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Kinky Choices, Dictators and Split Might

A Non-Cooperative Model for Household Consumption and Labor Supply*

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Abstract

It is unlikely that husbands and wives always agree on exactly what public goods to buy. Nor do they necessarily agree on how many hours to work with obvious consequences for the household budget. We therefore model consumption and labor supply behavior of a couple in a non-cooperative setting by adopting a Nash approach. Using minimal assumptions, we prove that demand for public goods is characterized by three regimes. It is either determined by the preferences of one of the partners only (Husband Dictatorship or Wife Dictatorship), or by both spouses' preferences where a partner's influence depends on the spouses' relative wage rates (Split Might). These regimes imply a kinked nature of the couple's aggregate demand curves. By imposing more structure on the model, we can derive testable implications on observed demand for public goods and labor supply that allow testing the model against the standard unitary model where a couple behaves as a single decision maker. The model is applied to a sample drawn from the Consumer Expenditure Survey (CEX) whereby we explicitly focus on expenses on children's goods that act as a public good in the spouses' preferences. We find that for couples with two or three children the standard unitary model is strongly rejected in favor of our non-cooperative model. Moreover, it turns out that for the majority of these couples, there is a Wife Dictatorship in the sense that the spending pattern is according to her preferences.

Key words: Consumption, labor supply, intra-household allocation, non-cooperative model, public goods.
JEL-classification: D11, D12, D13.

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1. Introduction

Common sense suggests that no two persons, and particularly not one man and one woman, are alike. And although it can be argued that two persons that voluntarily form a household will probably do so on the basis of shared interests and preferences, no one will maintain that married individuals have identical preferences on all accounts. This means that (economic) decisions taken jointly in a household will be more complicated than those taken by an individual.

One of the important insights from recent applied microeconometrics is that multi-person households indeed do not behave as single decision makers. This single decision maker assumption, which is fundamental to the standard unitary model of household behavior, is associated with the theoretical implication that observed demand or labor supply should satisfy the well-known Slutsky conditions. Slutsky symmetry and negativity, though, are usually rejected when confronted with consumption or labor supply data (see Fortin and Lacroix, 1997, Browning and Chiappori, 1998, Vermeulen, 2005, and Cherchye and Vermeulen, 2008, for some recent examples). Evidence thus suggests that intra-household bargaining aspects within multi-person households cannot be ignored.

A number of alternatives to the unitary model have been put forward. One strand in the literature assumes that household members only choose Pareto efficient allocations. This is either formalized by means of axiomatic bargaining theory (see, for example, Manser and Brown, 1980, and McElroy and Horney, 1981) or via the so-called collective model (see Chiappori, 1988, 1992, and Apps and Rees, 1988). Another strand of the literature assumes that household members behave non-cooperatively (see Leuthold, 1968, and Ashworth and Ulph, 1981, for seminal contributions and Browning, 2000, and Browning, Chiappori and Lechene, 2009, for more recent work).

One of the main implications of the models that recognize that households do not behave as single decision makers is that intra-household allocations may depend on individual resources of household members rather than on only the household's aggregate means. A leading empirical example in this respect is that children seem to benefit more when the mother (or grandmother) brings relatively more financial resources into the household. This has been consistently shown for both developed and developing nations (e.g. Lundberg, Pollak and Wales, 1997, for the U.K., Thomas, Contreras and Frankenberg, 1997, for Indonesia, and Duflo, 2003, for South Africa). From a policy point of view this dependence on who brings what to the table is very important. It implies that policy makers can alter the intra-household allocation of resources through targeted taxes or transfers.

The targeting issue has been investigated in a collective setting (see Blundell, Chiappori and Meghir, 2005) as well as in a non-cooperative setting. The usual approach in the latter is by considering models that focus on the private provision of public goods (see, among others, Bergstrom, Blume and Varian, 1986, Chen and Woolley, 2001, and Lechene and Preston, 2008). An important restriction in the above non-cooperative models is that the individual labor incomes, and thus the household's resources, are assumed exogenous. However, it is not difficult to come up with arguments in favor of more realistic models that not only describe the intra-household allocation of resources but also the generation of these resources themselves.

A first aim of the current study is to fill this gap by focusing on the private provision of public goods while also taking into account labor supply decisions and the consequent endogenous individual incomes. We will start out with the most general model and investigate what could happen to public goods demand in this set-up. We define three regimes and show that all Nash equilibria fall in either of these regimes. In the first regime, the husband is dictator (denoted by HD). More specifically, the household's demand for public goods fully reflects the husband's preferences given the household's aggregate resources. The second regime is associated with split might (SM): each spouse's preferences are reflected in the household's spending on public goods. The extent to which this happens depends on the spouses' relative wage rates. Finally, the third regime is associated with the wife being the dictator (WD).

Like in Browning, Chiappori and Lechene (2009), and contrary to Lundberg and Pollak (1993), the regimes are determined 'endogenously' by the spouses' relative wage rates. The main difference between our results and those obtained by Browning et al. (2009) is that we do not have a pure local income pooling phenomenon. This is due to the endogenous individual labor incomes in our model which rule out exogenous shifts in individual income sources that keep the household's aggregate budget fixed. Still, there is some sort of local income pooling in both dictatorship regimes: aggregate spending on public goods is as the most powerful spouse would want it to be.

A second contribution of the current study is that the theoretical model for the private provision of public goods with endogenous labor supply will be brought to the data. Contrary to collective models, of which a wide variety of empirical applications are available, empirical evidence for the private provision of public goods remains scarce. This gap will be partly filled in our paper by means of an empirical model that focuses on expenditures on children's goods (such as children's clothing, toys or tuition fees). These child related goods are considered to be public goods inside a household. Since the empirical model imposes more structure on the general model, we can derive testable implications of this model against the standard unitary model. The data for the application are drawn from the Consumer Expenditure Survey (CEX) collected by the Bureau of Labor Statistics of the U.S. Department of Labor. Taking the targeting literature into account, we suspect that preferences within the household regarding spending on children's goods will differ. We find that we can reject the standard unitary model in favor of our non-cooperative approach for couples with two or three children under the age of 18. Using the estimated preference parameters, we can then divide households into dictatorship and split might regimes. It turns out that 73% of two-child and 57% of three-child couples spend according to the wife's preferences.

The rest of this paper unfolds as follows. In Section 2, we propose a general private provision of public goods model with endogenous labor supply and discuss its implications. Section 3 focuses on the empirical specification and the estimation strategy, that will be based on a maximum likelihood approach. The data and estimation results are discussed in Sections 4 and 5. Section 6 concludes.

