CentER

Discussion Paper

No. 2004–69

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August 2004

ISSN 0924-7815

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Competition, Incomplete Discrimination and Versioning*

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23rd August 2004

Abstract

Producers of software viewers commonly offer basic versions of their products for free while more sophisticated versions are highly priced, thereby providing less attractive or lower valuations consumers with larger utility levels. We give some foundations to this outcome called versioning. We consider a duopoly in which firms offer differentiated goods to a representative consumer; the buyer has distinct marginal valuations for the quality of the products; each producer perfectly knows the consumer's taste for its own product, but remains uninformed about its taste for the rival's product.

When each product cannot be purchased in isolation of the other one, a phenomenon of endogenous preferences arises since a firm's offer to the consumer depends on the information unknown by the rival firm. Multiple equilibria emerge and the consumer's rent increases with his valuation for one product and decreases with the valuation for the other product. By contrast, when each product can be purchased in isolation of the other one, at the unique equilibrium consumers with larger valuations for a product earn higher rents.

The analysis is undertaken under two alternative pricing policies: in the partially-discriminatory case, producers make use of the known information only; in the fully-discriminatory case, each producer second-degree price discriminates the consumer according to the unknown information. We show that, sometimes, firms prefer partial to full discrimination, i.e., strategic ignorance of consumers' tastes for the rival brand softens competition.

Journal of Economic Literature Classification Number: L13, D82.
Keywords: Price Competition, Price Discrimination, Versioning.

*Particular thanks to Claude d'Aspremont, Bernard Caillaud, Jacques Crémer, Jean-Jacques Laffont, Maurice Marchand, Marc Ivaldi, David Martimort, Bernard Salanié and Jean Tirole for their support and help during this work. We gratefully acknowledge the help of Giacomo Calzolari at the early stages of the project. Part of this work has been done when the second author was research assistant at CORE-UCL (Marie Curie fellowship); he wishes to thank this institute for its hospitality.

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1 Introduction

Producers of software viewers commonly offer basic versions of their products for free while more sophisticated versions are highly priced. For instance, Apple offers its Quick Time Player for free and charges $29.99 for its Quick Time Pro. Real Networks offers its Real Player for free and charges $49.99 for its Real Player Plus. Shapiro and Varian (1998) have argued that this pricing behaviour, which they refer to as 'versioning', is intended to avoid competition in a market with intense competition, high fixed costs and marginal costs close to zero: without versioning, competition would drive prices to zero, making it impossible for firms to recoup their fixed costs. In fact, versioning consists in offering different price-version pairs.

There would be nothing to add to Shapiro and Varian's argument if the market for software viewers was highly competitive. However, Apple, Real Networks and Microsoft (with its free Media Player) enjoy some market power, which is actually needed to price discriminate, even imperfectly because of incomplete information. More strikingly, it even seems that in the versioning setting 'low-profile' buyers, i.e., buyers who are not willing to pay much for the product a priori, get higher surplus than the high-profile buyers.[1] If it is the case that low-profile users earn higher rents, then this way of making business by extracting surplus *mainly* from the high-profile users would stand in sharp contrast with the standard theory of second-degree price discrimination (see, e.g., Maskin and Riley, 1984). Shapiro and Varian further argue that 'The logic of the Free Version' can be explained by network effects, building product awareness, or foreclosing the market. Again, although these arguments may hold (especially the network aspect)[2], they do not fully explain the features of this market.

The purpose of this paper is to explain apparently 'odd' features in some markets, where the magnitude of rent extraction seems to go in the opposite sense of that predicted by the standard theory, the market for viewers being possibly one. We build a model of oligopoly competition under asymmetric information and price discrimination, which yields an outcome where consumers buying the lower qualities of a product can get more surplus or less surplus than those buying higher quality versions. As we discuss in Section 6, our analysis is closely related to recent empirical works on competition in nonlinear pricing: our model bears some resemblance to the structural models used in Ivaldi and Martimort (1994), who focus on the French energy distribution market, and in Miravete and Röller (2003, 2004), who focus on the U.S. cellular telephone industry.

Our results are obtained under the following main assumptions: (i) the consumer has distinct valuations for the imperfectly substitutable products; (ii) there exists an asymmetric information both between firms and consumers in the market, and between competing firms themselves.

The first assumption captures the fact that some consumers have inherent preferences over

[1] See Wang (2003). Of course, one cannot conclude that this is strictly the case, because consumers' valuations remain unobservable. However, by giving basic software for free to low-profile buyers, while charging a high price for a slightly more sophisticated one, it seems more likely, or at least possible, that the low-profile users are getting more surplus that the high-profile ones.

[2] Parker and Van Alstyne (2000) present an analysis of two-sided competition in software markets and show that firms might wish to distribute a software for free in order to boost the demand for complementary products.

particular brands;[3] hence, the consumer's heterogeneity is multi-dimensional. Moreover, unlike a CD player which can read any disc, a software viewer may not be compatible with all digital goods. Typically over the internet, a user is required to download a given viewer (possibly for free) if if he wants to visualize a digital good. The user who possesses only Media Player may then be obliged to get Real Player: a user may find it optimal to acquire both viewers. Although for a given set of digital files, Real Player and Media Player can be viewed as substitutes, there still is a substantial set of files for which both viewers are necessary.[4] Thus, we are led to consider two settings: when products are complements on the extensive margin[5], buying one product in isolation of the other good has no value to the consumer; by contrast, when products are substitutes on the extensive margin, the consumer can decide to shop exclusively with one firm.

The second assumption relates to one aspect about the market for digital goods, which can hardly be denied: a lot more information about consumers can be collected than through the traditional distribution channels. For example, in Shapiro and Varian (1998), we can read the following:

> Playboy, for example posts free images of its playmates on its Web site... Each image incorporates a digital watermark, enabling Playboy to track not only how many people view the image on its own site, but also how many people view it after it's been copied onto other sites. In this way, Playboy learns more about its on-line customers and how they use its products, further strengthening its ability to sell...on-line and print subscriptions

However, if there are other sites which offer similar products to the Playboy site (and there sure are...), Playboy may not have any idea about what the very same user who visited its site visited in the rival's sites. In other words, while a lot of information about a user can be collected from a product's own site, this information is very likely to be specific to the product. Therefore, the market for digital goods is very likely to be characterized by information asymmetry between firms.[6] Therefore, we consider that each firm is informed on the consumer's valuation for its own product but remains uninformed on the buyer's valuation for the rival good.

[3] In a sense, there is both horizontal and vertical differentiation in our model.

[4] As a matter of fact, several users have on their computers more than a viewer. One may argue that they do so because they they were offered for free (if the basic versions). However, such argument would be very doubtful as there are hundreds of free softwares on the internet, yet users do not acquire them all, but rather those that they need.

[5] This terminology is borrowed from Martimort and Stole (2003b). The case of complementarity on the extensive margin can be illustrated as follows: Operating systems and softwares are worth nothing one without the other, but a higher quality OS can be a substitute to a lower quality software. It is also intended to characterize different degree of competitive pressure.

[6] In most markets, firms (whether selling digital or more traditional products) can acquire information in two ways: directly, by repeated interaction (transaction or after-sale), telemarketing or direct mail survey, or indirectly, by credit cards reports, or a marketing firm; see Liu and Serfes (2003). The indirect way allows to acquire information about both consumer's preferences as a firm can always buy (e.g., from a marketing firm) information about consumer's preferences for any product, including rival's products. However, the direct way allows to have information about its own product only (in the case of firms selling online, a substantial amount of information can be collected which clearly may not say much about the consumer's characteristics that relate to rivals' products). Therefore, there are information sources available to all competitors, and sources available to single firms, which makes plausible that a firm is always more informed than its competitors about consumers'

As concerns firms' pricing policies, two possibilities are studied. In both cases, a firm's pricing policy depends on the known information: in this sense, there is always first-degree/direct discrimination along the known dimension of heterogeneity. In the first case, each firm offers the same price-quality pair to consumers with different unknown valuations: this pricing policy consists in a mix of uniform pricing along the unknown dimension of heterogeneity and first-degree/direct price discrimination along the known dimension. In the second case, each firm second-degree price discriminates the different types of consumer along the unknown dimension of heterogeneity; this pricing policy consists in a mix of first-degree and second-degree/indirect price-discrimination.[7] The analysis of these forms of competition between asymmetrically uninformed duopolists is, to the best of our knowledge, new to the literature. Strikingly, the equilibria that emerge under the different pricing policies bear some strong similarities.

In a first step, we analyze the case in which the consumer can threaten each competitor to buy only the rival's product. Each firm therefore needs to ensure that when purchasing both softwares, the consumer gets at least as much rent as what he could get if he were buying only the rival product. In this setting, we obtain a unique equilibrium; consumers with larger valuations for the products earn higher levels of surplus. Interestingly, equilibrium quality levels can be either upwards or downwards distorted with respect to the full information benchmark.

In a second step, we analyze the setting in which the consumer either acquires both products or none. There is now a phenomenon of endogenous preferences which can be explained as follows: a firm's ranking over the different unknown types of consumer now directly depends on the rival firm's firm offer to the consumer. Put differently, the information that a firm may want to extract from a user (by offering different price-version pairs) depends on the offer made by the rival firm which knows this information. The rival is then in position to manipulate the revelation of this information through its offer, and this gives rise to a multiplicity of equilibria; in each equilibrium, one firm ends up having a preference over the low type consumer[8] implying that the consumer's surplus is increasing in his valuation for one product but decreasing in the valuation for the other good. Therefore, in each equilibrium, one firm practices a 'standard' price discrimination by providing lower rents to lower types of consumer while the rival firm does the opposite by providing lower types with more rents than the higher types. In each equilibrium, the quality of one good is distorted upward w.r.t. the full information level whereas the quality of the other good is distorted downward.

