A NON-COOPERATIVE APPROACH TO THE COMPENSATION RULES FOR PRIMEVAL GAMES

By Yuan Ju, Peter Borm

September 2006

TILBURG ◆ UNIVERSITY

A Non-cooperative Approach to the Compensation Rules for Primeval Games[1]

Yuan Ju[2]   Peter Borm[3]

September 2006

[2]Corresponding author. School of Economic and Management Studies, Keele University, Keele, Staffordshire, ST5 5BG, UK. Tel: +44-1782-583095, Fax: +44-1782-717577, E-mail: Y.Ju@keele.ac.uk.
[3]CentER for Economic Research and Department of Econometrics and Operations Research, Tilburg University, the Netherlands.

Abstract

To model inter-individual externalities and analyze the associated compensation issue, Ju and Borm (2005) introduces a new game-theoretic framework, primeval games, and proposes, from a cooperative perspective, three compensation rules as solution concepts for primeval games: the marginalistic rule, the concession rule, and the primeval rule. In this paper, we provide a non-cooperative approach to address these problems more specifically. Inspired by the generalized bidding approach (Ju and Wettstein (2006)) for TU games, we design various bidding mechanisms to fit the model of primeval games and show that each implements the corresponding compensation rule in subgame perfect equilibrium. These mechanisms require nearly no condition on the game environment and obtain each solution itself rather than in expected terms. Moreover, since the various mechanisms share a common basic structure, this paper offers a non-cooperative benchmark to compare different axiomatic solutions, which, in return, may advance the axiomatic study of the issue by constructing alternative compensation rules.

1 Introduction

This paper provides a non-cooperative game theoretic approach to analyze the compensation issue in environments features by inter-individual externalities.

A negative externality arises when an (economic) agent undertakes an action that has an effect that turns out to be a cost imposed on another agent. When agents benefit from an activity in which they are not directly involved, the effect is called a positive externality. An associated fundamental question in real life is how to compensate the losses incurred by the negative externalities. Despite relatively less attention, the issue of paying for positive externalities is not trivial, as suggested by free-rider problems.

In economics, the issue of externality has been studied, to name a few, by Pigou (1920), Coase (1960)), Arrow (1970), and Varian (1994), which mainly aims to attain efficiency rather than focusing on normative standards.

The game theory literature on externality begins with Thrall and Lucas (1963) by the concept of *partition function form games*: a partition function assigns a value to each pair consisting of a coalition and a coalition structure which includes that coalition. Therefore, solving an externality-incurred compensation problem boils down to recommending rules or solutions for such games, which stresses the normative aspects of the issue. Some existing solution concepts can be found, among others, in Myerson (1977), Bolger (1986), Feldman (1994), Potter (2000), Pham Do and Norde (2002), Maskin (2003), Macho-Stadler, Pérez-Castrillo, and Wettstein (2004), and Ju (2004).

As one may observe, however, the framework of partition function form games does not model the externalities among individuals but restrict to specific coalitional effects. This is due to the fact, as is argued in Ju and Borm (2005), that both partition function form games and the standard TU (transferable utility) games always assume that all players in the player set N are present even if they do not form a coalition.

To deal with this open problem, i.e., how to compensate agents in the context of inter-individual externalities, Ju and Borm (2005) constructs a new class of games, *primeval games*. By considering a player's initial situation (no other players, in an absolute stand-alone sense) and other similar situations where only a subgroup of players are present (being active), primeval games model the externalities among individual players in all possible cases with respect to the active players. Employing an axiomatic approach, Ju and Borm (2005) proposes three compensation rules: the marginalistic rule, the concession rule, and the primeval rule, which may serve as specific benchmarks to solve the externality-associated compensation problems. Ju and Borm (2005) further characterizes the marginalistic rule and the concession rule. By discussing several desirable properties and comparing it with the first two rules, they argue that the primeval rule is more promis-

ing in the context of primeval games although a full axiomatic characterization remains open.

In this paper, we study the compensation problem within the framework of primeval games from a non-cooperative perspective. Inspired by the bidding mechanism first introduced by Pérez-Castrillo and Wettstein (2001) and the idea of generalized bidding approach (cf. Ju and Wettstein (2006)) to implement cooperative solutions for TU games, we construct a non-cooperative foundation to the above three compensation rules by designing various bidding mechanisms, adapting to primeval games, that differ in the role played (that is, the responsibility for externality effects assumed) by the proposer chosen through a bidding process. A desirable feature of these mechanisms is that they require nearly no condition on the game environment. Furthermore, these mechanisms obtain in subgame perfect equilibrium the prescribed outcome of each compensation rule itself rather than in expected terms.

To employ such a strategic approach not only helps to make the analysis of the compensation issue more specific, but also highlights the different "non-cooperative" rationales underlying the various compensation rules. Since these bidding mechanisms share a similar basic structure and same spirit in the game design but vary in details according to the specific compensation rule being implemented, they constitute a consistent benchmark for analyzing and comparing different normative solutions for primeval games from a strategic point of view.

In addition to this section introducing the paper briefly, the remaining part has the following structure. The next section presents the general model of primeval games and the compensation rules to be implemented. In Section 3, we describe the various bidding mechanisms and show that they implement the different compensation rules in subgame perfect equilibrium. The final section concludes the paper by briefly discussing the possible extensions of the mechanisms, which offers a direction to find new compensation rules.

2 Primeval games and the compensation rules

A primeval game, according to Ju and Borm (2005), is defined as follows. Let $N = \{1, 2, ..., n\}$ be the finite set of players. A subset S of N is called a *group of individuals* (in short, a group[1] S). A pair (i, S) that consists of a player i and a group S of N to which i belongs is called an *embedded player* in S. The set of embedded players is denoted by $\mathcal{E}(N) = \left\{ (i, S) \in N \times 2^N | i \in S \right\}$. A mapping $u : \mathcal{E}(N) \longrightarrow \mathbb{R}$ that assigns a real

---

[1]Here the term of group is used in order to be distinguished from the usual concept of coalition in the framework of TU games.

value $u(i, S)$ to each embedded player (i, S) is an *individual-group function* or a *primeval function*. The ordered pair (N, u) is called a *primeval game*. The set of primeval games with player set N is denoted by PRI^N.

