$ into the expression for the price for such a firm k. Hence we get

$$p_k(x_k) = 1 - \frac{2n}{1-\tau} x_k$$

Q.E.D.

For ease of later reference it is convenient to consider the symmetric case where each firm produces the same output level.

Lemma 12 If $x_1 = \ldots = x_m = x$ then the market equilibrium conditions (2) and (3) yield the following expressions for $x_j^I = x^I$ and $x_j^U = x^U$ for each $j \in \{1, \ldots, m\}$.

$$x^I = \frac{x}{\frac{1-\tau}{n}(1+\sigma(m-1))+\tau} \qquad (17)$$

$$= \frac{x}{\frac{1+\sigma(m-1)}{n}+\tau\frac{(n-1)(1-\sigma)+\sigma(n-m)}{n}} \qquad (18)$$

$$x^U = \frac{x}{\frac{1-\tau}{n}+\frac{\tau}{1+\sigma(m-1)}} \qquad (19)$$

$$= \frac{(1+\sigma(m-1))\,x}{\frac{(1+\sigma(m-1))}{n}+\tau\frac{(n-1)(1-\sigma)+\sigma(n-m)}{n}} \qquad (20)$$

Proof of lemma 6

• Effect of τ on β and γ.

Here it is convenient to start with $\frac{\partial\gamma}{\partial\tau}$. It is routine to verify that

$$\frac{\partial\gamma}{\partial\tau} = 2n^2\sigma\frac{(1-\sigma)(1+(m-1)\sigma)-(n+\sigma-1)(n-1-\sigma(m-1))\tau^2}{\{[\tau(n-1+\sigma)+(1-\sigma)][1+(m-1)\sigma(1-\tau)+\tau(n-1)]\}^2}$$

Hence we find that $\frac{\partial\gamma}{\partial\tau} \geq 0$ if and only if

$\tau^2 \leq \frac{(1-\sigma)(1+(m-1)\sigma)}{(n+\sigma-1)(n-1-\sigma(m-1))} = \frac{\sigma+\frac{1}{n-1}-\frac{n-m}{n-1}\sigma}{(\sigma+n-1)\left(1+\frac{\sigma}{1-\sigma}\frac{n-m}{n-1}\right)}$.

In order to prove that $\frac{\partial\beta}{\partial\tau} < 0$, we consider two cases:

(i) if $\frac{\partial\gamma}{\partial\tau} < 0$ then it is routine to verify that $\frac{\partial\beta}{\partial\tau} < 0$;

(ii) if $\frac{\partial\gamma}{\partial\tau} \geq 0$ then we prove by contradiction that $\frac{\partial\beta}{\partial\tau} < 0$: Suppose (by contradiction) that there exist values of m, n (with $m \leq n$), σ and τ such that $\frac{\partial\beta}{\partial\tau} \geq 0$. Then choose $x_1 = \ldots = x_m = x > 0$ and $x_{m+1} = \ldots = x_n = \bar{x}$ such that inequality (9) holds:

$$\bar{x} < \frac{mx}{m-1+\frac{1-\sigma}{\sigma}+\frac{n\tau}{\sigma(1-\tau)}} \qquad (21)$$

$$x > \frac{(m-1)\,x}{m-2+\frac{1-\sigma}{\sigma}+\frac{n\tau}{\sigma(1-\tau)}} \qquad (22)$$

and such that prices $p_1, \ldots, p_m$ are positive. This is always possible by choosing x and $\bar{x}$ small enough. Then as derived in equation (20) x^U can be written as

$$x^U = \frac{(1+\sigma(m-1))}{\frac{(1+\sigma(m-1))}{n} + \tau\frac{(n-1)(1-\sigma)+\sigma(n-m)}{n}} x$$

Hence for given value of x we find that $\frac{\partial x^U}{\partial \tau} < 0$ and hence $\frac{\partial p}{\partial \tau} > 0$ because $p(x^U) = 1 - 2x^U$. Also in the symmetric case we have $p(x) = 1 - (\beta + (m-1)\gamma)x$. Hence, $\frac{\partial p}{\partial \tau} > 0$ for given x implies

$$\frac{\partial(\beta + (m-1)\gamma)}{\partial \tau} < 0 \tag{23}$$

Because inequalities (21) and (22) hold with strict inequality, we have $\frac{\partial m}{\partial \tau} = 0$. Now $\frac{\partial \gamma}{\partial \tau} \geq 0$ together with $\frac{\partial \beta}{\partial \tau} \geq 0$ contradicts $\frac{\partial(\beta+(m-1)\gamma)}{\partial \tau} < 0$. Hence it must be the case that $\frac{\partial \beta}{\partial \tau} < 0$. Using a similar argument, we can rule out $\frac{\partial(\beta+(m-1)\gamma)}{\partial \tau} > 0$. This proves the result in line (c).

