Inflation Targets and Debt Accumulation in a Monetary Union*

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Abstract

This paper explores the interaction between centralized monetary policy and decentralized fiscal policy in a monetary union. Discretionary monetary policy suffers from a failure to commit. Moreover, decentralized fiscal policymakers impose externalities on each other through the influence of their debt policies on the common monetary policy. These imperfections can be alleviated by adopting state-contingent inflation targets (to combat the monetary policy commitment problem) and shock-contingent debt targets (to internalize the externalities due to decentralized fiscal policy).

Keywords: discretionary monetary policy, decentralized fiscal policy, monetary union, inflation targets, debt targets.

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1. Introduction

Although the Maatricht Treaty has laid the institutional foundations for European Monetary Union (EMU), how these institutions can best be operated in practice remains to be seen in the coming years. For example, the European Central Bank (ECB) has announced a two-tier monetary policy strategy based on a reference value for money growth and an indicator that is based on a number of other measures, such as output gaps, in‡ation expectations, etcetera (see European Central Bank, 1999). Over time the ECB may well shift to implicit targeting of in‡ation. Indeed, a number of economists has argued (e.g, see Svensson, 1998) that also the Bundesbank has pursued such a strategy. Furthermore, how the Excessive De…cit Procedure and the Stability and Growth Pact (see Beetsma and Uhlig, 1999) will work in practice is not yet clear.

This paper deals with the interaction between in‡ation targets and constraints on decentralized …scal policy in a monetary union. To do so, we extend our earlier work on the interaction between a common monetary policy and decentralized …scal policies in a monetary union. In particular, in Beetsma and Bovenberg (1999) we showed that monetary uni…cation raises debt accumulation, because in a monetary union countries only partly internalize the e¤ects of their debt policies on future monetary policy. This additional debt accumulation is actually welfare enhancing (if the governments share societal preferences). We showed that, in the absence of shocks, making the central bank su¢ciently conservative (in the sense of Rogo¤, 1985, that is by imposing on the central bank a loss function that attaches a su¢ciently high weight to price stability) can lead the economy to the second-best equilibrium. However, this is no longer the case in the presence of common shocks, as the economies are confronted with a trade o¤ between credibility and ‡exibility.

While Beetsma and Bovenberg (1999) emphasized the e¤ects of lack of commitment in monetary policy, this paper introduces another complication in the form of strategic interactions between decentralized …scal policymakers who have di¤erent views on the stance of the common monetary policy.[1] These di¤erent views originate in di¤erences among the economies in the monetary union. In particular, we allow for systematic di¤erences in labour and product market distortions, public spending requirements and initial public debt levels. We also allow for idiosyncratic stochastic shocks hitting the countries. In combination with the decentralization of …scal policy these di¤erences lead to con‡icts about the preferred future stance of the common monetary policy. In particular, countries that su¤er from severe distortions in labor and commodity markets, feature

[1] Our earlier model incorporated another potential distortion: the possibility that governments discount the future at a higher rate than their societies do. We ignore this distortion throughout the current paper.

higher public spending or initial debt levels or are hit by worse shocks prefer a laxer future stance of monetary policy. These con‡icts about monetary policy induce individual governments to employ their debt policy strategically, so as to induce the union's central bank to move monetary policy into the direction they prefer. This strategic behavior imposes negative externalities on other countries, thereby producing welfare losses.

In contrast to Beetsma and Bovenberg (1999), we do not address the distortions in the model by making the common central bank su¢ciently conservative. Instead, we focus on state-contingent in‡ation targets which, in contrast to a conservative central bank, can lead the economy to the second-best equilibrium if countries are identical. Hence, as stressed by Svensson (1997) in a model without …scal policy and debt accumulation, in‡ation targets eliminate the standard credibility-‡exibility trade-o¤. If …scal policy is decentralized to heterogeneous countries, however, the optimal state-contingent in‡ation targets need to be complemented by (country-speci…c) debt targets to establish the second best. In this way, in‡ation targets address the lack of commitment in monetary policy, while debt targets eliminate strategic interaction among heterogeneous governments with di¤erent views about the common monetary policy stance.

The remainder of this paper is structured as follows. Section 2 presents the model. Section 3 discusses the second-best equilibrium in which not only monetary but also …scal policy is centralized and in which monetary policy is conducted under commitment. This is the second-best optimum that can be attained under monetary uni…cation, assuming that the supranational authorities attach an equal weight to the preferences of each of the participating countries. Section 4 derives the equilibrium for the case of a common, discretionary monetary policy with decentralized …scal policies. Section 5 explores institutional arrangements (i.e. in‡ation targets and public debt targets) that may alleviate the welfare losses arising from the lack of monetary policy commitment and the wasteful strategic interaction among the decentralized governments. Finally, Section 6 concludes the main body of this paper. The derivations are contained in the appendix.

2. The model

A monetary union, which is small relative to the rest of the world, is formed by n countries.[2] A common central bank (CCB) sets monetary policy for the entire union, while …scal policy is determined at a decentralized, national level by the n governments. There are two periods.

[2] Monetary uni…cation is taken as given. Hence, we do not explore the incentives of countries to join a monetary union.

Workers are represented by trade unions who aim for some target real wage rate (e.g. see Alesina and Tabellini, 1987, and Jensen, 1994). They set nominal wages so as to minimize the expected squared deviation of the realized real wage rate from this target. Monetary policy (i.e., the inflation rate) is selected after nominal wages have been fixed. In each country, firms face a standard production function with decreasing returns to scale in labour. Output in period t is taxed at a rate τ_{it}. Therefore, output in country i in periods 1 and 2, respectively, is given by[3]

$$x_{i1} = \nu(\pi_1 - \pi_1^e - \tau_{i1}) - \mu - \varepsilon_i; \tag{2.1}$$
$$x_{i2} = \nu(\pi_2 - \pi_2^e - \tau_{i2}); \tag{2.2}$$

where μ represents a common union-wide shock, while ε_i stands for an idiosyncratic shock that solely hits country i. π_t^e denotes the inflation rate for period t expected at the start of period t (that is, before period t shocks have materialized, but after period $t-1, t-2, ...$ shocks have hit). We assume that $E[\varepsilon_i] = 0, \forall i$; $E[\mu] = 0$; $E[\varepsilon_i \varepsilon_j] = 0, \forall j \neq i$; and that $\bar{\varepsilon} \equiv \frac{1}{n}\sum_{i=1}^{n} \varepsilon_i = 0$.[4] The variances of μ and ε_i are given by σ_μ^2 and σ_ε^2, respectively. We abstract from shocks in the second period, because they would not affect debt accumulation.

Each country features a social welfare function which is shared by the government of that country. Hence, governments are benevolent. In particular, the loss function of government i is defined over inflation, output and public spending:

$$V_{S,i} = \frac{1}{2}\sum_{t=1}^{2} \beta^{t-1}\left[\alpha_\pi \pi_t^2 + (x_{it} - \tilde{x}_{it})^2 + \alpha_g (g_{it} - \tilde{g}_{it})^2\right], 0 < \beta \leq 1;\ \alpha_\pi, \alpha_g > 0. \tag{2.3}$$

Welfare losses increase in the deviations of inflation, (log) output and government spending (g_{it} is government spending as a share of output in the absence of distortions) from their targets (or first-best levels or "bliss points"). For convenience, the target level for inflation corresponds to price stability. The target level for output is denoted by $\tilde{x}_{it} > 0$. Two distortions reduce output below this optimal level. First, the output tax τ_{it} drives a wedge between the social and private benefits of additional output. Second, market power enables unions to drive the real wage above its level in the absence of distortions. Hence, even in the absence of taxes, output is below the first-best output level $\tilde{x}_{it} > 0$. The first-best

[3] Details on the derivations of these output equations can be found in Beetsma and Bovenberg (1999).

[4] Without this assumption, the mean $\bar{\varepsilon}$ of the ε's would play the same role as μ does. In the outcomes given below, μ would then be replaced by $\hat{\mu} \equiv \mu + \bar{\varepsilon}$. For convenience, we assume that $\bar{\varepsilon} = 0$.

level of government spending, $\tilde{g}_{it}$, can be interpreted as the optimal share of non-distortionary output to be spent on public goods if (non-distortionary) lump-sum taxes would be available (see Debelle and Fischer, 1994). The target levels for output and government spending can differ across countries. Parameters α_{π} and α_g correspond to the weights of the price stability and government spending objectives, respectively, relative to the weight of the output objective. Finally, β denotes society's subjective discount factor.

Government i's budget constraint can be approximated by (e.g., see Appendix A in Beetsma and Bovenberg, 1999):

$$g_{it} + (1+\rho)\, d_{i,t-1} = \tau_{it} + \chi\pi_t + d_{it}; \tag{2.4}$$

where $d_{i,t-1}$ represents the amount of public debt carried over from the previous period into period t, while d_{it} stands for the amount of debt outstanding at the end of period t. All public debt is real, matures after one period, and is sold on the world capital market against a real rate of interest of ρ. This interest rate is exogenous because the countries making up the monetary union are small relative to the rest of the world.[5] τ_{it} and χ (a constant) stand for, respectively, distortionary tax revenue and real holdings of base money as shares of non-distortionary output. All countries share equally in the seigniorage revenues of the CCB, so that the seigniorage revenues accruing to country i amount to $\chi\pi_t$.

We combine (2.4) with the expression for output, (2.1) or (2.2), to eliminate τ_{it}. The resulting equation can be rewritten to yield the government financing requirement of period t:

$$\begin{aligned} GFR_{it} &= \tilde{K}_{it} + (1+\rho)\, d_{i,t-1} - d_{it} + \delta_t(\mu + \epsilon_i)/\nu \\ &= [(\tilde{x}_{it} - x_{it})/\nu] + \chi\pi_t + (\tilde{g}_{it} - g_{it}) + (\pi_t - \pi_t^e); \end{aligned} \tag{2.5}$$

where δ_t is an indicator function, such that $\delta_1 = 1$ and $\delta_2 = 0$, and where

$$\tilde{K}_{it} \equiv \tilde{g}_{it} + \tilde{x}_{it}/\nu.$$

The government financing requirement, GFR_{it}, consists of three components. The first component, $\tilde{K}_{it}$, amounts to the government spending target, $\tilde{g}_{it}$, and an output subsidy aimed at offsetting the implicit output tax due to labor- or product-market distortions, $\tilde{x}_{it}/\nu$. The second component involves net debt-servicing costs,

[5] In the following, we will occasionally explore what happens when the number of union participants becomes infinitely large (i.e. $n \to \infty$) in order to strengthen the intuition behind our results. In these exercises the real interest rate remains beyond the control of union-level policymakers.

$(1+\rho)\, d_{i,t-1} - d_{it}$. The final component (in period 1 only) is the stochastic shock (scaled by ν), $(\mu + \varepsilon_i)/\nu$. The last right-hand side of (2.5) represents the sources of finance: the shortfall (scaled by ν) of output from its target (henceforth referred to as the output gap), $(\bar{x}_{it} - x_{it})/\nu$, seigniorage revenues, $\chi\pi_t$, the shortfall of government spending from its target (henceforth referred to as the spending gap), $\bar{g}_{it} - g_{it}$, and the inflation surprise, $\pi_t - \pi_t^e$.

All public debt is paid off at the end of the second period ($d_{i2} = 0; i = 1, ..., n$). Under this assumption, while taking the discounted (to period one) sums of the left- and right-hand sides of (2.5) ($t = 1, 2$), we obtain the intertemporal government financing requirement:

$$\begin{aligned} IGFR_i &= \bar{F}_i + (\mu + \varepsilon_i)/\nu \\ &= \sum_{t=1}^{2} (1+\rho)^{-(t-1)} \left[(\bar{x}_{it} - x_{it})/\nu + \chi\pi_t + (\bar{g}_{it} - g_{it}) + (\pi_t - \pi_t^e) \right], \end{aligned} \tag{2.6}$$

where $\bar{F}_i \equiv \bar{K}_{i1} + (1+\rho)\, d_{i0} + \bar{K}_{i2}/(1+\rho)$ stands for the deterministic component of the intertemporal government financing requirement.

Monetary policy is delegated to a common central banker (CCB), who has direct control over the union's inflation rate. One could assume that the CCB has certain intrinsic preferences regarding the policy outcomes. Alternatively, and this is the interpretation we prefer, one could assume that the CCB is assigned a loss function by means of an appropriate contractual agreement. More specifically, this agreement shapes the CCB's incentives in such a way (by appropriately specifying its salary and other benefits – for example, possible reappointment – conditional on its performance) that it chooses to maximize the following loss function:

$$V_{CCB} = \frac{1}{2} \sum_{t=1}^{2} \beta^{t-1} \left\{ \alpha_\pi (\pi_t - \pi_t^*)^2 + \frac{1}{n} \sum_{i=1}^{n} \left[(x_{it} - \bar{x}_{it})^2 + \alpha_g (g_{it} - \bar{g}_{it})^2 \right] \right\}, \tag{2.7}$$

where π_t^* is the inflation target in period t, which may be different from the socially-optimal inflation rate, which was set at zero.

If $\pi_1^* = \pi_2^* = 0$, the CCB's objective function corresponds to an equally-weighted average of the individual societies' objective functions. We assume that π_2^* is a linear function of $d_{i1}, i = 1, ..., n$. This linearity assumption suffices for our purposes: we will see later on that the optimal second-period inflation target is indeed a linear function of $d_{i1}, i = 1, ..., n$. The optimal first-period inflation target will be a function of d_{i0}, which is exogenous.

3. The second-best equilibrium

As a benchmark for the remainder of the analysis, we discuss the equilibrium resulting from centralized fiscal and monetary policies under commitment. Monetary policy is set by the CCB. Fiscal policy is conducted by a centralized fiscal authority, which minimizes:

$$V_U \equiv \frac{1}{n}\sum_{i=1} V_{S;i}; \tag{3.1}$$

where the $V_{S;i}$ are given by (2.3), $i = 1; \ldots; n$. Equation (3.1) assumes that countries have equal bargaining power as regards to the fiscal policy decisions taken at the union level. Government spending is residually determined, so that the CCB, when it selects monetary policy, internalizes the government budget constraints. The resulting equilibrium is Pareto optimal. In the sequel, we refer to this equilibrium as the second-best equilibrium. In the absence of first-best policies (such as the use of lump-sum taxation and the elimination of product- and labor-market distortions), it is the equilibrium with the smallest possible welfare loss (3.1), given monetary unification. The derivation of the second-best equilibrium is contained in Appendix A.

3.1. Inflation, the output gap and the public spending gap

Table 1 contains the outcomes for inflation, the output gap,[6] $\tilde{x}_{it} - x_{it}$, and the spending gap, $\tilde{g}_{it} - g_{it}$. We write each of these outcomes as the sum of two deterministic and two stochastic components. $\bar{F}_i^{\prime}$ is the deviation of country i's deterministic component of its intertemporal government financing requirement from the cross-country average, defined by $\bar{F}$. Formally, $\bar{F} \equiv \frac{1}{n}\sum_{j=1}^{n} \bar{F}_j$ and $\bar{F}_i^{\prime} \equiv \bar{F}_i - \bar{F}$. The factor between square brackets in each of the entries of Table 1 makes clear how, within a given period, the government financing requirement is distributed over the financing sources (seigniorage, the output gap, the spending gap and an inflation surprise). Indeed, for each period these factors add up to unity, both across the deterministic and across the stochastic components. For example, for the first period one has:

[6] Throughout, we present the outcome for the output gap instead of the outcome for the tax rate. The reason is that, in contrast to the latter, the former directly enters the welfare loss functions.

$$
\begin{aligned}
&\left[(\tilde{x}_{i1}-x_{i1})/\nu\right] + \chi\pi_1 + \left[(\tilde{g}_{i1}-g_{i1}) + (\pi_1-\pi_1^e)\right] \\
&= \frac{\beta^*(1+\rho)}{1+\beta^*(1+\rho)}\left[\bar{F}+F_i^{\cdot}\right] + \frac{\beta^*(1+\rho)(P^*/P)}{1+\beta^*(1+\rho)(P^*/P)}\left[\frac{\mu}{\nu}+\frac{\epsilon_i}{\nu}\right] \\
&= \tilde{K}_{i1} + (1+\rho)\,d_{i0} - d_{i1}^S + (\mu+\epsilon_i)/\nu; \qquad (3.2)
\end{aligned}
$$

where d_{i1}^S is the second-best debt level. The last equality can be checked by substituting (3.4)-(3.7) into (3.3) (all given below) and substituting the resulting expression into the last line of (3.2). For each of the outcomes, the terms that follow the factor in square brackets regulate the intertemporal allocation of the intertemporal government financing requirement.

The coefficients of the common stochastic shock $\frac{\mu}{\nu}$ (in the fourth column of Table 1, γ_2) differ in two ways from the coefficients of the common deterministic component of the intertemporal government financing requirement $\bar{F}$ (in the second column of Table 1, γ_0). The first difference is with respect to the first-period, intratemporal, allocation of the government financing requirement over the financing sources. The deterministic components of the government financing requirement are anticipated and thus correctly incorporated in expected inflation. The common shock, in contrast, is unanticipated and, hence, not taken into account when inflation expectations are formed. The predetermination of the inflation expectation is exploited by the central policymakers so as to finance part of this common shock through an inflation surprise. Indeed, whereas the coefficient of $\pi_1 - \pi_1^e$ is zero in the second column in Table 1, this coefficient is positive in the fourth column, indicating that part of the common shock is financed through an inflation surprise in the first period. With surprise inflation absorbing part of the common shock, the output gap and the spending gap have to absorb a smaller share of this shock.

