CentER

Discussion Paper

No. 2004–09

ON PROCEDURAL FREEDOM OF CHOICE

By R. Arlegi, D. Dimitrov

January 2004

ISSN 0924-7815

TILBURG ◆ UNIVERSITY

On Procedural Freedom of Choice*

Ritxar Arlegi
Department of Economics
Public University of Navarra, Spain
E-mail: rarlegi@unavarra.es

Dinko Dimitrov
CentER and Department of Econometrics and Operations Research
Tilburg University, The Netherlands
E-mail: d.a.dimitrov@uvt.nl

January 2004

Abstract

Numerous works in the last decade have analyzed the question of how to compare opportunity sets as a way to measure and evaluate individual freedom of choice. This paper defends that, in many contexts, external procedural aspects that are associated to an opportunity set should be taken into account when making judgements about the freedom of choice an agent enjoys. We propose criteria for comparing procedure-based opportunity sets that are consistent with both the procedural aspect of freedom and most of the standard theories of ranking opportunity sets.

*The authors would like to thank Martin van Hees, Clemens Puppe, Harrie de Swart, Walter Trockel, and Yongsheng Xu for helpful comments and suggestions. The work on this paper started while the authors were research fellows at ZiF (Bielefeld) for the project "Procedural Approaches to Conflict Resolution", 2002. We thank our hosts for their hospitality. The work of R. Arlegi was supported by the CICYT Government under Research Projects SEC2000-0838 and SEC2003-08105. The work of D. Dimitrov was supported by a Marie Curie Research Fellowship of the European Community programme "Improving the Human Research Potential and the Socio-Economic Knowledge Base" under contract number HPMF-CT-2002-02121.

Journal of Economic Literature Classification Number: D81.
Key Words: freedom of choice, opportunity sets, procedure-based opportunity sets.

1 Introduction and Motivation

According to the libertarian tradition rooted by J. S. Mill (1859) and more recently developed by R. Nozick (1974), the exercise of freedom of choice enriches human individual faculties and it is a necessary condition for a life to be considered as a "meaningful" life. In this sense, the value of freedom of choice is independent of individual preferences about the options to choose from and, therefore, it can only be captured by means of a non-welfaristic approach.

In particular, several authors have been concerned with the inadequacy of the standard theory of consumer behavior to display properly the value of freedom of choice. According to the standard view which evaluates budget sets by means of their indirect utility, an opportunity set of an arbitrary large number of alternatives is always declared by any agent as good as another opportunity set consisting uniquely of its best alternative in terms of that agent's preferences. Therefore, there is neither a positive nor a normative difference between choosing autonomously an alternative "x" from a large set of options, and being constrained (or even, obliged) to choose "x". This means that if we are concerned with measuring and evaluating the intrinsic value of freedom, opportunity sets should be evaluated in a different way. This has been the main motivation for many proposals in the last years to rank opportunity sets consistent with the assumption that freedom of choice is desirable farther away than due to welfaristic reasons (see, among many others Arlegi and Nieto (2001a,b), Bossert (1997, 2000), Bossert, Pattanaik, and

Xu (1994), Foster (1992), Gravel (1998), Jones and Sugden (1982), Klemisch-Ahlert (1993), Pattanaik and Xu (1990, 1998, 2000), Puppe (1995, 1996), and Sugden (1998)).

In all mentioned works the central object of formal analysis is that of an opportunity set: the set of (mutually exclusive) alternatives enjoyed by the decision maker. Then, comparisons and rankings over opportunity sets are defined, axiomatically characterized, and discussed on the basis of the information provided by the sets. In many of those works the comparisons are also made on the basis of the information given by the agent's preferences over the basic alternatives. Other works like Jones and Sugden (1982), Pattanaik and Xu (1998) or Sugden (1998) do not consider the actual preferences of the agent but the set of reasonable preferences: "those preferences that reasonable people would have in the position of the agent whose freedom we are trying to evaluate". In any case, the opportunity set availed by an agent, together with certain specification of his/her preferences (actual, potential or reasonable), are assumed to contain all the relevant information to make judgements about the individual's freedom.

However, there are numerous situations in which the set of available opportunities for the agent and his/her preferences do not capture certain circumstances that might be relevant in order to make positive or normative judgements about the freedom of choice that an agent enjoys when confronted with a certain opportunity set. Consider the following examples:

Example 1 *(Inspired by Gaertner and Xu (2004)) An agent enjoys a budget set which includes among the possible goods certain fancy sport shoes. In situation I it is known that those shoes have been produced by children under eleven in a South Asian country. In situation II the price of the shoes is the same, and so is the agent's budget set, but now it is known that the shoes have* *been fairly produced (whatever "fairly" is out of being produced by children).*

Example 2 *Two young persons i and j shall vote in the elections for president. Both have exactly the same preferences over the set of parties, and both will vote party X as their best option. Person i comes from an open-minded family that has provided to him a critical education; he has always had access to whatever information concerning politic issues, and has used to make use of such information. Person j is the son of X-radical activists: they have systematically forbidden and hidden to him any information not concordant with their ideology, so that his information about other parties different to X is inexistent or distorted.*

Example 3 *(Barberà, Bossert, and Pattanaik (2002)) Individual k has to decide about which career to follow. In scenario A, among k's options is not that of being a football player just because of some physical handicaps. In scenario B, k has exactly the same options as in scenario A (that do not include being a football player), but in this case k is not allowed to be a football player because he is black, and the low in the country forbids black people to play football.*

Example 4 *In country A divorce is legal, while in country B it is not. That is to say, in country A the choice of who to marry with is reversible, while in country B it is not.*

Example 5 *In situation I, person k lives in a country where everybody has access to eleven newspapers. In situation II, k can read the same eleven newspapers, but everybody else is not allowed to read newspapers at all.*

Example 6 *In two companies A and B the workers are allowed to choose the dates of their holidays. The workers in company A can make their choice effective by putting a sheet with the chosen dates and their signature in the* *director's mail box. In company B a tedious bureaucratic process must be followed: very complex formularies must be filled up, and an appointment with the director must be made two months in advance.*

Example 7 *In a certain department of Economics the allocation of different subjects among the professors has to be decided. Suppose there are more subjects than those that can be taught by the members of the department, so that some of them have to be rejected or postponed to later terms or years. In scenario I all the members join in one ore more meetings, expose their preferences, propose their arguments, and after this process each professor has the opportunity to choose one between one or two subjects. Professor Q, after being convinced by other members, agrees on choosing between giving a course on Game Theory or, otherwise, on Bounded Rationality, though these topics are not very close to her interests. In scenario II, Professor Q receives, without previous information, a rude letter by the head of the department compelling her to choose between teaching Game Theory or Bounded Rationality.*

These examples show certain decision circumstances that are not displayable either by the opportunity set enjoyed by the individual, or by his preferences (at least in a direct way). Let us briefly comment on each of the examples.

Example 1 says in an economic framework that it is *the procedure by which the opportunity sets has been generated* what seems to matter when evaluating the opportunity set. There, the technology applied to generate the options in the set is what can be regarded from an ethical perspective to affect the evaluation of the set.

Example 2 deals with the *way individual preferences have been formed* and with the Millian idea of individual autonomy: Sen (1993a) realizes about the crucial importance of the condition of autonomy for freedom of choice, and tries to capture it with the sole informational basis of the individual preferences and her opportunity set. However, neither the preferences nor the opportunity set are able to display the way in which the preferences were formed, the latter being a relevant aspect when evaluating individual autonomy.

In Example 3, the *negative freedom* of the individual is at stake, that is, what the decision maker cannot do due to others' encroachment. The concept of negative freedom is deeply explored by Berlin (1969) and recently developed by Van Hees (1998) in the context of opportunity sets comparisons. The mere opportunity set available by an agent cannot display any information about options that are not in the opportunity set, and even less, about which are the reasons why those options are not available. However, it seems reasonable that the options that one cannot choose, and the reasons why they are not choosable, are aspects to be considered when evaluating individual freedom.

