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PROCEDURAL GROUP IDENTIFICATION

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Abstract

In this paper we axiomatically characterize two recursive procedures for defining a social group. The first procedure starts with the set of all individuals who define themselves as members of the social group, while the starting point of the second procedure is the set of all individuals who are defined by everyone in the society as group members. Both procedures expand these initial sets by adding individuals

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who are considered to be appropriate group members by someone in the corresponding initial set, and continue inductively until there is no possibility of expansion any more. *Journal of Economic Literature* Classification Numbers: D63, D71.

Key Words: consensus, liberalism, procedure, social identity

1 Introduction

The problem of group identification serves as a background in many social and economic contexts. For example, when one examines the political principle of self-determination of a newly formed country, one would like to define the extension of a given nationality. Or when a newly arrived person in Atlanta chooses where to live, the person is interested in finding out a residential neighborhood that would suit her: "Are they my kind of people? Do I belong to this neighborhood?" In all those contexts, it is typically assumed that there is a well-defined group of people who share some common values, beliefs, expectations, customs, jargon, or rituals. Consequently, the questions like "how to define a social group" or "who belongs to the social group" arise.

In a recent paper, Kasher and Rubinstein (1997) provide an answer to the above questions from a social choice perspective. They view that each individual of a society has an opinion about every individual, including oneself, whether the latter is a member of a group to be formed. The collective identity of the group to be formed is then determined by aggregating opinions of all the individuals in the society. For this purpose, they provide, among others, an axiomatic characterization of a "liberal" aggregator whereby the group consists of those and only those individuals who each of them views oneself a member of the group.

The purpose of this paper is to extend the study of the group identification problem by adding a *procedural* view in the analysis. This procedural view allows us to see a collective as "a family of groups, subcollectives, each with its own view of who is a member of the collective, its own sense of tradition and its own underlying conceptual realm, but each bearing some resemblance to the other ones" (Kasher (1993, p. 70)). More specifically, we axiomatically characterize two recursive procedures for determining "who is a member of a social group": a *liberal-start-respecting* procedure which extends the "liberal" aggregator characterized by Kasher and Rubinstein (1997), and a *consensus-*

start-respecting procedure which is the one introduced by Kasher (1993).

The structure of both procedures consists of two components: an initial set of individuals and a rule according to which new individuals are added to this initial set. As the names of the procedures suggest, the initial set of the first procedure consists of all individuals who define themselves as members of the social group, while the initial set of the second procedure collects all individuals who are defined as group members by everyone in the society. The extension rule for both procedures is the same: only those individuals who are considered to be appropriate group members by someone in the corresponding initial set are added. The application of this rule continues inductively until there is no possibility of expansion any more.

The rest of the paper is organized as follows. In Section 2, we present the basic notation and definitions. Section 3 introduces the axioms used for characterization of the procedures. The main results are contained in Section 4, and Section 5 concludes with some final remarks.

2 Basic Notation and Definitions

Let $N = \{1, \ldots, n\}$ denote the set of all individuals in the society. The set of all subsets of N is denoted by $P(N)$. Each individual $i \in N$ forms a set $G_i \subseteq N$ consisting of all society members that in the view of i have the social identity G. For all $i \in N$, when $i \in G_i$, we also say that i considers himself as a G. A *profile* of views is an n-tuple of vectors $G = (G_1, \ldots, G_n)$ where $G_i \subseteq N$ for all $i \in N$. Let $\mathcal{G}$ be the set of all profiles of views, i.e. $\mathcal{G} = (P(N))^n$. A *Collective Identity Function* (CIF) $F : \mathcal{G} \rightarrow P(N)$ assigns to each profile $G \in \mathcal{G}$ a set $F(G) \subseteq N$ of socially accepted group members. Let $\mathcal{F}$ be the set of all collective identity functions.

For any $G \in \mathcal{G}$, define $L_0(G) = \{i \in N : i \in G_i\}$. Thus, $L_0(G)$ consists of all individuals in the society who consider themselves as Gs. For any $G \in \mathcal{G}$, with the help of $L_0(G)$, we now define a CIF being *liberal-start-respecting*, to be denoted by $L(G)$, as follows: for each positive integer t, let $L_t(G) = L_{t-1}(G) \cup \{i \in N : i \in G_k \text{ for some } k \in L_{t-1}(G)\}$; and if for some $t \geq 0$, $L_t(G) = L_{t+1}(G)$, then $L(G) = L_t(G)$.