In the second period, the allocation over the financing sources for the stochastic component $\frac{\mu}{\nu}$ is the same as for the deterministic component $\bar{F}$. The reason is that the first-period shock $\frac{\mu}{\nu}$ has materialized before second-period inflation expectations are formed. The effect of $\frac{\mu}{\nu}$ on the second-period outcomes will thus be perfectly anticipated. Indeed, the share of $\frac{\mu}{\nu}$ that is transmitted into the second period through debt policy becomes part of the deterministic component of the second-period government financing requirement (when viewed from the start of the second period).

The second way in which the coefficient of the stochastic shock $\frac{\mu}{\nu}$ differs from the coefficient of $\bar{F}$, involves the intertemporal allocation of the government financing requirement. In particular, the share of $\frac{\mu}{\nu}$ absorbed in the first period (relative to the second period) is larger than that of $\bar{F}$ ($\beta^*(P^*/P)\,c_1 > \beta^* c_0$ and $c_1 < c_0$, where c_0 and c_1 are defined in Table 1). The reason is again that first-period inflation expectations are predetermined when the stochastic shock hits. This enables the policymakers to absorb a relatively large share of the stochastic shock in the first period through an inflation surprise.

The responses of the output and government spending gaps to $\tilde{F}_i'$ and $\frac{\epsilon_i}{\omega}$ differ from the responses to $\tilde{F}$ and $\frac{\bar{\epsilon}}{\omega}$. Since inflation is attuned to cross-country averages, it cannot respond to country-specific circumstances as captured by $\tilde{F}_i'$ and $\frac{\epsilon_i}{\omega}$. Accordingly, taxes (the output gap) and the government spending gap have to fully absorb these country-specific components of the government financing requirements.

3.2. Public debt policy

The solution for debt accumulation in the second-best equilibrium can be written as:

$$d_{i1}^{S} = \bar{d}_1^{e;S} + d_{i1}^{\prime;e;S} + \bar{d}_1^{d;S} + d_{i1}^{\delta;S}; \tag{3.3}$$

where

$$\bar{d}_1^{e;S} = \frac{\left[\tilde{\kappa}_1 + (1+\rho)\bar{d}_0 - \tilde{\kappa}_2\right] + (1-\beta^*)\tilde{\kappa}_2}{1+\beta^*(1+\rho)}; \tag{3.4}$$

$$d_{i1}^{\prime;e;S} = \frac{\left[\tilde{\kappa}_{i1}' + (1+\rho)d_{i0}' - \tilde{\kappa}_{i2}'\right] + (1-\beta^*)\tilde{\kappa}_{i2}'}{1+\beta^*(1+\rho)}; \quad n > 1; \tag{3.5}$$

$$= 0; \quad n = 1;$$

$$\bar{d}_1^{d;S} = \left[\frac{1}{1+\beta^*(1+\rho)(P^*/P)}\right]\frac{\bar{\epsilon}}{\omega}; \tag{3.6}$$

$$d_{i1}^{\delta;S} = \left[\frac{1}{1+\beta^*(1+\rho)}\right]\frac{\epsilon_i}{\omega}; \quad n > 1; \tag{3.7}$$

$$= 0; \quad n = 1;$$

where the superscript "S" stands for "second-best equilibrium", the superscript "e" denotes the expectation of a variable, an upperbar above a variable indicates its cross-country average (except for variables carrying a tilde, like $\tilde{\kappa}_1$, where the cross-country average is indicated by dropping the country-index), a superscript " $'$ " denotes an idiosyncratic deviation of a deterministic variable from its cross-country average (for example, $\tilde{\kappa}_{i1}' \equiv \tilde{\kappa}_{i1} - \tilde{\kappa}_1$), a superscript "d" denotes the response to a common shock, a superscript "δ" indicates the response to an idiosyncratic shock, and where

$$\begin{aligned} \beta^* &\equiv \beta(1+\rho)\,; \qquad (3.8) \\ P &\equiv \chi^2/\alpha_\pi + 1/\nu^2 + 1/\alpha_g; \\ P^* &\equiv (\chi+1)^2/\alpha_\pi + 1/\nu^2 + 1/\alpha_g. \end{aligned}$$

Hence, optimal debt accumulation (3.3) is the sum of two deterministic components and two stochastic components. The component $\bar{d}_1^{e;S}$ optimally distributes over time the absorption of the cross-country averages of the deterministic components of the government financing requirements. Therefore, it is common across countries. The country-specific components $d_{i1}^{\Delta;e;S}$ intertemporally distribute the idiosyncratic deterministic components of the government financing requirements. The common (across countries) component $\bar{d}_1^{d;S}$ represents the optimal debt response to the common shock μ, while $d_{i1}^{\delta;S}$ stands for the optimal debt response to the country-specific shock, ε_i.

The debt response to the common shock is less active than the response to the idiosyncratic shock (since $P^*/P > 1$). The common inflation rate can exploit the predetermination of inflation expectations only in responding to the common shock, because the common inflation rate can not be attuned to idiosyncratic shocks. Hence, the share of the common shock that can be absorbed in the first period can be larger than the corresponding share of the idiosyncratic shock. Public debt thus needs to respond less vigorously to the common shock.

4. Discretionary monetary policy with decentralized fiscal policy

This section introduces two distortions compared with the second-best equilibrium explored in the previous section. First, the CCB is no longer able to commit to monetary policy announcements. Second, fiscal policy is decentralized to individual governments, which may result in wasteful strategic interaction among heterogeneous governments.

From now on, the timing of events in each period is as follows. At the start of the period, the institutional parameters are set. That is, an inflation target is imposed on the CCB for the coming period and, if applicable, the debt targets on the individual governments are set. The inflation target may be conditioned on the state of the world. In particular, the inflation target may depend on the average debt level in the union.[7] Furthermore, the debt target, which represents

[7] The optimal inflation target can either be optimally reset at the start of each period, or

the amount of public debt that a government has to carry over into the next period, may be shock-contingent.[8] After the institutional parameters have been set, inflation expectations are determined (through the nominal wage-setting process). Third, the shock(s) materialize. Fourth, taking inflation expectations as given, the CCB selects the common inflation rate and the fiscal authorities simultaneously select taxes and, in the absence of a debt target, public debt. Each of the players takes the other players' policies at this stage as given. Finally, public spending levels are residually determined. As a result, the CCB internalizes the effect of its policies on the government budget constraints.

This section explores the outcomes under pure discretion, i.e. in the absence of both inflation targets (i.e., $\pi_1^* = \pi_2^* = 0$) and debt targets. The complete derivation of the equilibrium is contained in Appendix B. The suboptimality of the resulting equilibrium compared to the second best motivates the exploration of inflation and debt targets in Section 5.

4.1. Inflation, the output gap and the public spending gap

Table 2 contains the solutions for the inflation rate, the output gap and the spending gap. The main difference compared to the outcomes under the second-best equilibrium (see Table 1) is that, for a given amount of debt d_{i1} to be carried over into the second period, expected first-period inflation (and, hence, seigniorage if $\chi > 0$) will be higher (compare the term between the square parentheses in the second column and the second row of Table 2 with the corresponding term in Table 1 and observe that $[\chi(\chi+1)/\alpha_\pi]/S > (\chi^2/\alpha_\pi)/P$, where $S \equiv \chi(\chi+1)/\alpha_\pi + 1/\eta^2 + 1/\alpha_g$). The source of the higher expected inflation rate under pure discretion is the inability to commit to a stringent monetary policy, which yields the familiar inflation bias (Barro and Gordon, 1983). The outcomes for inflation, the output gap and the spending gap deviate from the outcomes under the second-best equilibrium also because debt accumulation under pure discretion differs from debt accumulation under the second best. These differences are discussed below.

4.2. Public debt policy

Government i's debt can, analogous to (3.3), be written as:

$$d_{i1}^{D} = \bar{d}_1^{e,D} + d_{i1}^{\Delta,e,D} + \bar{d}_1^{d,D} + d_{i1}^{\delta,D}, \tag{4.1}$$

be determined according to a state-contingent rule selected at the beginning of the first period. These two alternative interpretations yield equivalent results.

[8] Debt at the end of the second period is restricted to be zero. Hence, the second period features a debt target of zero.

where the superscript "D" is used to indicate the solution of the purely discretionary equilibrium with decentralized fiscal policies and where

$$\bar{d}_1^{e,D} = \frac{\left[\bar{\kappa}_1 + (1+\rho)\,d_0 - \bar{\kappa}_2\right] + \left[1 - \beta^*\,(S^*/S)\right]\bar{\kappa}_2}{1 + \beta^*\,(1+\rho)\,(S^*/S)}, \tag{4.2}$$

$$d_{i1}^{\prime,e,D} = \frac{\left[\bar{\kappa}_{i1}' + (1+\rho)\,d_{i0}' - \bar{\kappa}_{i2}'\right] + \left[1 - \beta^*\,(Q/S)\right]\bar{\kappa}_{i2}'}{1 + \beta^*\,(1+\rho)\,(Q/S)}, \text{ if } n > 1; \tag{4.3}$$

$$= 0, \text{ if } n = 1,$$

$$\bar{d}_1^{d,D} = \left[\frac{1}{1 + \beta^*\,(1+\rho)\,(S^*/S)\,(P^*/S)}\right]\frac{1}{\nu}, \tag{4.4}$$

$$d_{i1}^{\delta,D} = \left[\frac{1}{1 + \beta^*\,(1+\rho)\,(Q/S)}\right]\frac{\epsilon_i}{\nu}, \text{ if } n > 1; \tag{4.5}$$

$$= 0, \text{ if } n = 1,$$

and where

$$S \equiv \chi\,(\chi+1)/\alpha_\pi + 1/\nu^2 + 1/\alpha_g, \tag{4.6}$$

$$S^* \equiv \chi\,(\chi+1)/\alpha_\pi + (\chi+1)/(n\alpha_\pi) + 1/\nu^2 + 1/\alpha_g,$$

$$Q \equiv [(n-1)/n]\,[\chi\,(\chi+1)/\alpha_\pi] + 1/\nu^2 + 1/\alpha_g.$$

4.2.1. Response to the common deterministic components of the government financing requirements

Positive analysis:

This subsection explores the solution for expected average debt $\bar{d}_1^{e,D}$ in (4.2). Whereas current inflation expectations are predetermined at the moment that debt is selected, future inflation expectations still need to be determined. A reduction in debt reduces the future government financing requirement and, thus, the tax rate in the future. This, in turn, weakens the CCB's incentive to raise future inflation in order to protect employment. Hence, by restraining debt accumulation, governments help to reduce future inflation expectations, which are endogenous from a first-period perspective. The reduction in future inflation expectations implies a lower inflation bias in the future. In other words, asset accumulation is an indirect way to enhance the commitment of a central bank to low future inflation.

Expected average debt $\bar{d}_1^{e,D}$ increases in the size of the union (because $\partial \bar{d}_1^{e,D} / \partial [\beta^{*}(S^{*}/S)] < 0$ and because S^{*}/S is decreasing in n – see Beetsma and Bovenberg, 1999): in a larger union, each individual government perceives a smaller effect of a unilateral reduction in public debt on the common future inflation rate. Hence, the incentive to restrain debt becomes weaker.[9] Indeed, in a monetary union, the credibility of the common monetary policy has the features of a public good.

Normative analysis:

Expected average debt accumulation is suboptimally low (because $S^{*}/S > 1$, $\beta^{*}(S^{*}/S) > \beta^{*}$ and, hence, $\bar{d}_1^{e,D} < \bar{d}_1^{e,S}$ – see also Beetsma and Bovenberg, 1999). The source of this underaccumulation of debt is the lack of commitment in monetary policy, which gives rise to an inflation bias. In order to strengthen the credibility of future monetary policy and thus reduce the inflation bias, governments try to exploit the predetermined nature of first-period inflation expectations by absorbing a relatively large part of the intertemporal government financing requirement in the first period. In equilibrium, these attempts to exploit the fixed nature of inflation expectations are ineffective, however, because the private sector anticipates the incentive facing the governments to exploit first-period inflation expectations and thus sets first-period wages (inflation expectations) higher than under commitment. Hence, the first-period equilibrium inflation rate, output gap and spending gap will be higher than under commitment. If the union becomes arbitrarily large (i.e. $n \to \infty$), individual governments are no longer able to affect the credibility of the common future monetary policy. This source of underaccumulation of debt thus disappears. Hence, as $n \to \infty$, $\bar{d}_1^{e,D} = \bar{d}_1^{e,S}$.

4.2.2. Response to country-specific deterministic components of the government financing requirements

Positive analysis:

This subsection investigates the response of public debt to the deviations of the deterministic components of the government financing requirements from the corresponding cross-country averages, $d_{i1}^{\prime,e}$. In equilibrium, these country-specific deviations of the financing requirements are fully absorbed by first- and second-period deviations of output and public spending from their targets. The reason is that monetary policy and, hence, inflation is attuned to the average conditions

[9] An analogous mechanism features in Cukierman and Lippi (1999), who explore how the (nominal) wage demands of trade unions change as a result of a switch from national monetary policymaking to a monetary union. In a monetary union, trade unions internalize the inflationary consequences of higher wage demands to a lesser extent. Hence, the incentive to restrain wages is weakened.

in the union and thus does not respond to the country-speci…c components of the government …nancing requirements.

Inspection of (4.3) and the de…nition of Q reveals that, if Â > 0, an increase in n weakens the (positive) response of debt to the …rst-period component $Ķ^{¢}_{i1} + (1 + ½)\, d^{¢}_{i0} > 0$, while it strengthens the negative response to $Ķ^{¢}_{i2} > 0$ (i.e., the response becomes more negative). To explain the intuition behind these effects, we …rst consider the case with $Ķ^{¢}_{i1} + (1 + ½)\, d^{¢}_{i0} > 0$ and $Ķ^{¢}_{i2} = 0$. In that case, expected debt accumulation in country i exceeds average expected debt accumulation in the union, i.e. $d^{¢;e;D}_{i1} > 0$. Hence, government i has a relatively large need for seigniorage revenues in the second period so that its preferred second-period in‡ation rate is relatively high (if Â > 0). This discrepancy between government i's preferred second-period in‡ation rate and the preferred second-period in‡ation rate of the average union member, as well as the fact that government i in the …rst period acts as a Stackelberg leader against the CCB in the second period, induces government i to strategically further raise its debt.[10] The resulting higher second-period tax rate forces the CCB to raise the second-period in‡ation rate (of course, in equilibrium, the in‡ation rate is una¤ected by country-speci…c factors, but this is neglected by the individual, optimizing government). However, in a larger union the e¤ect of such an increase in debt on the common in‡ation rate will be diluted and, hence, the incentive to use debt strategically is weaker.

Now, suppose that $Ķ^{¢}_{i2} > 0$ and $Ķ^{¢}_{i1} + (1 + ½)\, d^{¢}_{i0} = 0$. Debt accumulation by country i will be below average, while the need for seigniorage revenues in the second period will be relatively large. Hence, government i's preferred second-period in‡ation rate will be relatively high (if Â > 0). Therefore, it raises debt strategically (compared with the case in which seigniorage is absent) to force the CCB to bring the future in‡ation rate more in line with its own preferred future in‡ation rate. Again, in a larger union the e¤ect of unilateral changes in debt accumulation on the common in‡ation rate is weaker and, hence, the incentive to raise debt strategically is weaker. Hence, in a larger union, $d^{¢;e;D}_{i1}$ will be lower, ceteris paribus.[11]

Normative analysis:

[10] First-period debt and …rst-period in‡ation are simultaneously chosen, while the monetary and …scal policymakers take each other's decisions as given. Therefore, when selecting debt, governments ignore the e¤ect of …rst-period debt on …rst-period seigniorage.

[11] If = 0, preferences concerning the second-period in‡ation rate are the same across governments and, hence, governments will not strategically use debt. Indeed, if = 0, $d^{¢;e;D}_{i1}$ is una¤ected by n. Within the context of our simple model, therefore, the presence of seigniorage revenues is crucial for countries to engage in strategic debt accumulation. However, governments may di¤er in their preferences about future monetary policy for other reasons as well (for example, a di¤erent timing of business cycles). In that case, governments may also strategically use debt policy to a¤ect the future stance of monetary policy.

