Currency Hedging for International Stock Portfolios: A General Approach

Frans A. de Roon,[*] Theo E. Nijman,[†] and Bas J.M. Werker[‡]

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Abstract

This paper tests whether hedging currency risk improves the performance of international stock portfolios. We use a generalized performance measure which allows for investor-dependencies such as different utility functions and the presence of nontraded risks. In addition we show that an auxiliary regression, similar to the Jensen regression, provides a wealth of information about the optimal portfolio holdings for investors for the non mean-variance case. This is analogous to the information provided by the Jensen regression about optimal portfolio holdings for the mean-variance case. Our empirical results show that static hedging with currency forwards does not lead to improvements in portfolio performance for a US investor that holds a stock portfolio from the G5 countries. On the other hand, hedges that are conditional on the current interest rate spread do lead to significant performance improvements. Also, when an investor has a substantial exogenous exposure to one of the currencies, currency hedging clearly improves his portfolio performance. While these results hold for investors with power utility as well as with mean-variance utility functions, the optimal hedge ratios for these investors are different.

[*]Financial Management Department and RIFM, Erasmus University Rotterdam, PO Box 1738, 3000 DR Rotterdam, The Netherlands. E-mail: F.Roon@fbk.eur.nl

[†]CentER for Economic Research and Department of Econometrics, Tilburg University, PO Box 90153, 5000 LE Tilburg, The Netherlands. E-mail: Nyman@kub.nl

[‡]CentER for Economic Research and Department of Finance, Tilburg University, PO Box 90153, 5000 LE Tilburg, The Netherlands. E-mail: Werker@kub.nl

1 Introduction

When investing in international stock portfolios, investors are automatically exposed to exchange rate risk. The positions in the currencies resulting from investing in foreign stock markets without hedging the exchange rate risk are entirely determined by the size of the investments in foreign stock markets and may be suboptimal. It is well known that hedging the resulting currency risk with currency forwards or futures can improve the performance of international asset portfolios. For instance, Glen and Jorion (1993) analyze the performance of mean-variance efficient stock and bond portfolios from the G5 countries when hedging the associated currency risk with currency forwards. Similarly, DeSantis (1995), exploiting the duality between mean-variance frontiers and volatility bounds, finds strong evidence of hedging benefits for mean-variance efficient portfolios consisting of equities from 18 different countries.

The benefits from hedging international asset portfolios will no doubt be investor specific. For instance, there may be hedging benefits for investors with a mean-variance utility function, but not necessarily for investors with other utility functions. Similarly, for mean-variance investors the benefits of hedging may depend on their particular risk aversion. Also, some investors may face nontraded risks which they want to hedge and which will therefore affect the composition of their portfolio. Examples of nontraded risks are the position in foreign currency faced by an exporting firm, and the liabilities of insurance companies and pension funds.

In this paper we extend previous research by investigating the effect of currency hedging on portfolio performance for different groups of investors. Moreover, we consider the use of currency forwards as well options. From Huberman and Kandel (1987) it is well known how regression analysis can be used to test for mean-variance intersection and spanning, which is tantamount to measuring the performance of assets relative to mean-variance efficient portfolios (see, e.g., Jobson and Korkie (1989)). Chen and Knez (1996) derive a general performance measure as the price assigned to a security's payoff by a pricing kernel that is known to price the benchmark assets correctly. Their approach, which is also pursued in this paper, allows to measure performance for investors with both mean-variance and non mean-variance utility functions. For power utility investors this measure is also similar to the positive period weighting measure employed by Cumby and Glen (1990).

Our methodology builds on a regression framework to analyze whether the performance of international stock portfolios can be improved upon by hedging (part of) the associated currency risk. This regression framework allows for investor-dependency, because we can test the performance of currency forwards and options relative to mean-variance efficient portfolios for different risk aversions as well as relative to portfolios that are optimal for other utility functions. In using regression to measure the mean-variance performance of new securities, it is known from Jobson and Korkie (1984) and Stevens (1998) for instance, that the intercepts in a spanning or Jensen regression (i.e, Jensen's alphas) together with the covariance matrix of the error terms provide a wealth of information on the optimal portfolio holdings of the new securities in mean-variance efficient portfolios. We extend these results in showing that in a similar way the slope coefficients in the Jensen regression provide information on how to adjust the optimal holdings of the benchmark assets in the mean-variance efficient portfolio. In addition to this, we show that similar results hold approximately for the non mean-variance case. We assume in this paper that there are no market frictions such as short sales constraints or transaction costs. Tests for spanning in economies where such frictions are present are discussed in DeRoon, Nijman, and Werker (1999).

We analyze the benefits of currency hedging of stock portfolios from the G5 countries for US investors. We consider the case where investors have either a mean-variance utility function or a power utility function, with different levels of risk aversion. Also, we allow investors to have an exogenous exposure to some currency. We find that for investors that invest in the G5 stock markets, static hedging of the resulting currency risk with forwards or option contracts does not lead to significant improvements in portfolio performance, unless the investor has a relatively high risk aversion. On the other hand dynamic hedges that are conditional on the interest rate spread do lead to significant performance improvements in all cases. These findings are similar to the ones obtained by Glen and Jorion (1993) for the mean-variance case. For investors that have a sizeable nontraded position in a currency, adding currency forward contracts to their international stock portfolio leads to significant diversification benefits, except for investors with low risk aversions. These results hold for both power utility and for mean-variance investors. However, while the test results for outperformance are very similar for the power and mean-variance utility functions, we find that the optimal portfolio weights are different for both classes of utility functions.

The plan of the paper is as follows. In Section 2 we show how optimal mean-variance portfolio weights can be derived from the Jensen regression for outperformance. In Section 3 this is generalized to regression-based performance tests for other utility functions. In Section 4 we show how to test for outperformance in the presence of nontraded risks. Section 5 describes the data and Section 6 gives the empirical results for currency hedges with forwards for mean-variance investors. Section 7 provides results for the power utility case. Results for nontraded risks are presented in Section 8 and the paper ends with some concluding remarks.

2 Performance improvement and optimal portfolio weights: the mean-variance case

It is assumed that an investor initially holds an efficient portfolio of K assets with return vector R_t^x, $R_{i,t}^x = P_{i,t}/P_{i,t-1}$, plus a risk free asset with return R_{t-1}^f. Excess returns are denoted by r_t^x, i.e., $r_{i,t}^x = R_{i,t}^x - R_{t-1}^f$. If there would be no risk free asset available, then we can define returns $r_{i,t}^x$ in excess of the zero-beta rate η: $r_{i,t}^x = R_{i,t}^x - \eta$, in which case the analysis in the sequel still holds.[1] We analyze an investor that considers whether or not to add N securities with return vector R_t^y and excess return vector r_t^y to his portfolio. In the empirical analysis, the investor initially holds the stock indices from the G5 countries and the forward or option contracts on the currencies from these countries are the additional securities. From Jobson and Korkie (1982, 1989) and Huberman and Kandel (1987) it is known that mean-variance efficient portfolios from r_t^x are also efficient for the larger set (r_t^x, r_t^y) if in the regression

$$r_t^y = \alpha_J + Br_t^x + \varepsilon_t, \tag{1}$$

the vector of intercepts α_J, which contains the generalized Jensen measures, equals zero. In this case mean-variance investors cannot improve their portfolio performance by adding the new securities r_t^y to their optimal portfolio of the initial K assets r_t^x only. Tests for improvements in mean-variance portfolio performance therefore boil down to testing whether the generalized Jensen measures α_J are equal to zero.

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[1]The zero-beta rate η is the expected return on the mean-variance efficient portfolio that is uncorrelated with the mean-variance efficient portfolio of R_t^x for a given risk aversion γ.