For any $G \in \mathcal{G}$, define now $K_0(G) = \{i \in N : i \in G_k \text{ for all } k \in N\}$. Thus, $K_0(G)$ consists of all individuals who are considered to be Gs by everyone in the society. For any $G \in \mathcal{G}$, we define a CIF being *consensus-start-respecting*, to be denoted by $K(G)$, as follows: for each positive integer t, let

$K_t(G) = K_{t-1}(G) \cup \{i \in N : i \in G_k$ for some $k \in K_{t-1}(G)\}$; and if for some $t \geq 0$, $K_t(G) = K_{t+1}(G)$, then $K(G) = K_t(G)$.

To illustrate the above procedures for defining collectively accepted group members, consider the following example. Let $N = \{1, 2, 3\}$ and consider the profile $G = (G_1, G_2, G_3)$ with $G_1 = \{1, 2\}, G_2 = \{2, 3\}$ and $G_3 = \{2\}$. Then, for this profile, $L_0 = \{1, 2\}$, $L_1 = L_0 \cup \{3\} = \{1, 2, 3\}$, $L_2 = L_1$. Therefore, for the given profile of views, and as a result of the application of the liberal-start-respecting procedure we have $L = \{1, 2, 3\}$. For the same profile G of individual views we have $K_0 = \{2\}$, $K_1 = K_0 \cup \{3\} = \{2, 3\}$, $K_2 = K_1$. Therefore, the collectively accepted group members according to the consensus-start-respecting procedure are collected in the set $K = \{2, 3\}$.

The procedure L defined above is discussed in Kasher and Rubinstein (1997). For each $G \in \mathcal{G}$, it starts with $L_0(G)$ which consists of all members of the society who view themselves as Gs. Thus, the set $L_0(G)$ reflects a weak notion of *self-determination*: if one considers oneself a member of G, then one should be a member of G collectively[1]. In contrast, for each $G \in \mathcal{G}$, the procedure K starts with $K_0(G)$ which consists of all individuals who are viewed by everyone in the society N as group members. Kasher (1993) calls K_0 the "incontrovertible core" of a collective to be defined and he considers it as initial approximation to an appropriate definition of the group identity.

An "improved approximation" includes also the possibility of extending the above defined initial sets via an extension rule. For each $G \in \mathcal{G}$, the CIF L (resp., K) now expands the set $L_0(G)$ (resp., $K_0(G)$) as follows. If, according to some individual $i \in L_0(G)$ (resp., $i \in K_0(G)$) an individual $k \in N$ is viewed as a G, then k should be a G collectively. By adding all such ks to $L_0(G)$ (resp., to $K_0(G)$), we obtain the set $L_1(G)$ (resp., $K_1(G)$). We then repeat the above process with $L_1(G)$ (resp., with $K_1(G)$) by adding those individuals who are considered as Gs by some individuals in $L_1(G)$ (resp., in $K_1(G)$) to $L_1(G)$ (resp., to $K_1(G)$) to obtain $L_2(G)$ (resp., $K_2(G)$). Since n is finite, at a certain step t, we must have $L_t(G) = L_{t+1}(G)$ (resp., $K_t(G) = K_{t+1}(G)$), i.e. the set $L_t(G)$ (resp., $K_t(G)$) can no longer be expanded. The intuition behind each step of the expansion is in line with Kasher's argument (1993): every socially accepted G as being newly added brings a possibly unique new view of being a G collectively with him, and a collective identity function is supposed to aggregate those views and must pay attention to this new individual's G-concept in order to cover the whole

[1] In Kasher and Rubinstein (1997), the individuals in L_0 are called *liberals*.

diversity of views in the society about the question "what does it mean to be a G?".

3 Axioms

In order to present our axiomatic characterizations of the collective identity functions L and K we introduce the following axioms for a CIF to satisfy.

A CIF $F \in \mathcal{F}$ satisfies

Consensus (C) iff, for all $G \in \mathcal{G}$, $[j \in G_i$ for all $i \in N] \Rightarrow j \in F(G)$, and $[j \notin G_i$ for all $i \in N] \Rightarrow j \notin F(G)$.

Independence (I) iff, for all profiles $G, G' \in \mathcal{G}$, and all $i \in N$, if [for every $k \neq i$, $k \in F(G)$ iff $k \in F(G')$], and [for all $k \in N$, $i \in G_k$ iff $i \in G'_k$], then $i \in F(G)$ iff $i \in F(G')$.

