CentER Discussion Paper

No. 2004–94

INEQUALITY, REDISTRIBUTION AND GROWTH

By D.T. Haile, A.C. Meijdam

October 2004

ISSN 0924-7815

Tilburg University

Inequality, Redistribution and Growth*

Daniel T. Haile and Lex Meijdam†

October 13, 2004

Abstract

This paper analyzes the political economy of redistribution between two income groups in a dynamic economy and provides theoretical linkages between lobbying activities, income distribution and growth. It explains why the relationship between redistribution and growth is non-monotonic. Moreover, it is shown that in the presence of investment indivisibilities in human capital and missing capital markets both the pressure for redistribution and the initial wealth distribution affect economic growth. Persistence in inequality prevails when the political pressure by the low-skilled to increase redistribution is blocked by the lobby for lower taxes by the skilled, allowing for multiple steady-state equilibria. The resulting relationships are explored both theoretically and quantitatively.
Keywords: Economic Growth, Inequality, Labour Taxation, Lobbying and Redistribution.
JEL Classification: D31, D72, H23, O40.

1 Introduction

Traditionally, economists assumed a trade-off between equality and growth. This assumption was based on the fact that households with low incomes have a higher propensity to consume, so that lower inequality reduces savings and thus lowers growth. This conventional wisdom was challenged by a number of empirical studies showing that initial inequality is detrimental for growth (for a survey see Benabou, 1996). This evidence stimulated a new wave of literature on the relation between inequality and growth, leading to several explanations for a negative relationship between initial inequality and growth. Some of these explanantions are purely based on economic mechanisms. Saint Paul and Verdier (1993) and Aghion and Bolton (1997), for example, argue that in unequal societies redistribution stimulates growth as it enhances opportunities to invest in human capital accumulation and profitable projects. Many other explanations rely on political-economy arguments. For instance, Alesina and Perotti (1996)

*We are very grateful for helpful comments by Sjack Smulders and Henri de Groot.
†CentER & Department of Economics, Tilburg University, The Netherlands, email: d.haile@uvt.nl or a.c.meijdam@uvt.nl

contend that inequality leads to political instability which has a negative effect on investment and growth. And papers like Bertola (1993), Alesina and Rodrik (1994) and Persson, and Tabellini (1994), rely on the median voter theorem and show that greater skewness in the distribution of income leads to more redistribution in democratic societies. High redistribution in turn is asumed to generate disincentives for capital accumulation and thus to hamper growth. The first step in this reasoning is challenged empirically by Rodriguez (1999), who finds that pretax inequality has a significantly negative effect on transfers as well as on the capital tax rate, and by Figini (1999) who presents evidence that redistribution is lower when there is inequality. The empirical evidence for the second step in the argumentation, the negative relationship between growth and redistribution is weak too. In fact, the relation seems to be non-monotonic. Perotti (1996), for example, finds a positive relationship between redistribution and growth using marginal tax rates, social security/welfare spending and expenses on housing and health as redistribution measures.

In this paper, we present a political economy model of redistribution which allows for a non-monotonic relationship between redistribution as measured by the tax rate and growth. Unlike the above mentioned political economy models, our model does not rely on the median voter theorem. Instead, we use a pressure group model, assuming active participation of potential voters in the form of lobbying to influence policies. Such activities are a common practice in democracies and often require real resources which would otherwise be utilized in direct production (see Rodriguez, 2000). For example, politically active groups invest time in political activities such as strikes, working on campaigns, writing to members of parliament, lockouts et cetera. In line with this, we assume that individuals invest time to influence the political outcome, i.e. the level of redistribution.[1] While it is apparent that we cannot say too much about the details of political activity, this assumption allows us to link the level of redistribution to economic variables like differences in opportunity cost of time between groups of agents.

Our paper is not the first to examine the non-positive relationship between inequality, redistribution and growth. Benabou (2000), for example, assumes that there exists a wealth bias in the political system and that therefore the decisive voter has a higher income than the median voter. If his income is sufficiently high, then redistribution will raise his cost and hence he will oppose higher taxation. Benabou's idea is indeed a relevant point, but his model does not explain what determines the position of the decisive voter and how it changes over time. This is exactly what we want to solve in the present paper by investigating a model where the pressure for redistribution changes endogenously during the interaction of economic agents and the growth process over time. Bourguignon and Verdier (2000) also present a model of endogenous political participation. In their model agents can participate in political decision making only if they

[1] The studies of voter behaviour indicate that political activity depends on the state of the macro-economy. For example Radcliff (1992) and Filer et al. (1993) found that as real wage falls mobilization tend to rise and as a consequence voter turn out tend to increase.

are educated, i.e. political power is asymmetric by assumption.[2] Our model, in contrast, does not assume ex ante that the possibilities to influence political decision making are asymmetrical. The outcome of the political decision process may be asymmetrical ex post, however, for example as a result of differences in the investment of time in lobbying due to differences in opportunity costs, but also as a result of differences in the size of the groups.

The aim of our paper is to study the process by which support and opposition to redistribution emerges in a dynamic context. We develop a model of a small open economy with overlapping generations and inter-generational altruism (from parents to kids). Agents with two-period lives decide whether or not to invest in human capital, and all who invest acquire the same level of knowledge. Heterogeneity arises simply because of differences in inherited wealth which produces inefficiency in investing in human capital, as the unskilled can not afford to invest in becoming skilled due to capital market imperfections. This will lead to a situation with a rich dynasty which each period invests in human capital and leaves a large bequest, and a poor dynasty that remains unskilled and leaves less bequest to their children. It is important to note that investment in human capital is indivisible as in Galor and Zeira (1993), which in turn implies that in the long run there will be polarization of wealth between educated (rich) individuals and uneducated (unskilled) ones. Long-run income will depend on both the initial wealth distribution and on whether the redistribution scheme helps in mitigating the liquidity constraints of financing education. Initially, there may be a large group of unskilled who have low opportunity costs of time relative to the benefits of lobbying and therefore invest a relatively large amount of time in lobbying. This will result in a relatively high level of redistribution, which may enable some children of uneducated parents to pass the threshold cost of education. This will increase the relative size of the group of skilled and raise the opposition to redistribution. These two opposing forces, i.e. the *cost-benefit effect* and the *group size effect* may explain the nonmonotonicity of redistribution and growth (wealth distribution). A steady state prevails if the two effects cancel out. This steady state is not necessarily unique, however, but may depend upon the initial distribution of wealth.

We will show that, in the presence of investment indivisibilities in human capital and rigidities in capital markets, both the pressure for redistribution and the initial wealth distribution affect economic growth. Growth is mainly determined by human capital accumulation in both sectors.[3] Technical knowledge is endogenous, as the knowledge production exhibits both learning-by-doing and knowledge spill overs. Because of spill-over effects between the skilled and the unskilled sector, the long run-growth rate in both sectors is equal. We show that

[2] It implies that higher inequality could lead to lower redistribution as the rich people are the only participants.

[3] This is particularly true given that the genesis of growth seems to have changed substantially accross time. In the words of Williamson (1991, p.90) "the mode of accumulation in the nineteenth century appears to have been much more heavily directed towards conventional capital formation, while the mode of accumulation in the twentieth century seems to have been much more heavily directed towards human capital accumulation."

the long-run growth rate depends on the size of both sectors and therefore on redistribution. We derive a non-monotonic relation between the redistributive tax rate and long-run growth. The resulting relationships are explored both theoretically and quantitatively. In particular, we present numerical simulation experiments that illustrate the dynamic adjustment process towards a steady state after a technological shock that increases the wage gap between skilled and unskilled workers and after a redistribution of initial wealth. The simulations show that in both cases the number of skilled and growth eventually rise. This goes along with a lower tax and a larger wage gap in the long-run.

We proceed as follows. We first introduce the economy by specifying production, preferences, the extent of redistribution and how this redistribution is determined endogenously. In Section 2, we also analyze the immediate effects of an increase in the productivity of skilled workers on pressure for redistribution. Section 3 examines the dynamics of bequests and redistribution. Section 4 explains the non-monotonicity of the relation between redistribution and growth. In Section 5 we present some numerical simulation results. In particular, we illustrate how the economy reacts to an unanticipated increase in the productivity of skilled workers in the short run as well as in the long run and how a change in the initial wealth distribution affects long-run growth. The last section concludes. All proofs are gathered in the Appendix.

2 The Model

In this section we describe the economics, politics and the dynamic structure of the society at hand. We introduce a standard two-OLG model of a small open economy where parents are altruistic and leave bequests. We first present a brief description of the technology and behavior of households given the political institutions. Subsequently we elaborate on the political process.

2.1 The Technology

There is a single good that can be produced with a simple linear technology using skilled labour S_t and unskilled labour U_t :

$$Y_t = A_t^u U_t + A_t^s S_t$$

Each type of labour is paid its marginal product, that is, $w_t^u = A_t^u$ and $w_t^s = A_t^s$. It is assumed that high skilled labour is more productive than unskilled labour: $A_t^s > A_t^u$. We assume that there is a continuum of knowledge innovations in that increases production by skilled and unskilled via learning by doing. That is, knowledge creation is driven by production by skilled and unskilled workers and there are spill oversfrom one type of workers to the other:

$$\begin{cases} \Delta A_t^s = \theta^s \left(A_t^s S_t\right)^{\phi} \left(A_t^u U_t\right)^{1-\phi} \\ \Delta A_t^u = \theta^u \left(A_t^s S_t\right)^{\pi} \left(A_t^u U_t\right)^{1-\pi} \end{cases} \tag{1}$$

The parameters ϕ and π reflect the effect of production (and thus of the existing stock of knowledge) on the success of new knowledge production. We assume that production in the skilled sector is more important for knowledge production in this sector than production in the unskilled sector (i.e. $0.5 < \phi < 1$) and that the spillover from the skilled sector to the unskilled one is larger than the spillover from the unskilled sector to the skilled (i.e. $\pi > 1 - \phi$). Note that because of inter-sectoral spill overs, the growth rates in both sectors will be the same in the long run though the levels of A_t^s and A_t^u will be different. From this we can derive that in the steady state:

$$\frac{A_t^s}{A_t^u} = \left(\frac{\theta^s}{\theta^u}\right)^{\frac{1}{1+\pi-\phi}} \left(\frac{S}{U}\right)^{\frac{\phi-\pi}{1+\pi-\phi}} \tag{2}$$

Plugging this in equation (1) gives the following expression for long-run growth:

$$g = \theta^s S \left(\frac{\theta^s}{\theta^u}\right)^{\frac{\phi-1}{1+\pi-\phi}} \left(\frac{S}{U}\right)^{\frac{\phi-1}{1+\pi-\phi}}. \tag{3}$$

Note that the assumptions imply that both productivity by skilled workers and by unskilled workers raises long-run growth, but that the elaticity of growth with respect to an increase in skilled production is larger than the elasticity with respect to an increase in unskilled production.