For a mean-variance investor with risk aversion γ the optimal portfolio of the assets r_t^x and r_t^y is given by

$$\left(\begin{array}{c} w_x \\ w_y \end{array} \right) = \gamma^{-1} \left(\begin{array}{cc} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_{yy} \end{array} \right)^{-1} \left(\begin{array}{c} \mu_x \\ \mu_y \end{array} \right), \tag{2}$$

where $\mu_i = E[r_t^i]$, $\Sigma_{ij} = Cov[r_t^i, r_t^j]$, and w_x and w_y are the K and N-dimensional vector with optimal portfolio weights in r_t^x and r_t^y respectively. As shown in Jobson and Korkie (1984), Stevens (1998), and in Appendix A, using the partitioned inverse of the covariance matrix Σ the optimal portfolio weights w_y can be written as

$$w_y = \widetilde{\gamma}^{-1} \Sigma_{\varepsilon\varepsilon}^{-1} \alpha_J, \tag{3}$$

where α_J is the vector of generalized Jensen measures and $\Sigma_{\varepsilon\varepsilon}$ is the co-variance matrix of the error terms in the regression in (1). The risk-aversion parameter $\widetilde{\gamma}$ is the risk aversion associated with the portfolio of (R_t^x, R_t^y) that has the same zero-beta rate as the initial portfolio. Thus, up to a constant $\widetilde{\gamma}$, the optimal portfolio holdings w_y can easily be derived from the Jensen regression in (1). Notice that, when evaluating the performance of a set of securities, an individual negative (positive) Jensen measure in itself does not mean that the investor can improve his portfolio performance by taking short (long) positions in the new security. The structure of the covariance matrix $\Sigma_{\varepsilon\varepsilon}$ may be such that negative (positive) Jensen measures imply taking long (short) positions in this security.

In addition to the results in Jobson and Korkie (1984) and Stevens (1998) we show in Appendix A that the optimal portfolio holdings for the benchmark assets in (2) can be written as

$$\begin{aligned} w_x &= \frac{\gamma}{\widetilde{\gamma}} w_x^0 - \widetilde{\gamma}^{-1} B' \Sigma_{\varepsilon\varepsilon}^{-1} a \tag{4} \\ &= \frac{\gamma}{\widetilde{\gamma}} w_x^0 - B' w_y, \end{aligned}$$

where w_x^0 is the vector of optimal portfolio weights in the mean-variance efficient portfolio from the benchmark assets only, i.e.:

$$w_x^0 = \gamma^{-1} \Sigma_{xx}^{-1} \mu_x.$$

Thus, whereas for a mean-variance investor that initially invests in r_t^x only, the intercepts α_J together with the covariance matrix $\Sigma_{\varepsilon\varepsilon}$ in the regression

(1) yield the optimal holdings for r_t^y, the slope coefficients B together with w_y give information as to how the initial portfolio holdings for r_t^x must be adjusted in order to obtain the efficient portfolio for (r_t^x, r_t^y).

3 Performance improvement and optimal portfolio weights: the non mean-variance case

Following the ideas in Chen and Knez (1996) we can generalize the Jensen measure for non mean-variance utility functions by looking at the price assigned to the new security returns r_t^y by a stochastic discount factor associated with a particular utility function that prices the benchmark returns r_t^x correctly. Let m_t^x be a stochastic discount factor associated with a particular derived utility function u that prices both the benchmark returns R_t^x and the risk free rate R_{t-1}^f correctly:

$$E_{t-1}\left[m_t^x R_t^x\right] = \iota_K, \tag{5a}$$
$$E_{t-1}\left[m_t^x R_{t-1}^f\right] = 1, \tag{5b}$$

where ι_K is a K-dimensional vector of ones. The stochastic discount factor is known to be proportional to the marginal derived utility function u_t' evaluated in the optimal portfolio w_x^0:

$$m_t^x = c_0 \times u'\left(w_x^{0\prime} r_t^x + R_{t-1}^f\right), \tag{6}$$

where c_0 is a constant. It is well-known by now that for mean-variance utility functions m_t^x is linear in the returns R_t^x and R_{t-1}^f and is given by the linear projection $\widehat{m_t^x}$ of m_t^x on the returns R_t^x and R_{t-1}^f (see Hansen and Jagannathan (1991), DeSantis (1995), and Ferson, Foerster, and Keim (1993)). Since m_t^x assigns a price one to the gross returns $R_{i,t}^x$ and to R_{t-1}^f, it assigns a price zero to the excess returns r_t^x. If there is no risk free asset available, then we can always choose one of the initial assets to create excess returns that have price zero. This will not affect the analysis below. Following Balduzzi and Kallal (1997), for further use it is convenient to normalize the stochastic discount factor to have conditional expectation one:

$$q_t^x = \frac{m_t^x}{E_{t-1}\left[m_t^x\right]}, \tag{7}$$

which, in case there is a risk free rate, simplifies to $q_t^x = m_t^x \times R_{t-1}^f$. From (5) we immediately obtain:

$$E_{t-1}\left[q_t^x r_t^x\right] = 0. \tag{8}$$

Following Chen and Knez (1996) the performance of the new securities r_t^y for an investor with derived utility function u that initially invests in the benchmark assets only can be measured by the vector λ_{t-1} which is defined by

$$\lambda_{t-1} = E_{t-1}\left[q_t^x r_t^y\right]. \tag{9}$$

Taking unconditional expectations of (9) yields the unconditional performance measure λ

$$\lambda = E\left[\lambda_{t-1}\right] = E\left[q_t^x r_t^y\right]. \tag{10}$$

If the first order conditions of the investors portfolio problem are satisfied when the optimal portfolio consists of positions in the benchmark assets only, then the performance measure $\lambda = 0$. In this case there is *generalized spanning* of the new securities by the benchmark assets for the utility function u. If $\lambda \neq 0$ then the investor can increase his portfolio performance by also taking positions in the new securities. For a mean-variance investor, it follows immediately that $\lambda = \alpha_J$.

Alternatively, λ may be obtained from a Jensen-like regression of r_t^y on r_t^x and a constant, as well as on q_t^x scaled by its own second moment:

$$r_t^y = \alpha + \beta r_t^x + \gamma \frac{q_t^x}{E\left[q_t^{x2}\right]} + u_t. \tag{11}$$

This regression is similar to the Jensen regression in (1), except for the added pricing kernel. If there is no outperformance, then λ must be equal to zero. Using the fact that the error term u_t is orthogonal to q_t^x, Equations (8), (9), and (10) imply that

$$\begin{aligned} E\left[q_t^x r_t^y\right] &= \lambda = E\left[q_t^x \left(\alpha + \beta r_t^x + \gamma \frac{q_t^x}{E\left[q_t^{x2}\right]} + u_t\right)\right] \\ &= \alpha + \gamma. \end{aligned} \tag{12}$$

If we would test for outperformance for a mean-variance investor, i.e., a test for mean-variance spanning, the term involving q_t^x in (11) would be left out since it would be multicolinear with r_t^x and the constant, and we would simply

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test whether $\alpha = \alpha_J = 0$. In the non mean-variance case the test is whether or not $\alpha + \gamma = 0$. This illustrates again that λ is a generalization of the traditional Jensen measure.