• Effect of σ on β and γ.

It is routine to verify that

$$\frac{\partial \beta}{\partial \sigma} = -\frac{2(m-1)n^2\sigma(1-\tau)\tau[2+(1-\tau)(m-2)\sigma+2(n-1)\tau]}{(1+(m-1)\sigma(1-\tau)+\tau(n-1))^2(1-\sigma+(n+\sigma-1)\tau)^2} < 0$$

It is also routine to verify that

$$\frac{\partial \gamma}{\partial \sigma} = 2n^2\tau\frac{(m-1)\sigma^2(1-\tau)^2+(1+(n-1)\tau)^2}{(1+(m-1)\sigma(1-\tau)+\tau(n-1))^2(1-\sigma+(n+\sigma-1)\tau)^2} > 0$$

In order to show that $\frac{\partial(\beta+(m-1)\gamma)}{\partial \sigma} > 0$, we use again a proof by contradiction. Suppose (by contradiction) that there exist values of m, n (with $m \leq n$), σ and τ such that $\frac{\partial(\beta+(m-1)\gamma)}{\partial \sigma} < 0$. Then choose $x_1 = \ldots = x_m = x > 0$ and $x_{m+1} = \ldots = x_n = \bar{x}$ such that inequality (9) holds:

$$\bar{x} < \frac{mx}{m-1+\frac{1-\sigma}{\sigma}+\frac{n\tau}{\sigma(1-\tau)}} \tag{24}$$

$$x > \frac{(m-1)x}{m-2+\frac{1-\sigma}{\sigma}+\frac{n\tau}{\sigma(1-\tau)}} \tag{25}$$

and such that prices $p_1, \ldots, p_m$ are positive. This is always possible by choosing x and $\bar{x}$ small enough. Now we write the expression for x^U as in equation (19)

$$x^U = \frac{1}{\frac{1-\tau}{n} + \frac{\tau}{1+\sigma(m-1)}} x$$

Hence we see that $\frac{\partial x^U}{\partial \sigma} > 0$. Then it follows from $p(x^U) = 1 - 2x^U$ that $\frac{\partial p}{\partial \sigma} < 0$. Further, in a symmetric equilibrium we also find $p(x) = 1 - (\beta + (n-1)\gamma)x$. Then $\frac{\partial p}{\partial \sigma} < 0$ contradicts $\frac{\partial(\beta+(m-1)\gamma)}{\partial \sigma} < 0$. Hence it must be the case that

$$\frac{\partial(\beta + (n-1)\gamma)}{\partial \sigma} > 0$$

which proves the effect of σ in line (c) of the lemma.

Q.E.D.

Proof of lemma 7

- Effect of n on $\beta(\sigma, \tau, n, m)$ and $\gamma(\sigma, \tau, n, m)$.

Write the expression for $\gamma(\sigma, \tau, n, m)$ in equation (10) as

$$\gamma(\sigma, \tau, n, m) = \frac{2\sigma\tau}{\left(\frac{(1-\tau)(1-\sigma)}{n} + \tau\right)\left(\tau + \frac{(1-\tau)(1-\sigma+m\sigma)}{n}\right)}$$

Hence $\frac{\partial \gamma(\sigma,\tau,n,m)}{\partial n} > 0$ if $\tau < 1$.

Then writing equation (11) as

$$\beta(\sigma, \tau, n, m) = \gamma(\sigma, \tau, n, m) + \frac{2(1-\sigma)}{\frac{(1-\tau)(1-\sigma)}{n} + \tau}$$

we see that $\frac{\partial \beta(\sigma,\tau,n,m)}{\partial n} > 0$ if $\tau < 1$. From this it also follows that $\frac{\partial(\beta(\sigma,\tau,n,m)+(m-1)\gamma(\sigma,\tau,n,m))}{\partial n} > 0$.

- Effect of n on $\beta(\sigma, \tau, n, n)$ and $\gamma(\sigma, \tau, n, n)$.