2. The model

We focus on two adult households where f denotes the wife and m the husband.[1] Spouses have to decide on their demands for leisure $l^f, l^m \in [0,1]$ (normalized between zero and one; including the option to be out of the labor market) and on how the household's aggregate resources are allocated to a vector of private (denoted by the vectors $\mathbf{q}^f \in \mathbb{R}_+^{n_f}, \mathbf{q}^m \in \mathbb{R}_+^{n_m}$) and public goods inside the household (denoted by the vector $\mathbf{Q} \in \mathbb{R}_+^{n_p}$). The household's aggregate resources are assumed to be equal to the sum of the individual labor incomes $w^f\left(1-l^f\right)+w^m\left(1-l^m\right)$, where w^i is individual i's wage rate ($i=f,m$). This means we abstract away from non-labor income. Prices of private and public goods are denoted by respectively $\mathbf{p}$ and $\mathbf{P}$ (where with a slight abuse of notation we use the same notation $\mathbf{p}$ for both private good vectors).

In what follows, we assume that leisure is a private good in the sense that it does not entail intra-household externalities. Therefore, the wife's and husband's utility functions, representing their own

[1] It goes without saying that all results apply to same-sex couples as well. For notational reasons we stick to the traditional husband and wife terminology.

preferences, are given by respectively:

$$u^f\left(l^f,\mathbf{q}^f,\mathbf{Q}\right) \tag{1}$$

and

$$u^m\left(l^m,\mathbf{q}^m,\mathbf{Q}\right). \tag{2}$$

The question now is how the household comes to make decisions. Following, among others, Browning et al. (2009), we assume a non-cooperative setting by adopting a Nash approach. A Nash equilibrium is defined as follows:

Definition 1 *A* Nash equilibrium *consists of individual leisure, a vector of individual private consumption and individual contributions to public goods* $\left(l^{i*},\mathbf{q}^{i*},\mathbf{Q}^{i*}\right)$, $i=f,m$, *such that for each* i, $\left(l^{i*},\mathbf{q}^{i*},\mathbf{Q}^{i*}\right)$ *solves for* $i\neq j$*:*

$$\max_{l^i\in[0,1],\mathbf{q}^i,\mathbf{Q}^i\geq 0} u^i\left(l^i,\mathbf{q}^i,\mathbf{Q}^i+\mathbf{Q}^{j*}\right) \tag{3}$$

$$s.t.\ w^il^i+\mathbf{p}'\mathbf{q}^i+\mathbf{P}'\mathbf{Q}^i = w^i.$$

We make the following assumptions on the utility functions (deleting superscripts to ease notation if a condition holds for both partners). We denote marginal utility with respect to good x by u_x and assume that this derivative is well defined. The first four assumptions are fairly standard. The combination of the first and the third assumption seems to imply that both partners participate in the labor market, as one obtains infinite utility from consuming some private goods and as labor income is the only resource available to pay for these private goods. We come back to this below. The last assumption is most interesting as it implies a conflict within the household.

Assumption *For arbitrary values of* $l,\mathbf{q},\mathbf{Q}$ *we have that*

1. $\lim_{x\downarrow 0}u_x(l,\mathbf{q},\mathbf{Q})=+\infty$ *for leisure* ($x=l$)*, each private good* ($x=q_k$) *and each public good* ($x=Q_k$)*,*
2. $\lim_{x\to+\infty}u_x(l,\mathbf{q},\mathbf{Q})=0$ *for each private good* ($x=q_k$) *and each public good* ($x=Q_k$)*,*
3. $u_l(1,\mathbf{q},\mathbf{Q})<+\infty$*,*
4. $u(l,\mathbf{q},\mathbf{Q})$ *is concave in leisure, each private good and each public good and*
5. *there exist (at least) two public goods* $Q_k,Q_{k'}$ *such that*

$$\frac{u^f_{Q_k}}{u^f_{Q_{k'}}}>\frac{u^m_{Q_k}}{u^m_{Q_{k'}}}.$$

The first assumption says that for each good the marginal utility goes to infinity as the amount of the good goes to zero. The second assumption says that for each private and public good the marginal utility goes to zero as the amount of the good goes to infinity. These assumptions are made for ease of exposition. Since we are interested in corner solutions where one of the partners does not contribute to a public good, we want to avoid corner solutions in private goods and total contributions to public goods.[2] The third assumption implies that u_l is finite if the partner does not work at all ($l = 1$). This leaves us with two ways to capture non-participation in the labor market by a partner. First, $w = 0$ implies that someone does not participate. Second, in the empirical specification we assume that there are no private goods because of data limitations. Without private goods, non-participation arises for a range of wages $w > 0$. Concavity is sufficient to allow us to use stationary points to characterize a global maximum. Finally, we assume that there is a tension between the partners. They never agree on the overall contributions to the public goods. Under assumptions one to five, we can prove the following. The appendix contains the proof.

Lemma 1 *In Nash equilibrium*[3] *we have for both partners that*

$$\frac{u_l}{u_{q_k}} = \frac{w}{p_k}$$

for each private good q_k.

We next define three regimes and show below that all Nash equilibria fall in either of these regimes. In the first regime, the husband is dictator (denoted by HD). More specifically, the household's demand for public goods fully reflects the husband's preferences given the household's aggregate resources. The second regime is associated with split might (SM): each spouse contributes to public goods but not to all of them, and, moreover, has a say on how the household's aggregate resources are allocated. The resulting allocation is however not as any of the spouses would wish it to be. Finally, the third regime is associated with the wife being the dictator (WD). Like in Browning, Chiappori and Lechene (2009), and contrary to Lundberg and Pollak (1993), these regimes are determined 'endogenously' by the spouses relative wage rates. Note that one special case within the SM regime is the Separate Spheres regime of the latter two papers. Under Separate Spheres, each spouse contributes to strictly different sets of public goods (an example will be given in the empirical application in Section 3).

Formally, we have the following (where we use the convention on inequalities with vectors that $\mathbf{x} < \mathbf{y}$ implies that $x_k \leq y_k$ for all k where the inequality is strict for at least one k).

Definition 2 *The three regimes are defined as follows*

HD $u^m_{\mathbf{Q}^m} = \lambda^m \mathbf{P}$ *and* $u^f_{\mathbf{Q}^f} < \lambda^f \mathbf{P}$,

SM $u^m_{\mathbf{Q}^m} < \lambda^m \mathbf{P}$ *and* $u^f_{\mathbf{Q}^f} < \lambda^f \mathbf{P}$ *and*

WD $u^m_{\mathbf{Q}^m} < \lambda^m \mathbf{P}$ *and* $u^f_{\mathbf{Q}^f} = \lambda^f \mathbf{P}$.

where $\lambda^i = u^i_{q_k}/p_k$ *(from Lemma 1) is the Lagrange multiplier associated with partner i's* ($i = f, m$) *budget constraint.*

[2] Allowing for such corner solutions adds inequalities to the optimality conditions. This complicates notation without adding insight. The corner solutions for individual contributions to public goods are however interesting as we show below.

