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**MARKET EFFICIENCY AND PRICE FORMATION  
WHEN DEALERS ARE ASYMMETRICALLY  
INFORMED**

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**Discussion paper**

# Market Efficiency and Price Formation when Dealers are Asymmetrically Informed\*

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## Abstract

We consider the effect of asymmetric information on the price formation process in a quote-driven market where one market maker receives a private signal on the security fundamental. A model is presented where market makers repeatedly compete in prices: at each stage a bid-ask auction occurs and the winner trades the security against liquidity traders. We show that at equilibrium the market is not strong-form efficient until the last stage. We characterize a reputational equilibrium in which the informed market maker will affect market beliefs, and possibly misleads them. At this equilibrium, a price leadership effect arises, quotes are never equal to the expected value of the asset given the public information, the informed market maker expected payoff is positive and the rate of price discovery increases in the last stages of trade before the information becomes public.

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# 1 Introduction

Several empirical studies have shown that different market makers either have access to different levels of information, or at least differ in their understanding of market fundamentals. In the foreign exchange markets, Peiers (1997) and de Jong et al. (1999) have shown that some commercial banks are indeed commonly considered to have some informational advantage due to their preferential relation with the central bank. In the stock markets, Porter and Weaver (1998) have found some evidence that late trade reporting on the Nasdaq National Market System is normally associated with information contents: they interpreted this phenomenon by suggesting that “market makers choose to delay the reporting of those trades which either contain information about short-term price movements or reflect deviations from implicit quoting conventions”. Moreover, Cao, Ghysels and Hatheway (1998) have found evidence of leadership among market makers for a given stock looking at the pricing patterns in the Nasdaq pre-opening period. Similarly, Albanesi and Rindi (2000) have detected imitative pricing behavior in the Italian treasury bill market. These last studies suggest that dealers often know who among them are the best informed.

From a theoretical point of view, it is thus important to assess the characteristics of the price discovery process in a quote-driven market with transparent price competition among asymmetrically informed market makers. Indeed, in most of the above mentioned markets, the quotes posted by market makers are perfectly observable by all market participants<sup>1</sup>, whereas in models *à la* Kyle or Glosten and Milgrom, private information is held by floor traders who submit orders that are only partially observable by uninformed market makers.

This paper will give further inside on both financial microstructure theory and auction theory. From a financial market microstructure perspective, we will show that a highly transparent quote-driven market privately informed market maker optimally reveals his private signal through the choice of a noisy pricing strategy. The equilibrium we characterize explains empirically observed patterns such as price leadership (Peiers 1997), market manipulation (Cao et al. 1998) and excess quote volatility. Some of these phenomena cannot be explained by the standard microstructure models. As we model price competition between market makers as a sequential bid-ask auction,

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<sup>1</sup>See Figure 1 for Nasdaq

MMID	BID	SIZE	MMID	ASK	SIZE
LEHM	20	10	TORY	20 1/8	6
PRUS	20	10	SNDU	20 1/4	10
INCA	20	10	HMQT	20 1/4	10
MASH	20	10	SWCO	20 1/4	10
NITE	20	10	SALB	20 1/4	10
CHGO	20	10	MADF	20 1/4	10
DEAN	19 7/8	10	RSSF	20 1/4	10
ISLD	19 7/8	10	SWCO	20 1/4	10
BEST	19 7/8	10	HMQT	20 1/4	2
MONT	19 7/8	10	DEAN	20 3/8	10
SHWD	19 7/8	10	ISLD	20 3/8	10
DMGL	19 7/8	10	BEST	20 3/8	10
MSCO	19 7/8	10	MONT	20 3/8	10
SALB	19 7/8	10	SHWD	20 3/8	10
FBCO	19 3/4	10	DMGL	20 3/8	10
HRZG	19 3/4	10	MSCO	20 3/8	10

Figure 1:

we are able to contribute to this literature with the characterization of an equilibrium in a common value auction where  $T$  identical assets are traded sequentially. Contrary to what happens in static auctions, until the very last stages of trade, a market maker's equilibrium quotes only partially disclose his private information on the value of the asset. Indeed, bidding strategies are strictly monotone in market makers signals only in the last of the  $T$  auctions.

We study a model in which a risky asset is exchanged for a riskless asset between market makers and liquidity traders. In each period, market makers simultaneously set quotes and automatically execute liquidity traders' market orders. This is a close representation of the trading mechanism seen for example, in Nasdaq's screen-based order routing and execution systems as SelectNet and the Small Order Execution System (SOES), where clients' orders are automatically executed against market makers at the inside quotes. We quote from a document of NASD Department of Economic Research:

“Nasdaq market makers have also been subject to an increasing level of mostly affirmative obligations.

Market makers must continuously post firm two-sided quotes, good for 1000 shares [...]; they must report trades promptly; they

must be subject to automatic execution against their quotes via SOES; [...]” (J. W. Smith, J. P. Selway III, D. Timothy McCormick, 1998-01, page 2).

We assume that one of the market makers is informed about the liquidation value of the risky asset and, at some future date  $T + 1$ , this information will be publicly announced. The quantity exchanged in each period is constant and there is no exogenous shock coming from noise traders or from the arrival of new information. As quotes are observable, the uninformed market makers extract information on the value of the asset by observing the past quotes posted by the informed market maker. The latter takes into account the impact that his current quotes will have on the future uninformed dealers’ quoting strategy.

Our first result concerns the informational efficiency of the market. We show that in the last trading period, market maker’s private information is fully revealed by his quotes but the probability that this revelation occurs earlier in time is less than one. In other words, the market is strong form efficient in the long run but not in the short run.<sup>2</sup>

Moreover, we will show that in equilibrium, the informed market maker endogenously generates some “noise” in his quoting activity, that precludes the others to infer immediately his private information. In the literature, another example of noisy pricing comes from Gould and Verecchia (1985). In a static set up, they proved that when a monopolist specialist has private information on market fundamental, there exists a rational expectations equilibrium with noisy price. In order to obtain their result, Gould and Verecchia require that the specialist can precommit to add an exogenous noise to its price. By contrast, in our equilibrium this requirement does not exist as the distribution of noise corresponds to the equilibrium mixed strategy used by the informed dealer. Our “mixed strategy result” is in line with the literature that studies the effect of insiders’ disclosure rule in order-driven markets. In a Kyle’s framework, Huddart, Hughes and Levine (2001) proved that the imposition to the insider of a disclosure rule, induces him to play mixed strategy in order to avoid the market maker to infer his private information. Similarly, John and Narayanan (1997) proved that a regulation requiring in-

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<sup>2</sup>Combined with the result of Flood et al. (1998), where they show that efficiency is greatest in the most transparent trading mechanism, we argue that our result should extend to Nasdaq if dealers are given the option to submit anonymous quotes, and to anonymous markets as “*Telematico*” for fixed income securities.

siders to disclose their trades ex post creates incentives for them to create “endogenous noise” using mixed strategies and sometimes trading against their private signal.

The intuition of our result relies on two observations: (i) if the value of the asset is high it is worth buying it by setting high bid quotes, whereas if the value of the asset is low it is worth selling it by setting low ask quotes; and (ii) the more correct is the uninformed dealers’ belief, the smaller will be the profit for the informed market maker. On the one hand, when the informed market maker chooses the quotes that maximizes his current payoff, he reveals part of his information and decreases his future payoff. On the other hand, if he chooses quotes that make him lose money in the current trade, he will increase his future payoff by misleading the uninformed market makers. Thus, we will show that, as long as there are future trading rounds, it is optimal for the informed market makers to randomize between revealing his information and misleading the market trading against his signal.

We then provide some empirical implications of this equilibrium.

First, despite there is no exogenous shock during the trading process, quotes are volatile. Indeed, market makers’ quotes move because the uninformed dealers’ belief changes and because in every period they are the outcome of a mixed strategies.

Second, we can measure the speed of information revelation. We find that most of the information revelation in such markets happen in the very last trading sessions before a public announcement is revealed. This increases the winners curse during the last trading rounds and explains why in equilibrium the inside spread is positive and the average market spread increases as the game reaches its end. This last result explains the empirical observation that spread increases when the date of the public report approaches. This is in contrast with both Glosten and Milgrom (1985) and Kyle (1985) where in equilibrium the depth of the market is respectively decreasing or constant across time.

Third, we find that the equilibrium presents a positive serial correlation between the quotes set by the informed dealer at time  $t$  and the quotes set by the uninformed market maker at time  $t + 1$ . This is in tune with the empirical evidence obtained in Peiers (1997) and Cao et al. (1998), Albanesi and Rindi (2000) where some dealers appear to be price leaders.

The remainder of this paper is organized as follows. Section 2 presents the formal model. In section 3 we collect the construction of the equilibrium in the one, two, and  $T$ -steps cases, and we prove the short run information

inefficiency of the equilibrium. In section 4 we derive some empirical predictions from the properties of a numerical solution of the model. In section 5 we conclude, and all proofs are collected in the Appendix.

## 2 The model

Consider a market with  $N$  risk-neutral market-makers (MMs in the following) who trade a single security over  $T$  periods against liquidity floor traders. The liquidation value of the security is a random variable  $\tilde{V}$  which can, for simplicity, take two values,  $\{\bar{V}, \underline{V}\}$ , with  $\bar{V} > \underline{V}$ , according to a probability distribution  $(p, 1 - p)$  commonly known by all MMs, where  $p = \Pr(\tilde{V} = \bar{V})$ . We denote  $v = p\bar{V} + (1 - p)\underline{V}$  the expected value of the asset for any given  $p$ . The realization of  $\tilde{V}$  occurs at time 0 and at time  $T + 1$  a public report will announce it to all market participants. Time is discrete and  $T$  is finite.

### Information structure

At the beginning of the first period of trade, one<sup>3</sup> of the MMs,  $MM1$ , is privately informed about the realized liquidation value of the risky asset. We will refer to the realization  $(\bar{V}, \underline{V})$  as the “type” of  $MM1$ , and call  $MM1(\bar{V})$  (resp.  $MM1(\underline{V})$ ) the informed MM when  $\tilde{V} = \bar{V}$  (resp.  $\tilde{V} = \underline{V}$ ). The other  $N - 1$  market makers do not observe any private signals but they know that  $MM1$  has received a superior information; we will treat them as a unique dealer called  $MM2$ <sup>4</sup>.

### Market Rules

In each period the two MMs simultaneously<sup>5</sup> announce their ask and bid quotes which are firm for one unit of the asset<sup>6</sup>. Then, transactions take place

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<sup>3</sup>As in Kyle (1985) we assume that there is only one agent that receive private information on the realization of  $\tilde{V}$ .

<sup>4</sup>This assumption is made without loss of generality because the informed market maker only considers the probability of winning the auctions at a given price, no matter if this probability is the outcome of the strategy of one uninformed player or  $n$  equally uninformed players (see also Engelbrecht-Wiggans et al.(1982) and section 3.1).

<sup>5</sup>For simplicity we do not consider the timing problem arising when the bidding process is sequential, as in Cordella and Foucault (1998).

<sup>6</sup>It is standard in the literature to fix the traded quantity in each step (see O’Hara (1995)), and as we said before this assumption captures quite closely the rules of some markets.

between liquidity traders and the market makers. We assume that at each date, liquidity traders sell one unit of the asset to the market maker who sets the highest bid, and buy one unit of the asset from the market maker who sets the lowest ask<sup>7</sup> (i.e. price priority is enforced)<sup>8</sup>. If both market makers set the same quote, then liquidity traders will exchange with *MM2*.<sup>9</sup> Each MM can observe the past quotes of all market makers. Finally, we assume that market makers can not trade with each other and that short sales are permitted.

### Behavior of market participants and equilibrium concept

In each period a buy market order and a sell market order are proposed by floor traders who trade for liquidity reasons. It is worth stressing that in our model, traders do not act for informational motives, and so the flow of market orders neither incorporates nor depends on any information about the value of the asset. As price priority is enforced in any period, each market maker knows that he will buy (resp. sell) one asset only if he proposes the best bid (resp. ask) quote. We denote  $a_{i,t}$  and  $b_{i,t}$  the ask and bid price respectively set by market maker  $i$  in period  $t$ . Assuming that MMs are risk neutral, we can write the single period payoff functions for market makers as follows:

$$\Pi_{1,t}(\bar{V}) = (a_{1,t} - \bar{V}) \Pr(a_{2,t} > a_{1,t}) + (\bar{V} - b_{1,t}) \Pr(b_{2,t} < b_{1,t}) \quad (1)$$

$$\Pi_{1,t}(\underline{V}) = (a_{1,t} - \underline{V}) \Pr(a_{2,t} > a_{1,t}) + (\underline{V} - b_{1,t}) \Pr(b_{2,t} < b_{1,t}) \quad (2)$$

for *MM1*( $\bar{V}$ ) and *MM1*( $\underline{V}$ ) respectively, and for *MM2*

$$\begin{aligned} \Pi_{2,t} = & p(a_{1,t} - \bar{V}) \Pr(a_{1,t} \geq a_{2,t} | \tilde{V} = \bar{V}) + (1-p)(a_{1,t} - \underline{V}) \Pr(a_{1,t} \geq a_{2,t} | \tilde{V} = \underline{V}) + \\ & p(\bar{V} - b_{1,t}) \Pr(b_{1,t} \leq b_{2,t} | \tilde{V} = \bar{V}) + (1-p)p(\underline{V} - b_{1,t}) \Pr(b_{1,t} \leq b_{2,t} | \tilde{V} = \underline{V}) \end{aligned} \quad (3)$$

The overall payoff of each MM is simply the (non discounted) sum for

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<sup>7</sup>As market makers are risk neutral, this is equivalent to assume that in each period there is a constant probability of observing a buy order or a sell order.