The value $u(i, S)$ represents the payoff, or utility, of player i, given that all players in S are present while all players in $N \setminus S$ are absent. Hence, the model of primeval games does not consider the phenomenon of cooperation and, therefore, the individual numbers with respect to subgroups are not the result of internal negotiations among the players involved: They just model the consequences of individual externalities due to the presence of others. For a given group S and an individual-group function u, let $\bar{u}(S)$ denote the vector $(u(i, S))_{i \in S}$. We call $\bar{u}(N)$ the *status quo* of a primeval game u, and $u(i, \{i\})$ the absolute stand-alone payoff, or the *Rubinson Crusoe payoff* (in short, R-C payoff) of player i in game u.

The model of primeval games assumes that all players have the right to be in a game, which, however, does not necessarily mean that a player has the right to affect the others. Therefore, $\bar{u}(N)$ is the situation in question within the context of primeval games: Players have to accommodate each other but may not be satisfied with the status quo due to the presence of externalities. To smooth out the possible conflicts, allowing players to make compensations according to a reasonable rule would help.

A (compensation) rule on PRI^N is a function f, which associates with each primeval game (N, u) in PRI^N a vector $f(N, u) = (f_i(N, u))_{i \in N} \in \mathbb{R}^N$ of individual payoffs.

Following an axiomatic approach, Ju and Borm (2005) proposes and analyzes three compensation rules, the marginalistic rule, the concession rule, and the primeval rule, to solve the externality associated compensation problem.

For a primeval game $u \in PRI^N$, let $\Pi(N)$ be the set of all bijections $\sigma : \{1, ..., |N|\} \longrightarrow N$. For a given $\sigma \in \Pi(N)$ and $k \in \{1, ..., |N|\}$ we define $S_k^\sigma = \{\sigma(1), ..., \sigma(k)\}$ and $S_0^\sigma = \emptyset$. The *marginalistic rule* $\Phi(u)$ is defined as the average of the marginal vectors, i.e.,

$$\Phi(u) = \frac{1}{|N|!} \sum_{\sigma \in \Pi(N)} m^\sigma(u),$$

where the marginal vector $m^\sigma(u)$ is the vector in $\mathbb{R}^N$ defined by

$$m_{\sigma(k)}^\sigma(u) = \begin{cases} u(\sigma(1), \{\sigma(1)\}) & \text{if } k = 1 \\ u(\sigma(k), S_k^\sigma) + \sum_{j=1}^{k-1} \left(u(\sigma(j), S_k^\sigma) - u(\sigma(j), S_{k-1}^\sigma) \right) & \text{if } k \in \{2, ..., |N|\}. \end{cases}$$

To introduce the concession rule, we first define player $\sigma(k)$'s *concession payoff for the*

externalities on previous players as

$$\mathcal{P}^{\sigma}_{\sigma(k)}(u) = \sum_{j=1}^{k-1} \frac{u(\sigma(j), S^{\sigma}_k) - u(\sigma(j), S^{\sigma}_{k-1})}{2}$$

and his *concession payoff from the subsequent externalities* as

$$\mathcal{S}^{\sigma}_{\sigma(k)}(u) = \sum_{l=k+1}^{|N|} \frac{u(\sigma(k), S^{\sigma}_l) - u(\sigma(k), S^{\sigma}_{l-1})}{2}.$$

Then, we introduce the concession vector which is the vector in $\mathbb{R}^N$ defined by

$$C^{\sigma}_{\sigma(k)}(u) = \begin{cases} u(\sigma(1), \{\sigma(1)\}) + \mathcal{S}^{\sigma}_{\sigma(1)}(u) & \text{if } k = 1 \\ u(\sigma(k), S^{\sigma}_k) + \mathcal{P}^{\sigma}_{\sigma(k)}(u) + \mathcal{S}^{\sigma}_{\sigma(k)}(u) & \text{if } k = \{2, ..., |N| - 1\} \\ u(\sigma(|N|), N) + \mathcal{P}^{\sigma}_{\sigma(|N|)}(u) & \text{if } k = |N|. \end{cases}$$

The *concession rule* $\mathcal{C}(u)$ is defined as the average of the concession vectors, i.e.,

$$\mathcal{C}(u) = \frac{1}{|N|!} \sum_{\sigma \in \Pi(N)} C^{\sigma}(u).$$

Ju and Borm (2005) shows that the outcome prescribed by the concession rule turns out to be the average of the status quo payoff vector and the outcome of the marginalistic rule. That is, for any game $u \in PRI^N$,

$$\mathcal{C}_i(u) = \frac{1}{2} u(i, N) + \frac{1}{2} \Phi_i(u) \tag{1}$$

for all $i \in N$.

To introduce the primeval rule, we define player $\sigma(k)$'s *loss for compensating negative externalities* as

$$L^{\sigma}_{\sigma(k)}(u) = \sum_{j=1}^{k-1} \max \left\{ u(\sigma(j), S^{\sigma}_{k-1}) - u(\sigma(j), S^{\sigma}_k), 0 \right\}$$

and his *gain from subsequent positive externalities* as

$$G^{\sigma}_{\sigma(k)}(u) = \sum_{l=k+1}^{|N|} \max \left\{ u(\sigma(k), S^{\sigma}_l) - u(\sigma(k), S^{\sigma}_{l-1}), 0 \right\}.$$

Then, we define the primeval vector $B^{\sigma}(u)$, a vector in $\mathbb{R}^N$, by

$$B^{\sigma}_{\sigma(k)}(u) = \begin{cases} u(\sigma(1), \{\sigma(1)\}) + G^{\sigma}_{\sigma(1)}(u) & \text{if } k = 1 \\ u(\sigma(k), S^{\sigma}_k) - L^{\sigma}_{\sigma(k)}(u) + G^{\sigma}_{\sigma(k)}(u) & \text{if } k \in \{2, ..., |N| - 1\} \\ u(\sigma(|N|), N) - L^{\sigma}_{\sigma(|N|)}(u) & \text{if } k = |N|. \end{cases}$$

The *primeval rule* $\zeta(u)$ is defined as the average of the primeval vectors, i.e.,

$$\zeta(u) = \frac{1}{|N|!} \sum_{\sigma \in \Pi(N)} B^{\sigma}(u).$$

3 Implementing the compensation rules by bidding mechanisms

In this section, we will study the three compensation rules from a strategic perspective, inspired by the generalized bidding approach to construct the non-cooperative foundation for various cooperative solution concepts for TU games as proposed by Ju and Wettstein (2006). Below we will introduce three main bidding mechanisms and show that each implements in subgame perfect equilibrium (SPE) a specific compensation rule defined in the above section.