Countries with relatively severe product and labor market distortions, or high initial debt levels (i.e., countries with $\bar{F}_i^{\cdot} > 0$), feature a relatively high country-specific expected debt component. In fact, these countries overaccumulate debt (at least as far as the debt component $d_{i1}^{\cdot,e,D}$ is concerned, $d_{i1}^{\cdot,e,D} > d_{i1}^{\cdot,e,S}$).[12] The opposite holds true for countries with a relatively low intertemporal government financing requirement (i.e. $\bar{F}_i^{\cdot} < 0$). Governments featuring a high intertemporal government financing requirement (i.e. $\bar{F}_i^{\cdot} > 0$) overaccumulate debt in order to encourage the CCB to raise inflation in the second period, thereby bringing second-period inflation more in line with these governments' preferred inflation rate. These governments fail to internalize the negative externalities of this behavior on the governments with relatively small distortions or low initial debt. Similarly, governments with low intertemporal government financing requirements (i.e. $\bar{F}_i^{\cdot} < 0$) do not internalize the negative externalities on other governments associated with the underaccumulation of debt. Hence, although in equilibrium average expected debt and inflation are unaffected, this failure to internalize the negative externalities of suboptimal debt accumulation leads to wasteful strategic behavior by causing the country-specific deterministic components of public debt to deviate from their values in the second-best equilibrium. A larger union weakens the incentive for strategic behavior:

$$\frac{\partial d_{i1}^{\cdot,e,D}}{\partial n} = \left(\frac{\partial d_{i1}^{\cdot,e,D}}{\partial \beta^*(Q/S)}\right)\left(\frac{\partial \beta^*(Q/S)}{\partial n}\right) = -\frac{\beta^*(1+\rho)}{[1+\beta^*(1+\rho)(Q/S)]^2}\bar{F}_i^{\cdot}\left(\frac{\partial(Q/S)}{\partial n}\right); \text{ if } n \geq 2; \quad (4.7)$$

which is negative (positive) if $\bar{F}_i^{\cdot} > (<) 0$. Furthermore, as $n \to \infty$, $d_{i1}^{\cdot,e,D}$ converges to $d_{i1}^{\cdot,e,S}$. As n rises, the difference between $d_{i1}^{\cdot,e,D}$ and $d_{i1}^{\cdot,e,S}$ thus becomes smaller so that the welfare loss that originates in strategic behavior declines.

4.2.3. Response to the common shock.

Positive analysis:

This subsection turns to the response of debt policy to the common shock μ. As is the case for the second-best equilibrium, the share of the common stochastic shock μ that is absorbed in the first period is larger than the share of the common deterministic component of the intertemporal government financing requirement $\bar{F}$ that is absorbed in the first period (see Table 2 and note that $P^* > S$).

Monetary unification (i.e., an increase from $n = 1$ to $n > 1$) intensifies the response of public debt to unanticipated supply shocks (see (4.4) and note that

[12] Since $\beta^*(Q/S) < \beta^*$ and $\partial d_{i1}^{\cdot,e,D}/\partial[\beta^*(Q/S)] < (>) 0$ if $\bar{F}_i^{\cdot} > (<) 0$ (see also (4.7), below), $d_{i1}^{\cdot,e,D} > (<) d_{i1}^{\cdot,e,S}$ if $\bar{F}_i^{\cdot} > (<) 0$.

$\beta^*(S^*/S)$ is declining in n). Hence, in a monetary union, governments engage in more active debt stabilization policies to union-wide shocks than under national policy making.

The reason for more active debt stabilization is as follows. Fiscal authorities choose to absorb a relatively large part of an adverse first-period supply shock immediately in order to exploit the predetermination of first-period inflation expectations. As a result, second-period inflation expectations will to a lesser extent be affected by the common shock and, hence, inflation variability will be smaller in the second period. In a monetary union, however, this effect is smaller than under national monetary policymaking because each individual union member perceives a relatively small effect of its actions on the future variability of the union-wide inflation rate. A monetary union thus yields more variability of public debt and future inflation. Accordingly, even if shocks are shared by the countries, monetary unification results in more active debt stabilization.

Normative analysis:

Is the additional variability of debt produced by monetary unification excessive? Beetsma and Bovenberg (1999) show that, given that the CCB is not able to commit, the socially-optimal response of debt to common shocks amounts to:

$$d_1^{d,opt} = \left[\frac{1}{1 + \beta^* (1 + \xi) (P^*/S)^2} \right] \frac{\kappa}{o}; \tag{4.8}$$

This response is actually attained with national monetary policymaking (i.e., n = 1). As explained above, monetary unification (n > 1) leads to a more active response of debt to common shocks (i.e. for $\kappa > 0$, $d_1^{d,D} > d_1^{d,opt}$). The larger response of debt to uniform shocks is welfare reducing. Intuitively, reducing future inflation variability is a public good in a monetary union. Hence, individual countries freeride on each other when taking measures to reduce inflation variability.

The coefficient of $\frac{\kappa}{o}$ in (4.8) is smaller than the corresponding coefficient in (3.6) in the second-best equilibrium (since $S^2 < P^*P$). Hence, the response of debt to common shocks, as prescribed by (4.8), is too "conservative" when compared to the second best (i.e. for a bad shock ($\kappa > 0$), $d_1^{d,opt} < d_1^{d,S}$). By absorbing more of κ in the first period, this shock has less of an effect on future inflation expectations and, hence, the future inflation bias due to a lack of commitment in monetary policy.

Debt policy given by $d_1^{d,opt}$ thus deviates from $d_1^{d,S}$ as a result of the trade-off between future monetary policy credibility and a suboptimal distribution of welfare losses over time. This trade-off is a variant of the well-known "credibility-flexibility" trade-off. In this particular case, flexibility refers to activeness of the response of debt, rather than inflation, to shocks; to enhance the credibility of future monetary policy, the government reduces its flexibility in employing public debt to absorb shocks.

4.2.4. Response to the idiosyncratic shock.

Positive analysis:

Like the country-specific deterministic components of the intertemporal government financing requirement (see Subsection 4.2.2), the idiosyncratic shock ε_i is exclusively dealt with at the national level through fiscal policy. A bad idiosyncratic shock (i.e., $\varepsilon_i > 0$) raises the idiosyncratic debt component, $d_{i1}^{\delta,D}$.[13] Inspection of (4.5) and the definition of Q makes clear that, if $\chi > 0$, an increase in n weakens the response of $d_{i1}^{\delta,D}$ to ε_i (i.e., the coefficient of ε_i/ν in (4.5) becomes smaller). In other words, and in contrast to the response of debt to a common shock, debt responds less actively to idiosyncratic shocks as the union becomes larger. Hence, contrary to common wisdom, in a larger union a country will use less debt stabilization in response to country-specific shocks.

The explanation is the same as the earlier explanation for the dependency of $d_{i1}^{\Delta,e,D}$ on the number of union participants. If country i is hit by a bad shock $\varepsilon_i > 0$, it issues more debt than the average government in the union. Hence, if $\chi > 0$, government i has a relatively large need for seigniorage revenues in the second period and, hence, its preferred inflation rate in that period is relatively high. Government i thus strategically raises debt further to bring future inflation more in line with its preferred inflation rate. In a larger union, the perceived influence of government i's policies on the common monetary policy is reduced and, hence, the incentive to use debt strategically is weakened. If $\chi = 0$, all governments share the same preferences concerning the common second-period inflation rate.[14] Hence, governments have no reason to use debt strategically. Indeed, in that case, $d_{i1}^{\delta,D}$ does not depend on the number of countries.

[13] $d_{i1}^{\delta,D}$ is zero in the case of national monetary policymaking (i.e., n = 1). In that case, $\varepsilon_i = 0$ (as we have assumed that the cross-country average of the idiosyncratic shocks is zero). This constraint has explicitly been used in the derivation of the equilibrium. If we had assumed that $\varepsilon_i \neq 0$, then $\frac{1}{\nu}$ in (4.4) would have been replaced with $\frac{1+\varepsilon}{\nu}$, but $d_{i1}^{\delta,D}$ would have remained at zero.

[14] Although country i may have been hit by a relatively bad shock $\varepsilon_i > 0$, suggesting a larger need for inflation as a stabilizating tool, its preferred second-period inflation rate does not differ from the other governments' preferred inflation rate. The reason is that second-period inflation expectations adjust for the part of the shock that is transmitted into the second period. Hence, in the second period inflation no longer has a role in stabilizing the effects of ε_i on the economy.

Normative analysis:

A comparison of $d_{i1}^{\delta,D}$ and $d_{i1}^{\delta,S}$ reveals that the response of debt to idiosyncratic shocks is more vigorous than under the second-best equilibrium. A larger union renders the debt response less active. The associated reduction in wasteful strategic interaction boosts welfare. If the union becomes infinitely large (i.e., $n \to \infty$), $d_{i1}^{\delta,D}$ converges to $d_{i1}^{\delta,S}$ and the response of debt to idiosyncratic shocks becomes optimal. In that case, all wasteful strategic interactions are eliminated.

4.2.5. Summary of the effects of a larger union

If the number of countries goes to infinity and all strategic interactions among the governments disappear, the debt components $\bar{d}_1^{e,D}$, $d_{i1}^{\Delta e,D}$ and $d_{i1}^{\delta,D}$ all converge to the same response coefficients $1/(1+\beta^*(1+\rho))$. This response is in fact optimal (compare (4.2) with (3.4), (4.3) with (3.5) and (4.5) with (3.7) for $n \to \infty$). The response $\bar{d}_1^{d,D}$ to the common shock, in contrast, is optimal and equal to (4.8) if $n = 1$. It becomes excessive if $n > 1$, and the more so the larger the union becomes. Hence, only the response of debt to the common shock will be suboptimal for $n \to \infty$. Actually, the welfare loss associated with this response is largest in an infinitely large union. Even though for $n \to \infty$ the debt response to idiosyncratic shocks is more active than the response to common shocks, the former is optimal, while the latter is excessive.

5. Optimal institutional arrangements

This section investigates institutional adjustments that can help the monetary union to reach better equilibria. In particular, we allow for inflation targets. The second-period inflation target may depend on first-period debt levels and, thereby, indirectly, on first-period shocks. In addition, we allow for debt targets. These debt targets should be hit exactly. Hence, they act as debt ceilings when they prevent overaccumulation of debt, while they act as debt floors when they prevent underaccumulation of debt. The debt targets may depend on the shocks, i.e. one can write $d_{i1}^T = d_{i1}^T(\eta, \varepsilon_i)$, where the superscript "T" stands for "target". Inflation targets can be viewed as a contractual way to deal with the commitment problem in monetary policy. In the same way, debt targets are a contractual way to address externalities.

Inflation targets can be enforced by giving the central banker financial incentives to meet the target or by making reappointment of the central banker dependent upon meeting the target.[15] Reputational considerations may also help to

[15] This is the case for New Zealand – see Walsh (1995) for a detailed account of this arrangement.

enforce inflation targets. The announcement of an inflation target provides market participants with a benchmark against which the central banker can be held accountable. Failure to meet the target indicates a lack of willingness or ability on the side of the central banker to stick to the announcements. This information will be taken into account when future inflation expectations are formed.

Debt targets may be enforced through peer pressure (the loss of political prestige if the target is missed) and fines. Indeed, under the so-called Stability and Growth Pact (SGP), EMU participants who persistently violate the ceilings on public deficits will be fined (for more details on the SGP, see Artis and Winkler, 1998, and Eichengreen and Wyplosz, 1998).[16] In the sequel, we assume that both the inflation and the debt targets can be enforced.

This section explores how institutional rearrangements (inflation targets and debt targets) can induce optimal responses of debt to the various components of the intertemporal government financing requirement. In particular, it will investigate whether these arrangements ensure that debt accumulation mimics debt accumulation in the second-best equilibrium.

5.1. Identical economies ($\bar{\kappa}_{i1} = \bar{\kappa}_1$, $\bar{\kappa}_{i2} = \bar{\kappa}_2$, $d_{i0} = d_0$ and $\epsilon_i = 0, \forall i$)

This subsection assumes that the union participants are completely identical. Hence, the deterministic components of the government financing requirements are equal across the countries ($\bar{\kappa}_{i1} = \bar{\kappa}_1$, $\bar{\kappa}_{i2} = \bar{\kappa}_2$ and $d_{i0} = d_0$, $\forall i$), while idiosyncratic shocks are absent ($\epsilon_i = 0, \forall i$). As a result, all governments adopt identical policies. The solution for d_{i1} consists only of the response to the deterministic components of the average intertemporal government financing requirement and the response to the common shock. Potential strategic interactions arising from a disagreement about the common monetary policy are absent. Hence, optimal institutional design needs to address only the lack of commitment of monetary policy. Appendix C proves the following proposition:

Proposition 1. Suppose that $\bar{\kappa}_{i1} = \bar{\kappa}_1$, $\bar{\kappa}_{i2} = \bar{\kappa}_2$, $d_{i0} = \bar{d}_0$ and $\epsilon_i = 0, \forall i$. In that case, the following combination of state-contingent inflation targets

$$\pi_1^*(h^e) = -\left[\frac{1/\alpha\pi}{P}\right]\left[\bar{\kappa}_1 + (1+\rho)\, d_0 - h^e\right]; \quad (5.1)$$

$$\pi_2^*(h) = -\left[\frac{1/\alpha\pi}{P}\right]\left[\bar{\kappa}_2 + (1+\rho)\, h\right]; \quad (5.2)$$

with h equal to the cross-country average realized debt level $\bar{d}_1$, ensures that the decentralized, discretionary equilibrium coincides with the second-best equilibrium.

[16] Under exceptional circumstances (in particular, a large fall in GDP) the sanctions envisaged by the Pact may be waived. This aspect of the Pact to some extent resembles the contingent nature of the debt targets explored in this section.

Proposition 1 reveals that, in contrast to making the central banker conservative à la Rogo¤ (1985) – see Beetsma and Bovenberg (1999) –, the state-contingent in‡ation target succeeds in establishing the second best (see also Svensson, 1997, and Beetsma and Jensen, 1999). A conservative central bank addresses the commitment problem (the in‡ation bias), but at the same time distorts the stabilization of shocks. A state-contingent in‡ation target, in contrast, addresses the in‡ation bias without distorting stabilization. Thus, the contractual solution to the commitment problem (à la Svensson, 1997) dominates the solution of delegation (à la Rogo¤, 1985). The contract, however, needs to be quite rich. In particular, the in‡ation target needs to be state-contingent, because the size of the (second-period) in‡ation bias depends on the amounts of debt issued by the union participants. A state-independent in‡ation target would not be able to establish the second best.

By addressing the commitment problem through a state-contingent in‡ation target, one also eliminates the intertemporal distortions that originate in the lack of monetary policy credibility. In particular, by setting the second-period in‡ation target at the proposed level (5.2), the second-period in‡ation bias is eliminated. Accordingly, without a second-period in‡ation bias, governments in the …rst period no longer perceive the need to underaccumulate (for the purpose of enhancing the credibility of monetary policy in the second period) debt in response to the deterministic components of the government …nancing requirement. The absence of the second-period in‡ation bias also takes away the need to absorb an excessively large share of the common shock in the …rst period and thus ensures an optimal debt response to ^{1}. Finally, by setting the …rst-period in‡ation target conform (5.1), the …rst-period in‡ation bias is eliminated.

5.2. Di¤erences among countries

This subsection allows for cross-country di¤erences both in the deterministic components of the government …nancing requirements (i.e., $\bar{K}_{i1}; \bar{K}_{i2}$ and d_{i0}) and in the stochastic shocks (i.e., the $^{2}{}_{i}$'s are no longer assumed to be zero). As explained in Section 4, such di¤erences among countries produce con‡icts about the preferred future stance of the common monetary policy. The con‡icts result in wasteful strategic interactions between decentralized …scal policymakers. Debt targets eliminate these con‡icts of interest and the associated costly strategic interactions.

The following proposition is proven in Appendix D:

Proposition 2. *With deterministic differences in the government financing requirements and with idiosyncratic shocks, the combination of inflation and debt targets that minimizes V_U is obtained by setting the first- and second-period inflation targets at, respectively, (5.1) and (5.2), with h equal to the cross-country average realized debt level $\bar{d}_1$, and the (specific) debt target d_{i1}^T on country i $(i = 1, ..., n)$ at $d_{i1}^T = \bar{d}_1^{e,S} + d_{i1}^{\Delta,e,S} + \bar{d}_1^{d,S} + d_{i1}^{\delta,S}$. The resulting equilibrium coincides with the second-best equilibrium.*

By imposing debt targets attuned to each country's specific situation, the union can thus eliminate the externalities associated with the strategic behavior of the individual governments. Hence, with the proposed inflation and debt targets in place, the responses of public debt (and the other policy instruments) mimic their counterparts under the second-best equilibrium. In particular, compared with the purely discretionary equilibrium, debt targets restrict debt to be less active in response to the country-specific components of the government financing requirements ($\bar{F}_i^{\Delta}$ and $\frac{\varepsilon_i}{o}$). They thus operate as a ceiling on the associated debt responses if $\bar{F}_i^{\Delta} > 0$ or $\frac{\varepsilon_i}{o} > 0$, and as a floor if $\bar{F}_i^{\Delta} < 0$ or $\frac{\varepsilon_i}{o} < 0$. Only if $\chi = 0$ or $n \to \infty$, are debt targets redundant and are optimal inflation targets sufficient for the discretionary equilibrium to coincide with the second-best equilibrium. In that case, no debt targets are needed, because the effect of a unilateral change in debt on seigniorage revenues becomes negligible and any strategic effects disappear.

6. Conclusion

This paper investigated the interaction between fiscal policy and monetary policy in a monetary union. Our analysis has allowed for two imperfections. One is the lack of monetary policy commitment. The other involves spillovers among decentralized fiscal policymakers. We explored how inflation targets and debt targets can alleviate the welfare losses arising from these imperfections.

