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# On the determination of class-based storage assignments in an AS/RS having two I/O locations 

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#### Abstract

This paper presents the use and extension of a geometrical-based algorithmic approach for determining the expected $\mathrm{S} / \mathrm{R}$ machine cycle times, and therefore warehouse throughput, for class-based storage assignment layouts in an AS/RS. The approach was designed for the purpose of solving a practical storage assignment design problem for a major European manufacturer.

The algorithm may be used to layout racks that are either "square-in-time (SIT)" or "non-square in time (NSIT)" with single or multiple I/O points. It is believed that use of this approach can result in the expedient evaluation of warehouse throughput resulting from new layouts, or re-layouts, of AS/R system racks, thus making these systems more appealing for use in integrated supply chain systems in general.


## 1. Introduction

This paper presents the use and extension of a geometrical-based algorithmic approach for determining the expected $\mathrm{S} / \mathrm{R}$ machine cycle times, and therefore warehouse throughput, for class-based storage assignment layouts in an AS/RS. The approach was designed for the purpose of solving a practical storage assignment design problem for a major European manufacturer.

The manufacturer uses an AS/RS system to handle pallet loads of finished goods awaiting shipment. The system has two input/output (I/O) stations at the end of each aisle, located at floor level. Due to the frequent introduction of new products, there is a need to periodically reassign storage locations for classes of goods in the AS/RS rack structure in an effort to accommodate more SKU's and an ever growing number of pallet loads of each product.

The resulting rack layout may be either "square in time (SIT)" or "non-square in time (NSIT)". It is believed that use of this approach can result in the expedient evaluation of warehouse throughput resulting from re-layouts of the AS/R system racks, thus making these systems more appealing for use in integrated supply chain systems in general.

## 2. Literature review

There is a rich literature dealing with the operation of AS/RS systems. Researchers began with basic results such as computing the expected value and/or the distribution

[^0]of single- and dual-command cycle times for storage/retrieval machines [see Hausman et. al. (1976), Graves et. al. (1977), Bozer and White (1984), Foley and Frazelle (1991) Chang et. al. (1995), Kouvelis and Papanicolaan (1995), Sarker and Babu (1995), among others]. Then, operational issues such as $S / R$ machine dwell point strategies or storage/retrieval operation sequences received some attention [Egbelu and Wu (1993), Hwang and Lin (1993), Elsayed and Lee (1996), Lee and Schaefer (1996), Peters et. al. (1996), Chang and Egbelu (1997 a), Chang and Egbelu (1997 b),]. Later, twin-shuttle S/R machines [Keserla and Peters (1994), Sarker et. al. (1994)], multi-shuttle machines [Meller and Mungwattana (1997)], and storage and retrieval matching or AS/RS control/design strategies [Han et. al. (1987), Seidman (1988), Lim and Wysk (1990), Rosenblatt et. al. (1993), Wang and Yih (1997)] were considered. Simulation-based approaches have been employed to deal with random arrivals of storage and retrieval requests to an AS/RS [Lee (1997), Bozer and Cho (1998)]. Also, expected cycle time performance for AS/R systems having unequal sized cells, randomized storage assignments, and single- and dual-command cycles has been estimated [Lee et. al.(1999)]. One paper [Pan and Wang (1996)] takes a similar approach to that presented in this paper in developing a framework for the dual-command cycle, continuous travel, square-in-time model under class-based assignment.

Taken together, these studies have lead to a better understanding of the optimal operation of automated storage and retrieval systems. Furthermore, they have facilitated system design and performance evaluation of AS/R System.