The basic *bidding mechanism* can be described informally as follows: At stage 1 the players bid to choose a proposer. Each player bids by submitting an $(|N| - 1)$-tuple of numbers (positive or negative), one number for each player (excluding herself). The player for whom the net bid (the difference between the sum of bids made by the player and the sum of bids the other players made to her) is the highest is chosen as the proposer. Before moving to stage 2, the proposer pays to each player the bid she made. So at this stage, the net bid is used to measure a player' willingness to become the proposer (and therefore, to measure the willingness to make a proposal how to solve the compensation problem). As a reward to the chosen proposer for her effort (represented by her net bid), she has the right to make a scheme how to compensate among all the players in the next stage. At stage 2 the proposer makes such a scheme, i.e., offers a vector of payments to all other players. The offer is accepted if all the other players agree. In case of acceptance the game stops such that the proposer collects $\sum_{i \in N} u(i, N)$ and pays out the offers made. In case of rejection all the players other than the proposer play a new game which has the same structure and rule as the previous one whereas the only difference is that the remaining players bargain over their prescribed payoffs in the situation where the proposer does not exist, and the proposer gets her *status quo* payoff and pays all other players their payoff differences in these two situations, i.e., with the proposer and without the proposer, so as to compensate the externality effects caused by the existence of the proposer. Hence, the key feature of this mechanism is that when the offer is rejected, the proposer is required to assume the full responsibility for any externality, no matter whether it is negative or positive, on all other players to make sure that they get what they can obtain as if in the situation without the proposer.

We now formally describe the bidding mechanism (game) that implements the marginalistic rule.[2]

---

[2] Because of the relationship between the marginalistic rule and the Shapley value as noted by Ju and Borm (2005), one can find the similarity between this mechanism and the one implementing the Shapley

Mechanism A. If there is only one player $\{i\}$, she receives her R-C payoff, $u(i, \{i\})$. When there are two or more players, the mechanism is defined recursively. Given the rules of the mechanism for at most $|N| - 1$ players, the mechanism for $N = \{1, \ldots, n\}$ proceeds in three stages.

Stage 1: Each player $i \in N$ makes $|N| - 1$ bids $b_j^i \in \mathbb{R}$ with $j \neq i$. Hence, at this stage, a strategy for player i is a vector $(b_j^i)_{j \neq i}$.

For each $i \in N$, define the *net bid* to player i by $\mathcal{B}^i = \sum_{j \neq i} b_j^i - \sum_{j \neq i} b_i^j$. Let $i^* = \mathrm{argmax}_i(\mathcal{B}^i)$ where an arbitrary tie-breaking rule is used in case of a non-unique maximizer. Once the winner i^* has been chosen, player i^* pays every player $j \in N\backslash\{i^*\}$, $b_j^{i^*}$.

Stage 2: Player i^* makes a vector of offers $x_j^{i^*} \in \mathbb{R}$ to every player $j \in N\backslash\{i^*\}$.

Stage 3: The players other than i^*, sequentially, either accept or reject the offer. If a player rejects it, then the offer is rejected. Otherwise, the offer is accepted.

If the offer is accepted, which means that all players agree with the proposer on the scheme of compensating each other, then each player $j \in N\backslash\{i^*\}$ receives $x_j^{i^*}$ at this stage, and player i^* receives $\sum_{k \in N} u(k, N) - \sum_{j \neq i^*} x_j^{i^*}$. Hence, the final payoff to player $j \neq i^*$ is $x_j^{i^*} + b_j^{i^*}$; player i^* receives $\sum_{k \in N} u(k, N) - \sum_{j \neq i^*} x_j^{i^*} - \sum_{j \neq i^*} b_j^{i^*}$.

If the offer of the proposer i^* is rejected, i^* is requested to leave the game with her payoff $u(i^*, N)$ but pay each of the other players the difference between his or her payoff in the situation where i^* does not exist and the current payoff, i.e., $u(j, N\backslash\{i^*\}) - u(j, N)$ for all $j \in N\backslash\{i^*\}$. Meanwhile, all players other than i^* proceed to play a similar game with one player less, i.e., with the set of players $N\backslash\{i^*\}$ and the *status quo* as $\bar{u}(N\backslash\{i^*\})$. Formally, this game is defined as $(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$ where $u|_{N\backslash\{i^*\}}(j, S) = u(j, S)$ for all $S \subset N\backslash\{i^*\}$ and $j \in S$. Thus, player i^* receives $u(i^*, N) + \sum_{j \neq i^*} (u(j, N) - u(j, N\backslash\{i^*\}))$ from this stage. The final payoff to player i^* is then $u(i^*, N) + \sum_{j \neq i^*} (u(j, N) - u(j, N\backslash\{i^*\})) - \sum_{j \neq i^*} b_j^{i^*}$. The final payoff to any player $j \neq i^*$ is the payoff he obtains in the game played by $N\backslash\{i^*\}$ plus the bid $b_j^{i^*}$.

Below we show that for any primeval game, the subgame perfect equilibrium outcomes of Mechanism A coincide with the payoff vector as prescribed by the marginalistic rule.

---

value in Ju and Wettstein (2006). However, since the marginalistic rule has a different interpretation and due to the fact that no counterparts of the concession rule and the primeval rule exist in TU games, it is necessary to give a full description of this mechanism and provide the complete proof of Theorem 3.1, which will also help to save spaces when analyzing the other two compensation rules.

Theorem 3.1 *Mechanism A implements in SPE the outcome prescribed by the marginalistic rule of an arbitrary primeval game (N, u).*

Proof.

The proof proceeds by induction on the number of players $|N|$. It is easy to see that the theorem holds for $|N| = 1$. We assume that it holds for all $|M| \leq |N| - 1$ and show that it is satisfied for $|N|$.

First we show that the outcome of the marginalistic rule is an SPE outcome. We explicitly construct an SPE that yields the outcome prescribed by the marginalistic rule as an SPE outcome. Consider the following strategies:

At stage 1, each player i, $i \in N$, announces $b_j^i = \Phi_j(N, u) - \Phi_j(N\setminus\{i\}, u|_{N\setminus\{i\}})$, for every $j \in N\setminus\{i\}$.

At stage 2, a proposer, player i^*, offers $x_j^{i^*} = \Phi_j(N\setminus\{i^*\}, u|_{N\setminus\{i^*\}})$ to every $j \in N\setminus\{i^*\}$.

At stage 3, any player $j \in N\setminus\{i^*\}$ accepts any offer which is greater than or equal to $\Phi_j(N\setminus\{i^*\}, u|_{N\setminus\{i^*\}})$ and rejects any offer strictly less than $\Phi_j(N\setminus\{i^*\}, u|_{N\setminus\{i^*\}})$.