In Example 4 the *reversibility* of the choice is suggested to be taken into account as an external reference to the opportunity set.

Example 5 is concerned with *the distribution of freedom*. One could consider as relevant the degree of freedom of other agents when evaluating the degree of freedom a particular agent enjoys. In the welfaristic theory of choice, there are models that incorporate altruism considering others utilities as a component of the utility function for the altruistic agent. In an analogous way we could consider altruism in the freedom aspect: the perception of our freedom could be influenced by the fact that others share that freedom. Note however that the relationship could be the other way around: there could be agents that feel more free to choose in a situation like II in the example rather than in a situation like I. But, still, the mere opportunity set enjoyed by the decision maker does not provide enough information to make judgements about overall freedom. On the other hand, from a normative point of view, an observer that has to evaluate k's freedom in the example, would possibly measure it in a different way under scenarios I or II. Again, information of opportunity sets enjoyed by other agents has to be computed in order to evaluate k's freedom of choice.

Example 6 takes into consideration *the procedure for making a final choice among the alternatives.* The opportunity set is a helpful object to display the freedom about what to choose from, but not the freedom about *how* to make effective our choice: when to choose, who to choose with, how dressed, and in many other circumstances that might be a part of the choice act.

In Example 7, it is again *the procedure by which the opportunity sets has become available* what seems to matter. There, deliberation and argumentation seem to be *procedurally more free* than persuasion, and this more than threats. Note that we can perfectly be threatened to do what we want to do (for example, if the "threatener" is not aware of our preferences), but probably we would not feel equally free whether we make our choice under threats or not. Similarly, in the example, it is reasonable to assume that Professor Q does not feel equally free in both situations if she has certain value judgements about how free is the procedure that has generated her opportunity set. Also from a normative point of view, it seems defendable that we should not look uniquely at opportunity sets allocations regardless the procedure by which that allocations have been reached.

One could argue that we are trying to introduce aspects in the evaluation of freedom that are out of the sphere of what actually is freedom of choice. That is, one could defend that the freedom of choice enjoyed by an agent is uniquely determined by what the agent *can* choose. Therefore, how the set was generated or which is the set of non-available options, or how the final choice has to be made are all of them aspects that concern other things different to the actual freedom of choice provided by the opportunity set.

In a sense, this discussion is not new: Pattanaik and Xu (1990) propose the cardinalist rule to rank opportunity sets, that simply takes the number of alternatives in the set as an index of the freedom that it provides. They characterize the cardinalist ranking as an elegant consequence of very simple axioms. Their proposal can be considered as the barest measurement of freedom in the sense that, if we are concerned with freedom of choice, we should just count the number of options. Sen (1991, 1993a) defends that any evaluation of the overall freedom provided by a set should take into account also the preferences of the agent over the alternatives. From our point of view, the introduction of preferences implies a deviation from the purest description of freedom that Pattanaik and Xu (1990) provide. However, by following Sen, and considering the preferences over the alternatives (actual or potential) we gain a better accomplishment with the natural and practical meaning of *overall* freedom of choice.

In other words, our proposal to incorporate certain procedural aspects to evaluate freedom of choice also deviates from the purest account of freedom, but we believe that it contributes also to a better theory of comparing opportunity sets in terms of overall freedom.

In sum, we will distinguish between two aspects of freedom of choice:

- *objective freedom* - the freedom of choice provided by the mere availability of certain alternatives over which there are defined certain preferences. In this sense, all works mentioned above propose rules to compare opportunity sets that measure and evaluate objective freedom;

- *procedural freedom* - the freedom provided by any other external reference out of the set of alternatives and individual preferences like the procedure that generates the set, the way the final choice is made or any other external circumstance that we might consider relevant in terms of freedom of choice.

Another possible way to formally approach the problem could be by attaching the procedural information to the alternatives, rather than to the opportunity sets. That is, by considering any given opportunity (for example, "voting party A"), as a multiplicity of alternatives depending of the external reference. So that, one could consider "voting party A under coercion" as a different alternative of "voting party A under persuasion" or "voting party A under freely formed preferences". This is the kind of approach we find in some works as Sen (1993b), and might be adequate under certain meanings of the external reference. In this paper we consider external references that are associated to the act of choosing from any opportunity set, rather than to the specific alternatives. For that reason it seems more adequate to describe the reference as associated to the set, rather than to the particular alternatives.

In trying to capture both the objective and the procedural aspects of freedom we have organized the rest of the paper as follows. Section 2 is devoted to our basic notation and definitions, and it introduces the *procedure-based opportunity sets* as our basic analytical tool. In Section 3 we consider *freedom preserving relations of procedure-based opportunity sets*, i.e. relations which satisfy two axioms: *objective freedom preservation* and *procedural freedom preservation*. These relations are characterized as being extensions of a special partial ordering defined over the family of procedure-based opportunity sets. We introduce the notion of a *procedural extension* of a given binary relation over *non*-procedure-based opportunity sets in Section 4 and present the connection between such a procedural extension and the set inclusion relation defined over the set of basic alternatives. In Section 5 two particular classes of procedural extensions of freedom preserving relations are considered: a first one that prioritizes the procedural aspect over the ranking on (non-procedure-based) opportunity sets, and a second one that prioritizes the ranking on (non-procedure-based) opportunity sets over the procedure. We present two axioms which, together with the corresponding axioms characterizing the relations on (non-procedure-based) opportunity sets, produce a characterization of the relation of procedure-based opportunity sets. Procedural extensions of the cardinalist ranking (see Pattanaik and Xu (1990)) and of the leximax ranking (see Bossert, Pattanaik and Xu (1994)) of opportunity sets are exemplified as well. We present some final remarks and conclusions in Section 6.

2 Framework

Let X be a non-empty and finite set of alternatives, and $\mathcal{X}$ be the set of all non-empty subsets of X. We will denote the elements of $\mathcal{X}$ by $A, B \ldots$. The interpretation of each element of $\mathcal{X}$ is that of an (non-procedure-based) *opportunity set.*

Let Π be a non-empty finite set of *procedures.* The elements of Π represent the different states of a certain category (procedure) that affect the value of an opportunity set. According to the examples in the previous section, the set Π may collect the different technologies that generate an opportunity set; the different procedures by which the final option can be chosen; the different ways the preferences are formed; the different distributions of freedom over individuals; the different degrees of relevance of the final choice, or other states of any procedural aspect that we consider relevant to be taken into account in order to evaluate opportunity sets. We do not advocate to any particular interpretation. We will simply assume that such a set exists and that the elements in Π are linearly ordered according to their desirability in terms of the procedural freedom of choice they attach to an opportunity set. More precisely, we assume the existence of a complete, transitive and antisymmetric binary relation $R \subseteq \Pi \times \Pi$, reading, for all $i, j \in \Pi$, iRj as "*given an opportunity set, procedure i attaches to it at least as much procedural freedom as procedure j*". The corresponding strict preference and indifference are denoted by P and I, respectively[1].

As far as we are interested in evaluating both the objective and procedural freedom provided by an opportunity set, we are interested in elements (A, i) of $\mathcal{K} = \mathcal{X} \times \Pi$, which will be called *procedure-based opportunity sets.* Each element (A, i) of $\mathcal{K}$ will be interpreted as "opportunity set A associated to

[1] The foundations to establish the particular estates of a certain procedural category and the corresponding rankings over them are out of the scope of this paper. However, such a question is of big interest. For example, one could (roughly) approach the degree of freedom in the formation of preferences by means of the number of years of education, or the degree of negative freedom by means of the number of alternatives that are forbidden to the decision maker (see Van Hees (1998) and Steiner (1983)). Also, some works analyze rankings of social situations in terms of the equality in the distribution of opportunities they provide (see, among others, Arlegi and Nieto (1999), Bossert, Fleurbaey, and Van de Gaer (1999), Gravel, Laslier, and Trannoy (1998), Herrero, Iturbe-Ormaetxe, and Nieto (1998), or Ok and Kranich (1998)).

procedure i". According to the particular interpretation we may apply to Π, the term "associated to" can be substituted by more particular expressions such as "conditioned by" or, as in Gaertner and Xu (2004), "generated by". We will assume that all elements in $\mathcal{K}$ are potentially feasible, i.e. that any element in $\mathcal{X}$ can be associated to any element in Π.