[3] It follows from Theorem 1.2 in Fudenberg and Tirole (1991, p. 34) that a pure strategy Nash equilibrium exists in our case. Multiple equilibria cannot be ruled out.

The following proposition demonstrates that the three regimes above are the only ones that can occur in equilibrium. The proof can be found in the appendix.

Proposition 1 *If $w^m > 0$ and/or $w^f > 0$, then HD, SM and WD are the only possibilities. That is, the Nash equilibrium in (3) is characterized by the equalities in Lemma 1 and the conditions in either HD, SM or WD.*

The example in Browning et al. (2009) demonstrates that the regimes are ordered as HD, (our) SM and WD and that the ordering is a function of the wife's exogenous share of income. We present a similar result, with endogenous incomes, in the next section. Here we consider what we can say about this ordering in our general set-up. The next result (partially) characterizes the ordering of the three regimes in terms of relative wages $\rho = w^f/w^m$. See the appendix for a proof.

Proposition 2 *There exist critical values $\rho_0 > 0$ and $\rho_1 > \rho_0$ such that the household is in regime HD for each $w^f/w^m < \rho_0$ and in regime WD for each $w^f/w^m > \rho_1$.*

Our empirical specification in the next section imposes more structure on the utility functions u^m and u^f. This allows us to show that for given $w^m > 0$ we move through the regimes as w^f increases in the order HD, SM and finally WD. With the general set-up in this section we cannot rule out orderings like HD, SM, HD (again), WD. However, it is both surprising and insightful that we can prove a result like Proposition 2 given the few assumptions made.

The robust insight is that when one partner (potentially) has a sufficiently higher wage rate relative to that of the other partner, the household allocation to public goods is determined completely by this partner's preferences - given the household's aggregate resources. That is, one always starts with HD (for low ρ) and ends up with WD (for high ρ).

3. Empirical specification and estimation strategy

3.1. Empirical specification

We will illustrate the existence of dictatorship and split might regimes by means of a sample of couples with children drawn from the Consumer Expenditure Survey (CEX). This dataset contains not only detailed purchases by households but also information on wages and labor supply of each household member (see the next section for more details). Given the particular data at hand, we will focus on a special case of the general model described above. In this special case the only possible regimes continue to be Husband Dictatorship, Split Might and Wife Dictatorship. Moreover, in the empirical specification the ordering of the regimes is perfectly known. Firstly, it turns out that almost all prime age men in the selected sample work full time. Therefore, we will assume that men's labor supply is exogenously fixed. Wives, on the contrary, are assumed to be faced with a continuous hours choice (including non-participation). Wives' leisure is a private good in the model. Secondly, as is common in budget surveys, expenditures are recorded at the household level. This implies that, for most goods, one cannot observe the spouses' individual consumption of private goods. As a result, we will assume that all consumption is public inside the household. Two public goods will be distinguished: a composite good that relates to expenditures on children's goods (such as clothing, toys and tuition fees) and a composite good relating to other (nondurable) expenditures.

To obtain a tractable empirical specification, we will assume that spouses have preferences that can be represented by Cobb-Douglas utility functions. The utility functions of the husband and the wife are written as follows:

$$u^m = \beta \ln Q_1 + (1-\beta)\ln Q_2 \quad (4)$$

$$u^f = \omega \ln l^f + (1-\omega)(\alpha \ln Q_1 + (1-\alpha)\ln Q_2), \quad (5)$$

where Q_1 (Q_2) is the composite good related to non-children's (children's) goods and l^f is the wife's leisure. Let us denote the respective prices of these public goods by P_1 and P_2. The spouse-specific preference parameters α, β and ω are between zero and one.

Since we will let the data speak for themselves, we will not make any explicit assumption on the spouses' relative valuation of the public goods. In other words, we will not impose that, say, the wife values the child related public good relatively more than the husband. Therefore, two cases can be distinguished: if $\beta > \alpha$, then the husband values the child related public good less than the wife, while we have the reverse conclusion if $\beta \leq \alpha$.

Let us first focus on the $\beta > \alpha$ scenario. As shown below, this model is associated with three regimes like in the general model in Section 2. The first regime is characterized by the husband being the dictator, which implies that the household's aggregate resources are entirely allocated according to his preferences. The second regime is associated with split might. Given the assumption that $\beta > \alpha$, the child related public good is entirely financed by the wife, while the other public good is entirely financed by the husband. This situation corresponds to the Separate Spheres case in Browning, Chiappori and Lechene (2009). Finally, there is the regime where the wife will be the dictator. In this regime, the household's aggregate resources are allocated according to her preferences. The specific regime in which a couple will be located will depend on the wife's wage given her husband's wage and the spouses' preference parameters. Let us now explicitly characterize the three regimes when spouses have the above Cobb-Douglas utility functions.

Husband dictator First, for the wife's wage w^f rather small (to be made precise below) the husband dictates the entire allocation over Q_1 and Q_2. This implies that:

$$Q_1 = \frac{\beta y}{P_1} \quad (6)$$

$$Q_2 = \frac{(1-\beta)y}{P_2}, \quad (7)$$

where

$$y = w^m + (1-l^f)w^f. \quad (8)$$

Now consider the wife's labor supply decision in this situation:

$$\max_{l^f} \omega \ln l^f + (1-\omega)(\alpha \ln\left(\frac{\beta}{P_1}\right) + (1-\alpha)\ln\left(\frac{(1-\beta)}{P_2}\right) + \ln(w^m + (1-l^f)w^f)). \quad (9)$$

The first order condition for l^f can be written as

$$\frac{l^f w^f}{y} = \frac{\omega}{1-\omega}.$$

Solving for l^f, we get

$$l^f = \omega(1 + \frac{w^m}{w^f}). \quad (10)$$

Taking account of the fact that $l^f \leq 1$, the wife will not participate (i.e., $l^f = 1$) if

$$w^f \leq w^m \frac{\omega}{1-\omega}.$$

Hence, in this case we have

$$\begin{aligned} Q_1 &= \frac{\beta w^m}{P_1} \\ Q_2 &= \frac{(1-\beta) w^m}{P_2}. \end{aligned}$$

Next consider the case where $w^f \in [w^m \frac{\omega}{1-\omega}, w^m \frac{1-\beta(1-\omega)}{\beta(1-\omega)}]$. The household's aggregate resources are now given by

$$y = (1-\omega)(w^f + w^m). \tag{11}$$

In this case, the husband stays the dictator and the demand functions for public goods are

$$Q_1 = \frac{\beta(1-\omega)(w^f + w^m)}{P_1} \tag{12}$$

$$Q_2 = \frac{(1-\beta)(1-\omega)(w^f + w^m)}{P_2}. \tag{13}$$

Hence, we find that the husband's contribution to Q_2 is given by

$$\begin{aligned} Q_2^m &= \frac{(1-\beta)(1-\omega)(w^f + w^m) - w^f(1-l^f)}{P_2} \\ &= \frac{w^m - \beta(1-\omega)(w^m + w^f)}{P_2} \end{aligned} \tag{14}$$

as the wife spends her entire income on Q_2. This phase stops once $Q_2^m = 0$ which happens when $w^f \geq w^m \frac{1-\beta(1-\omega)}{\beta(1-\omega)}$.