With identical economies, imposing the optimal state-contingent inflation target on the common central bank is sufficient to establish the second-best equilibrium. If countries are heterogeneous, however, the inflation target needs to be complemented by debt targets. These debt targets eliminate the strategic, welfare-reducing interactions among governments arising from differences among the union participants on the preferred stance of monetary policy. The inflation and debt targets can be viewed as a contractual solution to the lack of commitment in monetary policy and the spillovers among the decentralized fiscal policymakers.

The analysis can be extended into a variety of directions. One extension would be to allow for a longer modelling horizon and to explore the optimal, dynamic paths for the inflation and debt targets. In particular, it would be interesting to explore whether debt targets converge over time for countries with di¤erent initial debt levels. Another extension is to investigate whether and how the debt targets can be enforced. Indeed, the enforcement of the Stability and Growth Pact is subject to some doubt. Such an analysis would require a dynamic framework that accounts for reputational considerations. The countries with the smallest product- and labor-market distortions and the countries with the lowest initial debt level can be expected to most favor strict enforcement of the targets. In a multiperiod context, these countries will take into account the e¤ects on the future behavior of governments (in terms of debt policies) of a failure to enforce the targets.

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Appendix

Notation:

We will use the following conventions: superscript e denotes the subjective expectation of a variable. When no confusion is possible, it is also used to denote the mathematical (model-induced) expectation. The deviation of some variable y_{it} from its expected value is denoted by superscript "d": $y_{it}^d = y_{it} - y_{it}^e$. An upperbar denotes the cross-country average of a variable: $\bar{y}_t = \frac{1}{n}\sum_{i=1}^n y_{it}$. The exception is a variable with a tilde, whose cross-country average is denoted by the omission of the country index: $\tilde{y}_t = \frac{1}{n}\sum_{i=1}^n \tilde{y}_{it}$. Note that $\bar{y}_t^e = \frac{1}{n}\sum_{i=1}^n y_{it}^e$ and that $\bar{y}_t^d = \bar{y}_t - \bar{y}_t^e$. Superscript "$\Delta$" denotes the deviation of a variable from its cross-country average: $y_{it}^{\Delta} = y_{it} - \bar{y}_t$. Note that $y_{it}^{\Delta;e} = y_{it}^e - \bar{y}_t^e$. Finally, superscript "$\delta$" is used to denote the deviation of a country-specific variable from the sum of its expected value and the cross-country average difference between this variable and its expected value: $y_{it}^{\delta} = y_{it} - \left(y_{it}^e + \bar{y}_t^d\right) = y_{it} - \bar{y}_t^e - y_{it}^{\Delta;e} - \bar{y}_t^d$. Hence, using our notation, we can decompose a variable y_{it} into $y_{it} = \bar{y}_t^e + y_{it}^{\Delta;e} + \bar{y}_t^d + y_{it}^{\delta}$, i.e. the sum of the cross-country average expectation, the difference between the expectation of y_{it} and the average expectation, the cross-country average prediction error, and the difference between y_{it} and its expectation plus the cross-country average prediction error.

Assumption:

As in the main text, the sum of the idiosyncratic shocks is assumed to be zero: $\sum_{i=1}^n \varepsilon_i = 0$.

1. Derivation of the second-best equilibrium

Both monetary and fiscal policymaking are centralized. Moreover, monetary policy is conducted under commitment. The supranational centralized fiscal authority (CFA) minimizes an equally-weighted average of the participating countries' social objectives. In solving for the equilibrium, we work backwards, starting with the second period.

1.1. Period 2

First, we solve for the second-period outcomes, given the first-period debt choices. The CCB's Lagrangian is:

$$2\mathcal{L}_2^{CCB} = \alpha_\pi \pi_2^2 +$$

$$\frac{1}{n}\sum_{i=1}^{n}\left\{[\nu(\pi_2 - \pi_2^e - \tau_{i2}) - \tilde{x}_{i2}]^2 + \alpha_g[-(1+\rho)d_{i1} + \tau_{i2} + \chi\pi_2 - \tilde{g}_{i2}]^2\right\} +$$
$$2\theta_2\{E[\pi_2] - \pi_2^e\};$$

where θ_2 is the Lagrange multiplier associated with the rational expectations constraint in period 2. The CCB's first-order conditions with respect to π_2 and π_2^e can be written as:

$$\alpha_\pi\pi_2 + \frac{1}{n}\sum_{i=1}\{\nu[\nu(\pi_2 - \pi_2^e - \tau_{i2}) - \tilde{x}_{i2}] + \chi\alpha_g(g_{i2} - \tilde{g}_{i2})\} + \theta_2 = 0;$$

$$E\left\{\frac{1}{n}\sum_{i=1}\nu[\nu(\pi_2 - \pi_2^e - \tau_{i2}) - \tilde{x}_{i2}]\right\} + \theta_2 = 0.$$

We combine these two equations to:

$$\begin{aligned}&\alpha_\pi\pi_2 + \frac{1}{n}\sum_{i=1}\{\nu[\nu(\pi_2 - \pi_2^e - \tau_{i2}) - \tilde{x}_{i2}] + \chi\alpha_g(g_{i2} - \tilde{g}_{i2})\} - \\ &E\left\{\frac{1}{n}\sum_{i=1}\nu[\nu(\pi_2 - \pi_2^e - \tau_{i2}) - \tilde{x}_{i2}]\right\} \\ =\ & 0;\end{aligned} \tag{1.1}$$

The loss function to be minimized by the CFA in the second period is:

$$\frac{1}{2n}\sum_{i=1}^{n}\left\{\alpha_\pi\pi_2^2 + [\nu(\pi_2 - \pi_2^e - \tau_{i2}) - \tilde{x}_{i2}]^2 + \alpha_g[-(1+\rho)d_{i1} + \tau_{i2} + \chi\pi_2 - \tilde{g}_{i2}]^2\right\}.$$

The first-order conditions with respect to the τ_{i2} can be written as:

$$\nu[\nu(\pi_2 - \pi_2^e - \tau_{i2}) - \tilde{x}_{i2}] = \alpha_g[-(1+\rho)d_{i1} + \tau_{i2} + \chi\pi_2 - \tilde{g}_{i2}];\ \forall i. \tag{1.2}$$

Imposing rationality of expectations (and noting that, in equilibrium, realizations coincide with expectations), we can write (1.1) and (1.2) as:

$$\alpha_\pi\pi_2 = \chi\alpha_g(\tilde{g}_2 - \bar{g}_2); \tag{1.3}$$

$$\left(\alpha_g + \nu^2\right)\left(\tau_{i2} + \frac{\tilde{x}_{i2}}{\nu}\right) = \alpha_g\left[\tilde{g}_{i2} + (1+\rho)d_{i1} - \chi\pi_2\right];\ \forall i. \tag{1.4}$$

Using the second-period government budget constraints, one can rewrite (1.3) as:

$$\left(\frac{1}{\alpha_g} + \frac{\chi^2}{\alpha_\pi}\right)\pi_2 = \frac{\chi}{\alpha_\pi}\left[\bar{K}_2 + (1+\rho)\,\bar{d}_1\right] - \frac{\chi}{\alpha_\pi}\left(\bar{\tau}_2 + \frac{\bar{x}_2}{\nu}\right). \tag{1.5}$$

Take cross-country averages of (1.4) and combine this with (1.5) to eliminate $\left(\bar{\tau}_2 + \frac{\bar{x}_2}{\nu}\right)$. The resulting equation can be rewritten to yield the solution for inflation, given average union debt, $\bar{d}_1$:

$$\pi_2 = \left[\frac{\chi/\alpha_\pi}{P}\right]\left[\bar{K}_2 + (1+\rho)\,\bar{d}_1\right], \tag{1.6}$$

where

$$P \equiv \chi^2/\alpha_\pi + 1/\nu^2 + 1/\alpha_g, \tag{1.7}$$

as defined in the main text. Combine this with (1.4) to eliminate π_2 and rewrite the resulting equation to yield the solution for $\tau_{i2} + \frac{\bar{x}_{i2}}{\nu}$:

$$\tau_{i2} + \frac{\bar{x}_{i2}}{\nu} = \left[\frac{1/\nu^2}{1/\nu^2 + 1/\alpha_g}\right]\left\{\left[\bar{K}_{i2} + (1+\rho)\,d_{i1}\right] - \left[\frac{\chi^2/\alpha_\pi}{P}\right]\left[\bar{K}_2 + (1+\rho)\,\bar{d}_1\right]\right\}, \; \forall i. \tag{1.8}$$

Combining this with (2.2) and using that $\pi_2 = \pi_2^e$, we obtain:

$$\bar{x}_{i2} - x_{i2} = \left[\frac{1/\nu}{1/\nu^2 + 1/\alpha_g}\right]\left\{\left[\bar{K}_{i2} + (1+\rho)\,d_{i1}\right] - \left[\frac{\chi^2/\alpha_\pi}{P}\right]\left[\bar{K}_2 + (1+\rho)\,\bar{d}_1\right]\right\}, \; \forall i. \tag{1.9}$$

Take expecations of (1.2), use that in period 2 realizations match expectations and combine the result with (1.8) to obtain the solution for $\bar{g}_{i2} - g_{i2}$:

$$\bar{g}_{i2} - g_{i2} = \left[\frac{1/\alpha_g}{1/\nu^2 + 1/\alpha_g}\right]\left\{\left[\bar{K}_{i2} + (1+\rho)\,d_{i1}\right] - \left[\frac{\chi^2/\alpha_\pi}{P}\right]\left[\bar{K}_2 + (1+\rho)\,\bar{d}_1\right]\right\}, \; \forall i. \tag{1.10}$$

The second-period loss of both the CCB and the CFA is given by:

$$\begin{aligned} L_2 \equiv\; & \frac{1}{2}\left[\frac{\chi^2/\alpha_\pi}{P^2}\right]\left[\bar{K}_2 + (1+\rho)\,\bar{d}_1\right]^2 + \\ & \frac{1}{2n}\left[\frac{1}{1/\nu^2 + 1/\alpha_g}\right]\sum_{j=1}\left\{\left[\bar{K}_{j2} + (1+\rho)\,d_{j1}\right] - \left[\frac{\chi^2/\alpha_\pi}{P}\right]\left[\bar{K}_2 + (1+\rho)\,\bar{d}_1\right]\right\}^2. \end{aligned}$$

1.2. Period 1

We now move back to solve for the first-period outcomes. The Lagrangian of the CCB in period 1 is:

$$2\varpi_1^{CCB} = \alpha_\pi \pi_1^2 + \frac{1}{n}\sum_{i=1}\left\{ \begin{array}{c} [\gamma(\pi_1 - \pi_1^e - \tau_{i1}) - \mu - \varepsilon_i - \tilde{x}_{i1}]^2 + \\ \alpha_g[-(1+\rho)d_{i0} + \tau_{i1} + \chi\pi_1 + d_{i1} - \tilde{g}_{i1}]^2 \end{array}\right\} + 2\beta L_2 + 2\theta_1\{E[\pi_1] - \pi_1^e\};$$

where θ_1 is the Lagrange multiplier associated with the rational expectations (of inflation) constraint in period 1. Note that L_2 is not affected by π_1. Therefore, the CCB's first-order conditions with respect to π_1 and π_1^e can be written as:

$$\alpha_\pi\pi_1 + \frac{1}{n}\sum_{i=1}\{\gamma[\gamma(\pi_1 - \pi_1^e - \tau_{i1}) - \mu - \varepsilon_i - \tilde{x}_{i1}] + \chi\alpha_g(g_{i1} - \tilde{g}_{i1})\} + \theta_1 = 0;$$

$$E\left\{\frac{1}{n}\sum_{i=1}\gamma[\gamma(\pi_1 - \pi_1^e - \tau_{i1}) - \mu - \varepsilon_i - \tilde{x}_{i1}]\right\} + \theta_1 = 0;$$

which can be combined to give:

$$\begin{aligned} &\alpha_\pi\pi_1 + \frac{1}{n}\sum_{i=1}\{\gamma[\gamma(\pi_1 - \pi_1^e - \tau_{i1}) - \mu - \varepsilon_i - \tilde{x}_{i1}] + \chi\alpha_g(g_{i1} - \tilde{g}_{i1})\} - \\ &E\left\{\frac{1}{n}\sum_{i=1}\gamma[\gamma(\pi_1 - \pi_1^e - \tau_{i1}) - \mu - \varepsilon_i - \tilde{x}_{i1}]\right\} \\ = \; & 0. \end{aligned} \qquad (1.11)$$

The loss function to be minimized by the CFA in the first period is:

$$\alpha_\pi \pi_1^2 + \frac{1}{n}\sum_{i=1}\left\{ \begin{array}{c} [\gamma(\pi_1 - \pi_1^e - \tau_{i1}) - \mu - \varepsilon_i - \tilde{x}_{i1}]^2 + \\ \alpha_g[-(1+\rho)d_{i0} + \tau_{i1} + \chi\pi_1 + d_{i1} - \tilde{g}_{i1}]^2 \end{array}\right\} + 2\beta L_2.$$

The first-order conditions with respect to τ_{i1} and d_{i1} $(i = 1, \ldots, n)$ can be written as:

$$-\gamma[\gamma(\pi_1 - \pi_1^e - \tau_{i1}) - \mu - \varepsilon_i - \tilde{x}_{i1}] + \alpha_g(g_{i1} - \tilde{g}_{i1}) = 0, \forall i; \qquad (1.12)$$

$$\left(\frac{1}{n}\right)\alpha_g(\tilde{g}_{i1} - g_{i1}) = \beta(\partial L_2/\partial d_{i1}), \forall i; \qquad (1.13)$$

where

$$
\begin{aligned}
\partial L_2/\partial d_{i1} = {} & \left[\frac{\chi^2/\alpha_\pi}{P^2}\right]\left[K_2 + (1+\rho)\bar{d}_1\right]\tfrac{1}{n}(1+\rho) + \\
& \tfrac{1}{n}\left[\frac{1}{1/\nu^2+1/\alpha_g}\right]\left\{\left[K_{i2} + (1+\rho)d_{i1}\right] - \left[\frac{\chi^2/\alpha_\pi}{P}\right]\left[K_2 + (1+\rho)\bar{d}_1\right]\right\}\cdot \\
& (1+\rho)\left[1 - \tfrac{1}{n}\left(\frac{\chi^2/\alpha_\pi}{P}\right)\right] - \\
& \tfrac{1}{n}\left[\frac{1}{1/\nu^2+1/\alpha_g}\right]\sum_{j\neq i}\left\{\left[K_{j2} + (1+\rho)d_{j1}\right] - \left[\frac{\chi^2/\alpha_\pi}{P}\right]\left[K_2 + (1+\rho)\bar{d}_1\right]\right\}\cdot \\
& \left[-(1+\rho)\tfrac{1}{n}\left(\frac{\chi^2/\alpha_\pi}{P}\right)\right] \\
= {} & \left[\frac{\chi^2/\alpha_\pi}{P^2}\right]\left[K_2 + (1+\rho)\bar{d}_1\right]\tfrac{1}{n}(1+\rho) + \\
& \tfrac{1}{n}\left[\frac{1}{1/\nu^2+1/\alpha_g}\right]\left\{\left[K_{i2} + (1+\rho)d_{i1}\right] - \left[\frac{\chi^2/\alpha_\pi}{P}\right]\left[K_2 + (1+\rho)\bar{d}_1\right]\right\}(1+\rho) - \\
& \tfrac{1}{n}\left[\frac{1}{1/\nu^2+1/\alpha_g}\right]\sum_{j=1}\left\{\left[K_{j2} + (1+\rho)d_{j1}\right] - \left[\frac{\chi^2/\alpha_\pi}{P}\right]\left[K_2 + (1+\rho)\bar{d}_1\right]\right\}\tfrac{1}{n}\left[\frac{\chi^2/\alpha_\pi}{P}\right]\cdot \\
& (1+\rho) \\
= {} & \tfrac{1}{n}\left[\frac{1}{1/\nu^2+1/\alpha_g}\right]\left\{\left[K_{i2} + (1+\rho)d_{i1}\right] - \left[\frac{\chi^2/\alpha_\pi}{P}\right]\left[K_2 + (1+\rho)\bar{d}_1\right]\right\}(1+\rho):
\end{aligned}
$$

Finally, we can rewrite the first-period government budget constraint as:

$$
K_{i1} + (1+\rho)d_{i0} = (\tau_{i1} + x_{i1}/\nu) + \chi\pi_1 + (\bar{g}_{i1} - g_{i1}) + d_{i1};\ \forall i: \tag{1.14}
$$

The system of first-order conditions to be solved is thus given by (1.11), (1.12), (1.13) and (1.14). Take some country-specific policy variable y_{i1} ($y_{i1} = \pi_{i1}; \tau_{i1}; g_{i1}$ or d_{i1}). We will solve for y_{i1} by solving for each of the components of the decomposition of y_{i1} into $y_{i1} = \bar{y}^e_1 + y^{\Delta;e}_{i1} + \bar{y}^d_1 + y^D_{i1}$, where $\bar{y}^e_1$ will be the response to the cross-country average of the deterministic components of the government financing requirements, $y^{\Delta;e}_{i1}$ will be the response to the country-specific deterministic components of the government financing requirements, $\bar{y}^d_1$ will be response to the common shock and y^D_{i1} will be the response to the idiosyncratic shock. We thus compute the solutions to these variables in four steps.