<sup>8</sup>This is the case, for example, in some Nasdaq's execution systems (see the introduction).

<sup>9</sup>This assumption simplifies the notation.



$t = 1, \dots, T$  of these payoffs:

$$\pi_1(V, T, p) = \sum_{t=1}^T \Pi_{1,t}(V) \quad \text{for } V = \{\bar{V}, \underline{V}\} :$$

$$\pi_2(T, p) = \sum_{t=1}^T E[\Pi_{2,t}]$$

For tractability, we restrict to equilibria where the MMs' strategy are Markov strategies, which depend only on the *state of the game*  $\gamma_t = (T - 1 + t, p_t)$ , that is defined by the number of trading rounds before the public report  $(T - 1 + t)$ , and the uninformed dealer's belief at beginning of period  $t$ ,  $p_t$ <sup>10</sup>. Given this restriction, a mixed strategy for *MM2* in period  $t$  can be defined with a function  $\sigma_2$  that maps the state of the game  $\gamma_t$  into a probability distribution over all couples of bid-ask quotes. As *MM1*'s strategy depends also on his private information, a mixed strategy for *MM1* in period  $t$  is a function  $\sigma_1$  that maps the value of the asset and the state of the game  $\gamma_t$  into a probability distribution over all couples of bid-ask quotes. For a given state of the game  $\gamma = (\tau, p)$  we denote  $\pi_1^*(V, \tau, p)$  and  $\pi_2^*(\tau, p)$  the expected equilibrium payoff for *MM1*, given  $\tilde{V} = V$ , and for *MM2* respectively.

We characterize the equilibrium strategies  $\sigma_1^*$  and  $\sigma_2^*$  solving the game by backward induction: at any time  $t$  MMs solve the following problems:

$$\begin{aligned} \sigma_1^*(\bar{V}, \tau, p_t) &= \arg \max_{\sigma_1(\bar{V})} \Pi_{1,t}(\bar{V}) + \pi_1^*(\bar{V}, \tau - 1, p_{t+1}) , \text{ given } \sigma_2^* \\ \sigma_1^*(\underline{V}, \tau, p_t) &= \arg \max_{\sigma_1(\underline{V})} \Pi_{1,t}(\underline{V}) + \pi_1^*(\underline{V}, \tau - 1, p_{t+1}) , \text{ given } \sigma_2^* \\ \sigma_2^*(\tau, p_t) &= \arg \max_{\sigma_2} \Pi_{2,t}(\bar{V}) + \pi_2^*(\tau - 1, p_{t+1}) , \text{ given } \sigma_1^* \end{aligned}$$

where  $\tau = T + 1 - t$  and  $p_{t+1} = \Pr(\tilde{V} = \bar{V} | a_{1,t}, b_{1,t})$  is determined by the Bayes' rule when this is possible and otherwise, it is arbitrarily chosen.

We denote  $\Gamma(T, p)$  the game representing the strategic interaction among MMs when there are  $T$  finite rounds of trade and  $\Pr(\tilde{V} = \bar{V}) = p$  at the beginning of the game ( $t = 0$ ).

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<sup>10</sup>MMs could use more complex strategies which depend on the whole set of past quotes, or at least on a bigger subset of them than in the Markov case. These strategies are extremely complex to analyze in our framework, and this puts a serious restriction to their actual implementability.

It is worth stressing that as market makers can alternatively buy or sell the security without inventory considerations, no matter the true value of the asset, there is always one of the two auctions that is profitable and one that is not. This suggests that what really matters for the equilibrium of the game is not the actual value of the asset,  $\bar{V}$  or  $\underline{V}$ , but how close is the belief of  $MM2$  to the truth: intuitively the more correct are  $MM2$ 's belief, the smaller is  $MM1$ 's profit. In the appendix we formally state this symmetry property of the game.

### 3 Equilibrium characterization

#### 3.1 One trading round

In this section we analyze the dealers' price competition when  $T = 1$ , which can also be interpreted as the last trading round. The bid auction alone has been studied by Engelbrecht-Wiggans, Milgrom and Weber (1983) (EMW henceforth) for an arbitrary distribution of the asset for sale. They showed that the equilibrium is unique and fully revealing, in the sense that  $MM2$  can infer unambiguously the value of the asset after observing  $MM1$ 's quotes.

Proposition 1 extends their result to the ask auction. Moreover it provides the equilibrium distribution of bid and ask quotes and market makers's equilibrium payoff for our specification of the traded asset's distribution.

**Proposition 1** *The equilibrium of the one shot game  $\Gamma(1, p)$  is unique and it is such that:*

(i)  *$MM2$  randomizes ask and bid prices according to*

$$\Pr(a_{2,1} < x) = F^*(x) = \begin{cases} 0 & \text{for } x \in ]-\infty, v] \\ \frac{x-v}{x-\underline{V}} & \text{for } x \in ]v, \bar{V}] \\ 1 & \text{for } x \in ]\bar{V}, \infty[ \end{cases}$$

$$\Pr(b_{2,1} < x) = G^*(x) = \begin{cases} 0 & \text{for } x \in ]-\infty, \underline{V}[ \\ \frac{\bar{V}-v}{\bar{V}-x} & \text{for } x \in [\underline{V}, v[ \\ 1 & \text{for } x \in [v, \infty[ \end{cases}$$

(ii) *If the value of the asset is  $\bar{V}$ , then  $MM1$  sets  $a_{1,1} = \bar{V}$  and he*

randomizes the bid price according to

$$\Pr(b_{1,1} \leq x | \tilde{V} = \bar{V}) = \bar{G}^*(x) = \begin{cases} 0 & \text{for } x \in ]-\infty, \underline{V}] \\ \frac{(1-p)(x-\underline{V})}{p(\bar{V}-x)} & \text{for } x \in ]\underline{V}, v] \\ 1 & \text{for } x \in ]v, \infty[ \end{cases}$$

(iii) If the value of the asset is  $\underline{V}$ , then MM1 sets  $b_{1,1} = \underline{V}$  and he randomizes the ask according to

$$\Pr(a_{1,1} \leq x | \tilde{V} = \underline{V}) = \underline{F}^*(x) = \begin{cases} 0 & \text{for } x \in ]-\infty, v] \\ \frac{x-v}{(1-p)(x-\underline{V})} & \text{for } x \in ]v, \bar{V}] \\ 1 & \text{for } x \in ]\bar{V}, \infty[ \end{cases}$$

(iv) Equilibrium payoffs are  $\pi_2^*(1, p) = 0$ ,  $\pi_1^*(\bar{V}, 1, p) = (1-p)(\bar{V} - \underline{V})$  and  $\pi_1^*(\underline{V}, 1, p) = p(\bar{V} - \underline{V})$ .

Just before the public report, the informed market maker has the last opportunity to gain from his private information and he does not care about MM2's posterior beliefs.<sup>11</sup> More concretely, if the liquidation value of the asset is  $\bar{V}$ , MM1 will try to buy the asset by winning the bid auction, whereas if the liquidation value of the asset is  $\underline{V}$ , he will try to sell the asset by winning the ask auction. Because the uninformed market maker does not know whether it is profitable to buy or to sell the asset, he will try to win both auctions.

The discrete distribution of  $\tilde{V}$  implies that the equilibrium is in mixed strategy<sup>12</sup>. This means that when a MM tries to buy (resp. sell) the asset he chooses his bid (resp. ask) price quotes using a lottery. In equilibrium bid quotes are distributed between  $\underline{V}$  and  $v$ , whereas ask quotes are distributed between  $v$  and  $\bar{V}$ <sup>13</sup>.

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<sup>11</sup>Indeed as MMs set simultaneously their quotes, MM2 will deduce the actual value of the asset from MM1's quotes only after having posted his own quotes, that is too late.

<sup>12</sup>Assuming the support of  $\tilde{V}$  is continuous would imply that the one-shot equilibrium is in pure strategies, but it would not affect the expected equilibrium payoff (see EMW (1982)).

<sup>13</sup>To understand why a pure strategy equilibrium does not exist notice first that, MM2 can always guarantee a zero profit by setting  $a_2 = \bar{V}$  and  $b_2 = \underline{V}$ . For this reason, he never posts bid greater than  $v$  or ask lower than  $v$ , as this would provide him with a strictly negative profit. This has two implications: first, MM1's equilibrium payoff is strictly positive as he can always guarantee it: for example, by setting  $a_1 = v - \varepsilon$  if  $\tilde{V} = \underline{V}$  and

To sum up, in the static game, the asymmetry of information between market makers leads to three important implications. First, the full revelation of information by *MM1* makes the market strong-form efficient at the last stage of trade. This follows from the fact that *MM1*'s quotes are observable.

Second, unlike the symmetric information case, given the information available to market makers, bid and ask market prices are different from the expected liquidation value of the asset. Indeed, market spread is typically positive and bid and ask quotes straddle  $v$ . However, there is no restriction over its width (up to  $\bar{V} - \underline{V}$ ) which depends on the output of the mixed strategies.

Third, although the uninformed market maker expected equilibrium payoff is zero, the best informed market maker obtains a positive expected payoff. More precisely, his informational rent is larger when the *MM2*'s belief is erroneous (i.e.  $|\tilde{V} - v|$  is large). Indeed, in this case, *MM1* can win the profitable auction at prices that are far from the true value of the asset.

## 3.2 Informational efficiency of the quote-driven market

In the last trading period *MM1* reveals to the market his private information through his posted quotes.

At a glance, given that the informed dealer's quotes are observable, it would seem that he is going to lose his informational advantage at the first trading round. However, this is not true for any period before the last one. More precisely, we will demonstrate that, at equilibrium, the probability that private information is completely conveyed into prices before the last period auction is less than one.

Consider an equilibrium of  $\Gamma(T, p)$  and let  $S_t(\tilde{V}) \subset R^2$  be the support<sup>14</sup> of bid and ask prices played in some period  $t$  by *MM1* given his information

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$b_1 = v + \varepsilon$  if  $\tilde{V} = \bar{V}$ , with  $\varepsilon > 0$ , his profit can be arbitrarily close to  $|\tilde{V} - v| > 0$ ; second, it is never optimal for *MM1* to post bid (resp. ask) strictly greater (resp. lower) than  $v$ , when  $\tilde{V} = \bar{V}$  (resp.  $\tilde{V} = \underline{V}$ ). Thus if a pure strategy equilibrium exists, then *MM1* would post  $a_1^* \geq v$  (with probability one) when  $\tilde{V} = \bar{V}$  and  $b_1^* \leq v$  when  $\tilde{V} = \underline{V}$ . But in this case *MM2*, best reply would be to post  $a_2^* = a_1^*$  and  $b_2^* = b_1^*$ , and *MM1*'s equilibrium payoff would be zero, that contradicts the observation that his payoff is positive.

<sup>14</sup> $S_t(\tilde{V})$  is the smallest subset of  $R^2$  such that in equilibrium  $\Pr((a_{1,t}, b_{1,t}) \in S_t(\tilde{V})) = 1$ , for  $\tilde{V} \in \{\bar{V}, \underline{V}\}$ .

$\tilde{V} \in \{\bar{V}, \underline{V}\}$ .

We say that a fully revealing phase occurs in period  $t$ , if  $S_t(\bar{V}) \cap S_t(\underline{V}) = \emptyset$ . In this case  $MM2$  unambiguously understands the value of the asset by observing whether  $(a_{1,t}, b_{1,t})$  belongs to  $S_t(\bar{V})$  or to  $S_t(\underline{V})$ .<sup>15</sup> After a fully revealing phase, the true value of the asset is commonly known and the  $MMs$ ' continuation payoffs are zero. Indeed, when  $MM2$  learns of the true value of the asset, then the asymmetric information vanishes, the market makers compete *à la* Bertrand, bid and ask quotes coincide with the true value of the asset and all market makers' payoffs are zero.

The following theorem shows that no fully revealing phase can occur before the end of the game.

**Theorem 2** *In any Bayesian-Nash equilibrium,  $\forall t < T$  the probability that time  $t$   $MM1$ 's quotes fully reveal his private information is less than 1.*

Theorem 2 states that private information is never revealed with probability one before  $T$  and thus, in the short run, it is not possible to infer  $MM1$ 's private information despite his quotes are perfectly observable.<sup>16</sup> This informational inefficiency result mimics the results obtained in the existing microstructure models. However, in Kyle's and Glosten and Milgrom's style models, market is not efficient because insider traders' action is confounded with the exogenous random demand that comes from noise traders. By contrast, our result does not rely on the existence of this exogenous noise. Theorem 2 shows that when an informed dealer cannot hide behind noise traders or anonymity of actions, he will endogenously generate some noise. The rationale of the proof of Theorem 2 is that before the last trading round a fully revealing phase is not credible. More precisely, if at some  $t < T$ ,  $MM1$ 's private information was surely fully revealed, then in period  $t$  market makers would play the unique equilibrium of the one shot game. However, in this case,  $MM1$  has at least one profitable deviation that consists in misleading  $MM2$  beliefs in period  $t$  and then profit from  $MM2$ 's completely wrong beliefs in the following trading periods.