Notice that each λ is associated with a particular derived utility function u, which implies that we can only make statements about improvements in portfolio performance for the particular class of investors to which this derived utility function applies. In Appendix B we show that a straightforward generalization of the procedure discussed for the mean-variance case can be used to approximate the new optimal portfolio holdings w_x and w_y associated with u. The procedure is based on the generalized performance measures λ and the second moment matrix of the error terms in the auxiliary regression

$$\widetilde{r}_t^y = D\widetilde{r}_t^x + \eta_t, \tag{13}$$

in which

$$\widetilde{r}_{i,t} = r_{i,t} \times \sqrt{-\frac{\partial q_t^x}{\partial w_x^{0\prime} r_t^x}} = r_{i,t} \times \sqrt{-u_t''/\overline{u}_t'},$$

where $\overline{u}_t'$ denotes the average of the marginal utility function evaluated in the optimal portfolio. Thus, (13) is essentially a Weighted Least Squares regression of the new securities returns on the benchmark returns, where the weights are determined by the second derivative of the utility function, u_t''. The weighting by the average marginal utility is simply a result of the fact that we use the normalized stochastic discount factor q_t^x. Using a first order Taylor series approximation, it is shown in Appendix B that the performance measures λ together with the auxiliary regression (13) imply that

$$w_y \approx E\left[\eta_t \eta_t'\right]^{-1} \lambda, \tag{14}$$

and

$$w_x \approx w_x^0 - D'w_y. \tag{15}$$

The intuition behind (14) and (15) is similar to the results for the mean-variance case in (3) and (4). From (14), the investor should choose his optimal positions in the new securities r_t^y according to the trade off between their performance λ and their riskiness relative to the benchmark assets r_t^x as measured by $E[\eta_t \eta_t']$. The error terms η_t are defined by the WLS-regression (13) of r_t^y on r_t^x, where the weights depend on the second derivative of the utility function. This weighting accounts for the risk aversion of the investor. Because of the weighting by u_t'', extreme portfolio returns yield a smaller

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weight in the regression. The positions in the new securities are financed by adjusting the positions in the benchmark assets from w_x^0 to w_x according to (15). Again, these adjustments are determined by the new weights w_y and the slope coefficients D. Since the slope coefficients D are essentially the portfolios of r_t^x that replicate r_t^y, it is natural that the adjustments in the benchmark assets are given by $D'w_y$. The attractiveness of r_t^y relative to r_t^x is determined by the 'abnormal returns' λ and the relative risk $E[\eta_t \eta_t']$. When the investor wants to benefit from this by taking positions in r_t^y, he wants to finance this by taking opposite positions in the replicating portfolios D.

In summary, using the prices λ assigned by a normalized pricing kernel q_t^x to new securities with returns r_t^y yields a generalization of the traditional Jensen measure which allows to measure the performance of r_t^y relative to the benchmark assets r_t^x investors with either mean-variance or non mean-variance utility functions. Using an auxiliary WLS-regression of the new securities returns on the benchmark returns r_t^x (where the weights depend on the second derivative of the utility function) in turn provides lots of information about how the investor should adjust his portfolio holdings. This is similar to the information provided by the traditional Jensen regression.

4   Performance improvement with nontraded assets

So far we treated all investors as if they had the same investment opportunity set. However, because investors can have positions in nontraded assets, i.e., they can face different *nonmarketable risks*, they may face different investment opportunity sets. For example, the investment opportunity set of an exporter or a corporation with a foreign subsidiary is affected by his exposure to foreign currency. Similarly, the investment opportunity sets of pension funds and insurance companies are affected by their liabilities. Consequently, when considering additional securities such as currency forwards, there may be outperformance relative to the benchmark assets for one agent, but not for others. The reason is that the presence of nontraded assets changes the net portfolio payoff for an investor. The effect of nontraded assets on portfolio choice and on expected asset and futures returns, have been analyzed extensively by Stoll (1979) and Hirshleifer (1988, 1989) for the mean-variance case. The empirical evidence of so-called hedging pressure effects on ex-

pected futures returns, which reflects the aggregate nontraded positions of agents in the economy, suggests that nontraded assets are indeed important for many agents (see, e.g., Carter, Rausser, and Schmitz (1983), Chang (1985), Bessembinder (1992), and De Roon, Nijman, and Veld (1999)).

Let W_{t-1} be the wealth invested in assets by an investor, excluding the positions in nontraded assets. The fraction invested in asset j is given by w_{Aj}, and w_A is the vector containing all w_{Aj}. Notice that $w'_A \iota = 1$. The returns on the assets are given by the vector R_t, which may or may not include a risk free asset. Besides investing in assets such as stocks and bonds, the investor can also take positions in zero-investment securities such as forward and futures contracts. The position in forward or futures contract k is also expressed as a fraction of wealth, W_{t-1}. The forward positions are given by the vector w_F. Since the forwards or futures are zero-investment contracts, we can treat their returns as excess returns, which are denoted by the vector r_t. Finally, the agent may have a position in a nontraded asset with a size z that yields a return R_t^z. The size of the nontraded position is also expressed as a fraction of W_{t-1}, implying that $w'_A \iota + z$ will not be equal to one if $z \neq 0$. The total return on his invested wealth for the investor is given by

$$R_{W,t} = w'_A R_t + w'_F r_t + z R_t^z. \tag{16}$$

Recall that the asset weights w_A must sum to one. Therefore, an equivalent way of writing the total return (16) is:

$$R_{W,t} = w'_A (R_t + z R_t^z \times \iota) + w'_F r_t = w'_A \widetilde{R}_t + w'_F r_t, \tag{17}$$

where $\widetilde{R}_t$ is the vector of returns adjusted for the position in the nontraded asset. Since there is only a portfolio restriction on the asset weights w_A and not on the forward and futures positions, only the asset weights must be adjusted for the position in the nontraded asset.

Since an investor will choose his portfolio taking the return on the nontraded asset into account, his interest will be in the adjusted returns $\widetilde{R}_t$, rather than the normal returns R_t. It is easy to see that this implies that for an investor that invests in the benchmark assets only, we must have

$$\begin{aligned} E\left[\widetilde{m}_t^x \widetilde{R}_t^x\right] &= \iota, \\ E\left[\widetilde{m}_t^x r_t^x\right] &= 0, \end{aligned}$$

where $\widetilde{m}_t^x$ is the stochastic discount factor that prices the adjusted returns correctly. In a similar way as before, the normalized stochastic discount

factor is

$$\tilde{q}_t^x = \frac{\widetilde{m}_t^x}{E[\widetilde{m}_t^x]}.$$

Tests for outperformance of the benchmark assets by other assets can now be based on the performance measure $\tilde{\lambda}$ which, for excess returns r_t^y is equal to

$$\tilde{\lambda} = E\left[\tilde{q}_t^x r_t^y\right].$$

If r_t^y is not the return on a forward or futures contract, but the return on a nonzero-investment asset like stocks and bonds, then we can create excess returns by subtracting one of the (unadjusted) benchmark returns. All the performance tests and interpretations of the regressions that we described before remain valid, provided that we replace the benchmark asset returns R_t^x by adjusted returns $\tilde{R}_t^x$, while the forward (or excess) returns r_t^x and r_t^y remain unchanged.

5 Data

To study the effects of currency hedging on international portfolios, we use a dataset that contains monthly returns on stock indices for the G5 countries as well as monthly returns on four forward currency contracts for the period February 1975 until December 1998. The stock indices are the MSCI indices for the US, France, Germany, Japan, and the United Kingdom. The forward contracts are contracts for the French Franc, German Mark, Japanese Yen, and British Pound with respect to the US Dollar, with a maturity of one month. These series are constructed from spot rates and one-month Eurodollar interest rates. All data are obtained from Datastream.

Table 1 contains summary statistics for the unhedged dollar returns on the five country indices and the four currency forwards. The summary statistics show that there are no big differences between the five countries in terms of risk as measured by the standard deviation, except that the US returns have a somewhat lower standard deviation because they are measured in their home currency. The four forward contracts have a similar amount of risk, which is about half the standard deviation of the index returns. The mean returns of the four forward contracts are close to zero, which suggests that hedging currency risk will not be very costly in terms of foregone expected returns. From the excess kurtosis it appears that the index returns are somewhat more leptokurtic than the forward returns. The Bera-Jarque

tests for which the associated p-values are reported in the last column of Table 1 also indicate that nonnormalities are more of an issue for the index returns than for the forward returns. Normality of the index returns can be rejected for all countries, whereas for the forwards it can be rejected in only two out of four cases. For the Japanese Yen forwards, the rejection appears to be due to a relatively high skewness, whereas for the British Pound forwards we find a rather high kurtosis. The one-but-last column presents the average correlation of each index or forward contract with the five stock indices. Here the correlation of each index with itself is excluded from the average. These average correlations show that, except for the US, the correlations between the indices are fairly high. The correlations of the forwards with the indices are somewhat lower, approximately 0.35 for all currencies.

