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CONTRACTING PRODUCTIVITY GROWTH

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Contracting Productivity Growth [¤]

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Abstract

In this paper, we analyze the interactions between growth and the contracting environment in the production sector. Allowing incompleteness in contracting implies that viable production relationships for firms and workers, and therefore the profitability of industries, depend on the rates of innovation and growth. The speed at which new innovations arrive in turn depends on the profitability of production, for the usual reasons examined in the endogenous growth literature. We show that these interactions can have important implications which are consistent with observed phenomena in both the micro and macro environment. In particular, we demonstrate that a technological shock (increasing productivity of research) can, through this interaction, lead to a productivity slowdown and a shift in labor market contracts away from firms providing implicit guarantees of lifetime employment and towards shorter-term "contractor" type arrangements. We show the consistency of an increase in the proportion of the labor force under short term employment, increased relative returns of workers in high productivity sectors, and increased income inequality, with a productivity slowdown of finite duration.

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1 Introduction

Endogenous growth theory suggests that the profitability of productive relationships matters for the rate of innovation, and therefore for sustaining growth. Innovation is encouraged when firms can successfully extract much of the surplus associated with a worker's effort, that is, when firms can obtain a rent from their ownership and use of a technology. In this way, innovation, and thus growth, are directly related to the feasible set of worker-firm relationships or contracts. Standard technology-based endogenous growth models abstract from this issue by implicitly assuming complete contracting in production.[1] Here, we explicitly model contracts as incomplete, and therefore we endogenously determine the form of worker-firm relationships. Not only do the feasible set of relationships determine the profitability of innovating firms and therefore the rate of growth, but it is also the case that macroeconomic variables determine which contracts can be offered in the economy. Specifically, the rates of innovation and growth affect the surplus to and the duration of relationships, and thus determine the set of feasible contracts.

We consider an environment where firms and workers cannot explicitly contract over worker effort in production, and analyze the form of "relational contracts" in their repeated interaction that will support a productive relationship.[2] One possible form of relational contract is for the firm to reward workers with delayed benefits that are paid only once the worker's effort is ascertained by the firm. We refer to such a relational contract as an "internal labor market" and its existence depends critically on the firm's valuation of its reputation. This, however, depends on the firm's expected life-time. In an environment with high potential growth and rapid firm turnover, firms cannot make credible promises of delayed reward to workers, so the internal labor market is necessarily limited. This is the sense in which the macroeconomy constrains feasible micro level contracts. On the other hand, incentives to innovate, and hence the economy's growth rate, depend on the returns in production, which are lower when labor costs are high, that is, when the internal labor market is limited. Thus, micro level contracting relationships also affect the macro environment.

Formally, we augment a standard Schumpeterian growth model, as in Grossman and Helpman (1991 Ch. 4) and Aghion and Howitt (1992), with incomplete contracting in production. We exclude the possibility of legally enforceable payments for worker effort, and consider only those effort/wage pairs that are self-enforcing. It has been widely argued that the non-verifiable particulars of a worker/firm relationship do not allow for explicit contracting over payments that are contingent upon worker performance (see Macleod and Malcomson 1989). In such a frame-

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[1] This contrasts with the more careful modelling of production relationships in both industrial organization and labor economics, where production under incomplete contracting has been more thoroughly examined. For surveys, see Hart (1995), Gibbons (1997) and Malcomson (1999).

[2] The term \relational contract" has been used by Baker, Gibbons and Murphy (1997) to denote these sorts of implicit contracts, a special case of these have also been called \e±ciency wage" contracts. These are discussed, and the related literature is reviewed in Malcomson (1999).

work, only self-enforcing payments and effort will be supplied in equilibrium. With sufficient surplus to production arrangements, however, it is well known that a multiplicity of self-enforcing wage/effort pairs is possible. It is in the creation of a surplus that innovation affects contracting. In an endogenous growth context, the surpluses required to sustain production arise naturally as the rewards to successful innovative activity. Thus, in our environment, monopoly profits to innovation serve two roles: in addition to the standard role of providing incentives for innovation they serve to provide sufficient surplus so that performance contingent implicit contracts can be self-enforcing.

These two roles of firm profits are potentially conflicting: a relationship-destroying role and a relationship-sustaining role. First, as in any standard endogenous growth model, firm profits motivate innovation: this innovation destroys the market power of current monopolists and destroys their relationships with their workers.[3] Second, in a model with incomplete contracting, firm rents allow a firm to develop a reputation for honoring its promises to its workers. In this way, they are a force that sustains worker-firm relationships. Here, these two roles introduce a tension – high firm profits sustain relationships through firm commitment and destroy relationships through encouraging rapid innovation. The introduction of this tension has important implications for the economy's contracting structure and ultimately for the rate of growth. To illustrate this, we use this model to offer an alternative theory of the productivity slowdown: a theory which is consistent with the marked changes in labor market relationships that were coincident with the observed slowdown in productivity.[4]

This model suggests an explanation for a set of macroeconomic changes which, though difficult to date precisely, seemed to begin in the early 1970s. These were: (1) the IT revolution,[5] (2) the productivity slowdown,[6] and (3) the commencement of the erosion of internal labor markets.[7] The information technology (IT) revolution initiated a period of high productivity

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[3]The standard notion of `creative destruction' refers to this first effect, destroying the market power of the existing monopolist. Here, we highlight this second type of destruction: worker-firm relationship destruction.

[4]Our explanation resembles previous attempts to explain the slowdown by labor market induced changes in innovation, or in the dissemination and implementation of new technology, i.e., Hornstein and Krusell (1996), Greenwood and Yorukoglu (1997) and Lloyd-Ellis (1999). However, since these approaches do not include a role for contracting at the micro level, they are clearly structurally very different to the one we present. We contrast these results with ours in the discussion section of the paper.

[5]Greenwood and Jovanovic (1999) date the IT revolution to the early 1970's.

[6]Since the early 1970s there has been a secular decline in the rate of productivity growth (both labor and total factor) in many western developed economies. Focusing on the US, from a trend rate of approximately 2.2% from 1950-1972, the rate of labor productivity growth (excluding agriculture) declined to about 1% from 1972 to 1987 and 1.2% from 1987 to 1994. Over the late 1990's growth in labour productivity has increased once again, from 1995-1999 output per hour in non-farm business grew at an average annual rate of 2.5%, see Oliner and Sichel (2000). Dolmas, Raj and Slottje (1999) show that the log level of productivity underwent a change in both level and slope of its linear trend in the 1970s. There has been some debate as the reliability of measured productivity growth in the light of the emergence of new sectors, but even adjusted estimates taking this into account, find a persistent slowdown over the relevant period, see for example R. Gordon (1996).

[7]There has been a large increase in the number of contingent workers, or those working without an expectation of ongoing employment. Recent estimates put the numbers in such positions at 12 million workers or over 10%

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of inventive effort. Ultimately, however, the returns to inventive effort depend on the degree to which this productivity can be profited from. This, in turn, depends on the form of contracting relationships between firms and employees, since employees will have to be hired to use the new technologies that are developed. Prior to the slowdown, these contracting relationships could best be characterized as an internal labor market;[8] firms rewarded workers with deferred benefits and were themselves disciplined to provide these rewards by their own labor market reputations. The increased innovative potential accompanying the IT revolution implied more rapid turnover in best-practice productive arrangements. These eventually reached a point (which may have differed by sector) where firms' labor market reputations provided insufficient discipline for the maintenance of the internal labor market. Therefore, as the IT revolution worked its way through the economy, within-industry labor markets underwent a fundamental change from being internal labor markets to what we term "contractor"-based labor markets. In a contractor labor market, the moral hazard is not borne by the employer but by labor, who are disciplined by their own reputations to provide effort. This leads to a shift in the distribution of returns away from firms towards labor since, as has already been realized in the theoretical literature, see Macleod and Malcomson (1989), incentive compatibility requires that the party standing to gain from opportunistic behavior must receive a surplus to continued honest participation. Each industry where the labor market undergoes such a restructuring experiences an industry wide decline in returns to innovation which offsets the potential productivity gains from the IT revolution. Thus, over this phase, the induced changes in the labor market heralded in by the IT revolution lead to the seemingly paradoxical possibility of lower productivity growth while the raw productivity of labor in research rises.[9]

There is already a large literature exploring the macroeconomic implications of microeconomic models of the employment relationship, in particular with respect to aggregate employment levels, (Macleod, Malcomson and Gomme 1994, Shapiro and Stiglitz 1984), and job destruction over the cycle, (Ramey and Watson 1997, Caballero and Hammour 1996). A close precursor of this work is Aghion and Howitt (1994), which explores causation from growth rates to unemployment through

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of the US labor force, see Cohany (1996). Approximately 8.3 million of the 12 million workers in non-traditional arrangements are independent contractors, 75% of whom work alone. There is no evidence that the increase in their numbers is attributable to employers cutting costs; rates of pay for independent contractors in business service firms are comparable to or higher than rates of pay to the permanent employees, (Abraham 1990, p. 102). These are generally relatively highly educated and highly paid individuals who experience considerable job satisfaction, see Cohany (1996, p. 35). This growth in contract work has been discussed by Segal and Sullivan (1997) and Abraham and Taylor (1996). Amongst these predominately white collar workers there has been a considerable decline in job security. As presented in Aaronson and Sullivan (1998), surveys of worker perceptions of job security in the General Social Survey, spanning 1977-1996, carried out by the National Opinion Research Center show that white collar and college educated workers experienced substantial increases in job insecurity in the 1990s.

[8]An internal labor market contract would be closely associated with something referred to as `a lifetime contract' in the media.

[9]Information possessing capacity, or the IT revolution can be thought of as a general purpose technology, which is how we model it. However nothing in the model requires that it be `general' in fact the results will obtain if different sectors are affected to differing degrees.

the destructive effects of new knowledge on existing job matches. At a theoretical level, Ramey and Watson (1997) also related the feasibility of the incomplete contracting relationship and its vulnerability to aggregate downturns. A major difference is that their work is concerned with cyclical aspects of this relationship and they do not explore the reverse direction of effect of micro contracting on the macroeconomy. The present paper is the first, to our knowledge, to explore the effects of the macro environment (in particular the innovation arrival, and hence growth, rate) on the possibility for contracting at the micro level, and also to explore the reverse causation of contracting at the micro level's effects on growth.

The structure of the paper is as follows. Section 2 develops the basic model. This is largely a standard Schumpeterian model of growth except that we allow for incomplete contracting in production. The severity of this contracting problem will be seen to vary with the expected productive lives of firms. We introduce sectoral heterogeneity, so that these lifetimes may vary across industries. Section 3 considers the steady state implications of the model. After establishing existence, we consider the effect of an exogenous increase in the productivity of research when this does not affect the contracting environment. It is then demonstrated that the contracting structure cannot remain impervious to sufficiently large increases in research productivity. The endogenous changes in contracting structure are shown to lead to the possibility of a growth slowdown. The model is simulated for reasonable parameter values in order to assess the likelihood of such a slowdown. We then briefly examine the model's implications for changes occuring elsewhere with the slowdown. Section 4 concludes.

2 The Model

Our analysis closely follows that of Aghion and Howitt's (1992) and Grossman and Helpman's (1991, Ch. 4) Schumpeterian models of growth through creative destruction. The numeraire final good $y(t)$ is produced by using intermediate goods, $x_j(t)$, that are distributed uniformly over an interval of measure $M > 1$, according to the following Cobb-Douglas technology:

$$\ln y(t) = \int_{j=0}^{M} \ln x_j(t)\, dj. \tag{1}$$

These intermediate good industries are further differentiated into M types or sectors each of measure 1, where these sectors are indexed by m. Sectors are differentiated by the step size in the quality ladder, γ_m, which represents the size of an incremental improvement or innovation in one of the industries in sector m. Without loss of generality, we order these sectors such that $m < m+1$ implies $\gamma_m < \gamma_{m+1}$. Further, we assume that the difference in step size across sectors is uniform. Specifically, the sector with smallest step size has a step of $\gamma_1 = \gamma > 1$, the next smallest has step size $\gamma_2 = A\gamma$, where $A > 1$; and the mth sector has step size, $\gamma_m = A^{m-1}\gamma$, up to the sector with the highest step size, sector M, with size $\gamma_M = A^{M-1}\gamma$. Figure 1 displays the

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structure of step sizes by sector. An industry is a point on the unit interval located within one of these sectors. We use the notation $m \in \{1, \ldots, M\}$ to denote sectors, and $j \in [0, M]$ to denote industries.

<center>INSERT FIGURE 1</center>

Let a measure $N \geq 1$ denote the amount of human capital in the economy. Each worker can be thought of as possessing a unit of human capital. These workers are endogenously allocated between research and production such that, during any period t, there are $L(t)$ production workers and $S(t)$ research workers which in aggregate must respect the economy's resource endowment:

$$L(t) + S(t) \cdot N: \tag{2}$$

Since our interest is in productivity growth, we focus on the allocation of that portion of the economy's labor stock with enough human capital to contribute to growth–generating activities (which we term research). We ignore other potential inputs, such as unskilled labor, and assume that all human capital in the model is perfectly substitutable between production and research.[10]

Intermediate industries are operated by monopolists using human capital as the only input in a constant returns to scale production process. At any time, the current monopolist in an industry is the firm which has developed the state of the art production technique, over which it has patent protection. Thus, if at time t there have been $n_j(t)$ innovations in industry j, the monopolist holding the $n_j(t)$ th innovation produces according to the production function:

$$x_j^s(t) = \circ_j^{n_j(t)} L_j(t): \tag{3}$$

Therefore, the monopolist's unit cost of producing $x_j(t)$, denoted $c_j(t) = \frac{w_j(t)}{\circ_j^{n_j(t)}}$, where $w_j(t)$ is the wage paid in industry j by the incumbent monopolist. Since labor is homogeneous, substitutable and freely mobile across industries, we denote the market clearing wage in period t by $w(t)$, without the subscript j, since it will not reflect industry level technologies: Of course, this is endogenously solved for in steady state.[11]

Due to the Cobb-Douglas final good production technology, monopolist j faces a unit elastic demand of

$$x_j^d(t) = \frac{y(t)}{M p_j(t)}: \tag{4}$$

As a result, monopolists wish to set the price of intermediate goods as high as possible. However they are constrained in their price setting by the redundant technology which, although strictly

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[10]We thereby abstract from issues of human capital accumulation and ability-biased technological change that may lead to a slowdown through diﬀerent channels as in Galor and Moav (1999), Galor and Tsiddon (1997), and Helpman and Rangel (1999).