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<sup>15</sup>For example, in the last repetition of the game  $S_T(\bar{V}) = \{(a_1, b_1) : a_1 = \bar{V}, b_1 \in ]\underline{V}, v]\}$ , whereas  $S_T(\underline{V}) = \{(a_1, b_1) : a_1 \in [v, \bar{V}[, b_1 = \underline{V}\}$ , and so it results  $S_T(\bar{V}) \cap S_T(\underline{V}) = \emptyset$ .

<sup>16</sup>Notice that theorem 2 can be extended to any choice of the distribution of  $\tilde{V}$ .

### 3.3 Manipulating strategies in equilibrium

Theorem 2 states that market is not efficient but it does not specify how in equilibrium the informed  $MM$  manages to hide and exploit his information. In this section we characterize a mixed strategy equilibrium of the dynamic auction where  $MM1$  generates endogenous noise in his quotes that allows him to profit from his informational advantage for several trading periods. Before providing the construction of this equilibrium, we shall describe the main economic forces that produce our results.

From the analysis of the one period case, we already know that in the last trading stage,  $MM1$  only competes in the profitable side of the market: he tries to sell the asset if  $\tilde{V} = \underline{V}$  or to buy it if  $\tilde{V} = \overline{V}$ . In the following we prove that during the trading periods before the last one,  $MM1$  “hides” his information by participating in the unprofitable side of the market with positive probability. In this way,  $MM2$  cannot unambiguously deduce  $MM1$ ’s information by observing whether  $MM1$  tried to buy or to sell the asset in the previous period. We define these type of strategies as *manipulating strategies* since there is a positive probability that the informed  $MM$  takes an action that aims to turn the uninformed  $MM$ ’s belief in the wrong direction.

In any period  $t$ ,  $MM1$ ’s incentive to mislead  $MM2$  by trying to win the unprofitable auction depends on two factors: the benefit that a misleading action has on the future payoff, and the current cost of misleading. Intuitively, the greater the number of remaining trading periods, the higher the weight of the future payoff will be, and so the greater will be  $MM1$ ’s benefit from misleading  $MM2$  in the current period. The cost of misleading depends on the correctness of  $MM2$ ’s belief, as underlined in John and Narayanan (1997). If we measure the *correctness of  $MM2$ ’s belief* with the random variable  $\tilde{c} = 1 - |\tilde{V} - v|/(\overline{V} - \underline{V})$ , (that is equal to 1, when  $MM2$  knows the true value of the asset, and close to 0 when his belief is completely wrong)<sup>17</sup>, we find that the cost of misleading decreases with  $\tilde{c}$ . To see this point, take for example the case  $\tilde{V} = \overline{V}$ . Loosely speaking, if  $MM1$  wants to mislead  $MM2$ , he has to post ask prices close to  $v$ , so that he will sell the asset with positive probability<sup>18</sup>. However, if  $\tilde{c}$  is close to 0, then  $v$  will be close to  $\underline{V}$  and so, if  $MM1$  misleads, he risks selling the asset at price that is much lower than its actual value,  $\overline{V}$ . Thus, the cost of misleading the market decreases

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<sup>17</sup>Notice that if  $\tilde{V} = \overline{V}$  the  $\tilde{c} = p$ , whereas  $\tilde{c} = 1 - p$  when  $\tilde{V} = \underline{V}$ .

<sup>18</sup>Intuitively,  $MM2$  will never accept to sell the asset at a price  $a_2 < v$  so that  $MM1$  can be sure to win the ask auction with an  $a_1$  sufficiently close to  $v$ .

with the correctness of  $MM2$ 's belief.

Let  $\tau$  be the number of trading stage before the public report. In the equilibrium we characterize, if  $\tilde{c} > 2^{1-\tau}$ , then  $MM1$  misleads the market with some probability, and tries to win the profitable auction with the complementary probability. The fact that the threshold  $2^{1-\tau}$  decreases with  $\tau$ , means that a misleading action is more likely to occur in the early stages of trade as it can be turned to account during several periods. Moreover, as  $2^{1-\tau}$  converges exponentially to 0 when  $\tau$  increases, implies that misleading occurs with positive probability for any given level of belief, provided that there are enough trading rounds before the public report. For this reason one should expect informativeness of  $MM1$ 's quotes to be low at time zero and to increase when  $T$  approaches.

The following proposition summarizes this qualitative description of the equilibrium:

**Proposition 3:** *Consider the game  $\Gamma(T, p)$ . Whenever a market maker tries to win the bid or the ask auction he randomizes his current bid on the support  $[b_{\min}, v]$  or his current ask on the support  $[v, a_{\max}]$  respectively, where  $b_{\min}$  and  $a_{\max}$  depend on the state of the game  $\gamma_t = (T - t, p_t)$ .*

*In the  $t$ -th trading round,  $MM2$  tries to win both auctions, bid and ask. If  $\tilde{c} < 2^{1-(T-t)}$ , then  $MM1$  tries to win only the profitable auction. If  $\tilde{c} > 2^{1-(T-t)}$ , then  $MM1$  randomizes between trying to win only the profitable auction and trying to win only the unprofitable auction.*

*A  $MM$ 's equilibrium expected payoff is zero if he is uninformed and it is positive if he is informed.*

The following section contains the constructive proof of proposition 3 for the case of two trading periods. The proof of the general case is in the appendix.

### 3.3.1 The two-periods game

This section contains the constructive proof of proposition 3 in the case  $T = 2$ . Notice that we are looking for an equilibrium of a particular kind, leaving the question of the existence of other equilibria unresolved.

To begin with, consider the game  $\Gamma(2, p)$  with  $p > 1/2$ . According to proposition 3, the equilibrium satisfies the following two characteristics:

**Feature A)** Take the first trading round,  $t = 1$ . In equilibrium,  $MM2$

tries both to buy and to sell the asset simultaneously by randomizing his bid and ask quotes on the support  $[b_{\min}, v] \times [v, a_{\max}]$ .

If the value of the asset is  $\underline{V}$ , then  $\tilde{c} = 1 - p < 1/2$  and the informed market maker competes only in the profitable auction. That is, he posts a bid price equal to  $b_{\min}$  and he randomizes the ask price in the interval  $[v, a_{\max}]$ .

If  $\tilde{V} = \bar{V}$ , then  $\tilde{c} = p > 1/2$  and *MM1* randomizes between trying to buy the asset, and misleading *MM2* by trying to sell the asset. If he tries to buy the asset he randomizes the bid price in  $]b_{\min}, v]$  and posts the ask equal to  $a_{1,1} = a_{\max}$ ; whereas he misleads, by posting a bid equal to  $b_{\min}$  and randomizing his ask in  $[v, a_{\max}]$ .

**Feature B)** In each period *MM2*'s expected payoff is zero in each of the bid and ask auctions.

Thus, in order to prove proposition 3 in the case  $T = 2$  and  $p > 1/2$ , it is sufficient to prove that there is an equilibrium that satisfies features (A) and (B). We proceed as follows: first, we derive some properties that *MMs*' quoting strategies must satisfy in an equilibrium with features (A) and (B). Second, we provide sufficient conditions on *MMs*' strategies so that the resulting strategies form an equilibrium that actually satisfies these features.

For convenience we use the following notation to describe the first period equilibrium distribution of market makers' quotes, and *MM2*'s posterior beliefs

**Notation:**  $\bar{G}(b) = \Pr(b_{1,1} \leq b | \tilde{V} = \bar{V})$ ,  $\underline{G}(b) = \Pr(b_{1,1} \leq b | \tilde{V} = \underline{V})$ ,  $\bar{F}(a) = \Pr(a_{1,1} < a | \tilde{V} = \bar{V})$ ,  $\underline{F}(a) = \Pr(a_{1,1} < a | \tilde{V} = \underline{V})$ . Furthermore,  $\bar{g}(b) = \bar{G}'(b)$  and  $\bar{f}(a) = \bar{F}'(a)$  and  $Post(a, b) = \Pr(\tilde{V} = \bar{V} | a_{1,1} = a, b_{1,1} = b)$ .

**Lemma 4:** *If  $p > 1/2$  and the equilibrium of the game  $\Gamma(2, p)$  satisfies features (A) and (B), then for any ask  $a \in [v, a_{\max}]$  and bid  $b \in [b_{\min}, v]$ , it results*

$$\bar{G}(b) = \frac{(1-p)(b-\underline{V})}{p(\bar{V}-b)} \underline{G}(b) \quad (4)$$

$$1 - \underline{F}(a) = \frac{p(\bar{V}-a)}{(1-p)(a-\underline{V})} (1 - \bar{F}(a)) \quad (5)$$



Furthermore, for any ask  $a \in [v, a_{\max}]$  or any bid  $b \in [b_{\min}, v]$ , it results

$$Post(a_{\max}, b) = \frac{\bar{g}(b)(b - \underline{V})^2}{(\bar{V} - \underline{V})(\bar{g}(b)(b - \underline{V}) - \bar{G}(b))} \quad (6)$$

$$Post(a, b_{\min}) = \frac{\bar{f}(a)(a - \underline{V})^2}{(\bar{V} - \underline{V})(\bar{f}(a)(a - \underline{V}) + 1 - \bar{F}(a))} \quad (7)$$

Expressions (4) and (5) provide the relation between  $MM1(\bar{V})$  and  $MM1(\underline{V})$  quotes distribution that guarantees that  $MM2$ 's payoff is zero for any ask and bid quotes in  $[v, a_{\max}]$  and  $[b_{\min}, v]$  respectively. Indeed, if in equilibrium  $MM2$  randomizes his quotes on this support (feature A), then his expected payoff must be constant and equal to his equilibrium payoff (that is zero for feature (B)) for any choice of  $(a_2, b_2)$  in  $[v, a_{\max}] \times [b_{\min}, v]$ .

From feature (A) we know that  $MM1$  never tries to simultaneously buy and sell the asset. Expressions (6) and (7) provide  $MM2$ 's posterior beliefs after observing that  $MM1$  tried to buy the asset at price  $b$  or to sell it at price  $a$  respectively. These expressions are obtained applying the Bayes' rule and using relations (4) and (5). Moreover, feature (A) implies that if in the first period  $MM1$  posts a bid quote that has a positive probability to win the bid auction, then he reveals that  $\tilde{V} = \bar{V}$ . Indeed, when  $p > 1/2$ ,  $MM1$  tries to win the bid auction only if  $\tilde{V} = \bar{V}$ . By contrast, if in the first period  $MM1$  posts an ask quote that has a positive probability to win the ask auction, then  $MM2$  cannot perfectly infer the value of the asset from  $MM1$ 's quotes<sup>19</sup>.

Now we derive the first period bid prices equilibrium distributions. According to feature (A), when  $\tilde{V} = \underline{V}$ , in equilibrium  $MM1$  never competes in the bid auction, that means  $\underline{G}(b) = 1$  for any  $b \geq b_{\min}$ . Substituting this expression in (4), it results

$$\bar{G}(b) = \frac{(1-p)(b - \underline{V})}{p(\bar{V} - b)} \quad (8)$$

Expression (8) and  $\underline{G}(b) = 1$  represent the distribution of  $MM1$ 's bid quotes for any bid  $b \in [b_{\min}, v]$  when  $\tilde{V} = \bar{V}$  and  $\tilde{V} = \underline{V}$  respectively.

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<sup>19</sup>Indeed, according to feature (A)  $MM1$  tries to win the ask auction with positive probability no matter his private information.

Now we derive *MM2*'s bid price distribution in equilibrium. *MM1*'s global equilibrium payoff when  $\tilde{V} = \bar{V}$  is equal to his expected payoff from posting any couple of quotes  $(a, b)$  that belongs to his equilibrium support in the first period:

$$\begin{aligned} \pi_1^*(\bar{V}, 2, p) &= \\ &= (\bar{V} - b) \Pr(b_{2,1} < b) + (a - \bar{V}) \Pr(a_{2,1} > a) + (1 - \text{Post}(a, b))(\bar{V} - \underline{V}) \end{aligned} \quad (9)$$

where the third term is the gain in the second period.<sup>20</sup> According to feature (A), any couple  $(a, b)$  such that  $a = a_{\max}$  and  $b \in ]b_{\min}, v]$  belongs to *MM1*'s equilibrium support. Moreover, if in the first period *MM1* tries to buy the asset and not to sell it, then he will fully reveal that  $\tilde{V} = \bar{V}$ , and his second period payoff will be zero.<sup>21</sup> Finally, according to feature (A), *MM2* never posts ask price greater than  $a_{\max}$  and so  $\Pr(a_{2,1} > a_{\max}) = 0$ . Thus, evaluating expression (9) for  $a = a_{\max}$  and  $b \in ]b_{\min}, v]$  we have

$$\pi_1^*(\bar{V}, 2, p) = (\bar{V} - b) \Pr(b_{2,1} < b) \quad (10)$$

Evaluating expression (9) for  $(a, b) = (a_{\max}, v)$ , and considering that  $\Pr(b_{2,1} < v) = 1$ , we have that

$$\pi_1^*(\bar{V}, 2, p) = (1 - p)(\bar{V} - \underline{V}) \quad (11)$$

Substituting (11) in (10) and solving for  $\Pr(b_{2,1} < b)$ , it results

$$\Pr(b_{2,1} < b) = G_2(b) = \frac{(1 - p)(\bar{V} - \underline{V})}{(\bar{V} - b)} \quad (12)$$

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<sup>20</sup>Remember that the equilibrium payoffs of the last stage game are:

$$\begin{aligned} \pi_1^*(\bar{V}, 1, p_T) &= (1 - p_T)(\bar{V} - \underline{V}) \\ \pi_1^*(\underline{V}, 1, p_T) &= p_T(\bar{V} - \underline{V}) \end{aligned}$$

and, considering two stages,  $p_T = \text{Post}(a, b)$ .