These results hold both when firms perform partial or full price discrimination. More precisely, the rent profiles are qualitatively similar under the two pricing policies but the distortions on the quality levels are different.

Finally, we conclude with a comparison of the two pricing policies. Full discrimination better allows firms to extract the consumer's rent; however, we also argue that it intensifies competition. We show that when the consumer can threaten each firm to buy only the rival product, then firms jointly prefer to stick to partially discriminatory pricing policies. Differently

preferences for its own product. For firms selling a large part of their product online, such as softwares, this asymmetry effect may be stronger.

[7] The distinction between direct and indirect price discrimination is borrowed from Stole (2001).

[8] Similar features arise in Bond and Gresik (1997, 1998).

stated, strategic ignorance of the consumer's taste for the rival's brands softens competition.

Our paper belongs to two distinct literatures. First, we borrow from the literature on common agency under adverse selection; see Martimort (1992) and Stole (1991) and more recently Martimort and Stole (2003a). Second, our paper relates with the literature on welfare effects of uniform vs. discriminatory pricing (see, e.g., Thisse and Vives, 1988, Holmes, 1989, and Corts, 1998, among others), which is extensively surveyed in Stole (2001). We bring to that literature a new comparison which incorporates different forms of price discrimination simultaneously.

The paper is organized as follows. The next section, Section 2, presents the building blocks of the model. In Section 3, we study the complete information benchmark. The partially-discriminatory and fully-discriminatory equilibria are derived in Sections 4 and 5 respectively. In Section 6, we compare the two pricing policies for a particular specification of the model. Section 7 briefly concludes.

2 The Model

Consider a duopoly in which two firms ($i = 1, 2$) sell one unit of variable quality products to a buyer.[9] The gross utility of the consumer who consumes good 1 with quality q_1 and good 2 with quality q_2 is given by (superscript 'g' stands for 'gross')

$$U^g(\theta, q_1, q_2) \equiv u_1(\theta_1, q_1) + u_2(\theta_2, q_2) + u(q_1, q_2), \tag{1}$$

where $\theta \equiv (\theta_1, \theta_2)$. Parameter θ_i relates to the consumer's intrinsic valuation for the quality of good i; that θ_i is (a priori) different from θ_j simply illustrates the fact that, for equal products' qualities, the consumer inherently has a larger preference for one good. However, the buyer's utility derived from the consumption of the products also depends on the attributes attached to the different goods, which is embodied in the quality levels. Products are assumed to be imperfect substitutes on the intensive margin, i.e., $U^g_{q_1 q_2} = u_{q_1 q_2} \leq 0$, for all quality levels: provided that the consumer buys both goods, a marginal increase in the quality of one product reduces the marginal valuation for the other good.

The following assumptions are assumed to hold for all strictly positive consumption levels: $U^g_{q_i} > 0$, $U^g_{q_i q_i} < 0$, $U^g_{\theta_i} > 0$, $U^g_{\theta_i q_i} > 0$. These conditions are interpreted as follows: for each good, marginal utility increases with quality, but at a decreasing rate; the larger the preference for good i is, the higher the consumer' utility and marginal utility levels. For tractability, we also assume that $u_i(\theta_i, 0) = 0$, $U^g(\theta, 0, 0) = 0$ and $|U^g_{q_i q_i}| \geq |U^g_{q_i q_j}|$.[10]

It is common knowledge that θ_i is independently distributed on the interval $\Theta_i \equiv [\underline{\theta}_i, \overline{\theta}_i]$

[9] Following Maskin and Riley (1984), another interpretation is that there is a continuum of buyers and that the taste parameters of each buyer are drawn independently from the same distribution.

[10] That $u_i(\theta_i, 0) = U^g(0, 0) = 0$ is assumed for analytical convenience only and does not affect our results as long as $u_i(\theta_i, 0)$ and $U^g(\theta, 0, 0)$ do not depend on θ_i; the last condition, namely $|U^g_{q_i q_i}| \geq |U^g_{q_i q_j}|$, is standard and implies that were firms offering purely linear prices the consumer's demand function for one good would be more sensitive to the price of that good than to the rival price.

according to the (strictly positive) density $f_i(.)$ with c.d.f. $F_i(.)$, $i = 1, 2$.[11] However, only the consumer perfectly knows his valuation for both goods. As regards firms, we assume they are asymmetrically un-informed as defined next.

Definition 1. *Asymmetrically un-informed duopolists: firm i is perfectly informed about θ_i but only knows the distribution of θ_j, $i \neq j$, $i, j = 1, 2$.*

This information structure underlines the fundamental asymmetry between competitors: each firm is better informed on the consumer's marginal valuation for its product than on the consumer's valuation for the rival product.

Firm i's profit is thus defined as follows

$$V_i = P_i - C_i(q_i),$$

where $C_i(q_i)$ is the strictly increasing and convex cost of producing good i with quality level q_i and P_i is the price paid by the consumer to firm i.

The consumer's net utility is the difference between his gross utility and the prices paid to the firms,[12] or

$$U = -P_1 - P_2 + U^g(\theta, q_1, q_2).$$

The timing goes as follows: first, firms make simultaneously and non-cooperatively their offers to the buyer; second, the consumer decides which offers he accepts. As argued in the Introduction, two cases are worth considering:[13]

- *Complementarity on the extensive margin.* In that case, the consumer attributes no value to product i without the purchase of good j. For instance, when software compatibility is very severe, the consumer prefers to acquire both softwares whatever their respective qualities. Therefore, to ascertain that the consumer will purchase its good, firm i must ensure that the consumer's utility when he accepts both firms' offers is larger that his utility if he decides to purchase none (which is normalized to 0), or

$$U \geq 0. \tag{PC_i^I}$$

- *Substitutability on the extensive margin.* In that setting, the consumer has the extra option to decide buying only one product. Therefore, from the perspective of firm i the participation constraint becomes

$$U \geq \max\left\{0; U_i^{out}(\theta)\right\}, \tag{PC_i^D}$$

where $U_i^{out}(\theta)$ is the consumer's outside opportunity with respect to firm i: this represents

[11]Notice that considering imperfectly correlated adverse selection parameters would not qualitatively change the main messages conveyed in this paper since this would only modify each firm's prior on the unknown piece of information.

[12]If $q_i = 0$ then the price paid by the consumer to firm i is null.

[13]The cases correspond to the intrinsic and delegated common agency settings, as coined by Bernheim and Whinston (1986).

the consumer's rent when he decides to buy only from the rival firm j. Notice that this outside opportunity is endogenous since it depends on the rival firm's offer.[14]

3 Complete Information Benchmark

As a useful benchmark, we consider (in this section only) that both firms perfectly know consumer's preferences, i.e., θ: in that case, firms can perfectly discriminate the different types of buyer. In order to determine the equilibrium in terms of price and quality levels offered by the firms, we consider the most general case in which firms offer the consumer price schedules $P_i(\theta, q_i)$, $i = 1, 2$; given the information at the disposal of firm i, this schedule links for all possible quality levels of good i chosen by the consumer the price paid by the buyer to firm i.

To determine firm i's best-response, we take as given the price schedule offered by the rival firm to the consumer in an equilibrium. Notice that different price schedules offered by firm j affect differently the consumer's behavior vis-à-vis firm i. To account for this effect, let us define the consumer's optimal quality choice for good j, and the corresponding utility level (gross of the price paid to firm i), for a fixed quality of good i as follows

$$\hat{q}_j(\theta, q_i) = \arg\max_{q_j}\{-P_j(\theta_j, q_j) + U^g(\theta, q_i, q_j)\},$$
$$\hat{U}^i(\theta, q_i) = \max_{q_j}\{-P_j(\theta_j, q_j) + U^g(\theta, q_i, q_j)\}.$$

$\hat{q}_j(\theta, q_i)$ highlights the linkage between the consumer's valuations for quality: a change in the quality of one good affects the consumer's marginal utility for the other good and therefore the level of quality of the latter product chosen by the consumer. When designing its offer for the consumer, each firm accounts for the impact of its rival's offer on the choice of quality levels by the consumer.

Moreover, let us assume that $\hat{q}_j(\theta, q_i)$ is characterized by the first-order condition

$$-P_{jq_j}(\theta, \hat{q}_j(\theta, q_i)) + U^g_{q_j}(\theta, q_i, \hat{q}_j(\theta, q_i)) = 0. \tag{2}$$

For $\hat{q}_j(\theta, q_i)$ to be indeed characterized by (2), it must be the case that the consumer's problem $\max_{q_1,q_2}\{-P_1(\theta, q_1) - P_2(\theta, q_2) + U^g(\theta, q_1, q_2)\}$ is concave at equilibrium. This is checked in Appendix A.1.

Similarly, if the consumer decides to buy only the rival product, then the quality purchased and the corresponding outside opportunity are defined as follows

$$q_j^{out}(\theta) = \arg\max_{q_j}\{-P_j(\theta, q_j) + U^g(\theta, q_i = 0, q_j)\} = \hat{q}_j(\theta, 0)$$
$$U_i^{out}(\theta) = \max_{q_j}\{-P_j(\theta, q_j) + U^g(\theta, q_i = 0, q_j)\} = \hat{U}^i(\theta, 0).$$

[14] Implicit in this formulation is that a firm's offer to the consumer cannot be contingent on the consumer's decision to buy only the rival product. Such a contracting possibility is likely to violate antitrust rules and is therefore discarded from our analysis.

If the consumer decides not to acquire good i, then his marginal utility for good j increases and he is thus willing to buy a higher quality of the latter good: therefore, $\hat{q}_j(\theta, q_i) \leq q_j^{out}(\theta)$. This intuitive result is shown in Appendix A.1.