1.2.1. Steps 1 and 2

Take expectations across (1.11), (1.12), (1.13) and (1.14) to yield:

$$
\alpha_\pi \pi^e_1 = \chi\alpha_g(\bar{g}_1 - \bar{g}^e_1); \tag{1.15}
$$

$$
\nu^2(\tau^e_{i1} + x_{i1}/\nu) = \alpha_g(\bar{g}_{i1} - g^e_{i1});\ \forall i; \tag{1.16}
$$

$$\left(\frac{1}{n}\right)\alpha_g\left(\bar{g}_{i1}-g^e_{i1}\right)=\beta\left(\partial L_2/\partial d_{i1}\right)^e;\ \forall i; \tag{1.17}$$

$$\bar{K}_{i1}+(1+\rho)\,d_{i0}=\left(\tau^e_{i1}+\bar{x}_{i1}/\nu\right)+\chi\pi^e_1+\left(\bar{g}_{i1}-g^e_{i1}\right)+d^e_{i1};\ \forall i: \tag{1.18}$$

Step 1: Computation of expectations of cross-country averages. Take cross-country averages across (1.15)-(1.18) to yield:

$$\alpha_\pi\pi^e_1=\chi\alpha_g\left(\bar{g}_1-\bar{g}^e_1\right); \tag{1.19}$$

$$\nu^2\left(\bar{\tau}^e_1+\bar{x}_1/\nu\right)=\alpha_g\left(\bar{g}_1-\bar{g}^e_1\right); \tag{1.20}$$

$$\left(\frac{1}{n}\right)\alpha_g\left(\bar{g}_1-\bar{g}^e_1\right)=\left(\frac{1}{n}\right)\beta(1+\rho)\left[\frac{1}{P}\right]\left[\bar{K}_2+(1+\rho)\,\bar{d}^e_1\right]; \tag{1.21}$$

$$\bar{K}_1+(1+\rho)\,\bar{d}_0=\left(\bar{\tau}^e_1+\bar{x}_1/\nu\right)+\chi\pi^e_1+\left(\bar{g}_1-\bar{g}^e_1\right)+\bar{d}^e_1: \tag{1.22}$$

Using (1.19), (1.20) and (1.22), one can solve for the expected cross-country averages of the outcomes, given $\bar{d}^e_1$:

$$\pi^e_1=\left[\frac{\chi/\alpha_\pi}{P}\right]\left[\bar{K}_1+(1+\rho)\,\bar{d}_0-\bar{d}^e_1\right]; \tag{1.23}$$

$$\bar{\tau}^e_1+\bar{x}_1/\nu=\left[\frac{1/\nu^2}{P}\right]\left[\bar{K}_1+(1+\rho)\,\bar{d}_0-\bar{d}^e_1\right];$$

$$\bar{g}_1-\bar{g}^e_1=\left[\frac{1/\alpha_g}{P}\right]\left[\bar{K}_1+(1+\rho)\,\bar{d}_0-\bar{d}^e_1\right]:$$

Combining this last equation with (1.21) to eliminate $\bar{g}_1-\bar{g}^e_1$, we can solve for the equilibrium value of $\bar{d}^e_1$ as:

$$\bar{d}^{e,S}_1=\frac{\left[\bar{K}_1+(1+\rho)\,\bar{d}_0-\bar{K}_2\right]+(1-\beta^*)\,\bar{K}_2}{1+\beta^*(1+\rho)}; \tag{1.24}$$

where

$$\beta^*=\beta(1+\rho); \tag{1.25}$$

and where (here and in the sequel) a superscript "S" stands for "second best".

Substitute (1.24) back into the expressions for π^e_1, $\bar{\tau}^e_1+\bar{x}_1/\nu$ and $\bar{g}_1-\bar{g}^e_1$ to give:

$$\pi_1^e = \left[\frac{\chi/\alpha_\pi}{P}\right]\left[\frac{\beta^*(1+\rho)}{1+\beta^*(1+\rho)}\right]\bar{F}; \tag{1.26}$$

$$\tau_1^e + \bar{x}_1/\nu = \left[\frac{1/\nu^2}{P}\right]\left[\frac{\beta^*(1+\rho)}{1+\beta^*(1+\rho)}\right]\bar{F}; \tag{1.27}$$

$$\bar{g}_1 - \bar{g}_1^e = \left[\frac{1/\alpha_g}{P}\right]\left[\frac{\beta^*(1+\rho)}{1+\beta^*(1+\rho)}\right]\bar{F}; \tag{1.28}$$

$$\bar{x}_1 - \bar{x}_1^e = \left[\frac{1/\nu}{P}\right]\left[\frac{\beta^*(1+\rho)}{1+\beta^*(1+\rho)}\right]\bar{F}; \tag{1.29}$$

where the last expression follows upon combining (1.27) and (2.1).

Step 2: Computation of country-specific expected deviations from cross-country expected averages. Subtract (1.19)-(1.22) from (1.15)-(1.18) to give the system:

$$\nu^2\left(\tau_{i1}^{\cdot;e} + x_{i1}^{\cdot}/\nu\right) = \alpha_g\left(\bar{g}_{i1}^{\cdot} - g_{i1}^{\cdot;e}\right);\ \forall i; \tag{1.30}$$

$$\alpha_g\left(\bar{g}_{i1}^{\cdot} - g_{i1}^{\cdot;e}\right) = \beta^*\left[\frac{1}{1/\nu^2+1/\alpha_g}\right]\left[\bar{K}_{i2}^{\cdot} + (1+\rho)\,d_{i1}^{\cdot;e}\right];\ \forall i; \tag{1.31}$$

$$\bar{K}_{i1}^{\cdot} + (1+\rho)\,d_{i0}^{\cdot} = \left(\tau_{i1}^{\cdot;e} + x_{i1}^{\cdot}/\nu\right) + \left(\bar{g}_{i1}^{\cdot} - g_{i1}^{\cdot;e}\right) + d_{i1}^{\cdot;e};\ \forall i; \tag{1.32}$$

where we note that the first equation has dropped out. Hence, from this system we obtain:

$$\tau_{i1}^{\cdot;e} + x_{i1}^{\cdot}/\nu = \left[\frac{1/\nu^2}{1/\nu^2+1/\alpha_g}\right]\left[\bar{K}_{i1}^{\cdot} + (1+\rho)\,d_{i0}^{\cdot} - d_{i1}^{\cdot;e}\right];\ \forall i;$$

$$\bar{g}_{i1}^{\cdot} - g_{i1}^{\cdot;e} = \left[\frac{1/\alpha_g}{1/\nu^2+1/\alpha_g}\right]\left[\bar{K}_{i1}^{\cdot} + (1+\rho)\,d_{i0}^{\cdot} - d_{i1}^{\cdot;e}\right];\ \forall i:$$

If we combine the last equation with (1.31), we can solve for $d_{i1}^{\cdot;e}$ as:

$$d_{i1}^{\cdot;e;S} = \frac{\left[\bar{K}_{i1}^{\cdot} + (1+\rho)\,d_{i0}^{\cdot} - \bar{K}_{i2}^{\cdot}\right] + (1-\beta^*)\,\bar{K}_{i2}^{\cdot}}{1+\beta^*(1+\rho)};\ \forall i: \tag{1.33}$$

1.2.2. Steps 3 and 4

Subtract the system (1.15)-(1.18) from the system (1.11), (1.12), (1.13) and (1.14). This yields:

$$\alpha_\pi \pi_1^d + \nu^2\left(\pi_1^d - \tau_1^d - \frac{1}{\nu}\right) + \chi\alpha_g \bar{g}_1^d = 0; \tag{1.34}$$

$$-\nu^2\left(\pi_1^d - \tau_{i1}^d - \frac{1+\varepsilon_i}{\nu}\right) + \alpha_g g_{i1}^d = 0;\ \forall i; \tag{1.35}$$

$$-\alpha_g g^d_{i\infty} = \beta^*(1+\rho)\left\{\left[\frac{1}{1/\nu^2+1/\alpha_g}\right] d^d_{i\infty} - \left[\frac{\chi^2/\alpha_\pi}{P}\right]\bar{d}^d_\infty\right\}, \forall i, \qquad (1.36)$$

$$0 = \tau^d_{i\infty} + \chi\pi^d_\infty - g^d_{i\infty} + d^d_{i\infty}, \forall i. \qquad (1.37)$$

Step 3: Responses to common shocks. Take cross-country averages across (1.34)-(1.37) to give:

$$\alpha_\pi\pi^d_\infty + \nu^2\left(\pi^d_\infty - \bar{\tau}^d_\infty - \frac{1}{\nu}\right) + \chi\alpha_g\bar{g}^d_\infty = 0, \qquad (1.38)$$

$$-\nu^2\left(\pi^d_\infty - \bar{\tau}^d_\infty - \frac{1}{\nu}\right) + \alpha_g\bar{g}^d_\infty = 0, \qquad (1.39)$$

$$-\alpha_g\bar{g}^d_\infty = \beta^*(1+\rho)\left[\frac{1}{P}\right]\bar{d}^d_\infty, \qquad (1.40)$$

$$0 = \bar{\tau}^d_\infty + \chi\pi^d_\infty - \bar{g}^d_\infty + \bar{d}^d_\infty, \qquad (1.41)$$

The solution for π^d_∞, $\bar{\tau}^d_\infty$, $\bar{g}^d_\infty$ and $\bar{x}^d_\infty$, given $\bar{d}_\infty$, is:

$$\pi^d_\infty = \left[\frac{(\chi+1)/\alpha_\pi}{P^*}\right]\left(\frac{1}{\nu} - \bar{d}^d_\infty\right),$$

$$\bar{x}^d_\infty = -\left[\frac{1/\nu}{P^*}\right]\left(\frac{1}{\nu} - \bar{d}^d_\infty\right),$$

$$\bar{g}^d_\infty = -\left[\frac{1/\alpha_g}{P^*}\right]\left(\frac{1}{\nu} - \bar{d}^d_\infty\right),$$

where

$$P^* = (\chi+1)^2/\alpha_\pi + 1/\nu^2 + 1/\alpha_g, \qquad (1.42)$$

as defined in the main text. Combine the expression for $\bar{g}^d_\infty$ with (1.41) to give the solution of $\bar{d}^d_\infty$ as:

$$\bar{d}^{d,S}_\infty = \left[\frac{1}{1+\beta^*(1+\rho)(P^*/P)}\right]\frac{1}{\nu}. \qquad (1.43)$$

Step 4: Computation of responses to idiosyncratic shocks. Substract the system (1.38)-(1.41) from (1.34)-(1.37) to give:

$$\nu^2\left(\tau^\delta_{i\infty} + \frac{\varepsilon_i}{\nu}\right) + \alpha_g g^\delta_{i\infty} = 0, \qquad (1.44)$$

$$-\alpha_g g_{i1}^{\delta} = \beta^{*}(1+\rho)\left[\frac{1}{1/\nu^2+1/\alpha_g}\right] d_{i1}^{\delta}; \tag{1.45}$$

$$0 = \tau_{i1}^{\delta} - g_{i1}^{\delta} + d_{i1}^{\delta}; \tag{1.46}$$

Using (1.44) and (1.46) we can solve $\tau_{i1}^{\delta} + \frac{\varepsilon_i}{\nu}$ and g_{i1}^{δ} for given d_{i1}^{δ}:

$$\tau_{i1}^{\delta} + \frac{\varepsilon_i}{\nu} = \left[\frac{1/\nu^2}{1/\nu^2+1/\alpha_g}\right]\left(\frac{\varepsilon_i}{\nu} - d_{i1}^{\delta}\right);$$

$$g_{i1}^{\delta} = -\left[\frac{1/\alpha_g}{1/\nu^2+1/\alpha_g}\right]\left(\frac{\varepsilon_i}{\nu} - d_{i1}^{\delta}\right):$$

Combine this with (1.45) to yield:

$$d_{i1}^{\delta,S} = \left[\frac{1}{1+\beta^{*}(1+\rho)}\right]\frac{\varepsilon_i}{\nu}: \tag{1.47}$$

2. Derivation of the decentralized equilibrium

We now solve the model for the case of a centralized, discretionary monetary policy and a decentralized fiscal policy. We allow for the possibility of a constant (possibly zero) inflation target or a state-contingent inflation target which is a linear function of the individual countries' debt choices. The case of pure discretion is obtained when the inflation target is restricted to zero in both periods. The model is solved through backwards induction.

2.1. Period 2

We compute the second-period policy outcomes, conditional on first-period debt choices. Substitute (2.2) and

$$g_{i2} = -(1+\rho)\,d_{i1} + \tau_{i2} + \chi\pi_2;$$

into (2.3). Hence, in period 2 the CCB minimizes over π_2:

$$\frac{1}{2}\left\{\begin{array}{c}\alpha_\pi(\pi_2 - \pi_2^{*})^2 + \\ \frac{1}{n}\sum_{i=1}^{n}\left[(\nu(\pi_2 - \pi_2^{e} - \tau_{i2}) - \tilde{x}_{i2})^2 + \alpha_g(-(1+\rho)\,d_{i1} + \tau_{i2} + \chi\pi_2 - \tilde{g}_{i2})^2\right]\end{array}\right\};$$

where π_2^{*} is a constant (possibly zero) or a linear function of the individual countries' debt choices. The CCB's first-order condition is:

$$\alpha_\pi(\pi_2 - \pi_2^*) + \frac{1}{n}\sum_{i=1}^{n}[\nu(\nu(\pi_2 - \pi_2^e - \tau_{i2}) - \tilde{x}_{i2}) + \chi\alpha_g(g_{i2} - \tilde{g}_{i2})] = 0. \quad (2.1)$$

The fiscal authority of country i minimizes over τ_{i2}:

$$\frac{1}{2}\left\{\alpha_\pi\pi_2^2 + [\nu(\pi_2 - \pi_2^e - \tau_{i2}) - \tilde{x}_{i2}]^2 + \alpha_g[-(1+\rho)d_{i1} + \tau_{i2} + \chi\pi_{i2} - \tilde{g}_{i2}]^2\right\}.$$

The first-order condition is:

$$-\nu[\nu(\pi_2 - \pi_2^e - \tau_{i2}) - \tilde{x}_{i2}] + \alpha_g(g_{i2} - \tilde{g}_{i2}) = 0, \forall i. \quad (2.2)$$

Furthermore, we can write the second-period government budget constraint as:

$$\tilde{K}_{i2} + (1+\rho)d_{i1} = (\tau_{i2} + \tilde{x}_{i2}/\nu) + \chi\pi_2 + (\tilde{g}_{i2} - g_{i2}). \quad (2.3)$$

Take (2.1), (2.2) and (2.3) together and impose that $\pi_2^e = \pi_2$. The system to be solved is then given by:

$$\alpha_\pi(\pi_2 - \pi_2^*) = \nu^2(\bar{\tau}_2 + \tilde{x}_2/\nu) + \chi\alpha_g(\tilde{g}_2 - \bar{g}_2); \quad (2.4)$$

$$\nu^2(\tau_{i2} + \tilde{x}_{i2}/\nu) = \alpha_g(\tilde{g}_{i2} - g_{i2}); \quad (2.5)$$

$$\tilde{K}_{i2} + (1+\rho)d_{i1} = (\tau_{i2} + \tilde{x}_{i2}/\nu) + \chi\pi_2 + (\tilde{g}_{i2} - g_{i2}). \quad (2.6)$$

Take the cross-country average of (2.5) and combine the resulting equation with (2.4) to eliminate $(\tilde{g}_2 - \bar{g}_2)$ from (2.4). We have, after rewriting the result:

$$\pi_2 = \pi_2^* + \left[\frac{(\chi+1)\nu^2}{\alpha_\pi}\right](\bar{\tau}_2 + \tilde{x}_2/\nu). \quad (2.7)$$

Take the cross-country average of (2.6) and combine this with (2.7) to eliminate π_2 from (2.6). Combine the result with the cross-country average of (2.5) to eliminate $\tilde{g}_2 - \bar{g}_2$. We end up with:

$$\bar{\tau}_2 + \tilde{x}_2/\nu = \left[\frac{1/\nu^2}{S}\right]\left[\tilde{K}_2 + (1+\rho)\bar{d}_1 - \chi\pi_2^*\right], \quad (2.8)$$

where

$$S = \chi(\chi+1)/\alpha_\pi + 1/\nu^2 + 1/\alpha_g S, \quad (2.9)$$

as in the main text. Combine (2.8) with (2.7) to give:

$$\pi_2 = \pi_2^* + \left[\frac{(\chi+1)/\alpha_\pi}{S}\right]\left[\tilde{K}_2 + (1+\rho)\bar{d}_1 - \chi\pi_2^*\right]$$
$$= \left[\frac{1/\nu^2 + 1/\alpha_g}{S}\right]\pi_2^* + \left[\frac{(\chi+1)/\alpha_\pi}{S}\right]\left[\tilde{K}_2 + (1+\rho)\bar{d}_1\right]. \qquad (2.10)$$

Furthermore, we can combine (2.5) and (2.6) to yield

$$\tilde{K}_{i2} + (1+\rho)\, d_{i1} = \left(1 + \frac{\nu^2}{\alpha_g}\right)(\tau_{i2} + \tilde{x}_{i2}/\nu) + \chi\pi_2.$$