[11]The reason we distinguish between $w_j(t)$ and $w(t)$ in notation is that, as will be seen, depending on the nature of contracts that are enforceable at the micro level, monopolists may have to pay a premium to workers over the market clearing wage to ensure incentive compatability, that is $w_j(t)$ need not equal $w(t):$
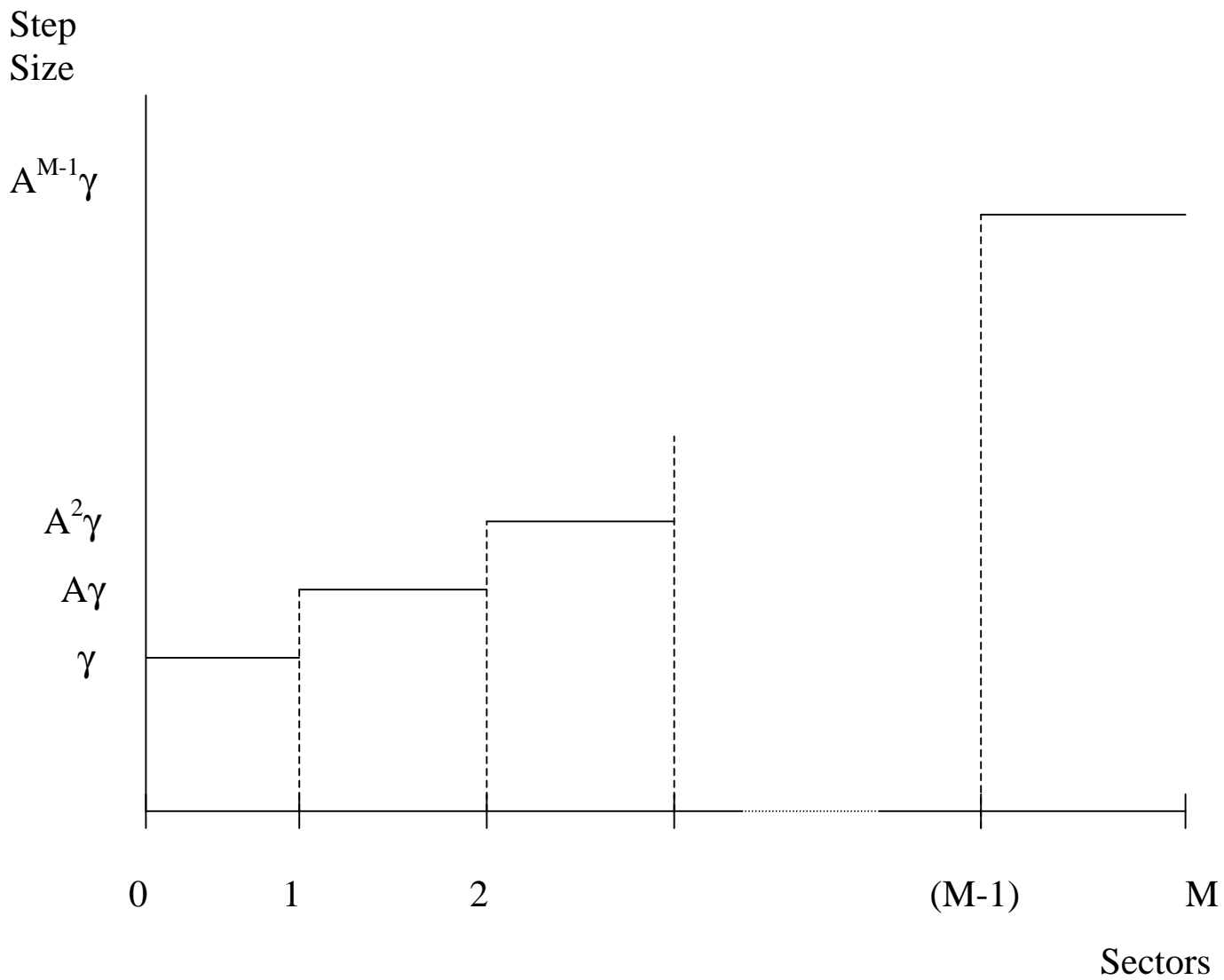
<center>5</center>

Figure 1. Step Size by Sector

less productive than their own and thus of zero value in equilibrium, is freely available to potential entrants. This threat of entry constrains the monopolist's price setting. As is standard (see, for example, Grossman and Helpman (1991)), we assume the threat of entry remains; so that, the monopolist must limit price at the marginal cost of production for a potential producer who were to enter using the previous technology. Only at this price (or lower) are entrants guaranteed non-positive profits, implying the monopolist sets the unit price of $x_j(t)$;

$$p_j(t) = \frac{w(t)}{\circ_j^{n_j(t)-1}}.$$ (5)

Therefore, the monopolist in industry j earns per period profit of:

$$
\begin{aligned}
\pi_j(t) &= [p_j(t) - c_j(t)]x_j^d(t) \\
&= [\frac{w(t)}{\circ_j^{n_j(t)-1}} - \frac{w_j(t)}{\circ_j^{n_j(t)}}]\frac{y(t)}{Mp_j(t)} \\
&= [\frac{w(t)}{\circ_j^{n_j(t)-1}} - \frac{w_j(t)}{\circ_j^{n_j(t)}}]\frac{\circ_j^{n_j(t)-1}y(t)}{Mw(t)} \\
&= \left[1 - \frac{w_j(t)}{w(t)\circ_j}\right]\frac{y(t)}{M}.
\end{aligned}
$$ (6)

Since $x_j(t) = \frac{\circ_j^{n_j(t)-1}y(t)}{Mw(t)}$, the amount of human capital used in production in industry j is

$$L_j(t) = \frac{y(t)}{\circ_j Mw(t)}.$$ (7)

2.0.1 Consumers

Individual consumer/workers are assumed to have isoelastic utility which depends only on their consumption of the final good.[12] There is a common discount factor, ρ; and each individual has a per period probability of dying $1-\pm$, in which case they obtain utility 0 from then on. A currently living individual's present discounted value of expected lifetime utility can thus be expressed as

$$u(t) = \sum_{\iota=t}^{\infty}\left(\frac{\pm}{1+\rho}\right)^{\iota-t}c(\iota)^{\theta};$$

where $\frac{1}{1-\theta}$ is an individual's intertemporal elasticity of substitution. Workers are replaced when they die to maintain a constant population size. Consumers maximize utility subject to a standard intertemporal budget constraint:

$$\sum_{\iota=t}^{\infty}\left(\frac{\pm}{1+r(\iota)}\right)^{\iota-t}c(\iota) \cdot B(t) + \sum_{\iota=t}^{\infty}\left(\frac{\pm}{1+r(\iota)}\right)^{\iota-t}w(\iota).$$

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[12]A worker's one unit of human capital is supplied inelastically so that, for simplicity, we do not factor it into the utility calculation. Note that even though moral hazard in effort is a critical feature of the problem in production, shirking does not involve substituting leisure for effort in our framework, since leisure generates no benefits. Moral hazard instead concerns the allocation of that inelastically supplied effort. This is made clear in section 2.3.

The net present value of expected lifetime consumption must be weakly less than the net present value of the consumer's asset stream, $B(t)$; and the net present value of expected lifetime earnings, where $r(¿)$ denotes the risk free interest rate on final output that is lent for one period.[13] In steady state with a growth rate g; the consumer's Euler equation implies the familiar first order condition:

$$(1+r) = \frac{(1+½)(1+g)^{1_i ¾}}{±}: \tag{8}$$

Note that a necessary and sufficient condition for the consumer's utility to be bounded and therefore for their optimization problem to be well defined is[14]

$$\frac{±}{(1+½)}(1+g)^{¾} < 1: \tag{9}$$

2.1 Innovation problem

Innovations of higher quality are produced by researchers. Let $S_j(t)$ denote the total number of researchers in industry j at time t: We assume the innovation technology generates an arrival probability with constant returns to scale in aggregate research, and therefore let $®S_j(t)$ be the probability of an innovation arriving in industry j in period t; where the parameter $®$ is a productivity parameter which we vary later to reflect changes in technological opportunities or innovative productivity: Innovations arriving during any given period are implemented at the start of the following period. We assume that each unit of the measure $S_j(t)$ total has an equal probability of being the one that realizes the innovation.[15]

There is free entry into research activities. At the start of each period, research entrepreneurs hire human capital to undertake research in a given sector and finance these research efforts by selling equity shares to the consumers.[16] Researchers enter until expected returns from research

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[13]Note that there is no capital in this model, but as in Grossman and Helpman (1991), the asset market simply trades claims to ¯nal output across periods, and r adjusts to equalize supply and demand for those claims in aggregate.

[14]This condition comes from considering the net present value of a consumer's consumption stream in a steady state growing at rate g: This is: $u(t) = c(t)^{¾} + \frac{±}{1+½}c(t+1)^{¾} + \left(\frac{±}{1+½}\right)^{2}c(t+2)^{¾} + :::::$

$= c(t)^{¾} + \frac{±}{1+½}[c(t)(1+g)]^{¾} + \left(\frac{±}{1+½}\right)^{2}\left[c(t)(1+g)^{2}\right]^{¾} + :::$ which is ¯nite and equals $c(t)\frac{1}{1_i \frac{±}{1+½}(1+g)^{¾}}$; given (9) holds.

[15]To avoid modelling the interaction between multiple innovators within a single period, we assume that innovators can observe the e®orts of others in their own sector but not the outcomes of others' innovations until after they are implemented in production. Thus, at the start of each period, innovative e®ort commences in each industry, after a single innovation arrives, research stops, since a further research success will yield for the innovator only zero pro¯t, due to the Bertrand interaction. Note also that, since $S_j(t)$ is allocated to research over a discrete time interval, the probability of an innovation $®S_j(t)$ does not re°ect the probability of an instantaneous arrival using the Poisson distribution, as in Grossman and Helpman (1991). Instead the arrival probability $®S_j(t)$ here is determined as 1 minus the probability of 0 innovations in the period using a binomial distribution.

[16]An equity share in research undertaken in industry j entitles the holder to a claim on pro¯ts arising from research undertaken in industry j. Since innovation successes are independent across industries, optimal trading of equity shares will lead to complete diversi¯cation of equity holdings over all industries.

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equal opportunity costs of human capital in production. Letting $V_j(t+1)$ be the lifetime expected value of a successful innovation occuring in period t, and $r(t)$ denote the interest rate for discounting from t to $t+1$; market clearing in research implies:

$$\frac{\circledR V_j(t+1)}{(1+r(t))} \cdot w(t): \tag{10}$$

Condition (10) holds with equality when there is a positive level of research in an industry: In steady state (as will be seen), per period profit grows at a constant rate (so that $\frac{1}{4}_j(t+1) = (1+g)\frac{1}{4}_j(t)$ for all t) and the interest rate is constant ($r(t) = r$ for all t) implying we can rewrite $V_j(t+1)$ as $V_j(t+1) = \frac{\frac{1}{4}_j(t+1)}{1_i(1_i \circledR S_j)\frac{(1+g)}{(1+r)}}:^{17}$ The stream of profits are discounted by not only r; but also by the expected future innovation rate, $\circledR S_j$. As in Aghion and Howitt (1992), this additional discount arises due to the possibility of a future innovator succeeding and usurping the incumbent's leading position (a creative destruction effect).

In steady state where there is positive research in sector j, S_j will solve

$$\frac{\circledR \frac{1}{4}_j(t+1)}{1+r_i(1_i \circledR S_j)(1+g)} = w_j(t): \tag{11}$$

The value of per period profits, $\frac{1}{4}_j(t)$; depend critically on the industry–specific wage bill and hence on contracting at the micro level, so we leave this until the next section. However it is possible to compute the economy wide growth implications of a given level of research in each industry, S_j: Even though each industry experiences growth probabilistically, since each sector is a unit density of industries, sectors have non-stochastic growth rates. Thus, given the allocations S_j; the economy's growth rate is also non-stochastic and is given by:[18]

$$g = \int_0^M \circledR S_j \ln \circ_j \, dj: \tag{12}$$

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[17]This is computed as follows:

$V_j(t+1) = \frac{1}{4}_j(t+1) + \frac{1}{1+r}(1_i \circledR S_j(t+1))\frac{1}{4}_j(t+2) + \left(\frac{1}{1+r}\right)^2 (1_i \circledR S_j(t+1))(1_i \circledR S_j(t+2))\frac{1}{4}_j(t+3) + \ldots$

$= \frac{1}{4}_j(t+1) + \frac{1+g}{1+r}(1_i \circledR S_j)\frac{1}{4}_j(t+1) + \left(\frac{1+g}{1+r}\right)^2 (1_i \circledR S_j)^2 \frac{1}{4}_j(t+1) + \ldots$

Provided that $(1_i \circledR S_j)\frac{(1+g)}{(1+r)} < 1$; this is bounded and implies $V_j(t+1) = \frac{\frac{1}{4}_j(t+1)}{1_i(1_i \circledR S_j)\frac{(1+g)}{(1+r)}}$: Substituting from the consumer's Euler condition $(1+r) = \frac{(1+\frac{1}{2})(1+g)^{1_i \frac{3}{4}}}{\pm}$; the condition required for the in¯nite stream of pro¯ts to be bounded yields $\frac{\pm}{(1+\frac{1}{2})}(1+g)^{\frac{3}{4}}(1_i \circledR S_j) < 1$; and condition (9) implies that this always holds:

[18]To derive this: $g = \ln y(t+1)_i \ln y(t)$: From the production function: $\ln y(t) = \int_0^M \ln x_j(t)\,dj = \int_0^M \ln \frac{y(t)}{Mw_j(t)} \circ_j^{n_j(t)_i 1}\,dj = \int_0^M \ln \frac{y(t)}{Mw_j(t)} + \int_0^M \circ_j^{n_j(t)_i 1}\,dj$: Since $\frac{y(t)}{Mw_j(t)}$ is constant in steady state, $\ln y(t) = \int_0^M \ln \frac{y(t)}{Mw_j(t)}\,dj + \int_0^M \circ_j^{n_j(t)_i 1}\,dj$; so that, $g = \int_0^M \ln \frac{y}{Mw_j}\,dj + \int_0^M \circ_j^{n_j(t+1)_i 1}\,dj_i \int_0^M \ln \frac{y}{Mw_j}\,dj_i \int_0^M \circ_j^{n_j(t)_i 1}\,dj = \int_0^M \circ_j^{n_j(t+1)_i 1}\,dj_i \int_0^M \circ_j^{n_j(t)_i 1}\,dj$: Substituting in that the probability of an innovation arriving in an industry equals $\circledR S_j$ yields: $g = \int_0^M [n_j(t) + \circledR S_j_i 1]\ln \circ_j\,dj_i \int_0^M [n_j(t)_i 1]\ln \circ_j\,dj$: Notice that in each industry we substituted in the expected number of innovation successes. This is correct since we have a unit measure of all industries $j \, 2 \, m$; for which $S_j = S_m$ for all m: Thus, by applying the law of large numbers on the unit interval, we obtain $n_m(t+1) = n_m(t) + \circledR S_m$: This reduces to: $g = \int_0^M \circledR S_j \ln \circ_j\,dj$:

2.2 Contractual Incompleteness

We assume that there is moral hazard in production and incomplete contracting. We model the contracting problem in a parsimonious way so that the standard Schumpeterian framework of growth in the macro economy can still be solved in closed form. A particularly simple form of contractual incompleteness is to assume a worker's output is non-observable to third parties, but known to both the worker and the firm after a lag of one period. This is the form of incompleteness analyzed in Macleod and Malcomson (1989), who were the first to provide a proper game theoretic foundation for an earlier literature in labor economics concerned with contracting incompleteness.[19]

Unlike the standard principal-agent model, the third party non-verifiability precludes the use of contracts linking worker payment to output produced. We assume that this contracting incompleteness only occurs in intermediate production, not in the research sector.[20] Formally, contracting incompleteness arises due to information limitations as follows.

INFORMATION AND TIMING:

Public information: All workers and firms know the identity of employers and their employees, in all previous periods. The particulars of the worker/firm relationship, in particular whether promised deferred payments are made by the firm, or promised effort is supplied by the worker, are not known publically. If a worker dies or a firm no longer holds the state of the art technology, this is also public information.

Worker's private information: At each period t; a worker knows her own wage payments, $w(\imath)$ for all previous periods $\imath < t$; and her own work performance for all periods $\imath < t$: In addition, she knows whether firms in which she was employed in any period $\imath < t$ delivered any promised deferred payments to her.

Firm's private information: At each period t; a firm knows the history of wage payments made to all of its past workers by it in all periods $\imath < t$: It also knows the effort contributed by all its employees in previous periods while they were employed with the firm:

A new firm or worker in period t has no private information, but has full access to all public information.

Within a period, the time line of events is as follows: At the start of a period a worker and a firm holding a valuable technology implicitly contract. As will be seen, such contracts involve

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[19]This literature was concerned with \efficiency wage" contracts, see for example (Shapiro and Stiglitz (1984), Carmichael (1985), Bull (1987)).

[20]It would also be natural to add contracting incompleteness to the research sector since this is at least as likely to be subject to moral hazard as production. We have not done this since the effects would not differ qualitatively from those in the simpler version of the model we follow here. However note that in the present version there is no expected rent obtained by the research entrepreneurs but, with incomplete contracts, rents play an important role in supporting incentive compatibility. Thus, with incomplete contracting in research, the market clearing in research condition, condition (10), would hold with a strict inequality, and would therefore admit less research effort. However, the qualitative results we obtain concerning changes in the steady state would be unaltered.

either the firm making an up front payment before effort is contributed to production by the worker, or the worker contributing effort before the firm makes a deferred payment. Thus, the timing is: (1) the firm chooses whether or not to pay up front; (2) then the worker chooses whether or not to contribute effort to production; (3) effort is inferred by the firm; (4) the firm chooses whether or not to pay any promised deferred payments; (5) both players, being aware of each other's actions during the period, choose independently whether to continue for another period or terminate the relationship. If either one chooses to terminate, the relationship ends; (6) the period ends and the worker continues to the next period with probability ±; and the firm continues to be the holder of a valuable technology (and thus to remain in existence) with a probability that is to be endogenously determined. This time line is summarized in Figure 2.

INSERT FIGURE 2

A worker's effort contribution in period t is not known by the firm in which it is employed until later in the period, and never verifiable.[21] Since contracts can only be written on verifiable, and hence public, information, it is not possible to write a contract tying a worker's payment to their effort contribution at a particular firm. In general, the assumption that there is no third party verifiability by the courts must then be accompanied by an assumption that other agents are also not able to verify the reasons for a termination, except when one of the parties "dies". As Macleod and Malcomson (1989) argue, this is a reasonable way to characterize an employment relationship. For example, even if the actual act of "firing" is observable, it cannot be inferred with certainty that the firm has violated the employment contract. The firm may have fired the worker for an unconscionable lack of effort. In that case, it was worker shirking that precipitated dismissal, and not the firm violating an implicit agreement of job security. Similarly a "quit" by a worker does not imply the worker broke an implicit agreement of continuation since the firm may have made conditions unbearable for the worker. In that case, it was the firm who violated the implicit contract. The general point is that without observing the actual actions undertaken by firms and workers, third parties cannot infer violations of promised behaviour just by observing a termination in the relationship. Thus, when they do observe the termination of a relationship, their beliefs, which must be consistent with the equilibrium strategies of all agents, play a critical role. Malcomson (1999) discusses the advantages of such an approach to analyzing real-world employment relationships, and provides a survey of this literature.