Monotonicity (M) iff, for all $G, G' \in \mathcal{G}$ and all $N_1 \subseteq N$ such that [for all $l \in N$, $G'_l = G_l$ or $G'_l = G_l - N'$ where $N' \subseteq N_1$], if $N_1 \cap F(G) = \emptyset$, then $N_1 \cap F(G') = \emptyset$.

Symmetry (SYM) iff, for all $G \in \mathcal{G}$, for all $j, k \in N$, if (i) $\forall i \in N - \{j, k\}$, $j \in G_i$ iff $k \in G_i$; (ii) $j \in G_j$ iff $k \in G_k$; (iii) $j \in G_k$ iff $k \in G_j$, then $j \in F(G) \Leftrightarrow k \in F(G)$.

Weak Liberalism I (WL(1)) iff, for all $G \in \mathcal{G}$, if $i \in G_i$ for some $i \in N$, then $F(G) \neq \emptyset$, and if there are disjoint subsets N_1 and N_2 of N such that $N_1 \cup N_2 = N$ and $N_2 \neq \emptyset$, $[G_i \subseteq N_1$ for all $i \in N_1$ and $i \notin G_i$ for all $i \in N_2]$, then $F(G) \neq N$.

Weak Liberalism II (WL(2)) iff, for all $G \in \mathcal{G}$, if $i \notin G_i$ for all $i \in N$, then $F(G) \neq N$, and if there are disjoint subsets N_1 and N_2 of N such that $N_1 \cup N_2 = N$, $N_1 \neq \emptyset$, $N_2 \neq \emptyset$, $[G_i \subseteq N_1$ for all $i \in N_1]$, then $F(G) \neq N$.

Extended Liberalism (EL) iff, for all $G \in \mathcal{G}$ and for all $k \in N$, if $[k \in G_i$ for some $i \in F(G)]$ and $[i \neq k]$, then $F(G) \neq \{i\}$.

Equal Treatment of Insiders' Views (ETIV) iff, for all $G, G' \in \mathcal{G}$, and all $i, k, m \in N$, if $[G_h = G'_h$ for all $h \in N - \{i, k\}]$, $[m \in G_k$ and $m \notin$

G_i], [$G'_k = G_k - \{m\}$], and [$G'_i = G_i \cup \{m\}$], then [$k \in F(G)$ and $i \in F(G')$] $\Rightarrow$ [$m \in F(G)$ iff $m \in F(G')$].

Equal Treatment of Outsider's Views (ETOV) iff, for all $G, G' \in \mathcal{G}$, and all $i, k, m \in N$, if [$G_h = G'_h$ for all $h \in N - \{i, k\}$, $m \in (G_i \cap G'_k)$, and $m \notin (G_k \cup G'_i)$, then [$\{i, k\} \cap F(G) = \emptyset$] $\Rightarrow F(G) = F(G')$.

Consensus and Independence are introduced and discussed in Kasher and Rubinstein (1997). Monotonicity, which is a stronger version of a monotonicity property introduced in Kasher and Rubinstein (1997), requires that, if, for a given profile G, none of the individuals in $N_1 \subseteq N$ are regarded as members of a social group, then, by changing G to another profile G' in which each individual's views regarding the individuals outside N_1 do not change, while possibly some individuals in N_1 are no longer considered as group members by each individual, none of the individuals in N_1 continue to be regarded as members of the social group. A similar monotonicity property is introduced in Samet and Schmeidler (forthcoming).

Symmetry is a slightly stronger version of the axiom Symmetry used by Kasher and Rubinstein. Weak Liberalism I and Weak Liberalism II are weaker versions of the axiom Liberalism introduced in Kasher and Rubinstein (1997), while Extended Liberalism requires that if, according to an individual i, an individual $k \neq i$ is a group member, and, if i is regarded as a group member collectively, then the set of group members consists of other individuals than i.

Equal Treatment of Insiders' Views requires that if an individual m is considered to be an appropriate group member by an individual k , $m \in G_k$ in a given profile, and if in a new profile k does not consider m as an appropriate group member anymore but a third individual i does, and nothing else has changed, then, when k is a G collectively in the original profile and i is a G collectively in the new profile, it must be true that m is a G collectively in the original profile if and only if m is a G collectively in the new profile. This axiom essentially requires that a CIF should treat the views of all the members who are considered to be Gs collectively equally.