Combine this with (2.10) to eliminate π_2. Rewrite the result to yield:

$$\tau_{i2} + \tilde{x}_{i2}/\nu$$
$$= \left[\frac{1/\nu^2}{1/\nu^2 + 1/\alpha_g}\right]\left\{\left[\tilde{K}_{i2} + (1+\rho)\, d_{i1}\right] - \left[\frac{\chi(\chi+1)/\alpha_\pi}{S}\right]\left[\tilde{K}_2 + (1+\rho)\bar{d}_1\right] - \frac{\chi(1/\nu^2 + 1/\alpha_g)}{S}\pi_2^*\right\}$$
$$= \left[\frac{1/\nu^2}{1/\nu^2 + 1/\alpha_g}\right]\left[\tilde{K}_{i2}^{\cdot} + (1+\rho)\, d_{i1}^{\cdot}\right] + \left[\frac{1/\nu^2}{S}\right]\left[\tilde{K}_2 + (1+\rho)\bar{d}_1 - \chi\pi_2^*\right]$$
$$= \left(\tau_{i2}^{\cdot} + \tilde{x}_{i2}^{\cdot}/\nu\right) + \left[\frac{1/\nu^2}{S}\right]\left[\tilde{K}_2 + (1+\rho)\bar{d}_1 - \chi\pi_2^*\right]. \qquad (2.11)$$

Combine this with (2.5) to find that:

$$\tilde{g}_{i2} - g_{i2}$$
$$= \left[\frac{1/\alpha_g}{1/\nu^2 + 1/\alpha_g}\right]\left\{\left[\tilde{K}_{i2} + (1+\rho)\, d_{i1}\right] - \left[\frac{\chi(\chi+1)/\alpha_\pi}{S}\right]\left[\tilde{K}_2 + (1+\rho)\bar{d}_1\right] - \frac{\chi(1/\nu^2 + 1/\alpha_g)}{S}\pi_2^*\right\}$$
$$= \left[\frac{1/\alpha_g}{1/\nu^2 + 1/\alpha_g}\right]\left[\tilde{K}_{i2}^{\cdot} + (1+\rho)\, d_{i1}^{\cdot}\right] + \left[\frac{1/\alpha_g}{S}\right]\left[\tilde{K}_2 + (1+\rho)\bar{d}_1 - \chi\pi_2^*\right]$$
$$= \left(\tilde{g}_{i2}^{\cdot} - g_{i2}^{\cdot}\right) + \left[\frac{1/\alpha_g}{S}\right]\left[\tilde{K}_2 + (1+\rho)\bar{d}_1 - \chi\pi_2^*\right]. \qquad (2.12)$$

Finally,

$$\tilde{x}_{i2} - x_{i2}$$
$$= \left[\frac{1/\nu}{1/\nu^2 + 1/\alpha_g}\right]\left\{\left[\tilde{K}_{i2} + (1+\rho)\, d_{i1}\right] - \left[\frac{\chi(\chi+1)/\alpha_\pi}{S}\right]\left[\tilde{K}_2 + (1+\rho)\bar{d}_1\right] - \frac{\chi(1/\nu^2 + 1/\alpha_g)}{S}\pi_2^*\right\}$$
$$= \left[\frac{1/\nu}{1/\nu^2 + 1/\alpha_g}\right]\left[\tilde{K}_{i2}^{\cdot} + (1+\rho)\, d_{i1}^{\cdot}\right] + \left[\frac{1/\nu}{S}\right]\left[\tilde{K}_2 + (1+\rho)\bar{d}_1 - \chi\pi_2^*\right]$$
$$= \left(\tilde{x}_{i2}^{\cdot} - x_{i2}^{\cdot}\right) + \left[\frac{1/\nu}{S}\right]\left[\tilde{K}_2 + (1+\rho)\bar{d}_1 - \chi\pi_2^*\right]. \qquad (2.13)$$

Using (2.10), (2.12) and (2.13), the CCB's and government i's second-period losses are, respectively:

$$
\begin{aligned}
L_2^{CCB} = {} & \tfrac{1}{2}\left[\frac{(\chi+1)^2/\alpha_\pi}{S^2}\right]\left[\bar{K}_2 + (1+\rho)\,\bar{d}_1 - \chi\pi_2^*\right]^2 + \\
& \frac{1}{2n}\left[\frac{1}{1/\nu^2+1/\alpha_g}\right]\sum_{j=1}^{n}\left\{\left[\bar{K}_{j2} + (1+\rho)\,d_{j1}\right] - \left[\frac{\chi(\chi+1)/\alpha_\pi}{S}\right]\left[\bar{K}_2 + (1+\rho)\,\bar{d}_1\right] - \frac{\chi\left(1/\nu^2+1/\alpha_g\right)}{S}\pi_2^*\right\}^2;
\end{aligned}
$$

$$
\begin{aligned}
L_{i2}^G = {} & \tfrac{1}{2}\alpha_\pi\left\{\left[\frac{1/\nu^2+1/\alpha_g}{S}\right]\pi_2^* + \left[\frac{(\chi+1)/\alpha_\pi}{S}\right]\left[\bar{K}_2 + (1+\rho)\,\bar{d}_1\right]\right\}^2 + \\
& \tfrac{1}{2}\left[\frac{1}{1/\nu^2+1/\alpha_g}\right]\left\{\left[\bar{K}_{i2} + (1+\rho)\,d_{i1}\right] - \left[\frac{\chi(\chi+1)/\alpha_\pi}{S}\right]\left[\bar{K}_2 + (1+\rho)\,\bar{d}_1\right] - \frac{\chi\left(1/\nu^2+1/\alpha_g\right)}{S}\pi_2^*\right\}^2.
\end{aligned}
\tag{2.14}
$$

2.2. Period 1

The CCB minimizes over π_1:

$$
\tfrac{1}{2}\left(\alpha_\pi(\pi_1 - \pi_1^*)^2 + \tfrac{1}{n}\sum_{i=1}\left[\begin{array}{c}(\nu(\pi_1 - \pi_1^e - \tau_{i1}) - \mu - \varepsilon_i - \bar{x}_{i1})^2 + \\ \alpha_g(-(1+\rho)\,d_0 + \tau_{i1} + \chi\pi_1 + d_{i1} - \bar{g}_{i1})^2\end{array}\right] + \beta L_2^{CCB}\right).
$$

Hence, the first-order condition is:

$$
\alpha_\pi(\pi_1 - \pi_1^*) + \tfrac{1}{n}\sum_{i=1}\left[\nu(\nu(\pi_1 - \pi_1^e - \tau_{i1}) - \mu - \varepsilon_i - \bar{x}_{i1}) + \chi\alpha_g(g_{i1} - \bar{g}_{i1})\right] = 0. \tag{2.15}
$$

The government of country i minimizes over τ_{i1} and d_{i1}:

$$
\tfrac{1}{2}\left(\begin{array}{c}\alpha_\pi\pi_1^2 + \left[\nu(\pi_1 - \pi_1^e - \tau_{i1}) - \mu - \varepsilon_i - \bar{x}_{i1}\right]^2 + \\ \alpha_g\left[-(1+\rho)\,d_0 + \tau_{i1} + \chi\pi_1 + d_{i1} - \bar{g}_{i1}\right]^2 + \beta L_{i2}^G\end{array}\right).
$$

The first-order conditions are:

$$
-\nu\left[\nu(\pi_1 - \pi_1^e - \tau_{i1}) - \mu - \varepsilon_i - \bar{x}_{i1}\right] + \alpha_g(g_{i1} - \bar{g}_{i1}) = 0; \tag{2.16}
$$

$$
\alpha_g(\bar{g}_{i1} - g_{i1}) = \beta\left(\partial L_{i2}^G/\partial d_{i1}\right); \tag{2.17}
$$

where

$$
\begin{aligned}
\partial L^G_{i2}/\partial d_{i1} = {} & \alpha_\pi \left\{ \left[\frac{1/\nu^2 + 1/\alpha_g}{S} \right] \pi^*_2 + \left[\frac{(\chi+1)/\alpha_\pi}{S} \right] \left[\bar{K}_2 + (1+\rho)\,\bar{d}_1 \right] \right\} * \\
& \left\{ \frac{1}{n}(1+\rho) \left[\frac{(\chi+1)/\alpha_\pi}{S} \right] + \left[\frac{1/\nu^2 + 1/\alpha_g}{S} \right] \frac{\partial \pi^*_2}{\partial d_{i1}} \right\} + \\
& \left[\frac{1}{1/\nu^2 + 1/\alpha_g} \right] * \\
& \left\{ \left[\bar{K}_{i2} + (1+\rho)\, d_{i1} \right] - \left[\frac{\chi(\chi+1)/\alpha_\pi}{S} \right] \left[\bar{K}_2 + (1+\rho)\,\bar{d}_1 \right] - \frac{\chi\left(1/\nu^2 + 1/\alpha_g\right)}{S} \pi^*_2 \right\} * \\
& \left\{ (1+\rho) \left[1 - \frac{1}{n} \frac{\chi(\chi+1)/\alpha_\pi}{S} \right] - \frac{\chi\left(1/\nu^2 + 1/\alpha_g\right)}{S} \frac{\partial \pi^*_2}{\partial d_{i1}} \right\} .
\end{aligned}
$$

Finally, we can rewrite the first-period government budget constraint as:

$$
\bar{K}_{i1} + (1+\rho)\, d_{i0} = (\tau_{i1} + \bar{x}_{i1}/\nu) + \chi\pi_1 + (\bar{g}_{i1} - g_{i1}) + d_{i1}. \tag{2.18}
$$

The system of first-order conditions to be solved is thus given by (2.15), (2.16), (2.17) and (2.18). Take some country-specific policy variable y_{i1} ($y_{i1} = \pi_{i1}, \tau_{i1}, g_{i1}$ or d_{i1}). We will solve for y_{i1} by solving for each of the components of the decomposition of y_{i1} into $y_{i1} = \bar{y}^e_1 + y^{\Delta,e}_{i1} + \bar{y}^d_1 + y^\delta_{i1}$, where $\bar{y}^e_1$ will be the response to the cross-country average of the deterministic components of the government financing requirements, $y^{\Delta,e}_{i1}$ will be the response to the country-specific deterministic components of the government financing requirements, $\bar{y}^d_1$ will be response to the common shock and y^δ_{i1} will be the response to the idiosyncratic shock. Note that $\pi^{\Delta,e}_{i1} = \pi^\delta_{i1} = 0$. We thus compute the solutions of the variables in four steps.

2.2.1. Steps 1 and 2

Take expectations across (2.15), (2.16), (2.17) and (2.18) to yield:

$$
\alpha_\pi (\pi^e_1 - \pi^*_1) = \nu^2 (\bar{\tau}^e_1 + \bar{x}_1/\nu) + \chi\alpha_g (\bar{g}_1 - \bar{g}^e_1); \tag{2.19}
$$

$$
\nu^2 (\tau^e_{i1} + \bar{x}_{i1}/\nu) = \alpha_g (\bar{g}_{i1} - g^e_{i1}); \tag{2.20}
$$

$$
\begin{aligned}
\alpha_g (\bar{g}_{i1} - g^e_{i1}) \\
= {} & -\alpha_\pi \left\{ \left[\frac{1/\nu^2 + 1/\alpha_g}{S} \right] \pi^{*,e}_2 + \left[\frac{(\chi+1)/\alpha_\pi}{S} \right] \left[\bar{K}_2 + (1+\rho)\,\bar{d}^e_1 \right] \right\} * \\
& \left\{ \frac{1}{n}(1+\rho) \left[\frac{(\chi+1)/\alpha_\pi}{S} \right] + \left[\frac{1/\nu^2 + 1/\alpha_g}{S} \right] \frac{\partial \pi^*_2}{\partial d_{i1}} \right\} + \\
& - \left[\frac{1}{1/\nu^2 + 1/\alpha_g} \right] * \\
& \left\{ \left[\bar{K}_{i2} + (1+\rho)\, d^e_{i1} \right] - \left[\frac{\chi(\chi+1)/\alpha_\pi}{S} \right] \left[\bar{K}_2 + (1+\rho)\,\bar{d}^e_1 \right] - \frac{\chi\left(1/\nu^2 + 1/\alpha_g\right)}{S} \pi^{*,e}_2 \right\} *
\end{aligned}
$$

$$\left\{(1+\rho)\left[1-\frac{1}{n}\frac{\chi(\chi+1)/\alpha_\pi}{S}\right]-\frac{\chi\left(1/\nu^2+1/\alpha_g\right)}{S}\frac{\partial\pi_2^*}{\partial d_{i1}}\right\}; \tag{2.21}$$

$$\bar K_{i1}+(1+\rho)\,d_{i0}=\left(\tau^e_{i1}+\bar x_{i1}/\nu\right)+\chi\pi_1^e+\left(\bar g_{i1}-g^e_{i1}\right)+d^e_{i1}. \tag{2.22}$$

Here, $\pi_2^{*,e}$ is the expectation about the second-period inflation target, formed before first-period shocks have occurred. Furthermore, we have used that $\frac{\partial\pi_2^*}{\partial d_{i1}}$ is constant (possibly zero), because π_2^* is a constant or a linear function of the individual countries' debt choices.

Step 1: Computation of expectations of cross-country averages. Take cross-country averages across (2.19)-(2.22) to yield the following system (again making use of the constancy of $\frac{\partial\pi_2^*}{\partial d_{i1}}$):

$$\alpha_\pi\left(\pi_1^e-\pi_1^*\right)=\nu^2\left(\bar\tau_1^e+\bar x_1/\nu\right)+\chi\alpha_g\left(\bar g_1-\bar g_1^e\right); \tag{2.23}$$

$$\nu^2\left(\bar\tau_1^e+\bar x_1/\nu\right)=\alpha_g\left(\bar g_1-\bar g_1^e\right); \tag{2.24}$$

$$\begin{aligned}\alpha_g\left(\bar g_1-\bar g_1^e\right)
&=\frac{-\alpha_\pi\left\{\left[\frac{1/\nu^2+1/\alpha_g}{S}\right]\pi_2^{*,e}+\left[\frac{(\chi+1)/\alpha_\pi}{S}\right]\left[\bar K_2+(1+\rho)\,\bar d_1^e\right]\right\}}{\left\{\frac{1}{n}(1+\rho)\left[\frac{(\chi+1)/\alpha_\pi}{S}\right]+\left[\frac{1/\nu^2+1/\alpha_g}{S}\right]\frac{\partial\pi_2^*}{\partial d_{i1}}\right\}}+\\
&\quad-\left[\frac{1}{1/\nu^2+1/\alpha_g}\right]\times\frac{\left\{\left[\bar K_2+(1+\rho)\,\bar d_1^e\right]-\left[\frac{\chi(\chi+1)/\alpha_\pi}{S}\right]\left[\bar K_2+(1+\rho)\,\bar d_1^e\right]-\frac{\chi\left(1/\nu^2+1/\alpha_g\right)}{S}\pi_2^{*,e}\right\}}{\left\{(1+\rho)\left[1-\frac{1}{n}\frac{\chi(\chi+1)/\alpha_\pi}{S}\right]-\frac{\chi\left(1/\nu^2+1/\alpha_g\right)}{S}\frac{\partial\pi_2^*}{\partial d_{i1}}\right\}};\end{aligned} \tag{2.25}$$

$$\bar K_1+(1+\rho)\,\bar d_0=\left(\bar\tau_1^e+\bar x_1/\nu\right)+\chi\pi_1^e+\left(\bar g_1-\bar g_1^e\right)+\bar d_1^e. \tag{2.26}$$

Combine (2.23) and (2.24) to eliminate $\alpha_g\left(\bar g_1-\bar g_1^e\right)$ and obtain:

$$\pi_1^e-\pi_1^*=\left[\frac{(\chi+1)\nu^2}{\alpha_\pi}\right]\left(\bar\tau_1^e+\bar x_1/\nu\right).$$

Combine this equation and (2.24) with (2.26) to eliminate both $\bar g_1-\bar g_1^e$ and π_1^e, to obtain after rewriting:

$$\left(\bar\tau_1^e+\bar x_1/\nu\right)=\left[\frac{1/\nu^2}{S}\right]\left[\bar K_1+(1+\rho)\,\bar d_0-\bar d_1^e-\chi\pi_1^*\right]. \tag{2.27}$$

Hence,

$$\pi^e_1 = \pi^*_1 + \left[\frac{(\chi+1)/\alpha_\pi}{S}\right]\left[\tilde{k}_1 + (1+\rho)\,\bar{d}_0 - \bar{d}^e_1 - \chi\pi^*_1\right]; \tag{2.28}$$

$$\bar{g}_1 - \bar{g}^e_1 = \left[\frac{1/\alpha_g}{S}\right]\left[\tilde{k}_1 + (1+\rho)\,\bar{d}_0 - \bar{d}^e_1 - \chi\pi^*_1\right]. \tag{2.29}$$