Comparisons of procedure-based opportunity sets will be represented by a binary relation $\succsim^+$ defined on $\mathcal{K}$. For all $(A, i), (B, j) \in \mathcal{K}$, $(A, i) \succsim^+ (B, j)$ should be read as "*set A, when associated to procedure i, is weakly preferred in terms of overall freedom to set B, when associated to procedure j*". The asymmetric and symmetric parts of $\succsim^+$ will be denoted by $\succ^+$ and $\sim^+$, respectively.

3 Freedom Preserving Relations over Procedure-Based Opportunity Sets

In this section, we introduce two axioms for $\succsim^+ \subseteq \mathcal{K} \times \mathcal{K}$ which can be seen as minimal requirements imposed on it in order to preserve the overall freedom.

- *Objective Freedom Preservation* (OF): For all $A, B \in \mathcal{X}$ and all $i \in \Pi$, $[B \subset A] \Rightarrow [(A, i) \succ^+ (B, i)]$.

- *Procedural Freedom Preservation* (PF): For all $i, j \in \Pi$ and all $A \in \mathcal{X}$, $[iRj] \Leftrightarrow [(A, i) \succsim^+ (A, j)]$.

(OF) establishes that a procedure-based opportunity set is always better than another procedure-based opportunity set provided that the first (non-procedure-based) opportunity set includes the second one, and that the associated procedure is the same. This axiom is a translation to our context of a common property in the works that consider the intrinsic value of freedom of choice. According to that property, any increasing in the number of the opportunities to choose from leads to an expansion of the freedom of choice. So that (OF) maintains, *ceteris paribus*, such a principle.

(PF) declares that an opportunity set associated to a given procedure as better in terms of overall freedom than the same set associated to another procedure if and only if the first procedure provides at least as much procedural freedom as the second one. In other words, (PF) states that, *ceteris paribus*, it is the procedural aspect that matters.

Definition 1 *Let* $\succsim^+ \subseteq \mathcal{K} \times \mathcal{K}$. *We will say that* $\succsim^+$ *is a* ***freedom preserving relation*** *if it satisfies (PF) and (OF).*

Clearly, (PF) and (OF) together only determine a partial ordering on $\mathcal{K}$, according to which we only know how to rank sets that are related by set inclusion when they are associated to the same procedure, and how to rank identical sets when they are associated to different procedures. But (OF) and (PF) do not allow to know anything about how to rank opportunity sets that are associated to the same procedure but not related by set inclusion; or opportunity sets that are associated to different procedures and related by set inclusion; or opportunity sets that are associated to different procedures but not related by set inclusion.

The way a relation on procedure-based opportunity sets solves the trade-offs mentioned above will depend on the particular values and importance that an individual or a society assigns to the procedures and to the availability of the different particular opportunities. We could think of individuals or societies for whom (which) the procedure is fundamental and there is no objective freedom that might compensate a better procedure. Or we can imagine a system of values that just considers the number of opportunities available, no matter the way the opportunity set is reached or generated. Several standard orderings of opportunity sets, such as the leximax rule (see Bossert, Pattanaik, and Xu (1994)) or the cardinalist one (see Pattanaik and Xu (1990)), are useful as a guide to possible solutions for the trade-offs of the first type (same procedure and opportunity sets not related by set inclusion), but at this point we do not know anything about how to manage the other kinds of trade-offs in which procedural aspects take place.

In what follows we propose a representation result for the class of freedom preserving relations on procedure-based opportunity sets. In order to elaborate on this point, we will make use of the following setup.

For the set X let σ_X be any fixed permutation of the elements of X the result of which is $X = \{x_1, \ldots, x_n\}$. Let $f : X \to \Re_{++}$ be any function assigning positive real numbers to the alternatives in X, and let $v : \Pi \to \Re_+$ be a *procedural value function* representing R (that is, such that for all $i, j \in \Pi$, $v(i) \geq v(j) \Leftrightarrow iRj$).

Let $u : \mathcal{X} \to \Re^n_{++}$ be such that for all $A \in \mathcal{X}$, $u(A) = (u_1, \ldots, u_n)$, where, for $i = 1, \ldots, n$,

$$u_i = \begin{cases} f(x_i) & \text{if} \quad x_i \in A, \\ 0 & \text{otherwise.} \end{cases}$$

Let $U : \mathcal{K} \to \Re^{n+1}_{++}$ be such that for all $(A, k) \in \mathcal{K}$, $U(A, k) = (U_1, \ldots, U_{n+1})$, where for $i \leq n$, $U_i = u_i$ and for $i = n + 1$, $u_i = v(k)$.

Let $\geq$ denote the "greater or equal" relation defined on $\Re^{n+1}_{++}$. Based on the construction above, we can define the following partial ordering $\succsim^*$ on $\mathcal{K}$:

$$\text{For all } (A, i), (B, j) \in \mathcal{K}, \ (A, i) \succsim^* (B, j) \Leftrightarrow U(A, i) \geq U(B, j).$$

We will denote the asymmetric and symmetric parts of $\succsim^*$ by $\succ^*$ and $\sim^*$, respectively.

A question that naturally arises is about the connection between $\succsim^*$ and relations on procedure-based opportunity sets. More specifically, our first result investigates the conditions under which a relation $\succsim^+$ over procedure-based opportunity sets can be seen as an *extension* of $\succsim^*$, that is, (1) $[(A,i) \succ^* (B,j)] \Rightarrow [(A,i) \succ^+ (B,j)]$, and (2) $[(A,i) \sim^* (B,j)] \Rightarrow [(A,i) \sim^+ (B,j)]$.

Theorem 1 *Let $\succsim^+ \subseteq \mathcal{K} \times \mathcal{K}$. Then, $\succsim^+$ is a freedom preserving relation if and only if it is an extension of $\succsim^*$.*

In order to prove this theorem we will need the following two lemmas.

Lemma 1 *For all $A, B \in \mathcal{X}$ and all $i, j \in \Pi$,*

$$[U(A,i) > U(B,j)] \Rightarrow [\{B \subset A \text{ and } iRj\} \text{ or } \{B = A \text{ and } iPj\}].$$

Proof. Suppose not. Then there are four logical possibilities:

(i) $\neg(B \subseteq A)$. Then, there exists $x_i \in B$ such that $x_i \notin A$, and therefore $\neg(U(A,i) > U(B,j))$, which is a contradiction.

(ii) jPi and $B \neq A$. But jPi implies $\neg(U(A,i) > U(B,j))$, which is a contradiction.

(iii) $\neg(B \subset A)$ and jRi. If $B = A$ then $(U(B,j) \geq U(A,i))$. If $B \neq A$ then there exists $x_i \in B$ such that $x_i \notin A$, and therefore $\neg(U(A,i) > U(B,j))$. In both cases we get a contradiction.

(iv) jPi. As in (ii) that implies $\neg(U(A,i) > U(B,j))$, again a contradiction. ■

Lemma 2 *For all $A, B \in \mathcal{X}$ and all $i, j \in \Pi$,*

$$[U(A,i) = U(B,j)] \Rightarrow [B = A \text{ and } iIj].$$

Proof. Suppose not. Then $B \neq A$ or $\neg(iIj)$. Both possibilities, by the construction of U, imply $\neg(U(A,i) = U(B,j))$, which is a contradiction. ■

Proof of Theorem 1. $\Rightarrow$ We have to prove that if $\succsim^+$ is a freedom preserving relation (i.e. it satisfies (PF) and (OF)), then for all $A, B \in \mathcal{X}$ and all $i, j \in \Pi$: (1) $[(A,i) \succ^* (B,j)] \Rightarrow [(A,i) \succ^+ (B,j)]$, and (2) $[(A,i) \sim^* (B,j)] \Rightarrow [(A,i) \sim^+ (B,j)]$.