2.2 The Households

The economy is populated by overlapping generations of individuals, each living for two periods. Each agent has one child, hence the population, N, is constant. Agents consume only in the second period of their life. Furthermore, we will assume that parents are altruistic, i.e. members of a dynasty are linked through bequests left to their children. Agents of a generation differ in the amount that they have inherited from their parents, but are the same in their preferences and abilities. Utility of an agent born in period t is assumed to be a logaritmic function of consumption in the second period of his life (c_{t+1}) and the bequest left to his child (b_{t+1}):

$$V_t = \alpha \log c_{t+1} + (1-\alpha) \log b_{t+1}, \text{where } 0 < \alpha < 1 \tag{4}$$

In the second period of their life, individuals are endowed with one unit of time each that they can spend either on working (l_t)or lobbying activities (γ_t). That is,

$$\gamma_t + l_t = 1 \tag{5}$$

Agents can work as a skilled labourer in the second period of their life if they invest in human capital during the first period. The investment in human capital is indivisible: that is either one invests $h_t > 0$ or one does not invest at all.[4] All investment must be financed out of inheritances, i.e. the capital market restrictions prevent agents from borrowing against future earnings to finance expenditures on education when young.[5] So only agents with an inheritance $b_t \geq h_t$ are able to become skilled. We denote by N_t^s the number of skilled individuals in period t and by $N_t^u = N - N_t^s$ the number of individuals of the same generation (i.e. born in period $t-1$) who remain unskilled. For all other variables (except b) we also distinguish between skilled and unskilled agents by the superscript s and u respectively. So the supply of skilled labour is $S_t = N_t^s l_t^s$ and the supply of unskilled labour is $U_t = N_t^u l_t^u$.

The government runs a redistribution scheme from the skilled to the unskilled of the same generation as a balanced budget scheme, financed by a tax on wage income of the skilled:

$$\lambda_t = (\frac{N_t^s}{N_t^u})\tau_t w_t^s l_t^s \tag{6}$$

where τ_t is the labour income tax.

In the second period of their life, the unskilled consume a part α of their wage income, their bequest plus returns and the transfer payments λ_t, while the rest is left as a bequest. So consumption and bequest for the unskilled is given by :

$$c_t^u = \alpha \left[b_{t-1}(1+r) + w_t^u l_t^u + \lambda_t\right], \tag{7}$$

$$b_t = (1-\alpha)\left[b_{t-1}(1+r) + w_t^u l_t^u + \lambda_t\right] \quad (b_{t-1} < h_{t-1}) \tag{8}$$

Unlike the unskilled, the skilled invest in human capital in the first period and receive a net wage of $(1-\tau_t)w_t^s$ in the second period. The consumption of a skilled agent is thus given by:

$$c_t^s = \alpha \left[(b_{t-1} - h_{t-1})(1+r) + (1-\tau_t) w_t^s l_t^s\right] \tag{9}$$

and the bequest they leave is:

$$b_t = (1-\alpha)\left[(b_{t-1} - h_{t-1})(1+r) + (1-\tau_t) w_t^s l_t^s\right] \quad (b_{t-1} \geq h_{t-1}) \tag{10}$$

We assume that an individual prefers to work as skilled, i.e. we assume $V^s > V^u$, which implies:[6]

$$w_t^s l_t^s > h_{t-1}(1+r) + w_t^u l_t^u + \tau_t w_t^s l_t^s \left(1 + \frac{N_t^s}{N_t^u}\right) \tag{11}$$

We assume this condition to hold.

[4] This model is perhaps best viewed as concerning investment in higher education as affordability is not generally an issue at the primary and secondary school of education. It is assumed that the cost of education is proportional to to the wage of skilled wokers, i..e. $h_t = \varkappa A_t^s$.

[5] Empirical evidence has shown the existence of market failures in financing education. We do not model this market imperfections as this would complicate the model with out changing the result. Moreover, credit on investment in human capital is constrained since embodied human capital is viewed as poor collateral by lenders. See Flug et al. (1998) for evidence.

[6] Note that this condition involves several endogenous variables. Consequently, it can not be assumed to hold ex ante. Instead, we chcked the simulation results to veryfy that the condition holds ex post throughout te whole time path.

2.3 Redistributive Lobbying

The purpose of this subsection is to link household heterogeneity to redistributive lobbying. This is accomplished by assuming that in the second period of their life, agents invest time to influence the government. In doing so they implicitly choose the tax rate of the redistribution schedule. It is important to note that we abstract from commitment and persistence. That is, each period the tax rate is determined independently from the tax rates in previous or future periods.

Of course, the notion of political lobbying is not new. A variety of static models that deal with lobbying and the related issue of endogenous policy exist, particularly in the field of international trade, for example Krueger (1974) and Das (1990), to name but few. Our model of the political process is similar to Becker (1983;1985) and Kristov et al.(1992)[7]. However, these papers typically neglect dynamic issues and focus on endogenous policy model with interest groups of a fixed size.

It is assumed that the government does not play an active role in the political process but is captured by the special interests of the skilled and the unskilled. We neglect within-group differences and assume that the common interests of each of the groups is represented by a lobby group. Moreover, we abstract from free riding effects and assume that each group chooses the amount of time that its members invest in influencing the redistributive policy so as to maximize the group members' lifetime utility. Thus at any time t, the redistributive tax on the skilled is determined by a political process that is implicitly described by:

$$\tau_t = \tau(N_t^u, N_t^s, \gamma_t^u, \gamma_t^s) \tag{12}$$

So the outcome of the political decision process depends not only on the individual investment of time in lobbying, but also on the size of the groups. The latter is in line with Cameron (1988), who noted that group size reflects a relevant resource in getting political power. Even though Olson (1965) has emphasized the free-rider problems affecting the organization of large groups, recently Acemoglu and Robinson (2001) have shown that larger groups have more influence in political decision making. Therefore, we assume that individual lobby time as well as group size have a positive effect on political influence.

It is assumed that the implicit tax function (15) is twice differentiable, with the following first-order and second-order derivatives:

$$\tau_t^u \equiv \frac{\partial \tau_t}{\partial \gamma_t^u} > 0, \; \tau_t^s \equiv \frac{\partial \tau_t}{\partial \gamma_t^s} < 0,$$

$$\tau_t^{uu} \equiv \frac{\partial^2 \tau_t}{(\partial \gamma_t^u)^2} < 0, \; \tau_t^{ss} \equiv \frac{\partial^2 \tau_t}{(\partial \gamma_t^s)^2} > 0, \tau_t^{su} \equiv \frac{\partial^2 \tau_t}{\partial \gamma_t^s \partial \gamma_t^u} \lesseqgtr 0 \tag{13}$$

[7] Most recent contributions that apply this influence function approach to the politics of inter generational redistribution are Becker and Mulligan (1998) and Mulligan and Sala-i-Martin (1999).

If one group invests more in lobbying, it is *ceteris paribus* more successful in achieving its objective, but the marginal effect of these rent-seeking investments is decreasing in absolute value. The sign of the cross partial derivative is not unambiguously clear. As we will see later on, this cross derivative is a determining factor for the reaction of one lobby group to changes in the rent seeking activities of the other group.

The following definition, which adopts the terminology of Aidt (1997), characterizes the lobby groups according to their best response towards a change in the political activities of their opponents.

Definition 1 *The type of a lobby group is determined by the slope of group j's best response function. There are three different cases:*

(i) A lobby group i is ***offensive*** *if its best response function is upward sloping,i.e.,* $\frac{d\gamma_t^i}{d\gamma_t^j} > 0, i, j = u, s,\ j \neq i$;

(ii) A lobby group i is ***defensive*** *if its best response function is downward sloping,i.e.,* $\frac{d\gamma_t^i}{d\gamma_t^j} < 0, i, j = u, s,\ j \neq i$;

(iii) A lobby group i is ***passive*** *if its best-response function is horizontal i.e.,* $\frac{d\gamma_t^i}{d\gamma_t^j} = 0, i, j = u, s,\ j \neq i$;

Consider, for example, the first case in the above definition. An increase in the political investment of group i will under these circumstances, increase the marginal lobbying activities of the other lobby group and it will be in the best interest of group j to exploit this and increase its lobbying and hence group j is offensive. A defensive group will (partially) retreat from the political arena if it encounters increased hostile activities. Passive groups do not react at all if the opposition alters its lobbying efforts.