Clearly these strategies yield the marginalistic rule outcome for any player who is not the proposer, since $b_j^{i^*} + x_j^{i^*} = \Phi_j(N, u)$, for all $j \neq i^*$. Moreover, given that following the strategies the offer is accepted by all players, the proposer also obtains her payoff specified by the marginalistic rule.

Here we want note that all net bids $\mathcal{B}^i$ equal to zero because for all $i, j \in N$

$$\Phi_i(N, u) - \Phi_i(N\setminus\{j\}, u|_{N\setminus\{j\}}) = \Phi_j(N, u) - \Phi_j(N\setminus\{i\}, u|_{N\setminus\{i\}}) \tag{2}$$

which can be readily verified along the same lines to prove the balanced contribution property (see Myerson (1980)) of the Shapley value for TU games.

To show that the previous strategies constitute an SPE, note first that the strategies at stages 2 and 3 are best responses:[3] In case of rejection a proposer i^* obtains $u(i^*, N) + \sum_{j \neq i^*} (u(j, N) - u(j, N\setminus\{i^*\}))$ and all other players play the bidding mechanism with player set $N\setminus\{i^*\}$. By the induction hypothesis, we have the marginalistic rule outcome as the equilibrium outcome of this game, i.e., each player $j \in N\setminus\{i^*\}$ receiving $\Phi_j(N\setminus\{i^*\}, u|_{N\setminus\{i^*\}})$. Consider now the strategies at stage 1. If a player i increases her total bid, then she will be chosen as the proposer with certainty, but her payoff will decrease. If she decreases her total bid another player will propose and player i's payoff would still

---

[3]We want to note that this result holds for any arbitrary primeval game (N, u) and does not require any special condition like the zero-monotonicity on the game environment.

equal to her payoff prescribed by the marginalistic rule. Finally, any change in her bids that leaves the total bid constant will influence the identity of the proposer but will not affect player i's payoff.

The proof that any SPE yields the marginalistic rule outcome proceeds by a series of claims.

Claim (a). In any SPE, at stage 3, all players other than the proposer i^* accept the offer if $x_j^{i^*} > \Phi_j(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$ for every $j \neq i^*$. Otherwise, if $x_j^{i^*} < \Phi_j(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$ for at least some $j \neq i^*$, then the offer is rejected.

Note that if an offer made by the proposer i^* is rejected at stage 3, by the induction hypothesis, the payoff to a player $j \neq i^*$ is $\Phi_j(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$. We denote the last player that has to decide whether to accept or reject the offer by β. If the game reaches β, i.e., there has been no previous rejection, her optimal strategy involves accepting any offer higher than $\Phi_\beta(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$ and rejecting any offer lower than $\Phi_\beta(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$. The second to last player, denoted by $\beta - 1$, anticipates the reaction of player β. So, $\beta - 1$ will accept the offer when the game reaches him with $x_{\beta-1}^{i^*} > \Phi_{\beta-1}(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$ and $x_\beta^{i^*} > \Phi_\beta(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$. If $x_{\beta-1}^{i^*} < \Phi_{\beta-1}(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$ and $x_\beta^{i^*} > \Phi_\beta(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$, player $\beta - 1$ will reject the offer. If $\beta - 1$ observes $x_\beta^{i^*} < \Phi_\beta(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$, he will be indifferent to accepting or rejecting any offer $x_{\beta-1}^{i^*}$. Following this argument till the first player, Claim (a) is constructed.

Claim (b). For the game that starts at stage 2 there exist two types of SPE. One is that at stage 2 player i^* offers $x_j^{i^*} = \Phi_j(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$ to all $j \neq i^*$ and, at stage 3, every player $j \neq i^*$ accepts any offer $x_j^{i^*} \geq \Phi_j(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$ and rejects the offer otherwise. The other is that at stage 2 the proposer offers $x_j^{i^*} \leq \Phi_j(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$ to some players $j \neq i^*$ and, at stage 3, any player $j \in N\backslash\{i^*\}$ rejects any offer $x_j^{i^*} \leq \Phi_j(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$.

To verify the first type of SPE, one can check that the proposer has no incentive to increase any offer, given that all offers are no lower than $\Phi_j(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$ for all $j \neq i^*$, to a level higher than $\Phi_l(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$ to any particular player $l \neq i^*$, and see that in all the SPE of this subgame the final payoffs to the proposer i^* and every other player $j \neq i^*$ are $\sum_{k \in N} u(k, N) - \sum_{j \in N\backslash\{i^*\}} u(j, N\backslash\{i^*\}) - \sum_{j \in N\backslash\{i^*\}} b_j^{i^*}$ and $\Phi_j(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}}) + b_j^{i^*}$, respectively. One can readily understand the second type of SPE by seeing no payoff difference, compared to the first type of SPE, actually caused on the proposer and all other

8

players when the offer is rejected following the proposed strategies.[4]

Claim (c). In any SPE, $\mathcal{B}^i = \mathcal{B}^j$ for all $i, j \in N$, and hence $\mathcal{B}^i = 0$ for all $i \in N$.

Denote $\Omega = \{i \in N | \mathcal{B}^i = \max_{j \in N}(\mathcal{B}^j)\}$. If $\Omega = N$ the claim is satisfied since $\sum_{i \in N} \mathcal{B}^i = 0$. Otherwise, we can show that any player i in Ω has the incentive to change her bids so as to decrease the sum of payments in case she wins. Furthermore, these changes can be made without altering the set Ω. Hence, the player maintains the same probability of winning and obtains a higher expected payoff. Take some player $j \notin \Omega$. Let player $i \in \Omega$ change her strategy by announcing $b_k'^i = b_k^i + \epsilon$ for all $k \in \Omega \backslash \{i\}$, and $b_j'^i = b_j^i - |\Omega|\epsilon$ for j, and $b_l'^i = b_l^i$ for all $l \notin \Omega \cup \{j\}$. Then, the new net bids are $\mathcal{B}'^i = \mathcal{B}^i - \epsilon$, $\mathcal{B}'^k = \mathcal{B}^k - \epsilon$ for all $k \in \Omega \backslash \{i\}$, $\mathcal{B}'^j = \mathcal{B}^j + |\Omega|\epsilon$ and $\mathcal{B}'^l = \mathcal{B}^l$ for all $l \notin \Omega \cup j$. If ϵ is small enough so that $\mathcal{B}^j + |\Omega|\epsilon < \mathcal{B}^i - \epsilon$, then $\mathcal{B}'^l < \mathcal{B}'^i = \mathcal{B}'^k$ for all $l \in \Omega$ (including j) and for all $k \in \Omega$. Therefore, Ω does not change. However, $\sum_{h \neq i} b_h^i - \epsilon < \sum_{h \neq i} b_h^i$.