## 3. Basic concepts

Veraart (1995) developed a geometry-based model to calculate the expected cycle time of an S/R machine that is the basis of this paper [Ashayeri, et al (1997) and Ashayeri, et al (2001)]. The assumptions underlying his model, together with the additional ones required for the model extension discussed in this paper are presented below. Most of the assumptions listed are common in the models of AS/RS operations cited in the preceding literature review section, i.e.:

1. A continuous approximation to the discrete rack face.
2. Each pallet holds only one part number or product type.
3. The system consists of a single $\mathrm{S} / \mathrm{R}$ machine serving a single aisle, providing access to two, single-deep storage rack structures on either side.
4. The S/R machine is capable of simultaneously moving both vertically and horizontally at constant speeds. Thus, the travel time required to reach any location in the rack is approximated by the Tchebyshev metric.
5. The S/R machine has a single shuttle and can operate only in single- or dualcommand modes.
6. The actual time required for the $\mathrm{S} / \mathrm{R}$ machine to load or unload a pallet at the I/O points or at a storage location is ignored, as is the time taken by the $\mathrm{S} / \mathrm{R}$ machine to travel from any external input/output device or hardware to the I/O points in the rack structure.
7. The fraction of single-command cycles, $f$, is known for a given planning horizon.
8. All pallets are randomly stored in empty locations within the appropriate zone assigned to the class of goods on the pallet.
9. Short-run dynamic considerations, like possible dependencies between successive retrieval and storage transactions, or seasonal demand distortions, are ignored.
10. Storage and retrieval requests are triggered independently of each other and are processed according to a first-come, first-served discipline.
11. There are two I/O stations per aisle, located at floor level on opposite ends.
12. The mean frequency of movements (probabilities), $p_{i}$, for each product to be stored in the AS/RS in each storage zone, assigned to each class of goods, can be determined using the data collected within the Warehouse Management System (WMS) and treated as a known constant for a specific time horizon.

It is routinely accepted that there are three ways of assigning products to storage locations in warehouses: random storage, class-based storage and dedicated storage. Class-based storage is effective when there are many products having different residence times in the storage system. The assignment of products to storage zones can be done with the help of an ABC analysis [Hausman, et al (1976)]. In this analysis all products are ranked by a criterion such as throughput per time period. The products on the resulting list are divided into groups or classes. Each class is then assigned to a zone of the warehouse consisting of a certain number of contiguous storage rack locations.

The general model requires the computation of the expected cycle time per storage/retrieval operation (transaction). Under the foregoing assumptions, the expected cycle time is as follows:

$$
\begin{aligned}
E(T) & =f \cdot E\left(T_{S C}\right)+(1-f) E\left(T_{D C}\right) \\
& =f \cdot \sum_{i} \sum_{k} \sum_{m} p_{k, i} \cdot p_{i, m}\left(E_{k, i}+E_{i, m}\right)+(1-f) \sum_{i} \sum_{k} \sum_{m} \sum_{j} p_{k, i} \cdot p_{i, j} \cdot p_{j, m}\left(E_{k, i}+E_{i, j}+E_{j, m}\right)
\end{aligned}
$$

where,
$f=$ percentage of single command transactions
$E\left(T_{S C}\right)=$ expected single-command time per transaction
$E\left(T_{D C}\right)=$ expected dual-command time per transaction and, the remaining terms in equation are defined in the following sections.

### 3.1. Calculation of $E\left(T_{S C}\right)$



Figure 1. Single-command transactions

As illustrated in Figure 1, a single-command (SC) transaction is the movement from either I/O port to a location in the rack to store or retrieve a pallet, plus the movement from this location back to an I/O port. Thus, the expected travel time for a single-command transaction is the weighted sum of the travel times of all possible single command transactions:

$$
\begin{align*}
& E\left(T_{S C}\right)=\sum_{i}\left[p_{I O 1, i} \cdot p_{i, I O 1}\left(E_{I O 1, i}+E_{i, I O 1}\right)+p_{I O 1, i} \cdot p_{i, I O 2}\left(E_{I O 1, i}+E_{i, I O 2}\right)+\right. \\
& \left.\quad p_{I O 2, i} \cdot p_{i, I O 1}\left(E_{I O 2, i}+E_{i, I O 1}\right)+p_{I O 2, i} \cdot p_{i, I O 2}\left(E_{I O 2, i}+E_{i, I O 2}\right)\right] \\
& =\sum_{i} \sum_{k} \sum_{m} p_{k, i} \cdot p_{i, m}\left(E_{k, i}+E_{i, m}\right) \tag{2}
\end{align*}
$$