(1) Note that, by the definition of $\succsim^*$, $(A,i) \succ^+ (B,j)$ implies $(U(A,i) > U(B,j))$, which by Lemma 1 implies (a) $B \subset A$ and iRj or (b) $B = A$ and iPj.

(1.a) If iIj then, by (OF), $(A,i) \succ^+ (B,j)$. If iPj then, by (PF), $(A,i) \succ^+ (A,j)$. Again, by (OF), $(A,j) \succ^+ (B,j)$. Finally, transitivity of $\succ^+$ implies $(A,i) \succ^+ (B,j)$.

(1.b) If $B = A$ and iPj then, by (PF) $(A,i) \succ^+ (B,j)$.

(2) In this case, by construction of $\succsim^*$ and by Lemma 2, $(A,i) \sim^* (B,j)$ implies $B = A$ and iIj. Since R is a linear ordering, by reflexivity of $\succsim^+$, $(A,i) \sim^+ (B,j)$.

$\Leftarrow$ We have to prove that if the implications $[(A,i) \succ^* (B,j)] \rightarrow [(A,i) \succ^+ (B,j)]$ and $[(A,i) \sim^* (B,j)] \rightarrow [(A,i) \sim^+ (B,j)]$ hold, then $\succsim^+$ satisfies (OF) and (PF).

Suppose $\neg$(OF). Then there exists $A, B \in \mathcal{X}$ and $i \in \Pi$ such that $B \subset A$ and $(B,i) \succsim^+ (A,i)$. But $B \subset A$, by construction of U, implies $U(A,i) > U(B,i)$. Therefore, by the definition of $\succsim^*$, $(A,i) \succ^* (B,i)$ and by hypothesis $(A,i) \succ^+ (B,i)$, a contradiction.

Suppose $\neg$(PF). Then there exists $A \in \mathcal{X}$ and $i, j \in \Pi$ such that (1) $(A,i) \succsim^+ (A,j)$ and jPi, or (2) iRj and $(A,j) \succ^+ (A,i)$.

(1) If jPi, then $(A,j) \succ^+ (A,i)$. Therefore $(A,j) \succ^* (A,i)$, which is a contradiction.

(2) In this case there are two possibilities: (a) iPj and $(A, j) \succ^+ (A, i)$, and (b) iIj and $(A, j) \succ^+ (A, i)$.

(2.a) By the definition of $\succsim^+$ and the construction of U, this case implies $U(A, j) > U(A, i)$. Therefore, by the definition of $\succsim^*$, $(A, j) \succ^* (A, i)$. Then, by hypothesis $(A, i) \succ^+ (A, j)$, a contradiction.

(2.b) By the construction of U this case implies $U(A, j) = U(A, i)$. Therefore, by the definition of $\succsim^*$, $(A, j) \sim^* (A, i)$. Then, by hypothesis $(A, i) \sim^+ (A, j)$, which is a contradiction. ■

We can interpret Theorem 1 as if the availability of any opportunity x had certain positive value $f(x)$, and the procedural aspect i associated to the set had also certain value $v(i)$ given by the linear ordering R defined over procedures. Then, (OF) and (PF) do not provide clues about how to aggregate such values, but they at least ensure that any relation that, ceteris paribus, preserve both the procedural aspect of freedom and the objective aspect of freedom, respects the dominance relation over vectors of the form given by the function U.

In order to know more about which are those values we would need to have a richer informational basis. For example, concerning the values of f and v, Theorem 1 just establishes that they are strictly positive. Very often in the literature on rankings over opportunity sets we find the assumption of the existence of a binary relation defined on X, representing the quality of the basic alternatives. In such a case, very plausibly, the information of such a relation could be incorporated in f. Moreover, some (complete) rankings of that literature fit well with particular ways to aggregate the values of f in order to obtain numerical representations of that rankings. A clear example would be Pattanaik and Xu's (1990) cardinalist ranking, which could be understood as if the decision maker made a simple addition of the (equal) values of the alternatives in the set.

However, the purpose of this work is neither to suggest and characterize particular ways of defining those values, nor to provide formulae to aggregate them in order to represent particular rankings. In the next sections we will make use of the characterization results of relations over (non-procedure-based) opportunity sets, that are well established in the literature, and provide a general guide that allows to import these results into our procedural framework.

4 Procedural Extensions of Relations over Opportunity Sets

As suggested in the Introduction, the rules for comparing opportunity sets that can be found in the literature ignore the procedural aspects or, put more positively, such rules are proposed as if the procedures associated to the different opportunity sets in $\mathcal{X}$ were the same. Keeping in mind this positive interpretation, any binary relation defined on $\mathcal{X}$ can be seen, for any $i \in \Pi$, as a binary relation defined on $\mathcal{X}_i = \{(A, i) \in \mathcal{K} : (A, i) \in \mathcal{X} \times \{i\}\}$. Then we naturally have $\mathcal{K} = \mathcal{X} \times \Pi = \cup_{i \in \Pi} \mathcal{X}_i$.

Related to the idea above, we introduce two new concepts by means of the following definitions.

Definition 2 *Let* $\succsim \subseteq X \times X$. *Let* $i \in \Pi$ *and* $\succsim_i \subseteq \mathcal{X}_i \times \mathcal{X}_i$. *We say that* $\succsim_i$ *is a* ***procedural replication*** *of* $\succsim$ *if for all* $A, B \in \mathcal{X}$, $[A \succsim B] \Leftrightarrow [(A, i) \succsim_i (B, i)]$.

Definition 3 *Let* $\succsim \subseteq \mathcal{X} \times \mathcal{X}$ *and* $\succsim^+ \subseteq \mathcal{K} \times \mathcal{K}$. *We will say that* $\succsim^+$ *is a*

procedural extension *of* $\succsim$ *if and only if for all* $i \in \Pi$ *and all procedural replications* $\succsim_i$ *of* $\succsim$*,* $\succsim_i \subset \succsim^+$.

Definition 2 just allows to replicate a given binary relation defined on (non-procedure-based) opportunity sets to a framework in which all those opportunity sets are associated to the same procedure. Concerning Definition 3, given a certain binary relation defined on opportunity sets, a procedural extension of it maintains such binary relation *across* procedures. Of course, the procedural extensions of a given relation over opportunity sets form a subclass of the possible relations over procedure-based opportunity sets. We could find examples in which the particular procedure affects the preferences over sets: for example, a too bureaucratic procedure might bias the preferences in favor of smaller sets. In other words, if $\succsim^+$ is a procedural extension of $\succsim$, then such kinds of framing effects are ruled out.

As we will see below, there is a natural relationship between $\succsim^+$ as being a procedural extension of $\succsim$ and satisfying (OF) on the one hand, and $\succsim$ as being an extension of the set inclusion relation on X on the other hand (i.e. for all $A, B \in X$ we have $[B \subset A] \Rightarrow [A \succ B]$).

Theorem 2 *Let* $\succsim$ *be a reflexive binary relation defined on* $\mathcal{X}$*. Let* $\succsim^+$ *be a procedural extension of* $\succsim$*. Then* $\succsim^+$ *satisfies (OF) if and only if* $\succsim$ *is an extension of the set inclusion relation.*

Proof. $\Rightarrow$ We have to prove that if $\succsim^+$ is a procedural extension of $\succsim$ and satisfy (OF) then $\succsim$ is an extension of the set inclusion relation.

Take $B \subset A$. By (OF) $(A, i) \succ^+ (B, i)$ for all $i \in \Pi$. Given that $\succsim^+$ is a procedural extension of $\succsim$, we have $A \succ B$.