<sup>21</sup>Notice that substituting expression (8) and its derivative in (6) it results  $\text{Post}(a_{\max}, b) = 1$ : when *MM2* observes that *MM1* tries to buy the asset in the first round, he infers that  $\tilde{V} = \bar{V}$ .

Expression (12) represents the distribution of  $MM2$ 's bid quotes for any bid  $b \in ]b_{\min}, v]$ . Distribution  $G_2(b)$  is such that when  $\tilde{V} = \bar{V}$ ,  $MM1$ 's payoff from posting  $a_{1,1} = a_{\max}$  and any  $b_{1,1} \in ]b_{\min}, v]$  is equal to  $(1-p)(\bar{V} - \underline{V})$ .

Now we compute the ask quotes distribution. According to feature (A), if in the first period  $MM1$  sets an ask  $a$  that has a positive probability of winning the ask auction (i.e.  $a_{1,1} \in [v, a_{\max}[$ ), then he stays out of the bid auction setting a bid  $b_{1,1} = b_{\min}$ . That means any couple  $(a, b)$  with  $a_{1,1} \in [v, a_{\max}[$  and  $b = b_{\min}$  belongs to the equilibrium support of all MMs no matter their information. Considering that  $\Pr(b_{2,1} < b_{\min}) = 0$ , then for any  $a \in [v, a_{\max}[$  and  $b = b_{\min}$ , it results

$$\pi_1^*(\bar{V}, 2, p) = (a - \bar{V}) \Pr(a_{2,1} > a) + (1 - Post(a, b_{\min}))(\bar{V} - \underline{V}) \quad (13)$$

$$\pi_1^*(\underline{V}, 2, p) = (a - \underline{V}) \Pr(a_{2,1} > a) + Post(a, b_{\min})(\bar{V} - \underline{V}) \quad (14)$$

Then:

$$\pi_1^*(\bar{V}, 2, p) + \pi_1^*(\underline{V}, 2, p) = (2a - \bar{V} - \underline{V}) \Pr(a_{2,1} > a) + (\bar{V} - \underline{V}) \quad (15)$$

Evaluating this expression for  $a = v$  and considering that  $\Pr(a_{2,1} > v) = 1$  it results

$$\pi_1^*(\bar{V}, 2, p) + \pi_1^*(\underline{V}, 2, p) = 2p(\bar{V} - \underline{V}) \quad (16)$$

Substituting expression (16) in expression (15) and solving for  $\Pr(a_{2,1} > a)$ , it results

$$\Pr(a_{2,1} > a) = 1 - F_2(a) = \frac{(p - 1/2)(\bar{V} - \underline{V})}{a - \frac{1}{2}(\bar{V} + \underline{V})} \quad (17)$$

Expression (17) represents the distribution of  $MM2$ 's ask quotes for any ask  $a \in [v, a_{\max}]$  for an equilibrium that satisfies features (A) and (B).

Expressions (11) and (16) lead to

$$\pi_1^*(\underline{V}, 2, p) = (3p - 1)(\bar{V} - \underline{V}) \quad (18)$$

Now we characterize the distribution of the informed market makers ask quotes.

Substituting expressions (7), in (13) and solving for  $\bar{f}(\cdot)$ , we obtain a first order differential equation in  $\bar{F}(a)$ :

$$\bar{f}(a) = \frac{(a - \underline{V} + (\bar{V} - a)F_2(a) - \pi_1^*(\bar{V}, 2, p))(1 - \bar{F}(a))}{(a - \underline{V})(\pi_1^*(\bar{V}, 2, p) - (\bar{V} - a)F_2(a))} \quad (19)$$

Where  $\pi_1^*(\cdot)$  and  $F_2(\cdot)$  are those in expressions (11) and (17) respectively. Solving differential equation (19) and using the initial condition  $\Pr(a_{1,1} \leq v | \tilde{V} = \bar{V}) = \bar{F}(v) = 0$ , we obtain the distribution function of the informed MM ask price when  $\tilde{V} = \bar{V}$ .<sup>22</sup> We use then (5) to find  $\underline{F}(a)$ , the distribution of MM1's ask prices when  $\tilde{V} = \underline{V}$ . This method provides the distribution function of MM's bid and ask quotes for any bid or ask that belong to MM's equilibrium support as described in feature (A).

To complete the characterization of the equilibrium for game  $\Gamma(2, p)$  when  $p > 1/2$ , what remains is to find the values of  $a_{\max}$  and  $b_{\min}$  and to show that there are no profitable deviations. These last conditions are shown in the proof of the following Proposition (see the Appendix).

**Proposition 5:** *Consider the game  $\Gamma(2, p)$  when  $p > 1/2$  and let  $F_2(a)$ ,  $G_2(b)$ ,  $\bar{G}(b)$ ,  $\underline{F}(a)$ ,  $\text{Post}(a, b_{\min})$  and  $\text{Post}(a_{\max}, b)$  be defined by expressions (17), (12), (8), (5), (7) and (6) respectively; let  $\bar{F}(a)$  be the solution of the differential equation (19) together with the initial condition  $\bar{F}(v) = 0$ ; let  $a_{\max} = \bar{V}$  and  $b_{\min}$  be the solution of  $\bar{G}(b_{\min}) = \bar{F}(\bar{V})$ .*

*Then the following strategies form a Bayesian equilibrium:*

*In the first trading round*

*(i) MM2 randomizes his ask and bid prices according to*

$$\Pr(a_{2,1} \leq x) = \begin{cases} 0 & \text{for } x \in ]-\infty, v] \\ F_2(x) & \text{for } x \in [v, a_{\max}[ \\ 1 & \text{for } x \in [a_{\max}, \infty[ \end{cases}$$

$$\Pr(b_{2,1} < x) = \begin{cases} 0 & \text{for } x \in ]-\infty, b_{\min}] \\ G_2(x) & \text{for } x \in ]b_{\min}, v] \\ 1 & \text{for } x \in ]v, \infty[ \end{cases}$$

*(ii) If the value of the asset is  $\bar{V}$  then, with probability  $(1 - \bar{F}(a_{\max}))$ , MM1 sets  $a_{1,1} = a_{\max}$  and randomizes his bid quotes on the support  $[v, b_{\min}[$ ; whereas with probability  $\bar{F}(a_{\max})$ , he sets  $b_{1,1} = b_{\min}$  and randomizes his ask*

<sup>22</sup>The boundary condition follows from feature (A). The resulting differential equation is of the form  $\bar{f}(x) = \frac{\alpha + \beta x}{\gamma + \delta x + \epsilon x^2 + \zeta x^3} (1 - \bar{F}(x))$  where  $\alpha, \beta, \gamma, \delta, \epsilon, \zeta$  are real numbers, and it has a closed form solution.

on the support  $[v, a_{\max}]$ ; furthermore it results

$$\Pr(a_{1,1} < x | \tilde{V} = \bar{V}) = \begin{cases} 0 & \text{for } x \in ]-\infty, v] \\ \bar{F}(x) & \text{for } x \in [v, a_{\max}] \\ 1 & \text{for } x \in ]a_{\max}, \infty[ \end{cases}$$

$$\Pr(b_{2,1} \leq x | \tilde{V} = \bar{V}) = \begin{cases} 0 & \text{for } x \in ]-\infty, b_{\min}] \\ \bar{G}(x) & \text{for } x \in [b_{\min}, v] \\ 1 & \text{for } x \in ]v, \infty[ \end{cases}$$

(iii) If the value of the asset is  $\underline{V}$ , then MM1 sets  $b_{1,1} = b_{\min}$  and randomizes his ask on the support  $[v, a_{\max}]$ ; furthermore it results

$$\Pr(a_{1,1} < x | \tilde{V} = \underline{V}) = \begin{cases} 0 & \text{for } x \in ]-\infty, v] \\ \underline{F}(x) & \text{for } x \in [v, a_{\max}[ \\ 1 & \text{for } x \in ]\bar{V}, \infty[ \end{cases}$$

(iv) MM2's posterior belief is  $p_2 = \Pr(V = \bar{V} | a_{1,1}, b_{1,1})$  with

$$p_2 = \begin{cases} 1 & \text{if } b_{1,1} > b_{\min} \text{ and } a_{1,1} = \bar{V} \\ \text{Post}(a_{1,1}, b_{\min}) & \text{if } a_{1,1} < [v, a_{\max}[ \text{ and } b_{1,1} = b_{\min} \end{cases}$$

(v) in the second trading round market makers' strategies correspond to the equilibrium of the game  $\Gamma(1, p_2)$ .

(vi) Equilibrium payoff are  $\pi_2^*(2, p) = 0$ ,  $\pi_1^*(\bar{V}, 2, p) = (1-p)(\bar{V} - \underline{V})$  and  $\pi_1^*(\underline{V}, 2, p) = (3p-1)(\bar{V} - \underline{V})$ .

By using the symmetry property of the game it is possible to characterize the equilibrium strategy in the first round of trade when  $p < 1/2$ . In this case, at  $t = 0$ ,  $MM1(\bar{V})$  always tries to buy the asset, while  $MM1(\underline{V})$  randomizes between trying to buy and to sell it. The equilibrium payoff are equal to 0 for MM2,  $(2-3p)(\bar{V} - \underline{V})$  for  $MM1(\bar{V})$  and  $p(\bar{V} - \underline{V})$  for  $MM1(\underline{V})$ .

Finally, if  $p = 1/2$ , then at  $t = 0$ , all market makers set bid and ask quotes equal to  $v = (\bar{V} + \underline{V})/2$  and posterior belief does not change.<sup>23</sup>

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<sup>23</sup>Such pure strategy equilibrium exists only for  $p = 1/2$  and it is sustained by the following out of equilibrium path belief:

$$\Pr(V = \bar{V} | a_{1,1}, b_{1,1}) = \begin{cases} 1 & \text{for } b_{1,1} > 1/2 \\ 0 & \text{for } a_{1,1} < 1/2 \end{cases}$$

We conclude this section with two observations. First, we point out an important characteristic that is peculiar to our model: the possibility to quantify the price-leadership effect of informed market makers in quote driven markets.

**Lemma 6:** *In the equilibrium of the game  $\Gamma(2, p)$  for  $p > 1/2$  an increase in the first period  $MM1$ 's ask quote increases  $MM2$ 's expected quote in the second period, whereas  $MM2$ 's first period quotes do not affect  $MM1$ 's second period quotes. More precisely*

$$\begin{aligned}\frac{\partial E[a_{2,2}]}{\partial a_{1,1}} &= -\ln(Post(a, b_{\text{inf}})) \frac{(2p-1)(\bar{V}-\underline{V})^2}{(2a_{1,1}-\bar{V}-\underline{V})^2} > 0 \\ \frac{\partial E[b_{2,2}]}{\partial a_{1,1}} &= -\ln(1-Post(a, b_{\text{inf}})) \frac{(2p-1)(\bar{V}-\underline{V})^2}{(2a_{1,1}-\bar{V}-\underline{V})^2} > 0 \\ \frac{\partial E[a_{1,2}]}{\partial a_{2,1}} &= \frac{\partial E[b_{1,2}]}{\partial a_{2,1}} = \frac{\partial E[a_{1,2}]}{\partial b_{2,1}} = \frac{\partial E[b_{1,2}]}{\partial b_{2,1}} = 0\end{aligned}$$

Lemma 6 proves that a high ask price for  $MM1$  in the first trading round increases the expected quotes for  $MM2$  in the second round.

Simulations suggest that the covariance between two successive ask quotes of  $MM1$  and  $MM2$  is roughly 15% of  $(\bar{V}-\underline{V})$  that represents a significant price effect of  $MM1$  over  $MM2$ . The effect that  $MM1$ 's first period bid price has on  $MM2$  second period price is even sharper. Indeed, any  $b_{1,1} > b_{\text{min}}$  moves posterior belief to 1, and so in the second stage quotes jump to  $\bar{V}$ .