where,
$E_{k, i}=\quad E_{k, i}\left(t_{i}\right)=$ expected travel time between input port $k$ and a random location in zone $i$
$E_{i, m}=E_{i, m}\left(t_{i}\right)=$ expected travel time between a random location in a storage zone and an output port $m$
$p_{k, i}=$ probability that a movement from input port $k$ to zone $i$ takes place
$p_{i, m}=$ probability that a movement from zone $i$ to output port $m$ takes place $k, m=\mathrm{I} / \mathrm{O} 1, \mathrm{I} / \mathrm{O} 2$.

Note that the expected travel time in a warehouse with only one I/O port located at one end of the aisle can be found by setting $p_{i, m}$ in equation (2) to 1 and $E_{k, i}=E_{i, m}$.

The expected travel times in equation (2) are computed using a modification of the Turbo Pascal computer code developed by Veraart (1995) and Valkenburg (1997), due to the tedious mathematical evaluations required by this methodology. The resulting code requires only a few seconds to compute the expected $\mathrm{S} / \mathrm{R}$ machine cycle time when the coordinates of the storage zones in a rack layout design are given. The determination of these zones will be discussed in sections 3.4 and 3.5 below.

To apply equation (2) in a more general form for a warehouse with $p$ input ports and $q$ output ports, allow the indices of k and m to range over all $\mathrm{I} / \mathrm{O}$ ports, i.e., $k=$ $\mathrm{I} / \mathrm{O} 1,2, . ., p$, and $m=\mathrm{I} / \mathrm{O} 1,2, . ., q$.

### 3.2. Calculation of $E\left(T_{D C}\right)$

As illustrated in Figure 2, a dual-command (DC) transaction is a crane movement from an I/O port to a location to store a product, and, without returning to an I/O port, a movement to a new location to retrieve a product, and then a movement to an I/O port to deposit the retrieved load.


Figure 2. Dual-command transactions

The equation for computing the expected travel time of a dual-command transaction is composed in a similar fashion to that presented for single-command transactions, i.e.:

$$
\begin{align*}
E\left(T_{D C}\right)= & \sum_{i}\left[\sum_{j} p_{I O 1, i} \cdot p_{i, j} \cdot p_{j, I O 1}\left(E_{I O 1, i}+E_{i, j}+E_{j, I O 1}\right)+\right. \\
& \sum_{j} p_{I O 2, i} \cdot p_{i, j} \cdot p_{j, I O 2}\left(E_{I O 2, i}+E_{i, j}+E_{j, I O 2}\right)+ \\
& \sum_{j} p_{I O 1, i} \cdot p_{i, j} \cdot p_{j, I O 2}\left(E_{I O 1, i}+E_{i, j}+E_{j, I O 2}\right)+ \\
& \left.\sum_{j} p_{I O 2, i} \cdot p_{i, j} \cdot p_{j, I O 1}\left(E_{I O 2, i}+E_{i, j}+E_{j, I O 1}\right)\right] \\
= & \sum_{i} \sum_{k} \sum_{m} \sum_{j} p_{k, i} \cdot p_{i, j} \cdot p_{j, m}\left(E_{k, i}+E_{i, j}+E_{j, m}\right) \tag{3}
\end{align*}
$$

where,

$$
\begin{aligned}
E_{i, j} & =E\left(t_{i j}\right)=\text { expected travel time between a random point in zone } i \text { and } \\
& \text { a random point in zone } j
\end{aligned}
$$