$\Leftarrow$ Let $\succsim^+$ be a procedural extension of $\succsim$, and let $\succsim$ be an extension of the set inclusion relation. Then $[B \subset A] \Rightarrow [A \succ B] \Rightarrow [(A, i) \succ^+ (B, i)]$, as required in order to demonstrate that $\succsim^+$ satisfies (OF). ■

In the next section we investigate two particular types of freedom preserving relations that are procedural extensions of a given underlying relation $\succsim$. Since any freedom preserving relation satisfies (OF) (in addition to (PF)), Theorem 2 implies that the underlying relation $\succsim$ has to be an extension of the set inclusion relation.

5 Some Particular Procedural Extensions

Given a relation $\succsim$ defined on the set of non-empty opportunity sets, we will concentrate on two particular procedural extensions of $\succsim$. The first rule prioritizes the value of the procedure over the objective value given by $\succsim$. The second rule ranks procedure-based opportunity sets according to the value given by $\succsim$, and only when the sets are indifferent according to $\succsim$, considers better the set which is associated to a better procedure.

In order to introduce these rules more precisely, let us consider the following axioms:

- *Procedure Priority* (PP): For all $i, j \in \Pi$, and for all $A, B \in \mathcal{X}$, $[iPj] \Rightarrow [(A, i) \succ^+ (B, j)]$.

- *Objective Relation Priority* (ORP): Let $\succsim$ be a binary relation defined on $\mathcal{X}$. Then for all $A, B \in \mathcal{X}$ and all $i, j \in \Pi$, $[A \succ B] \Rightarrow [(A, i) \succ^+ (B, j)]$.

(PP) says that if a certain procedure i is strictly better than another procedure j, then that is a sufficient condition to establish that any set associated to i is better than any set associated to j. (ORP) says that whenever an opportunity set A is considered strictly better than another opportunity set B according to $\succsim$, that is a sufficient condition to assert that, when ranking procedure-based opportunity sets, A is going to be better than B, regardless the procedures which are associated to these sets.

In the previous section we suggested that the way of solving the trade-offs between the procedural aspect of choice and the objective relation over sets may depend on the values the individual or the society attaches to procedural freedom and objective freedom in the particular context to be analyzed. (PP) and (ORP) simply display two particular ways of managing the trade-offs between the procedural aspect of choice and the information of the objective freedom given by $\succsim$. According to (PP) procedural aspects are always prioritized in order to evaluate overall freedom, while according to (ORP) always objective freedom aspects are prioritized.

Since we are interested in freedom preserving relations (i.e. relations that satisfy (OF) and (PF)), a question that immediately arises concerns the compatibility of such relations with (PP) and (ORP), respectively.

On the one hand, a relation on procedure-based opportunity sets can be freedom preserving and satisfy (ORP). In such a case we just have a relation that respects the set inclusion relation not only under a given procedure, but whatever the procedures associated to the sets are. This follows from Theorem 2, which requires $\succsim$ to be an extension of the set inclusion relation. Moreover, such a relation would respect the procedural aspect when the opportunity sets to be compared are the same.

On the other hand, there is neither tension between (PF) and (PP); actually, the following proposition shows that (PP) is a stronger version of (PF).

Proposition 1 *Let $\succsim^+$ be a reflexive binary relation defined on $\mathcal{K}$. If $\succsim^+$ satisfies (PP), then it also satisfies (PF).*

Proof. Since (PF) is an "if and only if" condition, we have to prove that, if $\succsim^+$ satisfies (PP), then: (1) $[iRj] \Rightarrow [(A,i) \succsim^+ (A,j)]$ and (2) $[(A,i) \succsim^+ (A,j)] \Rightarrow [iRj]$.

(1) There are two cases: If iPj, then, by (PP), $(A,i) \succ^+ (B,j)$ holds for all $A, B \in \mathcal{X}$. Taking $A = B$ we have $(A,i) \succ^+ (A,j)$. On the other hand, if iIj, then $(A,i) \sim^+ (A,j)$ follows from the fact that R is a linear order and by reflexivity of $\succsim^+$.

(2) Suppose not, i.e. we have $(A,i) \succsim^+ (A,j)$ but jPi. Then, by (PP), $(A,j) \succ^+ (B,i)$ holds true for all $A, B \in \mathcal{X}$. Take $A = B$. Then we have $(A,j) \succ^+ (A,i)$, which is a contradiction. ■

Now, we will define our two particular classes of procedural extensions: the first one prioritizes the procedural aspect, while the second one prioritizes the objective aspect.

Definition 4 *Let $\succsim$ be a binary relation defined on $\mathcal{X}$. The* ***procedural extension $\succsim^+_{PP}$ of $\succsim$ that prioritizes the procedural aspect*** *is defined by:*

$$\text{For all } (A,i),(B,j) \in \mathcal{K},\ (A,i) \succsim^+_{PP} (B,j) \text{ iff } [iPj \text{ or } (iIj \text{ and } A \succsim B)].$$

Definition 5 *Let $\succsim$ be a binary relation defined on $\mathcal{X}$. The* ***procedural extension $\succsim^+_{ORP}$ of $\succsim$ that prioritizes the objective aspect*** *is defined by:*

$$\text{For all } (A,i),(B,j) \in \mathcal{K}, (A,i) \succsim^+_{ORP} (B,j) \text{ iff } [A \succ B \text{ or } (A \sim B \text{ and } iRj)].$$

The rule $\succsim^+_{PP}$ pays attention first to the procedure that is associated to the corresponding opportunity set. Thus, the set that is associated to a better procedure is declared better, and if the procedure that is associated to both sets is the same, then the comparison of the procedure-based opportunity sets is directly given by the comparison of the (non-procedure-based) opportunity sets. The rule $\succsim^{+}_{ORP}$ is somehow the dual of $\succsim^{+}_{PP}$: it respects first the relation over the (non-procedure-based) opportunity sets, and only if two opportunity sets are indifferent then the better procedure counts.

Note that the structure of both $\succsim^{+}_{PP}$ and $\succsim^{+}_{ORP}$ is constrained by the structure of the primitive relation $\succsim$. For example, $\succsim^{+}_{PP}$ ($\succsim^{+}_{ORP}$) will be a complete preorder (i.e. a reflexive, complete and transitive binary relation) if and only if $\succsim$ is a complete preorder.

The main purpose of this section is to provide axiomatic characterizations of $\succsim^{+}_{PP}$ and $\succsim^{+}_{ORP}$. Such axiomatic characterizations will be, obviously, linked closely to the axiomatic structure of the original relation $\succsim$. In order to formalize this link we need to introduce some additional notation and definitions.

Let $i \in \Pi$, $\succsim \subseteq \mathcal{X} \times \mathcal{X}$, and let $\succsim_i \subseteq \mathcal{X}_i \times \mathcal{X}_i$ be the corresponding procedural replication of $\succsim$. Let ξ be a set of axioms that characterizes $\succsim$. For each $(\bullet) \in \xi$ construct a *replication* $(\bullet)_i$ to be imposed on $\succsim_i$ as follows:

(1) whenever $(\bullet)$ refers to any $A \in \mathcal{X}$, substitute A by $(A, i) \in \mathcal{X}_i$,

(2) whenever $(\bullet)$ refers to $\succsim$ $(\succ, \sim)$, substitute it by $\succsim_i$ $(\succ_i, \sim_i)$.

For a given $i \in \Pi$, ξ_i will denote the set of replications of axioms in ξ.

Note that the construction of $\mathcal{X}_i$ preserves the characteristics of a set, that is, any function or binary relation that is well defined for sets can be applied to the elements of $\mathcal{X}_i$ (for example, it makes sense to talk about the cardinality of (A, i), or to consider the set $(A, i) \cup (B, i)$). This guarantees that all the syntactic elements of any axiom that is applied on $\mathcal{X}$ can be reproduced on $\mathcal{X}_i$, and that the only difference when *replicating* an axiom is merely semantic.