In general, there are two types of implicit contract (sometimes also termed relational contracts) that can solve the incomplete contracting problem and yield incentive compatible self–enforcing contracts: one allows for the worker to be subject to moral hazard in effort, and the other allows

---

[21]This is the reason why we model time as discrete in contrast to, for example, the Schumpeterian framework developed by Grossman and Helpman (1991). With production in continuous time, the cost of non-observability until the end of the production period becomes negligible.

Worker
And firm
Implicitly
contract

Worker
Chooses
Effort
Contribution

Firm
Decides to
Pay deferred
Payments or
Not

Period Ends:
Worker continues,
Prob. δ
Firm continues,
Prob. endogenous

Period starts

Firm
Chooses
Amount
To pay
Up front

Firm
Observes
Effort
Contribution

Both
Worker and
Firm decide
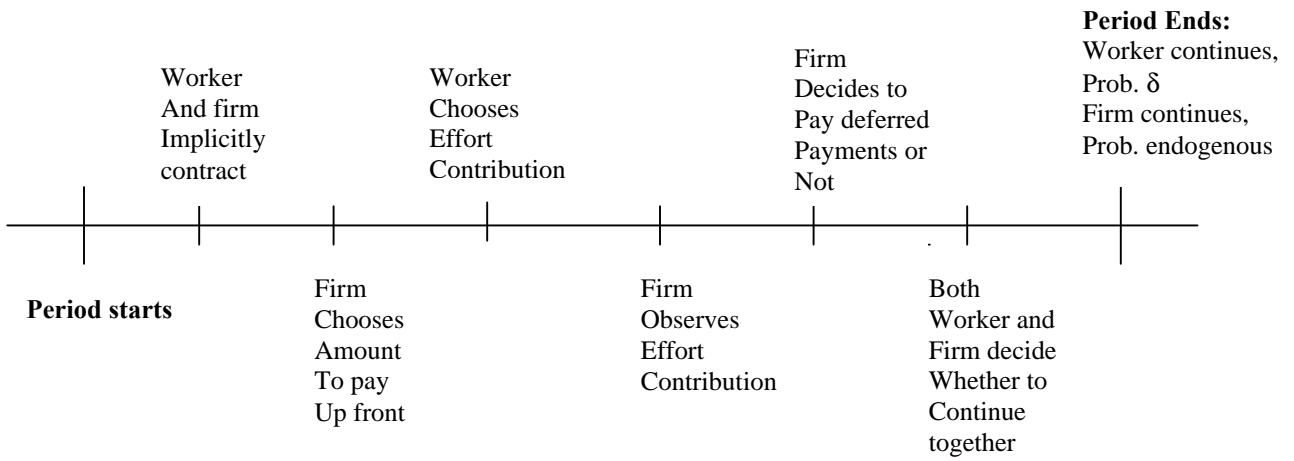Whether to
Continue
together

Figure 2: Time Line of Events

for the firm to be subject to moral hazard in payment. We shall refer to the first case as that of "contractors"; and the second case as an "internal labor market". Both sorts of self-enforcing contracts involve play by agents that would not constitute Nash equilibria in a one shot meeting, but which are supported as equilibria of the continuation game when agents have high enough valuations of the future.

2.3   The contractor case

When firms hire contractors, the moral hazard problem resides with the workers. These workers contract their labor services to the firm on a per period basis. The contractual relationship specifies a payment to the contractor, which is made by the firm at the start of the period; and hence before the contractor's effort has been applied to production, and before the firm knows effort. Later in the period, after the firm observes the correct effort was exerted, the implicit contract specifies the firm will choose to re-hire the contractor for the next period, under the same sort of implicit contract. If the correct effort is not exerted, the firm terminates the relationship.[22] Since effort cannot be inferred by the firm until later in the period, nothing stops the worker from contributing insufficient effort in the firm, and obtaining returns to her effort elsewhere. Since we do not model individual labor supply decisions here (individuals inelastically supply one unit of labor each period and utility is not a function of leisure), such shirking by a contractor does not involve consuming leisure, but instead involves obtaining the opportunity cost of labor supplied to its best alternative use. This is the market wage $w(t)$:[23] It is the value of the contractor's reputation, the potential for future above–market clearing returns, that provides incentives to contribute the promised effort in equilibrium.

For it to be incentive compatible for these contractors to supply correct effort, this reputation must be sufficiently valuable – the wage paid must be sufficiently higher than their opportunity cost. We denote this incentive compatible contractor's wage by $w^c(t)$; and recall that contractors (like all agents in the model) live another period with probability ±. The contractor's incentive compatibility condition simply requires that the contractor receive higher expected lifetime income from contributing the correct effort to the firm, than from shirking and working elsewhere.[24] The present discounted value of contributing the correct effort is the net present value of the income

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[22]We keep discussion of the strategies which support this implicit contract informal in the text and provide a more formal treatment of the equilibrium strategies in Appendix A. We could easily assume that the probability that shirking is detected is less than one; however, this would not qualitatively a®ect the results. This type of implicit contract corresponds to an e±ciency wage as denoted by Shapiro and Stiglitz (1982).

[23]Alternatively, $w(t)$ could be the return to home production if the household labor allocation decision were endogenously modeled, since equilibrium would ensure this return equals the market wage. A small re{speci¯cation of the model would allow this interpretation, and once again, nothing important would change.

[24]Strictly, the IC constraint states that the contractor receive higher expected lifetime utility (not income) from contributing correct e®ort than from shirking. These are equivalent here. The contractor has access to perfect capital markets, at rate of interest r. Since he/she will always choose the optimal consumption path for a given income stream, in order to work out which one of two action paths leads to higher utility we need only to compute the net present value of the income stream.

stream under the implicit contract:

$$w^c(t) + \left(\frac{\pm}{1+r}\right) w^c(t+1) + \left(\frac{\pm}{1+r}\right)^2 w^c(t+2) + \ldots\ldots = \Sigma_{\xi=t}^1 \left(\frac{\pm}{1+r}\right)^{\xi\mathbin{\text{\i}} t} w^c(\xi).$$

If a contractor produces the correct amount, he continues to receive the payment $w^c(\xi)$ each period that he remains in the labor force. This is because, even if his current employer turns over, his reputation as a reliable contractor in that industry persists so that new producers will hire him as well. Note the important role of the public information assumption here; when a firm turns over and a relationship ends, other agents know this was not a termination due to malfeasance by one of the parties.

If shirking in period t; the contractor receives the payment $w^c(t)$ at the start of period t but then contributes no effort to this firm and instead obtains the market return for her effort elsewhere, $w(t)$: From then on, under all other agents' equilibrium strategies, she is never again employed as a contractor, and thus receives the lower wage, $w(\xi)$ in subsequent periods.[25] That is, shirking yields an income stream of net present value:

$$w^c(t) + w(t) + \left(\frac{\pm}{1+r}\right) w(t+1) + \left(\frac{\pm}{1+r}\right)^2 w(t+2) + \ldots\ldots = w^c(t) + \Sigma_{\xi=t}^1 \left(\frac{\pm}{1+r}\right)^{\xi\mathbin{\text{\i}} t} w(\xi)$$

Combining these, a contractor's incentive compatibility constraint in period t is:

$$\Sigma_{\xi=t}^1 \left(\frac{\pm}{1+r}\right)^{\xi\mathbin{\text{\i}} t} w^c(\xi) \; \_, \; w^c(t) + \Sigma_{\xi=t}^1 \left(\frac{\pm}{1+r}\right)^{\xi\mathbin{\text{\i}} t} w(\xi): \tag{13}$$

In a stationary steady state of an economy growing at rate g; condition (13) reduces considerably. In any such steady state, all endogenous variables must also grow at rate g; so that $(1+g) = \frac{y(t+1)}{y(t)} = \frac{w(t+1)}{w(t)} = \frac{w^c(t+1)}{w^c(t)}$:[26] Therefore, the LHS of (13) equals:

$$\Sigma_{\xi=t}^1 \left(\frac{\pm(1+g)}{1+r}\right)^{\xi\mathbin{\text{\i}} t} w^c(t) = w^c(t) \left[\frac{1}{1 \mathbin{\text{\i}} \frac{\pm(1+g)}{1+r}}\right] \tag{14}$$

where the infinite series converges since $\frac{\pm(1+g)}{1+r} < 1$:[27] Similarly the RHS of (13) can be re-expressed as:

$$w^c(t) + \Sigma_{\xi=t}^1 \left(\frac{\pm(1+g)}{1+r}\right)^{\xi\mathbin{\text{\i}} t} w(\xi) = w^c(t) + w(t) \left[\frac{1}{1 \mathbin{\text{\i}} \frac{\pm(1+g)}{1+r}}\right] : \tag{15}$$

---

[25] See Appendix A for the statement of the equilibrium strategies that clarify this sequence of payments. It is also possible to construct equilibria in which individuals can, with some probability, again transition into employment with a ¯rm as a contractor. This does not change the qualitative nature of the equilibrium but does raise the equilibrium incentive compatible wage, since this lowers the punishment cost of dismissal.

[26] If one of these variables is growing at rate not equal to g; it either becomes arbitrarily small relative to y or larger than y, but this then violates the stationarity of the model. To verify this, the value of wages in steady state is computed after existence is established.

[27] To see that this condition does hold, recall the condition that is required to bound the consumer's problem (9). Substituting into this condition using the consumer's Euler equation, $(1 + r) = \frac{(1+\frac{1}{2})(1+g)^{1\mathbin{\text{\i}}\frac{3}{4}}}{\pm}$, yields: $\frac{(1+g)}{(1+r)} < 1$: Recalling that $\pm < 1$; this necessarily implies that $\frac{\pm(1+g)}{1+r} < 1$:

Thus, incentive compatibility holds if and only if (14) ¸ (15); that is, if and only if:

$$w^c(t) \left[\frac{1}{1 - \frac{\pm(1+g)}{1+r}} \right] \; \gtrless \; w^c(t) + w(t) \left[\frac{1}{1 - \frac{\pm(1+g)}{1+r}} \right]$$

$$\Rightarrow \quad w^c(t) \gtrless w(t)\frac{(1+r)}{\pm(1+g)} \tag{16}$$

Therefore, the binding incentive compatible wage is given by:[28]

$$w^c(t) = \frac{1+r}{\pm(1+g)}w(t). \tag{17}$$

The appendix formally states the strategies played by workers and firms supporting this as an equilibrium outcome, but we discuss them briefly here. Incentive compatibility depends not only on the firm at which the worker currently works punishing shirking by not re-hiring, but also other firms doing so as well. However, it is assumed that third parties cannot observe the reason for a termination between a worker and a firm. Unless one of them "dies". If a relationship ends between a contractor and a firm due to a termination, it is impossible for third parties to tell whether it was in fact due to shirking. Thus, agents' beliefs about the likely cause of the termination (that is, beliefs off the equilibrium path) play a critical role. In particular, in the "contractor" type solution to the moral hazard problem, all other firms believe that, when a termination occurs, the worker involved will shirk if hired as a contractor in the future. These off–equilibrium path beliefs suggest to firms that they should not hire such a worker in the future and thus support the equilibrium by imposing large costs on shirkers. Moreover, given these beliefs, as the appendix shows, it is optimal for workers who have experienced a termination in the past to shirk in the future at $w^c(t)$; so that these beliefs are self-fulfilling.

The incentive compatible wage from condition (17) can also be expressed in terms of the worker's discount rate, using the Euler condition $(1+r) = \frac{(1+\tfrac{1}{2})(1+g)^{1-\tfrac{3}{4}}}{\pm}$; as:

$$w^c(t) = \frac{(1+\tfrac{1}{2})}{\pm^2(1+g)^{\tfrac{3}{4}}}w(t). \tag{18}$$

In a faster growing economy, the wage premium required to ensure incentive compatibility is lower since the future leads to relatively higher rewards.[29] In contrast, in a slowdown, there is an increase in the disparity between contractor wages and those of regular employees, a point to which we return later.

---

[28]Note that, in steady state, the contractor's payment will bind at this incentive compatible wage in industries with worker moral hazard. For higher contractor wages, workers currently working at $w(t)$ can credibly o®er their services as a contractor for an amount greater than $w(t)$; but below the wage of existing contractors, thus upsetting the equilibrium. We return to this in section 3.2.

[29]Note that this condition seems to imply that the incentive compatible wage goes to zero as $g \to 1$: However, this is not possible since g is bounded by the need to have a well de¯ned consumer's problem. See also footnote 27.

2.4 Internal labor markets

Recall that intermediate production occurs in firms owning the state of the art technology. Usually, these firms do not have a significant role in this type of model, since their profits are distributed to the dispersed shareholders of the economy, and it is assumed they simply hire labor at the going wage, see Grossman and Helpman (1991). Here, however, we identify firms with their incumbency as owners of the state of the art technology. This has already been done implicitly in calculating the discounted value of future firm profits, V_j, which depends on the firm's expected lifetime, $\frac{1}{\circledR S_j}$: Firms hold a patent ensuring them the exclusive right to produce with the technology. The value of a patent is zero once the technology is no longer the state of the art, and then the firm disbands.[30]

If firms can credibly commit to paying employees in the future for effort exerted today, there exists another solution to the non-contractibility problem which we term an "internal labor market'. The implicit contract between worker and firm, in this case, specifies that the worker contributes effort up front in promise of deferred payment. Later in the period, when the firm can infer the worker's contribution, the worker is paid. The worker undertakes to continue with the firm only if the firm meets its promised payment. For this solution to work, it is necessary that firms who renege on promised payments are punished in their future dealings in the labor market. Once again, punishments are the equilibrium actions of other players when observing actions that are not supposed to occur on the equilibrium path. These depend critically on agents' beliefs in equilibrium, which are specified formally in Appendix A. Intuitively, punishment here consists of future workers not believing that this firm will meet their promised payments if the firm has been involved in a termination in the past. Given these beliefs, future workers optimally choose to reject offers of deferred payment in future, since they do not believe the firm's promise of rewards to follow. Given these beliefs, and that strategy of workers, the best response for firms who have been involved in a termination in the past is to, in fact, cheat any worker who does accept a job. Since their reputation has already been destroyed, they suffer no additional reputational loss, and equilibrium strategies do not allow a firm to re-establish their reputation.

This relational contract solves the moral hazard problem in production if firms place enough value on their reputations, which enable them to continue hiring workers through deferred payments. This valuation depends on their discount rate and their length of expected incumbency. Since a firm only lives profitably until the next innovation arrives in its industry, it discounts future profits by the probability of an innovation arriving, $\circledR S_j$, as well as the discount factor, r.

A critical difference between this solution to the moral hazard problem, and the previous contractor solution, is that the firm need only promise to pay workers the going wage, $w(t)$: There is no need for a wage premium since the moral hazard resides with the firm. If the firm

---

[30]Firms are indi®erent to remaining in existence if it is costless to do so; however, with even an arbitrarily small cost of maintaining the ¯rm, shutting down and disbanding is strictly preferred.

cheats and fails to pay a worker after effort is contributed, they obtain $\frac{y(t)}{M}$ today without cost in the current period, since the optimal form of cheating will be to renege on all payments. The cost of cheating is that they can only continue to produce in the future by contracting out production, hence by paying $w^c(t)$, determined in equation (18). This is because no worker will believe that a firm who cheated in the past will meet deferred payment obligations in the future. From equation (6), the per period profits of an incumbent with its reputation in tact and who is able to pay the going market wage, i.e., $w_j(t) = w(t)$, are given by

$$
\begin{aligned}
\pi_j(t) &= [p_j(t) - c_j(t)]x_j(t) \\
&= [\frac{w(t)}{\sigma_j^{n_j(t)-1}} - \frac{w(t)}{\sigma_j^{n_j(t)}}]\frac{\sigma_j^{n_j(t)-1}y(t)}{Mw(t)} \\
&= \left[1 - \frac{1}{\sigma_j}\right]\frac{y(t)}{M}. \tag{19}
\end{aligned}
$$

In contrast, an incumbent who must resort to hiring contractors has to pay the higher wage, $w_j(t) = w^c(t)$. Thus, per period profits are:

$$
\begin{aligned}
\pi_j^c(t) &= [p_j(t) - c_j(t)]x_j(t) \\
&= [\frac{w(t)}{\sigma_j^{n_j(t)-1}} - \frac{w^c(t)}{\sigma_j^{n_j(t)}}]\frac{\sigma_j^{n_j(t)-1}y(t)}{Mw(t)} \\
&= [1 - \frac{w^c(t)}{\sigma_j w(t)}]\frac{y(t)}{M}. \tag{20}
\end{aligned}
$$

Note that $\pi_j^c(t) < \pi_j(t)$.[31] The incentive compatibility constraint for an incumbent firm in sector j is:

$$
\pi_j(t) + \left(\frac{1-\delta S_j}{1+r}\right)\pi_j(t+1) + \left(\frac{1-\delta S_j}{1+r}\right)^2\pi_j(t+2) + \ldots
$$
$$
\geq \frac{y(t)}{M} + \left(\frac{1-\delta S_j}{1+r}\right)\pi_j^c(t+1) + \left(\frac{1-\delta S_j}{1+r}\right)^2\pi_j^c(t+2) + \ldots \tag{21}
$$

---

[31]Here, resorting to sub-optimal hiring behaviour cannot help firms in satisfying their own incentive constraints. Such behaviour was the solution to a similar commitment problem arising in Greif, Milgrom and Weingast (1994). The logic of their solution works when firms have a concave production function, and when cheating a worker implies that only the relationship with the single worker is severed, not the whole workforce. In that situation, if firms hire up to the optimal amount, that is where they are indifferent between using the last unit or not, they have strictly positive incentive to cheat the last unit, since they obtain the benefit of cheating, but only lose the service of the last unit about whom they are indifferent. To solve this problem, less labor than the profit maximizing level must be employed so that when the labor that is cheated is withdrawn, it imposes a positive cost on the firm. Here, however, as production is linear in L, firms are not able to affect their own incentive compatibility conditions by adjusting their labor hiring away from the optimal one that is implied by (20). Nor do they need to since, when a firm cheats one worker, and a separation occurs, in the internal labor market equilibrium, all other workers rationally refuse to accept deferred payments from the firm. So, the firm is punished by being forced to pay a wage premium to all future workers. This is also the reason why a firm's best form of cheating involves violating payment promises to all workers.