Claim (d). In any SPE, each player's payoff is the same regardless of whom is chosen as the proposer.

This claim can be readily proved by contradiction. If some player can get extra payoff given a specific identity of the proposer, then this player will have incentive to adjust her bids accordingly, which contradicts Claim (c).

Claim (e) In any SPE, the final payment received by each of the players coincides with the payoff prescribed by the marginalistic rule of the game.

We know that player i's final payoff will be $\sum_{k \in N} u(k, N) - \sum_{j \in N \backslash \{i\}} u(j, N \backslash \{i\}) - \sum_{j \neq i} b_j^i$ if she is the proposer, and will be $\Phi_i(N \backslash \{j\}, u|_{N \backslash \{j\}}) + b_i^j$ in case of player $j \neq i$ becoming the proposer. Then, the sum of the payoffs to player i over all possible choices of the proposer is

$$\left(\sum_{k \in N} u(k, N) - \sum_{j \in N \backslash \{i\}} u(j, N \backslash \{i\}) - \sum_{j \neq i} b_j^i \right) + \sum_{j \in N \backslash \{i\}} \left(\Phi_i(N \backslash \{j\}, u|_{N \backslash \{j\}}) + b_i^j \right)$$

$$= \sum_{k \in N} u(k, N) - \sum_{j \in N \backslash \{i\}} u(j, N \backslash \{i\}) + \sum_{j \in N \backslash \{i\}} \Phi_i(N \backslash \{j\}, u|_{N \backslash \{j\}}) - \mathcal{B}^i$$

---

[4]Note that the first type of SPE implies an agreement will form among all players in N whereas no such an agreement will emerge in the second type of SPE. However, each player's final payoff remains the same in both type of SPE.

which, by Equation (2) and the fact that $\mathcal{B}^i = 0$, equals $|N| \cdot \Phi_i(N, u)$. What remains is obvious due to Claim (d). ∎

As one can see, in order to obtain the marginalistic rule outcome in SPE the above mechanism takes a rather extreme treatment on the proposer and the remaining players with respect to the responsibilities of externalities: It requires the proposer chosen through the bidding stage to fully assume the responsibility of the externalities (i.e., pays the other players if she causes negative externalities to them and receives from the others if she imposes positive externalities to them) in case her offer is rejected.

Then, we might be curious about the possible result of an opposite argument: In case of rejection the proposer is completely free from the responsibility for the externalities but simply gets her *status quo* payoff and all other players continue in a similar fashion. This leads to the following mechanism.

Mechanism B. This mechanism is the same as Mechanism A except the rule when an offer is rejected at stage 3. Here, if the offer is rejected, the proposer i^* leaves the game with her *status quo* payoff $u(i^*, N)$ from this stage, which implies that the proposer is not supposed to be responsible for the externalities on the other players. Consequently, any player $j \neq i^*$ will have to receive the externality $u(j, N) - u(j, N \backslash \{i^*\})$ besides the payoff $u(j, N \backslash \{i^*\})$. That is, in case of proposer i^*'s offer being rejected, any player $j \neq i^*$ will still get $u(j, N)$. Thus, all players other than i^* proceed to play the subgame $(N \backslash \{i^*\}, u^{-i^*})$ defined by $u^{-i^*}(j, S) = u(j, N)$ for $S = N \backslash \{i^*\}$ and for all $j \in S$, and $u^{-i^*}(j, S) = u(j, S)$ for all $S \subsetneqq N \backslash \{i^*\}$ and $j \in S$. The final payoff to player i^* is then $u(i^*, N) - \sum_{j \neq i^*} b_j^{i^*}$. The final payoff to any player $j \neq i^*$ is the payoff he obtains in the game $(N \backslash \{i^*\}, u^{-i^*})$ plus the bid $b_j^{i^*}$.

Lemma 3.2 *Mechanism B implements in SPE the status quo payoff vector of an arbitrary primeval game (N, u).*

Proof. The proof can be constructed along the same lines of that of Theorem 3.1. The main difference lies in the construction of an SPE that yields the *status quo* payoff vector as an SPE outcome:

At stage 1, each player $i \in N$ announces $b_j^i = u(j, N) - u(j, N) = 0$, for every $j \in N \backslash \{i\}$. At stage 2, a proposer, player i^*, offers $x_j^{i^*} = u(j, N)$ to every $j \in N \backslash \{i\}$.

At stage 3, any player $j \in N \backslash \{i^*\}$ accepts any offer greater than or equal to $u(j, N)$ and rejects any offer strictly smaller than $u(j, N)$.

One can readily verify that these strategies yield the *status quo* payoff vector and they constitute an SPE. ∎

Comparing Mechanism A and Mechanism B, one can find that both of them take an extreme treatment in terms of externality responsibility on the proposer (then also on the other players) in case her offer is rejected. Then, making a "fair" compromise between the two seems reasonable and might be practical in reality, which leads to the following option.

Mechanism C. The mechanism is the same as Mechanism A except for the details related to the case when an offer is rejected at stage 3. Now, suppose an offer is rejected at stage 3. Then, the proposer i^* leaves the game and receives her status quo payoff $u(i^*, N)$ plus half of the externality effect of all other players, i.e., $\frac{1}{2} \sum_{j \neq i^*} (u(j, N) - u(j, N \backslash \{i^*\}))$, from this stage. Meanwhile, each of the other players firstly get half of the externalities, i.e., $\frac{1}{2} (u(j, N) - u(j, N \backslash \{i^*\}))$ for all $j \neq i^*$, and then all of them proceed to play in the same way a subgame $(N \backslash \{i^*\}, u|_{N \backslash \{i^*\}})$. Hence, the final payoff to player i^* is $u(i^*, N) + \frac{1}{2} \sum_{j \neq i^*} (u(j, N) - u(j, N \backslash \{i^*\})) - \sum_{j \neq i^*} b_j^{i^*}$, and the final payoff to any player $j \neq i^*$ is the payoff he obtains in the game $(N \backslash \{i^*\}, u|_{N \backslash \{i^*\}})$ plus half of the externality generated by i^*, i.e., $\frac{1}{2} (u(j, N) - u(j, N \backslash \{i^*\}))$, and the bid $b_j^{i^*}$. To make the rule clearer, suppose that the next offer by j^* from the set of players $N \backslash \{i^*\}$ is also rejected. Then, j^* will get $u(j^*, N \backslash \{i^*\})$ plus the half externality from i^*, i.e., $\frac{1}{2} (u(j^*, N) - u(j^*, N \backslash \{i^*\}))$ and half of the externality effect to all other players $\frac{1}{2} \sum_{k \in N \backslash \{i^*, j^*\}} (u(k, N \backslash \{i^*\}) - u(k, N \backslash \{i^*, j^*\}))$.