Theorem 3 *Let* $\succsim$ *be a binary relation defined on* $\mathcal{X}$*. Let* ξ *be a set of axioms that characterize* $\succsim$*. Let* $\succsim^+$ *be a freedom preserving relation on* $\mathcal{K}$ *that is a procedural extension of* $\succsim$*, and let* $\xi^+ = \cup_{i\in\Pi}\xi_i$ *. Then* $\succsim^+ = \succsim^+_{PP}$ *if and only if* $\succsim^+$ *satisfies* $\xi^+ \cup \{(PP)\}$*.*

Proof. $\Rightarrow$ Let $\succsim \subseteq \mathcal{X} \times \mathcal{X}$, and let ξ be a set of axioms that characterizes $\succsim$. Note that, for all $i \in \Pi$, the models $(\xi, \succsim)$ and $(\xi_i, \succsim_i)$ are different only in semantic terms and that, by definition, the replications of the axioms capture all the semantic differences between the two models. In terms of model theory, $(\xi, \succsim)$ and $(\xi_i, \succsim_i)$ are *isomorphic* models. Therefore, the equivalence [ξ characterize $\succsim$] $\Leftrightarrow$ [ξ_i characterize $\succsim_i$] implies that, if $\succsim$ satisfies ξ, then, for each $i \in \Pi$, the procedural replication $\succsim_i$ satisfies all axioms in ξ_i. Given that $\succsim^+$ is a procedural extension of $\succsim$, by definition of procedural extension, $\succsim_i \subset \succsim^+$ for all $i \in \Pi$. Therefore $\succsim^+$ satisfies all axioms in $\xi^+ = \cup_{i\in\Pi}\xi_i$. By definition of $\succsim^+_{PP}$, for all $A, B \in \mathcal{X}$ and for all $i, j \in \Pi$ such that iPj, we have $(A, i) \succ^+_{PP} (B, j)$. In other words, $\succsim^+_{PP}$ satisfies (PP) as well.

$\Leftarrow$ Suppose that $\succsim^+$ satisfies $\xi^+ \cup (PP)$ and $\succsim^+ \neq \succsim^+_{PP}$. Then, there exist $(A, i), (B, j)$ such that [iPj or (iIj and $A \succsim B$)], but not [$(A, i) \succsim^+ (B, j)$]. If iIj, given that R is a linear ordering, $i = j$. Then, given that $\succsim^+$ satisfies ξ^+ and the models $(\xi, \succsim)$ and $(\xi_i, \succsim_i)$ are isomorphic, we have $A \succsim_i B \Leftrightarrow A \succsim B$ for all $i \in \Pi$ and all $A, B \in \mathcal{X}$. Therefore, [iIj and $A \succsim B$] implies [$(A, i) \succsim_{i(=j)} (B, j)$] which, because $\succsim^+$ is a procedural extension of $\succsim$, implies $(A, i) \succsim^+ (B, j)$, getting into a contradiction. If iPj, then, by (PP), $(A, i) \succsim^+ (B, j)$, which is a contradiction. ■

Theorem 4 *Let* $\succsim$ *be a binary relation defined on* $\mathcal{X}$*. Let* ξ *be a set of axioms that characterize* $\succsim$*. Let* $\succsim^+$ *be a freedom preserving transitive relation* *defined on $\mathcal{K}$ that is a procedural extension of $\succsim$, and let $\xi^+ = \cup_{i\in\Pi}\xi_i$. Then $\succsim^+ = \succsim^+_{ORP}$ if and only if $\succsim^+$ satisfies $\xi^+ \cup \{(ORP)\}$.*

Proof. The proof of the only if part is similar to the corresponding part for Theorem 3. We will prove the if part:

Suppose that $\succsim^+$ satisfies $\xi^+ \cup (ORP)$ and $\succsim^+ \neq \succsim^+_{ORP}$. Then, there exist $(A, i), (B, j)$ such that $(A \sim B$ and $iRj)$ or $(A \succ B)$ but not $(A, i) \succsim^+ (B, j)$. Given that, for all $i \in \Pi$ the models $(\xi, \succsim)$ and $(\xi_i, \succsim_i)$ are isomorphic, we have $A \succsim_i B \Leftrightarrow A \succsim B$. If $(A \sim B$ and $iRj)$ or $(A \succ B)$ we can distinguish three cases:

(1) $(A \sim B$ and $iIj)$: Given that R is a linear ordering, iIj implies $i = j$. Then, $A \sim B$ implies $(A, i) \sim_{i=j} (B, i)$ for all $i \in \Pi$. Therefore, by definition of $\succsim^+$, $(A, i) \sim^+ (B, i)$, which is a contradiction.

(2) $(A \sim B$ and $iPj)$. By the implication above $(A, i) \sim^+ (B, i)$. Given that $\succsim^+$ is a freedom preserving relation, it satisfies (PF). By (PF) $(B, i) \succ^+ (B, j)$, and by transitivity of $\succsim^+$, $(A, i) \succsim^+ (B, j)$, which is a contradiction.

(3) $A \succ B$, in which case, by (ORP), $(A, i) \succ^+ (B, j)$, again a contradiction. ■

According to Theorems 3 and 4, given an axiomatic characterization ξ of $\succsim$, we can characterize two plausible relations over procedure-based opportunity sets: the procedural extension of $\succsim$ that prioritizes the procedural aspect, which is characterized by the set of procedural replications of axioms in ξ plus (PP); and the procedural extension of $\succsim$ that prioritizes the objective aspect, which is characterized by the same set of axioms, but substituting (PP) by (ORP).

Moreover, we should remark that Theorem 3 does not make any assumption on the formal structure of $\succsim$ and $\succsim^+$. It applies for the case in which both are complete preorders, but in general, as noticed before, the structure of $\succsim^+$ will be constrained by the structure of $\succsim$. However, in order to prove Theorem 4 we needed the assumption that $\succsim^+$ is transitive.

The next results concern the independence of the axioms.

Theorem 5 *Let $\succsim$ be a binary relation defined on $\mathcal{X}$. Let ξ be a non-empty set of axioms that characterize $\succsim$, and let $\xi^+ = \cup_{i\in\Pi}\xi_i$. Then, the axioms in $\xi^+ \cup (PP)$ are independent if and only if the axioms in ξ are independent.*

Proof. $\Rightarrow$ If $\xi^+ \cup (PP)$ is a set of independent axioms, then any subset of them is also a set of independent axioms. In particular, for all $i \in \Pi$, ξ_i is a set of independent axioms. Then, by the isomorphism of the models $(\xi, \succsim)$ and $(\xi_i, \succsim_i)$, ξ is a set of independent axioms.

$\Leftarrow$ Suppose not. Then there exists at least one axiom in $\xi^+ \cup (PP)$ that is implied by a subset of the other axioms in $\xi^+ \cup (PP)$. Assume, without loss of generality, that $(\bullet)_k \in \xi_k \subseteq \xi^+$ is the implied axiom. Let $(\bullet) \in \xi$ be the axiom imposed on $\succsim$ of which $(\bullet)_k$ is a replication. Given that axioms in ξ are independent, then there is $\succsim^1 \subseteq \mathcal{X} \times \mathcal{X}$ such that $\succsim^1$ satisfies $\xi \setminus \{(\bullet)_k\}$ and does not satisfy $(\bullet)_k$. Now, let $\succsim^2 \subseteq \mathcal{K} \times \mathcal{K}$ be defined as follows:

For all $(A, i), (B, j) \in \mathcal{K}$,

$$(A,i) \succsim^2 (B,j) \text{ iff } \begin{cases} iPj \\ \text{or} \\ iIj \quad \text{and} \begin{cases} A \succeq B & \text{if } i \neq k \text{ or } j \neq k, \\ A \succeq^1 B & \text{otherwise.} \end{cases} \end{cases}$$

Then, $\succsim^2$ satisfies all the axioms in $\xi^+ \cup (PP)$ except $(\bullet)_k$. Hence, $(\bullet)_k$ can not be an implied axiom.