Finally, the next lemma turns out to be useful.

Lemma 1. *When goods are substitutes on the intensive margin and firms compete in price schedules, both outside opportunities are positive.*

Proof. See Ivaldi and Martimort (1994). □

To build the intuition, consider the extreme scenario in which products are close substitutes. In that case, if the consumer decides not to acquire product i, then his loss of utility can be almost fully offset by increasing the level of quality chosen for good j; differently stated, the consumer can threaten firm i to shop exclusively with its rival and derive a strictly positive utility level. Lemma 1 shows that this reasoning holds even when products are weakly substitutable, whatever the information at firms' disposal.

Let us start with the case of substitutability on the extensive margin. Under complete information, firm i's best-response is to implement a quality level $q_i(\theta)$ and a price $p_i(\theta) = P_i(\theta, q_i(\theta))$ which maximize its profit while satisfying the corresponding consumer's participation constraint, or using Lemma 1

$$\max_{\{q_i(\theta), p_i(\theta)\}} V_i(\theta_i, q_i(\theta)) \quad \text{s.t. } U(\theta) = -p_i(\theta) + \hat{U}^i(\theta, q_i(\theta)) \geq U_i^{out}(\theta).$$

The participation constraint must bind at equilibrium since firm i seeks to set as high a price as possible. This defines the value of the optimal price set by firm i, or equivalently the rent earned by the consumer

$$p_i^{FB}(\theta) \text{ s.t. } U^{FB}(\theta) = U_i^{out}(\theta).$$

Then, optimizing with respect to $q_i(\theta)$ yields the following first-order condition, which is necessary and sufficient in our context,

$$\hat{U}^i_{q_i}(\theta, q_i(\theta)) = C_{iq_i}(q_i(\theta)). \tag{3}$$

Equation (3) is a usual 'marginal benefit equals marginal cost' rule from firm i's perspective, which accounts for the fact that the buyer optimally chooses his quality level of good j.

At equilibrium, consistency imposes that $\hat{q}_j(\theta, q_i^{FB}(\theta)) = q_j^{FB}(\theta)$ and the equilibrium quality levels are characterized by the following conditions[15]

$$U^g_{q_i}(\theta, q_i^{FB}(\theta), q_j^{FB}(\theta)) = C_{iq_i}(q_i^{FB}(\theta)) \quad i \neq j \quad i, j = 1, 2. \tag{4}$$

Differentiating (4) w.r.t. θ_i and θ_j, we obtain that $\frac{\partial q_i^{FB}}{\partial \theta_i}(\theta) \geq 0$ and $\frac{\partial q_i^{FB}}{\partial \theta_j}(\theta) \leq 0$, for $i, j = 1, 2$, $i \neq j$. Intuitions are straightforward. First, the lower the consumer's valuation θ_i for the quality

[15] To focus on the interesting cases, equilibrium qualities are assumed to be strictly positive.

of the good produced by firm i is, the lower is firm i's equilibrium quality level. Second, applying a similar argument, the smaller θ_j is, the smaller is quality q_j; such a decrease in q_j leads to an increase in q_i since the marginal utility of the consumer for firm i's good is increased.

Consider now the case of complementarity on the extensive margin. With respect to the previous case, the unique difference is that the participation constraint becomes

$$U(\theta) = -p_i(\theta) + \hat{U}^i(\theta, q_i(\theta)) \geq 0.$$

Therefore, the equilibrium quality levels are not affected by the level of utility that must be given up by each firm to the consumer in order to ensure that this consumer will not shop only with the other firm. The intuition is immediate: under full information, each firm perfectly (first-degree) discriminates the consumer, but might be constrained, in terms of surplus given up to the consumer, by its competitors. The optimal way for a firm to provide the consumer with sufficiently high a rent to induce his participation is to reduce its price, which has a first-order impact of the buyer's rent, without distorting its quality level. Under complete information on the firms' side, the competitive pressure which is channeled through the buyer's outside opportunities only leads to a reallocation of the total surplus between the consumer and the firms.

In Appendix $A.1$ we show that two-part price schedules enable to decentralize the equilibrium allocation. The variable part of the price schedule is used to provide the buyer with the correct incentive as regards his quality decision (i.e., to internalize the cost of quality), the fixed part enables to capture as much rent as consistent with the buyer's participation. In line with the intuition, we also prove that the outside opportunity $U_i^{out}(\theta)$, which coincides with the consumer's equilibrium rent when products are substitutes on the extensive margin, is increasing with θ_j: under complete information, from the viewpoint of firm i, the larger the buyer's valuation for the rival good is, the larger is the rent that must be given up to that consumer to prevent him from shopping exclusively with the rival firm.

Moreover, under complete information, it is straightforward to show that the same equilibrium allocation would emerge had we assumed that firms compete in price-quality pairs (or singleton contracts $\{p_i(\theta), q_i(\theta)\}$, $i = 1, 2$) instead of price schedules.

We summarize this benchmark in the next proposition.

Proposition 0. *Under complete information, equilibrium quality levels are uniquely defined and are not affected by the possibility for the consumer to shop exclusively with one firm. Only under the possibility of exclusive shopping does the consumer earn a rent; this rent is increasing in his valuations for the products.*

We now come back to our initial framework in which each firm suffers from incomplete information on the consumer's valuation for the rival good. As we shall see, many of the previously-mentioned results do not carry over to the situation of incomplete information.

4 Competition in Partially-Discriminatory Pricing

In this section, we analyze a simple form of competition in which both firms do not discriminate the consumer along the unknown dimension of heterogeneity. Firm i being informed on the consumer's valuation for its own product, its offer to the consumer still depends on this piece of information. Hence, the firms' pricing policy consists in a mix of first-degree price discrimination along the known dimension of the consumer's preference and uniform pricing along the unknown dimension: firms only partially discriminate the different types of buyer.

Substitutability on the extensive margin. Firm i offers a price-quantity pair which depends only on the known information, $\{p_i(\theta_i), q_i(\theta_i)\}$,[16] in order to maximize its expected profit while ensuring that the buyer does not shop exclusively with its rival, or

$$\max_{\{q_i(\theta_i),p_i(\theta_i)\}} \mathbb{E}_{\theta_j}\left\{V_i(\theta_i, q_i(\theta_i))\right\} \quad \text{s.t. } \forall\theta_j \quad U(\theta_i,\theta_j) \geq \max\left\{0; U_i^{out}(\theta_j)\right\},$$

where the outside opportunity is now given by

$$U_i^{out}(\theta_j) = -p_j(\theta_j) + U^g(\theta_j, q_i = 0, q_j(\theta_j)).$$

Again, we need to determine whether the threat to shop exclusively with the rival is credible, i.e., if outside opportunities are positive when firms compete in partially-discriminatory pricing policies. The following result is reminiscent of our previous analysis.

Lemma 2. *When goods are substitutes on the intensive margin and firms compete in partially-discriminatory pricing policies, both outside opportunities are positive.*

Proof. See Appendix A.2. □

The main difficulty that arises in solving firm i's problem stems from the fact that, *a priori*, firm j's offer to the consumer depends on firm i's unknown information, θ_j. As we argue next, this implies that the behavior of the consumer's rent requires to be studied with particular attention. To highlight this point, we split the firms' optimization problem into two sub-problems: first, we study the duopolists' best-responses assuming that the buyer's rent behaves monotonically w.r.t. the adverse selection parameters; second, we focus on the variation of the buyer's rent w.r.t. the private information parameters.

Best-responses for given indifferent types. Given Lemma 2, a relevant variable in our context is the difference between the rent $U(\theta)$ and the outside opportunity $U_i^{out}(\theta_j)$ that we call the 'net rent'. Indeed, assume in a first time that $\frac{\partial}{\partial\theta_j}[U(\theta) - U_i^{out}(\theta_j)]$ has a constant sign.[17] This implies that, from firm i's viewpoint, there exists a type $\theta_j^* \in \{\underline{\theta}_j, \overline{\theta}_j\}$ of buyer, called the *indifferent type for firm i*, such that if the participation constraint is satisfied for this type, then it is satisfied

[16] In a sense, firm i's offer consists in a price schedule which is degenerated to a single point.

[17] This property is satisfied at equilibrium.

for all the other types of buyer. Since firm i dislikes giving up excessive rent to the buyer, the individual rationality constraint binds at equilibrium for $\theta_j = \theta_j^*$, or

$$p_i(\theta_i) \text{ s.t. } U(\theta_i, \theta_j^*) = U_i^{out}(\theta_j^*). \tag{5}$$

Two points are worth highlighting: first, since we consider participation of all the different types of consumers, everything happens as if firm i was considering that the buyer is of type θ_j^*; second, the price offered by firm i is uniquely defined by the binding participation constraint.

Replacing the value of $p_i(\theta_i)$ defined by (5) in firm i's problem and optimizing with respect to $q_i(\theta_i)$ yields the following first-order condition[18]

$$U^g_{q_i}(\theta_i, \theta_j^*, q_i(\theta_i), q_j(\theta_j^*)) = C_{iq_i}(q_i(\theta_i)). \tag{6}$$

We can perform similar computations for firm j and obtain the following condition

$$U^g_{q_j}(\theta_i^*, \theta_j, q_i(\theta_i^*), q_j(\theta_j)) = C_{jq_j}(q_j(\theta_j)). \tag{7}$$

Considering (6) evaluated in $\theta_i = \theta_i^*$ and (7) in evaluated in $\theta_j = \theta_j^*$ forms a system of two equations, whose solution determines $q_i(\theta_i^*)$ and $q_j(\theta_j^*)$. Then, plugging back $q_i(\theta_i^*)$ and $q_j(\theta_j^*)$ in (6) and (7) respectively, and solving this new system yields the quality profiles $q_i(\theta_i)$ and $q_j(\theta_j)$ in the PDP equilibrium. We note that, using our assumptions, total differentiation of (6) with respect to θ_i immediately yields $q_i'(\theta_i) \geq 0 \ \forall \theta_i$.