Table 2 provides the Bera-Jarque test-statistics for normality of portfolio returns. The first column of Table 2 gives the risk aversions for the power-utility functions of the form

$$u(W_t) = \frac{W_t^{1-\rho}}{1 - \rho}, \tag{18}$$

for which we test for intersection in the subsequent sections. The second column presents the p-value associated with the Bera-Jarque test for normality of the returns of the portfolio that maximizes this utility function. Except for the very low risk aversion, $\rho = 1$, the hypothesis that the portfolio returns are normal is strongly rejected. If returns would be normally distributed then for each power utility function there would be an equivalent mean-variance utility function yielding the same optimal portfolio. For a given utility function u, the risk aversion in the equivalent mean-variance utility function

$$E[W_t] - \gamma Var[W_t], \tag{19}$$

that would yield the same optimal portfolio under normality, can be derived using a generalized version of Stein's Lemma as

$$\gamma = -\frac{E[u''(W_t)]}{E[u'(W_t)]}. \tag{20}$$

The derivation of the generalized version of Stein's Lemma is given in Appendix C.

Based on the power utility functions, using the sample-equivalent of (20), the third column shows the risk aversions for the equivalent mean-variance

functions. These equivalent mean-variance risk aversions γ are close to the risk aversions ρ in the power utility functions. The last column shows the p-values of the Bera-Jarque test statistics for the hypothesis that the returns on the mean-variance efficient portfolio are normally distributed. Again, the tests show that this hypothesis is easily rejected except for a very low risk aversion. It is important to note that we do not assume normality in our tests for outperformance. The calculated risk aversion γ yields the mean-variance portfolio that would be identical to the optimal portfolio of the power-utility investor if returns would be normally distributed. However, to the extent that returns are not normally distributed the optimal portfolios may be very different and the results of the performance tests for the power-utility and the mean-variance case may be different as well.

These summary statistics in Table 1 and 2 set the stage for the analysis of currency hedging in the next sections. Since the currency forwards and the indices have correlations that are not too high and the currency forward contracts have mean returns close to zero, Table 1 and 2 suggest that at least mean-variance investors can obtain diversification benefits from adding currency forwards to their stock portfolio, i.e., they can benefit from currency hedging. In addition, to the extent that nonnormalities are important for power-utility investors, currency hedging may be important for them, given the nonnormalities in the index and portfolio returns. Whether these suggested benefits are reliable or simply due to sampling error, and whether they exist for mean-variance investors only or for power utility-investors as well, will be analyzed in the next sections.

6 The benefits of currency hedging for mean-variance investors

6.1 Currency hedging with forwards

Our starting point is a US-investor that holds an international stock portfolio consisting of the G5 indices. We analyze whether hedging the currency risk associated with this portfolio improves his portfolio performance. Table 3 shows the results for tests whether the mean-variance performance of international stock portfolios can be improved upon by hedging the currency risk with the four available forward contracts, using static and dynamic hedges. The first three columns present test results for the hedging benefits of for-

ward contracts using static hedges. The first column shows the values of the risk aversion γ as reported in Table 2. The test-statistics in the second column are the Wald test-statistics for mean-variance intersection, i.e., for the hypothesis that the Jensen measures of the forward contracts are equal to zero. Since there are four forward contracts, the limiting distribution of the test-statistic under the null-hypothesis of no outperformance is χ^2_4. The test-statistics and the p-values show that the hypothesis of no outperformance can only be rejected for very high values of the risk aversion parameter (> 30). Even though the low (absolute) average forward returns and the low correlations of the forwards with the indices in Table 1 suggest that the forward contracts can offer diversification benefits, Table 3 shows that static hedges with currency forwards do not lead to a significant improvement of portfolio performance for a US mean-variance investor, unless he is very risk averse.

The last two columns of Table 3 show the test results for outperformance of a dynamic hedging strategy. Since there is ample evidence that currency forward returns are at least to some extent predictable, dynamic strategies that exploit this predictability may lead to a better portfolio performance. Currency forward returns are known to be related to the current interest rate spread between the two currencies (see, e.g., Fama (1984) and Banzal and Dahlquist (1999)). Therefore, it seems natural to use the current interest rate spread as the conditioning variable in our dynamic hedging strategy. Specifically, define z_{t-1} as the beginning-of-period interest rate spread, $z_{i,t-1} = (1 + R^f_{t-1})/(1 + R^{f*}_{i,t-1}) - 1$, where R^f_{t-1} and $R^{f*}_{i,t-1}$ are the home and foreign risk free interest rates. We consider a dynamic strategy that takes each period a position $z_{i,t}$ in forward contract i. The returns on our dynamic hedging strategy can then be written as:

$$r^z_{i,t} = z_{i,t-1} \times r^y_{i,t}. \tag{21}$$

The results for the dynamic hedges are presented in the last two columns of Table 3. Here we include both the dynamic forward returns $r^z_{i,t}$ as well as the simple forward returns $r^y_{i,t}$, implying that the optimal forward position may consist of a constant part and a part that is linear in $z_{i,t}$. Therefore, the limiting distribution of the test-statistic under the null-hypothesis of no outperformance is χ^2_8. These results leave no doubt about the value of conditional hedging. Unlike the unconditional case, where there was no evidence of outperformance, in the conditional case the hypothesis of no outperformance is rejected for every value of the risk aversion parameter. These findings are in line with the ones obtained by Glen and Jorion (1993).

To get some intuition about these results, Table 4 contains some summary statistics for the conditional forward returns. The summary statistics are calculated for $(z_{i,t-1} - \overline{z}_{i,t-1})r_{i,t}^y/\sigma(z_{i,t-1})$, i.e., forward returns scaled by a normalized spread. Comparing the statistics in Table 4 with the statistics for the unconditional forward returns and the indices in Table 1, we see that the conditional forward returns have much higher (absolute) means than the unconditional ones, but that they are also somewhat riskier. In addition, the correlations of the conditional returns with the G5 indices are lower than the correlations of the unconditional returns. The diversification benefits of the conditional forward strategies may therefore be driven by the lower correlations with the indices and the higher (absolute) mean returns relative to the unconditional strategies.

7 The benefits of currency hedging for power-utility investors

7.1 Currency hedging with forwards

The previous section showed that for mean-variance investors, static currency hedges with forwards do not improve the performance of international stock portfolios, whereas dynamic currency hedges do improve the portfolio performance. Since the results in Table 1 and 2 showed that the index returns as well as the portfolio returns are clearly non-normal, the benefits of hedging may be different for power utility investors. Therefore, to see whether the conclusions in the previous section also hold for investors with a power utility function, Table 5 presents tests whether the performance of international stock portfolios can be improved upon with currency forwards, for such investors. The risk aversions that are used are the ones presented in Table 2. Table 5 can therefore be compared with Table 3.

The first three columns of Table 5 present test results for the benefits of static hedging with forward contracts for investors with a power utility function. The test-statistics in the second column are the Wald test-statistics for the hypothesis that the performance measure λ as defined in (10) are equal to zero:

$$W = T \times \widehat{\lambda}' Var\left[\widehat{\lambda}\right]^{-1} \widehat{\lambda}. \tag{22}$$

This gives a test for the hypothesis that the stochastic discount factor asso-

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ciated with a power utility function that prices the five G5 indices correctly also prices the four forward contracts correctly. This test is similar to the test for outperformance as proposed by Chen and Knez (1996). Under the null-hypothesis of no outperformance of the four forwards and standard regularity conditions this statistic will asymptotically be χ^2_4-distributed. In estimating the covariance matrix $Var[\widehat{\lambda}]$ we account for the fact that $\widehat{\lambda}$ is based on the estimated pricing kernel $\widehat{q}_t$ rather than the true kernel q_t (see Hall and Knez (1995)). As for the mean-variance case, the test-statistic and the p -values show that the hypothesis of no outperformance can only be rejected for very high values of the risk aversion parameter (≥ 30). This similarity of the test results for mean-variance and power utility investors is analogous to the findings of Cumby and Glen (1990) for the performance of international mutual funds.