If reneging on its commitment to meet promised payments (the RHS), the firm will optimally choose to pay all workers an amount 0; so that $\frac{1}{4}_j(t) = \frac{y(t)}{M}$; for the period in which the firm cheats: Substituting from (19) and (20) into (21) yields:

$$
\left(1 - \frac{1}{\degree_j}\right) \frac{1}{M} \sum_{\iota=t} \left(\frac{1 - {}^{\circledR}S_j}{1+r}\right)^{\iota - t} y(\iota) = \frac{y(t)}{M} + \frac{1}{M} \sum_{\iota=t+1} \left(1 - \frac{w^c(\iota)}{\degree_j w(\iota)}\right) \left(\frac{1 - {}^{\circledR}S_j}{1+r}\right)^{\iota - t} y(\iota):
\tag{22}
$$

By once again imposing stationarity and using (17) to express $w^c(t)$; we can analyze the implications of (22) in a stationary steady state. It reduces to a remarkably simple necessary and sufficient condition:

Lemma 1 A necessary and su±cient condition for a ¯rm in industy j^0s incentive compatibility condition to hold is that their industry's per period probability of receiving a successful innovation, ${}^{\circledR}S_j$ satis¯es:

$$
{}^{\circledR}S_j \cdot 1_i \pm:
\tag{23}
$$

Any industry with an innovation arrival probability greater than $1_i \pm$ cannot maintain an internal labor market and must hire contract workers. The condition implies a dependence of firms' incentive compatibility on workers' continuation probabilities. Rearranging the condition as $\pm \cdot 1_i {}^{\circledR}S_j$; it says that the probability of a firm staying in the relationship for one more period $(1_i {}^{\circledR}S_j)$ must exceed the probability of the worker doing so $(\pm):$ If the worker has a higher probability of continuing than the firm, then the firm will cheat.[32] This is due to the fact that the cost to the firm of reneging on payments is relatively low since high ± implies a relatively low contractor wage w^c; and the benefit to the firm of cheating is relatively high, since a low value of $(1_i {}^{\circledR}S_j)$ implies a relatively low probability of continuing for another period as incumbent and

3 Steady State Analysis

We now analyze the implied macroeconomic steady state fixing the contracting solution at the microeconomic level within an industry. If an industry has a "contractor" solution, as analyzed in section 2.3, then, for existence of a steady state with positive growth, necessarily, condition (16) must hold. If an industry has an "internal labor market" solution as analyzed in section 2.4, then necessarily condition (22) must hold. We first proceed by calculating the steady state when all sectors have an "internal labor market" solution. We then show that when technological

---

[32]Note that if (23) fails there is no possibility of satisfying incentive compatibility for the ¯rm by resorting to some other division of the surplus. To see why, note that, since the worker only receives $w(t)$; the ¯rm is already receiving the full amount of the rent from the relationship and this is not high enough given their low valuation of the future. Lowering this further by giving some of the surplus to the worker, i.e. by raising w only makes the ¯rm's incentive compatibility condition worse. However, there is still the possibility of the relationship changing to a \contractor" solution and the worker's incentive compatibility condition holding, as in Section (2:3):

possibilities increase ($\circledR$ rises) the economy's growth rate monotonically rises provided that all industries continue with an internal labor market structure. This is standard in Schumpeterian growth models, since an increase in $\circledR$ raises expected returns to research, and thus induces greater research effort, higher innovation rates, and consequently, higher growth. However, for high enough $\circledR$, we show that the "internal labor market" contracting structure is not sustainable (in particular, condition (23) is violated).

3.1 Stationary Steady State with Internal labor Markets

We first solve for a steady state in which all hiring is done through internal labor markets. All steady states that we analyze will involve each sector undertaking positive research, and thus contributing to growth, in equilibrium.[33] Since industries are heterogeneous, they vary in $\circ_j$; a sufficient condition for existence will be necessary to ensure that there is enough incentive for research in equilibrium. That is, incentives for research, which depend upon $\circ$ and $\circledR$, need to be high enough to ensure that positive research is undertaken in each sector. We derive a sufficient condition for existence here. Recall again that in such a stationary steady state g must also be the rate of growth in wages w, profits $\frac{1}{4}$ and consumption c; which we shall verify:

Formally, a stationary steady state, is an allocation of research workers, S_j; and a corresponding labor allocation to production, L_j for each industry, j; such that:

(I) All labor is employed: equation (2) holds with equality.

(II) Monopolists demand labor according to (7).

(III) The equilibrium growth rate is given by (12).

(IV) Consumers are optimizing: r is determined by (8):

(V) The labor market clears: S_j solves (11).

(VI) Employment contracts are incentive compatible: condition (22) holds,

In addition to these conditions, firms' and employees' strategies must support honest fulfillment of implicit contracts, that is, the equilibrium strategies support condition (22); as elaborated in Appendix A.

Since monopolists in all sectors compete for homogeneous labor; and since we consider a steady state in which all industries hire through an internal labor market, $w_j(t) = w(t)$ for all j:

---

[33]This is without loss of generality. With lower productivity of research, steady states with growth coming from research in only the high $\circ$ sectors will exist. In such steady states, the low $\circ$ sectors play no real role other than to constantly absorb a fraction of the labour force. This can be easily allowed for here but is not of extra interest. As in Aghion and Howitt (1992), there also exists another equilibrium in which the economy cycles between high and low growth, driven by expectations, we focus solely on the stationary steady state. In addition, our framework always also admits equilibria in which research and production shuts down in all sectors. This is possible here because of the contracting incompleteness in production which always admit the possibility of a bad equilibrium in which no agent trusts others to ful⁻ll their side of delayed commitment. This corresponds to agents simply playing the Nash equilibrium of the single shot game, which is always an equilibrium of the repeated game. In that case both output and growth are zero in steady state. These are also equilibria which we do not consider here since they seem of limited applicability.

Thus, equation (6) yields per period profit for successful innovators of $\left[\frac{\circ_{j-1}}{\circ_j}\right]\frac{y(t)}{M}$ for each sector j: Substituting this into the labor market clearing condition (11) and using $(1+r)$ from (8); we obtain S_j as:

$$S_j(t) = \frac{\mu\left(\frac{\circ_j - 1}{\circ_j}\right)^{\P} \frac{y(t)}{Mw(t)} + \frac{1}{\circledR} i}{\pm\circledR(1+g)^{\frac{3}{4}}} \qquad 8j: \tag{24}$$

In stationary steady state, $\frac{y(t)}{w(t)}$ is necessarily a constant, which we can solve for using the market clearing condition in intermediate production. For each intermediate industry j; the Cobb-Douglas final good production function implies that demand for final goods $x_j^d(t) = \frac{y(t)}{Mp_j(t)}$: This unit elasticity of demand function implies limit pricing is optimal for producers so that $p_j(t) = \frac{w(t)}{\circ_j^{n_j(t)-1}}$: On the supply side, $x_j^s(t) = L_j(t)^{\circ n_j(t)}$: Thus, $x_j^s(t) = x_j^d(t)$ implies that

$$\frac{y(t)}{w(t)} = ML_j{}^{\circ}{}_j; \tag{25}$$

due to $L_j(t) = L_j$ in stationary steady state. The equation also implies that the term $L_j{}^{\circ}{}_j$ must also be the same for all industries j: Therefore, without loss of generality, we use $L_1{}^{\circ}$ where L_1 denotes the allocation of production labor to industries with lowest step size, i.e. sector 1 with step size $\circ$:[34] Thus (24) can be expressed as:

$$S_j(t) = \frac{\mu\left(\frac{\circ_j - 1}{\circ_j}\right)^{\P} L_1{}^{\circ} + \frac{1}{\circledR} i}{\pm\circledR(1+g)^{\frac{3}{4}}} \qquad 8j: \tag{26}$$

This equation computes the allocation of labor in research that is consistent with labor market clearing given the economy's growth rate, g, that sector's profitability of research, $\frac{\circ_j - 1}{\circ_j}$; and the amount of labor allocated to production, L_1.

Note that for each industry j in sector m, $\circ_j = \circ_m$ so that S_j is equivalent for all j in a sector:

$$S_j(t) = \frac{\mu\left(\frac{\circ_m - 1}{\circ_m}\right)^{\P} L_1{}^{\circ} + \frac{1}{\circledR} i}{\pm\circledR(1+g)^{\frac{3}{4}}} = S_m(t) \qquad 8j\, 2\, m; \tag{27}$$

where $S_m(t)$ denotes the allocation of labor to research in a single industry of sector m: Note that this is increasing in $\circ_m$ so that $S_m(t)$ is higher for higher m: Since, in steady state, $S_m(t)$ is time invariant, we drop the t notation.

From (12); the innovation technology also yields an expression for g as a function of sectoral allocations to research, S_m:

$$
\begin{aligned}
g &= \int_0^M \circledR S_j \ln \circ_j \, dj \\
&= \int_0^1 \circledR S_1 \ln \circ_1 + \int_1^2 \circledR S_2 \ln \circ_2 + \cdots \int_{M-1}^M \circledR S_M \ln \circ_M
\end{aligned}
$$

[34]Recall that sectors $m > 1$ all have step size $A^{m-1}\circ$: In computing the steady state, we work with L_1 but we could equivalently have worked with any L_m; since once we know A; $\circ$ and one L_m; all the others are uniquely determined through (25):

$$= \sum_{m=1}^{M} \beta S_m \ln \gamma_m;$$

where the second step follows from the fact that $S_j = S_m; \forall j \geq m$: Since $\gamma_m = A^{m-1}\gamma$ for all m, we have

$$g = \sum_{m=1}^{M} \beta S_m \ln \left(A^{m-1}\gamma \right)$$
$$= \beta \sum_{m=1}^{M} S_m \left((\ln A) m - \ln A + \ln \gamma \right) \tag{28}$$

Substituting (27) into (28), we obtain the following expression that implicitly defines g in terms of L_1:

$$g = \beta \sum_{m=1}^{M} \left[\frac{\gamma_{m-1}}{\gamma_m} L_1 \gamma + \frac{1}{\beta} - \frac{(1+\eta)}{\rho \beta (1+g)^{\theta}} \right] \left((\ln A) m - \ln A + \ln \gamma \right): \tag{29}$$

We can solve for L_1 using the economy's resource constraint, equation (2), which holds with equality in steady state:

$$N = L + S$$
$$= \int_0^M L_j \, dj - \int_0^M S_j \, dj$$
$$= \sum_{m=1}^{M} L_m - \sum_{m=1}^{M} S_m$$

Since $L_m \gamma_m$ is a constant, this can be expressed as:

$$N = \sum_{m=1}^{M} L_1 \left(\frac{1}{A^{m-1}} \right) - \sum_{m=1}^{M} S_m$$
$$= L_1 \frac{A - A^{-M+1}}{A - 1} - \sum_{m=1}^{M} S_m$$

From (27), $S_m = \frac{\gamma_{m-1}}{\gamma_m} L \gamma - \frac{(1+\eta)(1+g)^{-\theta}}{\beta \rho} + \frac{1}{\beta}$ implying that:

$$N = L_1 \frac{A - A^{-M+1}}{A - 1} - \sum_{m=1}^{M} \left[\frac{\gamma_{m-1}}{\gamma_m} L \gamma - \frac{(1+\eta)(1+g)^{-\theta}}{\beta \rho} + \frac{1}{\beta} \right]$$

Rearranging leads to:

$$L_1 = \frac{N - 1 - \frac{(1+\eta)(1+g)^{-\theta}}{\rho} - \frac{M}{\beta}}{\gamma M}: \tag{30}$$

We let the term $\left[1 - \frac{(1+\hbar)(1+g)^{-\frac{3}{4}}}{\pm}\right]^{\frac{1}{\circledR}} \cdot \Omega$; so that $L_1 = \frac{N - \Omega M}{\circ M}$: Substituting (30) into (29) yields an expression in one unknown, g; that is, the economy's growth rate in a steady state where all sectors undertake positive research, and all sectors hire in an internal labor market:

$$g = \circledR \sum_{m=1}^{M} \cdot \left[\frac{\circ_m - 1}{\circ_m}\right]^{\P} \left[\frac{N - \Omega M}{\circ M}\right]^{\P} + \Omega^{\P} \left((\ln A)(m - 1) + \ln \circ\right) \tag{31}$$

Evaluating the summation on the RHS over m yields the following expression which implicitly determines g :

$$g = \circledR N \left[\ln \circ + \frac{1}{2}\ln A (M - 1)\right]^{\P}$$

$$+\frac{\circledR \frac{N - \Omega M}{\circ M}}{(A - 1)^2} \left[(\ln \circ)(A - 1) \left[A^{1-M} - A\right]^{\mathcal{C}} + (\ln A) \left[A^{1-M}(M(A - 1) + 1) - A\right]^{\mathcal{C}¤}\right] \tag{32}$$

where g enters the RHS of this expression through Ω only. Provided incentives for research are high enough and firms live long enough to support incentive compatibility of contracts, a positive valued solution exists and is unique:

Proposition 1 If

$$\circledR \frac{N}{M} (\circ - 1) > \left[\frac{1 + \hbar}{\pm} - 1\right]^{\P};$$

there exists a unique steady state satisfying conditions (I) to (VI) with corresponding growth rate $g^{¤}$; and sectoral allocation of research, $fS_m^{¤}g$; provided that the necessary condition for suporting internal labour markets $\circledR S_M^{¤} \cdot 1 - \pm$ holds.

The existence condition is intuitive. It simply states that the returns from research, $\circ$; and the likelihood of research success $\circledR$ have to be high enough relative to the discount rate $\frac{1+\hbar}{\pm}$ (which is the opportunity cost of research since research costs labor effort today and produces returns only in the future). Moreover the economy's stock of human capital relative to the number of sectors undertaking research, $\frac{N}{M}$ must be high enough for there to exist incentives for research in even the least productive sector.

In this steady state, each sector experiences ongoing research but at differing levels of intensity, $\circledR S_j$; reflecting the differing profit opportunities available. Firms have sufficiently long expected lives to be able to offer a credible commitment of deferred payments to their workers, so hiring occurs through an internal labor market with moral hazard on the firm's side. Workers therefore contribute effort to production in advance of payment, and firms reward effort once it has been inferred. Since there is no moral hazard on the worker's side, there is no need for a wage premium and workers are simply paid the market clearing wage in all sectors, $w(t)$: The sectoral allocations to production and research are pinned down once g is determined from (32): That is, given g; L_1 is obtained from (30); L_1 then pins down the value of L_j in all other sectors j, since $L_j \circ_j$ is

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a constant. Given L_1 and g, research in each sector, S_j is given by (27): Prices within a period are completely pinned down by the technology, the evolution of which is determined by S_j: The market clearing wage, $w(t)$ is computed from the production function and determined by the technology so that this grows proportionately with y and $\frac{1}{4}$.[35]

We now consider the effect of an increase in the productivity of research on the economy's growth in steady state.

3.2   Information Technology

There are many effects that the introduction of computers and the general revolution in information can have on production: it can increase the need for training, lead to a substitution of capital for labor, improve distribution, directly change production efficiency, etc. We wish to abstract from all of these and focus purely upon the increase in the availability of information and therefore in the speed of dissemination of new ideas. In this way, we focus purely on the effect of the information technology revolution in speeding up the arrival of new ideas (or equivalently, in reducing the cost of research). We posit that this is a truly general purpose technology that improves the research productivity of all sectors, though qualitatively similar results will obtain if only some sectors are affected, or if sectors are affected to differing degrees.

To consider the consequences of such a change, we consider the effect of increasing the technological parameter, $\circledR$; capturing the marginal effect of research on the innovation arrival probability. If the contracting structure within industries remains unchanged, we have the same steady state as in Proposition 1, and the rate of growth simply increases with the increased research effectiveness.

Corollary 1 With a ¯xed structure of contracts in the economy, increasing $\circledR$ increases the economy's growth rate.