Finally, Equal Treatment of Outsider's Views basically says that a CIF should treat the views of all the members who are considered to be non-Gs collectively equally. This axiom is in the spirit of the exclusive self-determination axiom introduced in Samet and Schmeidler (forthcoming).

4 Characterizations

In this section, we give axiomatic characterizations of a CIF being L and of a CIF being K as defined in Section 2.

Theorem 1 *A CIF $F \in \mathcal{F}$ satisfies the axioms (C), (M), (I), (SYM), (WL(1)) and (ETIV) if and only if $F = L$.*

Proof. It can be verified that the CIF $L \in \mathcal{F}$ satisfies the axioms (C), (M), (I), (SYM), (WL(1)) and (ETIV). Let F be a CIF satisfying the six axioms. We show that for all $G \in \mathcal{G}$,
(i) $L(G) \subseteq F(G)$, and
(ii) $F(G) = L(G)$.

(i) In the following, in order to prove $L(G) \subseteq F(G)$, we prove $L_t(G) \subseteq F(G)$ for all profiles $G \in \mathcal{G}$ by induction on t.

We first show $L_0(G) \subseteq F(G)$ for all $G \in \mathcal{G}$. Suppose to the contrary that there exists a profile $G \in \mathcal{G}$ such that $i \in L_0(G)$ but $i \notin F(G)$ for some $i \in N$. From the definition of L_0, we have $i \in G_i$. Let us consider the profile G' defined as follows:

$$\text{for all } l \in N, G'_l = \begin{cases} G_l - \{i\} & \text{if } l \neq i, \\ G_l & \text{if } l = i. \end{cases}$$

By (M), it follows that $i \notin F(G')$. Note that $i \in L_0(G')$. Consider the profile G'' defined as follows:

$$\text{for all } l \in N, G''_l = \begin{cases} \{l\} & \text{if } l \in (F(G') \cup \{i\}), \\ \emptyset & \text{if } l \in (N - \{F(G') \cup \{i\}\}). \end{cases}$$

By (C), $j \notin F(G'')$ for all $j \in (N - \{F(G') \cup \{i\}\})$. By (SYM), either $F(G'') = F(G') \cup \{i\}$ or $F(G'') = \emptyset$ is true to hold. Suppose $F(G'') = F(G') \cup \{i\}$. Then, for all $k \in N - \{i\}$, $k \in F(G'')$ iff $k \in F(G')$. Note that, for all $k \in N$, $i \in G''_k$ iff $i \in G'_k$. Thus, by (I) and noting that $i \notin F(G')$, we have $i \notin F(G'')$, which contradicts $F(G'') = F(G') \cup \{i\}$. It must be true that $F(G'') = \emptyset$. However, by (WL(1)), $F(G'') \neq \emptyset$, a contradiction. Therefore, $L_0(G) \subseteq F(G)$ for all $G \in \mathcal{G}$.

Next, we assume that $L_t(G) \subseteq F(G)$ for all $G \in \mathcal{G}$ and show that $L_{t+1}(G) \subseteq F(G)$ for all $G \in \mathcal{G}$. From the definition of L, it is sufficient to show that $i \in F(G)$ if $i \in G_k$ for some $k \in L_t(G)$. If $i \in L_t(G)$, then we are done. Assume therefore $i \notin L_t(G)$. Note that $k \in F(G)$, which follows from $k \in L_t(G)$ and our assumption, and that $i \neq k$, which is due to $i \notin L_t(G)$ and $k \in L_t(G)$. Let $G' \in \mathcal{G}$ be a profile defined as follows:

$$\text{for all } l \in N, G'_l = \begin{cases} G_l \cup \{i\} & \text{if} \quad l = i, \\ G_l - \{i\} & \text{if} \quad l = k, \\ G_l & \text{if} \quad l \in (N - \{i,k\}) \,. \end{cases}$$

From the definition of L_0, $i \in L_0(G')$. From $L_0(G') \subseteq F(G')$, it follows that $i \in F(G')$. Noting that $k \in F(G)$ and $i \in F(G')$, by (ETIV), we obtain $i \in F(G)$.

Therefore, $L(G) \subseteq F(G)$ for all $G \in \mathcal{G}$.