As mentioned above, we assume that each lobby group maximizes the lifetime utility of its group members by choosing the optimal amount of lobbying effort. Moreover, we assume that it takes the decision of its political opponent as given when deciding on the lobby effort. The first-order condition for the lobby group representing the skilled in period t is

$$-\tau_t^s l_t^s = 1 - \tau_t \tag{14}$$

The analogous first-order condition for the lobby group representing the unskilled is

$$\frac{N_t^s}{N_t^u}\tau_t^u w_t^s l_t^s = w_t^u \tag{15}$$

Hence, both lobby groups invest in political influence until the marginal effect of lobbying equals the opportunity costs of time, i.e. the (after-tax) wage. For the unskilled, the marginal benefit of lobbying depends on the wage of the skilled as well as on the number of skilled relative to the number of unskilled. As a result, the unskilled will, for example, lobby more if the number of skilled rises. Of course, one group's response to such a change may incite the other group to change its behavior as well, depending on the response functions. As mentioned above, the best response of a group depends on its character, which in turn depends on τ_t^u and τ_t^{su}. The following lemma relates the character of the lobby group to the partial derivatives.

Lemma 1 *The character of the opposing lobby groups and the partial derivatives τ_t^u and τ_t^{su} are related as follows:*

i) $\tau_t^{su} > \tau_t^u \Rightarrow$ the lobby representing the skilled is defensive and the lobby representing the unskilled is offensive.

ii)$\tau_t^{su} < \tau_t^u \Rightarrow$ the lobby representing the skilled is offensive and the lobby representing the unskilled is defensive.

Proof. See appendix A1 ■

If for example τ_t^{su} is negative, this means that an increase in the lobbying activities by the lobby group representing the skilled will make the lobby group of the unskilled marginally less successful. The best response of the lobby group representing the unskilled to increasing political activity by the other group is, therefore, to invest less in rent seeking. The unique Nash equilibrium is stable if (see Appendix $A2$)

$$\frac{\tau_t^{su} l_t^s - \tau_t^u}{2\tau_t^s - l_t^s \tau_t^{ss}} \quad < \quad -\frac{\tau_t^{uu} l_t^s}{\tau_t^{su} l_t^s - \tau_t^u} \tag{16}$$

The above condition shows that the absolute value of the slope of the best response functions of the rich should be smaller than the absolute value of the slope of the reaction function of the unskilled. The following subsection discusses the effects of a change in wage of the skilled to illustrate the working of the political model.

An increase in the wage of skilled workers

In this subsection we illustrate the working of the lobby model by analyzing the effects of an increase in inequality that results from a change in technology that increases the productivity of skilled workers (relative to the productivity of unskilled), i.e. an increase in A^s.

Assume that initially, at $t = 0$, the economy is in the steady state and unanticipated increase in A^s occurs. Note that the number of skilled and unskilled is predetermined and thus will not change initially. The immediate effect of an increase in A^s is reflected in a higher gross wage of skilled workers w^s, leading to an increase in the wage gap between skilled and unskilled workers. As can be seen from the first-order condition (18), an increase in the wage gap incites unskilled to lobby more. This is due to the increase in the tax base automatically implying an increase in the rent the unskilled seeks, while the costs of lobbying do not change. This so-called *cost-benefit effect* will - *ceteris paribus* - have a positive effect on the tax rate. However, the skilled will react to the increase in the rent seeking activity of the unskilled. The effect on the lobby effort of the skilled depends on the character of the lobby group. If $\tau_t^{su} > \tau_t^u > 0$, the lobby group representing the skilled is defensive and will react to the increased political activity of the unskilled by reducing its lobby effort. This is illustrated in Figure 1A.

INSERT FIGURES 1A AND 1B

The wage increase for the skilled workers implies a parallel shift rightward of the best response function of the lobby group representing unskilled workers. Consequently, the political equilibrium moves from E_o to E_1 and the tax rate will unambiguously rise. It is evident that the unskilled's welfare will increase due to positive technological shock as they will receive a higher benefit, for any given level of lobbying activity, that is, the extra time lost in lobbying will be offset by a generous redistributive policy. The effect on the welfare of the skilled is ambiguous: the wage increase as well as the decrease in lobbying have a positive effect on utility, but this may be offset by an increase in the tax rate if the extra lobby efforts of the unskilled are very successful. If the lobby group of the skilled is offensive (i.e. $\tau_t^{su} < 0$ or $\tau_t^u > \tau_t^{su} > 0$) (see Figure 1B) the skilled react to the increased lobby effort of the young by raising their activity in the political arena. As a result more resources are lost in the political struggle, but the effect on the tax rate is ambiguous. For the increase in the rent seeking activities of the unskilled to dominate the increase in the lobbying effort of the skilled, and thus for the tax rate and the welfare of the unskilled to rise, it is required that

$$\frac{\tau_t^{su} l_t^s - \tau_t^u}{2\tau_t^s - l_t^s \tau_t^{ss}} < 1 \tag{17}$$

The effect on the utility of the skilled will be ambiguous again: the wage increase will positively affect their welfare, but the extra time lost in lobbying as well as the increase in the tax rate will have a negative effect.

So far, we only looked at the effects of the increase in productivity of the skilled in the first period and thus on the welfare of the current generation of skilled and unskilled workers. However, it is evident that such a shock will have effects on the welfare of future generations also. Firstly, there is a direct effect on the inheritance of the children of the current workers: assuming that the increased rent seeking activities by the unskilled do not completely offset the welfare gain for the skilled due to the higher wage, all bequest will be higher. This may enable some children of unskilled parents to afford education and thus raise their lifetime income by more than just the increased inheritance. Secondly, the increase in the number of skilled that results if more kids can afford education, will change the political equilibrium and lead to a lower tax rate. Thirdly, a change in the number of skilled may affect the technological spill overs and thus influence economic growth. Finally, these effects are of course not restricted to the first period, but will propagate themselves to the following periods, leading to a gradual transition to a new steady growth path. In the subsequent sections, these dynamic effects are studied. In particular, Section 3 discusses the dynamics of wealth and redistribution, i.e. the first and the second effect. Section 4 elaborates on the relation between number of (un)skilled and the growth rate. Assuming a simple specification for the tax function (15), this section also derives the long-run relation between the tax and the growth rate. It is shown that this relation is typically non-monotonic. In Section 5 we present the results of some numerical simulation experiments illustrating the adjustment process towards a new steady growth path.

3 Dynamics of Wealth and Redistribution

As already noted, individuals who inherit an amount larger than h_t are able (and willing) to invest in human capital and become skilled in the second period of their life. Consequently, the distribution of inheritances in period t determines the number of skilled in period $t+1$. Let D_t be this distribution: $\int_0^\infty dD_t\left(b_t\right) = N$, then the number of skilled in the next period is:

$$N_{t+1}^s = \int_h^\infty dD_t\left(b_t\right) \tag{18}$$

and the number of unskilled:

$$N_{t+1}^u = \int_0^h dD_t\left(b_t\right) \tag{19}$$

The distribution of wealth at time t not only determines the numbers of skilled and unskilled in period $t+1$, but also affects redistribution, technological spill overs and growth in both sectors, and thus indirectly the distribution of inheritances in future periods. These dynamic effects are quite complex. In this section we neglect the effects on technology and growth (i.e. we assume $\theta^s = \theta^u = 0$ so that w^s and h are constant) and discuss the dynamic relation between redistribution and wealth. These dynamics can be described by

$$b_{t+1} = (1-\alpha)\left(1+r\right)b_t + q_t \tag{20}$$

where q_t is

$$q_t = \begin{cases} (1-\alpha)\left[l_t^u w^u + \lambda_t\right] \equiv q_t^u \text{ if } b_t < h \\ (1-\alpha)\left[\left(1-\tau_t\right) l_t^s w^s - (1+r)\,h\right] \equiv q_t^s \text{ if } b_t \geq h \end{cases} \tag{21}$$

We assume that the dynamic equation is stable,

$$(1-\alpha)\left(1+r\right) < 1$$

Moreover, we assume that initially $q_t^u < [1-(1-\alpha)(1+r)]h$ and $q_t^s < [1-(1-\alpha)(1+r)]h$. That is, we assume that polarization prevails and children of unskilled parents will be unskilled and children of skilled parents will become skilled. This case is illustrated by the lines AB and CD in Figure 2.

INSERT FIGURE 2

In this case, the number of skilled and unskilled is constant, and hence (given that the wages are also assumed to be constant) the tax rate and q^u and q^s will be constant. As a result, equation (22) is a piecewise linear function that intersects the 45^o-line two times, at the equilibria, $b_u^* \equiv \frac{q^u}{1-(1-\alpha)(1+r)}$ and

$b_s^* \equiv \frac{q^s}{1-(1-\alpha)(1+r)}$. So, in the long run, wealth levels within the groups converge, but there is complete dichotomy between the two groups. The long-run level of average wealth can be expressed as $\bar{b} = b_u^* + (b_s^* - b_u^*)$,which is increasing with $\frac{N^s}{N}$ if b_u^* and b_s^* are taken to be constant. However, a change in the number of skilled will shift the political equilibrium and thus affect b_u^* and b_s^*. In particular, an increase in the number of skilled will decrease the tax rate, and lower b_u^* and raise b_s^*. So the relation between redistribution and average wealth is not straightforward.

In order to further investigate the relation between redistribution and wealth, the next subsection presents the dynamic effects of an increase in the wage of the skilled.

An increase in the wage of the skilled

The analysis starts from the initial situation as illustrated by the lines AB and CD in Figure 2. We analyze the effect of a once-and-for-all increase in w^s. As discussed in Section 2.3, this affects lobbying and thus redistribution, but the exact effects depend on the form of the tax function. The analysis in this subsection is based on the assumption that the tax rate and the net income of both the skilled and the unskilled rises.