The indifferent types. It remains to determine θ_i^* and θ_j^*. To this purpose and to build the intuition, let us introduce the 'net prices' defined as follows: $\tilde{p}_i(\theta_i) \equiv p_i(\theta_i) - u_i(\theta_i, q_i(\theta_i))$, $i = 1, 2$. Since firm i is perfectly informed on the buyer's valuation for its product, the part $u_i(\theta_i, q_i(\theta_i))$ of the gross utility is perfectly observed by that firm and hence cannot be a source of rent for the consumer. This is the basic motivation to focus on net prices. With this new notations in hand, the buyer's rent writes as $U(\theta) = -\tilde{p}_i(\theta_i) - \tilde{p}_j(\theta_j) + u(q_i(\theta_i), q_j(\theta_j))$, while his outside opportunity becomes $U_i^{out}(\theta_j) = -\tilde{p}_j(\theta_j) + u(0, q_j(\theta_j))$. The binding participation constraint (5) defines the equilibrium net price $\tilde{p}_i(\theta_i)$.

Now consider a hypothetical marginal increase in the buyer's valuation for good j. First, the buyer is led to pay a larger net price to firm j: $\tilde{p}_j'(\theta_j)d\theta_j = u_{q_j}(q_i(\theta_i^*), q_j(\theta_j))q_j'(\theta_j)d\theta_j \geq 0$. Note that the increase in the net price affects in a similar way the consumer's rent and the outside opportunity. Second, this also affects the part of the buyer's rent which cannot be fully captured by the firm; this in turn impacts differently the buyer's rent and his outside opportunity since the former increases by $u_{q_j}(q_i(\theta_i), q_j(\theta_j))q_j'(\theta_j)d\theta_j$ whereas the latter increases by $u_{q_j}(0, q_j(\theta_j))q_j'(\theta_j)d\theta_j$; since products are substitutable on the intensive margin from the consumer's viewpoint, the former increase is weaker than the latter, thereby implying that the net rent is decreasing in θ_j, i.e.,

$$\frac{\partial}{\partial \theta_j}\left[U(\theta) - U_i^{out}(\theta_j)\right] = q_j'(\theta_j)\int_0^{q_i(\theta_i)} u_{q_iq_j}(x, q_j(\theta_j))dx \leq 0. \tag{8}$$

[18]The associated second-order condition is satisfied.

An implication of (8) is that $\theta_i^* = \overline{\theta}_i$; using symmetry we obtain $\theta_j^* = \overline{\theta}_j$. Another implication of (8) is that, at equilibrium, the buyer's rent can be expressed as follows

$$U(\theta) = U_i^{out}(\theta_j) + \int_{\theta_j}^{\overline{\theta}_j} q_j'(\hat{\theta}_j) \int_0^{q_i(\theta_i)} -u_{q_iq_j}(x, q_j(\hat{\theta}_j))dxd\hat{\theta}_j. \tag{9}$$

Equation (9) illustrates that the consumer derives utility from two distinct sources. The first one is incomplete information on the firms' side, which prevents competitors from fully extracting the consumer' surplus. The second one stems from the implicit threat put by the consumer on each of the non-cooperating firms to shop exclusively with the rival firm when products are substitutes on the extensive margin. Firm i provides the consumer with the highest valuation for the rival good j (i.e., $\overline{\theta}_j$) with a utility level equal to his outside opportunity; consumers with lower valuations for good j earn more than their outside opportunities. But, the larger the valuation for one product is, the larger the consumer's rent is. Indeed, we have

$$\begin{aligned}\frac{\partial U}{\partial \theta_j}(\theta) &= -\tilde{p}_j'(\theta_j) + u_{q_j}(q_i(\theta_i), q_j(\theta_j))q_j'(\theta_j) \\ &= q_j'(\theta_j) \int_{q_i(\theta_i)}^{q_i(\overline{\theta}_i)} -u_{q_iq_j}(x, q_j(\theta_j))dx \geq 0.\end{aligned}$$

Here, competition forces firms to provide higher valuations consumer with a larger utility level. The profile of rents is illustrated in Figure 1.[19]

We focus now on the pattern of quality levels in a partially-discriminatory equilibrium. The first-order conditions (6) and (7) can be rewritten as follows

$$U_{q_i}^g(\theta, q_i(\theta_i), q_j(\theta_j)) - C_{iq_i}(q_i(\theta_i)) = -\int_{q_j(\theta_j)}^{q_j(\overline{\theta}_j)} u_{q_iq_j}(q_i(\theta_i), y)dy \geq 0, \tag{10}$$

$$U_{q_j}^g(\theta, q_i(\theta_i), q_j(\theta_j)) - C_{jq_j}(q_j(\theta_j)) = -\int_{q_i(\theta_i)}^{q_i(\overline{\theta}_i)} u_{q_iq_j}(x, q_j(\theta_j))dx \geq 0. \tag{11}$$

Equations (10) and (11) implicitly define the firms' best-responses, denoted by $R_i(q_j)$ for firm i, in the partially-discriminatory pricing setting. With respect to the complete information benchmark, where the best-responses were such that marginal utility equals marginal cost for each product, asymmetric information implies that the best-responses curves are downward shifted. Everything happens here as if firm i anticipates that the consumer's valuation for product j is given by $\overline{\theta}_j$; since quality levels are increasing in the consumer's valuations, firm i anticipates too high a quality level purchased by the consumer for product j; products being substitutes, firm i's quality level tends to decrease w.r.t. the full information benchmark. However, there is a feedback effect to consider in order to fully apprehend the equilibrium pattern of quality levels. Suppose indeed that θ_j is close enough to $\overline{\theta}_j$; then firm i's quality level is not much distorted; suppose simultaneously that θ_i is close to $\underline{\theta}_i$; then firm j's quality level is much distorted; since

[19]To draw the utility curves in Figure 1, we use the fact that at equilibrium $\frac{\partial^2 U}{\partial \theta_i \partial \theta_j}(\theta) \leq 0$.

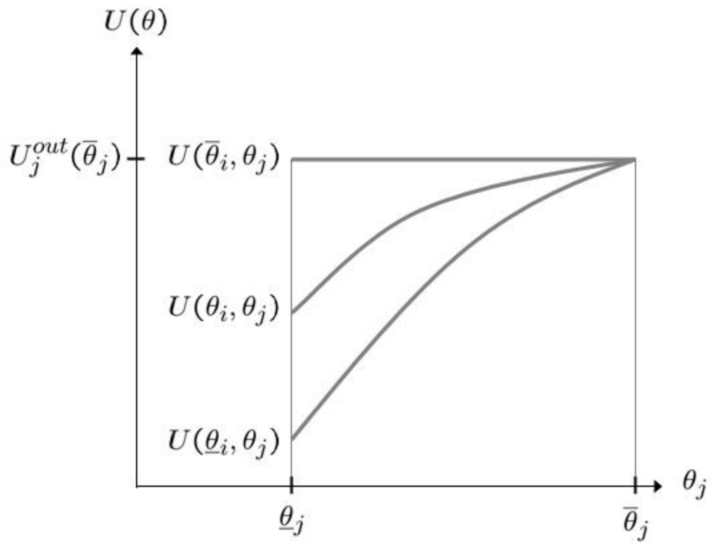

Figure 1: Rent profiles in the PDP equilibrium with substitutes on the extensive margin.

products are substitutes, firm i over-supplies and firm j under-supplies quality at equilibrium. The pattern of distortion is represented graphically in Figure 2.

Summarizing, we get the following proposition.

Proposition 1. *There exists a unique partially-discriminatory pricing equilibrium, with indifferent types $\theta_j^* = \overline{\theta}_j$ and $\theta_i^* = \overline{\theta}_i$. With respect to the complete information situation, there is either over-supply of quality for one good and under-supply of quality for the other good, or under-supply of quality for both goods; the larger the consumer's valuation for a product is, the higher is his rent.*

For future references, in the PDP equilibrium with substitutability on the extensive margin, consumers with higher valuations earn larger rents; w.r.t. the full information benchmark, buyers who value much more product i than product j are offered a lower quality good i and a higher quality good j. To illustrate, consider a consumer who highly likes Real Player (from Real Networks); this consumer purchases a sophisticated version of Real Player although incomplete information leads to some distortion on the quality level. Because goods are substitutes on the intensive margin, such a consumer hardly benefit from acquiring at the same time the high quality version of, say, Quick Time (from Apple). Hence, Apple has to tailor his offer by increasing the quality of its good (w.r.t. the complete information setting) to ensure that this buyer will not use exclusively the other player. Intuitively, when goods are substitutes on the extensive margin, so that the consumer can in equilibrium choose to get the sophisticated Real Player only, then it becomes very costly for Apple to 'convince' that consumer to take even the

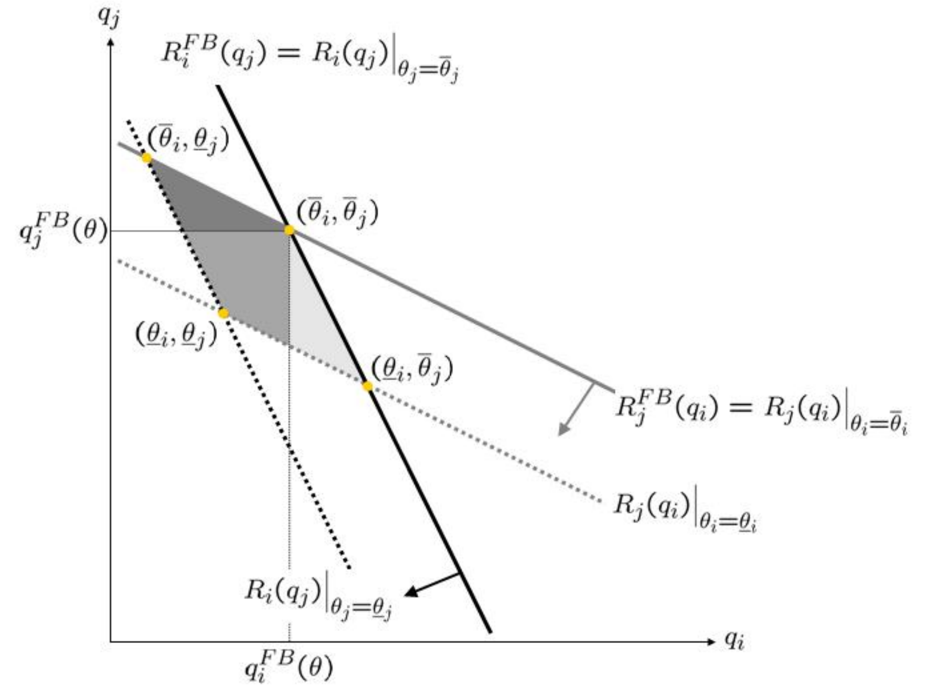

Figure 2: Best-responses and the set of quality levels in the PDP equilibrium.

low quality Quick Time.