Concerning the independence of (PP), let $\succsim^+_{ORP}$ be the relation that prioritizes the objective relation associated to $\succsim$. Then $\succsim^+_{ORP}$ satisfies all the axioms in ξ^+ but not (PP). ■

Theorem 6 *Let* $\succsim$ *be a binary relation defined on* $\mathcal{X}$. *Let* ξ *be a non-empty set of axioms that characterize* $\succsim$, *and let* $\xi^+ = \cup_{i \in \Pi} \xi_i$. *Then, the axioms in* $\xi^+ \cup (ORP)$ *are independent if and only if the axioms in* ξ *are independent.*

Proof. The proof is similar to the proof of Theorem 5. ■

We conclude this section with two examples. The *first example* is that of the *cardinalist ranking of opportunity sets* $\succsim_{\#}$ introduced by Pattanaik and Xu (1990) and defined as follows: $\forall A, B \in \mathcal{X}$, $A \succsim_{\#} B$ iff $\#A \geq \#B$ (where $\#A(\#B)$ denotes the cardinality of $A(B)$).

Pattanaik and Xu (1990) characterize the cardinalist ranking by means of the following three axioms:

- $\forall x, y \in X$, $\{x\} \sim \{y\}$;
- for all distinct $x, y \in X$, $\{x, y\} \succ \{x\}$;
- $\forall A, B \in \mathcal{X}$, and $\forall x \in X \setminus (A \cup B)$, $A \succsim B$ iff $A \cup \{x\} \succsim B \cup \{x\}$.

Taking $\succsim_{\#}$ as a reference, and considering Theorems 3 and 4, we can define and characterize, in a procedural framework, the following two rankings of procedure-based opportunity sets.

Definition 6 *The* ***procedural extension*** $\succsim^+_{\#PP}$ ***of the cardinalist ranking that prioritizes the procedural aspect*** *is defined as follows:*

For all $(A, i), (B, j) \in \mathcal{K}$, $(A, i) \succsim^+_{\#PP} (B, j)$ *iff* $[iPj$ *or* $(iIj$ *and* $A \succsim_{\#} B)]$.

According to Theorem 3, the rule $\succsim^+_{\#PP}$ is characterized by the following axioms:

- $\forall x, y \in X$, $\forall i \in \Pi$, $(\{x\}, i) \sim^+ (\{y\}, i)$;

- for all distinct $x, y \in X$, $\forall i \in \Pi$, $(\{x, y\}, i) \succ^{+} (\{x\}, i)$;
- $\forall A, B \in \mathcal{X}$, $\forall i \in \Pi$, and $\forall x \in X \setminus (A \cup B)$, $[(A, i) \succsim^{+} (B, i)$ iff $(A \cup \{x\}, i) \succsim^{+} (B \cup \{x\}, i)]$;
- (PP).

Definition 7 *The **procedural extension** $\succsim^{+}_{\#ORP}$ **of the cardinalist ranking that prioritizes the objective aspect** is defined as follows:*

For all $(A, i), (B, j) \in \mathcal{K}$, $(A, i) \succsim^{+}_{\#ORP} (B, j)$ *iff* $[A \succ_{\#} B$ *or* $(A \sim_{\#} B$ *and* $iRj)]$.

Similarly, we can axiomatically characterize the rule $\succsim^{+}_{\#ORP}$ by means of the same axioms that characterize $\succsim^{+}_{\#PP}$ but substituting (PP) by (ORP).

The *second example* is that of the *leximax ranking of opportunity sets* of Bossert, Pattanaik and Xu (1994). The leximax ranking of opportunity sets ($\succsim_{LM}$) is defined in the following way.

Let R^* be a complete preorder defined on $X \times X$ (P^* and I^* denoting respectively its asymmetric and symmetric factors). For all $S \in \mathcal{X}$, $\#S = r$, let $S = \{s_1, s_2, \ldots, s_r\}$ be such that $s_1 R^* s_2 R^* \ldots R^* s_r$. Then, for all $A, B \in \mathcal{X}$, $A \succsim_{LM} B$ iff $(A = B)$ or $(\exists k \in \{1, \ldots, max\{\#A, \#B\}\}$ such that $a_i I^* b_i$ for all $i < k$ and $[(a_k P^* b_k)$ or (a_k exists and b_k does not exist)]).

Bossert, Pattanaik and Xu (1994) characterize the leximax ranking by means of the four following axioms:

- $\forall x, y \in X$, $xP^*y \Rightarrow \{x\} \succ \{y\}$;
- for all distinct $x, y \in X$, $\{x, y\} \succ \{x\}$;
- $\forall A, B \in \mathcal{X}$, and $\forall x \in X \setminus (A \cup B)$, $A \succsim B$ iff $A \cup \{x\} \succsim B \cup \{x\}$;

- $\forall A, B \in \mathcal{X}$, and $\forall x \in X \setminus (A \cup B)$, $[A \succ B$ and yP^*x for all $y \in A$ and zP^*x for all $z \in B] \Rightarrow [A \succ B \cup \{x\}]$.

As before, taking $\succsim_{LM}$ as a reference, and considering Theorems 3 and 4, we can define and characterize, in a procedural framework, the following two rankings of procedure-based opportunity sets:

Definition 8 *The* ***procedural extension*** $\succsim^+_{LMPP}$ ***of the leximax ranking that prioritizes the procedural aspect*** *is defined as follows:*

For all $(A, i), (B, j) \in \mathcal{K}$, $(A, i) \succsim^+_{LMPP} (B, j)$ *iff* $[iPj$ *or* $(iIj$ *and* $A \succsim_{LM} B)]$.

Definition 9 *The* ***procedural extension*** $\succsim^+_{LMORP}$ ***of the leximax ranking that prioritizes the objective aspect*** *is defined as follows:*

For all $(A, i), (B, j) \in \mathcal{K}$, $(A, i) \succsim^+_{LMORP} (B, j)$ *iff* $[A \succ_{LM} B$ *or* $(A \sim_{LM} B$ *and* $iRj)]$.

According to Theorem 4, the rule $\succsim^+_{LMPP}$ is characterized by the following axioms:

- $\forall x, y \in X$, $\forall i \in \Pi$, $xP^*y \Rightarrow (\{x\}, i) \succ^+ (\{y\}, i)$;

- for all distinct $x, y \in X$, $\forall i \in \Pi$, $(\{x, y\}, i) \succ^+ (\{x\}, i)$;

- $\forall A, B \in \mathcal{X}$, $\forall i \in \Pi$, and $\forall x \in X \setminus (A \cup B)$, $(A, i) \succsim^+ (B, i)$ iff $(A \cup \{x\}, i) \succsim^+ (B \cup \{x\}, i)$;

- $\forall A, B \in \mathcal{X}$, $\forall i \in \Pi$, and $\forall x \in X \setminus (A \cup B)$, $[(A, i) \succ^+ (B, i)$ and yP^*x for all $y \in A$ and zP^*x for all $z \in B] \Rightarrow [(A, i) \succ^+ (B \cup \{x\}, i)]$;

- (PP).

Similarly, we can axiomatically characterize $\succsim^+_{LMORP}$ by means of the same axioms that characterize $\succsim^+_{LMPP}$ but substituting (PP) by (ORP).

6 Final Remarks and Conclusion

In this paper we have considered the question of how to compare procedure-based opportunity sets in terms of overall freedom, i.e. how to rank sets of options when there is some procedural aspect associated to the sets that matters. We have studied the links between comparisons of (non-procedure-based) opportunity sets and comparisons of procedure-based opportunity sets.