The increase in w^s raises q^s. This results in an upward shift of the line CD in Figure 2 to C'D'. The subsequent increase in redistribution raises q^u and lowers q^s, but we assume that q^s will still be larger than in the initial situation. So C'D' shifts down to C"D" and AB shifts up. Now there are two possibilities: the increase in q^u may or may not be large enough to allow the children of the unskilled with the highest inheritance to become skilled. In the latter case, AB shifts up to A'B', but the number of skilled does not change and the lines in the figure do not shift anymore. In the former case, however, the dynamic process is much more complicated. In this case AB shifts up further, for example to A"B". As a consequence, children with an inheritance just below h will be able to afford investing in human capital and consequently, in the next period the number of skilled will be higher. This shifts the political equilibrium. In particular, the unskilled will lobby more as the higher number of skilled implies a larger tax base and thus an increase in the marginal benefit from lobbying for a higher tax rate while the marginal costs of lobbying will not change. This *dependency ratio effect*[8] will have an upward effect on the tax rate. At the same time however, the lobby of the skilled will gain influence relative to that of the unskilled because of the increase in their relative number. This *group size effect* will - *ceteris paribus* - lower the tax rate. We assume the dynamic process to be stable, i.e. it assumed that the group size effect dominates the dependency ratio effect so that the tax rate will fall again. This will in the next period shift A"B" downward again while C"D" shifts up again. Once more, there are two possibilities: either the number of skilled remains constant from then on and curves do not shift anymore, or the number of skilled rises further and the

[8] In fact, the *dependency ratio effect* is a special case of what we labbeled the *cost-benefit effect.*

process goes on shifting AB future down and CD further up. This continues until AB is so far down that a new equilibrium is reached where the number of skilled does not rise anymore. In this new equilibrium, the number of skilled will be larger than in the initial situation, and the number of unskilled will therefore be lower. Moreover, redistribution and the amount of time both groups spend on political activities will have changed.

Note that in this subsection, we abstracted from technological spill-overs and growth. However, it is evident that if we take growth into account, the growth rate in the new steady state will be different from the initial growth rate. It is not evident what the exact effect on growth is, however. Therefore, in the next section, we analyze the relation between redistribution, the size of both sectors and long-run growth.

4 Redistribution, Spill-overs, and Growth in the Long Run

The aim of this section is to illustrate the relation between redistribution, technological spill-overs, and economic growth. We first analyze the relation between the long-run growth rate and the number of skilled and unskilled, for given amounts of time spent on lobbying by both groups. Subsequently, we assume a specific functional form of the tax influence function and analyze the relation between growth and the tax rate.

Increasing the number of skilled *ceteris paribus* increases skilled production which, due to spill-over effects, leads to higher growth rates in both sectors. However, given total population size, an increase in the number of skilled implies a decrease in the number of unskilled which exerts downward pressure on the growth rates in both sectors. As a result, an increase in the number of skilled only increases growth if the number of unskilled is relatively large, i.e. if the number of skilled is below its optimal level. This is summarized in the following proposition.

Proposition 1 *An increase in the number of skilled leads to higher long-run growth if and only if* $N^s < \left(\frac{\pi}{1+\pi-\phi}\right) N$.

Proof. See Appendix B1 . ■

This proposition assumes the amount of time individuals spend on lobbying to be constant. However, we know from the political model that, in general, changes in the size of both lobby groups affect lobby efforts. Consequently, given wages of skilled and unskilled, there is a relation between group size and the tax rate. In order to be able to analyze this relation, we simplify the model by introducing a specific functional form for the tax influence function (13). We assume this function to be of the following form:

$$\tau_t = \tau_o \left(\frac{N_t^u}{N_t^s}\right)^{\mu} \left(\frac{\gamma_t^u}{\gamma_t^s}\right)^{\rho}, \mu > 0, \rho < 1, \tag{22}$$

Note that this functions satisfies the conditions with respect to the first- and second order derivatives and that the cross derivative $\tau^{us} < 0$. The latter implies that the lobby group representing the unskilled is defensive while the group representing the skilled is offensive. Using this function and the corresponding first-order conditions to substitute out γ_t^u and γ_t^s, we can specify the relation between groups size and the tax rate. Combining this with the relation between group size and long-run growth as summarized in Proposition 1, we are able to derive a relation between the tax rate and long-run growth state, i.e. a relation between redistribution and growth as shown in the following equation (see Appendix B2 for details).[9]

$$g = \Theta \tau^{\left(\frac{\phi-1}{\rho-u(1+\pi-\phi)}\right)} (1-\tau)^{\left(\frac{\rho(\phi-1)}{\rho-u(1+\pi-\phi)}\right)} \tag{23}$$

This relation is depicted in Figure 3[10].

INSERT FIGURE 3

This figure illustrates what is already evident from equation (24): the relation between growth and the tax rate is typically non-monotonic. This is the result of the interplay between several effects. First, the political process plays an essential role. An increase in τ increases redistribution which enables some children of unskilled to join the pool of skilled. This raises the benefit of lobbying for the unskilled, and thus the leads to aditional political presure to further increase the the tax rate. Howevere, via the group-size effect the higher number of skilled individuals will also lead to a stronger lobby to lower the tax rate. Moreover, an increase in τ distorts the labour supply decision and leads to a reduction in effective labour supply - and thus an increase in lobbying activity - by the skilled. In a long-run equilibrium, the tax rate is stable, which is only posible if the political forces to increase and to decraese the redistribution exactly balance. Second, the process of knowledge creation is important. Given the level of the wage of skilled workers relative to that of unskilled, the requirement that the political forces should exactly balance implies a certain size of both groups and an allocation of time, and thus a certain level of production of skilled workers relative to that of unskilled. But, given that the growth rates of skilled and unskilled wages are equal in a steady state, this in turn determines the relative wages (cf. equation (5)). Of course, in order to have a long-run equilbrium, these relative wages should be consitent with the ones assumed in the political process. This requirement results in a unique relation between the tax rate on the one hand and relative wages, group size and allocation of time

[9] It should be noted that both the tax rate and the rate of growth are endogenous variables. The relation betwee these two variables results from comparing steady states with different number of skilled.

[10] It should be noted that, for analytical convenience, equation (24) was derived using the relation between the number of skilled and the tax rate that underlies on Proposition 1. Consequently, the amount of time spend on lobbying is still not fully endogenized. However, numerical simulation experiments show that the relation between the tax rate and long-run growth is non-monotonic when political activity is fully endogenous (see Appendix C1 and Figure 6).

on the other. This implies a relation between the tax rate and the level of production of skilled workers relative to that of unskilled: low tax rates go along with a relatively large production by skilled workers, high tax rates with relatively low production of skilled. Differences in relative production go along with differences in the growth rate. As already suggested by Proposition 1, there is an optimum for the relative production by skilled workers and thus an optimum tax rate. As long as the tax rate is below this growth-maximizing level, raising the tax rate increases growth, above this level a higher tax rate leads to lower growth. The following proposition summarizes.

Proposition 2 *A non-monotonic relationship prevails between long-run growth and tax rate: for low values of the tax rate more redistribution goes along with higher growth, for high tax rates a further increase of redistribution goes along with lower growth.*

5 The Dynamics of Wealth, Redistribution and Growth

When the economy is not in the steady state, it converges via an adjustment process towards an equilibrium. During this adjustment process, wages of skilled and unskilled will grow at different rates. This adds additional effects to the relations discussed so far. In the next section, we illustrate the full dynamic adjustment process by some numerical simulation experiments. As already noted, the adjustment process as well as the eventual steady state depends on the initial wealth distribution. The basic simulation experiment starts from a uniform distribution of wealth. We present two variants on this basic simulation. First, we illustrate the effect of the initial wealth distribution on the outcome of the dynamic process. Second, we discuss the full dynamic effects of a technological shock that increases the wage of the skilled. This complements the discussion of the partial effects of this shock in Sections 2 and 3.

5.1 The effect of the initial wealth distribution

Figures 4A-D present the results of the basic simulation and the effect of a change in the initial wealth distribution.

INSERT FIGURES 4A-D

The parameters for the basic simulation can be found in Appendix C1. This basic simulation starts with 100 individuals with wealth levels uniformly distributed on the interval $[1, 100]$. The starting level of the investment in human capital h is 60, and in the subsequent periods h is assumed to grow at the same growth rate as the wage of the skilled. Given the initial value of h and the distribution of wealth, it is evident that the initial number of unskilled is 60. However, given the initial wage levels and the tax rate that initially results from the political process, some children of the initial generation of unskilled will be able to afford education and hence the number of unskilled drops to 56. This reduces the political influence of the unskilled and hence the tax rate is lower in the second period (despite the fact that the gap between the wages of the skilled and the unskilled has increased a bit due to the growth rate of skilled wages being higher than that of unskilled wages). This development continues at with decreasing intensity in periods 3 and 4. By then the number of unskilled has decreased to 53 and the tax rate is more than 2 percent below its initial value. From then on, no children of unskilled are able to pass the treshold h and there is complete dichotomy between skilled and unskilled. As a result, the decrease in the tax rate that results from the growth in size of the lobby group of the skilled stops. From then on, the tax rate slightly increases again. The reason for this is that the growth rate of the wage of the skilled is still larger than that of wage of the unskilled, implying a growing wage gap and increasing investment in political activity by the unskilled. Due to spillover effects, the difference in the growth rates of wages of skilled and unskilled decreases over time and eventually the wage gap stabilizes and the economy reaches a steady state.

To illustrate the effect of a change in the initial distribution of wealth, we ran the same simulation starting from a flatter wealth distribution. That is, we took away 80 percent of the wealth above 75 and redistributed this wealth to the individuals with wealth below 75, in such a way that a continuous but kinked distribution results.[11] The redistribution of initial wealth enables one additional child to invest in human capital, leading to a lower number of unskilled in period one. The adjustment process that follows is qualitatively comparable to the one described above. However, the number of skilled will permanently be larger than in the simulation that starts from a uniform distribution of wealth, so the economy stabilizes in another steady state. In particular, the steady state that is reached starting from a flatter distribution is characterized by a larger wage gap, a lower tax rate and higher growth rate. This numerical example shows that, if the number of skilled is inefficiently low due to imperfect capital markets, redistribution of wealth may decrease inefficiency and foster growth. At the same time, however, this reinforces the political influence of the skilled and leads to more inequality and less redistribution in the long run. Still, due to the higher growth rate, also future generations of unskilled will be better off.