The last two columns of Table 5 show the benefits of dynamic hedging with forward contracts, where the positions in forward contracts are again conditional on the interest rate spread. As in the mean-variance case, we now find that there is a significant performance improvement for every value of the risk-aversion parameter. Unlike static hedges, our dynamic hedges always create significant benefits for investors, irrespective of the risk aversion. Comparing the results in Table 3 and Table 5, we see that the test-statistics for outperformance and their p-values are somewhat different for the two classes of utility functions, but that the picture that emerges is identical, although according to Table 2 the portfolio returns are clearly non-normal: In case of static hedges, it is again only for very risk investors that a static hedge of the associated currency risk yields a significant improvement of their portfolio performance[2]. For dynamic hedges there are significant benefits for all investors considered here.

---

[2]It may be conjectured that in case of non-normalities currency options provide a better hedge than currency forwards. Although not reported here, we also analyzed the performance of static hedges with one-month at-the-money currency put and call options. This leads to the same conclusion as with forward contracts: Static hedges with currency options do not lead to a significant portfolio performance improvement for mean-variance and power utility investors, unless they are very risk averse. These results can be obtained from the authors upon request.

7.2 Optimal forward positions

Even though the results for the mean-variance case and the power-utility case appear to be similar in terms of the test results, it may be the case that the optimal forward positions for power-utility investors are very different form the optimal mean-variance hedges. As explained in Section 2 and 3, the optimal forward positions can be derived from a regression of the forward returns on the benchmark returns. Table 6 presents the estimated optimal forward positions for an investor with a power utility function that invests in the G5 countries, with a risk aversion parameter $\rho = 1, 5$, or 15.

For each risk aversion, the first row shows the (exact) optimal forward positions, i.e., the forward positions for which

$$\frac{1}{T} \sum_{t=1}^{T} \widehat{m}_t \left(\begin{array}{c} r_t^x \\ r_t^y \end{array} \right) = 0_{K+N},$$

where $\widehat{m}_t$ is the estimated stochastic discount factor for a power utility function, that prices both the indices and the forward contracts correctly. These estimated positions show that investors with a low risk aversion ($\rho = 1$, i.e., log-utility investors) want to take extreme speculative positions in the currency forward contracts. For investors with higher risk aversions these estimated positions become much more realistic as the other two panels of Table 6 show. For instance, for a risk averse investor with $\rho = 15$, the positions taken in currency forwards as a percentage of invested wealth, are always smaller than 40% in absolute value. Still, these positions can be rather large relative to the position taken in the corresponding country. The country weights themselves are given in the fourth row of each of the three panels.

The second row of each of the three panels shows the approximate forward position that are derived from a WLS regresssion of the forward returns on the stock indices, where the derivative of the stochastic discount factor are used as the weights in the regression as in (13). Thus, we run the following Weighted Least Squares regression:

$$r_t^y = d_1(r_t^{Fra} - r_t^{US}) + d_2(r_t^{Ger} - r_t^{US}) + d_3(r_t^{Jap} - r_t^{US}) + d_4(r_t^{UK} - r_t^{US}) + \varepsilon_t$$

where r_t^y is the vector of forward returns on currency i and the weights are $\sqrt{-u_t''/\overline{u}_t'}$. Because we do not include a risk free asset in the analysis, we use returns in excess of the returns on the US index. The approximate portfolio weights for the four currencies can now be obtained using (14).

Comparing the regression-based weights with the weights in the first row of the three panels, we see that the approximation works well. Especially for the high risk aversion, the absolute difference in the two weights is at most two percentage points. Alternatively, we might use the equivalent mean-variance portfolio weights to approximate the portfolio weights of the power utility function. The third row of each of the three panels in Table 6 shows the portfolio weights for the equivalent mean-variance function that has a risk aversion γ calculated using Stein's Lemma in (20). These weights are obtained using (3) and the regression

$$r_t^y = \alpha + \beta_1(r_t^{Fra} - \eta) + \beta_2(r_t^{Ger} - \eta) + \beta_3(r_t^{Jap} - \eta) + \beta_4(r_t^{UK} - \eta) + \beta_5(r_t^{US} - \eta) + \varepsilon_t,$$

where η is the zero-beta rate of the mean-variance efficient portfolio of the five benchmark indices for the risk aversion γ.

Comparing the (approximate) weights for the power utility functions with the mean-variance weights shows that mean-variance efficient portfolios can be rather different from power utility portfolios. For instance, in the last panel of Table 6, risk averse investors with a power utility function take a short position of 26% in British Pound forward contracts, whereas a mean-variance investor would take almost no position in these forward contracts.

Apart from the static hedges, here the interest is also in the optimal forward positions for the dynamic hedges that are conditional on the interest rate spread. In case of dynamic hedges, basically we add eight different forward strategies: the four static forward hedges that were analyzed in the previous section with return $r_{i,t}^y$ and the four dynamic hedges that have return $r_{i,t}^z = (z_{i,t-1} - \overline{z}_i) r_{i,t}^y / \sigma(z_{i,t-1})$. As in Table 4, here we use again the normalized spread in order to make the results easier to interpret. Table 7 presents the four weights for $r_{i,t}^y$, w_i^y and the four weights for $r_{i,t}^z$, w_i^z, for both the power utility-functions (based on the Weighted Least Squares regression) and for the equivalent mean-variance functions. Since the weights w_i^z are based on the normalized interest rate spread, they can be interpreted as the change in the forward position per standard deviation of the interest rate spread. Also note that the normalized spread has mean zero, implying that the average position in the forward contract on currency i is given by w_i^y.

Similar to the static hedges in Table 6, investors with a low risk aversion ($\rho = 1$) have relatively big estimated positions in the forward contracts, both on average and in relation to the interest rate spread. Again, as the risk aversion increases, the estimated forward positions become more realistic.

In case $\rho = 15$, all positions w_i^y and w_i^z are between zero and one. This is true for both the power utility and the mean-variance investors. Notice that the optimal forward positions are sensitive to the value of the interest rate spread. As with the static hedges, there can be substantial deviations between the optimal power utility positions and the mean-variance positions, even though the tests for outperformance for both utility functions behave quite similar.

In order to analyze whether the reported differences in optimal forward positions for mean-variance and power-utility investors are statistically significant, Table 8 presents the results for tests whether stock portfolios hedged with mean-variance forward positions are also optimal for power-utility investors. Here, for a power-utility investor with risk aversion ρ we calculate the optimal mean-variance forward positions for the equivalent risk-aversion γ. We then test whether the power-utility investor that uses these mean-variance hedges for his stock portfolio can improve the performance of his portfolio by changing the forward positions. Thus, the test is whether the forward contracts outperform the mean-variance hedged portfolio for investors with a power utility function.

The first three columns of Table 8 show the results for static hedges. Here we see that we can not reject the hypothesis that the mean-variance portfolio is optimal for power-utility investors unless he is very risk avers. These results are consistent with the results in Table 3 and 5, where we concluded that there are no significant benefits from static currency hedges unless the investor is very risk averse. For very risk-averse investors the optimal forward positions are not only different from zero but the mean-variance positions are also significantly different from the optimal power utility positions.

For the dynamic hedges, Tables 3 and 5 already showed that they improve the portfolio performance for both mean-variance and power utility investors, with any level of risk aversion. The last two columns of Table 8 show that we can reject the hypothesis that the mean-variance forward positions are optimal for the power-utility investors for any level of the risk aversion parameter. Therefore, we can conclude that the optimal forward positions in the dynamic hedges are different for the two classes of utility functions.