As usually occurs in a Schumpeterian growth model with increasing productivity of research, greater returns to research imply more research and higher growth. However, it is not always possible for the contracting structure to remain unchanged. Starting in a situation in which all sectors have internal labor markets (that is, there is no contract work), increasing $\circledR$ eventually makes hiring contractors the only solution to the moral hazard problem:

Proposition 2 For the highest sector, M; there exists a unique value of $\circledR$; denoted $\circledR^0_M$ such that for $\circledR > \circledR^0_M$; sector M cannot hire workers in an internal labour market. Consequently, for $\circledR > \circledR^0_M$; the steady state de¯ned by conditions (I) to (VI) no longer exists.

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[35]To see this, simply substitute from (4) into (1) using limit pricing to obtain: $\ln y(t) = \int_0^1 \ln \left(\frac{y(t)}{w(t)M} \circ n_j^{i-1} \right)^3 dj$:
Rearranging yields: $w(t) = \frac{e^{\int_0^1 \ln \circ n_j^{i-1} dj}}{M}$: Since this depends only on n_j; it grows proportionately with both y and $\frac{1}{4}$ in steady state.

The contracting structure is not impervious to changes in ® because the increase in arrival rates, ®S$_j$; eventually renders firms' expectations of the length of a productive relationship with their employees short. If short enough (leading to a violation of condition (23)), firms cannot credibly commit to providing deferred benefits to their employees. This problem arises earlier (i.e. for lower values of ®) for industries with higher °; since these are the industries that attract proportionately more research, and hence have higher arrival rates, holding all else equal.

As ® increases, and a sector becomes unable to hire employees in the internal labor market, a possible solution is for it to hire contractors. If this is possible, then not only does this sector change the structure of its hiring, but also the economy's steady state changes. Formally, if condition (VI) cannot hold since (22) is violated, the existence of a steady state in which all sectors grow depends on the existence of a "contractor" solution to the moral hazard problem in these high ° sectors. Let m^0 denote the lowest sector for which internal labor markets cannot operate; that is, the lowest value of m such that ®$_m^0$ < ®: We now look for a steady state in which the sectors that are unable to hire with internal labor markets, instead solve the moral hazard problem through incentive compatible "contractor" solutions. Therefore, we replace condition (VI) in our steady state conditions with:

(VI^0): Employment contracts are incentive compatible: therefore, for sectors m < m^0 condition (22) holds, and for sectors m ¸ m^0 condition (16) holds.

The other conditions describing a steady state are unaltered since, even though they are affected by employment contracts, they must still hold as stated.[36]

The following proposition establishes the existence of a unique steady state satisfying conditions (I) to (VI^0).

Proposition 3 If

$$\circledR \frac{N}{M} \left(^{\circ} - \frac{1+\frac{1}{2}}{\pm^2}\right) > \left(\frac{1+\frac{1}{2}}{\pm} - 1\right)\frac{(1+\frac{1}{2})}{\pm^2}; \tag{33}$$

there exists a unique steady state satisfying conditions (I) to (VI^0) with corresponding growth rate g$^¤$ and sectoral allocation of research, S$_m^¤$, provided that for m < m^0; internal labor markets can be supported: ®S$_m^¤$ · 1 ¡ ± holds:

This existence condition is of the same structure as the previous one for the steady state with internal labor markets. Now, however, the condition is stricter, the term $\left(^{\circ} - \frac{1+\frac{1}{2}}{\pm^2}\right)$ on the LHS implies that the step size in the smallest industry ° now has to exceed $\frac{1+\frac{1}{2}}{\pm^2} > 1$ instead of 1 as before. This is required since it is more difficult to guarantee that it is possible to operate profitably when wage costs are higher. As before, the condition is more likely to hold the higher are ®; N and °; and the smaller is $\frac{1+\frac{1}{2}}{\pm}$ and M: Note also that, when m^0 = M +1; that is, all sectors can

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[36]The other conditions are market clearing and optimization conditions for consumers and ¯rms, so though the details of these are a®ected, the conditions as stated in (I) to (V), that is conditions (2); (7); (12); (8); and (11) do not change.

hire in an internal labor market, the condition reduces to $\mathbb{R}\frac{N}{M}^{\circ} \mathbf{i} \frac{1+\frac12}{\pm^2} > \frac{1+\frac12}{\pm} \mathbf{i} 1 \frac{(1+\frac12)}{\pm^2}$:
In that case, we have the same existence condition as in proposition (1) since $\frac{1+\frac12}{\pm^2}$ is replaced by 1 since all employers can pay w.

This steady state features sectors $m \, {}_¸ \, m^0$ hiring contractors, and sectors $m < m^0$ hiring through an internal labor market.[37] Notice that the necessity part only depends on the incentive compatibility conditions in sectors $m < m^0$: For ® high enough, it is impossible to satisfy these conditions in any sectors. In that case, $m^0 = 1$ and all sectors hire with contractors. Existence of the contractor solution is, of course, not adversely affected by increasing ® as can be seen immediately from condition (33): The upshot is that as ® increases, the cut-off sector m^0 is decreasing. In other words, the number of sectors able to sustain internal labor markets is falling. However, the capacity to solve the moral hazard problem with a contractor solution is unaffected by increasing ®:

3.3 The slowdown

We now demonstrate that the economy must experience a slowdown as ® increases beyond certain critical values. Beyond these critical ® values, m^0 falls and fewer sectors can sustain internal labor markets. The effect of a fall in m^0 on the growth rates is shown in the next proposition:

Proposition 4 Consider an economy in steady state as described by conditions (I) to (V I^0), with a contracting structure such that $m^0 \, {}_¸ \, 2$. If a small increase in the productivity of research, ®, implies that a sector is now unable to hire in an internal labour market and must switch to the use of contractors. That is,

$$\mathbb{R} S_{m^0}^{\pi} \cdot 1 \mathbf{i} \pm \tag{34}$$

no longer holds, then m^0 falls, and the economy's growth rate falls.

When a sector loses the ability to hire workers in an internal labor market, its production costs rise thus lowering the profitability of research. This definitely lowers research in that sector; but by releasing labor, it raises research efforts in the rest of the economy. This reallocation of researchers is generally from the switching sectors with high returns in research toward stable sectors with lower returns, since the higher ones are less able to support internal labor markets. This lowers the economy's growth rate, which, in turn, further lowers incentives for research. The

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[37]Note that the existence condition ensures that workers live long enough for it to be possible to satisfy the worker's incentive compatibility condition in steady state. If this were not the case then, when ¯rms' incentive compatibility conditions did not hold, i.e for $m > m^0$; there would no longer be any possibility of either production, or research in those sectors, since neither worker's nor ¯rm's incentive compatibility conditions would hold. This would imply that the slowdown result, which we establish in the next section, would be even more pronounced, since instead of a reduction in growth in some sectors we obtain a complete cessation of growth. Though the qualitative behaviour of the model is una®ected, we do not analyze this case since a complete halt of production in some sectors seems somewhat less realistic.

proposition shows that when a sector changes in this way, growth always falls provided $m^0 \geq 2$; that is, provided the sector changing is not the last one.

Thus, in addition to the well known positive effect of an increase in ® on the economy's growth rate, the induced change in contracting at the micro level exerts a negative effect. Which effect dominates depends on the magnitude of the change in ®. The positive effect works continuously to increase growth; but for continual increase in ®, it is punctuated by discrete downward jumps in growth as sectors' contracting structures endogenously change. Can such downward jumps actually lead to an overall growth slowdown? To obtain an idea of the likely magnitude of the relative effects, we simulate the model.

3.4 Simulation of the model

Since we have already analytically established that growth falls when a sector is no longer able to sustain an internal labor market, for small increases in ®, the simulations allow us to investigate whether such falls are simply downward perturbations that are swamped by the positive effects of increasing ® over longer ranges, or whether these falls are significant relative to the increases.

We normalize the economy's endowment of human capital to 1 and start with the number of sectors equal to 5 for baseline simulations: Parameter values must satisfy the existence conditions, ensuring that there is sufficient incentive for positive research in each sector (this will be true as long as it holds in the lowest ° sector) and must also ensure that the arrival probability of innovations is well defined, $®S_j < 1$: The parameter values are chosen relative to our definition of a period. In the model, a period is the time elapsed between a worker's poor performance and the firm's capacity to infer that ex post. Thus, a day seems too short, for then the cost of shirking would be small, but alternatively a year is probably too long, firms should generally be able to infer unreliability more quickly than that. So, we choose a baseline of 6 months, implying that the approximate 15 year average survival probability within an occupation requires a ± value of 0:97. Similarly a 2% discount, or $½ = :02$ is suitable as a baseline case for a 6 month period. We set $¼ = :25$; so that the consumer's intertemporal elasticity of substitution equals 1.33.[38] There is a degree of freedom in our choice of the parameters ° and ®: Recall that ° is the smallest productivity increment for innovations arriving in the economy (that is in sector 1); and ® reflects the marginal productivity of effort targetted at innovations (i.e. the likelihood of research effort yielding success). Any given level of target growth rate in the model can be generated by inversely varying ° and ®: Relatively high ° and low ® implies low chances of large productivity enhancements with research, whereas, low ° and high ® implies higher likelihood of success which is of less impact. The variable ®, which is a parameter of the research production function,

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[38]Our choice of utility function does not allow examining of cases where this elasticity is less than one. Beaudry and van Wincoop (1996) argue, using disaggregated data, for elasticities exceeding one, although more macro based estimates reject such high values. The principal e®ect of lowering ¼ is that starting growth rates fall, but this does not greatly a®ect the simulated slowdown results we report below.

does not have directly observable correlates and must be varied with our choice of normalization for labor market size so it is not useful to calibrate from. In contrast, $°$, which measures the productivity advantage of new innovations over incumbents, is reflected in markups about which there is considerable evidence. We take as a reasonable benchmark a starting value of $° = 1.2$; and then choose $®$ to yield reasonable growth rates. This is a sensible productivity increment in the least productive sectors as it implies that innovations in the lowest sector are 20% more productive than incumbents. It implies a gross markup (i.e. the ratio of price to marginal cost)

$$\mu = \frac{\frac{w}{°·n_i·1}}{\frac{w}{°·n}} = ° = 1.2;$$

which is about the middle of the range estimated by Norrbin (1993) and Basu (1996).

Finally, we need to specify the productivity increment differences across sectors. We take a low value for the differences across sectors, $A = 1.01$. Increasing A and M implies larger differences between returns to research in the high and low $°$ sectors since the productivity increment of an innovation in the highest research sector is $A^{M_i\,1}$ times greater than that in the lowest.[39] Given these values, since we wish to generate a reasonable growth rate, we solve out a starting value for $® = 1.5$; to generate growth of about 1.7% per period. This is quite high for 6 month periods, but recall that we have an economy where all sectors are contributing to growth, so appended to an economy with some non-growing sectors this is not unreasonable.

Figure 3 depicts the corresponding steady state growth rates, for increasing values of $®$; in this baseline case of the model. The discrete downward jumps in the figure occur due to the falls in m^0; as sectors shift from internal labor market to contractor solutions, as expected from Proposition 4. The last downward jump occurs at around $® = 2$; when the lowest sector is then unable to hire on an internal labor market. From then on, increases in $®$ monotonically increase the economy's growth rate. Overall, the falls in growth accompanying contractual changes are large in comparison with the standard positive effects of increased $®$: Taking $® = 1.5$ as our initial starting point, the figure shows that growth does not reach its pre-slowdown levels again until $®$ is at least 2.3. That is, an increase in $®$; and hence research productivity of $\frac{0.8}{1.5} = 53\%$; is required before growth rates eventually reach the starting level of 1.7%. Note that without the change in contracting such an increase in $®$ of 53% would have lead to a growth rate of 5.44%, i.e. over a 300% increase in the growth rate.[40] This baseline case thus suggests a possible explanation of the slowdown. If the IT revolution is interpreted as a GPT that increases $®$ at the economy wide level, the corresponding contractual changes induced at the micro level require a massive increase in research productivity (around 50%) for the economy to sustain growth rates of the same level.

The result in this baseline case persists for variations in parameters provided the value of $°$

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[39]As will be seen, a consequence of large differences is that in order to obtain existence of steady states in which even the lowest $°$ sector undertakes research requires a relatively severe restriction on the values of $°$: A point which we discuss subsequently.

[40]The reason for the more than proportional effect of the increases in research productivity is the large induced increase in research activity occuring in response to the increased productivity.

Figure 3
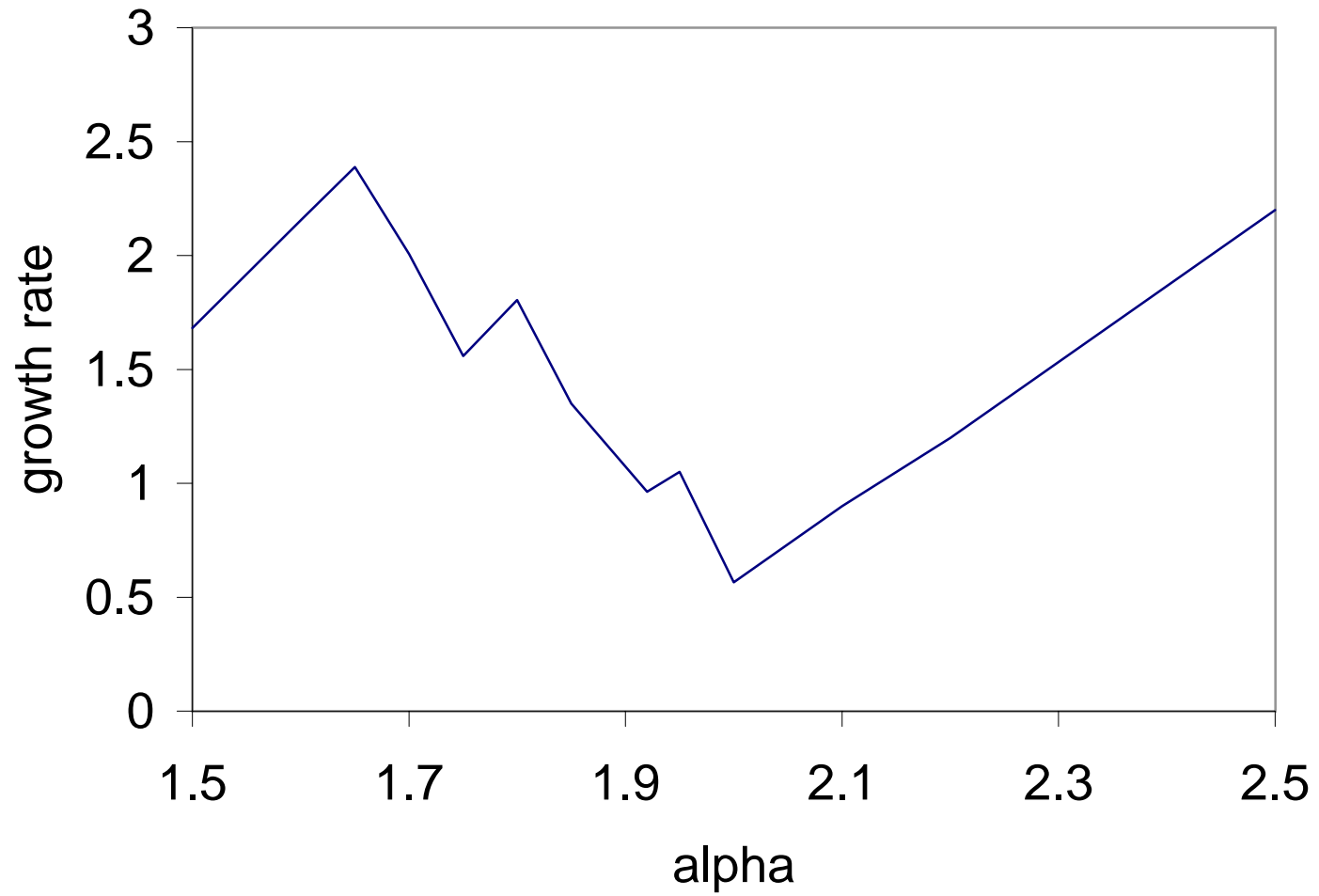
does not reach implausibly high levels. Varying the consumer's parameters (¾, ½ and ±) does not change the qualitative result of large downward effects of contracting structure changes relative to the productivity enhancements. Lowering the value of ° towards the bottom of the range, ° = 1:05; increases the slowdown range, however the effect becomes weaker as the value of ° is higher. For instance comparing with the benchmark case of ° = 1:2 and its 53% increase in ® required for the economy to be back at the starting growth rate, when ° = 1:3 then only a 33% increase in ® is required for the economy's growth rate to be the same when all sectors are unable to hire on an internal labor market. This value of ° is still within the range of markups estimated by Norrbin (1993) and Basu (1996) who find a range of ° = 1:05 to 1:4: However, even for implausibly high values of °; for instance ° = 1:5; the range is significant, though lower still, approximately 15%. The reason for the size of the range falling with the value of ° is that the direct positive effect of an increase in ® is greater for higher °, whereas the decline in growth due to the negative effect is relatively unchanged. For very high values of °, even small changes in ® have large positive effects on the growth rate since any induced increases in research, and increases in the probability of success, serve to raise the chance of a very large increment to productivity, and thus increase the economy's rate of growth. For example, a slowdown will not occur in this baseline case when values of ° exceed 1.6. Growth rates still fall when a sector changes contracting structure, but these declines are only downward spikes which are quickly outweighed by even small further increases in ®: Encouragingly, only such implausibly high values of ° eliminate the slowdown range altogether.[41]

In the simulation, each increase in ® is solved for as a new steady state. The actual ® increments accompanying the introduction of a GPT like the IT revolution are probably not smoothly occurring as in the figures of the simulation, but rather discrete jumps along the horizontal axis. Therefore, the range of ® values for which the slowdown occur are best understood as an indication of how likely such positive increments to research productivity are to generate an explanation for the slowdown. A small range suggests this is very unlikely, but the large range shown in the baseline case, which persists for reasonable values of the mark-up, is supportive of the slowdown possibility.