The second observation is that  $MM1$  ex-interim total equilibrium payoffs for the game  $\Gamma(2, p)$  are continuous piecewise-linear monotone function in  $p$ . The same can be seen in the equilibrium payoff of the one shot game.

$$\begin{aligned}\pi_1^*(\bar{V}, 2, p) &= \begin{cases} (2-3p)(\bar{V}-\underline{V}) & \text{if } p \leq 1/2 \\ (1-p)(\bar{V}-\underline{V}) & \text{if } p > 1/2 \end{cases} \\ \pi^*(\underline{V}, 2, p) &= \begin{cases} p(\bar{V}-\underline{V}) & \text{if } p \leq 1/2 \\ (3p-1)(\bar{V}-\underline{V}) & \text{if } p > 1/2 \end{cases}\end{aligned}$$

This suggests that we can apply recursively the same method used in this section to obtain the equilibrium when the market makers interactions last an arbitrary number of periods  $T$  (see the Appendix).

## 4 Equilibrium properties and empirical implications

In this section, we will compute numerical solutions of the equilibrium described in Lemma 9 in the appendix, using  $\bar{V} = 1$  and  $\underline{V} = 0$  and varying the initial belief  $p$  and the length of the game  $T$ .

The purpose is to assess the properties in terms of informational efficiency and liquidity of our equilibrium.

### 4.1 Informational efficiency

One of the appealing properties of auction mechanisms is that it is possible to extract the bidders' private information on the value of the auctioned object by observing the bidders' bid. Not surprisingly, this is confirmed by the analysis of our one shot auction. Indeed, in the last period, quotes fully reveal  $MM1$  information. However, Theorem 2 shows that this is not the case when identical assets are traded sequentially with an auction mechanism. Indeed, the probability that  $MM1$ 's quotes fully reveal his information before  $T$  is less than one. The equilibrium we have characterized in proposition 3 allows us to characterize the minimum necessary number of trading rounds to observe fully revealing quotes with positive probability.

**Lemma 7:** *If  $T$  is the number of periods before the public report, then the probability of observing fully revealing quotes in the current stage is positive if, and only if,  $\tilde{c} > 1 - 2^{1-T}$ .*

$MM1$ 's quotes do not reveal completely his information as long as  $MM2$ 's belief is sufficiently incorrect. Thus, the minimum amount of time required to have a strong form efficient market corresponds to the minimum amount of time required to have  $MM2$ 's belief sufficiently correct. In Figures 2 and 3 we consider a game where the public report occurs after 20 rounds of trade. These figures represent the maximum and the minimum levels that equilibrium posterior belief can reach after  $t$  rounds of trade for  $p = 0.07$  and  $p = 0.4$  respectively.

Figure 4 plots the same variables for  $p = 0.4$  when there are only 10 rounds of trade before the public report.

Consider first Figure 2. When  $p = 0.07$  and  $\tilde{V} = \underline{V}$ , then  $\tilde{c}$  is high. Still, a fully revealing price will be observed between the 4-th and the 20-th

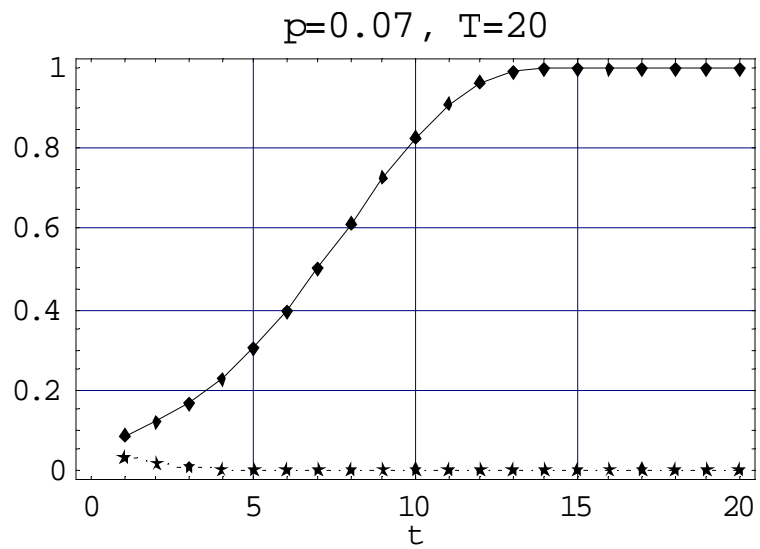


Figure 2:

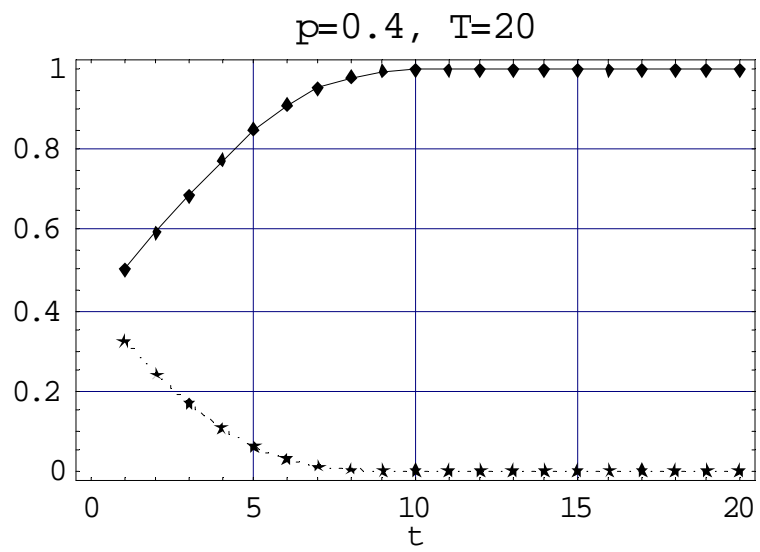


Figure 3:



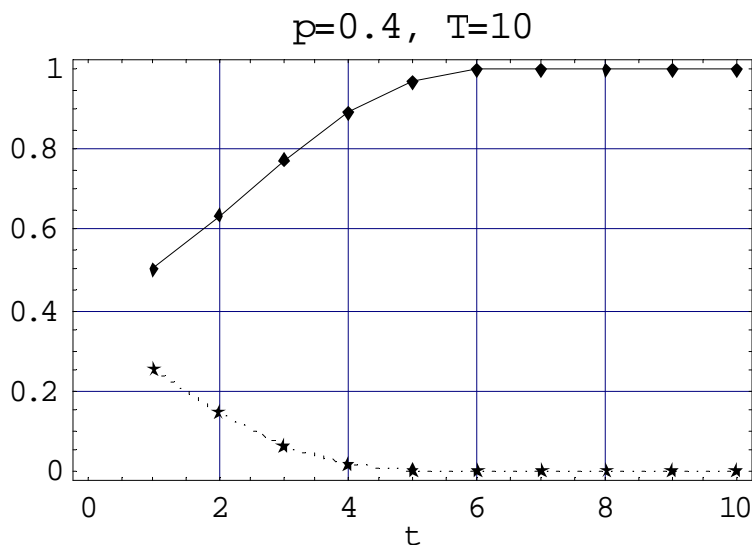


Figure 4:

round, but not before. If  $\tilde{c}$  is low (i.e.  $\tilde{V} = \bar{V}$ ), then one has to wait at least to the 13-th rounds for a fully revealing price. This suggests that private information is incorporated into quotes faster when uninformed MM beliefs are correct. Comparing Figures 3 and 4, we can also see that MM1 has more incentives to quickly reveal his signal when the date of the public report is closer. Indeed, the threshold  $1 - 2^{1-T}$  that  $\tilde{c}$  must reach for having a positive probability to observe fully revealing quotes, decreases when the end of the game approaches.

Alternatively, we can measure the informational efficiency of the market with the evolution of the variance of the true value conditioned on all relevant public information,  $\Sigma_t$ . The closer we are to the end of the game the lower is  $\Sigma_t$ , that drops to zero when the quotes of *MM1* signal his actual information. The faster the convergence of  $\Sigma_t$  to zero, the better the properties of the market.

Figure 5 plots the expected rate of change of  $\Sigma_t$  after each trading round for a game repeated 5 times and two different levels of the initial prior. The variance of the value of the risky asset decreases at a rate that depends on the level of the initial prior belief. When this prior belief is close to 1 or 0 (thick line), the initial variance of  $\tilde{V}$  decreases more slowly than

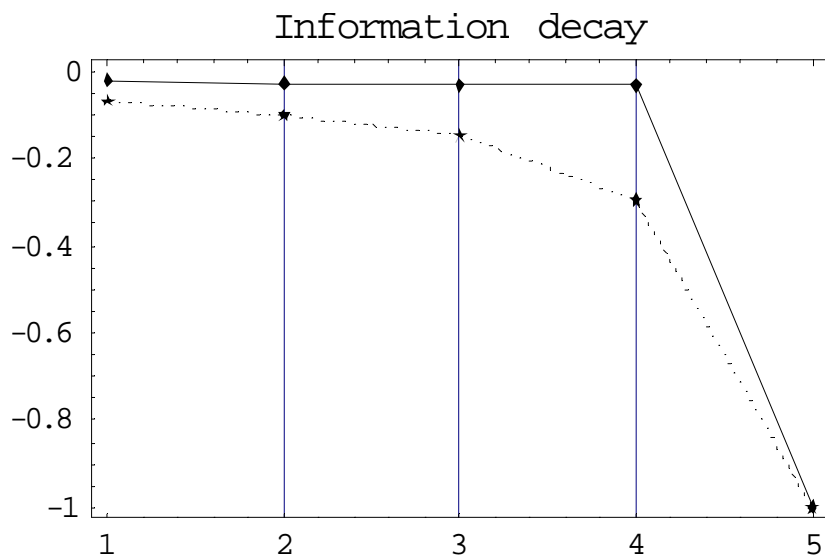


Figure 5:

when the prior is close to  $1/2$  (dotted line). In both cases, however,  $\Sigma_t$  reduces at an increasing rate, which means that less information is revealed at the early stages and  $MM1$ 's quotes reveal more during the last rounds of trade. It is interesting to compare this with the known results on the rate of price discovery in models of order-driven markets (Kyle (1985), Holden and Subrahmanyam (1992), Foster and Viswanathan (1996), Huddart, Hughes and Levine (2001)). In all these models  $\Sigma_t$  is either constant or decreases at a dwindling rate, implying that most of the private information is conveyed into the prices relatively early on in the game. Our result suggests that in a repeated auction framework where dealers compete in prices the first stages of the game are “waiting” stages with a relatively low signalling activity, while most of the information is released in the very last trading stages.

## 4.2 The expected cost of trading

Some empirical and experimental evidence have shown that the inside spreads usually widen as the moment some public announcement is supposed to be released approaches. Indeed, in figure 6 we show that for a fixed level of  $p$ , the expected inside spread in the first round of trade increases as the date of

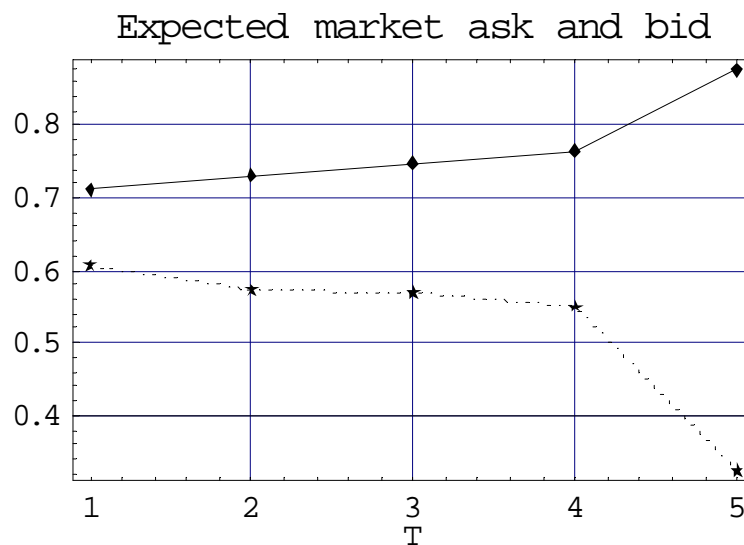


Figure 6:

public report approaches. In the last stages game, the spread is maximum.

This finding is in tune with the description of equilibrium. In the early phases of trading rounds, the  $MM1$ 's incentive to mislead is strong. This implies that the sign of  $MM1$ 's information affects slightly his quoting strategy during the initial trading rounds. However, as  $T$  approaches the incentive to mislead decreases and private information strongly affects  $MM1$  strategies. In other words, at the beginning of the game, the winner's curse is weak as being the buyer or the seller of the asset does not tell much about what the true value of the asset is. Thus, initially bid ask quotes are on the average concentrated around the ex-ante expected value of the asset. However, when a value-relevant information is drawing near,  $MM1$ 's strategy will depend sharply on his private information and competition between specialists will be heavily affected by the winner's curse. This forces the uninformed to quote quite "conservatively", and thus on the average the spread will increase. To sum up, at the end of the game more private information is released, and the winners' course effect is indeed stronger.