In short, while the results in Table 3 and 5 showed that currency hedging for mean-variance investors and power-utility investors is similar in the sense that hedging can improve portfolio performance for both utility functions in the same situations, this does not mean that the investors behave in the same way. The results in this section show that in case hedging is beneficial, the

optimal forward positions for both utility functions are materially different.

8 Currency hedging in the presence of non-traded risks

In the previous sections we analyzed the benefits of static and dynamic currency hedges for investors with different utility functions. Apart from their utility function, investors may also have different exogenous risk exposures, which effectively changes their investment opportunity set. As explained in Section 4, the presence of an exogenous risk exposure, or nontraded risk, changes the investors portfolio payoff, which can be accounted for in tests for performance improvement by adjusting the returns on the stock positions, but not the returns on the forward positions.

In this section we impose a relatively simple exogenous exposure upon the investment portfolio, which is a nontraded position in one of the four currencies. It is assumed that the investor, which may be an exporting or importing firm, or a foreign pension fund e.g., has a long position in one of the four currencies which equals 67% of his total wealth.

Table 9 shows the results for tests of performance improvements when there is a 67% nontraded currency positions. The four panels show the results for an exposure in each of the four currencies. The left hand part of the table shows the results for investors with a power utility function, whereas the right hand part shows the results for the equivalent mean-variance functions. For the nontraded positions in each of the four currencies the test results are very similar. First, for the power-utility investors, hedging currency risk does not lead to a significant improvement of portfolio performance for investors with a very low risk aversion, but for all other investors ($\rho \geq 5$) it does lead to a significant performance improvement. Thus, unlike the simple case, once there exists a sizeable exposure to one of the currencies, there are obvious benefits to currency hedging, at least for risk averse investors.

The results for the equivalent mean-variance investors are comparable to the results for the power-utility investors. First of all notice that the equivalent mean-variance risk aversions γ are again very close to the risk aversions of the power-utility function, ρ. As with the power-utility functions, currency forwards lead to significant outperformance, except for low values of the risk aversion parameter ($\gamma = 1$). However, notice that the diversification benefits

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for the risk averse investors are much more pronounced in the mean-variance case, as can be judged from the values of the Wald test-statistics which are much higher in the mean-variance case than in the power-utility case. This may be due to the linear relation between the forward contracts and the underlying currency exposures, which may cause the diversification benefits of currency forwards to be much more pronounced in the (linear) marginal utility of the mean-variance investors than in the (nonlinear) marginal utility of the power-utility investors.

In summary, in case of a sizeable nontraded currency exposure, hedging with currency forwards leads to clear diversification benefits. Having a 67% exposure to currency risk, it is not sufficient for investors to adjust their international stock portfolio, but additional positions in currency forwards are needed. This conclusion holds for both power-utility and mean-variance investors, except when they have a very low risk aversion. Also, although both classes of investors clearly benefit from currency hedging, the benefits are much more pronounced for the mean-variance investors than for the power-utility investors.

9   Summary and conclusions

We analyze the benefits of currency hedging for US investors that hold a diversified portfolio of the G5 stock indices. Our methodoly to test for outperformance of the G5 stock indices by the respective forward contracts allows for investor dependencies such as different utility functions and the presence of nontraded risk. Moreover, we show that the Jensen regression employed in the mean-variance case as well as a similar auxiliary regression for the non mean-variance case, provide a lot of informaton as to how to adjust the optimal portfolio holdings and forward positions in order to obtain the new optimal portfolio.

Our empirical analysis shows that for both mean-variance and power utility investors, static hedges do not improve portfolio performance unless the investor is very risk averse. On the other hand, dynamic hedges that are conditional on the interest rate spread do significantly improve portfolio performance for both classes of utility functions, irrespective of the risk aversion of the investor. Moreover, the optimal forward positions for power utility investors are significantly different from the optimal mean-variance positions. Finally, investors that have a sizeable exogenous exposure to one of the cur-

rencies can significantly improve their portfolio performance by using static currency hedges.

A Deriving optimal mean-variance portfolio weights from the Jensen regression

In this appendix we show how the optimal mean-variance portfolio weights can be derived from the Jensen regression in (1):

$$r_t^y = \alpha_J + Br_t^x + \varepsilon_t.$$

Recall that the slope coefficients B and the intercepts α_J in this regression can be written as

$$
\begin{align}
B &= \Sigma_{yx}\Sigma_{xx}^{-1}, \tag{23a}\\
\alpha_J &= \mu_y - B\mu_x. \tag{23b}
\end{align}
$$

The mean-variance efficient portfolio weights for an investor that has a risk aversion $\widetilde{\gamma}$ and that invests in r_t^x as well as in r_t^y are given in (2):

$$
\begin{pmatrix} w_x \\ w_y \end{pmatrix} = \widetilde{\gamma}^{-1} \begin{pmatrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_{yy} \end{pmatrix}^{-1} \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}.
$$

Since

$$\Sigma_{\varepsilon\varepsilon} = \Sigma_{yy} - \Sigma_{yx}\Sigma_{xx}^{-1}\Sigma_{xy},$$

the partitioned inverse of the covariance matrix of r_t^x and r_t^y can be written as

$$
\begin{pmatrix} \Sigma_{xx}^{-1} + \Sigma_{xx}^{-1}\Sigma_{xy}\Sigma_{\varepsilon\varepsilon}^{-1}\Sigma_{yx}\Sigma_{xx}^{-1} & -\Sigma_{xx}^{-1}\Sigma_{xy}\Sigma_{\varepsilon\varepsilon}^{-1} \\ -\Sigma_{\varepsilon\varepsilon}^{-1}\Sigma_{yx}\Sigma_{xx}^{-1} & \Sigma_{\varepsilon\varepsilon}^{-1} \end{pmatrix}
$$
$$
= \begin{pmatrix} \Sigma_{xx}^{-1} + B'\Sigma_{\varepsilon\varepsilon}^{-1}B & -B'\Sigma_{\varepsilon\varepsilon}^{-1} \\ -\Sigma_{\varepsilon\varepsilon}^{-1}B & \Sigma_{\varepsilon\varepsilon}^{-1} \end{pmatrix}.
$$

Multiplying with $\widetilde{\gamma}^{-1}$ and postmultiplying with the expected return vector yields

$$
\gamma^{-1} \begin{pmatrix} \Sigma_{xx}^{-1}\mu_x + B'\Sigma_{\varepsilon\varepsilon}^{-1}B\mu_x - B'\Sigma_{\varepsilon\varepsilon}^{-1}\mu_y \\ -\Sigma_{\varepsilon\varepsilon}^{-1}B\mu_x + \Sigma_{\varepsilon\varepsilon}^{-1}\mu_y \end{pmatrix},
$$

which, combined with (23) gives

$$\begin{pmatrix} w_x \\ w_y \end{pmatrix} = \widetilde{\gamma}^{-1} \begin{pmatrix} \Sigma_{xx}^{-1}\mu_x + B'\Sigma_{\varepsilon\varepsilon}^{-1}\alpha_J \\ \Sigma_{\varepsilon\varepsilon}^{-1}\alpha_J \end{pmatrix}.$$

This is the desired result.