The next corollary lists the qualitative nature of any changes that accompany a slowdown in

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[41]The model can be solved for M and A that are large but it implies implausibly high values of °: This is because the perfect inter-sectoral substitutability of research e®ort implies that to start in a steady state where the highest sector is able to maintain an internal labour market it is necessary for ® to be relatively small, otherwise ®S$_M$ will be too large. But then a relatively high value of ° is required to maintain su±cient incentives for positive research in the lowest sector. For instance with A = 1:04, M =10 and ® = 0:6; it is necessary that ° > 1:58 in order to ensure that S$_1$ > 0: This requires for the highest sector a productivity increment of °$_M$ = (1:58) (1:04)^9 = 2:248; or a markup exceeding 100% of costs. Even in the lowest sector, the markup of 1.58 greatly exceeds the upper bound for average gross markups in Norrbin (1993) and Basu (1996). But it is not possible to lower the value of ° to more reasonable levels by increasing ® since then the internal labour market constraint in the high ° sectors is violated. To see this note that, for these parameters, the di®erence 1 ¡ ± ¡ ®S$_M$ = 9:14 £ 10$^{¡3}$; so that any small increase in ® will make this fail:

this model.

Corollary 2 If an increase in ® causes a slowdown, then: (i) the proportion of the labor force without guarantee of ongoing employment increases, (ii) relative returns of workers in high ° sectors rise, (iii) income inequality rises and (iv) sectors with the largest declines in productivity growth will be the ones that initially had the highest rates of growth.

There can be no slowdown without a restructuring of labor contracts in some sectors. Without such a change, the arrival of innovations simply increases with a rise in ®; and growth rises.

A lower growth rate lowers individuals' valuations of the future; and therefore, as can be seen from equation (18), contractors must be compensated more immediately to maintain incentives. Since contractors start at higher wages than employees, further changes in the contracting structure along the slowdown worsen inequality in the earnings distribution by lowering returns to workers while simultaneously increasing the premium to contractors. Since the increase here would not be attributable to observable worker characteristics (for example, education and training levels), unless earnings equations were estimated with information about contract structure, it would be picked up in the residual of an earnings equation.[42] This corresponds well to the findings of Juhn, Murphy and Pierce (1993) where such earnings equations (which are estimated without information about the form of employment contracts) attributed much of the growth in inequality to unmeasured components.

Furthermore, the increases in inequality that occur happen in the high ° sectors. That is, these will tend to occur in industries where, for given contracting structure, there are higher growth rates. A positive correlation between industry wages and technological change has been observed in many studies, Hodson and England (1986), Dickens and Katz (1987) and Loh (1992). Though, it should be noted that Bartel and Sicherman (1999), by controlling for individual fixed effects, argue that most of this is due to sorting of high "ability" individuals into sectors with higher rates of technological change.

The model's implications for turnover are similar to a standard growth model, with industries' growth rates positively correlated with turnover. An interesting implication, however, is the model's results regarding workers' perceptions of job security. During the slowdown, the change in labor markets from internal labor market hiring, with promises of ongoing employment, to contractor situations, where there is no such commitment by the employer, imply less job security for employees.[43] This is consistent with perceptions of greater job insecurity over the period. As presented in Aaronson and Sullivan (1998), surveys of worker perceptions of job security in the General Social Survey, spanning 1977-1996, show that white collar and college educated workers

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[42]This serves to make this model testably di®erent than Galor and Moav's (1999) model with technological change which is ability biased, where ability is unobservable.

[43]Though not necessarily employment or income security, since these depend on re-employment possibilities.

experienced substantial increases in job insecurity in the 1990s. The period also corresponded with a "sea-change" in the norms and expectations of professional careers, caused by the erosion of internal labor markets, see Smith (1997) and D. Gordon (1996). This downward trend in job security is also reported for the same period by Schmidt (1999).

An implication of the slowdown generated by an increasing $\circledR$ is that it is of finite duration. In this framework, productivity can only fall with $\circledR$ increasing if contracts change. In particular, when labor hiring practices in all sectors have changed towards contracts, there will no longer be the possibility of a slowdown precipitated by further increases in $\circledR$; although, for $\circledR$ fixed, productivity growth rates will remain at lower levels.

In this model, we have assumed that the population is fixed. However if instead of increasing the technological parameter we increased the population, we could generate similar results. Fixing the contracting structure, a population increase will increase the amount of research being undertaken and therefore the arrival rates of new innovations through a standard scale effect. This interacts with the contracting structure in approximately the same manner as described above. And so, in this model, we can generate a short-term negative relationship between population growth and the growth rate. A series of newer versions of technology-based endogenous growth already allow for the possibility of a negative relationship between population growth and productivity growth. However, these models (Young 1998, Howitt 1999, and Segerstrom 1999) do not suggest why the slowdown should have occurred when it did. In contrast, our model even when change is motivated by population growth (not technology) relates the macro changes to observed contemporaneous changes in the labor market, and thus, links the timing of the slowdown to the period of those changes.[44]

A final point to note is that a firm's productive life is assumed to end when a new innovation arrives. However, in reality, many firms produce more than one type of good. These firms may still be able to provide a form of commitment to employees by shifting them from newly redundant processes to other productive roles. In fact, within any one firm, there will be employees with differing levels of job security, depending on how wedded their employment is to the technology they are using in production. However, it will still be the case that the firm's commitment to any particular employee will vary with that employee's expected productive life with the firm. A firm's capacity to commit to lengthy employment will not necessarily end with the arrival of faster innovations at the industry level, but should still be negatively affected by them. A similar flavor of effect has been explored by Bertrand (1999). She examines whether employment relationships adjust under increased product market competition and found that increased financial pressure (proxied through increased import competition) transformed the employment relationship from

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[44]Note that the insights we obtain from the current model translate fully into the framework of this newer generation of Schumpeterian growth models. The steady state with contracting incompleteness will persist in those frameworks and will similarly lead to an increase in the proportion of sectors that are unable to sustain internal labour markets as the productivity of research rises.

one governed by implicit agreements to one governed by the market. An interpretation she forwards, which is consistent with our hypothesis, is that increased competition weakens the enforceability of implicit wage agreements, so that the spot market governs the relationship.

4   Conclusion

The paper has developed a framework that explores a somewhat subtle two way interaction between contracting at the micro level and the growth of the macroeconomy. An implication of this interaction is that an increase in the productivity of research can actually lower the economy's growth rate, in contrast to all previous Schumpeterian growth models. We have argued that the slowdown generated here by an increase in productivity of research, can provide some insight into the actual slowdown that hit the US economy from the early 1970's, corresponding to the IT revolution. Other explanations of the slowdown, such as Helpman and Trajtenberg's (1996) explanation based on the costs associated with dissemination of such a General Purpose Technology, or Lloyd-Ellis's (1999) explanation, arising from a fall in supply of qualified workers and hence a reduction in absorptive capacity, are not mutually exclusive. Certainly such supply side effects would similarly slow down growth in our model.

A promising feature of our approach is that it suggests a set of contemporaneous changes which seemed to occur with the slowdown: for example; that the source of increased earnings dispersion is in the residual of earnings regressions; that relatively large increases in earnings should occur in the sectors with most innovativeness; and that along the slowdown workers should have increased perceptions of job insecurity. However, we recognize that the extreme parsimony of the model precludes a more careful analysis of the detailed inter-sectoral changes that actually did occur. In our opinion a model which could be seriously taken to the data would at least also require solutions to be provided for corner cases, where innovation possibilities stop in some sectors altogether, and the introduction of a meaningful role for capital. We leave the building of such an extension to future work.

5   Appendix A:

5.0.1   Strategies supporting the incentive compatible incomplete contracts

Information sets:

A strategy maps from each player's information set to the set of actions.

Consistent with the information assumptions, at any time t; all workers and firms know the past history of all firm/worker employment pairs, referred to as their public information. Importantly, public information sets do not include the reason for a worker/firm relationship to terminate, except if a worker died or a firm ended production. A "termination" refers to

the ending of a relationship for reasons other than death or closing down. The agents involved themselves know the reason for a termination, other agents do not know whether it was due to firm, worker, or both parties not fulfilling promised obligations. A worker, i^0s, public history at time t is denoted $h^i(t)$; with $h^i(t) = 1$ if the worker has not been involved in a termination for any $\iota \cdot t$, and $h^i(t) = 0$; otherwise. Similarly, a firm, j's, public history at time t is denoted $h^j(t)$; with $h^j(t) = 1$ if the firm has not been involved in a termination for any $\iota \cdot t$, and $h^j(t) = 0$; otherwise. In addition, both workers and firms have some private information. A worker knows her own effort contribution upto time t. For worker i; this is denoted $e_i(t)$; with $e_i(t) = 1$ if the worker has contributed promised effort for all $\iota \cdot t$; and $e_i(t) = 0$; otherwise. As well, the worker knows the payment history of any firm with which it has been involved. Thus, for firm j the worker i knows whether j has paid the promised amounts to i in all previous interactions between i and j, denoted $p_i^j(t)$: If the firm has paid all amounts that were promised then $p_i^j(t) = 1$; otherwise, $p_i^j(t) = 0$: In the case of no previous interactions, $p_i^j(t) = 1$: Firms know their own private histories and the histories of the workers in their interactions with them. Thus, firm j knows whether it has paid promised amounts to all its workers, if it has then $p^j(t) = 1$; otherwise $p^j(t) = 0$: If a firm has never before promised payments, then $p^j(t) = 1$: Similarly, if a firm has employed an employee i, it knows whether the employee has contributed the promised amounts of effort when working for j, if it has, for all $\iota \cdot t$ then $e_i^j(t) = 1$; otherwise $e_i^j(t) = 0$: If they have never before interacted, $e_i^j(t) = 1$:

A worker's information set in period t comprises the public histories of all firms and workers upto and including period t_i 1; $h^W(t_i 1) [h^F(t_i 1)$; where W is the set of all workers and F the set of all firms, as well as the private information they have from their own employment history, $e_i(t_h 1)$; and the information they have on the set of firms, denoted F_i; for whom they have worked $p_i^j(t_i 1)$ for all $j 2 F_i$: A firm's information set comprises the public histories of all firms and workers up to and including period t_i 1; $h^W(t_i 1) [h^F(t_i 1)$ as well as the private information they have from their own history as an employer. In particular, they know $p^j(t_i 1)$ and the information they have on the set of workers who have worked for them in the past, denoted E^j: Where for all $i 2 E^j$ they know $e_i^j(t_i 1)$:

Strategies:

Denote a worker's strategy by $\frac{3}{4}^W(t)$. It has two parts: firstly, it specifies a decision of whether to accept or reject every level of wage offer from every firm, these wage offers can be either up front offers, or offers that a firm promises to pay at the end of the period. Secondly, where up front wage offers have been accepted from a given firm, it specifies a decision of whether to work ($e_i = 1$) or shirk ($e_i = 0$) for a given firm and whether to continue in the relationship or terminate it.

Denote a firm's strategy by $\frac{3}{4}^f(t)$: It has two parts: firstly, whether to offer up front payments to workers or to offer payments at the end of the period, and the amounts to offer. Then, for end

of period payments offered to a particular worker, it specifies whether to honor those payments, ($p_j = 1$) or not ($p_j = 0$) and whether to continue in the relationship or terminate it.

5.0.2 Equilibrium strategies for a `contractor' outcome:

Denote these strategies by $\tilde{\sigma}^f(t)$ and $\tilde{\sigma}^w(t)$:

$\tilde{\sigma}_j^f(t)$ for firm j - For any worker i with $h^i(t-1) = 1$ and $p^j(t-1) : e_i^j(t-1) = 1$; offer w^c from equation (18) upfront, for all other workers make no offer.

If $e_i^j(t) = 0$; i.e., the worker shirks, terminate the relationship. If not, continue with the relationship.

If starting production, first make offers to the workers who worked for the previous incumbent, provided they satisfy $h^i(t-1) = 1$.

Do not honor commitments to make deferred payments.

$\tilde{\sigma}_i^w(t)$ for individual i - accept any non-negative wage offer made up front, do not accept offers of deferred wage payments.

If the up front wage w is such that $w \geq w^c$ and $h^i(t-1) = 1$ and $p_i^j(t-1) : e_i(t-1) = 1$; then set $e_i = 1$; otherwise set $e_i = 0$:

These strategies induce, as an equilibrium wage in that sector, the wage w^c with no workers shirking and all firms rehiring the same employees if and only if they do not shirk and they remain in the labor market. If a worker dies, firms then choose randomly from the pool of applicants (some of whom are new workers, but this is irrelevant since they all have $h^w(t-1) = 1$); and if a firm turns over new firms hire their employees (since their reputations are in tact, i.e. $h^w(t-1) = 1$.

To see that this is an equilibrium consider the returns to agents choosing actions deviating from the equilibrium strategies. Consider first the incentives of a worker to deviate from $\tilde{\sigma}_i^w(t)$. If the worker shirks under a $w \geq w^c$; and works elsewhere for the alternative wage of $w(t)$ she cannot be made better off, since under $\tilde{\sigma}_j^f(t)$ she will never again be hired as a contractor so that incentive compatible wages are given by equation (18); which defines w^c. Consider the optimality of the strategies for paths that are off the equilibrium play. For instance, suppose that a worker shirks in period t; and thus, according to $\tilde{\sigma}_j^f(t)$ is dismissed by the firm. The worker's strategy $\tilde{\sigma}_i^w(t)$ states that the worker will shirk again at $w^c(t)$: This is optimal for the worker. To see this, note that for all future $\iota > t$, public information over the worker will now be $h^i(\iota) = 0$: Thus, under $\tilde{\sigma}_j^f(t)$; even if the worker is currently working, she anticipates never again being hired at wage w^c in future, that is, her employment will be terminated at the end of the period. Thus her optimal action is to shirk. Suppose alternatively that the firm offers the worker a wage $w < w^c$; the worker's strategy says the worker should accept the wage and shirk. This is clearly optimal since $w < w^c$ is not incentive compatible. Consider a firm's incentive to deviate from $\tilde{\sigma}_j^f(t)$: Suppose, for instance, that in period $t + 1$ the firm decides not to dismiss a worker that shirked in period t. Since the worker shirked in period t then $e_i(t) = 0$: Thus under the worker's

strategy, $\hat{\sigma}_i^w(t)$; the worker will shirk again, and the firm will suffer a loss. So non-employment of a shirker is optimal. Suppose that the firm decides to terminate the employment of a worker that has not shirked. This does not increase the firm's profits, thus retaining a non-shirker is a weak best response. It is also a weak best response for new firms to hire workers who did not shirk for the previous incumbent. Suppose that a firm tried to deviate to a hiring type equilibrium. That is, it offered workers no payments up front, but promised to pay them after work is completed. Under $\hat{\sigma}_i^w(t)$ no worker would accept the offer. Clearly for the firm to deviate to any $w < w^c$ is also not optimal, since $\hat{\sigma}_i^w(t)$ specifies workers will take the wage and shirk.

Note the form of beliefs induced by these strategies. Though workers and firms do not know the precise reason for a termination in this equilibrium they believe that, once one has occured, the worker will shirk again at w^c: Moreover, these beliefs will be true, the worker will find it optimal to shirk from then on, so the beliefs are consistent. Of course, along the equilibrium play, shirking paths are not realized.