Using equations (2) and (3), the expected cycle time per transaction $\mathrm{E}(\mathrm{T})$ in equation (1) can be computed for the general model. Here again, the expected travel times between random points in the storage zones in each class in equations (2) and (3) are computed using a modification of the Turbo Pascal computer code developed by Veraart (1995) and Valkenburg (1997), due to the tedious mathematical evaluations required by this methodology. For more details, we refer to Ashayeri, et al. (2001). The resulting code requires only a few seconds to compute the expected $\mathrm{S} / \mathrm{R}$ machine cycle time when given the coordinates of the storage zones in a rack layout design. The determination of these zones will be discussed in sections 3.4 and 3.5 below.

The probabilities of movement required in equations (2) and (3) are computed by the methods in the following section.

### 3.3. Determination of probabilities

The probability of a crane movement from input port $k$ to a product class storage zone $i$ is the probability that a movement starts at input port $k$ and (times) the conditional probability of a movement to zone $i$ given that the movement starts at input port $k$ :
$p_{k, i}=p_{k} \cdot p_{i \mid k}$
where,
$p_{k}=\quad$ percentage of movements that start at input port $k$ in period $P$ with respect to the total number of movements at both input ports in period $P$. $p_{i \mid k}=$ percentage of movements to zone $i$, from input port $k$ in period $P$ with respect to the total number of movements starting at input port $k$ in period $P$.
$P$ is the planning horizon, which is fixed at one year in this case study discussed in the next section. Note, in applications where the order pattern is quickly changing, the period $P$ should be reset more often. If needed, the model can easily be adjusted to incorporate the effect of time by having an extra index to represent the demand pattern in changing periods.

Similarly, the probability of a movement from storage zone $i$ to output port $m$ is equal to:

$$
p_{i, m}=p_{m \mid i}
$$

where,
$p_{m \mid i}=$ percentage of outgoing movements from zone $i$ to output port $m$ in period $P$ with respect to the total number of outgoing movements from zone $i$ in period $P$.

To determine the expected travel time of a dual-command transaction in equation (3), the probability of a movement from zone $i$ to zone $j$ must be calculated. The probability of a movement to a location in storage zone $j$ is independent of the storage zone from which the movement originates:

$$
p_{i, j}=p_{j}=\sum_{k} p_{k, j}
$$

where,
$k \in i=$ are the storage locations belonging to zone $i$.

### 3.4. Determination of the zone layout of the rack face

The algorithm used to determine the number of storage zones for classes of products, and their sizes is presented in this section. The ranking that was found by performing an ABC analysis on the product demand is utilized. The products in this ranking are assigned to locations in the warehouse in the order of their appearance on the ranking list. For each product, the mean required number of storage locations in the warehouse per day must be known (a priori).

## Algorithm

Step 1. Initialization: $i=1, l=1$ and $E(T)=\infty$.
Step 2. Form the smallest possible zone $i$ from unassigned storage locations (see also below the extension of a zone). The new zone should form a square-intime zone together with any zones defined in prior iterations. Compare the number of available storage locations in the new zone with the required number of locations for the product with index $l$ in the ranking list. STOP, if all locations in the warehouse have been assigned.
Step 3a. If the required number of locations for storage of product $l$ is larger than the number of available locations, then enlarge zone $i$ in a diagonal direction until the number of available locations in the zone is sufficient to accommodate the storage of all pallets of product with index $l$. STOP if all locations in the warehouse have been assigned.
Step 3b. If product $l$ fits into the new zone and there are still locations available, then assign the next products in the ranking list to the zone as long as locations are available. If the last product assigned to the new zone has index $l+n$, then $l:=(l+n)+1$.
Step 4. Calculate the expected cycle time per transaction, $E\left(T_{\text {new }}\right)$ and compare it with the current cycle time, $E(T)$.
Step 5a. If the cycle time $E\left(T_{\text {new }}\right)<E(T)$, set $E(T)=E\left(T_{\text {new }}\right)$, then proceed with step 3 a .
Step 5b. If the cycle time increased from the previous iteration, then $i:=i+1, E(T)=$ $\infty$ and go to step 2.