Split might Assume that $w^f \in [w^m \frac{1-\beta(1-\omega)}{\beta(1-\omega)}, w^m \frac{1-\alpha(1-\omega)}{\alpha(1-\omega)}]$. This is the situation in which $Q_2^m = 0$ and $Q_1^f = 0$. Now we have

$$Q_1 = Q_1^m = \frac{w^m}{P_1} \tag{15}$$

$$Q_2 = Q_2^f = \frac{(1-\omega)\, w^f - \omega w^m}{P_2}. \tag{16}$$

This will last until

$$\frac{Q_2}{Q_1} = \frac{1-\alpha}{\alpha} \frac{P_1}{P_2}$$

or, equivalently,

$$w^f = w^m \frac{1-\alpha(1-\omega)}{\alpha(1-\omega)}. \tag{17}$$

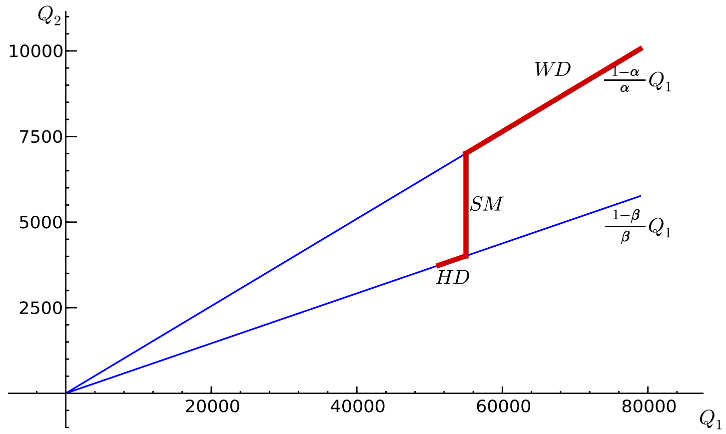

Figure 1: The kinky nature of demand for public goods when $\beta > \alpha$. As w^f increases demand moves along the thick line. (Here $w^m = 55.000$, $\alpha = 0.887$, $\beta = 0.932$ and $\omega = 0.191$.)

Wife dictator For $w^f \geq w^m \frac{1-\alpha(1-\omega)}{\alpha(1-\omega)}$, the allocation of the household's aggregate resources is entirely determined by the wife's preferences:

$$Q_1 = \frac{\alpha(1-\omega)(w^f + w^m)}{P_1} \tag{18}$$

$$Q_2 = \frac{(1-\alpha)(1-\omega)(w^f + w^m)}{P_2}. \tag{19}$$

The above characterization of the three regimes applies when $\beta > \alpha$ (i.e., when the wife values the child related public good relatively more than the husband). Figure 1 summarizes what happens in this situation for given values of w^m, α, β and ω.[4] A similar characterization can be derived for the situation when $\beta \leq \alpha$ (i.e., when the husband values the child related good relatively more than the wife). The characterizations of the different regimes for both situations are summarized as follows:

Situation 1: $\beta > \alpha$

$$l^f = \begin{cases} 1 & \texttt{if} \quad w^f \leq \frac{\omega}{1-\omega} w^m \\ \omega(1 + \frac{w^m}{w^f}) & \texttt{if} \quad w^f > \frac{\omega}{1-\omega} w^m \end{cases}$$

$$Q_1 = \begin{cases} \frac{\beta w^m}{P_1} & \texttt{if} \quad w^f \leq \frac{\omega}{1-\omega} w^m \\ \frac{\beta(1-\omega)(w^f + w^m)}{P_1} & \texttt{if} \quad w^f \in [w^m \frac{\omega}{1-\omega}, w^m \frac{1-\beta(1-\omega)}{\beta(1-\omega)}] \\ \frac{w^m}{P_1} & \texttt{if} \quad w^f \in [w^m \frac{1-\beta(1-\omega)}{\beta(1-\omega)}, w^m \frac{1-\alpha(1-\omega)}{\alpha(1-\omega)}] \\ \frac{\alpha(1-\omega)(w^f + w^m)}{P_1} & \texttt{if} \quad w^f > w^m \frac{1-\alpha(1-\omega)}{\alpha(1-\omega)} \end{cases} \tag{20}$$

[4] The parameters in the figure are based on the data that we introduce below.

$$Q_2 = \begin{cases} \frac{(1-\beta)w^m}{P_2} & \texttt{if} \quad w^f \leq \frac{\omega}{1-\omega} w^m \\ \frac{(1-\beta)(1-\omega)(w^f+w^m)}{P_2} & \texttt{if} \quad w^f \in [w^m \frac{\omega}{1-\omega}, w^m \frac{1-\beta(1-\omega)}{\beta(1-\omega)}] \\ \frac{(1-\omega)w^f - \omega w^m}{P_2} & \texttt{if} \quad w^f \in [w^m \frac{1-\beta(1-\omega)}{\beta(1-\omega)}, w^m \frac{1-\alpha(1-\omega)}{\alpha(1-\omega)}] \\ \frac{(1-\alpha)(1-\omega)(w^f+w^m)}{P_2} & \texttt{if} \quad w^f > w^m \frac{1-\alpha(1-\omega)}{\alpha(1-\omega)}. \end{cases}$$

Situation 2: $\beta \leq \alpha$

$$l^f = \begin{cases} 1 & \texttt{if} \quad w^f \leq \frac{\omega}{1-\omega} w^m \\ \omega(1 + \frac{w^m}{w^f}) & \texttt{if} \quad w^f > \frac{\omega}{1-\omega} w^m \end{cases}$$

$$Q_1 = \begin{cases} \frac{\beta w^m}{P_1} & \texttt{if} \quad w^f \leq \frac{\omega}{1-\omega} w^m \\ \frac{\beta(1-\omega)(w^f+w^m)}{P_1} & \texttt{if} \quad w^f \in [w^m \frac{\omega}{1-\omega}, w^m \frac{1-(1-\beta)(1-\omega)}{(1-\beta)(1-\omega)}] \\ \frac{(1-\omega)w^f - \omega w^m}{P_1} & \texttt{if} \quad w^f \in [w^m \frac{1-(1-\beta)(1-\omega)}{(1-\beta)(1-\omega)}, w^m \frac{1-(1-\alpha)(1-\omega)}{(1-\alpha)(1-\omega)}] \\ \frac{\alpha(1-\omega)(w^f+w^m)}{P_1} & \texttt{if} \quad w^f > w^m \frac{1-(1-\alpha)(1-\omega)}{(1-\alpha)(1-\omega)} \end{cases} \tag{21}$$