5.2 An increase in the wage of the skilled

Figures 5A-D present the results of the adjustment process if we start from an initial situation with a higher wage for the skilled as compared to the results of the basic simulation.

INSERT FIGURES 5A-D

[11] The resulting distribution is: $b(i) = 75 + \frac{0.2*13}{37}(i - 75)$ for $i = 1, ..., 75$
$b(i) = 75 + 0.2(i - 75)$ for $i = 76, ..., 100$

The higher wage of the skilled will not affect the initial number of unskilled. However, the increased wage gap makes them lobby more. The skilled react to this by also slightly increasing their investment in political activity, but this is not sufficient to prevent a higher tax rate in the first period. The adjustment process is again similar to the one described above. However, the increased redistribution enables more unskilled to become skilled in the course of time. So, in the steady state, the number of skilled is higher. As a result, the tax rate is lower and growth is higher in the long run than in the simulation starting with a lower w^s. So, if the investment in human capital is inefficiently low due to capital market constraints, a technological shock that initially only benefits the skilled will eventually benefit all individuals. Via the political redistribution process, the unskilled will appropriate part of the gains which allows more unskilled to become skilled and thus fosters growth.

6 Concluding Remarks

We have analyzed the consequences of endogenous political decision making on redistribution in a two-sector endogenous growth model where growth is driven by human capital, investment in human capital is indivisible and capital markets are missing. Clearly, in such an economy, investment in human capital tends to be inefficiently low and redistribution may have significant effects on growth.

The political-economy model used does not assume ex ante asymmetry in political influence[12], but the outcome may be asymmetrical due to differences in time invested in lobbying. These differences may be the result of *cost-benefit effects*, i.e. differences in the balance between the costs and benefits of lobbying. If technological progress increases the wage gap between skilled and unskilled, for example, this will result in more active political support for social assistance, minimum wage laws and other types of intra-generational redistribution. On the other hand, when the political activity of unskilled workers increases, the dynamics of redistribution may cause the number of unskilled to shrink and the group of skilled workers to grow. If *group size effects* are important, that is, if the relative size of a lobby group matters for influencing government policy decisions, this may lead to a countervailing power of the skilled lobby for their sheer number. As a result of these two opposing forces persistence in inequality may prevail when redistribution is blocked, leading to multiple steady-state equilibria.

We have shown that the equilibrium that is actually realized in the long run is dependent upon the initial distribution of wealth. In particular, we have illustrated that effective redistribution of wealth can promote growth.

As another example, we analyzed the effects of a technological shock that increases the wage gap between skilled and unskilled workers. This example illustrated that political decision making may be crucial for the effects of economic shocks. In particular, we demonstrated that if the unskilled, via the

[12] That is, abstracting from group-size effects, the possibilities to affect the outcome of the political process may be the same for skilled and unskilled individuals.

political redistribution process manage to appropriate part of the gains of such technological shock, this may enable some of them to invest in human capital and thus foster growth.

In the examples we presented, higher long-run growth is accompanied by a lower tax rate, i.e. less redistribution. This is not necessarily the case, however. We have shown that the relation between redistribution and long-run growth is non-monotonic: for low values of the tax rate, more redistribution can go along with higher growth, but for higher values of the tax rate the relationship is opposite. Moreover, due to the evolution of the opposing forces in the political process, the time path of the tax rate towards its steady-state level may be non-monotonic.

The analysis presented in this paper allows for many useful extensions. For example, we did not take into account the accumulation of physical capital. This may overestimate the benefit that the unskilled derive from redistributive taxation as the income tax may reduce the pace of physical capital accumulation. Another useful extension of the model could be to allow for nonlinear redistributive schemes and progressivity in income tax rates. However, the most promising route for further research seems to be further elaboration of the model of the political process. In particular, in our model, the effective pressure of a lobby group only depends upon its size and its political activity, and not on the economic position of its members. In reality, money also seems to play a role in influencing government decision making. Therefore, it would be interesting to see whether our results change if we allow for investment of wealth in addition to the investment of time in order to influence political decision making on redistribution.

References

[1] Acemoglu,D. and Robinson,J.A., 2001, "Inefficient Redistribution" American Political Science Review 95: 649-661.

[2] Aidt, T.S., 1997 "Strategic Entry, Rent Seeking and Transfers" University of Aarhus Working Paper 1997-2021.

[3] Aghion,P. and Bolton,P.,1997,"A Theory of Trickle-Down Growth and Development" Review of Economic Studies 64: 151-172.

[4] Alesina,A. and Rodrik,D.,1994,"Distributive Politics and Economic Growth" Quarterly Journal of Economics 108: 465-490.

[5] Alesina,A. and Perotti,R., 1996,"Income Distribution, Political Instability, and Investment" European Economic Review 40: 1203-28.

[6] Becker, G.S. 1983 "A Theory of Competition among Pressure Groups for Political Influence" Quarterly Journal of Economics 98: 371-400.

[7] Becker, G.S. 1985 "Public Policies, Pressure Groups and Dead Weight Costs" Journal of Public Economics 28 : 329-347.

[8] Becker, G.S. 1998 ”Deadweight Costs and the Size of Government” NBER Working Paper 6789.

Benabou,R.2000, ”Unequal Societies: Income Distribution and the Social Contract” American Economic Review 90: 96-129.

[9] Bertola,G.,1993,”Factor Shares and Savings in Endogenous Growth”American Economic Review 83: 1184-1198.

[10] Bourguignon,F. and Verdier T.,2000, ”Oligarchy,democracy,inequality and growth” Journal of Development Economics 62: 285-313.

[11] Cameroon, D.R.,1988,”Distributional Coalitions and other Sources of Economic Stagnation: On Olson’s Rise and Decline of Nations” International Organization 42: 561-603.

[12] Canton, E.J.F., H.L.F. de Groot and R. Nahuis , 2002, ”Vested Interests and Resistance to Technology Adoption” Journal of European Political Economy

[13] Conway, Margaret M. 1991,”Political Participation in the United States. Washington DC: U.S. Congressional Quarterly Press.

[14] Figini,P.,1999,”Inequality and Growth Revisited”,Trinity and College,Trinity Economic Paper Series,Paper No.99/2.

[15] Galor,O. and Zeira, J.,1993,”Income Distribution and Macroeconomics”Review of Economic Studies 60: 35-52.

[16] Filer, J., Kenny, L, and Morton B. 1993, "Redistribution, Income and Voting" American Journal of Political Science 37: 63-87.

[17] John E.Filer, Lawrence W. Kenny and Rebecca B. Morton,1993 ” Redistribution, Income and Voting” American Journal of Political Science 37: 63-87.

[18] Mulligan, C.B. and X.Sala-i-Martin, 1999 ”Gerontocracy, Retirement and Social Security” NBER Working Paper 7117.

[19] Olson, M., 1965, ”The logic of Collective Action” MA: Harvard University Press.

[20] Olson,M.,1982, ”The rise and decline of Nations” New Haven: Yale University Press.

[21] Perotti,R.,1996,”Democracy,income Distribution and Growth: What the Data Says.” Journal of Economic Growth 1:149-187.

[22] Peresson,T. and Tabellini,G.,1994,”Is Inequality Harmful for Growth?American Economic Review 84: 600-21.

[23] Radcliff, Benjamin.,1992, " The Welfare State, Turnout, and the Economy: A Comparative Analysis" The American Political Science Review 86: 444-454.

[24] Rodriguez,F.,1999."Does Inequality lead to Redistribution?Evidence from The United States" Economics and Politics,11: 2.

[25] Rosenstone, Steven J. and Hansen, John M., 1993," Mobilization,Participation and Democracy in America. New York: MacMillan.

[26] Saint-Paul,G., and Verdier,G.,1993,"Education,Democracy and Growth" Journal of Development Economics 42: 399-407.

[27] Sala-i-Martin,X.,1996,"A positive Theory of Social Security"Journal of Economic Growth 1: 277-304.

[28] Williamson, J.G. 1999. "Inequality,Poverty, and History, Oxford: Blackwell.

APPENDIX

Appendix A1: Proof of Lemma 1

It is straight forward to derive the best response functions for both lobby groups. After differentiation of first-order condition (15) one gets,

$$-\tau_t^{su} l_t^s d\gamma_t^u - l_t^s \tau_t^{ss} d\gamma_t^s + \tau_t^s d\gamma_t^s = -\tau_t^u d\gamma_t^u - \tau_t^s d\gamma_t^s \tag{A1}$$

Rearranging, gives the slope of the reaction function for the skilled

$$d\gamma_t^s = \frac{\tau_t^{su} l_t^s - \tau_t^u}{2\tau_t^s - \tau_t^{ss} l_t^s} d\gamma_t^u \tag{A2}$$

As noted from (14), since $(\tau_t^s < 0, \tau_t^{ss} > 0, \tau_t^u > 0)$ the slope of the reaction curves is determined by the sign of τ_t^{su} and also the size of τ_t^u and τ_t^{su}. If $\tau_t^{su} > \tau_t^u > 0$ holds, then the slope will be negative. However, if $\tau_t^{su} < 0$ or $\tau_t^u > \tau_t^{su} > 0$ holds, then the slope of the reaction function of the lobby group representing the skilled will be positive and hence the lobby group is offensive.