### 4.3 Price leadership

The manipulating equilibrium of proposition 3 can explain the price leadership phenomena that has been documented in the empirical literature on foreign exchange, OTC markets and Nasdaq. Indeed, at equilibrium there is a positive correlation between the quotes posted by the uninformed MM and the quotes that the informed MM posted in the previous trading stage. The explanation is simple: the informed MM is more likely to post relatively high quotes when he knows  $\tilde{V} = \bar{V}$  rather than when  $\tilde{V} = \underline{V}$ . Thus, the higher are the informed MM quotes, the more the uninformed MM will be induced to believe that  $\tilde{V} = \bar{V}$  and to increase on average his own quotes in the following trading stage. Indeed, in equilibrium,  $MM2$ 's posterior belief is an increasing function of  $MM1$ 's last quotes, and  $MM2$ 's expected quotes are increasing functions of his prior belief. Thus, we can conclude that

$$\begin{aligned} \frac{\partial E[a_{2,t+1}]}{\partial a_{1,t}} > 0 & \quad \frac{\partial E[a_{2,t+1}]}{\partial b_{1,t}} > 0 \\ \frac{\partial E[b_{2,t+1}]}{\partial a_{1,t}} > 0 & \quad \frac{\partial E[b_{2,t+1}]}{\partial b_{1,t}} > 0 \end{aligned}$$

One should expect that this leadership effect increases as the date of the public report approaches as  $MM1$ 's quotes become more informative.

### 4.4 The value of information

Finding the value of private information has been a central issue in financial economics. In most of the microstructure literature the existence of equilibria where the information has a positive value seems to be related to the presence of exogenous noise in the economy. For example, in Kyle (1985) the profit of the insider trader is proportional to the volatility of noise traders' demand. We show that this is not the case in a quote driven market, as a market maker can derive a positive profit from superior information even without exogenous noise in the market. This result is known in the auction literature (EMW (1981), Milgrom and Weber (1982)) for auctions where a single object is sold and one bidder is privately informed on the value of the object. We contribute to this literature showing that in our bid-ask, repeated auction there are two factors that affect the value of the private information: the ex-ante volatility of  $\tilde{V}$  and the length of the game  $T$ .

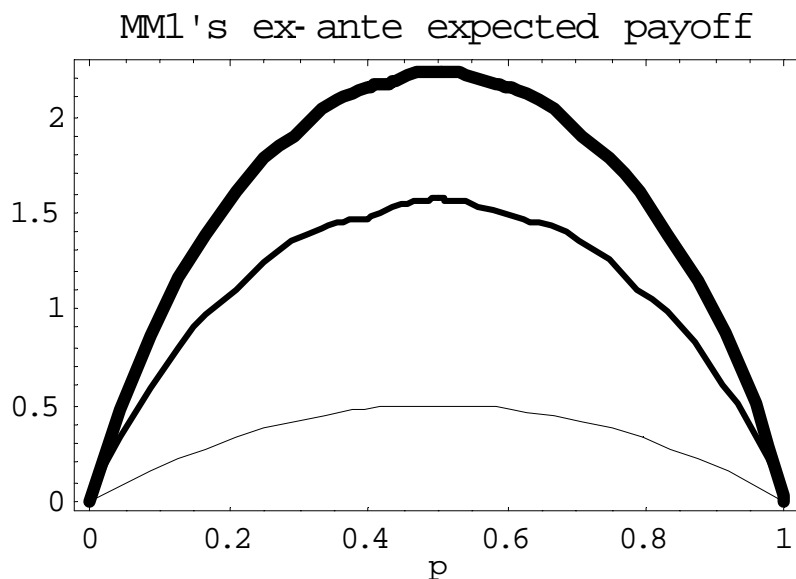


Figure 7:

The volatility of the fundamental is measured by the unconditional variance of  $\tilde{V} = p(1-p)(\bar{V} - \underline{V})^2$ . Figure 7 plots *MM1*'s ex-ante equilibrium payoff as a function of  $p$  for the game repeated once (thin curve), 15 times, and 30 times (thick curve). The ex-ante payoff is maximum when the uncertainty in the market is high, that corresponds to  $p$  close to  $1/2$ . Not surprisingly, private information is more valuable in markets where little is known about large shocks on the fundamentals.

Figure 7 shows that the informed *MM*'s payoff increases with the number of trading rounds available before the public report occurs. However, the increment in *MM1*'s payoff from one additional trading round decreases with  $T$ . Figure 8 plots *MM1*'s ex-ante expected marginal profit from adding two more trading rounds when  $p$  is around 0.5.

## 5 Conclusion

When there is asymmetric information between market makers in a quote driven market, quotes fully incorporate private information in the long run but not in the short run. Despite the highest possible transparency of the

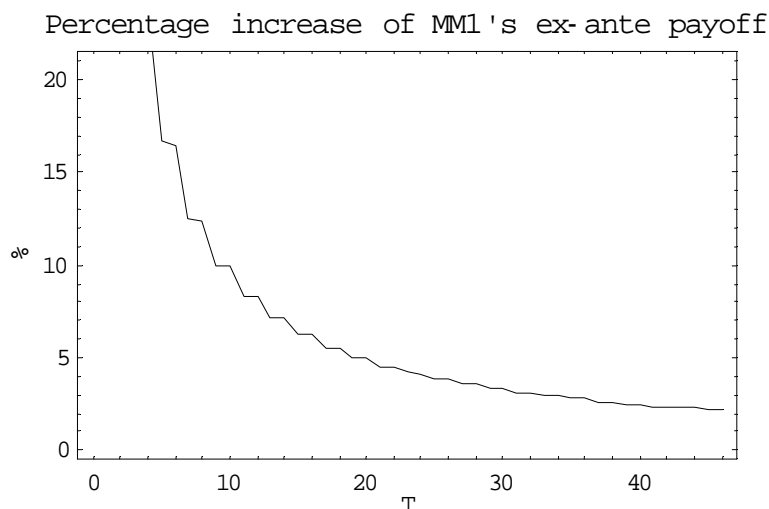


Figure 8:

market, that allows all dealers and floor traders to observe the best informed agent's actions (i.e. his bid and ask quotes), the market is not strong-form efficient. Indeed, at equilibrium the informed market maker strategically releases his private information with mixed strategies with the purpose to create some endogenous noise. This equilibrium behavior has at least four important empirical implications: first, trading prices are different from the expected value of the risky asset given market makers' information in any period; second, despite the lack of noise trading and new shocks in the fundamentals, quotes are volatile ; third, there is a positive correlation between the informed market makers quotes at  $t$  and the uninformed market maker quotes at  $t + 1$  and finally, the private information has a positive value even in such a highly transparent markets, this justifies the activity of costly collection of information by institutional dealers.

## 6 Appendix

**Symmetry:** The game  $\Gamma(T, p)$  is symmetric with respect to the following transformation:

$$\tilde{V}' = \bar{V} + \underline{V} - \tilde{V} \quad (20)$$

$$a'_{i,t} = \bar{V} + \underline{V} - b_{i,t} \quad (21)$$

$$b'_{i,t} = \bar{V} + \underline{V} - a_{i,t} \quad (22)$$

$$p' = 1 - p \quad (23)$$

**Proof:** It is sufficient to write MMs' payoffs substituting to  $a_{i,t}$  the expression  $\bar{V} + \underline{V} - b'_{i,t}$  and to  $b_{i,t}$  the expression  $\bar{V} + \underline{V} - a'_{i,t}$ ,  $i = 1, 2$ . Once MMs types are changed following (20), we obtain payoffs that differ from the original ones just for the use of the new variables ( $a'_{i,t}$ ,  $b'_{i,t}$ ,  $p'$ ) and types  $\tilde{V}'$ . Thus, one can derive the equilibrium of the game  $\Gamma(T, p - 1)$  using the equilibrium strategies of the game  $\Gamma(T, p)$ . For example if at equilibrium of the game  $\Gamma(T, p)$  it results that  $\Pr(b_{1,t} \leq x | \tilde{V} = V) = G(x, V)$ , then there is an equilibrium of the game  $\Gamma(T, p - 1)$  where  $\Pr(a_{1,t} > x | \tilde{V} = \bar{V} + \underline{V} - V) = G(x, V)$ , and similarly for the strategies of the other players.

**Proof of proposition 1:** The one shot game is a first price bid-ask auction with proprietary of information. The bid auction has been studied in EMW, considering that the ask auction is homomorphic to a bid auction, this proposition follows from their result. For expositional completeness, we show that the described strategy profile is an equilibrium while we leave uniqueness as a consequence of EMW result.

Substituting the expression  $\underline{F}^*(x)$  and  $\bar{G}^*(x)$  in expression (3), it results that  $MM2$ 's payoff is 0 for any  $b_2 \leq v$  and any  $a_2 \geq v$ . If  $MM2$  sets  $b_2 > v$ , then he is sure to win the bid auction with an expected profit of  $v - b_2 < 0$ . Similarly, any  $a_2 < v$  would lead to a loss in the ask auction. Therefore, there does not exist any profitable deviation for  $MM2$ . Substituting the  $\bar{G}^*(x)$  in (1), it follows that  $MM1(\bar{V})$ 's payoff is equal to  $(1 - p)(\bar{V} - \underline{V})$  for any  $b_1 \in ]\underline{V}, v]$ ; if  $b_1 \leq \underline{V}$ , then  $MM1(\bar{V})$  does not win the bid auction and his payoff is 0; if  $b_1 > v$ , then  $MM1(\bar{V})$  wins the bid auction and his payoff is  $\bar{V} - b_1 < \bar{V} - v = (1 - p)(\bar{V} - \underline{V})$ . This means that  $MM1(\bar{V})$  does not have a profitable deviation on the bid auction. On the ask auction any  $a_1 < \bar{V}$  (resp.  $a_1 > \bar{V}$ ) would lead to negative profit (resp. 0 profit), so that  $a_1 = \bar{V}$  is a best reply. A symmetric argument applies for  $MM1(\underline{V})$ . ■

**Proof of Theorem 2.** The proof contains one lemma.

**Lemma 8:** *If in equilibrium the private information is revealed with probability one at  $t \leq T$ , then time  $t$  equilibrium strategies are those of the one shot game equilibrium described in proposition 1.*

**Proof:** Let  $(\sigma_1(\overline{V}), \sigma_1(\underline{V}), \sigma_2)$  be some fully revealing equilibrium strategy profile that is played in  $t$ . After time  $t$  there is no asymmetry of information and each player will set bid and ask prices equal to the true value of the asset. Using standard backward induction argument, it results that players' equilibrium payoff after  $t$  is equal to zero. Thus players total equilibrium payoff from time  $t$  to  $T$  is equal to the stage  $t$  payoff.

To prove the lemma, suppose that  $(\sigma_1(\overline{V}), \sigma_1(\underline{V}), \sigma_2)$  is different from the unique equilibrium of the one shot game. Therefore, there is some player  $i$  ( $i = MM1(\overline{V})$  or  $MM1(\underline{V})$  or  $MM2$ ) that could deviate in time  $t$  increasing his stage  $t$  payoff; furthermore he could set  $a_{i,\tau} = \overline{V}$  and  $b_{i,\tau} = \underline{V}$  for any  $\tau > t$  providing a continuation payoff not smaller than 0. This is a profitable deviation as it increases his time  $t$  payoff and does not decrease his continuation payoff; thus a contradiction.  $\square$

Suppose that there exists an equilibrium where in some period  $t < T$  the probability of full revelation is one. Then, after time  $t$ , there will be no asymmetry of information, each MM will set bid and ask prices equal to the true value of the asset and MMs will make no profits.

From lemma 8, at time  $t$  all agents behave as if they were in the last repetition of the game whose unique equilibrium is described in Proposition 1. From proposition 1,  $MM1(\overline{V})$ 's equilibrium payoff is equal to  $(1-p_t)(\overline{V}-\underline{V})$ .