Write the expression for $\gamma(\sigma, \tau, n, n)$ in equation (10) as

$$\gamma(\sigma, \tau, n, n) = \frac{2\sigma\tau}{\left(\frac{(1-\tau)(1-\sigma)}{n} + \tau\right)\left(1 - (1-\tau)(1-\sigma) + \frac{(1-\tau)(1-\sigma)}{n}\right)}$$

Hence $\frac{\partial \gamma(\sigma,\tau,n,n)}{\partial n} > 0$ if $\tau < 1$ and $\sigma < 1$.

Then writing equation (11) as

$$\beta(\sigma;\tau;n;n) = \gamma(\sigma;\tau;n;n) + \frac{2(1-\sigma)}{\frac{(1-\tau)(1-\sigma)}{n} + \tau}$$

we see that $\frac{\partial \beta(\sigma;\tau;n;n)}{\partial n} > 0$ if $\tau < 1$ and $\sigma < 1$. From this it also follows that $\frac{\partial(\beta(\sigma;\tau;n;n)+(n-1)\gamma(\sigma;\tau;n;n))}{\partial n} > 0$.

Q.E.D.

Proof of proposition 9

For the Cournot Nash price we have
$p^N = p_i(x^N;\ldots;x^N) = 1 - (\beta + (n-1)\gamma)\, x^N$: Hence

$$\begin{aligned}\frac{dp^N}{d\tau} &= -\left[x^N \frac{d(\beta+(n-1)\gamma)}{d\tau} + (\beta+(n-1)\gamma)\frac{dx^N}{d\tau}\right] \\ &= -\frac{x^N}{2\beta+\gamma(n-1)+c''(x^N)} \times \\ &\quad \left[-(\beta+(n-1)\gamma)\frac{d(2\beta+(n-1)\gamma)}{d\tau} + (2\beta+\gamma(n-1)+c''(x^N))\frac{d(\beta+(n-1)\gamma)}{d\tau}\right]\end{aligned}$$

Now using equation (11) to write β as $\beta = \gamma + \varepsilon$, where $\varepsilon \equiv \frac{2n(1-\sigma)}{(1-\tau)(1-\sigma)+n\tau}$; we can write $\frac{dp^N}{d\tau}$ as

$$\frac{dp^N}{d\tau} = -\frac{x^N}{2\beta+\gamma(n-1)+c''(x^N)} \times \left[\left(c''(x^N) - (n-1)\gamma\right)\underbrace{\frac{\partial\varepsilon}{\partial\tau}}_{<0} + \underbrace{\left(c''(x^N)n + \varepsilon(n-1)\right)}_{>0}\frac{\partial\gamma}{\partial\tau}\right]$$

Hence we see that $\frac{dp^N}{d\tau} > 0$ if $c''(x^N) - (n-1)\gamma > 0$ and $\frac{\partial\gamma}{\partial\tau} < 0$. Also $\frac{dp^N}{d\tau} < 0$ if $c''(x^N) - (n-1)\gamma < 0$ and $\frac{\partial\gamma}{\partial\tau} > 0$. Using lemma 6 (c), the condition on the sign of $\frac{\partial\gamma}{\partial\tau}$ translates into the condition whether τ is bigger or smaller than $\sqrt{\frac{\sigma+\frac{1}{n-1}}{\sigma+n-1}}$.

Similarly, the effect of σ on p^N is determined as follows

$$\frac{dp^N}{d\sigma} = -\frac{x^N}{2\beta + \gamma(n-1) + c''(x^N)} \times \left[\left(c''(x^N) - (n-1)\gamma\right) \underbrace{\frac{\partial \varepsilon}{\partial \sigma}}_{<0} + \underbrace{\left(c''(x^N) n + \varepsilon (n-1)\right)}_{>0} \underbrace{\frac{\partial \gamma}{\partial \sigma}}_{>0} \right]$$

Hence, a sufficient condition for $\frac{dp^N}{d\sigma} < 0$ is $c''(x^N) - (n-1)\gamma < 0$.