$$Q_2 = \begin{cases} \frac{(1-\beta)w^m}{P_2} & \texttt{if} \quad w^f \leq \frac{\omega}{1-\omega} w^m \\ \frac{(1-\beta)(1-\omega)(w^f+w^m)}{P_2} & \texttt{if} \quad w^f \in [w^m \frac{\omega}{1-\omega}, w^m \frac{1-(1-\beta)(1-\omega)}{(1-\beta)(1-\omega)}] \\ \frac{w^m}{P_2} & \texttt{if} \quad w^f \in [w^m \frac{1-(1-\beta)(1-\omega)}{(1-\beta)(1-\omega)}, w^m \frac{1-(1-\alpha)(1-\omega)}{(1-\alpha)(1-\omega)}] \\ \frac{(1-\alpha)(1-\omega)(w^f+w^m)}{P_2} & \texttt{if} \quad w^f > w^m \frac{1-(1-\alpha)(1-\omega)}{(1-\alpha)(1-\omega)}. \end{cases}$$

3.2. Estimation strategy

As is clear from above, the household demand system for female leisure and both public goods has a kinked nature. Figure 1 highlights this. Moreover, the kinks are determined endogenously since they depend on the spouses' preference parameters and relative wage rates. In what follows, we will lay down a maximum likelihood estimation strategy that is combined with an iteration method to obtain estimates of the unknown α, β and ω parameters. Note that we do not include any observable demographic variables or taste shifters in the model; the reasons for this are twofold. First, we have a sample that is already selected on the basis of variables that are typically included in demand analyses (think about demographic composition; see the next section for details); a generalization, though, is feasible. Second, our estimation strategy in which we allow for the possibility that $\beta > \alpha$ or $\beta \leq \alpha$ complicates such a generalization substantially.

It is easily seen that in each regime within the situations (20) and (21) there is adding up: adding expenditures on the two composite public goods and female leisure always equals the household's full budget (i.e., $w^f l^f + \mathbf{P}'\mathbf{Q} = w^f + w^m$). This implies that, as in standard demand analysis, one of the goods in the three-good demand system may be deleted. We opt to model the wife's leisure and the child related public good.

The estimation strategy can be summarized as follows. Firstly, optimization error with respect to the demand equations, translated in item specific disturbance terms, and measurement error in observed female wages are introduced. Secondly, for some starting values of the preference parameters α, β and ω, the location of the kinks is determined on the basis of the right-hand sides of equations (20) and (21).[5] These initial boundaries determine in which region each household is initially located; this

[5]Which set of equations is used depends on whether $\beta > \alpha$ or not. To save on notation, we will not explicitize this in what follows.

is done by comparing the wife's wage with the respective boundaries. Thirdly, a likelihood function is maximized which involves both the probability of observing a couple at a particular location and the densities of the demand system disturbance terms. The obtained maximum likelihood estimates for α, β and ω are then used to determine an updated location of the kinks, which in its turn determines an updated location of each household on the kinked demand curves. The whole procedure is iterated until convergence of the estimated preference parameters (which, at the same time, obtains converged locations of the kinks). Let us now explain the estimation strategy in more detail.

Let Q_{i2} denote the demand for the child related public good in couple i and let l_i^f be the wife's leisure. These observed demands are assumed to be equal to $f_1^{r,s}\left(w_i^m, w_i^f, P_{i1}, P_{i2}\right) + \epsilon_{i1}$ and $f_2^{r,s}\left(w_i^m, w_i^f, P_{i1}, P_{i2}\right) + \epsilon_{i2}$, where $f_k^{r,1}$ (for goods k =1,2, and for regions r =1,2,3,4) correspond to the left-hand side equations in (20) and $f_k^{r,2}$ to those in (21). Optimization errors are captured by the disturbances ϵ_{ik} (k =1,2). Let us further assume that the true wage rate of the wife in couple i is observed with some additive measurement error η_i independent from the true wage. The vector containing both optimization error and measurement error $\begin{pmatrix} \epsilon_{i1} & \epsilon_{i2} & \eta_i \end{pmatrix}'$ is assumed to be drawn from a multivariate normal distribution with mean $\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}'$ and a covariance matrix $\begin{pmatrix} \sigma_{\epsilon_1}^2 & 0 & 0 \\ 0 & \sigma_{\epsilon_2}^2 & 0 \\ 0 & 0 & \sigma_\eta^2 \end{pmatrix}$.

For given starting values for the unknown preference parameters, denoted by $\widehat{\alpha}_0$, $\widehat{\beta}_0$ and $\widehat{\omega}_0$, the (couple-specific) location of the kinks in the demand system can be calculated. This happens on the basis of the right-hand sides of equations (20) and (21) where the former is used when $\widehat{\beta}_0 > \widehat{\alpha}_0$ and the latter when $\widehat{\beta}_0 \leq \widehat{\alpha}_0$. Next, the (initial) location of a couple in the demand system can be determined by comparing the wife's relative wage rate with these boundaries.

Assume that $\beta > \alpha$. Then the probability that couple i is located before the first kink in the demand curves can be written as $P\left(w_i^f + \eta_i \leq w_i^m \frac{\omega}{1-\omega}\right)$. As the scale of the measurement error is not identified, we normalize it by assuming that it is equal to the variance of observed wages. Given the distributional assumptions made, this implies that the above probability equals $\Phi\left(w_i^m \frac{\omega}{1-\omega} - w_i^f\right)$. In a similar way, the probabilities associated with the other kinks can be derived. In summary, we have the following probabilities:

$$\begin{aligned}
P_i^{1,1} &= P\left(w_i^f + \eta_i \leq w_i^m \frac{\omega}{1-\omega}\right) = \Phi\left(w_i^m \frac{\omega}{1-\omega} - w_i^f\right) \\
P_i^{2,1} &= P\left(w_i^m \frac{\omega}{1-\omega} < w_i^f + \eta_i \leq w_i^m \frac{1-\beta(1-\omega)}{\beta(1-\omega)}\right) \\
&= \Phi\left(w_i^m \frac{1-\beta(1-\omega)}{\beta(1-\omega)} - w_i^f\right) - \Phi\left(w_i^m \frac{\omega}{1-\omega} - w_i^f\right) \\
P_i^{3,1} &= P\left(w_i^m \frac{1-\beta(1-\omega)}{\beta(1-\omega)} < w_i^f + \eta_i \leq w_i^m \frac{1-\alpha(1-\omega)}{\alpha(1-\omega)}\right) \\
&= \Phi\left(w_i^m \frac{1-\alpha(1-\omega)}{\alpha(1-\omega)} - w_i^f\right) - \Phi\left(w_i^m \frac{1-\beta(1-\omega)}{\beta(1-\omega)} - w_i^f\right) \\
P_i^{4,1} &= 1 - \Phi\left(w_i^m \frac{1-\alpha(1-\omega)}{\alpha(1-\omega)} - w_i^f\right).
\end{aligned} \tag{22}$$

In a similar way, probabilities associated with the situation where $\beta \leq \alpha$ (denoted by $P_i^{r,2}$ for $r = 1, 2, 3, 4$) can be derived.