Theorem 3.3 *Mechanism C implements in SPE the outcome prescribed by the concession rule of an arbitrary primeval game (N, u).*

Proof. The proof is also analogous to that of Theorem 3.1. Therefore, below we only explicitly construct an SPE that yields the concession rule outcome as an SPE outcome for illustration:

At stage 1, each player $i \in N$, announces

$$ b_j^i = \mathcal{C}_j(N, u) - \left(\mathcal{C}_j(N \backslash \{i\}, u|_{N \backslash \{i\}}) + \frac{1}{2} (u(j, N) - u(j, N \backslash \{i\})) \right) $$

for every $j \in N\backslash\{i\}$.[5]

At stage 2, a proposer, player i^*, offers

$$x_j^{i^*} = \mathcal{C}_j(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}}) + \frac{1}{2}(u(j, N) - u(j, N\backslash\{i^*\}))$$

to every $j \in N\backslash\{i\}$.

At stage 3, any player $j \in N\backslash\{i^*\}$ accepts any offer greater than or equal to

$$\mathcal{C}_j(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}}) + \frac{1}{2}(u(j, N) - u(j, N\backslash\{i^*\}))$$

and rejects otherwise. ∎

As is argued in Ju and Borm (2005), the primeval rule seems to fit the framework of primeval games best. This can also be verified by the bidding approach in this paper. For example, one might see a difficulty in applying Mechanism A into reality. That is, when an offer is rejected, the proposer i^* will take the full responsibility for the externalities. It is easy to accept that the proposer will compensate the negative externalities to the other players. However, one might find hard to force the others to transfer the payoffs incurred by the proposer's positive externalities back to her. Although the non-proposer players will not pay such transfers to the proposer by Mechanism B, they will not be compensated for negative externalities by the proposer either. To a certain extent, the concession rule may help to overcome this problem as it makes a compromise. However, it seems more desirable to have a mechanism that in case of an offer being rejected the proposer will compensate the others for negative externalities but the other need not give back the benefits from the positive externalities to her, which might be well accepted and applied in practice.

The following mechanism, which looks more complicated, indeed adopts the above idea and implements the primeval rule.

Mechanism D. When an offer is rejected at stage 3, the proposer i^* will leave the game whereas all remaining players proceed. In more detail, any player $j \in N\backslash\{i^*\}$ will get $\max\{u(j, N) - u(j, N\backslash\{i^*\}), 0\}$ and then get the outcome of the subgame $(N\backslash\{i^*\}, u|_{N\backslash\{i^*\}})$ played by the remaining players $N\backslash\{i^*\}$ using the same rule as for the case with player set N. Thus, the proposer i^* leaves the game and receives $u(i^*, N) + \sum_{j \in N\backslash\{i^*\}} \min\{u(j, N) - u(j, N\backslash\{i^*\}), 0\}$ from this stage. To further illustrate the mechanism. Suppose that the next offer by the second proposer j^* is rejected again. Then, any player $k \in N\backslash\{i^*, j^*\}$ gets

---

[5]Note that by Equation (1) one can simplify the equilibrium bid to be $b_j^i = \frac{1}{2}\left(\Phi_j(N, u) - \Phi_j(N\backslash\{i\}, u|_{N\backslash\{i\}})\right)$. Then, together with Equation (2), one can readily check that all net bids equal to zero.

$\max\{u(k, N) - u(k, N\backslash\{i^*\}), 0\} + \max\{u(k, N\backslash\{i^*\}) - u(k, N\backslash\{i^*, j^*\}), 0\}$ plus the outcome of the subgame $(N\backslash\{i^*, j^*\}, u|_{N\backslash\{i^*, j^*\}})$ whereas the proposer j^* leaves the game and receives $u(j^*, N\backslash\{i^*\}) + \max\{u(j^*, N) - u(j^*, N\backslash\{i^*\}), 0\} + \sum_{k \in N\backslash\{i^*, j^*\}} \min\{u(k, N\backslash\{i^*\}) - u(k, N\backslash\{i^*, j^*\}), 0\}$ from this stage.

Lemma 3.4 *For any game $u \in PRI^N$ we have*

$$\sum_{j \in N\backslash\{i\}} \zeta_j(u) - \sum_{j \in N\backslash\{i\}} \left(\zeta_j(u|_{N\backslash\{i\}}) + \max\{u(j, N) - u(j, N\backslash\{i\}), 0\} \right)$$

$$= (|N| - 1)\zeta_i(u) - \sum_{j \in N\backslash\{i\}} \left(\zeta_i(u|_{N\backslash\{j\}}) + \max\{u(i, N) - u(i, N\backslash\{j\}), 0\} \right)$$

for all $i, j \in N$.

Proof. By efficiency of the primeval rule, we know that $\sum_{j \in N\backslash\{i\}} \zeta_j(u) = \sum_{k \in N} u(k, N) - \zeta_i(u)$ and $\sum_{j \in N\backslash\{i\}} \zeta_j(u|_{N\backslash\{i\}}) = \sum_{j \in N\backslash\{i\}} u(j, u|_{N\backslash\{i\}})$. Therefore, it suffices to show that

$$\sum_{k \in N} u(k, N) - |N|\zeta_i(u)$$

$$= \sum_{j \in N\backslash\{i\}} u(j, u|_{N\backslash\{i\}}) + \sum_{j \in N\backslash\{i\}} \max\{u(j, N) - u(j, N\backslash\{i\}), 0\}$$

$$- \sum_{j \in N\backslash\{i\}} \left(\zeta_i(u|_{N\backslash\{j\}}) + \max\{u(i, N) - u(i, N\backslash\{j\}), 0\} \right).$$

By definition of the primeval rule, we have that

$$|N|\zeta_i(u) = \frac{1}{(|N| - 1)!} \sum_{\sigma \in \Pi(N)} B_i^\sigma(u)$$

$$= \frac{1}{(|N| - 1)!} \sum_{\sigma \in \Pi(N),\ \sigma(|N|)=i} B_i^\sigma(u) + \frac{1}{(|N| - 1)!} \sum_{\sigma \in \Pi(N),\ \sigma(|N|)\neq i} B_i^\sigma(u).$$