Finally, the effect of n on p^N is determined by

$$\frac{dp^N}{dn} = -x^N \underbrace{\frac{d\left(\beta + (n-1)\gamma\right)}{dn}}_{>0} - \left(\beta + (n-1)\gamma\right) \underbrace{\frac{dx^N}{dn}}_{<0} - \gamma x^N$$

where the sign of $\frac{dx^N}{dn}$ follows from differentiating equation (12) with respect to n:

$$\frac{dx^N}{dn} = -\frac{x^N}{2\beta + \gamma(n-1) + c''(x^N)} \frac{d\left(2\beta + (n-1)\gamma\right)}{dn} < 0$$

substituting this into the expression for $\frac{dp^N}{dn}$ we get

$$\begin{aligned} \frac{dp^N}{dn} &= \frac{x^N}{2\beta + \gamma(n-1) + c''(x^N)} \times \\ &\quad \left[\left(\beta + (n-1)\gamma\right) \frac{d\beta}{dn} - \left(\beta + c''(x^N)\right) \frac{d\left(\beta + (n-1)\gamma\right)}{dn} - \left(2\beta + \gamma(n-1) + c''(x^N)\right)\gamma \right] \\ &= \frac{x^N}{2\beta + \gamma(n-1) + c''(x^N)} \times \\ &\quad \left[\left((n-1)\gamma - c''(x^N)\right) \frac{d\beta}{dn} - (n-1)\left(\beta + c''(x^N)\right) \frac{d\gamma}{dn} - \left(3\beta + \gamma(n-1) + 2c''(x^N)\right)\gamma \right] \end{aligned}$$

Since $\frac{d\beta}{dn} > 0$ and $\frac{d\gamma}{dn} > 0$, we see that a sufficient condition for $\frac{dp^N}{dn} < 0$ is $(n-1)\gamma - c''(x^N) < 0$.

Q.E.D.

Proof of proposition 11

In the symmetric equilibrium, we can write

$$\begin{aligned} u^I &= n\left(x^I - \left(x^I\right)^2 - \sigma(n-1)\left(x^I\right)^2 + M\right) \\ &= n\left(x^I - \left(x^I\right)^2 - \sigma(n-1)\left(x^I\right)^2 - p x^I\right) + y \\ &= n\left(x^I - \left(x^I\right)^2 - \sigma(n-1)\left(x^I\right)^2 - \left[x^I - 2\left(1 + \sigma(n-1)\right)\left(x^I\right)^2\right]\right) + y \end{aligned}$$

where we have used that the price for each good k equals $p_k = 1 - 2x_k - 2\sigma\sum_{j\neq k} x_j$. Hence we find

$$u^I = y + n\left(1+\sigma(n-1)\right)\left(x^I\right)^2$$

Similarly, we find

$$\begin{aligned} u_j^U &= x^U - \left(x^U\right)^2 + y - \left(1 - 2x^U\right)x^U \\ &= y + \left(x^U\right)^2 \end{aligned}$$

Further, in a symmetric equilibrium we have from equations (20) and (18)

$$\begin{aligned} x^U &= \frac{\left(1+\sigma(n-1)\right)x}{\frac{1+\sigma(n-1)}{n} + \tau\frac{(n-1)(1-\sigma)}{n}} \\ x^I &= \frac{1}{\left(1+\sigma(n-1)\right)}x^U \end{aligned}$$

Hence we can write welfare as

$$W = \tau g\left(y + \frac{n}{1+\sigma(n-1)}\left(x^U\right)^2\right) + (1-\tau)g\left(y + \left(x^U\right)^2\right)$$

Consequently,

$$\frac{dW}{d\tau} = \underbrace{g(u^I) - g(u^U)}_{(*)} + \left[\tau g'(u^I)\frac{n}{1+\sigma(n-1)} + (1-\tau)g'(u^U)\right]2x^U\underbrace{\frac{dx^U}{d\tau}}_{(**)}$$

We look at the terms (*) and (**) in turn.

(*): Taking a first order Taylor approximation of $g(u^I)$ around u^U we get

$$\begin{aligned} g(u^I) &= g(u^U) + g'(\xi)\left(u^I - u^U\right) \\ &= g(u^U) + g'(\xi)\left(\frac{n}{1+\sigma(n-1)} - 1\right)\left(x^U\right)^2 \\ &= g(u^U) + g'(\xi)\frac{(n-1)(1-\sigma)}{1+\sigma(n-1)}\left(x^U\right)^2 \end{aligned}$$

for some value $\xi \in \langle u^U; u^I\rangle$. Because $g(.)$ is either convex or concave, we also find that $g'(\xi)$ lies between $g'(u^I)$ and $g'(u^U)$.