Complementarity on the extensive margin. In this setting competition is less severe since the consumer's outside opportunities are now null. How is the analysis affected?

As previously, consider firm i and assume that the consumer's rent behaves monotonically w.r.t. θ_j. Then, we can define the indifferent type θ_j^* such that the participation constraint is exactly binding, or $U(\theta_i, \theta_j^*) = -\tilde{p}_i(\theta_i) - \tilde{p}_j(\theta_j^*) + u(q_i(\theta_i), q_j(\theta_j^*)) = 0$. Then, we immediately see that the first-order condition that characterizes firm i's optimal quality level is still given by (6): if the indifferent types were the same, then the option to shop exclusively with one of the competitors would have no impact on the quality levels and may only lead to a reallocation of total surplus between the consumers and the firms.

Simple computations lead to $\frac{\partial}{\partial \theta_j} U(\theta) = -\tilde{p}_j'(\theta_j) + u_{q_j}(q_i(\theta_i), q_j(\theta_j))q_j'(\theta_j)$, which can no longer be signed a priori: *firm i's ranking over the different unknown types of consumer is endogenous as it depends now directly on firm j's offer to the consumer.* Suppose that firm i expects that firm j's offer is such that consumer with larger valuations for product j earn smaller rents, or $\frac{\partial}{\partial \theta_j} U(\theta) \leq 0$. Firm i's offer is then such that $\tilde{p}_i(\theta_i) = -\tilde{p}_j(\overline{\theta}_j) + u(q_i(\theta_i), q_j(\overline{\theta}_j))$. Straightforward manipulations then show that the consumer's rent is such that $\frac{\partial}{\partial \theta_i} U(\theta) \geq 0$; therefore, the indifferent types are either $(\underline{\theta}_i, \overline{\theta}_j)$ or $(\overline{\theta}_i, \underline{\theta}_j)$: multiple equilibria emerge, each equilibrium leading to rent and quality levels which differ than those obtained in the substitutability on the extensive margin setting. In particular, adapting the previous analysis, we observe that best-response are shifted in opposite directions: at equilibrium, there is now over-supply of quality for one good and under-supply for the other product. The equilibrium quality levels and rent profiles in that case are represented in Figure 3 and Figure 4 respectively. Summarizing, we

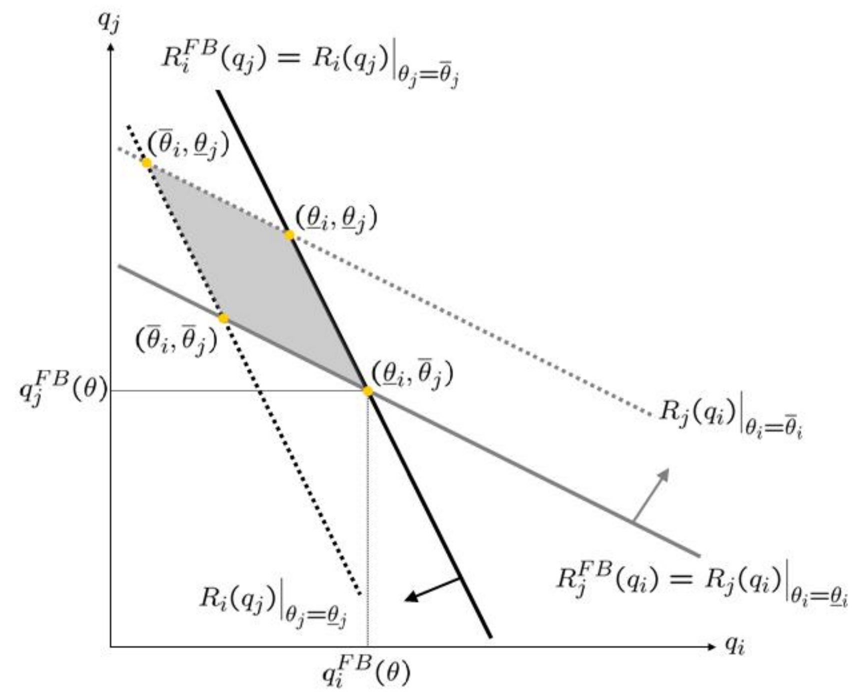

Figure 3: Best-responses and the set of quality levels in the PDP equilibrium: the case of bundling with indifferent types $(\underline{\theta}_i, \overline{\theta}_j)$.

obtain the following proposition.

Proposition 2. *When products are complements on the extensive margin, there exist two symmetric equilibria: one with indifferent types $(\underline{\theta}_i, \overline{\theta}_j)$, the other with indifferent types $(\overline{\theta}_i, \underline{\theta}_j)$. In both equilibria, there is over-supply of quality for one good and under-supply for the other; the consumer's rent is increasing in the valuation for one good and decreasing in the valuation for the other product.*

In these equilibria, consumers who value much more one good than the other product earn larger rents. Proposition 2 explains why a consumer may get a higher surplus by acquiring a basic version of a product, rather than the sophisticated ones.[20] This may be applied to other markets than simply the market for software viewers. The informational asymmetry between firms is likely to exist in many markets (see footnote 6). Within the car industry for example, our model may explain the evidence that price markups are much lower in base models than in models with more options (for more, see Verboven, 1999). In the same paper, a citation from Louis Phlips concludes that 'one has the impression that extra options are overpriced, to extract

[20]Remember that the quality level of good i is increasing in θ_i.

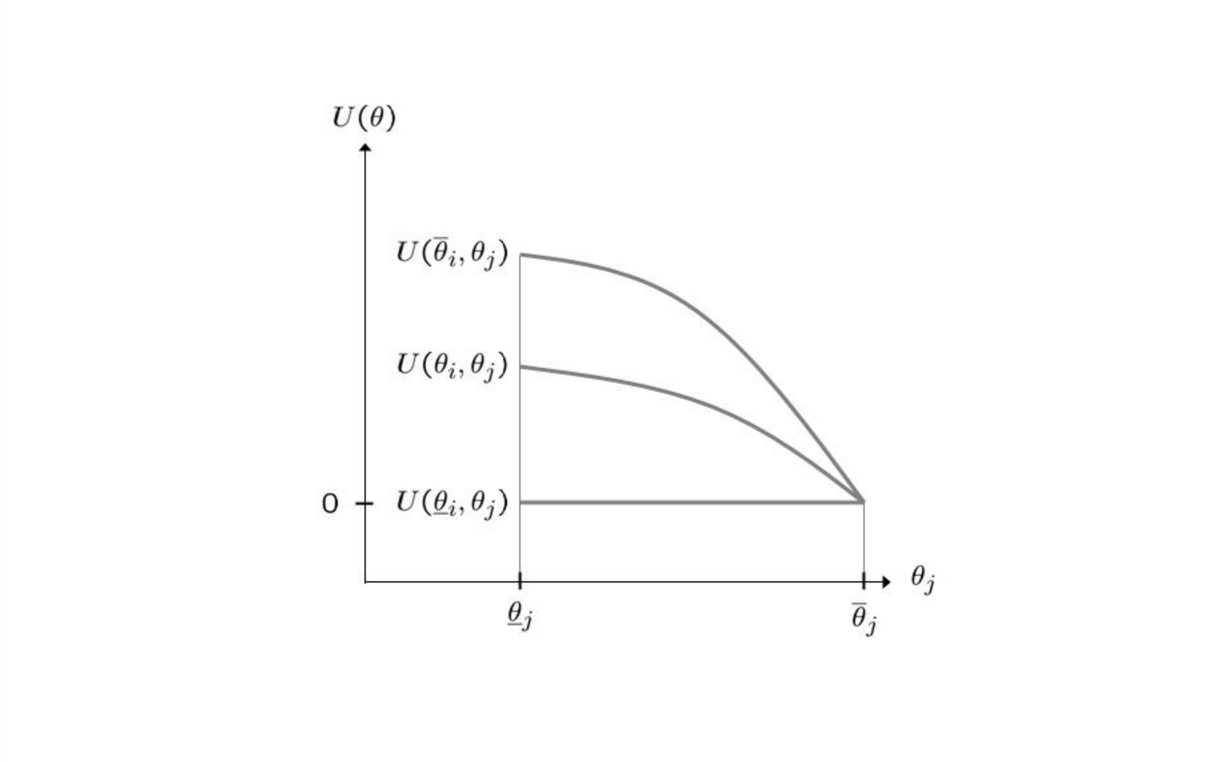

Figure 4: Rent profiles in the PDP equilibrium with complements on the extensive margin.

the highest possible price from those who want fancy tires or extra horsepower'. Phlips seems to suggest that consumers with preference for the basic models may actually get a higher surplus than those who prefer the sophisticated versions.