(ii) We now show that $L(G) = F(G)$ for all $G \in \mathcal{G}$. Let $G \in \mathcal{G}$ be given. Consider the profile G' defined as follows:

$$\text{for all } i \in N, G'_l = \begin{cases} G_l & \text{if } l \in L(G), \\ G_l \cup (N - L(G)) - \{l\}) & \text{if } l \in (N - L(G)). \end{cases}$$

Clearly, $L(G) = L(G')$. If $L(G) = N$, from (i), $L(G) \subseteq F(G)$. $F(G) = L(G) = N$ then follows easily. In the following, consider $L(G) \neq N$ (and therefore $L(G') \neq N$). Noting that, from (i), $L(G') \subseteq F(G')$, a straightforward application of (SYM) implies $F(G') = L(G')$ or $F(G') = N$. Consider $N_1 = L(G')$ and $N_2 = N - L(G')$. Noting that $N_2 \neq \emptyset$, $G'_i \subseteq N_1$ for all $i \in N_1$, and $i \notin G'_i$ for all $i \in N_2$, by (WL(1)), it follows that $F(G') \neq N$. Therefore, $F(G') = L(G')$. Note that the profile G can be obtained by appropriately deleting elements in $N - L(G)$ from each G'_i, where $i \in (N - L(G'))$. Since $i \notin F(G')$ for all $i \in (N - L(G))$, by (M), we obtain $F(G) = L(G)$. ■

Theorem 2 *A CIF $F \in \mathcal{F}$ satisfies the axioms (C), (I), (M), (SYM), (WL(2)), (EL) and (ETOV) if and only if $F = K$.*

Proof. It can be verified that the CIF $K \in \mathcal{F}$ satisfies the axioms (C), (I), (M), (SYM), (WL(2)), (EL) and (ETOV). Let F be a CIF satisfying the seven axioms. We show that for all $G \in \mathcal{G}$,
(i) $K(G) \subseteq F(G)$, and
(ii) $F(G) = K(G)$.

(i) In the following, in order to prove $K(G) \subseteq F(G)$, we prove $K_t(G) \subseteq F(G)$ for all profiles $G \in \mathcal{G}$ by induction on t.

To begin with, we note that, for all $G \in \mathcal{G}$, by (C), $K_0(G) \subseteq F(G)$. When $K_0(G) = \emptyset$, $K_t(G) = \emptyset$ for all t, and $K(G) \subseteq F(G)$ follows easily.

When $K_0(G) = N$, by (C), $F(G) = K_0(G)$ $(= K(G))$ follows immediately. Therefore, let $K_0(G) \neq \emptyset$ and $K_0(G) \neq N$ for the subsequent proof.

Next, we show the following:

$$\text{If } G_j = \{i\} \text{ for all } j \in N - \{i\} \text{ and } i \in G_i, \text{ then } F(G) = G_i. \qquad (*)$$

If $G_j = \{i\}$ for all $j \in N$, then, $F(G) = G_i = \{i\}$ follows from (C) directly. Let $G_i \neq \{i\}$ and $i \in G_i$. By (C), $i \in F(G)$. By (EL), $F(G) \neq \{i\}$, i.e., $k \in F(G)$ for some $k \in N$ with $k \neq i$. From (C), it must be true that such a $k \in F(G)$ belongs to G_i. By (SYM), it follows that for all $l \in (G_i - \{i\})$, $l \in F(G)$; that is $F(G) = G_i$.

We are ready to show that for if $K_t(G) \subseteq F(G)$, then $K_{t+1}(G) \subseteq F(G)$; that is, we need to show that $i \in F(G)$ if $i \in G_k$ for some $k \in K_t(G)$. If $i \in K_t(G)$, then we are done. Assume therefore $i \notin K_t(G)$. Suppose to the contrary that $i \notin F(G)$. Consider the profile G' defined as follows:

$$G'_l = \begin{cases} G_l & \text{if } l = k, \\ G_l - \{i\} & \text{otherwise.} \end{cases}$$

By (M), it follows that $i \notin F(G')$. Note that $i \in K_{t+1}(G') = K_{t+1}(G)$. Consider the profile G'' defined below:

$$G''_l = \begin{cases} F(G') \cup \{i\} & \text{if } l = k, \\ \{k\} & \text{otherwise.} \end{cases}$$

Note that, by (*), $F(G'') = F(G') \cup \{i\}$; that is, for all $l \in (N - \{i\})$, $l \in F(G')$ iff $l \in F(G'')$. Note further that for all $l \in N$, $i \in G'_l$ iff $i \in G''_l$. By (I), $i \in F(G')$ iff $i \in F(G'')$. But $i \in F(G'')$ and $i \notin F(G')$, a contradiction. Therefore, $i \in F(G)$. Thus, we have shown that $[K_t(G) \subseteq F(G)] \Rightarrow [K_{t+1}(G) \subseteq F(G)]$. Therefore, $K(G) \subseteq F(G)$.