Now consider the following deviation for  $MM1(\overline{V})$  :

$$\begin{aligned} b_{1,t} &= \underline{V} \\ a_{1,t} &= \overline{V} - \varepsilon \end{aligned}$$

with  $\varepsilon > 0$ .  $MM1(\overline{V})$ 's stage  $t$  deviation payoff is equal to  $-\varepsilon \Pr(a_2 > \overline{V} - \varepsilon)$ ; this can be set arbitrarily close to 0 by choosing  $\varepsilon$  small. In the one shot equilibrium the quotes  $b_{1,t} = \underline{V}$  and  $a_{1,t} = \overline{V} - \varepsilon$  are played with positive probability only when the state of nature is  $\underline{V}$ ; therefore, when  $MM2$  observes  $b_{1,t} = \underline{V}$  and  $a_{1,t} = \overline{V} - \varepsilon$ , he believes that the value of the asset is  $\underline{V}$  and his posterior belief in  $t+1$  will be  $p_{t+1} = 0$ . Thus, in  $t+1$  the



uniformed market maker will set  $a_{2,t+1} = b_{2,t+1} = \underline{V}$ . Consequently, in  $t + 1$ ,  $MM1(\bar{V})$  can reach a payoff arbitrarily close to  $(\bar{V} - \underline{V})$  by playing  $a_{1,t+1} = \bar{V}$  and  $b_{1,t+1} = \underline{V} + \varepsilon$ . It follows that  $MM1(\bar{V})$ 's overall deviation payoff can be arbitrarily close to  $(\bar{V} - \underline{V})$  that is greater than his equilibrium payoff  $(1 - p_t)(\bar{V} - \underline{V})$ , thus a contradiction. ■

**Proof of lemma 4:** We provide the proof for equations (5) and (7), a similar argument applies to the bid side. From feature(B), the  $MM2$  equilibrium payoff in the first period for the ask auction is zero, thus for any ask  $a$  belonging to  $MM2$ 's equilibrium support, his current payoff on the ask auction is

$$p(a - \bar{V})(1 - \bar{F}(a)) + (1 - p)(a - \underline{V})(1 - \underline{F}(a)) = 0$$

Solving for  $(1 - \underline{F}(a))$ , it results

$$(1 - \underline{F}(a)) = \frac{p(a - \bar{V})}{(1 - p)(a - \underline{V})}(1 - \bar{F}(a))$$

that is expression (5). Differentiating both sides with respect to  $a$ , we have

$$-\underline{f}(a) = \frac{p[(a - \underline{V})(\bar{V} - a)\bar{f}(a) - (1 - \bar{F}(x)(\bar{V} - \underline{V}))]}{(1 - p)(a - \underline{V})^2} \quad (24)$$

where  $\underline{f}(a) = \underline{F}'(a)$ . If  $MM1$  randomizes ask prices according to the lotteries with densities  $\bar{f}(\cdot)$ ,  $\underline{f}(\cdot)$ , then it results by Bayes' rule that

$$\Pr(V = \bar{V} | a_{1,1} = a, b_{1,1} = b_{\min}) = \frac{p\bar{f}(a)}{p\bar{f}(a) + (1 - p)\underline{f}(a)}$$

substituting (24) in this expression and simplifying, equation (7) follows. ■

**Proof of proposition 5:** From the construction of the equilibrium, we know that if market makers follow the strategies described in the proposition, then their payoff are those provided in (iv). Still, we need to prove that there are no profitable deviations in the first trading stage. First, consider  $MM2$ . If he sets  $b_{2,1} \leq v$ , then his current payoff is zero. If he sets  $b_{2,1} > v$ , then he is sure to win the bid auction and his current expected payoff is equal to  $v - b_{2,1} < 0$ . Thus, the uninformed MM has no profitable deviation in the bid

auction. A similar argument applied to the ask auction proves that  $MM2$  has no profitable deviations. Consider now  $MM1(\underline{V})$ , a possible deviation is to set  $b_{1,1} = b_{\min} + \varepsilon$ ,  $a_{1,1} = \bar{V}$ ,  $b_{1,2} = \underline{V}$  and  $a_{1,2} = \bar{V} - \varepsilon$ . After observing  $MM1$ 's quotes in the first stage,  $MM2$  will believe that  $\tilde{V} = \bar{V}$  and he will set  $a_{2,2} = b_{2,2} = \bar{V}$ . Thus,  $MM1(\underline{V})$ 's expected payoff from this deviation can be made arbitrarily close to

$$(\underline{V} - b_{\min})G_2(b_{\min}) + (\bar{V} - \underline{V})$$

where the first term is the loss in the first period and the second term is the gain in the second period. Considering (8) it results that this expression is not greater than  $\pi_1^*(\underline{V}, 2, p) = (3p - 1)(\bar{V} - \underline{V})$  if  $b_{\min} \geq (\bar{V} + \underline{V})/2$ . Another possible deviation for both  $MM1(\bar{V})$  and  $MM1(\underline{V})$  is to propose bid and ask prices that have a positive probability to win both bid and ask auctions (i.e.  $b_{1,1} > b_{\min}$  and  $a_{1,1} < \bar{V}$ ). This is not profitable if there exist an out of equilibrium belief  $post(a_1, b_1)$  such that

$$\begin{aligned} (1 - p)(\bar{V} - \underline{V}) &\geq (a_1 - \bar{V})(1 - F_2(a_1)) + (\bar{V} - b_1)G_2(b_1) + (1 - post(a_1, b_1))(\bar{V} - \underline{V}) \\ (3p - 1)(\bar{V} - \underline{V}) &\geq (a_1 - \underline{V})(1 - F_2(a_1)) + (\underline{V} - b_1)G_2(b_1) + post(a_1, b_1)(\bar{V} - \underline{V}) \end{aligned}$$

Where  $F_2(\cdot)$  and  $G_2(\cdot)$  are given by (17) and (12). Easy computation shows that such a belief exists whenever  $b_{\min} \geq (\bar{V} + \underline{V})/2$ . We can conclude that if  $b_{\min} \geq (\bar{V} + \underline{V})/2$ , then  $MM1$  has no profitable deviations as cross quotes and huge spread are clearly dominated.

Finally, if  $MM1(\bar{V})$  never tries to simultaneously buy and sell the asset, then the probability of bidding on the ask side must be equal to the probability of not bidding on the bid side this is true when  $\Pr(b_{1,1} = b_{\min} | \bar{V}) = \bar{G}(b_{\min}) = \bar{F}(\bar{V}) = \Pr(a_{1,1} < \bar{V})$ . Solving numerically this equation we find  $b_{\min} > (\bar{V} + \underline{V})/2$ , and this complete the proof. ■

**Proof of lemma 6:** Let  $p_2 = \Pr(V = \bar{V} | a_{1,1}, b_{1,1} = b_{\inf})$  and let  $v_2 = p_2\bar{V} + (1 - p_2)\underline{V}$

$$\begin{aligned} E[a_{2,2}] &= \int_{v_2}^{\bar{V}} x dF^*(x) + \bar{V}(1 - F^*(\bar{V})) = v_2 - p_2 \ln(p_2) (\bar{V} - \underline{V}) \\ E[b_{2,2}] &= \int_{\underline{V}}^{v_2} x dG^*(x) + \underline{V}G^*(\underline{V}) = v_2 - (1 - p_2) \ln(1 - p_2) (\bar{V} - \underline{V}) \end{aligned}$$

where  $F^*(.)$  and  $G^*(.)$  are given in Proposition 1. Deriving this expression with respect to  $p_2$  we have

$$\begin{aligned}\frac{\partial E[a_{2,2}]}{\partial p_2} &= -(\bar{V} - \underline{V}) \ln(p_2) > 0 \\ \frac{\partial E[b_{2,2}]}{\partial p_2} &= -(\bar{V} - \underline{V}) \ln(1 - p_2) > 0\end{aligned}$$

Rearranging expression (14) , we have

$$p_2 = Post(a_{1,1}, b_{\min}) = \frac{\pi_1^*(\underline{V}, 2, p) - (a_{1,1} - \underline{V})(1 - F_2(a_{1,1}))}{(\bar{V} - \underline{V})}$$

Using the expression of  $F_2(a)$  provided by (17), and deriving with respect to  $a_{1,1}$  we have

$$\frac{\partial p_2}{\partial a_{1,1}} = \frac{(2p - 1)(\bar{V} - \underline{V})}{(2a_{1,1} - \bar{V} - \underline{V})^2}$$

That is positive as  $p > 1/2$ . The result follows from  $\frac{\partial E[a_{2,2}]}{\partial a_{1,1}} = \frac{\partial E[a_{2,2}]}{\partial p_2} \frac{\partial p_2}{\partial a_{1,1}}$  and  $\frac{\partial E[b_{2,2}]}{\partial a_{1,1}} = \frac{\partial E[b_{2,2}]}{\partial p_2} \frac{\partial p_2}{\partial a_{1,1}}$ . To prove that  $MM1$  quotes in the second period do not depend on  $MM2$ 's quotes in the first period it is sufficient to observe that the distribution of  $(a_{1,2}, b_{1,2})$  is only affected by  $p_2$  that does not change with  $MM2$ 's quotes. ■

**Proof of lemma 7:** First note that  $\tilde{c} > 1 - 2^{1-T}$  when  $p > 1 - 2^{1-T}$  and  $\tilde{V} = \bar{V}$ , or when  $p < 2^{T-1}$ , and  $\tilde{V} = \underline{V}$ . Take  $T > 1$  and suppose  $p > 1 - 2^{1-T}$ . If  $\tilde{V} = \bar{V}$ , then  $\tilde{c} = p > 1 - 2^{1-T} > 2^{1-T}$  and for Proposition 3 the informed market maker will randomize between trying to buy and trying to sell the asset. However, if  $\tilde{V} = \underline{V}$  then  $\tilde{c} = 1 - p < 2^{1-T}$  and  $MM1$  will only try to sell. As a result, if  $p > 1 - 2^{1-T}$ , then  $MM1$  tries to buy the asset if and only if  $\tilde{V} = \bar{V}$ , and so if  $MM2$  observes  $b_{1,1} > b_{\min}$  he infers that  $\tilde{V} = \bar{V}$ .<sup>24</sup> Similarly when  $T > 1$  and  $\tilde{c} < 1 - 2^{1-T}$  (i.e. when  $2^{1-T} < p < 1 - 2^{1-T}$ ), the probability that  $MM1$ 's current quotes fully reveal his information is zero. Indeed, in this case,  $\tilde{c} > 2^{1-T}$  no matter the realization of  $\tilde{V}$ , and so  $MM1$  randomizes between trying to buy and trying to sell the asset. Thus,  $MM2$  cannot fully infer  $MM1$ 's information.□

<sup>24</sup>A perfectly symmetric argument applies to the case  $p < 2^{1-T}$ .

## 6.1 The $T$ -stages game

In this section we describe the equilibrium of the  $T$ -stages game  $\Gamma(p, T)$ . As we focus on Markov equilibria, at each stage  $t$  of trade, players' strategy will depend only on the state of the game  $\gamma_t = (T - t + 1, p_t)$ .

To characterize the whole equilibrium bidding strategies, it is sufficient to provide the equilibrium bidding strategies profile and the equilibrium payoff for the first round of the game  $\Gamma(T, p)$  for any  $T$  and  $p$ . Indeed, the MMs' strategies in the following round will correspond to the equilibrium strategy of the first round of the game  $\Gamma(T - 1, p_2)$ , where  $p_2 = \Pr(\tilde{V} = \bar{V} | a_{1,1}, b_{1,1})$ .

To begin with, we introduce the building blocks we will use to describe the equilibrium strategies. For any natural number  $t \geq 1$  and for any natural number  $j \leq t$ , we define the numbers  $r_{j,t}$ ,  $\bar{\mu}_{j,t}$ ,  $\bar{\eta}_{j,t}$ ,  $\underline{\mu}_{j,t}$ , and  $\underline{\eta}_{j,t}$  as follows:

$$\begin{aligned}
 r_{j,t} &= \begin{cases} 0 & \text{if } j \leq 0 \\ 1 & \text{if } j \geq t \\ \frac{r_{j-1,t-1} + r_{j,t-1}}{2} & \text{elsewhere} \end{cases} \\
 \bar{\mu}_{j,t} &= \begin{cases} \frac{r_{j,t} + r_{j-1,t-2}}{r_{j,t} - r_{j-1,t}} & \text{for } j \leq t \\ 0 & \text{for } j > t \end{cases} \\
 \bar{\eta}_{j,t} &= \begin{cases} 1 & \text{for } j \leq 0 \\ 0 & \text{for } j \geq t \\ \frac{(1 + \bar{\eta}_{j,t-1})r_{j,t}(1 - r_{j-1,t}) + (1 - \bar{\eta}_{j-1,t-1})r_{j-1,t}(1 - r_{j,t}) + (\underline{\eta}_{j,t} - \underline{\eta}_{j-1,t})(1 - r_{j-1,t})(1 - r_{j,t})}{r_{j,t} - r_{j-1,t}} & \text{elsewhere} \end{cases} \\
 \underline{\mu}_{j,t} &= \begin{cases} 0 & \text{for } j \leq 0 \\ \frac{r_{j,t} + r_{j-1,t}}{r_{j,t} - r_{j-1,t}} & \text{for } j > 0 \end{cases} \\
 \underline{\eta}_{j,t} &= \begin{cases} 0 & \text{for } j \leq 0 \\ 1 & \text{for } j \geq t \\ \frac{\underline{\eta}_{j-1,t-1}r_{j,t}(1 - r_{j-1,t}) + \underline{\eta}_{j,t-1}r_{j-1,t}(1 - r_{j,t}) + (\bar{\eta}_{j-1,t} - \bar{\eta}_{j,t} - 2)r_{j-1,t}r_{j,t}}{r_{j,t} - r_{j-1,t}} & \text{elsewhere} \end{cases}
 \end{aligned}$$

For any state of the game we can now formally describe MMs' equilibrium payoff and MMs' quoting strategies during the first trading stage.

Let

$$i = \min_{j \leq T} \{r_{j,T} \geq p\} \quad (25)$$

in other words  $i$  is such that  $p \in ]r_{i-1,T}, r_{i,T}]$ .