As mentioned above, the likelihood function that will be maximized takes into account both the above probabilities of observing a couple at the respective locations in the demand system and the densities of the disturbance terms. Let us introduce the binary variable d_0 which equals 1 if the starting values for the preference parameters imply that $\widehat{\beta}_0 > \widehat{\alpha}_0$ and 0 otherwise. Let us further introduce four couple-specific dummy variables d_{i0}^r (r =1,2,3,4) that indicate the region in which a couple locates on the basis of these starting values.

Combining all probabilities and densities, and taking account of the above distributional assumptions, one obtains the following loglikelihood function:

$$\begin{aligned}
\log L_0 &= d_0 \sum_{i=1}^{n} \left(\sum_{r=1}^{4} \left\{ d_{i0}^{r,1} (\ln P_i^{r,1} + \ln(\frac{1}{\sqrt{2\pi\sigma_{\epsilon_1}^2}} \exp(-\frac{1}{2}\frac{(Q_{i2} - f_1^{r,1})^2}{\sigma_{\epsilon_1}^2}))) \right\} \right. \\
&+ \left. \sum_{r=2}^{4} \left\{ d_{i0}^{r,1} (\ln(\frac{1}{\sqrt{2\pi\sigma_{\epsilon_2}^2}} \exp(-\frac{1}{2}\frac{(l_i^f - f_2^{r,1})^2}{\sigma_{\epsilon_2}^2}))) \right\} \right) \\
&+ (1 - d_0) \sum_{i=1}^{n} \left(\sum_{r=1}^{4} \left\{ d_{i0}^{r,2} (\ln P_i^{r,2} + \ln(\frac{1}{\sqrt{2\pi\sigma_{\epsilon_1}^2}} \exp(-\frac{1}{2}\frac{(Q_{i2} - f_1^{r,2})^2}{\sigma_{\epsilon_1}^2}))) \right\} \right. \\
&+ \left. \sum_{r=2}^{4} \left\{ d_{i0}^{r,2} (\ln(\frac{1}{\sqrt{2\pi\sigma_{\epsilon_2}^2}} \exp(-\frac{1}{2}\frac{(l_i^f - f_2^{r,2})^2}{\sigma_{\epsilon_2}^2}))) \right\} \right)
\end{aligned}$$

Maximizing this loglikelihood function obtains the first-round estimates $\widehat{\alpha}_1$, $\widehat{\beta}_1$ and $\widehat{\omega}_1$. On the basis of these estimates, updated couple-specific locations of the kinks in the demand system can be derived. Furthermore, updated locations of couples on the demand system can be determined in a similar way as above. This also results in updated binary variables d_1 and d_{i1}^r that are defined in a similar way as d_0 and d_{i0}^r, while updated probabilities (22) can be calculated. Finally, all these variables are brought together in the updated loglikelihood function $\log L_1$, which has the same form as $\log L_0$ but this time constructed on the basis of the estimates $\widehat{\alpha}_1$, $\widehat{\beta}_1$ and $\widehat{\omega}_1$. The loglikelihood function $\log L_1$ is again maximized to obtain new estimates $\widehat{\alpha}_2$, $\widehat{\beta}_2$ and $\widehat{\omega}_2$. The iteration procedure is repeated until convergence of the parameter estimates. I.e., until the final maximum likelihood estimates equal the second to last ones which were used to construct the boundaries and probabilities. Note that the parameters α, β and ω have to be between zero and one to have a coherent system. This requirement turns out to be satisfied in the application below. It should further be remarked that the loglikelihood function is continuous in the parameters to be estimated within a given situation (depending on which spouse values the child's expenses most). Given that convergence of the iteration procedure also implies the same situation when the parameters are about to converge, potential discontinuities do not pose any problem here. As a final remark, we should stress that several local maxima may exist, which is taken into account in the estimation process.

4. Data

For the empirical analysis in this paper we have used the Consumer Expenditure Survey (CEX) collected by the Bureau of Labor Statistics of the U.S. Department of Labor. Like most authors in the consumption literature that use the CEX (e.g., Deaton and Paxson, 1994, Attanasio and Weber, 1995, and Attanasio and Davis, 1996) we have compiled a dataset from the quarterly Interview Survey (IS)

Table 1: Summary statistics for couples

	One child	Two children	Three children
Number of households	2,596	3,312	1,168
Mean direct child expenditures	\$2,212	\$2,517	\$3,106
Median direct child expenditures	\$800	\$1,270	\$1,588
Mean hourly wage husband	\$25.29	\$28.63	\$30.63
Median hourly wage husband	\$21.06	\$23.42	\$23.67
Mean hourly wage wife	\$17.39	\$18.23	\$17.91
Median hourly wage wife	\$14.92	\$15.44	\$14.58
Wife not in labor force	23.4%	29.8%	34.5%

that collects data through a recall questionnaire rather than from the biweekly Diary Survey (DS) that collects data through a daily purchase questionnaire.[6] This was done for three reasons. First, the IS contains more observations. Second, the IS was especially designed to collect data on major items of expense. The most substantial elements of child expenses, such as tuition, classify as such. Third, the expenditure component directly related to children is larger - both in terms of absolute and relative value - in the IS than in the DS.

Our CEX dataset comprises of quarterly household observations from the first quarter of 2005 until the first quarter of 2008. The IS has a rotating panel setup in which one household is interviewed a maximum of four times. This means that 89 percent of households in the CEX enters the dataset more than once and 54 percent of households are observed for a full year. We aggregate all family expenses to the quarterly level so as to not lose observations of families that we do not observe for a full year. There are 90,955 observations in our waves of the CEX.

We construct a sample that is best suited for our structural approach. It includes observations of all married couples in which the husband works at least 25 hours a week for at least 40 weeks a year and in which neither of the spouses is enrolled in a college or university nor is self-employed. Those families with one or both spouses currently attending college or university are excluded because we would like to be sure that tuition expenses can be classified as direct child expenses. Households that included other adults were dropped. Furthermore, those households in which male wages or female wages fell in the 1st or 99th percentile of the male income distribution were disregarded. Also, those families in which the wife worked more hours than the average male were not included in the sample. Finally, we divide our sample on the basis of the number of children under the age of 18 living in the household. In order for the direct children's expenses to be comparable on an absolute level we differentiate between families with one, two or three children. Because of the many constraints we impose on the data, our three samples are considerably smaller than the total CEX. They consist of 2,596, 3,312 and 1,168 observations for one, two and three child families respectively. Table 1 displays summary statistics for the three groups.