What happens in the algorithm is that a storage zone is expanded and products are assigned to it as long as the expected cycle time in the warehouse decreases. As soon as the expected cycle time increases, the borders of the zone will be fixed and the algorithm begins the determination of the next zone in a similar way. This process continues until all products have been assigned to storage zones and the entire warehouse has been divided into zones.

The extension of a zone in a diagonal direction can be done in several ways, e.g.:

1. Expand one storage location in the horizontal direction and $y$ locations in the vertical direction.
2. Expand one storage location in the vertical direction and $x$ locations in the horizontal direction.
3. Expand $x$ locations in the horizontal direction and $y$ locations in the vertical direction, simultaneously.

In all three cases, the number of storage locations $x$ and/or $y$ should be chosen so as to preserve the square-in-time (SIT) nature of the resulting storage zone. The numbers $x$ and $y$ are integer numbers of storage locations.

Note that $x$ and $y$ are not equal, given the slope of the SIT line. For example a pallet in the high bay warehouse of the case study discussed in the next section is 1.24 m width and 2.20 m hight. The horizontal and vertical speeds of cranes are respectively 2.33 $\mathrm{m} / \mathrm{s}$ and $2.20 \mathrm{~m} / \mathrm{s}$. Therefore, for a crane to move one pallet width it takes 0.53 seconds while one pallet hight it takes 3.28 seconds. Given the fact that minimum $x$ and $y$ is one pallet position, in our case while in the vertical direction a zone is extended by one pallet position, 3.28 seconds movement time, a crane moves in the horizontal direction a distance of $3.28 * 2.33=7.64$ meters. Thus after rounding, this is equal to 6 pallet positions in the horizontal direction.

A zone that is square-in-time means that as much time is necessary for the $\mathrm{S} / \mathrm{R}$ machine to reach the end of the zone as is necessary to reach the top of the zone when a movement starts at the a corner of the zone. The algorithm attempts to prescribe the formation of a smallest possible square-in-time zone to accommodate storage of a product at each iteration. This is achieved by expanding the zone by the first or second method cited above, depending upon the horizontal and vertical speeds of the crane. The third zone expansion method can be applied when the number of zones that must be formed is known in advance.

### 3.5. Determination of the shapes of the storage zones

The algorithm places the products that generate the largest number of movements into storage zones closest to an I/O point. As shown in Figure 3, in a warehouse with only one I/O port, the zones are formed around each other. The first zone will be square in time, the following zones have an L-shape, but, taken together with zones in previous iterations, they form either a square or rectangular zone.


Figure 3. Storage rack layout with the one I/O port in the lower left corner

In Figure 3, zones 1 and 2, taken together, constitute a square-in-time zone; whereas, zones 1,2 and 3 taken together constitute a rectangular, non-square-in-time zone. That is, beginning in the lower left corner, the time required by the crane to reach a storage location at the extreme right side of the combined zones and the time required by the crane to reach a storage location at the top of the combined zones are not equal.

In a warehouse with I/O ports at opposite ends of the aisle, the zones are formed, in turn, around each I/O port. Zones with irregular shapes can result. Again, the first zones will be square in time, then the following zones will be either square or rectangular. This continues until the borders of the zones approach the mid-section of the warehouse. From this point, subsequent iterations of the algorithm may possibly assign locations in a zone that have already been assigned to another zone. Such intersecting locations must be subtracted from all but one zone. The result of this process may be the formation of T-shaped zones, as illustrated in Figure 4.