Then, one can readily verify that

$$\sum_{k \in N} u(k, N) - \frac{1}{(|N| - 1)!} \sum_{\sigma \in \Pi(N),\ \sigma(|N|)=i} B_i^\sigma(u)$$

$$= \sum_{j \in N\backslash\{i\}} u(j, u|_{N\backslash\{i\}}) + \sum_{j \in N\backslash\{i\}} \max\{u(j, N) - u(j, N\backslash\{i\}), 0\}.$$

Thus, it remains to show that

$$\frac{1}{(|N| - 1)!} \sum_{\sigma \in \Pi(N),\ \sigma(|N|)\neq i} B_i^\sigma(u) = \sum_{j \in N\backslash\{i\}} \left(\zeta_i(u|_{N\backslash\{j\}}) + \max\{u(i, N) - u(i, N\backslash\{j\}), 0\} \right).$$

13

This follows from the fact that

$$\frac{1}{(|N|-1)!} \sum_{\sigma \in \Pi(N),\ \sigma(|N|) \neq i} B_i^\sigma(u)$$

$$= \sum_{j \in N\setminus\{i\}} \left(\frac{1}{(|N|-1)!} \sum_{\sigma \in \Pi(N\setminus\{j\})} \left(B_i^\sigma(u|_{N\setminus\{j\}}) + \max\{u(i,N) - u(i,N\setminus\{j\}), 0\} \right) \right)$$

$$= \sum_{j \in N\setminus\{i\}} \left(\zeta_i(u|_{N\setminus\{j\}}) + \max\{u(i,N) - u(i,N\setminus\{j\}), 0\} \right).$$

∎

Theorem 3.5 *Mechanism D implements in SPE the outcome prescribed by the primeval rule of an arbitrary primeval game (N, u).*

Proof.

The proof is, again, analogous to that for Theorem 3.1. the difference lies in the construction of the SPE strategies and in Claim (e). Here, we explicitly construct an SPE that yields the primeval rule outcome and show that the Claim (e) (that payoffs must coincide with the payoffs prescribed by the primeval rule) holds as well.

To construct an SPE yielding the primeval rule outcome consider the following strategies.

At stage 1, each player $i \in N$ announces

$$b_j^i = \zeta_j(N, u) - \left(\zeta_j(N\setminus\{i\}, u|_{N\setminus\{i\}}) + \max\{u(j,N) - u(j,N\setminus\{i\}), 0\} \right),$$

for every $j \in N\setminus\{i\}$.

At stage 2, a proposer, player i^*, offers

$$x_j^{i^*} = \zeta_j(N\setminus\{i^*\}, u|_{N\setminus\{i^*\}}) + \max\{u(j,N) - u(j,N\setminus\{i^*\}), 0\}$$

to every $j \in N\setminus\{i\}$.

At stage 3, any player $j \in N\setminus\{i\}$ accepts any offer that is greater than or equal to $\zeta_j(N\setminus\{i^*\}, u|_{N\setminus\{i^*\}}) + \max\{u(j,N) - u(j,N\setminus\{i^*\}), 0\}$ and rejects any offer strictly smaller than $\zeta_j(N\setminus\{i^*\}, u|_{N\setminus\{i^*\}}) + \max\{u(j,N) - u(j,N\setminus\{i^*\}), 0\}$.

To show that in any SPE each player's final payoff must coincide with her payoff prescribed by the primeval rule, we note that if i is the proposer her final payoff is given by

$$\sum_{k \in N} u(k,N) - \left(\sum_{j \in N\setminus\{i\}} u(j,N\setminus\{i\}) + \sum_{j \in N\setminus\{i\}} \max\{u(j,N) - u(j,N\setminus\{i\}), 0\} \right) - \sum_{j \neq i} b_j^i,$$

and in case of player $j \neq i$ becoming the proposer her final payoff will be

$$\zeta_i(N\backslash\{j\}, u|_{N\backslash\{j\}}) + \max\{u(i, N) - u(i, N\backslash\{j\}), 0\} + b_i^j.$$

Then, the sum of the payoffs to player i over all possible choices of the proposer is

$$\sum_{k \in N} u(k, N) - \left(\sum_{j \in N\backslash\{i\}} u(j, N\backslash\{i\}) + \sum_{j \in N\backslash\{i\}} \max\{u(j, N) - u(j, N\backslash\{i\}), 0\} \right) - \sum_{j \neq i} b_j^i$$
$$+ \sum_{j \in N\backslash\{i\}} \left(\zeta_i(N\backslash\{j\}, u|_{N\backslash\{j\}}) + \max\{u(i, N) - u(i, N\backslash\{j\}), 0\} + b_i^j \right)$$
$$= \sum_{k \in N} u(k, N) - \sum_{j \in N\backslash\{i\}} u(j, N\backslash\{i\}) - \sum_{j \in N\backslash\{i\}} \max\{u(j, N) - u(j, N\backslash\{i\}), 0\}$$
$$+ \sum_{j \in N\backslash\{i\}} \left(\zeta_i(N\backslash\{j\}, u|_{N\backslash\{j\}}) + \max\{u(i, N) - u(i, N\backslash\{j\}), 0\} \right) - \mathcal{B}^i$$

which, by the fact that $\mathcal{B}^i = 0$ and Lemma 3.4, can be shown to equal $|N| \cdot \zeta_i(N, u)$. What remains is obvious due to Claim (d). ∎

Below we provide an extension on the game design to implement these compensation rules. In the mechanism to implement the outcome prescribed by the marginalistic rule, holding other details unchanged, we reduce the payoff to the rejected proposer i^* from $u(i^*, N) + \sum_{j \neq i^*} (u(j, N) - u(j, N\backslash\{i^*\}))$ to any arbitrary level $\theta^{i^*} \leq u(i^*, N) + \sum_{j \neq i^*} (u(j, N) - u(j, N\backslash\{i^*\}))$ by taking the decrease as a punishment to i^* for making an unacceptable offer, while the rest of the players still get $\sum_{j \neq i^*} (u(j, N)$ from this stage if they reach an agreement among them. A more general description is that if the game continues, after all preceding offers being rejected, to the set of players $S \subset N$, and the corresponding offer made by the proposer i_S^* chosen among S is rejected, then i_S^* will get $\theta^{i_S^*} \leq u(i_S^*, S) + \sum_{k \in S\backslash\{i_S^*\}} (u(k, S) - u(k, S\backslash\{i_S^*\}))$ from this stage and the remaining players continue in the same fashion. This mechanism also implements the marginalistic rule of an arbitrary primeval game (N, u) in SPE. However, we note that in this case, only the first type of SPE exists in Claim (b) if we have $\theta^{i^*} < u(i^*, N) + \sum_{j \neq i^*} (u(j, N) - u(j, N\backslash\{i^*\}))$. One can readily adapt this idea to other mechanisms and implement the corresponding compensation rules.