Hence, combining this last equation with (2.25) to eliminate $\bar{g}_1 - \bar{g}^e_1$ and rewriting yields:

$$\begin{aligned}
&\tilde{k}_1 + (1+\rho)\,\bar{d}_0 - \bar{d}^e_1 - \chi\pi^*_1 \\
= &-\beta^* \left\{ \left[\frac{\frac{1}{n}\left(\frac{1}{\sigma^2}+\frac{1}{\alpha_g}\right) - \left(\frac{n-1}{n}\right)\chi S}{S}\right]\pi^{*,e}_2 + \frac{S^*}{S}\left[\tilde{k}_2 + (1+\rho)\,\bar{d}^e_1\right]\right\} + \\
&- \beta\frac{\alpha_\pi\left(1/\sigma^2+1/\alpha_g\right)}{S}\left\{\left[\frac{1}{\sigma^2}+\frac{1}{\alpha_g}\right]\pi^{*,e}_2 + \left[\frac{\chi+1}{\alpha_\pi}\right]\left[\tilde{k}_2 + (1+\rho)\,\bar{d}^e_1\right]\right\}\frac{\partial\pi^*_2}{\partial d_{i1}} - \\
&- \beta\frac{\chi\left(1/\sigma^2+1/\alpha_g\right)}{S}\left[\tilde{k}_2 + (1+\rho)\,\bar{d}^e_1 - \chi\pi^{*,e}_2\right]\frac{\partial\pi^*_2}{\partial d_{i1}};
\end{aligned} \tag{2.30}$$

where

$$S^* = \chi(\chi+1)/\alpha_\pi + (\chi+1)/(n\alpha_\pi) + 1/\sigma^2 + 1/\alpha_g; \tag{2.31}$$

as defined in the main text. Hence,

$$\begin{aligned}
\bar{d}^e_1 = &\frac{\left[\tilde{k}_1+(1+\rho)\bar{d}_0-\tilde{k}_2\right]+\left[1-\beta^*(S^*/S)\right]\tilde{k}_2}{1+\beta^*(1+\rho)(S^*/S)} - \\
&\left[\frac{1}{1+\beta^*(1+\rho)(S^*/S)}\right]\chi\pi^*_1 + \left[\frac{\beta^*}{1+\beta^*(1+\rho)(S^*/S)}\right]\chi\pi^{*,e}_2 - \\
&\frac{1}{n}\beta^*\left[\frac{(\chi+1)(P/S)}{1+\beta^*(1+\rho)(S^*/S)}\right]\pi^{*,e}_2 - \\
&\left[\frac{\beta}{1+\beta^*(1+\rho)(S^*/S)}\right]\frac{\alpha_\pi\left(1/\sigma^2+1/\alpha_g\right)}{S} \ast \\
&\left\{\left[\frac{1}{\sigma^2}+\frac{1}{\alpha_g}\right]\pi^{*,e}_2 + \left[\frac{\chi+1}{\alpha_\pi}\right]\left[\tilde{k}_2+(1+\rho)\,\bar{d}^e_1\right]\right\}\frac{\partial\pi^*_2}{\partial d_{i1}} + \\
&\left[\frac{\beta}{1+\beta^*(1+\rho)(S^*/S)}\right]\frac{\chi\left(1/\sigma^2+1/\alpha_g\right)}{S}\left[\tilde{k}_2+(1+\rho)\,\bar{d}^e_1-\chi\pi^{*,e}_2\right]\frac{\partial\pi^*_2}{\partial d_{i1}}.
\end{aligned} \tag{2.32}$$

One obtains $\bar{d}^{e,D}_1$ by imposing a zero inflation target in both periods. Hence, the terms involving π^*_1, $\pi^{*,e}_2$ and $\frac{\partial\pi^*_2}{\partial d_{i1}}$ in (2.32) all drop out.

Step 2: Computation of country-specific expected deviations from cross-country expected averages. Subtract (2.24)-(2.26) from (2.20)-(2.22). This gives the following system:

$$\sigma^2\left(\tau^{\cdot,e}_{i1} + \varepsilon^{\cdot}_{i1}/\sigma\right) = \alpha_g\left(\bar{g}^{\cdot}_{i1} - g^{\cdot,e}_{i1}\right); \tag{2.33}$$

$$\alpha_g\left(\bar{g}^{\cdot}_{i1} - g^{\cdot;e}_{i1}\right) = -\left[\frac{1}{1/\nu^2+1/\alpha_g}\right]\left[\bar{K}^{\cdot}_{i2} + (1+\rho)\, d^{\cdot;e}_{i1}\right] \times$$
$$\left\{(1+\rho)\left[1 - \frac{1}{n}\frac{\chi(\chi+1)/\alpha_\pi}{S}\right] - \frac{\chi\left(1/\nu^2+1/\alpha_g\right)}{S}\frac{\partial\pi^*_2}{\partial d_{i1}}\right\}; \quad (2.34)$$

$$\bar{K}^{\cdot}_{i1} + (1+\rho)\, d^{\cdot}_{i0} = \left(\tau^{\cdot;e}_{i1} + \bar{x}^{\cdot}_{i1}/\nu\right) + \left(\bar{g}^{\cdot}_{i1} - g^{\cdot;e}_{i1}\right) + d^{\cdot;e}_{i1}. \quad (2.35)$$

Equations (2.33) and (2.35) can be combined to yield outcomes conditional on $d^{\cdot;e}_{i1}$;

$$\tau^{\cdot;e}_{i1} + \bar{x}^{\cdot}_{i1}/\nu = \left[\frac{1/\nu^2}{1/\nu^2+1/\alpha_g}\right]\left[\bar{K}^{\cdot}_{i1} + (1+\rho)\, d^{\cdot}_{i0} - d^{\cdot;e}_{i1}\right]; \quad (2.36)$$
$$\bar{g}^{\cdot}_{i1} - g^{\cdot;e}_{i1} = \left[\frac{1/\alpha_g}{1/\nu^2+1/\alpha_g}\right]\left[\bar{K}^{\cdot}_{i1} + (1+\rho)\, d^{\cdot}_{i0} - d^{\cdot;e}_{i1}\right]. \quad (2.37)$$

Combining the latter equation with (2.34) gives:

$$\bar{K}^{\cdot}_{i1} + (1+\rho)\, d^{\cdot}_{i0} - d^{\cdot;e}_{i1} = -\left[\bar{K}^{\cdot}_{i2} + (1+\rho)\, d^{\cdot;e}_{i1}\right] \times$$
$$\left\{(1+\rho)\left[1 - \frac{1}{n}\frac{\chi(\chi+1)/\alpha_\pi}{S}\right] - \frac{\chi\left(1/\nu^2+1/\alpha_g\right)}{S}\frac{\partial\pi^*_2}{\partial d_{i1}}\right\}. \quad (2.38)$$

Setting $\frac{\partial\pi^*_2}{\partial d_{i1}} = 0$, one can easily solve for $d^{\cdot;e;D}_{i1}$ given in the main text.

2.2.2. Steps 3 and 4

Subtract equations (2.19)-(2.22) from equations (2.15)-(2.18), respectively, to give the following system in terms of deviations of realizations of variables from their expectations:

$$\alpha_\pi \pi^d_1 + \nu^2\left(\pi^d_1 - \bar{\tau}^d_1 - \frac{\bar{x}}{\nu}\right) + \chi\alpha_g \bar{g}^d_1 = 0; \quad (2.39)$$

$$-\nu^2\left(\pi^d_1 - \tau^d_{i1} - \frac{1+\epsilon_i}{\nu}\right) + \alpha_g g^d_{i1} = 0; \quad (2.40)$$

$$-\alpha_g g^d_{i1} = -\alpha_\pi\left\{\left[\frac{1/\nu^2+1/\alpha_g}{S}\right]\pi^{*;d}_2 + \left[\frac{(\chi+1)/\alpha_\pi}{S}\right](1+\rho)\,\bar{d}^d_1\right\} \times$$
$$\left\{\frac{1}{n}(1+\rho)\left[\frac{(\chi+1)/\alpha_\pi}{S}\right] + \left[\frac{1/\nu^2+1/\alpha_g}{S}\right]\frac{\partial\pi^*_2}{\partial d_{i1}}\right\} +$$
$$-\left[\frac{1}{1/\nu^2+1/\alpha_g}\right] \times$$
$$\left\{(1+\rho)\, d^d_{i1} - \left[\frac{\chi(\chi+1)/\alpha_\pi}{S}\right](1+\rho)\,\bar{d}^d_1 - \frac{\chi\left(1/\nu^2+1/\alpha_g\right)}{S}\pi^{*;d}_2\right\} \times$$
$$\left\{(1+\rho)\left[1 - \frac{1}{n}\frac{\chi(\chi+1)/\alpha_\pi}{S}\right] - \frac{\chi\left(1/\nu^2+1/\alpha_g\right)}{S}\frac{\partial\pi^*_2}{\partial d_{i1}}\right\}; \quad (2.41)$$

$$0 = \tau^d_{i1} + \chi\pi^d_1 - g^d_{i1} + d^d_{i1}; \tag{2.42}$$

where $\pi^{*,d}_2 = \pi^*_2 - \pi^{*,e}_2$.

Step 3: Responses to common shocks. In step 3 we solve for π^d_1, $\bar{\tau}^d_1 = \bar{\tau}_1 - \bar{\tau}^e_1$, $\bar{g}^d_1 = \bar{g}_1 - \bar{g}^e_1$ and $\bar{d}^d_1 = \bar{d}_1 - \bar{d}^e_1$, which are the policy responses to the common shock μ. We take the cross-country averages of the system we just obtained and use the assumption that the cross-country average of the ε_i's equals zero. This yields:

$$\left(\alpha_\pi + \gamma^2\right)\pi^d_1 = \gamma^2\left(\bar{\tau}^d_1 + \mu/\gamma\right) - \chi\alpha_g\bar{g}^d_1; \tag{2.43}$$

$$-\gamma^2\left(\pi^d_1 - \bar{\tau}^d_1\right) + \gamma\mu + \alpha_g\bar{g}^d_1 = 0; \tag{2.44}$$

$$\begin{aligned}
-\alpha_g\bar{g}^d_1 = \; & -\beta\alpha_\pi\left\{\left[\frac{1/\gamma^2+1/\alpha_g}{S}\right]\pi^{*,d}_2 + \left[\frac{(\chi+1)/\alpha_\pi}{S}\right](1+\rho)\,\bar{d}^d_1\right\} \ast \\
& \left\{\frac{1}{n}(1+\rho)\left[\frac{(\chi+1)/\alpha_\pi}{S}\right] + \left[\frac{1/\gamma^2+1/\alpha_g}{S}\right]\frac{\partial\pi^*_2}{\partial d_{i1}}\right\} + \\
& -\beta\left(\frac{1}{S}\right)\left[(1+\rho)\,\bar{d}^d_1 - \chi\pi^{*,d}_2\right] \ast \\
& \left\{(1+\rho)\left[1 - \frac{1}{n}\frac{\chi(\chi+1)/\alpha_\pi}{S}\right] - \frac{\chi\left(1/\gamma^2+1/\alpha_g\right)}{S}\frac{\partial\pi^*_2}{\partial d_{i1}}\right\};
\end{aligned} \tag{2.45}$$

$$0 = \bar{\tau}^d_1 + \chi\pi^d_1 - \bar{g}^d_1 + \bar{d}^d_1. \tag{2.46}$$

One can solve (2.43), (2.44) and (2.46) to obtain the solutions for the variables for given $\bar{d}^d_1$:

$$\begin{aligned}
\pi^d_1 &= \left[\frac{(\chi+1)/\alpha_\pi}{P^*}\right]\left(\frac{1}{\gamma} - \bar{d}^d_1\right); \bar{\tau}^d_1 + \frac{1}{\gamma} = \left[\frac{(\chi+1)/\alpha_\pi + 1/\gamma^2}{P^*}\right]\left(\frac{1}{\gamma} - \bar{d}^d_1\right); \\
\bar{x}^d_1 &= -\left[\frac{\alpha_\pi}{(\chi+1)\gamma}\right]\pi^d_1; \bar{g}^d_1 = -\left[\frac{\alpha_\pi}{(\chi+1)\alpha_g}\right]\pi^d_1.
\end{aligned} \tag{2.47}$$

Finally, we can solve for $\bar{d}^d_1$ by combining (2.45) and (2.47). We can solve for $\bar{d}^{d,D}_1$ if we set $\pi^*_2 = 0$ (hence, $\pi^{*,d}_2 = \frac{\partial\pi^*_2}{\partial d_{i1}} = 0$).

Step 4: Computation of responses to idiosyncratic shocks. These responses are defined as $\tau^\delta_{i1} \equiv \tau^d_{i1} - \bar{\tau}^d_1$, $g^\delta_{i1} \equiv g^d_{i1} - \bar{g}^d_1$ and $d^\delta_{i1} \equiv d^d_{i1} - \bar{d}^d_1$. The relevant system that needs to be solved is obtained by subtracting (2.43)-(2.46) from (2.39)-(2.42), respectively. This yields (note that the first equation drops out):

$$-\gamma\left[\gamma\left(-\tau^\delta_{i1}\right) - \varepsilon_i\right] + \alpha_g\left(g^\delta_{i1}\right) = 0; \tag{2.48}$$

$$\alpha_g\left(-g^{\delta}_{i1}\right) = -\beta^*\left\{\left[\frac{1}{1/\kappa^2+1/\alpha_g}\right](1+\rho)\left[\frac{Q}{S}\right] - \frac{\chi\left(1/\kappa^2+1/\alpha_g\right)}{S}\frac{\partial \pi_2^*}{\partial d_{i1}}\right\} d^{\delta}_{i1}, \qquad (2.49)$$

$$0 = \tau^{\delta}_{i1} - g^{\delta}_{i1} + d^{\delta}_{i1}. \qquad (2.50)$$

where

$$Q = [(n-1)/n]\,[\chi\,(\chi+1)/\alpha_\pi] + 1/\kappa^2 + 1/\alpha_g, \qquad (2.51)$$

as defined in the main text. Combine (2.48) and (2.50) to eliminate τ^{δ}_{i1} and solve for g^{δ}_{i1} to yield:

$$g^{\delta}_{i1} = \left[\frac{1/\alpha_g}{1/\kappa^2+1/\alpha_g}\right]\left(d^{\delta}_{i1} - \frac{\varepsilon_i}{\kappa}\right). \qquad (2.52)$$

By substituting the right-hand side of (2.52) into (2.49), we can solve for d^{δ}_{i1}. In addition, if we set $\pi_2^* = 0$ (hence, $\frac{\partial \pi_2^*}{\partial d_{i1}} = 0$), we obtain the solution for $d^{\delta,D}_{i1}$.

3. Proof of Proposition 1

Let $\bar{k}_{i1} = \bar{k}_1$, $\bar{k}_{i2} = \bar{k}_2$, $d_{i0} = d_0$ and $\varepsilon_i = 0$, $\forall i$. Hence, $d_{i1} = \bar{d}_1$. We pursue the following strategy in proving Proposition 1. First, we derive the value $\pi_2^*(\bar{d}_1)$ for π_2^*, which can be written as $\pi_2^*(\bar{d}_1) = \pi_2^{*,e}(\bar{d}_1^e) + \pi_2^{*,d}(\bar{d}_1^d)$, that replicates the second-period policy outcomes under the second best, for given $\bar{d}_1$.[17] Then, we derive the value $\pi_1^*(\bar{d}_1^e)$ for π_1^* that replicates the first-period policy outcomes under the second best, for given $\bar{d}_1^e$. We plug $\pi_2^{*,e}(\bar{d}_1^e)$ and $\pi_1^*(\bar{d}_1^e)$ for $\pi_2^{*,e}$ and π_1^*, respectively, into (2.30), which is then solved for $\bar{d}_1^e$. Finally, we plug $\pi_2^{*,d}(\bar{d}_1^d)$ into (2.45) to solve for $\bar{d}_1^d$. The resulting solutions for $\bar{d}_1^e$ and $\bar{d}_1^d$ turn out to coincide with the corresponding outcomes in the second-best equilibrium.

For given $\bar{d}_1$, the optimal inflation target $\pi_2^*(\bar{d}_1)$ is the one that yields the second-best outcome for π_{i2}, i.e. $\pi_{i2} = \left[\frac{\chi/\alpha_\pi}{P}\right]\left[\bar{k}_2 + (1+\rho)\,\bar{d}_1\right]$. Hence, we solve for $\pi_2^*(\bar{d}_1)$ from the following equation:

$$\left[\frac{\chi/\alpha_\pi}{P}\right]\left[\bar{k}_2 + (1+\rho)\,\bar{d}_1\right] = \left[\frac{1/\kappa^2+1/\alpha_g}{S}\right]\pi_2^* + \left[\frac{(\chi+1)/\alpha_\pi}{S}\right]\left[\bar{k}_2 + (1+\rho)\,\bar{d}_1\right],$$

which is obtained by equating the right-hand side of the first equation in (1.6) with the right-hand side in (2.10). The solution is:

$$\pi_2^*(\bar{d}_1) = -\left[\frac{1/\alpha_\pi}{P}\right]\left[\bar{k}_2 + (1+\rho)\,\bar{d}_1\right]. \qquad (3.1)$$

[17] These are the outcomes that would have been obtained from the computations in Appendix B had we constrained debt accumulation in country i to $\bar{d}_1$.