Market makers equilibrium payoffs are:

$$\pi_1^*(\bar{V}, T, p) = (\bar{\mu}_{i,T}p + \bar{\eta}_{i,T}) (\bar{V} - \underline{V}) \quad (26)$$

$$\pi_1^*(\underline{V}, T, p) = (\underline{\mu}_{i,T}p + \underline{\eta}_{i,T}) (\bar{V} - \underline{V}) \quad (27)$$

$$\pi_2^*(T, p) = 0 \quad (28)$$

$MM2$ 's quotes distributions are:

$$G_2(b) = \frac{r_{i,T}(\pi_1^*(\bar{V}, T, p) - \bar{\eta}_{i,T-1}(\bar{V} - \underline{V})) + (1 - r_{i,T})(\pi_1^*(\underline{V}, T, p) - \underline{\eta}_{i,T-1}(\bar{V} - \underline{V}))}{r_{i,T}\bar{V} + (1 - r_{i,T})\underline{V} - b} \quad (29)$$

$$1 - F_2(a) = \frac{r_{i-1,T}(\pi_1^*(\bar{V}, T, p) - \bar{\eta}_{i-1,T-1}(\bar{V} - \underline{V})) + (1 - r_{i-1,T})(\pi_1^*(\underline{V}, T, p) - \underline{\eta}_{i-1,T-1}(\bar{V} - \underline{V}))}{a - r_{i-1,T}\bar{V} - (1 - r_{i-1,T})\underline{V}} \quad (30)$$

$MM2$ 's posterior belief after observing that  $MM1$  quotes a bid price equal to  $b$  and sets an ask price that is certain to lose the auction is given by

$$Post_{bid}(b) = \frac{\bar{g}(b)(b - \underline{V})^2}{(\bar{V} - \underline{V})(\bar{g}(b)(b - \underline{V}) - \bar{G}(b))} \quad (31)$$

Where  $\bar{G}(b)$  is  $MM1$ 's bid price distribution in equilibrium when  $\tilde{V} = \bar{V}$ , and it can be obtained as the solution of the differential equation implicitly defined by the following system:

$$\begin{cases} \pi_1^*(\bar{V}, T, p) = (\bar{V} - b)G_2(b) + (\bar{\mu}_{i,T-1}Post_{bid}(b) + \bar{\eta}_{i,T-1})(\bar{V} - \underline{V}) \\ \bar{G}(v) = 1 \end{cases} \quad (32)$$

Where  $G_2(\cdot)$  is given by expression (29).

If  $\tilde{V} = \underline{V}$ , then  $MM1$ 's bid price distribution in equilibrium is given by the following relation

$$\underline{G}(b) = \frac{p(\bar{V} - b)}{(1 - p)(b - \underline{V})} \bar{G}(b) \quad (33)$$

$MM2$ 's posterior belief after observing that  $MM1$ , tries to sell the asset at price  $a$  and sets a bid price that surely loses the auction is

$$Post_{ask}(a) = \frac{\bar{f}(a)(a - \underline{V})^2}{(\bar{V} - \underline{V})(\bar{f}(a)(a - \underline{V}) + 1 - \bar{F}(a))} \quad (34)$$

Where  $\bar{F}(a)$  is  $MM1$ 's ask price distribution in equilibrium when  $\tilde{V} = \bar{V}$ , and it can be obtained as the solution of the differential equation implicitly defined by the following system:

$$\begin{cases} \pi_1^*(\bar{V}, T, p) = (a - \bar{V})(1 - F_2(a)) + (\bar{\mu}_{i-1, T-1} Post_{ask}(a) + \bar{\eta}_{i-1, T-1})(\bar{V} - \underline{V}) \\ \bar{F}(v) = 0 \end{cases} \quad (35)$$

where  $F_2(\cdot)$  is given by expression (30).

If  $\tilde{V} = \underline{V}$ , then  $MM1$ 's ask price distribution in equilibrium is given by the following relation

$$1 - \underline{F}(a) = \frac{p(\bar{V} - a)}{(1 - p)(a - \underline{V})}(1 - \bar{F}(a)) \quad (36)$$

The maximum ask  $a_{\max}$  and the minimum bid  $b_{\min}$  that are posted with positive probability in the first round are the solution to the following system:

$$\begin{cases} \bar{F}(a_{\max}) = \bar{G}(b_{\min}) \\ \underline{F}(a_{\max}) = \underline{G}(b_{\min}) \end{cases} \quad (37)$$

**Proof of proposition 3:** The proof contains one lemma:

**Lemma 9:** Consider the game  $\Gamma(T, p)$ . Let  $i = \min_{j \leq T} \{r_{j, T} \geq p\}$  and let  $F_2(\cdot)$ ,  $G_2(\cdot)$ ,  $\bar{F}(\cdot)$ ,  $\bar{G}(\cdot)$ ,  $\underline{F}(\cdot)$ ,  $\underline{G}(\cdot)$ ,  $Post_{bid}(\cdot)$ ,  $Post_{ask}(\cdot)$ ,  $a_{\max}$  and  $b_{\min}$  as defined by (30), (29), (35), (32), (36), (33), (34), (31) and (37) respectively.

Then in equilibrium the bidding strategy in the first trading round are

(i)  $MM2$  randomizes his ask and bid prices according to

$$\Pr(a_{2,1} \leq x) = \begin{cases} 0 & \text{for } x \in ]-\infty, v] \\ F_2(x) & \text{for } x \in ]v, a_{\max}] \\ 1 & \text{for } x \in ]a_{\max}, \infty] \end{cases}$$

$$\Pr(b_{2,1} < x) = \begin{cases} 0 & \text{for } x \in ]-\infty, b_{\min}] \\ G_2(x) & \text{for } x \in ]b_{\min}, v] \\ 1 & \text{for } x \in ]v, \infty] \end{cases}$$

(ii) If the value of the asset is  $\bar{V}$  then, with probability  $(1 - \bar{F}(a_{\max}))$ ,  $MM1$  sets  $a_{1,1} = a_{\max}$  and randomizes his bid quotes on the support  $[v, b_{\min}[$

; whereas, with probability  $\overline{F}(a_{\max})$ , he sets  $b_{1,1} = b_{\min}$  and randomizes his ask on the support  $[v, a_{\max}]$ ; moreover it results

$$\Pr(a_{1,1} < x | \tilde{V} = \overline{V}) = \begin{cases} 0 & \text{for } x \in ]-\infty, v] \\ \overline{F}(x) & \text{for } x \in ]v, a_{\max}] \\ 1 & \text{for } x \in ]a_{\max}, \infty] \end{cases}$$

$$\Pr(b_{1,1} \leq x | \tilde{V} = \overline{V}) = \begin{cases} 0 & \text{for } x \in ]-\infty, b_{\min}[ \\ \overline{G}(x) & \text{for } x \in [b_{\min}, v[ \\ 1 & \text{for } x \in ]v, \infty] \end{cases}$$

(iii) If the value of the asset is  $\underline{V}$ , then with probability  $(1 - \underline{F}(a_{\max}))$ , MM1 sets  $a_{1,1} = a_{\max}$  and randomizes his bid quotes on the support  $[v, b_{\min}[$ ; whereas with probability  $\underline{F}(a_{\max})$ , he sets  $b_{1,1} = b_{\min}$  and randomizes his ask on the support  $[v, a_{\max}]$ ; furthermore it results

$$\Pr(a_{1,1} < x | \tilde{V} = \underline{V}) = \begin{cases} 0 & \text{for } x \in ]-\infty, v] \\ \underline{F}(x) & \text{for } x \in ]v, a_{\max}] \\ 1 & \text{for } x \in ]a_{\max}, \infty] \end{cases}$$

$$\Pr(b_{1,1} \leq x | \tilde{V} = \underline{V}) = \begin{cases} 0 & \text{for } x \in ]-\infty, b_{\min}[ \\ \underline{G}(x) & \text{for } x \in [b_{\min}, v[ \\ 1 & \text{for } x \in ]v, \infty] \end{cases}$$

(iv) MM2's posterior belief is  $p_2 = \Pr(\tilde{V} = \overline{V} | a_{1,1}, b_{1,1})$  with

$$p_2 = \begin{cases} Post_{bid}(b_{1,1}) & \text{if } b_{1,1} > b_{\min} \text{ and } a_{1,1} = a_{\max} \\ Post_{ask}(a_{1,1}) & \text{if } a_{1,1} < a_{\max} \text{ and } b_{1,1} = b_{\min} \end{cases}$$

(v) MMs' equilibrium payoffs are  $\pi_2^*(T, p) = 0$ ,  $\pi_1^*(\overline{V}, T, p) = (\overline{\mu}_{i,T} p + \overline{\eta}_{i,T})(\overline{V} - \underline{V})$  and  $\pi_1^*(\underline{V}, T, p) = (\overline{\mu}_{i,t} + \underline{\eta}_{i,t})(\overline{V} - \underline{V})$ .

**Proof of lemma 9:** We provide here only a sketch of the proof. Indeed the equilibrium can be characterized recursively applying the method used for the two period case (see section 3.3.1).<sup>25</sup>

First, we give an intuition of the recursive construction of equilibrium supports. Fixing a date  $t$ , for all natural numbers  $j \leq t$  we generate the numbers  $r_{j,t}$  recursively starting from  $r_{0,T} = 0$  and  $r_{1,T} = 1$ . In this way, we

<sup>25</sup>The complete proof is available upon request from the authors.

partition the interval  $[0, 1]$  in successively many  $j$  sub-intervals  $[r_{j,\cdot}, r_{j+1,\cdot}]$  as the end of the game  $T$  gets further in time. For each of this sub-intervals, we can compute the vector  $(\bar{\mu}_{j,t}, \bar{\eta}_{j,t}, \underline{\mu}_{j,t}, \underline{\eta}_{j,t})$  that gives us  $MM1$ 's expected payoff if  $p_t \in ]r_{j-1,t}, r_{j,t}]$ , as described in (26) and (31). Within each sub-interval, then,  $MM1(\bar{V})$  and  $MM1(\underline{V})$ 's equilibrium payoff are still linear in the initial  $p$  as it is the case in the one shot game.<sup>26</sup> This allows us to construct the equilibrium strategies exactly in the same way we constructed the equilibrium for the twice repeated game. The only difference is that now the belief  $p_t$  follows a process that makes it jump into different sub-intervals at each stage. Namely if  $p \in [r_{i-1,T}, r_{i,T}]$  and  $MM1$  tries to buy (resp. to sell) the asset, then posterior belief will belong to the interval  $[r_{i-1,T-1}, r_{i,T-1}]$  (resp.  $[r_{i-2,T-1}, r_{i-1,T-1}]$ ). Thus one has to take into account the piecewise linearity of  $MM$ 's continuation payoff when writing differential equations (32) and (36). Apart from this, the characterization of  $MM$ 's equilibrium strategies is analogous to that given in section 3.3.1.  $\square$

In order to complete the proof of proposition 3, it is sufficient to prove that the equilibrium described in lemma 8 satisfies the properties of the equilibrium described in proposition 3. Firstly we show that when  $\tilde{c} < 2^{1-T}$ , then in the first trade, the informed  $MM$  tries only to win the profitable auction of the market. Indeed if  $\tilde{c} < 2^{1-T}$ , then either  $\tilde{V} = \bar{V}$  and  $p < 2^{1-T}$ , or  $\tilde{V} = \underline{V}$  and  $p > 1 - 2^{1-T}$ . Take the case  $\tilde{V} = \underline{V}$  and  $p > 1 - 2^{1-T}$ , a similar argument applies to the other case. We want to prove that when  $\tilde{V} = \underline{V}$  and  $p > 1 - 2^{1-T}$ , in the first round of traded  $MM1$  only tries to sell the asset. Considering expression (25) and that  $r_{T-1,T} = 1 - 2^{1-T}$ , we have that  $i = T$  as  $p \in [r_{T-1,T}, r_{T,T}]$ , and so  $r_{i,T} = 1$ . Substituting such  $r_{i,T}$  in (29) and considering that  $\bar{\eta}_{T,T-1} = 0$ , we have  $G_2(b) = \pi_1^*(\bar{V}, T, p)/(\bar{V} - b)$ . Substituting this expression for  $G_2(b)$  in (32), it results that the system (32) is satisfied if, and only if,  $MM1(\bar{V})$ 's continuation payoff is zero for any  $b \in ]b_{\min}, v]$ , i.e.  $(\bar{\mu}_{i,T-1} Post_{bid}(b) + \bar{\eta}_{i,T-1})(\bar{V} - \underline{V}) = 0$ . However, this happens only if  $MM1$  fully reveals that  $\tilde{V} = \bar{V}$  whenever he tries to buy the asset in the first round. But this is possible if, and only if, in the first round  $MM1$  does not try to buy the asset when  $\tilde{V} = \underline{V}$ .

To see that when  $\tilde{c} > 2^{1-T}$ ,  $MM1$  participates to the unprofitable auction with positive probability consider the case  $\tilde{V} = \underline{V}$  and  $p < 1 - 2^{1-T}$ .<sup>27</sup> Then

<sup>26</sup>Moreover,  $MM1(\bar{V})$  and  $MM1(\underline{V})$ 's equilibrium payoff are continuous in  $p \in [0, 1]$ .

<sup>27</sup>A similar argument applies to  $\tilde{V} = \bar{V}$  and  $p > 1 - 2^{1-T}$ .



$i < T$  and  $r_{i,t} < 1$ . Therefore  $G_2(b) \neq \pi_1^*(\bar{V}, T, p)/(\bar{V} - b)$  for (29), and so  $MM1(\bar{V})$ 's continuation payoff must be positive for (32). But this only happens if  $MM1(\bar{V})$ 's private information is not completely revealed when he tries to buy the asset. This means that  $MM1(\underline{V})$  also tries to buy the asset with positive probability. ■

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