5 Competition in Fully-Discriminatory Pricing

As regards the previous section, one may object that firms might try to better tailor their offers to the different characteristics of the consumer. Under incomplete information, firm i can achieve such a second-degree price discrimination by facing the consumer with a price schedule $P_i(\theta_i, q_i)$ such that different types of a consumer choose different quality levels. In that context, the firms' pricing policy consists in a mix of first-degree price discrimination along the known dimension of heterogeneity and second-degree price discrimination along the unknown dimension of the consumer's preference: firms fully discriminate the different types of buyers.

For a given price schedule offered by firm j, there is no loss of generality in applying the Revelation Principle[21] to determine firm i's optimal price and quality levels. However, the consumer's behavior and therefore his incentive vis-à-vis firm i depend on the price schedule offered by the rival firm j. Hence, let us again define $\hat{U}_i(\theta_j, q_i)$ the indirect utility function which gives the maximal gain of the consumer with type $\theta = (\theta_i, \theta_j)$ for a given consumption level q_i when that buyer chooses optimally his the quality level for product j. $\hat{q}_j(\theta_j, q_i)$ is assumed

[21]See Green and Laffont (1979) or Myerson (1981) for instance.

to be defined through the following first-order condition

$$-P_{jq_j}(\theta_j, \hat{q}_j) + U^g_{q_j}(\theta, q_i, \hat{q}_j) = 0. \tag{12}$$

Now, from the viewpoint of firm i, everything happens as if it were facing a buyer with rent given by

$$U(\theta) \equiv \max_{q_i} \left\{ -P_i(\theta_i, q_i) + \hat{U}_i(\theta_j, q_i) \right\}.$$

We can now apply the standard methodology[22] to find the conditions for (local) incentive compatibility from the viewpoint of firm i. These conditions are, as usual, expressed in terms of rent-quality pairs instead of price-quality pairs and stated in the next lemma.

Lemma 3. *A pair $\{U(\theta_i, .), q_i(\theta_i, .))\}$ satisfies the local incentive compatibility constraints if and only if, for all $\theta_j \in \Theta_j$, the following conditions are satisfied:*

$$\frac{\partial U}{\partial \theta_j}(\theta) = \frac{\partial \hat{U}_i}{\partial \theta_j}(\theta_j, q_i(\theta)) = -P_{j\theta_j}(\theta_j, \hat{q}_j) + U^g_{\theta_j}(\theta_j, \hat{q}_j), \tag{$FOIC_i$}$$

$$\frac{\partial^2 \hat{U}_i}{\partial q_i \partial \theta_j}(\theta_j, q_i(\theta)) \times \frac{\partial q_i}{\partial \theta_j}(\theta) \geq 0. \tag{$SOIC_i$}$$

Proof. The proof is standard and hence omitted. □

From ($FOIC_i$), we observe that firm i can obtain the revelation of the unknown information only in an indirect way: the quality of its product q_i does not affect directly the buyer's rent. As in the partially-discriminatory pricing case, ($FOIC_i$) also shows that one cannot sign a priori the derivative of the consumer's rent w.r.t. to firm i's unknown information θ_j since this derivative depends on the endogenous price schedule offered by firm j.

In order to focus on the most economically meaningful situations, we shall now make some additional assumptions:

- **The first-order approach.** As in the complete information benchmark, the determination of the fully-discriminatory pricing equilibrium assumes that the consumer's behavior can be characterized by the first-order condition (12); this amounts to assuming that the consumer's problem is concave.

- **The Spence-Mirrlees condition**. As usual, the Spence-Mirrlees condition plays a crucial role in determining the impact of asymmetric information on the equilibrium quality profiles. This condition, which ensures that the problem faced by each firm is well-behaved, can be stated as follows

  $$\forall (\theta_j, q_i),\ \frac{\partial^2 \hat{U}_i}{\partial q_i \partial \theta_j}(\theta_j, q_i) \leq 0$$

  Provided that this condition is satisfied, local incentive compatibility of an allocation ensures that it is also globally incentive compatible and the local second-order condition

[22] See, e.g., Guesnerie and Laffont (1984).

for incentive compatibility reduces to a standard monotonicity constraint on the quality profile of good i.

These assumptions on the endogenous price schedules are needed to obtain a simple characterization of the equilibrium in fully-discriminatory pricing policies. In our general setting, we have not been able to check that they were always ascertained at equilibrium.[23] It should nonetheless be emphasized that with the particular specification used in the next section, the necessary conditions are indeed sufficient.

We now show that some important features of the PDP equilibria carry over to the FDP equilibria. More precisely, we show that the indifferent types are the same in the FDP and the PDP equilibria.

As previously, the analysis proceeds in two steps: first, determination of the equilibrium given some assumptions on the behavior of the consumer's rent w.r.t. the heterogeneity parameters; second, determination of the indifferent types.

Equilibrium quality profiles given the indifferent types. We assume first that the equilibrium rent profile is monotonic in the adverse selection parameters. Therefore, from the viewpoint of firm i there exists an indifferent type $\theta_j^* \in \{\underline{\theta}_i, \overline{\theta}_i\}$ such that the participation constraint is binding, or

$$U(\theta_i, \theta_j^*) = \begin{cases} U_i^{out}(\theta_j^*) & \text{with substitutability on the extensive margin,} \\ 0 & \text{with complementarity on the extensive margin.} \end{cases}$$

Denote by $\tilde{\theta}_j$ the boundary of Θ_j different from θ_j^*. In Appendix A.3, we show that the optimal quality of product i for a given price schedule offered by firm j is given by

$$\hat{U}^i_{q_i}(\theta, q_i(\theta)) - C_{iq_i}(q_i(\theta)) = -\frac{\int_{\tilde{\theta}_j}^{\theta_j} f_j(x)dx}{f_j(\theta_j)} \frac{\partial^2 \hat{U}_i}{\partial q_i \partial \theta_j}(\theta_j, q_i(\theta)). \tag{13}$$

Under our assumptions, and as in the PDP equilibrium case, the distortions on the best-responses (and hence on the equilibrium quality profiles) created by asymmetric information only depend on the indifferent types.[24] For further references, we note that the consumer's rent can be

[23]This is symptomatic of the common agency framework. Our presumption is that as soon as one deviates from the perfectly symmetric standard common agency framework it becomes often impossible to ensure that these second-order conditions are satisfied without putting more structure on the model; this issue appears in Laffont and Pouyet (2004) or Olsen and Osmundsen (2002) for instance, who assume the existence of a common agency differentiable equilibrium and focus on necessary conditions. Since our framework is not symmetric (e.g., w.r.t. the adverse selection parameters), it is not surprising that we encounter the same difficulty. Moreover, the equilibrium quality profiles are characterized by nonlinear partial differential equations, which are not amenable to apply the methodology proposed by Martimort (1992) to check that the different optimality conditions are satisfied at equilibrium.

[24]In Appendix A.3, we show that equilibrium quality levels are characterized by a set of nonlinear partial differential equations.

rewritten as follows

$$U(\theta) = \begin{cases} U_i^{out}(\theta_j^*) + \int_{\theta_j^*}^{\theta_j} \frac{\partial \hat{U}^i}{\partial \theta_j}(y, q_i(\theta_i, y))dy & \text{with substitutability on the extensive margin,} \\ \int_{\theta_j^*}^{\theta_j} \frac{\partial \hat{U}^i}{\partial \theta_j}(y, q_i(\theta_i, y))dy & \text{with complementarity on the extensive margin.} \end{cases}$$

The indifferent types with substitutability on the extensive margin. Remember that the buyer's outside opportunity w.r.t. firm i is defined by $U_i^{out}(\theta_j) \equiv \max_{q_j} \{-P_j(\theta_j, q_j) + U^g(\theta, 0, q_j)\} = \hat{U}_i(\theta_j, 0)$. Therefore, the derivative of the net rent is given by

$$\frac{\partial}{\partial \theta_j}\left[U(\theta) - U_i^{out}(\theta_j)\right] = \int_0^{q_i(\theta)} \frac{\partial^2 \hat{U}_i}{\partial \theta_j \partial q_i}(\theta_j, x)dx \leq 0.$$

This implies that the indifferent types are $(\overline{\theta}_i, \overline{\theta}_j)$, as in the PDP equilibrium.

The indifferent types with complementarity on the extensive margin. Using the corresponding writing of the firm's rent, it is immediate to show that if $\frac{\partial U}{\partial \theta_j}(\theta) \geq 0$ then it must be the case that $\frac{\partial U}{\partial \theta_i}(\theta) \leq 0$: the indifferent types are either $(\underline{\theta}_i, \overline{\theta}_j)$ or $(\overline{\theta}_i, \underline{\theta}_j)$.

Discussion. As shown in the previous analysis, the indifferent types in the FDP equilibria are the same as the indifferent types in the corresponding PDP equilibria. The best-responses in a FDP equilibrium are therefore shifted in the same direction than in the corresponding PDP equilibrium. However, in terms of equilibrium quality levels, there is a difference between the fully-discriminatory and partially-discriminatory pricing cases. Indeed, in a FDP equilibrium, there is no distortion on the quality levels for a consumer with type $(\tilde{\theta}_i, \tilde{\theta}_j)$, whereas in the PDP equilibrium, there is no distortion on the quality levels for a consumer with type (θ_i^*, θ_j^*). Consequently, the pattern of distortions on the equilibrium quality levels in the FDP case is 'opposite' to the one obtained in the PDP case. To illustrate this point, let us consider two examples:

- *Buyers with large valuations for both goods.* When θ_i and θ_j are high, then, as regards the full information benchmark, there will be weak distortions on the quality levels in the PDP case, but large downward distortions in the FDP case. A reverse conclusion in the case the buyer has low valuations for both products.