B   Deriving optimal portfolio weights from performance measures and regression parameters

Recall from (6) that the normalized stochastic discount factor that prices r_t^x correctly is given by

$$q_t^x = u'\left(w_x^{0\prime} r_t^x + R_{t-1}^f\right)/\overline{u}_t'.$$

This is the discount factor for an agents that invests in r_t^x only, i.e., $w^0 = (w_x^{0\prime}\ w_y^{0\prime})' = (w_x^{0\prime}\ 0_N')'$. If the agent also invests in r_t^y, then his optimal portfolio is $w = (w_x'\ w_y')'$ and the normalized stochastic discount factor is given by

$$q_t = \frac{c}{c_0 \overline{u}_t'} \times u'(w_x' r_t^x + w_y' r_t^y + R_{t-1}^f). \tag{24}$$

This stochastic discount factor assigns a price zero to all the returns r_t^x and r_t^y. Using a first order Taylor series approximation around the weights w^0, we get

$$\begin{aligned}
E\left[\frac{c_0}{c}q_t \begin{pmatrix} r_t^x \\ r_t^y \end{pmatrix}\right] &= \begin{pmatrix} 0_K \\ 0_N \end{pmatrix} \\
&\approx E\left[q_t^x \begin{pmatrix} r_t^x \\ r_t^y \end{pmatrix}\right] + E\left[\frac{1}{\overline{u}_t'}u_t'' \begin{pmatrix} r_t^x \\ r_t^y \end{pmatrix} \begin{pmatrix} r_t^{x\prime} & r_t^{y\prime} \end{pmatrix} \begin{pmatrix} w_x - w_x^0 \\ w_y \end{pmatrix}\right] \\
&= \begin{pmatrix} 0_K \\ \lambda \end{pmatrix} + E\left[\begin{pmatrix} \widetilde{r}_t^x \\ \widetilde{r}_t^y \end{pmatrix} \begin{pmatrix} \widetilde{r}_t^{x\prime} & \widetilde{r}_t^{y\prime} \end{pmatrix}\right] \begin{pmatrix} w_x - w_x^0 \\ w_y \end{pmatrix}.
\end{aligned}$$

From this we get

$$\begin{pmatrix} w_x - w_x^0 \\ w_y \end{pmatrix} = E\left[\begin{array}{cc} \widetilde{r}_t^x \widetilde{r}_t^{x\prime} & \widetilde{r}_t^x \widetilde{r}_t^{y\prime} \\ \widetilde{r}_t^y \widetilde{r}_t^{x\prime} & \widetilde{r}_t^y \widetilde{r}_t^{y\prime} \end{array}\right]^{-1} \begin{pmatrix} 0_K \\ \lambda \end{pmatrix},$$

which, using the partitioned inverse of the second moment matrix of $\widetilde{r}_t^x$ and $\widetilde{r}_t^y$ and the regression

$$\widetilde{r}_t^y = D\widetilde{r}_t^x + \eta_t,$$

can be rewritten as

$$\begin{pmatrix} w_x - w_x^0 \\ w_y \end{pmatrix} = \begin{pmatrix} -D'E\left[\eta_t\eta_t'\right]^{-1}\lambda \\ E\left[\eta_t\eta_t'\right]^{-1}\lambda \end{pmatrix}.$$

This shows the result in (14) and (15).

C A multivariate extension of Stein's lemma

This Appendix shows how Stein's lemma can be generalized to functions of a vector of normally distributed random variables X.

Theorem 1 *Let X be a K-dimensional vector of normally distributed variables, $X\widetilde{\ }N(\mu, \Sigma)$ and $g: I\!R^K \to \dot{I\!R}$. Then we have*

$$E\left[g'(X)\right] = E\left[g(X)\Sigma^{-1}\left(X - \mu\right)\right] = \Sigma^{-1}E\left[g(X)\left(X - \mu\right)\right].$$

Proof. *Let ϕ denote the density function of X and note that $\phi'(x) = -\phi(x)\Sigma^{-1}(x - \mu)$. By partial integration we find*

$$\begin{aligned}
E\left[g'(X)\right] &= \int \ldots \int_x g'(x)\phi(x)dx \\
&= -\int \ldots \int_x g(x)\phi'(x)dx \\
&= \int \ldots \int g(x)\Sigma^{-1}(x - \mu)\phi(x)dx \\
&= E\left[g(X)\Sigma^{-1}\left(X - \mu\right)\right].
\end{aligned}$$

∎

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Table 1: Summary statistics

The table contains summary statistics for monthly returns on the MSCI indices and forward contracts in our sample. Mean returns and standard deviations are in percentages. 'c(ind)' gives the average correlation of each index or forward contract with the five indices, where the correlation of each index with itself is excluded. 'BerJar' gvies the p-value associated with the Bera-Jarque test for normality of the returns. Returns are calculated for the period February 1977 until December 1998.

Index returns						
	mean	stdev	skew	kurt	c(ind)	BerJar
Fra	1.59	6.70	-0.14	1.41	0.54	0.000
Ger	1.36	5.93	-0.23	1.16	0.49	0.000
Jap	1.15	6.78	0.28	0.69	0.40	0.025
UK	1.60	5.69	-0.14	1.32	0.49	0.000
US	1.32	4.14	-0.54	3.20	0.21	0.000
Forward returns						
	mean	stdev	skew	kurt	c(ind)	BerJar
FF	0.13	3.38	-0.11	0.25	0.34	0.624
DM	0.03	3.51	-0.01	0.53	0.33	0.302
JY	0.10	3.70	0.35	0.45	0.35	0.037
BP	0.22	3.37	-0.05	1.54	0.35	0.000

Table 2: Risk aversions and normality tests

The table provides Bera-Jarque test-results for the hypothesis that efficient portfolio returns are normally distributed. The first column contains the risk-aversion parameter ρ for a power-utility investor with utility function $U = \frac{W^{1-\rho}}{1-\rho}$. The second column shows the p-value associated with the Bera-Jarque test for normality of the optimal portfolio returns for the power utility function. The third column shows the mean-variance risk aversion γ that corresponds to ρ based on Stein's lemma. The last column shows the p-values associated with the Bera-Jarque test-statistic for normality of the optimal G5-portfolio returns for the Mean-Variance utility functions. Returns are calculated for the period February 1977 until December 1998.

$\rho(power)$	$BJ(pow)$	$\gamma(MV)$	$BJ(MV)$
1.00	(0.465)	0.99	(0.412)
2.00	(0.000)	1.98	(0.000)
5.00	(0.000)	4.99	(0.000)
10.00	(0.000)	10.14	(0.000)
15.00	(0.000)	15.65	(0.000)
30.00	(0.045)	36.59	(0.000)

Table 3: Tests for mean-variance hedging with currency forwards

The table provides test-results for the benefits of currency hedging, for a US-investor that has a mean-variance utility function and that initially invests in G5 stock markets. The first column shows the mean-variance risk aversion γ that corresponds to ρ in Table 2 based on Stein's lemma. The next two columns show the Wald test-statistic and the associated p-value for mean-variance intersection based on the calculated risk aversion. The last two columns provide test-results for the benefits of dynamic currency hedging with forward contracts, for a US-investor that initially invests in the G5 stock markets only. Positions in the forward contracts are conditional upon the current interest rate spread, which is implemented using scaled returns: $z_{t-1}r_t^y$ where r_t^y is the return on the forward contract, and z_{t-1} is the interest rate spread between currency i and the US Dollar. Returns are calculated for the period February 1977 until December 1998.

$\gamma(MV)$	$Wald(static)$	$p\ (static)$	$Wald(dynam)$	$p(dynam)$
0.99	3.34	(0.502)	31.80	(0.000)
1.98	3.04	(0.552)	31.22	(0.000)
4.99	2.44	(0.655)	29.45	(0.000)
10.14	2.53	(0.639)	26.66	(0.001)
15.65	3.72	(0.446)	24.41	(0.002)
36.59	9.43	(0.051)	21.52	(0.006)

Table 4: Summary statistics for conditional forward returns

The table contains summary statistics for monthly conditional forward returns. Positions in the forward contracts are conditional upon the current interest rate spread, which is implemented using scaled returns: $(z_{t-1} - \overline{z}_{t-1}) r_t^y / \sigma(z_{t-1})$ where r_t^y is the return on the forward contract, and z_{t-1} is the interest rate spread between currency i and the US Dollar. The conditional returns are scaled by the standardized spread to make them comparable with the unconditional returns. Mean returns and standard deviations are in percentages. 'c(ind)' gives the average correlation of each index or forward contract with the five indices, where the correlation of each index with itself is excluded. Returns are calculated for the period February 1977 until December 1998.

	mean	stdev	c(ind)
FF	-0.69	3.65	0.07
DM	-0.51	4.15	0.09
JY	-1.06	3.97	0.08
BP	-0.82	3.75	-0.07

Table 5: Tests for hedging with forward contracts for investors with power utility functions

The table provides test-results for the benefits of currency hedging, for a US-investor that has a power utility function $U = \frac{W^{1-\rho}}{1-\rho}$ and that initially invests in the G5 stock markets. The first column shows the risk aversion ρ. The next two columns show the Wald test-statistic and the associated p-value for outperformance of the G5 markets by static currency hedges. The last two columns provide test-results for the benefits of dynamic currency hedging with forward contracts, for a US-investor that initially invests in the G5 stock markets only. Positions in the forward contracts are conditional upon the current interest rate spread, which is implemented using scaled returns: $z_{t-1}r_t^y$ where r_t^y is the return on the forward contract, and z_{t-1} is the interest rate spread between currency i and the US Dollar. Returns are calculated for the period February 1977 until December 1998.