For the lobby group representing unskilled, totally differentiating (16) gives

$$\frac{N_t^s}{N_t^u} w_t^s l_t^s \tau_t^{uu} d\gamma_t^u + \frac{N_t^s}{N_t^u} w_t^s l_t^s \tau_t^{su} d\gamma_t^s - \frac{N_t^s}{N_t^u} \tau_t^u w_t^s d\gamma_t^s + \frac{N_t^s}{N_t^u} \tau_t^u l_t^s = 0 \tag{A3}$$

Re-arranging and eliminating terms, the following equation gives reaction function for the unskilled

$$d\gamma_t^u = -\frac{\tau_t^{su} l_t^s - \tau_t^u}{\tau_t^{uu} l_t^s} d\gamma_t^s - \frac{\tau_t^u}{w_t^s \tau_t^{uu}} \tag{A4}$$

Given that $\tau_t^{uu} < 0$, it is straight forward to derive the sign of best response function of the unskilled. If $\tau_t^{su} > \tau_t^u > 0$ holds, then the slope will is positive and hence the lobby group is offensive. However if $\tau_t^{su} < 0$ or $\tau_t^u > \tau_t^{su} > 0$, the slope will be negative and hence the lobby group will be defensive. This proves Lemma 1.

Appendix A2: Proof of the Stability Condition

In this appendix we proof the stability condition (17).

If the pressure of u and s deviate in period t from their equilibrium levels by the small amounts $d\gamma_t^s$ and $d\gamma_t^u$, then the optimizing conditions for skilled and unskilled is given by equations (15) imply that

$$\begin{cases} \left(2\tau_t^s - \tau_t^{ss} l_t^s\right) d\gamma_t^s - \left(\tau_t^{su} l_t^s - \tau_t^u\right) d\gamma_{t-1}^u = 0 \\ \left(\tau_t^{su} l_t^s - \tau_t^u\right) d\gamma_{t-1}^s + \left(\tau_t^{uu} l_t^s\right) d\gamma_{t-1}^u = \tau_t^u l_t^s \end{cases} \tag{A5}$$

Hence by substitution

$$d\gamma_t^s = \left[\frac{\left(\tau_t^{su} l_t^s - \tau_t^u\right)}{\left(2\tau_t^s - \tau_t^{ss} l_t^s\right)}\right] \left[\frac{-\left(\tau_t^{su} l_t^s - \tau_t^u\right)}{\left(\tau_t^{uu} l_t^s\right)}\right] d\gamma_{t-1}^s + \left[\frac{\tau_t^{su} l_t^s - \tau_t^u}{2\tau_t^s - \tau_t^{ss} l_t^s}\right] \frac{\tau_t^u l_t^s}{w_t^s} \tag{A6}$$

which implies that $d\gamma_{t+1}^s$ and $d\gamma_{t+1}^u$ return to their equilibrium values if the absolute value of the determinant is positive i.e.

$$|A| = \left(-\left(2\tau_t^s - \tau_t^{ss} l_t^s\right)\left(\tau_t^{uu} l_t^s\right) - \left(\tau_t^{su} l_t^s - \tau_t^u\right)^2\right) > 0 \tag{A7}$$

This proves the stability condition in (17).

Appendix B1: Proof of Proposition 1

From equation (5) we have that

$$g_A = \theta^s S \left(\frac{\theta^s}{\theta^u}\right)^{\frac{\phi-1}{1+\pi-\phi}} \left(\frac{S}{U}\right)^{\frac{\phi-1}{1+\pi-\phi}} \tag{A8}$$

Rewriting the above equation and letting $\alpha = \frac{\pi}{1+\pi-\phi}$ and $1-\alpha = \frac{1-\phi}{1+\pi-\phi}$ we find

$$g_A = \theta^s \left(\frac{\theta^s}{\theta^u}\right)^{\alpha-1} \left(N^s l^s\right)^{\alpha} \left(\left(N - N^s\right) l^u\right)^{1-\alpha} \tag{A9}$$

The effect of an increase in the supply of skilled people N^s. on growth is determined by partially differentiating the above function w.r.t N^s and applying the product rule, which gives

$$\frac{\partial g_A}{\partial N^s} = \left[\theta^s \left(\frac{\theta^s}{\theta^u}\right)^{\alpha-1}\right] \left[\frac{N^s l^s}{\left(N - N^s\right) l^u}\right]^{\alpha} l^u \left[\alpha \frac{\left(N - N^s\right)}{N^s} - (1-\alpha)\right] \tag{A10}$$

The sign $\frac{\partial g_A}{\partial N^s}$ is positive if and only if $\left(\frac{\pi}{1+\pi-\phi}\right) N > N^s$. This proves Proposition 1.

Appendix B2: Proof of Proposition 2

The tax influence function as specified in equation (23) has the following characteristics:

$$\begin{aligned}
\tau^u &= \rho\frac{\tau}{\gamma_{t+1}^u} > 0, \tau^s = -\rho\frac{\tau}{\gamma_{t+1}^s} < 0 \\
\tau^{uu} &= \rho(\rho-1)\frac{\tau}{\left(\gamma_{t+1}^u\right)^2} < 0, \tau^{ss} = \rho(\rho+1)\frac{\tau}{\left(\gamma_{t+1}^s\right)^2} > 0, \\
\tau^{su} &= \tau^{us} = -\rho^2\tau/(\gamma_{t+1}^u\gamma_{t+1}^s) < 0
\end{aligned} \tag{A11}$$

From the first-order conditions (15) and (16) we have that

$$\frac{\tau_t^u}{\tau_t^s} = -\frac{N_t^u w_t^s}{N_t^s w_t^u\left(1-\tau_t\right)} \tag{A12}$$

and consequently

$$\frac{\gamma_t^s}{\gamma_t^u} = \frac{N_t^u w_t^u}{N_t^s w_t^s\left(1-\tau_t\right)} \tag{A13}$$

Solving for γ^u and γ^s, we find

$$\gamma_t^u = \frac{\rho\tau_t}{1+\tau_t\left(\rho-1\right)}\frac{w_t^s}{w_u^o}\frac{N_t^s}{N_t^u} \tag{A14}$$

$$\gamma_t^s = \frac{\rho\tau_t}{1+\tau_t\left(\rho-1\right)} \tag{A15}$$

Using (A10) to substitute out γ_t^u and γ_t^s from (23) gives

$$\frac{N_t^s}{N_t^u} = \left[\frac{\tau_t}{\tau_o\left(1-\tau_t\right)^\rho}\left(\frac{w_t^s}{w_t^u}\right)^{-\rho}\right]^{1/(\rho-\mu)} \tag{A16}$$

Inserting the wage gap equation (3) in the above equation and re-arranging we have that

$$\frac{N_t^s}{N_t^u} = \left[\left(\frac{\tau_t}{\tau_o\left(1-\tau_t\right)^\rho}\right)^{\frac{1+\pi-\phi}{\rho-\mu(1+\pi-\phi)}}\left(\frac{\theta^s}{\theta^u}\right)^{\frac{-\rho}{\rho-\mu(1+\pi-\phi)}}\left(\frac{l_t^s}{l_t^u}\right)^{\frac{\rho(\pi-\phi)}{\rho-\mu(1+\pi-\phi)}}\right] \tag{A17}$$

Substituting (A14) into (A5) and rearranging, we have that

$$g_t = \Theta\tau_t^{\left(\frac{\phi-1}{\rho-u(1+\pi-\phi)}\right)}\left(1-\tau_t\right)^{\left(\frac{\rho(\phi-1)}{\rho-u(1+\pi-\phi)}\right)} \tag{A18}$$

where $\Theta = N_t^s\left(\frac{\theta^s}{\tau_o}\right)\left(\frac{\theta^s}{\theta^u}\right)^{\frac{-\mu(\phi-1)}{\rho-\mu(1+\pi-\phi)}}\left(l_t^s\right)^{\frac{\rho\phi-\mu\pi}{\rho-\mu(1+\pi-\phi)}}\left(l_t^u\right)^{-\left[\frac{(\phi-1)(\rho-\mu)}{\rho-\mu(1+\pi-\phi)}\right]}$. Loosely speaking if Θ is taken to be constant, equation (A18) says that the relation betwen g and τ_t is non-monotonic. Θ can be fully endogenized if we eliminate l_t^s and l_t^u and express them in terms of γ_t^u and γ_t^s . That is, equations (A14)-(A18) can be used in combination with $N^u+N^s=N$ to solve for γ_t^u, γ_t^s, $N^u, N^s\frac{w_t^s}{w_t^u}$

Parameter	Definition	Value
h_0	Initial cost of becoming skilled	60
A_0^u	Initial wage of unskilled workers	50
A_0^s	Initial wage of skilled workers	140
μ	Group size effect	1.2
ρ	Opportunity cost effect	0.1
α	Share of consumption	0.5
r	Interest rate	0.2
N	Total number of individuals	100
τ_0	Transfer base	0.05
θ_u	Shift parameter	0.0008
θ_s	Shift parameter	0.002
π	Effect of production on creation of knowledge of unskilled	0.35
ϕ	Effect of production on creation of knowledge of skilled	0.80

Table 1: Parameter Values

and g as a function of the tax rate. Unfortumately, the resulting expressions do not provide much insight. However, Figures 3 and 6 summarize the behaviour of growth and tax variables. That a growth-maximizing tax rate exists can be shown by setting $\frac{\partial g_t}{\partial \tau} = 0$ and checking $\frac{\partial^2 g_t}{\partial \tau^2} < 0$.

Appendix C1: Numerical Simulation.

In this appendix, we provide some information on the numerical simulation examples we performed. The table below presents the parameter settings used in the experiments. The simulations are performed by Compaq Visual Fortran. The relevant program is available from the authors.