It is easy to check that for this value of the inflation target, the other second-period variables attain their second-best values, given $\tilde{d}_1$. Further, note that,

$$\frac{\partial \pi_2^*}{\partial d_{i1}} = -\frac{1}{n}\left[\frac{1/\alpha_\pi}{P}\right](1+\rho). \tag{3.2}$$

For given $\tilde{d}_1^e$, the optimal inflation target $\pi_1^*(\tilde{d}_1^e)$ is the one that yields $\pi_1^e = \left[\frac{\chi/\alpha_\pi}{P}\right]\left[\bar{K}_1 + (1+\rho)\, d_0 - \tilde{d}_1^e\right]$. Hence, we solve $\pi_1^*(\tilde{d}_1^e)$ from the following equation:

$$\left[\frac{\chi/\alpha_\pi}{P}\right]\left[\bar{K}_1 + (1+\rho)\,\tilde{d}_0 - \tilde{d}_1^e\right] = \left[\frac{1/\nu^2 + 1/\alpha_g}{S}\right]\pi_1^* + \left[\frac{(\chi+1)/\alpha_\pi}{S}\right]\left[\bar{K}_1 + (1+\rho)\,\tilde{d}_0 - \tilde{d}_1^e\right];$$

which is derived upon equating the right-hand sides of (1.23) and (2.28). The solution is:

$$\pi_1^*(\tilde{d}_1^e) = -\left[\frac{1/\alpha_\pi}{P}\right]\left[\bar{K}_1 + (1+\rho)\,\tilde{d}_0 - \tilde{d}_1^e\right]. \tag{3.3}$$

It is easy to check that for this value of the inflation target, the other first-period variables attain their second-best equilibrium values, given $\tilde{d}_1^e$.

Having noticed that for these inflation targets the intratemporal allocation of the government financing requirements is optimal (i.e. according to the second-best equilibrium), for given debt policy, we now need to check that for these inflation targets debt policy coincides with debt policy in the second-best equilibrium. To this end, substitute π_1^*, given by (3.3), $\frac{\partial \pi_2^*}{\partial d_{i1}}$, given by (3.2), and

$$\pi_2^{*;e} = -\left[\frac{1/\alpha_\pi}{P}\right]\left[\bar{K}_2 + (1+\rho)\,\tilde{d}_1^e\right];$$

into (2.30). It is easy to see that the final two terms in (2.30) cancel out against each other. It is then straightforward to solve the resulting equation to yield $\tilde{d}_1^e = \tilde{d}_1^{e;S}$.

As the final step, substitute

$$\pi_2^{*;d} = -\left[\frac{1/\alpha_\pi}{P}\right](1+\rho)\,\tilde{d}_1^d;$$

(obtained by substracting the expectation of (3.1) from (3.1)) into (2.45). Let's work out the various terms after this substitution. For the left-hand side we have:

$$-\alpha_g \tilde{g}_1^d = \frac{1}{P^*}\left(\frac{1}{\nu} - \tilde{d}_1^d\right).$$

For the right-hand side we have:

$$\beta\alpha_\pi\left\{\left[\frac{1/\nu^2+1/\alpha_g}{S}\right]\pi_2^{*;d} + \left[\frac{(\chi+1)/\alpha_\pi}{S}\right](1+\rho)\,\tilde{d}_1^d\right\} = -\beta^*\alpha_\pi\left[\frac{\chi S/\alpha_\pi}{PS}\right]\tilde{d}_1^d;$$

$$\frac{1}{n}(1+\rho)\left[\frac{(\chi+1)/\alpha_\pi}{S}\right] + \left[\frac{1/\nu^2+1/\alpha_g}{S}\right]\frac{\partial \pi_2^*}{\partial d_{i1}} = \frac{1}{n}(1+\rho)\left[\frac{\chi S/\alpha_\pi}{PS}\right];$$

$$\beta^*\left(\frac{1}{S}\right)\left[(1+\rho)\,\bar{d}_1^d - \chi\pi_2^{*,d}\right] = \beta^*\left(\frac{1}{P}\right)\bar{d}_1^d,$$

$$(1+\rho)\left[1-\frac{1}{n}\frac{\chi(\chi+1)/\alpha_\pi}{S}\right] - \frac{\chi\left(1/\nu^2+1/\alpha_g\right)}{S}\frac{\partial\pi_2^*}{\partial d_{i1}} = (1+\rho)\left(\frac{R}{P}\right),$$

where

$$R \equiv [(n-1)/n]\left[\chi^2/\alpha_\pi\right] + 1/\nu^2 + 1/\alpha_g. \tag{3.4}$$

Hence, we obtain the following equation to be solved for $\bar{d}_1^d$:

$$\frac{1}{P^*}\left(\frac{1}{\nu}-\bar{d}_1^d\right) = \beta^*(1+\rho)\frac{1}{n}\left[\frac{\chi^2/\alpha_\pi}{P^2}\right]\bar{d}_1^d + \beta^*\left(\frac{1}{P}\right)\bar{d}_1^d(1+\rho)\left(\frac{R}{P}\right).$$

The right-hand side of this equation can be written as $\beta^*(1+\rho)\,\bar{d}_1^d/P$. Using this, we can solve the equation to give $\bar{d}_1^d = \bar{d}_1^{d,S}$. This completes the proof.

4. Proof of Proposition 2

First, we derive the equilibrium when a debt target d_{i1}^T is imposed on country i, $i = 1,...,n$. This debt target is the exact amount of debt that country i has to carry over into the second period. After having derived the equilibrium, we prove Proposition 2.

4.1. Derivation of the equilibrium with debt targets d_{i1}^T, $i = 1,...,n$.

In deriving the equilibrium, we closely follow the derivations of the decentralized equilibrium in Appendix B.

4.1.1. Period 2

We replace d_{i1} with d_{i1}^T in the derivation of the second-period outcomes in Appendix B and, consistent with the notation we used so far, we use $\bar{d}_1^T$ to denote the cross-country average of the individual countries' debt targets. The second-period policy outcomes can then be written as:

$$\pi_2 = \pi_2^* + \left[\frac{(\chi+1)/\alpha_\pi}{S}\right]\left[\bar{\kappa}_2 + (1+\rho)\,\bar{d}_1^T - \chi\pi_2^*\right], \tag{4.1}$$

$$\tilde{g}_{i2} - g_{i2} = \left[\frac{1/\alpha_g}{1/\nu^2+1/\alpha_g}\right] \times \left\{ \kappa_{i2} + (1+\rho)\,d_{i1}^T - \left[\frac{\chi(\chi+1)/\alpha_\pi}{S}\right]\left[\bar{\kappa}_2 + (1+\rho)\,\bar{d}_1^T\right] - \frac{\chi\left(1/\nu^2+1/\alpha_g\right)}{S}\pi_2^* \right\}, \tag{4.2}$$

$$\bar{x}_{i2} - x_{i2} = \left[\frac{1/\nu}{1/\nu^2 + 1/\alpha_g}\right] \ast \left\{ \left[\bar{K}_{i2} + (1+\rho)\, d^T_{i1}\right] - \left[\frac{\chi(\chi+1)/\alpha_\pi}{S}\right]\left[\bar{K}_2 + (1+\rho)\, d^T_1\right] - \frac{\chi\left(1/\nu^2 + 1/\alpha_g S\right)}{S}\, \pi^*_2 \right\}; \qquad (4.3)$$

analogous to (2.10), (2.12) and (2.13), respectively.

4.1.2. Period 1

Because first-period debt is no longer a choice variable, the relevant first-order conditions for government i are:

$$\alpha_\pi (\pi_1 - \pi^*_1) + \frac{1}{n} \sum_{i=1}^{n} \left[\nu\left(\nu\left(\pi_1 - \pi^e_1 - \tau_{i1}\right) - \mu - \varepsilon_i - \bar{x}_{i1}\right) + \chi\alpha_g\left(g_{i1} - \bar{g}_{i1}\right)\right] = 0; \qquad (4.4)$$

$$-\nu\left[\nu\left(\pi_1 - \pi^e_1 - \tau_{i1}\right) - \mu - \varepsilon_i - \bar{x}_{i1}\right] + \alpha_g\left(g_{i1} - \bar{g}_{i1}\right) = 0; \qquad (4.5)$$

$$\bar{K}_{i1} + (1+\rho)\, d_{i0} = \left(\tau_{i1} + \bar{x}_{i1}/\nu\right) + \chi\pi_1 + \left(\bar{g}_{i1} - g_{i1}\right) + d^T_{i1}; \qquad (4.6)$$

analogous to (2.15), (2.16) and (2.18), respectively.

We solve now for the first-period outcomes. As before, we solve for these in four steps. Take expectations across the first-order conditions (4.4), (4.5) and (4.6), to yield:

$$\alpha_\pi (\pi^e_1 - \pi^*_1) = \nu^2 \left(\bar{\tau}^e_1 + \bar{x}_1/\nu\right) + \chi\alpha_g \left(\bar{g}_1 - \bar{g}^e_1\right); \qquad (4.7)$$

$$\nu^2 \left(\tau^e_{i1} + \bar{x}_{i1}/\nu\right) = \alpha_g \left(\bar{g}_{i1} - g^e_{i1}\right); \qquad (4.8)$$

$$\bar{K}_{i1} + (1+\rho)\, d_{i0} = \left(\tau^e_{i1} + \bar{x}_{i1}/\nu\right) + \chi\pi^e_1 + \left(\bar{g}_{i1} - g^e_{i1}\right) + d^{T,e}_{i1}. \qquad (4.9)$$

Step 1: Solution in terms of cross-country averages. Take cross-country averages across the previous three equations. This yields:

$$\alpha_\pi (\pi^e_1 - \pi^*_1) = \nu^2 \left(\bar{\tau}^e_1 + \bar{x}_1/\nu\right) + \chi\alpha_g \left(\bar{g}_1 - \bar{g}^e_1\right); \qquad (4.10)$$

$$\nu^2 \left(\bar{\tau}^e_1 + \bar{x}_1/\nu\right) = \alpha_g \left(\bar{g}_1 - \bar{g}^e_1\right); \qquad (4.11)$$

$$\bar K_1 + (1+\rho)\,\bar d_0 = (\bar\tau_1^e + \bar x_1/\sigma) + \chi\bar\pi_1^e + (\bar g_1 - \bar g_1^e) + \bar d_1^{T,e}. \qquad (4.12)$$

Following the steps in Appendix B, we solve for the outcomes as a function of $\bar d_1^{T,e}$:

$$\bar x_1 - \bar x_1^e = \left[\frac{1/\sigma}{S}\right]\left[\bar K_1 + (1+\rho)\,\bar d_0 - \bar d_1^{T,e} - \chi\bar\pi_1^*\right]. \qquad (4.13)$$

$$\bar\pi_1^e = \bar\pi_1^* + \left[\frac{(\chi+1)/\alpha_\pi}{S}\right]\left[\bar K_1 + (1+\rho)\,\bar d_0 - \bar d_1^{T,e} - \chi\bar\pi_1^*\right], \qquad (4.14)$$

$$\bar g_1 - \bar g_1^e = \left[\frac{1/\alpha_g}{S}\right]\left[\bar K_1 + (1+\rho)\,\bar d_0 - \bar d_1^{T,e} - \chi\bar\pi_1^*\right]. \qquad (4.15)$$

Step 2: Country-specific expected deviations from cross-country expected averages. Subtract (4.11) and (4.12) from (4.8) and (4.9), respectively, which gives the following pair of equations:

$$\sigma^2\left(\tau_{i1}^{\prime,e} + \bar x_{i1}^{\prime}/\sigma\right) = \alpha_g\left(\bar g_{i1}^{\prime} - g_{i1}^{\prime,e}\right),$$

$$\bar K_{i1}^{\prime} + (1+\rho)\,d_{i0}^{\prime} = \left(\tau_{i1}^{\prime,e} + \bar x_{i1}^{\prime}/\sigma\right) + \left(\bar g_{i1}^{\prime} - g_{i1}^{\prime,e}\right) + d_{i1}^{T,\prime,e}.$$

These are solved to give the outcomes conditional on $d_{i1}^{T,\prime,e}$,

$$\bar x_{i1}^{\prime} - x_{i1}^{\prime,e} = \left[\frac{1/\sigma}{1/\sigma^2+1/\alpha_g}\right]\left[\bar K_{i1}^{\prime} + (1+\rho)\,d_{i0}^{\prime} - d_{i1}^{T,\prime,e}\right],$$
$$\bar g_{i1}^{\prime} - g_{i1}^{\prime,e} = \left[\frac{1/\alpha_g}{1/\sigma^2+1/\alpha_g}\right]\left[\bar K_{i1}^{\prime} + (1+\rho)\,d_{i0}^{\prime} - d_{i1}^{T,\prime,e}\right].$$

Step 3: Responses to common shocks. Subtract (4.7)-(4.9) from (4.4)-(4.6), respectively, to give the system:

$$\alpha_\pi\pi_1^d + \sigma^2\left(\pi_1^d - \bar\tau_1^d - \frac{1}{\sigma}\right) + \chi\alpha_g\bar g_1^d = 0, \qquad (4.16)$$

$$-\sigma^2\left(\pi_1^d - \tau_{i1}^d - \frac{1+\epsilon_i}{\sigma}\right) + \alpha_g g_{i1}^d = 0, \qquad (4.17)$$

$$0 = \tau_{i1}^d + \chi\pi_1^d - g_{i1}^d + d_{i1}^{T,d}. \qquad (4.18)$$

Next, take cross-country averages of this system, to yield:

$$\alpha_\pi\pi_1^d + \sigma^2\left(\pi_1^d - \bar\tau_1^d - \frac{1}{\sigma}\right) + \chi\alpha_g\bar g_1^d = 0, \qquad (4.19)$$

$$-\gamma^2\left(\pi_1^d - \bar{\tau}_1^d - \frac{1}{\gamma}\right) + \alpha_g \bar{g}_1^d = 0; \tag{4.20}$$

$$0 = \bar{\tau}_1^d + \chi\pi_1^d - \bar{g}_1^d + \bar{d}_1^{T;d}. \tag{4.21}$$

The solution of this system is:

$$\pi_1^d = \left[\frac{(\chi+1)/\alpha_\pi}{P^*}\right]\left(\frac{1}{\gamma} - \bar{d}_1^{T;d}\right); \bar{x}_1^d = -\left[\frac{\alpha_\pi}{(\chi+1)\gamma}\right]\pi_1^d; \bar{g}_1^d = -\left[\frac{\alpha_\pi}{(\chi+1)\alpha_g}\right]\pi_1^d. \tag{4.22}$$

Step 4: Computation of responses to idiosyncratic shocks. The relevant system to be solved is obtained by subtracting (4.20) and (4.21) from (4.17) and (4.18), respectively, to give:

$$-\gamma\left[\gamma\left(-\tau_{i1}^\delta - \varepsilon_i\right)\right] + \alpha_g\left(g_{i1}^\delta\right) = 0;$$

$$0 = \tau_{i1}^\delta - g_{i1}^\delta + d_{i1}^{T;\delta}.$$

The solution of this system is:

$$x_{i1}^\delta = \left[\frac{1/\gamma}{1/\gamma^2 + 1/\alpha_g}\right]\left(d_{i1}^{T;\delta} - \frac{\varepsilon_i}{\gamma}\right); \tag{4.23}$$

$$g_{i1}^\delta = \left[\frac{1/\alpha_g}{1/\gamma^2 + 1/\alpha_g}\right]\left(d_{i1}^{T;\delta} - \frac{\varepsilon_i}{\gamma}\right). \tag{4.24}$$

4.2. Proof of Proposition 2

We show that by setting the individual countries' debt targets at $d_{i1}^T = \bar{d}_1^{e;S} + d_{i1}^{\Delta;e;S} + \bar{d}_1^{d;S} + d_{i1}^{\delta;S}$ $(i = 1; \ldots; n)$, and the inflation targets at

$$\pi_1^* = -\left[\frac{1/\alpha_\pi}{P}\right]\left[\bar{K}_1 + (1+\rho)\,d_0 - \bar{d}_1^T\right]; \tag{4.25}$$

$$\pi_2^* = -\left[\frac{1/\alpha_\pi}{P}\right]\left[\bar{K}_2 + (1+\rho)\,\bar{d}_1^T\right]; \tag{4.26}$$

as proposed in Proposition 2, the second-best equilibrium is attained. Hence, this combination of inflation and debt targets mimimizes V_U.

Substitute expression (4.26) into (4.1) in order to eliminate π_2^*. The resulting expression can be simplified to:

$$\pi_2 = \left[\frac{\chi/\alpha_\pi}{P}\right]\left[\bar{K}_2 + (1+\rho)\,\bar{d}_1^T\right].$$

Because we set $\tilde{d}_1^{T} = \tilde{d}_1^{e,S} + \tilde{d}_1^{d,S}$ in Proposition 2, we substitute the solutions for $\tilde{d}_1^{e,S}$ and $\tilde{d}_1^{d,S}$ obtained in Section 3 into this expression for π_2, to yield:

$$\pi_2 = \left[\frac{\chi/\alpha\pi}{P}\right]\left[\frac{1+\rho}{1+\beta\Lambda(1+\rho)}\right]\tilde{F} + \left[\frac{\chi/\alpha\pi^S}{P}\right]\left[\frac{1+\rho}{1+\beta\Lambda(1+\rho)(P^{\Lambda}/P)}\right]\left(\frac{1}{o}\right);$$

which is the expression for π_2 given in Table 1.

We can proceed in a similar fashion to show that under the proposed combination of targets $\tilde{g}_{i2} - g_{i2}$, $\tilde{x}_{i2} - x_{i2}$, π_1, $\tilde{g}_{i1} - g_{i1}$ and $\tilde{x}_{i1} - x_{i1}$ all coincide with their second-best counterparts. This completes the proof.