For the estimation of the structural parameters in our model we need four variables: l^f, the wife's leisure; Q_2, which contains an estimate of total yearly child expenses per household; w^m, which is the husband's total yearly net labor income; and w^f, which is the wife's full budget being her net wage rate times the maximum number of hours (normalized to one) she could have worked. Not all child expenses are separately observed in the CEX. As we do not know who consumes what it is unclear whether expenses such as sweets or cinema tickets were intended for children or for adults. For

[6] For a detailed comparison of the Interview Survey and the Diary Survey see Battistin (2004).

some categories it can however be ruled out that the goods were intended for adults. These expense categories include school meals, infant furniture, boys apparel, girls apparel, boys and girls footwear, infants apparel, toys, educational books and supplies, and elementary school, high school and college tuition and fees. Note that we have quarterly expenditure information and that this amount is then multiplied by four to obtain an estimate of yearly expenses in the household.

Price information is also obtained from the Bureau of Labor Statistics. Seasonally adjusted Urban Consumer Price Indices that were reported per calendar month have been used. This means that we do not take regional price variation into account. We compute a monthly CPI for child expenses by averaging the available separate product CPIs and taking the weights that these products have in total child expenses into account. Separate CPIs were available for all components of Q_2, except for baby furniture and school meals. As the quarterly IS interviews take place during all 12 months of the year, our dataset contains 40 monthly values of Q_2's CPI. Because households report expenditures over the previous three months, we have chosen to assign the CPI that pertains to the third month prior to the interview. Cumulative inflation on our basket of child expenses has been 11.9 percent from November 2004 until February 2008. This amounts to a yearly average inflation level of 3.3 percent on Q_2.

The CEX documents total yearly household net income as well as individuals' gross labor incomes. We have employed two alternative ways to estimate the husband's net labor income, w^m. If the household received no non-labor income we derived it as the percentage of household's net income corresponding to his gross labor income share. If the household did receive some non-labor income, we computed his net wage on the basis of his gross wage using an estimated spline relation between gross and net income in the households that did not receive any non-labor income.

Potential female wage income, w^f, is based on the same gross to net conversion as male wage income. The hourly wage rate is moreover computed for all women that are employed but imputed for all women that are currently unemployed or out of the labor force. To do the latter, we estimate a Heckman selection model in which the overidentifying variable is the number of children each women has (the wage imputation is done on the total CEX dataset). The Heckman model corrects for the wife's education level, age group, her state of residence, and for the year of the survey. The potential female wage income, w^f, is then computed by multiplying the (imputed) wage rate times the average weekly number of hours worked by men times 52 (the number of weeks). The female wage rate was imputed in respectively 23.4%, 29.8% and 34.5% of the households in our samples. In other words, these are the percentages of mothers that did not participate in the labor market.

5. Estimation results

5.1. Stylized facts about child expenses of singles

Using the CEX we also construct similar datasets of single - fulltime employed, non-college going - parents with one, two or three children in order to obtain a first impression of male and female preferences for child expenses. As can be seen in Table 2 it turns out that if we do not control for income, single mothers of one child spend on average more on their children than single fathers. Given that the hourly wage rate of the single women is on average lower than that of single men, this is remarkable. In the two and three children samples, the single fathers spend unconditionally more.

If we assume that labor supply of single mothers is exogenous[7] (as it is assumed for single men), then we can estimate the Cobb-Douglas model with a gender taste shifter. Singles are necessarily

[7] The selection of parents is such that they should all work 25 hours or more a week, for at least 40 weeks a year.

Table 2: Summary statistics for singles

	One child	Two children	Three children
Single fathers	212	124	29
Single mothers	965	591	193
Mean child expenditures men	$1,181	$1,786	$2,565
Median child expenditures men	$528	$1,264	$1,696
Mean hourly wage men	$20.30	$24.67	$19.56
Mean child expenditures women	$1,200	$1,731	$1,758
Median child expenditures women	$600	$1,080	$948
Mean hourly wage women	$15.20	$15.04	$14.49

Table 3: Estimation results for singles

	One child		Two children		Three children	
	Est.	St. err.	Est.	St. err.	Est.	St. err.
β^f	0.964	0.003	0.949	0.009	0.939	0.013
β^m	0.974	0.007	0.971	0.004	0.949	0.015

dictators in this set-up - public good demand is according to their preferences alone. Hence, we estimate a β^f for single women and a β^m for single men using their respective labor incomes. Table 3 presents the estimates. It can be seen that in all groups, the female taste for spending on children is higher than the male taste. The difference between the two estimates of β is however only significant at a five percent level in the two children group. The small number of single fathers with three children could account for the lack of significance in their sample.

5.2. Estimation results couples with children

Let us now focus on the estimation results for couples with respectively one, two and three child(ren). Preference parameter estimates were obtained by means of the estimation strategy outlined above. The iterated maximum likelihood method worked well: estimates were obtained after only a few iterations. Moreover, the unconstrained preference parameters were between zero and one as theory predicts they should be.[8]

Table 4 summarizes the estimation results for the three sets of couples. It is clear from the results that on average wives value the child related public good more than their husbands. This is in line with the results obtained for singles. The table further shows that, ceteris paribus, the marginal utility of the child related public good increases with the number of children. This is the case for both husbands and wives. This makes sense since the data show that the expenditures on children increase with their number.[9] Also the wife's marginal utility of leisure increases with the number of children. This is also reflected by the data given that women on average work less when they have more children.

As is clear from Section 3, our non-cooperative model nests the standard unitary model. If α equals

[8] A few local maxima were found. We retained the highest maximum. Note that the results discussed below are qualitatively robust for the different maxima.

[9] This may be remarkable given the public good connotation of the child related expenditures. Still, for a given family constellation, this does not rule out that children's expenses are public in the sense that they appear in both spouses' utility functions.

Table 4: Estimation results for couples

	One child		Two children		Three children	
	Est.	St. err.	Est.	St. err.	Est.	St. err.
α	0.922	0.024	0.887	0.009	0.823	0.020
β	0.966	0.040	0.932	0.006	0.894	0.012
ω	0.140	0.006	0.191	0.005	0.222	0.010

Table 5: Proportion of couples in the different regimes

	One child	Two children	Three children
Husband dictator	6.82	19.62	32.11
Split might	5.20	7.28	11.30
Wife dictator	87.98	73.10	56.59

β, then the household behaves as if it consists of a single decision maker with preferences that are equal to those of the wife (given that preferences also depend on the wife's leisure). We tested the null hypothesis that α is equal to β for the three sets of couples by means of a Wald test. Interestingly, the null hypothesis is strongly rejected for the couples with two and three children (p-values are close to 0). Their observed behavior can clearly not be captured by a single decision maker's rational preferences. Note though that the null hypothesis could not be rejected for couples with one child (p-value equals 0.47). The expenditures on child related goods are relatively low for these households. It could well be the case that the data therefore do not allow for a sharp distinction between the spouses' preference parameters.