FIGURES

Figure 1A : A sketch of the reaction of the political system with $0 < \tau^u < l^s\tau^{su}$

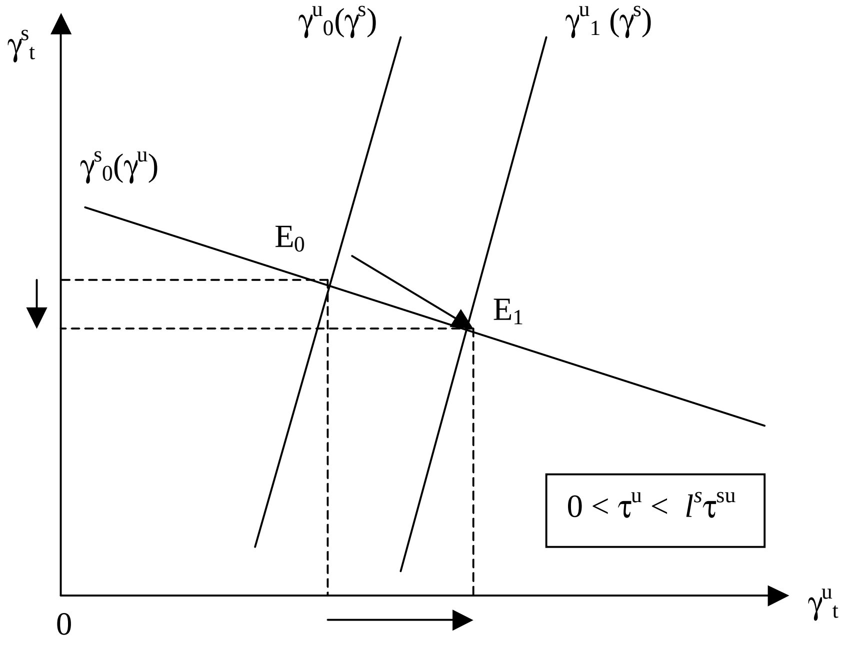


Figure 1B. Effects of an increase in wage gap with $\tau^{su} < 0$ or $\tau^{u}_{t} > \tau^{su}_{t} > 0$.

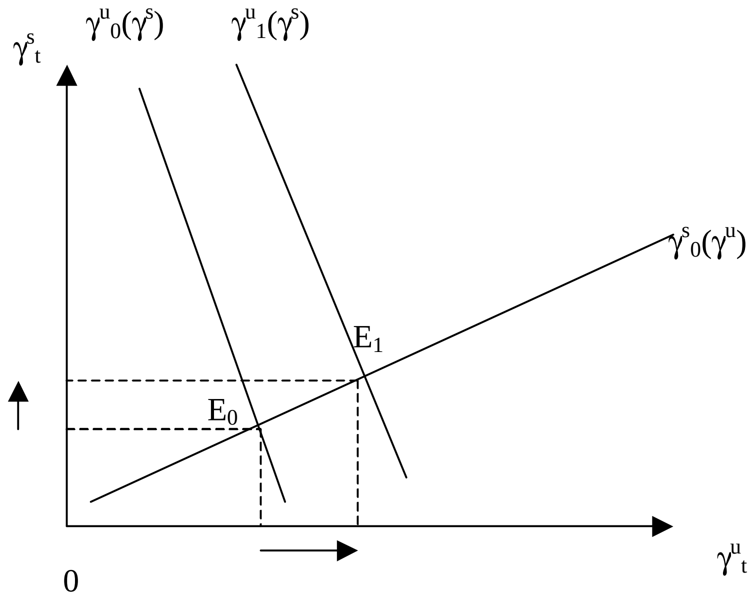


Figure 2. Dynamics of Wealth Distribution

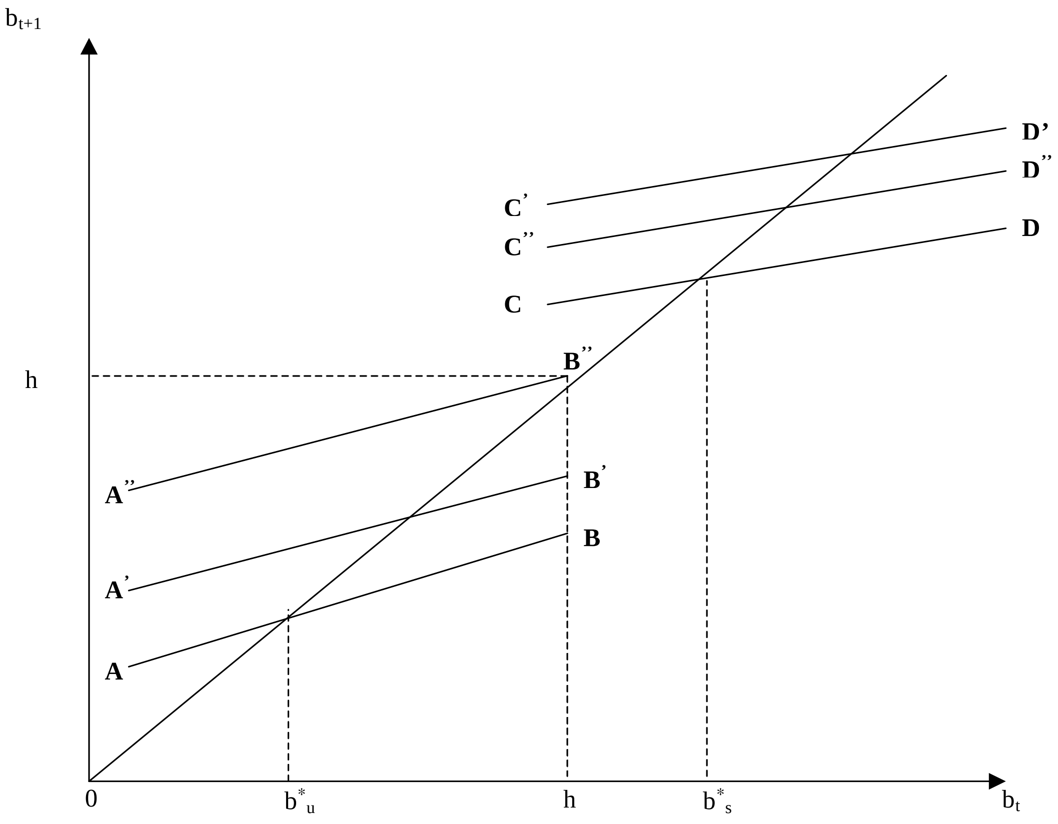


Figure 3. Non-monotonic relationship between growth and tax rate.

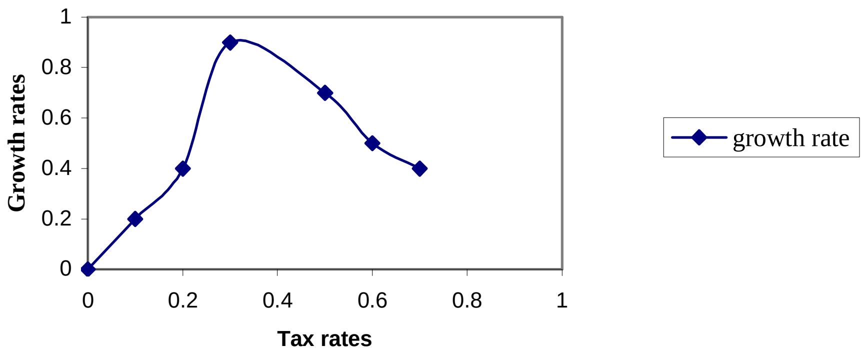
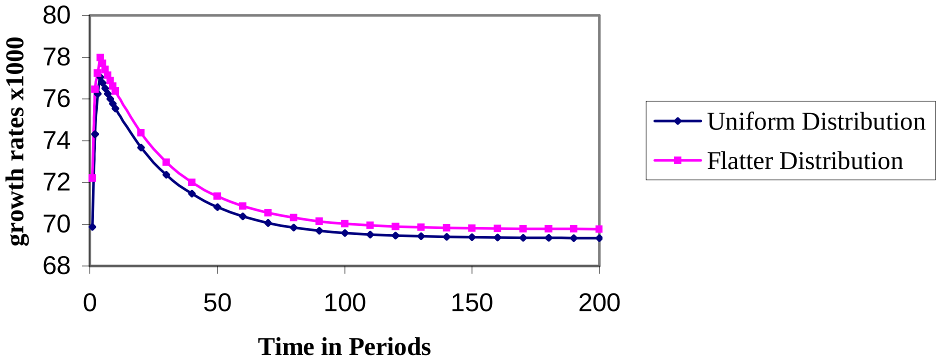


Figure 4A. Growth rates of skilled sector under different Distribution of Wealth



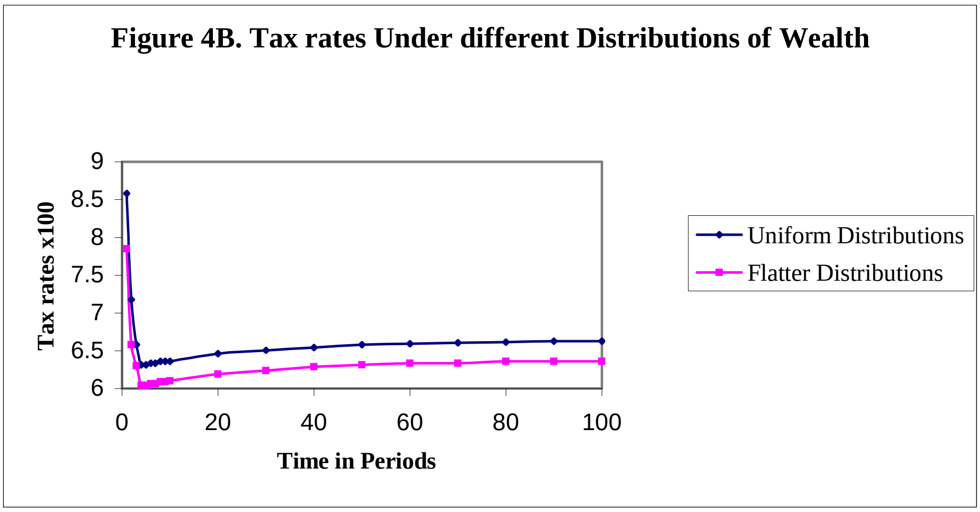


Figure 4B. Tax rates Under different Distributions of Wealth
Tax rates x100
9
8.5
8
7.5
7
6.5
6
0
20
40
60
80
100
Time in Periods
Uniform Distributions
Flatter Distributions