(ii) We now show that $F(G) = K(G)$ for all $G \in \mathcal{G}$. Let the profile $G \in \mathcal{G}$ be given.

If $K_0(G) = \emptyset$ (so that $K(G) = \emptyset$ as well), then for all $h \in N$, there exists at least one $l \in N$ such that $h \notin G_l$. Consider the following profile G': for all $h \in N$, $G'_h = N - \{h\}$. By (SYM), $F(G') = N$ or $F(G') = \emptyset$. From (WL(2)), noting that $h \notin G'_h$ for all $h \in N$, we cannot have $F(G') = N$. Therefore, $F(G') = \emptyset$. Notice that there exists a G'' in which, for all $h \in N$, $G_h \subseteq G''_h$ and there exists exactly one $l \in N$ such that $h \notin G''_l$. It follows from (ETOV) that $F(G'') = F(G') = \emptyset$. By applying (M) with $N_1 = N$, we have $N \cap F(G) = \emptyset$, i.e., $F(G) = \emptyset$. Hence, $F(G) = \emptyset$ whenever $K_0(G) = \emptyset$.

Next, consider $K_0(G) \neq \emptyset$. From (i), we must have $K(G) \subseteq F(G')$. If $K(G) = N$, $K(G) = F(G)$ follows easily. Consider $K(G) \neq N$. Let G' be a profile defined as follows:

$$\text{for all } l \in N, G'_l = \begin{cases} G_l & \text{if } l \in K(G), \\ G_l \cup (N - K(G)) & \text{if } l \in (N - K(G)). \end{cases}$$

Note that $K(G') = K(G)$. Suppose to the contrary that $F(G') \neq K(G')$. Then, given that $K(G') \subseteq F(G')$, it must be true that for some $j \in (N - K(G'))$, $j \in F(G')$. By (SYM), it follows that for all $k \in (N - K(G)), k \in F(G')$. Hence, we must have $F(G') = N$. On the other hand, consider $N_1 = K(G)$ and $N_2 = N - K(G)$. Note that $G'_l \subseteq N_1$ for all $l \in N_1$ and $N_2 \neq \emptyset$. By (WL(2)), we obtain $F(G') \neq N$, a contradiction. Therefore, $k \notin F(G')$ for all $k \in (N - K(G'))$. Hence, $K(G') = F(G')$. Note that $G'_l - G_l \subseteq (N - K(G))$. From applying (M) with $N_1 = N - K(G)$, we have $F(G) \subseteq K(G)$. Noting that $K(G) \subseteq F(G)$, therefore, $K(G) = F(G)$. ■

5 Conclusion

In this paper, we have axiomatically characterized the procedures that define the collective identify functions L and K in the framework proposed by Kasher and Rubinstein (1997). Though it is not our intention to advocate these procedures, we note some interesting features of them. Note that the procedure L starts with all those individuals who are "self-claimed" members of a social group and then expands accordingly. Therefore, the procedure L reflects a strong liberal view[2] of collective identity. On the other hand, the procedure K starts with all those individuals who are considered as group members by *everyone* in the society and then expands accordingly. It therefore suggests that the procedure K is a "consensus-building" liberal view of collective identity.

References

[1] A. Kasher, 1993. Jewish collective identity, in: D.T. Goldberg and M. Krausz (eds.), Jewish Identity, Temple University Press, Philadelphia,

[2] See, for example, Kasher and Rubinstein (1997), and Samet and Schmeidler (forthcoming). If the determination of the membership of a social group is a personal matter, there is indeed some reason to call individuals in L_0 as liberals (see Sen (1970)).

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[2] A. Kasher and A. Rubinstein, 1997. On the Question "Who is a J?": A Social Choice Approach, Logique & Analyse 160 385-395.

[3] D. Samet and D. Schmeidler, Between Liberalism and Democracy, Journal of Economic Theory, forthcoming.

[4] A. K. Sen, 1970. The Impossibility of a Paretian Liberal, Journal of Political Economy 78 152-157.