5.0.3 Equilibrium strategies for an `internal labor markets' outcome:

Denote these strategies by $\tilde{\sigma}^f(t)$ and $\tilde{\sigma}^w(t)$:

$\tilde{\sigma}_j^f(t)$ for firm j - If $^{\circledR}S_j^{\pi} \cdot 1 \mathbf{i} \pm$ as defined in (23); then, provided $h^j(t \mathbf{i} 1) = 1$; and, for worker i; $h^i(t \mathbf{i} 1) = 1$; and $p^j(t \mathbf{i} 1):e_i^h(t \mathbf{i} 1) = 1$ offer deferred payment of $w(t)$ and honor the payment if and only if the worker sets $e_i(t) = 1$: If $^{\circledR}S_j^{\pi} \cdot 1 \mathbf{i} \pm$ and $h^j(t \mathbf{i} 1) = 0$; $h^i(t \mathbf{i} 1) = 1$ and $p^j(t \mathbf{i} 1):e_i^j(t \mathbf{i} 1) = 1$; then offer an upfront payment of w^c: If $^{\circledR}S_j^{\pi} \cdot 1 \mathbf{i} \pm$ and $h^j(t \mathbf{i} 1) = 1$; and $h^i(t \mathbf{i} 1) = 0$; and $p^j(t \mathbf{i} 1):e_i^j(t \mathbf{i} 1) = 1$; make no offer to i. If $^{\circledR}S_j^{\pi} \cdot 1 \mathbf{i} \pm$ and $h^j(t \mathbf{i} 1) = 1$; and $h^i(t \mathbf{i} 1) = 1$; and $p^j(t \mathbf{i} 1):e_i^j(t \mathbf{i} 1) = 0$; make no offer to i. If $^{\circledR}S_j^{\pi} > 1 \mathbf{i} \pm$ and $h^j(t \mathbf{i} 1) = 0$; $h^i(t \mathbf{i} 1) = 1$ and $p^j(t \mathbf{i} 1):e_i^h(t \mathbf{i} 1) = 1$; then offer an upfront payment of w^c; as defined in (18): If $^{\circledR}S_j^{\pi} > 1 \mathbf{i} \pm$ and not $[h^j(t \mathbf{i} 1) = 0; h^i(t \mathbf{i} 1) = 1$ and $p^j(t \mathbf{i} 1):e_i^h(t \mathbf{i} 1) = 1]$; then make no offer.

$\tilde{\sigma}_i^w(t)$ for individual i - accept any non-negative wage offer made upfront from a firm j. If an up front wage offer is such that $w \geq w^c$ and $h^w(t \mathbf{i} 1) = 1$ and $p^j(t \mathbf{i} 1):e_i^j(t \mathbf{i} 1) = 1$; then set $e_i = 1$; otherwise set $e_i = 0$: If $^{\circledR}S_j^{\pi} \cdot 1 \mathbf{i} \pm$ and $h^j(t \mathbf{i} 1) = 1$; and $h^i(t \mathbf{i} 1) = 1$; and $p^j(t \mathbf{i} 1):e_i^j(t \mathbf{i} 1) = 1$ accept a promise of deferred payment of $w(t)$ and set $e_i = 1$: Otherwise accept no offers of deferred payment. Terminate any relationship with a firm if deferred payments are not made, otherwise continue.

This induces an equilibrium in which workers accept a work offer with deferred payment of $w(t)$; set $e_i = 1$; and firms pay workers only if $e_i = 1$: In equilibrium, no workers shirk and all firms honor their payment commitments. To see this, note that equilibrium strategies state that a firm who has reneged on payments (and hence had a termination) will continue to do so. And note that it is a best response for them to do so. Given this strategy, a worker's best response is not to trust such deferred payments and to instead work only for payments made up front. Then,

however, payments satisfying workers' incentive compatibility conditions, w^c, will be required to induce effort.

Under $\hat{\omega}^f(t)$ firms make deferred payments only if the worker contributes correct effort in production. If a worker were not to contribute correct effort, the firm knows that under the worker's equilibrium strategy $\hat{\omega}^w(t)$, the worker will terminate the relationship at the end of the period, and the firm will then be punished by other agents, playing $\hat{\omega}^w(t)$, as they will no longer accept deferred payments from this firm. However, the firm still finds it optimal not to pay because, under $\hat{\omega}^w(t)$, once a worker has not contributed correct effort in an earlier period, the worker will continue to do so. Note also that this is rational for the worker to do, given that firms are playing $\hat{\omega}^f(t)$: If a firm itself has lost its reputation, $h^j(t-1) = 0$, or a firm's incentive compatibility condition does not hold, (23) fails, then it offers w^c to workers. Under $\hat{\omega}^w(t)$, workers will work at this wage, and this is the best the firm can do since the strategy also specifies that no worker will accept a lower wage and work, or accept a wage as deferred payment. The other deviations are ruled out by similar reasoning.

6 Appendix B:

6.0.4 Proofs

Proof of Lemma 1: Multiplying both sides by M, condition (22) is:

$$\left(1 - \frac{1}{\sigma_j}\right) \sum_{\iota=t}^{\infty} \left(\frac{1 - \beta S_j}{1+r}\right)^{\iota-t} y(\iota) \geq y(t) + \sum_{\iota=t+1}^{\infty} \left(1 - \frac{w^c(\iota)}{\sigma_j w(\iota)}\right) \left(\frac{1 - \beta S_j}{1+r}\right)^{\iota-t} y(\iota):$$

Since $y(\iota+1) = y(\iota)(1+g)$ for all ι the expression becomes:

$$\left(1 - \frac{1}{\sigma_j}\right) y(t) \sum_{\iota=t}^{\infty} \left(\frac{1 - \beta S_j}{1+r}\right)^{\iota-t} (1+g)^{\iota-t} \geq y(t)\left(1 + \sum_{\iota=t+1}^{\infty} \left(1 - \frac{w^c(t)}{\sigma_j w(t)}\right) \left(\frac{1 - \beta S_j}{1+r}\right)^{\iota-t} (1+g)^{\iota-t}\right):$$

Cancelling $y(t)$ from both sides and computing the infinite sums the LHS becomes:

$$\left(1 - \frac{1}{\sigma_j}\right)\left[\frac{1}{1 - \frac{(1-\beta S_j)(1+g)}{1+r}}\right]:$$

Since, in steady state the ratio $\frac{w^c}{w}$ is a constant, the RHS can similarly be computed as:

$$1 + \left(1 - \frac{w^c}{\sigma_j w}\right)\left(\frac{1 - \beta S_j}{1+r}\right)(1+g)\left[\frac{1}{1 - \frac{1-\beta S_j}{1+r}(1+g)}\right]:$$

Combining the two sides and putting on a common denominator the condition reduces to:

$$\left(1 - \frac{1}{\sigma_j}\right) \geq 1 - \frac{(1-\beta S_j)(1+g)}{1+r} + \left(1 - \frac{w^c}{\sigma_j w}\right)\frac{(1-\beta S_j)(1+g)}{1+r}$$

$$\Rightarrow \left(1 - \frac{1}{\sigma_j}\right) \geq 1 - \frac{w^c}{\sigma_j w}\frac{(1-\beta S_j)(1+g)}{1+r}$$

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Since from (17) $\frac{w^c}{w} = \frac{1+r}{\pm(1+g)}$; we have:

$$\dot{} \; 1 \quad , \quad \dot{} \; \frac{(1 \; \dot{} \; ®S_j)}{\pm}$$
$$) \; (1 \; \dot{} \; ®S_j) \quad , \quad \pm;$$

which rearranges to the expression in the lemma. $\bowtie$

Proof of Proposition 1:

Proof: Consider the two sides of equation (32): With g on the horizontal axis, Figure 3 shows the LHS is simply a 45 degree line through the origin. The RHS is given by the term
$®N \; \left[\ln ° + \frac{1}{2} \ln A (M \; \dot{} \; 1) \right]^{\mathbb{C}} +$

$\frac{® \; \frac{N_{\dot{}} \; \Omega M}{° M}}{(A_{\dot{}} \; 1)^2} \; \left[(\ln °)(A \; \dot{} \; 1) \; \dot{} \; A^{1 \dot{} M} \; \dot{} \; A \right]^{\mathbb{C}} + (\ln A) \; \dot{} \; A^{1 \dot{} M} \; (M (A \; \dot{} \; 1) + 1) \; \dot{} \; A \right]^{\mathbb{C} \bowtie}$

Note that g enters RHS through the term Ω only. Recalling that Ω is defined as $\Omega = 1 \; \dot{} \; \frac{(1+½)(1+g)^{\dot{} \; ¾}}{\pm} \; \frac{1}{®}$; then $\frac{d\Omega}{dg} > 0$: Thus the sign of the term, $\frac{dRHS}{dg}$; must equal the sign of $\frac{dRHS}{d\Omega}$: Recall that A and M exceed 1, so it is easy to verify that the term in square brackets < 0 implying that $\frac{dRHS}{d\Omega} > 0$ which in turn implies that $\frac{dRHS}{dg} > 0$. Further, the second derivative, $\frac{d^2RHS}{dg^2} = \frac{dRHS}{d\Omega} \frac{d^2\Omega}{dg^2}$; but since $\frac{d^2\Omega}{dg^2} < 0$; the second derivative is also < 0: Thus plotting both sides of equation (31) together yields two curves (solid lines) of the following generic shapes:
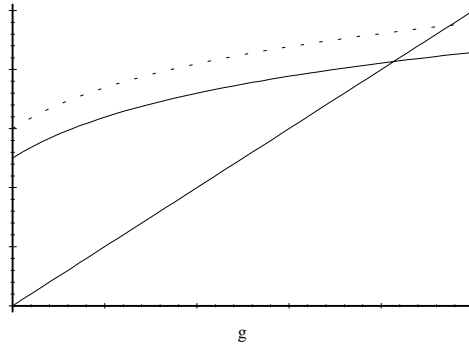

Figure 4

Existence and uniqueness of a positive intersection point are thus guaranteed provided $RHS > 0$ when $g = 0$: Since the RHS is simply the expression $\left[\sum_{m=1}^{M} ®S_m \ln \; \dot{} \; A^{m \dot{} \; 1 \circ} \right]^{\mathbb{C}}$; a sufficient condition for this is that $S_1 > 0$ when $g = 0$; since $S_m > S_1$ for all $m > 1$: That is, a sufficient condition is $\frac{° \dot{} \; 1}{°} \; L_1 ° + \Omega > 0$: Substituting in for L_1 and Ω and setting $g = 0$ yields:

$$® \frac{N}{M} (° \; \dot{} \; 1) > \left[\frac{1 + ½}{\pm} \; \dot{} \; 1 \right]:$$

Finally the steady state equations are calculated on the assumption that firms' incentive compatibility conditions hold. A necessary condition is that, for all m; $®S_m^{\bowtie} \cdot \; 1 \; \dot{} \; \pm$: Since S_m is

increasing in m; provided this holds for the highest m; it will hold for all lower ones, therefore an additional necessary condition is: $^{®}S_M^{¤} \cdot 1 \text{¡} ±: ¤$

Proof of Corollary 1

Consider again both sides of equation (32) that implicitly defines the growth rate. The LHS is clearly independent of $®$. For existence of a steady state, the previous proof showed that the RHS must exceed 0. Recall that the first term on the RHS >0 while the second <0. Thus an implication of existence is that the absolute value of the first term must exceed the second. By inspection, note that the first term is proportionally increasing in $®$: The second term is, however, increasing less than proportionately with $®$ since it is multiplied by $®$

$$\left(\frac{N \text{¡} ΩM}{°M} \right)^3 = ® @ \frac{N \text{¡} 1 \text{¡} \frac{(1+½)(1+g)^{i-¾}}{±} \frac{1}{®}M}{°M} A =$$

$$@ \frac{^{®}N \text{¡} 1 \text{¡} \frac{(1+½)(1+g)^{i-¾}}{±} M}{°M} A^{\left(\frac{μ}{1} \right)} :$$ Thus, since existence implies the positive term exceeds the negative

term and the positive term increases proportionately with $®$ whereas the negative term falls less than proportionately, the RHS is increasing in $®$: For given g; then, the RHS expression is higher, as depicted by the dashed line in Figure 4, so that, necessarily, the new intersection is at a higher g. ¤

Proof of Proposition 2.

From Corollary (1) the economy's growth rate increases in $®$: From (28); this requires that S_m increases in all sectors. Since S_m is endogenous, we specify $S_m (®)$ to denote the value of S_m under a given value of $®$: There must therefore exist a critical level of $®$, solving:

$$^{®}S_M (®) = 1 \text{¡} ±:$$

Denote this critical value $^{®}_M^{0}$: This is the value of $®$ such that, for $® = ^{®}_M^{0}$ condition (22) just holds in sector M: For $® > ^{®}_M^{0}$; (22) can no longer hold in sector M, so that internal labor markets cannot solve the moral hazard problem there. Steady state condition (VI) cannot be satisfied: Notice that, from (27) $S_m (®) < S_M (®)$; since $°_m < °_M$ for all other $m ⊖ M$: Thus (22) continues to hold there. ¤

Proof of Proposition 3

For $m < m^0$ from (27) we have, as before

$$S_m = \frac{°_m \text{¡} 1}{°_m} L_1° \text{¡} \frac{(1+½)(1+g)^{i-¾}}{®±} + \frac{1}{®} = \frac{A^{m \text{¡} 1°} \text{¡} 1}{A^{m \text{¡} 1°}} L_1° + Ω: \tag{35}$$

For $m ¸ m^0$; (27) does not apply. The wage is now given by the binding incentive compatible wage, from equation (18): Clearly the wage cannot be below this and still satisfy worker incentive compatibility in these sectors. The reason the wage cannot be above this is usual one in efficiency wage models: Given that not all human capital obtains w^c; since some work in research and others work in $m < m^0$ obtaining w; all individuals in $m < m^0$ would strictly prefer to take a

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job there. If the steady state wage in $m > m^0$ were to exceed w^c workers in the other sectors could credibly commit to incentive compatible employment at a wage strictly below the steady state wage, and incumbent monopolists would clearly raise profit by employing them at such a wage, thus upsetting the equilibrium. The only wage at which such undercutting by outside workers is not feasible is at $w_j = w^c$; for all $j > m^0$. Note that even though the monopolist must pay the higher wage, w^c; to ensure incentive compatibility of their workers, in order to dissuade entrants using the old technology from stealing their market, they must limit price taking account of the market clearing wage, w. Since the old technology is worthless and the previous incumbent disbanded, entrants with free access to that technology will profitably enter at any price above their marginal costs, therefore the monopolist must set $p_j = \frac{\circ \eta_{j,1}}{w}$; which is independent of contracts operating at the industry level. This level of per-period profit is already computed in equation (20). Substituting for w^c using (18) and using $L_1^\circ = \frac{Y}{M}$, per period profit is $\Psi_m^c = \frac{\pm^2 \circ m_i 1_i \frac{1}{2}}{\pm^2 \circ m} L_1^\circ$: Substituting this into equation (11); using $(1+r)$ from (8) and re-arranging yields:

$$S_m^c = \frac{\pm^2 \circ m_i 1_i \frac{1}{2}}{\pm^2 \circ m} L_1^\circ \left[\frac{(1+\frac{1}{2})(1+g)^{i \frac{3}{4}}}{\circledR \pm} + \frac{1}{\circledR} \right] = \frac{\left(\pm^2 A^{m_i 1 \circ}\right)_i 1_i \frac{1}{2}}{\pm^2 A^{m_i 1 \circ}} L^\circ + \Omega; \qquad (36)$$

where the last equality utilizes the definitions of $\circ_m$ and Ω: Note we denote the research labor in the sectors $m > m^0$ as S_m^c where the c denotes hiring occurs through a "contractor" relationship. Now consider the labor resource constraint: $\sum_{m=1}^M L_m = N_i \sum_{m=1}^M S_m$: Computing the finite sum $\sum_{m=1}^M L_m = \sum_{m=1}^M L_1 \left(\frac{1}{A^{m_i 1}}\right) = L_1 \frac{A_i A_i^{M+1}}{A_i 1}$; and substituting this into the resource constraint yields

$$L_1 = \frac{N_i \sum_{m=1}^M S_m}{\frac{A_i A_i^{M+1}}{A_i 1}}: \qquad (37)$$

The summation in the numerator of this expression is complicated by the fact that now we have to account for S_m differently for low and high m sectors. In particular, re-write the sum as $\sum_{m=1}^M S_m = \sum_{m=1}^{m^0_i 1} S_m + \sum_{m=m^0}^M S_m^c$ where the first summation uses (35) and the second must use (36):

This summation then becomes: $\sum_{m=1}^{m^0_i 1} \left[\frac{A^{m_i 1 \circ}_i 1}{A^{m_i 1 \circ}} L^\circ + \Omega \right] + \sum_{m=m^0}^M \left[\frac{(\pm^2 A^{m_i 1 \circ})_i 1_i \frac{1}{2}}{\pm^2 A^{m_i 1 \circ}} L^\circ + \Omega \right]$

Again, computation of these finite summations yields:
$\frac{(\Omega+L_1^\circ)M\pm^2(A_i 1)+L_1 \left[A^{1_i M}(1+\frac{1}{2})+\pm^2 A + A^{2_i m^0}\left(1+\frac{1}{2}_i \pm^2\right)\right]}{(A_i 1)\pm^2}$; which when substituted back into (37) gives:

$$L_1 = \frac{N_i @ \frac{(\Omega+L_1^\circ)M\pm^2(A_i 1)+L_1 \left[A^{1_i M}(1+\frac{1}{2})+\pm^2 A + A^{2_i m^0}\left(1+\frac{1}{2}_i \pm^2\right)\right]}{(A_i 1)\pm^2}}{\frac{A_i A_i^{M+1}}{A_i 1}}.$$ Solving this for L_1 yields:

$$L_1 = \frac{(A_i 1)(N_i \Omega M)}{\left(A^{1_i M}_i A^{2_i m^0}\right)\frac{1+\frac{1}{2}}{\pm^2}_i 1 + \circ M(A_i 1)}:$$ Notice that, when $m^0 = M+1$; that is when all sectors can hire through an internal labor market, this expression reduces to (30):