- *Buyers with a large valuation for one product and a low valuation for the other good.* Consider for instance that θ_i is low and θ_j is large. In the PDP equilibrium, the quality of good i is distorted upwards whereas the quality of product j is distorted downwards; in the corresponding FDP equilibrium, the reverse holds.

6 Comparisons of Pricing Policies

In this section, we perform a welfare analysis of the different equilibria. In order to derive explicit solutions, we use a quadratic specification of the consumer's gross utility function[25], quadratic and identical cost functions[26] and assume that the adverse selection parameters are uniformly distributed on their respective support. Computing the equilibrium quality and price levels is straightforward in a PDP equilibrium. However, it becomes more involved in a FDP equilibrium; in that case, we focus attention on solutions of the nonlinear partial differential equations which are linear in the adverse selection parameters. The determination of the price schedules is not inherently difficult but requires to perform cumbersome calculus that will not be detailed here.[27]

To make sensible comparisons, we compute the expected welfare, firm's profit and consumer's rent, where the expectation is taken over both adverse selection parameters. Before presenting our results, let us emphasize that the PDP and the FDP equilibrium perfectly coincide when goods are independent from the consumer's viewpoint (i.e., when $U^g_{q_1 q_2} = 0$). Indeed, in that case firm i cannot affect the consumer's incentive to reveal θ_j since the choices of q_i and q_j are independent.

Result 1. *With substitutability on the extensive margin, from the firms' viewpoint, partially-discriminatory pricing dominates fully-discriminatory pricing.*

To trace out fully the origin of this result, it is worth mentioning that in both equilibria the profit of each firm (gross of the quality cost) can be decomposed into two parts. The first (positive) part corresponds to the standard surplus extracted from the consumer (modulo the information rent given up to that consumer for incentive reason) and the second (negative) part corresponds to the outside opportunity of consumers which has to be given up by the firm.

Fully-discriminatory pricing enable firms to finely tailor their offers to the characteristics of the buyer. Hence, discriminatory pricing performs better in extracting surplus from the different types of consumer. However, discriminatory pricing also triggers an intense competition between firms, which provides the consumer with higher outside opportunities, thereby leading to higher equilibrium rents. As said earlier, the 'threat' to consume only one good is credible and a uniform pricing strategy turns out to be preferred by firms. Although it may appear at first sight inefficient, the decision not to tailor offers to both consumer's characteristics allows the duopolists to somehow refrain themselves from competing: *Strategic ignorance of consumer's taste for the rival's brand softens competition*!

Finally, Result 1 is also in the spirit of those obtained in the literature on price discrimination under complete information (e.g., Thisse and Vives, 1988, Holmes, 1989, and Corts, 1998, among others). In these papers, uniform pricing may dominate third-degree price discrimination. Our result extends this argument to settings with both first- and second-degree price discrimination

[25] $U^g = (a+\theta_1)q_1 - \frac{1}{2}q_1^2 + (a+\theta_2)q_2 - \frac{1}{2}q_2^2 - \lambda q_1 q_2$ with $\lambda \in (-1, 0]$. See Ivaldi and Martimort (1994) or Miravete and Röller (2003, 2004) for theoretical and applied analysis using a similar specification for the utility function.

[26] $C_i(q_i) = \frac{1}{2}q_i^2$, $i = 1, 2$.

[27] These computations are available at the following url: `http://www.enpc.fr/ceras/pouyet/working_papers.htm`.

by arguing that partial discrimination dominates full discrimination when the consumer can threaten each firm not to consume its good.

Besides, Result 1 has also some empirical implications. Indeed, it argues, first, that the 'nature' of asymmetric information (i.e., whether firms are symmetrically or asymmetrically un-informed) is important and should be tested and, second, that attention should not be restricted exclusively to nonlinear tariffs since 'simpler contracts' might be used by firms.

For instance, Miravete and Röller (2003, 2004) consider competition in the cellular telephone industry; their structural model follows the lines of Ivaldi and Martimort (1994) and therefore is identical to our model except that firms are un-informed about both private information parameters; however, they also consider that firms have a priori different knowledge about the consumer's characteristics and argue that one possible reason is that a firm has a better knowledge than its rival about some of the consumer's characteristics. Ivaldi and Martimort (1994) focus on the French energy distribution market; analyzing the data, they argue that asymmetric information is multi-dimensional but that "there is not too much asymmetric information" (p. 96) and that there is evidence of discrimination between groups of buyers; moreover, they argue that an energy supplier usually observes the technology used by a client and therefore might be informed on the client's willingness-to-pay for its energy. This suggests that in these contexts it would be relevant to test the nature of incomplete information.

Moreover, these papers focus mainly on nonlinear price schedules. Our analysis suggests that simpler contracts can emerge as a strategic response in a competitive environment when consumers have the option to shop exclusively with one competitors at equilibrium. This may explain why fully nonlinear tariffs are seldom observed in practice.[28]

7 Conclusion

In this paper, we have developed a model which allows to explain situations in which competing firms price-discriminate, yet sometimes provide lower valuations consumers with higher utility levels. Our model also offers a rich environment that incorporates both direct and indirect price discrimination and that could serve as a building block for more applied work.

Some interesting questions have been left aside. For instance, we have assumed that firms can credibly commit to a partially-discriminatory pricing policy. It would be worth investigating if firms manage to reach their preferred outcome when they can choose (prior to the competition stage) either to discriminate or not. In the same vein, if firms are able to credibly disclose their information to their competitors prior the competition stage, it would be interesting to study whether they decide to share their information of if some prisoner's dilemma occurs at equilibrium.

Recent research in mechanism design has started to examine the impact of information sharing between principals. In a setting with one informed and one uninformed principals, Bond and

[28]Miravete (2004) considers a monopoly under incomplete information and argues that the firm should use 'simple' mechanisms since the gains to offer fully discriminatory contracts are negligible. Our model shows that a qualitatively similar result holds in a competitive environment under certain conditions, but for different reasons.

Gresik (1998) show that the possibility for the informed principal to communicate his information to the uninformed principal leads to no effective information transmission at equilibrium.[29] In a sequential common agency context, Calzolari and Pavan (2004) study the possibility for the first-mover principal to disclose the information learned from contracting with the agent to the last-mover principal and show that, under certain conditions, information transmission between principals occurs at equilibrium. Coming back to our setting of asymmetrically un-informed principals, it would be worth investigating the possibility for one principal to extract the piece of information commonly known by the rival principal and the agent through, for instance, Maskinian mechanisms.[30]

These extensions are left for future research.

A Appendix

A.1 The Complete Information Equilibrium

Determination of the Equilibrium Price Schedules. Let us start with the case of substitutability on the extensive margin. Consider that firms offer price schedules of the form $P_i(\theta, q_i) = \gamma_i(\theta) + \tilde{P}_i(\theta, q_i)$. In order to provide the consumer with the correct incentive, firm i should offer a two-part tariff with the variable part equal to the marginal cost corresponding to the equilibrium quality level: $\tilde{P}_i(\theta, q_i) = C_{iq_i}(q_i^{FB}(\theta)) \times q_i$. Let consider the determination of the fixed part. Using the fact that, first, outside opportunities are positive and, second, that the participation constraints are binding, we obtain that

$$\gamma_i(\theta) = \max_{q_1,q_2} \left\{ -\tilde{P}_1(\theta, q_1) - \tilde{P}_2(\theta, q_2) + U^g(\theta, q_1, q_2) \right\} - \max_{q_j} \left\{ -\tilde{P}_j(\theta, q_j) + U^g(\theta, q_i = 0, q_j) \right\}.$$

In the case of complementarity on the extensive margin, only the sum of the fixed part of the price schedules is determined at equilibrium.

The Validity of the First-Order Approach The consumer's problem is

$$\max_{q_1,q_2} -P_1(\theta, q_1) - P_2(\theta, q_2) + U^g(\theta, q_1, q_2).$$

Since firms' equilibrium price schedules consist in two-part prices schedules, it is immediate to check that this problem is locally and globally concave.

[29] More precisely, they show that the most efficient equilibrium when principals can communicate is equivalent to a pooling equilibrium that emerges in the game with no direct communication between principals.

[30] See Maskin (1979).

Behavior of the Outside Opportunities. We can rewrite the outside opportunities as follows

$$
\begin{aligned}
U_i^{out}(\theta_j) &\equiv -\gamma_j(\theta) + \max_{q_j} \left\{ -\tilde{P}_j(\theta, q_j) + U^g(\theta, q_i = 0, q_j) \right\}, \\
&= \max_{q_1, q_2} \left\{ -\tilde{P}_1(\theta, q_1) - \tilde{P}_2(\theta, q_2) + U^g(\theta, q_1 = 0, q_2) + U^g(\theta, q_1, q_2 = 0) \right\} \\
&\qquad - \max_{q_1, q_2} \left\{ -\tilde{P}_1(\theta, q_1) - \tilde{P}_2(\theta, q_2) + U^g(\theta, q_1, q_2) \right\}.
\end{aligned}
$$

Therefore, we obtain the following expression of the derivative of the outside opportunity w.r.t. the relevant valuation (we omit some arguments for the sake of clarity)

$$
\begin{aligned}
\frac{dU_i^{out}}{d\theta_j}(\theta_j) &= \int_{q_i^{FB}}^{q_i^{out}} -\tilde{P}_{i\theta_j q_i}(\theta, x) dx + \int_{q_j^{FB}}^{q_j^{out}} \left[-\tilde{P}_{j\theta_j q_j}(\theta, x) + U^g_{\theta_j q_j}(\theta_j, x) \right] dx, \\
&= \int_{q_i^{FB}}^{q_i^{out}} -C_{iq_iq_i}(q_i^{FB}) \frac{\partial q_i^{FB}}{\partial \theta_j} dx + \int_{q_j^{FB}}^{q_j^{out}} \left[-C_{jq_jq_j}(q_j^{FB}) \frac{\partial q_j^{FB}}{\partial \theta_j} + U^g_{\theta_j q_j}(\theta_j, x) \right] dx.
\end{aligned}
$$

Under our assumptions the terms in the first and the second integral are positive. Finally, notice that by construction, we have $q_j^{FB} = \hat{q}_j(\theta, q_j^{FB}(\theta))$ and $q_j^{out} = \hat{q}_j(\theta, 0)$. Differentiating (12) w.r.t. q_i and using the fact that the consumer's problem is concave, we immediately obtain that $\partial \hat{q}_j / \partial q_i \leq 0$, thereby implying that $q_j^{out} \geq q_j^{FB}$. This finally enables to conclude that $\partial U_i^{out}(\theta_j)/\partial \theta_j \geq 0$.