4 Concluding remarks

In this paper, we discuss a non-cooperative approach to the compensation rules for primeval games. Having the same basic bidding stage, we design different ending rules such that in

case of an offer being rejected, the proposer is required to assume different responsibilities for the externalities and make the corresponding compensations. In order to implement the marginalistic rule, we require the proposer to assume the full responsibility of the externalities, which implies that if she is rejected all other players should get what they would have as if the proposer does not exist. An opposite view will lead to implementing the *status quo* payoff vector. Making a compromise between the two results in a mechanism to implement the concession rule. Finally, the primeval rule is implemented via a mechanism following a practical principle: In case of a proposal being rejected, the proposer will have to compensate all other players if she causes negative externalities on them but cannot receive compensation from the others even if she generates positive externalities. We show that these results hold for any primeval game.

On the applied side, the mechanisms proposed in the paper can help to resolve the real-life compensation problems. Theoretically, this study highlights the difference between the compensation rules from a strategic viewpoint and makes the analysis of the issue more specific. This issue can be further addressed by constructing alternative mechanisms to implement these rules. For a primeval game with all positive externalities among the players and that their total payoffs are monotonic with respect to the size of player group, one can adapt the "re-negotiation" bidding mechanism proposed by Ju and Wettstein (2006) to obtain alternative mechanisms to implement these rules. In Ju, Borm and Ruys (2004), a two-level bidding mechanism is discussed to implement the consensus value for TU games by introducing an exogenously given probability parameter. One can also readily apply this idea to construct a mechanism to implement the concession rule.

As is seen from the above, different ending rules may result in different equilibrium outcomes. Therefore, following the basic bidding game, one can look into other reasonable possibilities in terms of externality compensation when the proposer is rejected and construct new bidding mechanisms. That would lead to new compensation rules, which, in return, calls for the axiomatic study. Hence, the approach in the paper can help to bridge the two different perspectives about the issue and gain further insights.

We want to point out that a primeval game usually assumes that the status quo, i.e., all players exist in the game, is the final state of a game. If we relax such a condition and allow players to negotiate with each other by compensation to form efficient group structures, then the existing approaches are not adequate, which suggests a future research topic.

References

[1] Arrow, K. (1970), The Organization of Economic Activity: Issues Pertinent to the Choice of Market versus Non-market Allocation, In: Haveman, R.H. and J. Margolis (Eds.), *Public Expenditures and Policy Analysis*, Chicago: Markham, pp. 59-73.

[2] Bolger, E. (1989), A Set of Axioms for a Value for Partition Function Games, *International Journal of Game Theory*, 18, 37-44.

[3] Coase, R. (1960), The Problem of Social Cost, *Journal of Law and Economics*, 3, 1-44.

[4] Feldman, B. (1994), *Value with Externalities: the Value of the Coalitional Games in Partition Function Form*, Ph.D. Thesis, Department of Economics, State University of New York, Stony Brook, NY, USA.

[5] Ju, Y. (2004), The Consensus Value for Games in Partition Function Form, *CentER Discussion Paper*, 2004-60, Tilburg University, Tilburg, the Netherlands. To appear in *International Game Theory Review*.

[6] Ju, Y. and P. Borm (2005), Externalities and Compensation: Primeval Games and Solutions, *Keele Economics Research Paper*, 2005-05, Keele University, UK.

[7] Ju, Y., P. Borm and P. Ruys (2004), The Consensus Value: a New Solution Concept for Cooperative Games, *CentER Discussion Paper*, 2004-50, Tilburg University. To appear in *Social Choice and Welfare*.

[8] Ju, Y. and D. Wettstein (2006), Implementing Cooperative Solution Concepts: a Generalized Bidding Approach, *Keele Economics Research Papers*, 2006-06, Keele University.

[9] Luce, R. and H. Raiffa (1957), *Games and Decisions: Introduction and Critical Survey*, New York: John Wiley.

[10] Macho-Stadler, I., D. Pérez-Castrillo, and D. Wettstein (2004), Sharing the Surplus: a Just and Efficient Proposal for Environments with Externalities (new title "Sharing the Surplus: an Extension of the Shapley Value for Environments with Externalities"), *CREA Working Paper*, 119, Universitat Autònoma de Barcelona, Spain. To appear in *Journal of Economic Theory*.

[11] Maskin, E. (2003), Bargaining, Coalitions, and Externalities, *Presidential Address to the Econometric Society*, Econometrc Society European Meeting, Stockholm, Sweden.

[12] Myerson, R. (1977), Values for Games in Partition Function Form, *International Journal of Game Theory*, 6, 23-31.

[13] Myerson, R. (1980), Conference Structures and Fair Allocation Rules, *International Journal of Game Theory* 9: 169-182.

[14] Pérez-Castrillo, D. and D. Wettstein (2001), Bidding for the Surplus: A Non-Cooperative Approach to the Shapley Value, *Journal of Economic Theory*, 100, 274-294.

[15] Pham Do, K.H. and H. Norde (2002), The Shapley Value for Partition Function Form Games, *CentER Discussion Paper*, No. 2002-04, Tilburg University, Tilburg, the Netherlands. To appear in *International Game Theory Review*.

[16] Pigou, A. (1920), *The Economics of Welfare*, London: Macmillan.

[17] Potter, A. (2000), A Value for Partition Function Form Games, *Working Paper*, Dept. of Mathematics, Hardin-Simmons University, Abilene, Texas, USA.

[18] Shapley, L. (1953), A Value for n-Person Games, in Kuhn, H., Tucker, A.W. (Eds.), *Contributions to the Theory of Games*, Vol. II, Princeton: Princeton University Press, pp. 307-317.

[19] Thrall, R. And W. Lucas (1963), n-Person Games in Partition Function Form, *Naval Research Logistics Quarterly*, 10, 281-293.

[20] Varian, H. (1994), A Solution to the Problem of Externalities When Agents are Well-informed, *American Economic Review*, 84, 1278-1293.