For example, our first rule $\succsim^+_{PP}$ classifies the procedure-based opportunity sets into clusters such that: (1) the sets associated to the same procedure are in the same cluster; (2) within each cluster the ranking respects the ranking over the corresponding opportunity sets; (3) a cluster is ranked higher that another cluster if the first procedure is better that the second one. Our second rule $\succsim^+_{ORP}$ also classifies the procedure-based opportunity sets into clusters such that: (1) a cluster consists of procedure-based opportunity sets such that the corresponding opportunity sets are indifferent; (2) within each cluster the ranking respects the linear order of the procedures; (3) a procedure-based opportunity set in a cluster is ranked higher than a procedure-based opportunity set in another cluster by respecting the ranking of the corresponding opportunity set. With respect to the characterizations of these rankings, our results allow for taking the corresponding characterization of the ranking over the opportunity sets and for adding two axioms: when adding (PP) we have an axiomatic characterization of $\succsim^+_{PP}$, and when adding (ORP) we have an axiomatic characterization of $\succsim^+_{ORP}$.

As pointed out before, (PP) and (ORP) display particular ways to solve the trade-offs between the procedural aspect and the information given by the primitive relation $\succsim$ over opportunity sets. Indeed, they propose a rather rough solution. Other procedural extensions of $\succsim$ that do not prioritize systematically any of both aspects are indeed conceivable.

Presumably, a way to approach those other possible formulae to manage with the trade-offs is by means of the representation result proposed in Theorem 1. In such a case, the way a particular ranking over procedure-based sets solves the trade-offs could be captured by the values of functions f and v, and by the particular operations to aggregate them. For example, $\succsim^{+}_{\#PP}$ could be interpreted as the case in which $f(x) = 1/n$ for all $x \in X$, $v(i) > 1$ for all $i \in \Pi$, and the ranking is represented by the sum of all those values. $\succsim^{+}_{\#ORP}$ could correspond to the case in which $f(x) = 1$ for all $x \in X$, $v(i) < 1$ for all $i \in \Pi$ and, again, we sum all those values. $\succsim^{+}_{LMPP}$ could be understood as the case in which, for all $x_i \in X$, $f(x_i) > \sum_{x_i P^* x_j} f(x_j)$; $v(i) > \sum_{x_i \in X} f(x_i)$ and the ranking is represented by the sum of all those values. And $\succsim^{+}_{LMORP}$ could be represented by the sum of values such that for all $x_i \in X$, $f(x_i) > \sum_{x_i P^* x_j} f(x_j)$, and $v(i) < f(x_i)$ for all $i \in \Pi$.

Note that our understanding of a procedure in this paper is quite general and abstract, i.e. our work provides a general formal framework for the research on particular procedural aspects and the implications of incorporating their own ethical and philosophical foundations. On the other hand, the richness of the informational basis could also refer to the nature of the procedure. Procedural circumstances pointed out in the Introduction suggest that some of them have important ethical implications that should be captured by the way of comparing opportunity sets. In this sense, the reader is referred to Hansson (1996), Suzumura (1999), and Suzumura and Xu (2001, 2003).

References

[1] Arlegi, R. and J. Nieto (2001a): Ranking Opportunity Sets: An Approach Based on the Preference for Flexibility, Social Choice and Welfare 18, 23-36.

[2] Arlegi, R. and J. Nieto (2001b): Incomplete Preferences and the Preference for Flexibility, Mathematical Social Sciences 41, 151-165.

[3] Arlegi, R. and J. Nieto (1999): Equality of Opportunity: Cardinality-based Criteria, in: de Swart, H. (ed.), Logic, Game Theory and Social Choice, Tilburg University Press, Tilburg, 458-481.

[4] Barberà, S., W. Bossert, and P. Pattanaik (2003): Ranking Sets of Objects, in: Barberà, S., P. Hammond, and Ch. Seidl (eds.), Handbook of Utility Theory, Vol. 2, Kluwer Academic Publishers, forthcoming.

[5] Berlin, I. (1969): Two Concepts of Liberty, in: Berlin, I. Four Essays on Liberty, Oxford University Press, London, 118-172.

[6] Bossert, W. (2000): Opportunity Sets and Uncertain Consequences, Journal of Mathematical Economics 33, 475-496.

[7] Bossert, W. (1997): Opportunity Sets and Individual Well-Being, Social Choice and Welfare 14, 97-112.

[8] Bossert, W., M. Fleurbaey, and D. Van de Gaer (1999): Responsibility, Talent and Compensation: A Second-best Analysis, Review of Economic Design 4, 35-55.

[9] Bossert, W., P. Pattanaik, and Y. Xu (1994): Ranking Opportunity Sets: An Axiomatic Approach, Journal of Economic Theory 63, 326-345.

[10] Foster, J. (1992): Notes on Effective Freedom, Mimeo.

[11] Gaertner, W. and Y. Xu (2004): Procedural Choice, Economic Theory 24, 335-349.

[12] Gravel, N. (1998): Ranking Opportunity Sets on the Basis of Their Freedom of Choice and Their Ability to Satisfy Preferences: A Difficulty, Social Choice and Welfare 15, 371-382.

[13] Gravel, N., J.-F. Laslier, and A. Trannoy (1998): Individual Freedom of Choice in a Social Setting, in: Laslier, J-F., M. Fleurbaey, N. Gravel, and A. Trannoy (eds.), Freedom in Economics: New Perspectives in Normative Analysis, Routledge, London, 76-92.

[14] Hansson, S. O. (1996): Social Choice with Procedural Preferences, Social Choice and Welfare 13, 215-230.

[15] Herrero, C., I. Iturbe-Ormaetxe, and J. Nieto (1998): Ranking Opportunity Profiles on the Basis of the Common Opportunities, Mathematical Social Sciences 35, 273-289.

[16] Jones, P. and R. Sugden (1982): Evaluating Choice, International Review of Law and Economics 2, 47-69.

[17] Klemisch-Ahlert, M. (1993): Freedom of Choice: A Comparisom of Different Rankings of Opportunity Sets, Social Choice and Welfare 10, 189-207.

[18] Mill, J.S. (1859, 1959): On Liberty. Liberal Arts Press, New York.

[19] Nozick, R. (1974): Anarchy, State, and Utopia. Blackwell, Oxford.

[20] Ok, E. and L. Kranich (1998): The Measurement of Opportunity Inequality: A Cardinality-based Approach, Social Choice and Welfare 15, 263-287.

[21] Pattanaik, P. and Y. Xu (1990): On Ranking Opportunity Sets in Terms of Freedom of Choice, Recherches Economiques de Louvain 56, 383-390.

[22] Pattanaik, P. and Y. Xu (1998): On Preference and Freedom, Theory and Decision 44, 173-198.

[23] Pattanaik, P. and Y. Xu (2000): On Diversity and Freedom of Choice, Mathematical Social Sciences 40, 123-130.

[24] Puppe, C. (1995): Freedom of Choice and Rational Decisions, Social Choice and Welfare 12, 137-153.

[25] Puppe, C. (1996): An Axiomatic Approach to 'Preference for Freedom of Choice', Journal of Economic Theory 68, 174-199.

[26] Sen, A. (1991): Welfare, Preference, and Freedom, Journal of Econometrics 50, 15-29.

[27] Sen, A. (1993a): Markets and Freedom, Oxford Economic Papers 45, 519-541.

[28] Sen, A. (1993b): Internal Consistency of Choice, Econometrica 61, 495-521.

[29] Steiner, H. (1983): How Free: Computing Personal Freedom, in: Phillips-Griffiths, A. (ed.), Of Liberty, Cambridge University Press, Cambridge, 73-89.

[30] Sugden, R. (1998): The Metric of Opportunity, Economics and Philosophy 14, 307-337.

[31] Suzumura, K. (1999): Consequences, Opportunities, and Procedures, Social Choice and Welfare 16, 17-40.

[32] Suzumura, K. and Y. Xu (2003): Consequences, Opportunities, and Generalized Consequentialism and Non-consequentialism, Journal of Economic Theory 111, 293-304.

[33] Suzumura, K. and Y. Xu (2001): Characterizaitons of Consequentialism and Non-consequentialism, Journal of Economic Theory 101, 423-436.

[34] Van Hees, M. (1998): On the Analysis of Negative Freedom, Theory and Decision 45, 175-197.