A.2 Proof of Lemma 2

We adapt the proof of Ivaldi and Martimort (1994) to the PDP equilibrium.

First, consider that both outside opportunities strictly negative at equilibrium. Then we have $-p_i(\theta_i) + u_i(\theta_i, q_i(\theta_i)) + u(0, q_i(\theta_i)) < 0$ for $i = 1, 2$, implying that $-p_1(\theta_1) - p_2(\theta_2) + u_1(\theta_1, q_1(\theta_1)) + u_2(\theta_2, q_2(\theta_2)) + u(0, q_2(\theta_2)) + u(q_1(\theta_1), 0) < 0$. With demand substitutes, the following inequality holds: $u(q_1(\theta_1), 0) + u(0, q_2(\theta_2)) > u(0, 0) + u(q_1(\theta_1), q_2(\theta_2))$, implying in turn that the equilibrium rent of the agent must be strictly negative, a contradiction.

Second, consider that goods are substitutes and that $U_2^{out}(\theta_1) > 0$. Since the net rent is decreasing in θ_1, we have $U(\overline{\theta}_1, \theta_2) = U_2^{out}(\overline{\theta}_1)$, or $p_2(\theta_2) = u_2(\theta_2, q_2(\theta_2)) + u(q_1(\overline{\theta}_1), q_2(\theta_2)) - u(q_1(\overline{\theta}_1), 0)$. Then, simple manipulations show that $U_1^{out}(\theta_2) = -\int_0^{q_1(\overline{\theta}_1)} \int_0^{q_2(\theta_2)} u_{q_1 q_2}(x, y) dy dx > 0$.

Third, consider that goods are complements and that $U_2^{out}(\theta_1) > 0$. Since the net rent is increasing in θ_1, we have $U(\underline{\theta}_1, \theta_2) = U_2^{out}(\underline{\theta}_1)$, or $p_2(\theta_2) = u_2(\theta_2, q_2(\theta_2)) + u(q_1(\underline{\theta}_1), q_2(\theta_2)) - u(q_1(\underline{\theta}_1), 0)$. Then, simple manipulations show that $U_1^{out}(\theta_2) = -\int_0^{q_1(\underline{\theta}_1)} \int_0^{q_2(\theta_2)} u_{q_1 q_2}(x, y) dy dx < 0$.

A.3 The Fully Discriminatory Pricing Equilibrium

The Hamiltonian associated to firm i's problem is

$$H_i = f_j(\theta_j)\left\{\hat{U}^i(\theta, q_i(\theta)) - C_i(q_i(\theta)) - U(\theta)\right\} + \mu_j(\theta_j)\frac{\partial \hat{U}^i}{\partial \theta_j}(\theta_j, q_i(\theta)),$$

where $\mu_j(\theta_j)$ is the co-state variable. Since there is no transversality condition in $\theta_j = \tilde{\theta}_j$, we have $\mu_j(\tilde{\theta}_j) = 0$. The Maximum Principle implies that $-\frac{\partial H_i}{\partial U} = \dot{\mu}_j(\theta_j)$. Therefore, we obtain that $\mu_j(\theta_j) = \int_{\tilde{\theta}_j}^{\theta_j} f_j(x)dx$. Then, optimizing w.r.t. the control variable $q_i(\theta)$, we obtain (13).

We can even further simplify (13). At equilibrium, consistency imposes that $\hat{q}_j(\theta_j, q_i(\theta)) = q_j(\theta)$; therefore, the equilibrium consumption profiles satisfy the following relations

$$-P_{jq_j}(\theta_j, q_j(\theta)) + U^g_{q_j}(\theta_i, \theta_j, q_i(\theta), q_j(\theta)) = 0. \tag{14}$$

Differentiation of (14) w.r.t. θ_i and θ_j enables to simplify (13) as follows (we omit arguments for simplicity)

$$U^g_{q_i}(\theta, q_i, q_j) - C_{iq_i}(q_i) = \frac{\int_{\tilde{\theta}_j}^{\theta_j} f_j(x)dx}{f_j(\theta_j)} \frac{\frac{\partial q_i}{\partial \theta_j}\frac{\partial q_j}{\partial \theta_i} - \frac{\partial q_i}{\partial \theta_i}\frac{\partial q_j}{\partial \theta_j}}{\frac{\partial q_i}{\partial \theta_i}} u_{q_i q_j}(q_i, q_j)$$

with initial conditions $q_i(\theta_i, \tilde{\theta}_j) = q_i^{FB}(\theta_i, \tilde{\theta}_j)$.

B References

Bernheim, D. and M. Whinston, 1986, "Common Agency", *Econometrica*, 54: 923-943.

Bond, E. and T. Gresik, 1997, "Competition Between Asymmetrically Informed Principals", *Economic Theory*, 10: 227-240.

Bond, E. and T. Gresik, 1998, "Incentive Compatible Information Transfer between Asymmetrically Informed Principals", *Mimeo*.

Calzolari, G. and A. Pavan, 2004, "On the Optimality of Privacy in Sequential Contracting", *Working Paper*, Northwestern University.

Corts, K., 1998, "Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment", *RAND Journal of Economics*, 30: 441-455.

Green, J. and J.J. Laffont, 1977, "Characterization of Satisfactory Mechanisms for the Revelation of Preferences for Public Goods", *Econometrica*, 45, 427-35.

Guesnerie, R. and J.-J. Laffont, 1984, "A Complete Solution to a Class of Principal-Agent Problems with an Application to the Control of a Self-Managed Firm", *Journal of Public Economics*, 25: 329-369.

Holmes, T., 1989, "The Effects of Third-Degree Price Discrimination in Oligopoly", *American Economic Review*, 79: 244-250.

Ivaldi, M. and D. Martimort, 1994, "Competition under Nonlinear Pricing", *Annales d'Economie et Statistiques*, 34, 72-114.

Laffont, J.J. and J. Pouyet, 2004, "The Subsidiarity Bias in Regulation", *Journal of Public Economics*, 88(1-2): 255-283.

Liu, Q. and K. Serfes, 2003, "Customer Information Sharing Among Rival Firms", *Mimeo*, University of Massachusetts.

Martimort, D., 1992, "Multiprincipaux avec Anti-Sélection", *Annales d'Economie et Statistiques*, 28: 1-38.

Martimort, D. and L. Stole, 2003a, "Contractual Externalities and Common Agency Equilibria", *Advances in Theoretical Economics*, 3(1), Article 4.

Martimort, D. and L. Stole, 2003b, "Market Participation under Delegated and Intrinsic Common Agency Games", *Mimeo.*

Maskin, E. and J. Riley, 1984, "Monopoly with Incomplete Information", *RAND Journal of Economics*, 15: 171-196.

Miravete, E., 2004, "Are all those Calling Plans Really Necessary? The Limited Gains from Complex Tariffs", CEPR DP 4237.

Miravete, E. and L.-H. Röller, 2003, "Competitive Nonlinear Pricing in Duopoly Equilibrium: The Early U.S. Cellular Telephone Industry", *Working Paper*, University of Pennsylvania.

Miravete, E. and L.-H. Röller, 2004, "Estimating Markups under Nonlinear Pricing Competition", Journal of the European Economic Association, 2, 526-535.

Myerson, R., 1981, "Optimal Auction Design", *Mathematics of Operations Research*, 6: 58-63.

Olsen, T.E. and P. Osmundsen, 2002, "Strategic Tax Competition; Implications of National Ownership", *Journal of Public Economics*, 81(2): 253-277.

Parker, G. and M. Van Alstyne, 2000, "Information Complements, Substitutes, and Strategic Product Design", mimeo Tulan University.

Shapiro, C. and H. Varian, 1998, *Information rules: A strategic guide to the network economy*, Harvard Business School Press.

Spulber, D., 1989a, "Product Variety and Competitive Discounts", *Journal of Economic Theory*, 48: 510-525.

Spulber, D., 1989b, *Regulation and markets*, Cambridge, MA: The MIT Press.

Stole, L., 1991, "Mechanism Design under Common Agency", *Mimeo*, Department of Economics, MIT.

Stole, L., 2001, "Price Discrimination and Imperfect Competition", *Mimeo*, University of Chicago, Graduate School of Business.

Thisse, J.-F. and X. Vives, 1988, "On the Strategic Choice of Spatial Price Policy", *American Economic Review*, 70: 651-659.

Wang, Y., 2003, "A joint Application of Third-Degree Price Discrimination and Network Effect on Software Pricing", *Communication of the International Information Management Association*, 3(3): 103-106.