On the basis of the above preference parameters, the expected number of couples in each of the three regimes can be calculated. Since $\widehat{\beta} > \widehat{\alpha}$, we derive these numbers on the basis of the right-hand sides of equation (20). The obtained proportions are presented in Table 5. Interestingly, all possible regimes contain a significant proportion of the couples in the sample. Moreover, it turns out that most of the households behave as if the wife was the dictator in the household. More specifically, for these families, the allocation of the household's aggregate resources is according to the wife's preferences. Note though that the proportion of households with the wife acting as the dictator decreases with the number of children. Equation (20) shows that the wife is the dictator in the household if the condition $w^f > w^m \frac{1-\alpha(1-\omega)}{\alpha(1-\omega)}$ is satisfied. This condition becomes harder to meet if, ceteris paribus, α decreases, ω increases or the ratio w^f/w^m decreases. A closer look at the data shows that that the latter ratio is indeed lower for higher numbers of children. At the same time the increase in ω counteracts the decrease in α, which implies a more restrictive condition to obtain female dictators in couples with higher numbers of children.

6. Conclusion

In this paper, we model the consumption and labor supply behavior of a couple in a non-cooperative setting by adopting a Nash approach. Using minimal assumptions, we prove that demand for public goods is defined by only three regimes. Demand for public goods is either determined by the preferences of one of the partners only (Husband Dictatorship or Wife Dictatorship), or by both spouses having a say on the allocation of income to public goods (Split Might). The particular regime in which a couple locates is shown to depend on the spouses' relative wage rates, which resembles the endogenous regimes (depending on exogenous individual incomes) in Browning, Chiappori and Lechene (2009).

By imposing more structure on the general model, we can derive testable implications on observed demand for public goods and labor supply that allow testing the model against the standard unitary model where a couple behaves as a single decision maker. The model is applied to a sample of couples drawn from the Consumer Expenditure Survey (CEX) whereby we focus on expenses on children's goods that act as a public good in the spouses' preferences. We find that for couples with two or three children the standard unitary model is strongly rejected in favor of our non-cooperative model. Women apparently like to spend more on goods for their children than men, which would explain findings in the literature that indicate that children benefit more when household resources are owned by women (e.g., Lundberg, Pollak and Wales, 1997, and Duflo, 2003). Using the estimated preference parameters, we can divide households into dictatorship and split might regimes. It turns out that 73% of two-child and 57% of three-child couples spend according to the wife's preferences.

Appendix A. Proofs of results

Proof of Lemma 1
We define

$$\begin{aligned} dl &= \varepsilon \\ dq_k &= -\frac{w}{p_k}\varepsilon \\ u(\varepsilon) &= u(l + dl, \mathbf{q} + dq_k \iota_k, \mathbf{Q}) \end{aligned}$$

where ι_k denotes a vector which equals 1 (one) at position k and is zero everywhere else. Note that dl and dq_k are defined in such a way that $\varepsilon \neq 0$ is feasible in terms of the budget restriction.

It follows that

$$u'(\varepsilon) = u_l - u_{q_k}\frac{w}{p_k}. \tag{A.1}$$

First, consider the case where $w > 0$. Then we prove by contradiction that the equality in the lemma holds. Note that $\frac{u_l}{u_{q_k}} > \frac{w}{p_k}$ implies that utility increases with ε contradicting equation (3). We need to be careful though as $\varepsilon > 0$ is not possible with $l = 1$. However at $l = 1$ we have $q_k = 0$ (as there is no income to spend on private goods) and thus $u_{q_k} = +\infty$ (by assumption 1). Since u_l is finite at $l = 1$ (by assumption 3), we cannot have $\frac{u_l}{u_{q_k}} > \frac{w}{p_k}$. Further, note that $\frac{u_l}{u_{q_k}} < \frac{w}{p_k}$ implies $\varepsilon < 0$ would raise utility again contradicting equation (3). It is not possible to have $\varepsilon < 0$ at $l = 0$. But due to assumption 1 we cannot have $\frac{u_l}{u_{q_k}} < \frac{w}{p_k}$ at $l = 0$.

Second, consider $w = 0$. Then we have $l = 1, q_k = 0$. Due to assumption 1 and 3 we then have

$$\frac{u_l}{u_{q_k}} = 0 = \frac{w}{p_k}$$

and the equality in the lemma holds. *Q.E.D.*

Proof of proposition 1
The proposition implies that we need to rule out two cases. First, we need to rule out that $u^j_{Q_k} > \lambda^j \mathbf{P}_k$ for any partner $j \in \{f, m\}$ and any public good Q_k. Second, we need to rule out that $u_{\mathbf{Q}} = \lambda \mathbf{P}$ for both partners.

Suppose (by contradiction) that $u^j_{Q_k} > \lambda^j \mathbf{P}_k$. Since we assumed $w^m > 0$, the husband will contribute to every public good (if the wife does not contribute to any public good). If $w^f > 0$ the wife can contribute to public goods as well. In either case we have $Q_k > 0$ for each public good Q_k and hence $u^j_{Q_k}$ is finite for both partners. Given the Nash assumption that $Q_k^{(-j)*}$ is given, partner j can raise utility by increasing Q^j_k which contradicts equation (3).

Second, assume (by contradiction) that $u_{\mathbf{Q}} = \lambda \mathbf{P}$ for both partners. This would imply

$$\frac{u^f_{Q_k}}{u^f_{Q_{k'}}} = \frac{P_k}{P_{k'}} = \frac{u^m_{Q_k}}{u^m_{Q_{k'}}}$$

for each pair of public goods $Q_k, Q_{k'}$. However, this contradicts assumption 5. *Q.E.D.*

Proof of proposition 2
Consider $w^m > 0$ and $w^f = 0$. Hence $\mathbf{Q}^f = 0$ as the wife earns no income. Given assumption 1 it

is optimal for the husband to contribute to each public good Q_k. Hence $u^f_{Q_k}$ is finite for each public good Q_k. Since $w^f = 0$ implies that $\mathbf{q}^f = 0$, we have $\lambda^f = +\infty$. Hence, indeed we are in regime HD. By continuity this also holds for $w^f > 0$ close enough to zero.

The proof of the existence of ρ_1 is done in the same way but then starting from $w^f > 0, w^m = 0$. This gives us regime WD. By continuity we are also in regime WD for $w^m > 0$ close enough to zero. *Q.E.D.*

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