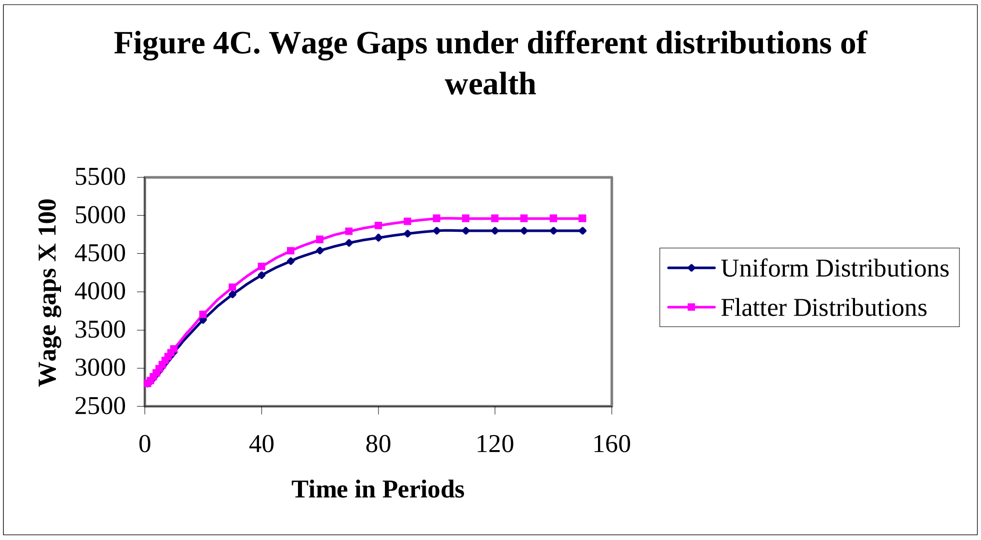


Figure 4C. Wage Gaps under different distributions of wealth
Wage gaps X 100
5500
5000
4500
4000
3500
3000
2500
0
40
80
120
160
Time in Periods
Uniform Distributions
Flatter Distributions

Figure 4D. Number of Unskilled individuals under different Distribution of Wealth

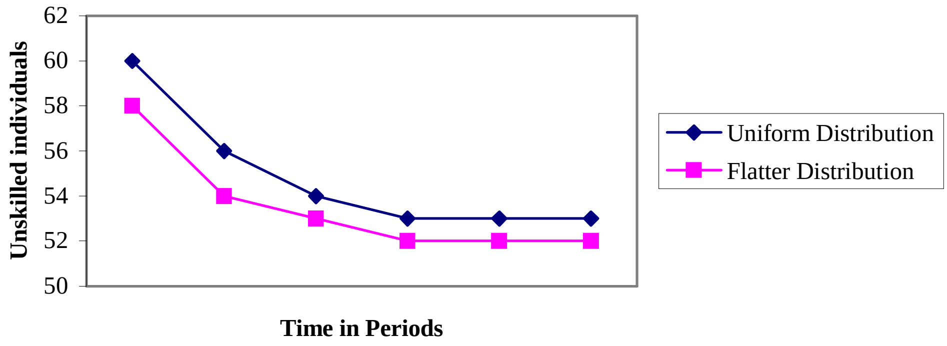
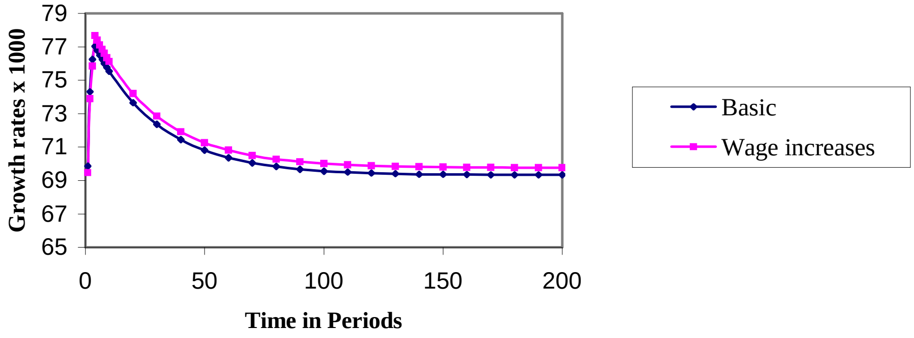


Figure 5A. Effect of wage increase on growth rates under uniform Distribution



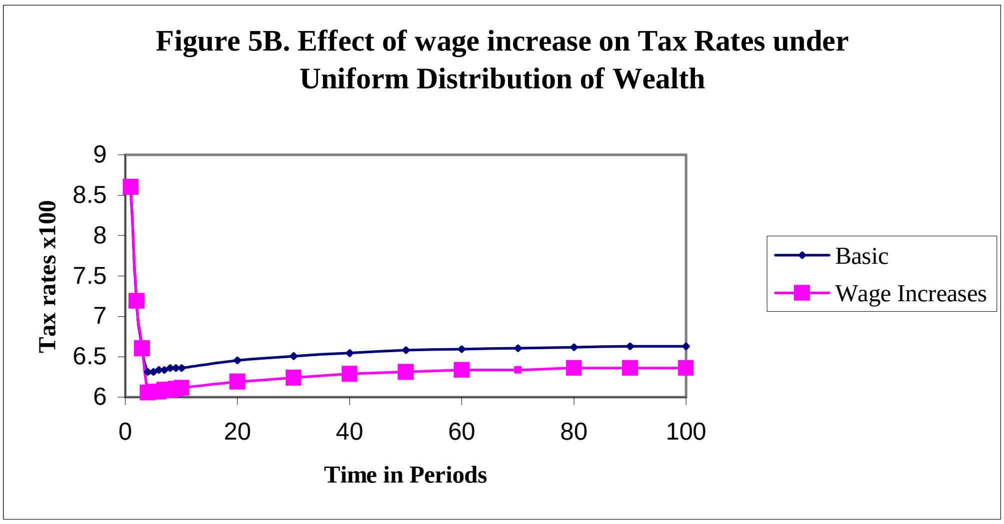


Figure 5B. Effect of wage increase on Tax Rates under Uniform Distribution of Wealth
Tax rates x100
9
8.5
8
7.5
7
6.5
6
0
20
40
60
80
100
Time in Periods
Basic
Wage Increases

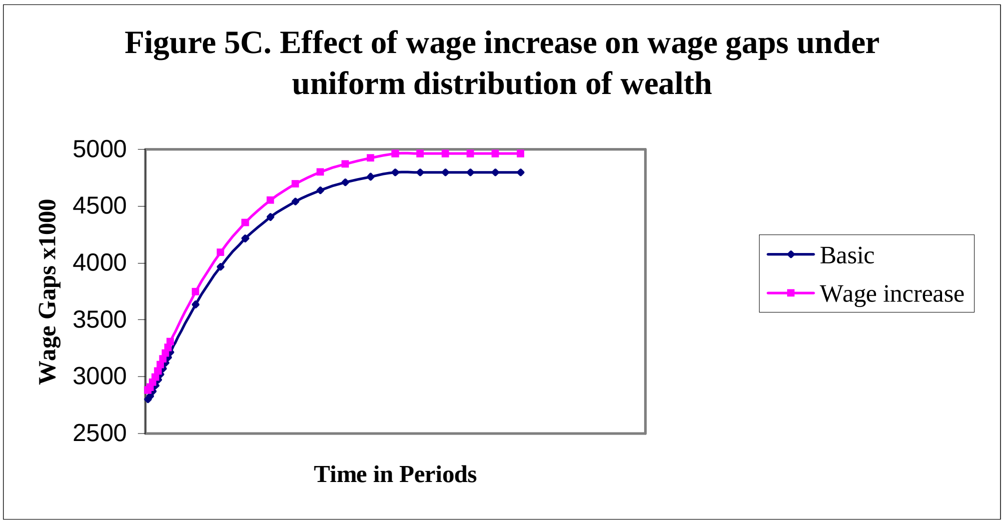


Figure 5C. Effect of wage increase on wage gaps under uniform distribution of wealth
Wage Gaps x1000
5000
4500
4000
3500
3000
2500
Time in Periods
Basic
Wage increase

Figure 5D. The Effect of wage increase on the evolution of Long run # of unskilled individuals under uniform distribution

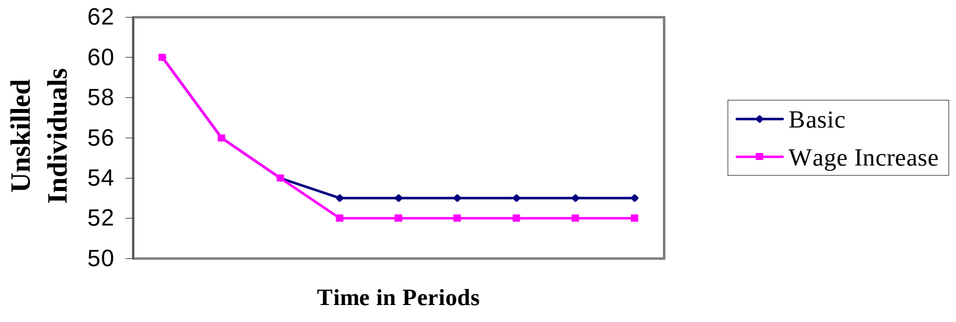


Figure 6. Non-monotonic relationship between growth and tax rate