Figure 4. Storage rack layout with two I/O ports located at opposite ends of the rack

The geometric model developed by Veraart (1995) calculates the expected cycle time for a class-based storage assignment layout for rectangular zones. Therefore, in order to use his method, all storage zones that do not have a rectangular shape are divided into rectangular components. It is assumed that products assigned to a zone are uniformly distributed over the storage zone. The probabilities of a crane movement to a location in any zone can be determined by dividing the area of this part by the total area of the original zone, and multiplying this number by the original probability.

## 4. Application of the algorithm and results

This algorithm was applied in the redesign of the storage assignments in the AS/RS of a major manufacturer in Europe, which had an I/O point in the front and rear of an aisle, with both I/O stations at floor level. Figure 5 shows the initial layout of storage rack. Table 1 provides information about the number of zones in the initial layout, the types of products stored in the zones, and the number of storage locations assigned to each zone.


Figure 5. Initial zone layout of the AS/RS

Table 1. Information concerning the products stored in the layout of Figure 5

| Zone <br> number | Classification of products | Number of locations <br> in each zone |
| :---: | :--- | :---: |
| 1 | A brands: Fast movers | 8187 |
| 2 | B brands: Average movers | 2868 |
| 3 | C brands: Slow movers | 1500 |
| 4 | Flammable materials | 1260 |
| 5 | Specials | 72 |
| 6 | Others | 156 |
| $7 / 8$ | Emergency A, B, C, Specials, others | 12 |

Figure 6 shows the re-layout of the rack resulting from application of the algorithm presented herein. And, Table 2 provides the storage data for the revised layout in Figure 6. Note that the zone numbering in Figure 6 does not correspond to the same number of pallet positions as depicted in Figure 5 due to a new ranking of product movements.

The expected cycle time per transaction for the revised layout is 52.61 seconds. Since no data was available regarding the cycle time of the initial layout, the expected cycle time per transaction for random storage assignment in the rack was taken as an approximation. The expected cycle time per transaction under that approximation assumption is 70.10 seconds. Thus, the revised layout from application of the classbased algorithm discussed in this paper results in a reduction in expected cycle time of $24.95 \%$ below that for a warehouse using random storage, i.e., no product class storage zones.


Figure 6. Revised storage zone layout of the AS/RS

Table 2. Information concerning the products stored in the layout of Figure 6

| Zone | Number <br> of locations | Percentage <br> of locations | Number <br> of products | Percentage <br> of products | Percentage of total <br> activity in zone |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 4704 | $33.62 \%$ | 89 | $24.05 \%$ | $33.42 \%$ |
| 2 | 1800 | $12.86 \%$ | 132 | $35.68 \%$ | $51.54 \%$ |
| 3 | 1236 | $8.83 \%$ | 56 | $15.14 \%$ | $6.35 \%$ |
| 4 | 3708 | $26.50 \%$ | 66 | $17.84 \%$ | $5.34 \%$ |
| 5 | 732 | $5.23 \%$ | 20 | $5.41 \%$ | $1.33 \%$ |
| 6 | 540 | $3.86 \%$ | - | - | - |
| 7 | 1272 | $9.09 \%$ | 7 | $1.89 \%$ | $2.02 \%$ |
| Total | 13992 | $100 \%$ | 370 | $100 \%$ | $100 \%$ |

## 5. Conclusions

A heuristic approach has been developed for use in storage location assignments for class-based layouts in AS/R systems. The algorithm determines the size, number of zones, and their relative layout in AS/RS storage racks.

The advantage of the class-based storage layout was illustrated in the results of an application of this algorithm to an AS/RS having two I/O ports at floor level on opposite ends of each aisle. After applying the algorithm the expected cycle time per transaction was reduced by almost $25 \%$ over that computed by assuming random storage of product loads throughout the rack structure.

Use of this approach has enabled a major European manufacturer to increase production of existing product lines and to accommodate the storage of new products in its existing AS/RS system. Another indirect benefit of the use of such a storage layout reassignment algorithm is the reduction of maintenance cost and down times of the $\mathrm{S} / \mathrm{R}$ machines by reducing their unnecessary movements.

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