ρ(power)	$Wald(static)$	$p(static)$	$Wald(dynam)$	$p \ (dynam)$
1.00	3.24	(0.518)	30.93	(0.000)
2.00	2.95	(0.566)	30.14	(0.000)
5.00	2.26	(0.688)	27.96	(0.000)
10.00	1.68	(0.794)	25.32	(0.001)
15.00	2.09	(0.719)	24.63	(0.002)
30.00	12.56	(0.014)	50.65	(0.000)

Table 6: Optimal and approximate weights of forward contracts for static hedges

The table gives the optimal forward positions for a US investor that invests in the G5 countries and that statically hedges his currency risk. The weights are the optimal positions for a power utility function with risk aversion ρ, w(optimal), the regression based approximations of these weights, w(approximate), the optimal position for the equivalent mean-variance portfolio, w(mv), and the associated position in the country-indices themselves, w(country).

$\rho = 1$	FF	DM	JY	BP
w(optimal)	8.17	-10.32	1.34	2.59
w(approximate)	9.77	-11.74	1.21	2.61
w(mv)	10.54	-12.49	1.28	2.53
w(country)	0.50	0.46	-0.84	0.53
$\rho = 5$	FF	DM	JY	BP
w(optimal)	1.71	-2.02	0.02	0.45
w(approximate)	1.84	-2.14	0.01	0.45
w(mv)	2.02	-2.32	0.01	0.33
w(country)	0.07	0.22	0.06	0.15
$\rho = 15$	FF	DM	JY	BP
w(optimal)	0.30	-0.27	-0.17	0.26
w(approximate)	0.31	-0.29	-0.17	0.27
w(mv)	0.58	-0.61	-0.20	-0.03
w(country)	-0.01	0.10	0.28	-0.02

Table 7: Optimal weights of forward contracts in dynamic strategies
The table gives the optimal forward positions for a US investor that invests in the G5 countries and that hedges his currency risk using a dynamic strategy. The weights are the optimal positions for a power utility function with risk aversion ρ, w^y (pow) and w^z (pow), and the optimal position for the equivalent mean-variance portfolio, w^y (mv) and w^z (mv). The weights w^y are the static positions in the forward contracts, and w^z are the dynamic positions, i.e., the positions in $(z_{t-1} - \overline{z}_{t-1})r_t^y/\sigma(z_{t-1})$ where r_t^y is the return on the forward contract, and z_{t-1} is the interest rate spread between currency i and the US Dollar.

$\rho = 1$	FF	DM	JY	BP
w^y (pow)	8.58	-11.35	5.05	0.97
w^z (pow)	-7.20	5.31	-5.44	-4.99
w^y (mv)	8.48	-11.54	3.72	2.09
w^z (mv)	-4.43	2.64	-6.28	-3.46
$\rho = 5$	FF	DM	JY	BP
w^y (pow)	1.72	-2.19	0.70	0.22
w^z (pow)	-1.38	0.93	-1.37	-0.80
w^y (mv)	1.59	-2.11	0.50	0.26
w^z (mv)	-0.92	0.48	-1.27	-0.59
$\rho = 15$	FF	DM	JY	BP
w^y (pow)	0.35	-0.38	0.10	0.15
w^z (pow)	-0.57	0.36	-0.53	-0.16
w^y (mv)	0.43	-0.53	-0.04	-0.05
w^z (mv)	-0.33	0.12	-0.42	-0.10

Table 8: Tests whether mean-variance hedges are optimal for power utility investors

The table provides test-results for the benefits of currency hedging, for a US-investor that has a power utility function $U = \frac{W^{1-\rho}}{1-\rho}$ and that initially invests in G5 stock markets hedged with mean-variance optimal forward positions. The first column shows the risk aversion ρ. The next two columns show the Wald test-statistic and the associated p-value for outperformance of the mean-variance hedged G5 markets by static currency hedges. The last two columns provide test-results for the benefits of dynamic currency hedging with forward contracts, for a US-investor that initially invests in the G5 stock markets and that uses dynamic mean-variance hedges. Positions in the forward contracts are conditional upon the current interest rate spread, which is implemented using scaled returns: $z_{t-1}r_t^y$ where r_t^y is the return on the forward contract, and z_{t-1} is the interest rate spread between currency i and the US Dollar. Returns are calculated for the period February 1977 until December 1998.

$\rho(power)$	$Wald(static)$	$p(static)$	$Wald(dynam)$	$p\ (dynam)$
1.00	0.54	(0.969)	30.93	(0.000)
2.00	0.25	(0.993)	30.14	(0.000)
5.00	0.22	(0.994)	27.96	(0.001)
10.00	0.82	(0.936)	25.32	(0.001)
15.00	1.77	(0.779)	24.63	(0.001)
30.00	32.68	(0.000)	50.65	(0.000)

Table 9: Tests for static hedging with currency forwards when there are nontraded assets

The table provides test-results for the benefits of currency hedging with currency forwards, for a US-investor that initially invests in the G5 stock markets only and that has a nontraded exposure to one of the four associated currencies. The exposure or nontraded currency position is 67% of his wealth. The first column contains the risk-aversion parameter ρ for a power-utility investor with utility function $U = \frac{W^{1-\rho}}{1-\rho}$. The next two columns show the Wald test-statistic and the associated p-value for outperformance of the G5 markets by the currency forwards. The fourth column shows the mean-variance risk aversion γ that corresponds to ρ based on Stein's lemma. The last two columns show the Wald test-statistic and the associated p-value for mean-variance intersection based on the calculated risk aversion. Returns are calculated for the period February 1977 until December 1998.

$\rho(power)$	$Wald(pow)$	$p\ (pow)$	$\gamma(MV)$	$Wald(MV)$	$p\ (MV)$
Exposure to the French Franc					
1.00	3.16	(0.531)	1.00	3.41	(0.492)
5.00	10.64	(0.031)	5.05	32.37	(0.000)
10.00	28.24	(0.000)	10.29	115.76	(0.000)
15.00	46.28	(0.000)	15.71	205.13	(0.000)
Exposure to the German Mark					
1.00	3.96	(0.412)	1.00	4.86	(0.301)
5.00	12.62	(0.013)	5.04	42.94	(0.000)
10.00	25.63	(0.000)	10.28	136.43	(0.000)
15.00	44.15	(0.000)	15.75	231.63	(0.000)
Exposure to the Japanese yen					
1.00	3.05	(0.550)	1.00	3.12	(0.538)
5.00	11.41	(0.022)	5.03	36.38	(0.000)
10.00	29.18	(0.000)	10.24	134.80	(0.000)
15.00	38.81	(0.000)	15.64	238.56	(0.000)
Exposure to the British Pound					
1.00	2.91	(0.573)	1.00	2.97	(0.563)
5.00	9.11	(0.058)	5.05	28.64	(0.000)
10.00	24.34	(0.000)	10.30	110.46	(0.000)
15.00	41.75	(0.000)	15.75	200.32	(0.000)