(**): Writing x^U as $x^U = \frac{nx}{\tau\frac{(n-1)(1-\sigma)}{1+\sigma(n-1)}+1}$, the term $\frac{dx^U}{d\tau}$ can be written as

$$\frac{dx^U}{d\tau} = -\frac{x^U}{\tau\frac{(n-1)(1-\sigma)}{1+\sigma(n-1)}+1}\frac{(n-1)(1-\sigma)}{1+\sigma(n-1)} + \frac{x^U}{x}\underbrace{\frac{dx}{d\tau}}_{\approx 0}$$

where $\frac{dx}{d\tau}$ is close to zero because of the assumption that $c''(x^N)$ is big.

Substituting the expressions for $(*)$ and $(**)$ into the equation for $\frac{dW}{d\tau}$, one finds

$$
\begin{aligned}
\frac{dW}{d\tau} &= g'(\xi)\frac{(n-1)(1-\sigma)}{1+\sigma(n-1)}\left(x^U\right)^2 - \\
&\quad 2\left(x^U\right)^2 \frac{\left[\tau g'(u^I)\frac{n}{1+\sigma(n-1)} + (1-\tau)\, g'(u^U)\right]}{\tau\frac{(n-1)(1-\sigma)}{1+\sigma(n-1)}+1}\,\frac{(n-1)(1-\sigma)}{1+\sigma(n-1)} \\
&= \frac{(n-1)(1-\sigma)}{1+\sigma(n-1)}\left(x^U\right)^2 \left[g'(\xi) - 2\underbrace{\frac{\left[\tau\frac{n}{1+\sigma(n-1)}g'(u^I) + (1-\tau)\, g'(u^U)\right]}{\tau\frac{n}{1+\sigma(n-1)} + (1-\tau)}}_{(***)} \right]
\end{aligned}
$$

As noted above, $g'(\xi)$ lies between $g'(u^I)$ and $g'(u^U)$, hence one can choose a value $\tilde{\tau}$ so that the term labelled $(***)$ equals $g'(\xi)$ and $\frac{dW}{d\tau} < 0$. Further, if $g(.)$ is concave, putting more weight on $g'(u^U)$ (i.e. choosing a lower value of τ than $\tilde{\tau}$) ensures that $\frac{dW}{d\tau} < 0$. Similarly, if $g(.)$ is convex, putting more weight on $g'(u^I)$ (i.e. choosing a higher value of τ than $\tilde{\tau}$) ensures that $\frac{dW}{d\tau} < 0$.

Finally, if $g(.)$ is linear, $g'(\xi) = g'(u^I) = g'(u^U)$ and $\frac{dW}{d\tau} < 0$.

Q.E.D.

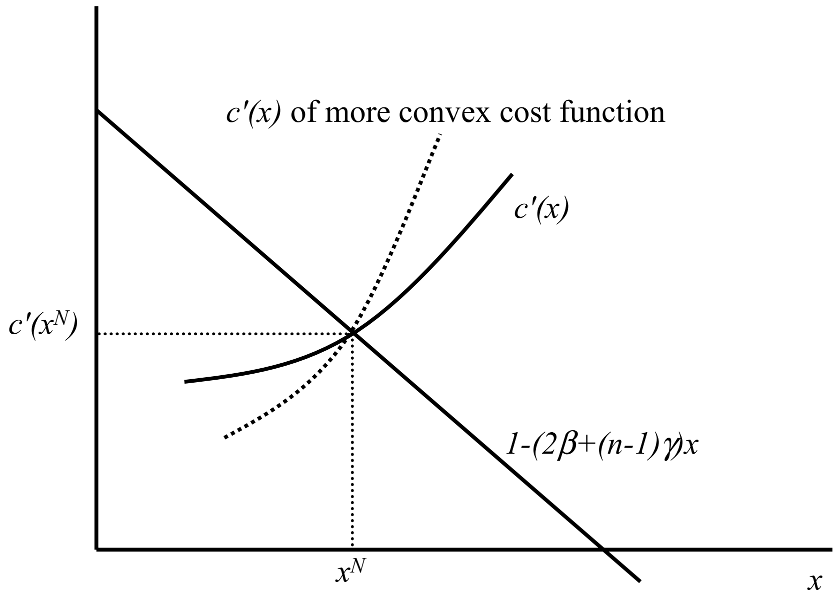

Figure 1: making costs *c(x)* more convex without changing the Nash equilibrium