Now consider the growth equation, equation (28) which must also now be split up between sectors below and above m^0:

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$$g = \sum_{m=1}^{m^0_i 1} \circledR S_m \ln\left(A^{m_i 1\circ}\right) + \sum_{m=m^0}^{M} \circledR S_m^c \ln\left(A^{m_i 1\circ}\right)$$
$$= \circledR \sum_{m=1}^{m^0_i 1} S_m ((\ln A) m_i \ln A + \ln\circ) + \circledR \sum_{m=m^0}^{M} S_m^c ((\ln A) m_i \ln A + \ln\circ)$$

From (35)$_3$; the first summation becomes:

$$\circledR \sum_{m=1}^{m^0_i 1} \frac{A^{m_i 1\circ_i 1}}{A^{m_i 1\circ}} L_1\circ + \Omega ((\ln A) m_i \ln A + \ln\circ)$$
$$= \circledR \sum_{m=1}^{m^0_i 1} (L_1\circ + \Omega)((\ln A) m_i \ln A + \ln\circ)_i \circledR \sum_{m=1}^{m^0_i 1} \frac{1}{A^{m_i 1\circ}} L\circ ((\ln A) m_i \ln A + \ln\circ)$$

when this finite sum is computed we obtain:

$$\circledR (L_1\circ + \Omega) (\ln A) \frac{1}{2} \left(m^{02}_i 3m^0 + 1 \right) + (\ln\circ) \left(m^0_i 1 \right)$$
$$+\circledR L_1 \frac{(\ln\circ) A^{2_i m^0}_i A (A_i 1) + (\ln A) A^{2_i m^0}(A(m^0_i 1) + 2_i m^0)}{(A_i 1)^2} \tag{38}$$

From (36)$_i$ the second summation becomes:

$$\circledR \sum_{m=m^0}^{M} \frac{(\pm^2 A^{m_i 1\circ})_i 1_i \frac{1}{2}}{\pm^2 A^{m_i 1\circ}} L\circ + \Omega ((\ln A) m_i \ln A + \ln\circ) =$$
$$\circledR \sum_{m=m^0}^{M} 1_i \frac{1+\frac{1}{2}}{\pm^2 A^{m_i 1\circ}} L\circ + \Omega ((\ln A) m_i \ln A + \ln\circ) =$$
$$\circledR \sum_{m=m^0}^{M} [L\circ + \Omega]((\ln A) m_i \ln A + \ln\circ)_i \circledR \sum_{m=m^0}^{M} \frac{1+\frac{1}{2}}{\pm^2 A^{m_i 1\circ}} L\circ ((\ln A) m_i \ln A + \ln\circ) =$$
$$\circledR (L_1\circ + \Omega) (\ln A) \frac{3}{2} \left(m^0_i (M + 1) \right)_i \frac{1}{2} \left(m^{02}_i (M + 1)^2 \right) + (\ln\circ) \left(M + 1_i m^0 \right)$$
$$+ \frac{\circledR (1 + \frac{1}{2}) L_1:}{\pm^2 (A_i 1)^2} \left[(\ln\circ) A^{1_i M}_i A^{2_i m^0} (A_i 1) + (\ln A) A^{1_i M}(1 + A(M_i 1))_i A^{2_i m^0}(A(m^0_i 1) + 2_i m^0) \right] \tag{39}$$

Combining the two parts (38) and (39) yields the analogous expression to (32) but for the case when sectors m^0 and beyond must hire contractors:

$$g = \circledR (L_1\circ + \Omega) (\ln A) \frac{1}{2} \left(m^{02}_i 3m^0 + 1 \right) + (\ln\circ) [m^0_i 1]$$
$$+\circledR L_1 \frac{(\ln\circ) A^{2_i m^0}_i A (A_i 1) + (\ln A) A^{2_i m^0}(A(m^0_i 1) + 2_i m^0)}{(A_i 1)^2}$$
$$+ \circledR (L_1\circ + \Omega) (\ln A) \frac{3}{2} (m^0_i (M + 1))_i \frac{1}{2} \left(m^{02}_i (M + 1)^2 \right) + (\ln\circ) (M + 1_i m^0)$$
$$+\circledR (1 + \frac{1}{2}) L_1 \frac{(\ln\circ) A^{1_i M}_i A^{2_i m^0} (A_i 1) + (\ln A) A^{1_i M}(1 + A(M_i 1))_i A^{2_i m^0}(A(m^0_i 1) + 2_i m^0)}{\pm^2 (A_i 1)^2}.$$

This simplifies to:

$$g = (\circ L_1 M + \Omega M) \circledR (\ln\circ) + \frac{1}{2} \circledR (\ln A) (M_i 1)$$
$$+\frac{\circledR L_1}{(A_i 1)^2} \left[(\ln\circ) (A_i 1) A^{2_i m^0} 1_i \frac{1+\frac{1}{2}}{\pm^2} + \frac{1+\frac{1}{2}}{\pm^2} A^{1_i M}_i A \right.$$
$$\left. + (\ln A) A^{2_i m^0}(A(1_i m^0) + m^0_i 2) \frac{1+\frac{1}{2}}{\pm^2}_i 1 + \frac{1+\frac{1}{2}}{\pm^2} A^{1_i M} [M (A_i 1) + 1]_i A \right] \tag{40}$$

where Ω is defined as previously and now $L_1 = \frac{(A_i 1)(N_i \Omega M)}{\left(A^{1_i M}_i A^{2_i m^0}\right) \frac{1+\frac{1}{2}}{\pm^2}_i 1 + \circ M(A_i 1)}$: This is more complicated than (32) due to the terms involving m^0. Note that when $m^0 = M + 1$; (when all sectors hire on an internal labor market) the expression simply reduces to (32):

Unlike the case for (32) it is not possible to compute the sign of the slope of the RHS function with g unambiguously. However, existence can still be proved by noting that the g terms only enter RHS through Ω once again: Thus, the sign of the derivative $\frac{dRHS}{dg} = \frac{dRHS}{d\Omega}$ since $\frac{d\Omega}{dg} > 0$: Recall also that $\frac{d^2\Omega}{dg^2} < 0$: Thus also because the RHS is linear in Ω; the sign of $\frac{d^2RHS}{dg^2} = \frac{dRHS}{d\Omega}\frac{d^2\Omega}{dg^2}$; which is therefore opposite in sign to $\frac{dRHS}{dg}$: The first derivative thus always has opposite sign to the second derivative. We thus either have: (1) $\frac{dRHS}{dg} > 0$ and $\frac{d^2RHS}{dg^2} < 0$ or (2): $\frac{dRHS}{dg} < 0$ and $\frac{d^2RHS}{dg^2} > 0$: Therefore, given that the LHS is a 45 degree line through the origin, a sufficient condition for both existence and uniqueness is that, for $g = 0$, RHS > 0: To see this, consider Figure 5 below. The RHS is represented by one of the two curved lines. The upper one corresponding to situation (1) and the lower to situation (2). Even though we do not know which one holds, in either case, a crossing point is assured provided that the point at which RHS meets the y axis exceeds zero. Moreover the crossing points are unique, from the fact that the sign of the first derivative, in either case, does not change.
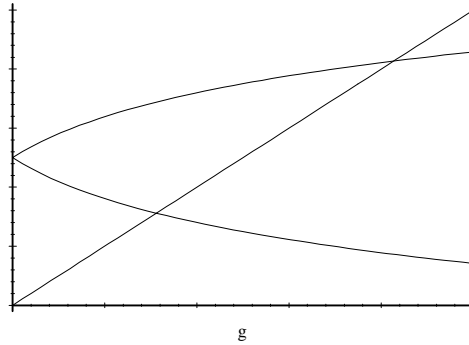

g

Figure 5

A sufficient condition for the RHS>0 when $g = 0$ is that S_1 computed using (36) exceeds 0. This implies that existence is ensured for any m since S_m for $m > 1$ will strictly exceed S_1: A sufficient condition for $S_1 > 0$ is:

$$\circledR\frac{N}{M}^{\mu}\circ_i \left(\frac{1+\frac12}{\pm^2}\right)^{\P} > \frac{\mu\left(\frac{1+\frac12}{\pm}_i 1\right)^{\P}}{\tilde{A}}$$
$$\frac{(1+\frac12)}{\pm^2} + \frac{A^{1_i M}_i A^{2_i m^0}}{(A_i 1)M}\left(\frac{1+\frac12}{\pm^2}_i 1\right)^{\!\!\P!}:$$

Since the last term is negative, the simpler condition in the statement of the Proposition is sufficient to ensure this.

Finally, note that the worker's incentive compatibility condition (18) is satisfied by construction and has been used in computing (20) and thus in (36); so this holds immediately provided (33) is satisfied. Thus, all that is required is for the firms' incentive compatibility conditions to hold in sectors $m < m^0$; as stated in the proposition.¤

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Proof of Proposition 4:

We consider the effect of an arbitrarily small increase in $\circledR$ starting from a point at which (34) holds with equality. Such an increase in $\circledR$ will imply that m^0 necessarily falls. The proof here establishes that, as m^0 falls, g necessarily also falls. The variable m^0 is integer valued, but nothing in the existence equation, equation (40) requires this, so that here we establish that g falls for a continuous decline in m^0. To obtain the discrete version we can integrate over the continuous changes considered here, which, since all of the same sign, will yield a continuous change of the same sign. Re-arranging equation (40) slightly yields:

$$g = \begin{bmatrix} \frac{\circledR L_1}{(A_i 1)^2} \end{bmatrix} \left[\frac{(\circ LM + \Omega M)_3 \circledR (\ln \circ)_3 + \frac{1}{2}\circledR(\ln A)(M_i 1)}{(\ln \circ)(A_i 1) A^{2_i m^0}_3 [1_i \frac{1+\frac{1}{2}}{\pm^2} + \frac{1+\frac{1}{2}}{\pm^2}A^{1_i M}_i A] + (\ln A) A^{2_i m^0}(A(1_i m^0)+m^0_i 2) [\frac{1+\frac{1}{2}}{\pm^2}_i 1] + \frac{1+\frac{1}{2}}{\pm^2}A^{1_i M}[M_3(A_i 1)+1]_i A} \right]^{\frac{1}{3}} A$$

where $L_1 = \frac{(N_i \Omega M)(A_i 1)}{\left(A^{1_i M}_i A^{2_i m^0}\right) \frac{1+\frac{1}{2}}{\pm^2}_i 1 + \circ M(A_i 1)}$ as solved previously and recall $\Omega = \frac{1}{\circledR}\left[1_i \frac{(1+\frac{1}{2})(1+g)^{i \frac{3}{4}}}{\pm}\right]$.

Substituting in for L_1, the right hand side then becomes a function of Ω, m^0 and exogenous parameters where g enters only through the term Ω. Now consider the right hand side as m^0 increases. This corresponds to an increase in the number of sectors that can hire using internal labor markets. We show that this derivative $\frac{dRHS}{dm^0} > 0$ which implies that, since the LHS is independent of m^0, the steady state level of g increases with m^0. Since m^0 enters both through L_1 and directly into RHS, this derivative is complicated but straightforward, though tedious, to compute. It is composed of the following 5 parts:

$$\frac{dRHS}{dm^0} = {}_i \frac{1}{2}\circledR A^{2_i m^0}(\ln A)L_1 \frac{\circ M(2\ln \circ_i \ln A + M\ln A)(\pm^2_i 1_i \frac{1}{2})}{(A^{1_i M}_i A^{2_i m^0})(1+\frac{1}{2}_i \pm^2)+\circ \pm^2 M(A_i 1)}$$

$$+\circledR L_1\left\{ \frac{\ln \circ(A_i 1) A^{1_i M}_i A^{2_i m^0} + (\ln A) A^{1_i M}(1_i M+AM)+A^{2_i m^0}(A(1_i m^0)+m^0_i 2)}{(A_i 1)^2} \right\}$$

$${}_i L_1^\circledR \left[\frac{\ln \circ A^{2_i m^0}_i A (1_i A)+(\ln A) A^{2_i m^0}(A(1_i m^0)+m^0_i 2)+A}{(A_i 1)^2} \right] A^{2_i m^0}(\ln A)\frac{\pm^2_i \frac{1}{2}_i 1}{(A^{1_i M}_i A^{2_i m^0})(1+\frac{1}{2}_i \pm^2)+\circ \pm^2 M(A_i 1)}$$

$${}_i L_1^\circledR (\ln A) A^{2_i m^0} \frac{(\ln \circ)(A_i 1)+(\ln A)(2_i m^0+A(m^0_i 1))_i(A_i 1)}{(A_i 1)^2}$$

$${}_i (\ln A)^\circledR (1+\frac{1}{2})L_1 A^{2_i m^0} \frac{\ln \circ(1_i A)+\ln A(A(1_i m^0)+m^0_i 2)+(A_i 1)}{(A_i 1)^2 \pm^2}.$$

In signing these expressions recall that, $M > 1$, $A > 1$, $(1+\frac{1}{2}) > \pm^2$. Consider the last two terms, these reduce to:

$$(\ln A)^\circledR (1+\frac{1}{2}) L_1 A^{2_i m^0} \left[\frac{1+\frac{1}{2}}{\pm^2}_i 1\right] \frac{(\ln \circ)(A_i 1)+(\ln A)(A(m^0_i 1)+2_i m^0)_i(A_i 1)}{(A_i 1)^2}.$$

Provided that $m^0 \geq 2$ this exceeds 0 since then, always, $(\ln A)(A(m^0_i 1)+2_i m^0)_i (A_i 1) > 0$.

Now consider the first term which can be expressed as:

$$\circledR A^{2_i m^0}(\ln A) L_1 \frac{(\pm^2_i \frac{1}{2}_i 1)}{(A^{1_i M}_i A^{2_i m^0})(1+\frac{1}{2}_i \pm^2)+\circ \pm^2 M(A_i 1)} \frac{1}{2}(\circ M(_i 2\ln \circ + \ln A_i M\ln A)).$$

Since the first bracketed expression < 0 and, since $M > 1$, the second is < 0, this first term is positive. From now on denote this first bracketed expression by $(_i ve)$, since the same term enters into the remaining two terms. These can be combined and re-written as:

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$$+(-\text{ve})(\ln\sigma)\left[\frac{h}{2}\sigma^2\,A^{2-m'}\left(A_3(1-A)+(A-1)^3\,A^{1-M}-A^{2-m'}\right)(1+\mu)\right]$$

$$+(-\text{ve})(\ln A)\left[\frac{h}{4}\left(-\sigma^2\,A^{2-m'}(A(1-m')-2+m')+A\,+\right.\right.$$
$$\left.\left.(1+\mu)\,A^{1-M}(1-M+AM)+A^{2-m'}((A(1-m')-2+m'))\right)\right]$$

Expanding the first term yields:

$$(-\text{ve})(\ln\sigma)\left[A^{2-m'}(A-1)^i\sigma^2-(1+\mu)^\complement+(A-1)^i(1+\mu)A^{1-M}-A\sigma^2{}^{\complement i}\right]:$$

Adding and subtracting the term $(A-1)^i-A^{1-M}\sigma^2{}^\complement$ yields:

$$(-\text{ve})(\ln\sigma)\left[A^{2-m'}(A-1)^i\sigma^2-(1+\mu)^\complement+(A-1)^i(1+\mu)A^{1-M}-A^{1-M}\sigma^2{}^\complement+(A-1)^iA^{1-M}\sigma^2-A\sigma^2{}^{\complement i}\right]$$

which rearranges to:

$$(-\text{ve})(\ln\sigma)\left[A^{2-m'}-A^{1-M}\right](A-1)^i\sigma^2-(1+\mu)^\complement+(A-1)^iA^{1-M}-A^\complement\sigma^2{}^i \text{ which is clearly}$$
> 0.

Finally consider the second of the 2 terms above, this can be expressed as:

$$(-\text{ve})(\ln A)\left[A^{2-m'}\,1+\mu-\sigma^2{}^\complement(A(1-m')-2+m')-\sigma^2A+A^{1-M}(1+\mu)(1+AM-M)\right]:$$

Add and subtract $-\sigma^2 A^{1-M}(1+AM-M)$ yielding:

$$A^{2-m'}\,1+\mu-\sigma^2{}^\complement(A(1-m')-2+m')+A^{1-M}\,1+\mu-\sigma^2{}^\complement(1+AM-M)-\sigma^2A+\sigma^2A^{1-M}(1+AM-M)$$

which equals

$$A^{2-m'}\,1+\mu-\sigma^2{}^\complement(A(1-m')-2+m')+A^{1-M}\,1+\mu-\sigma^2{}^\complement(1+AM-M)-\sigma^2{}^\pounds A-A^{1-M}(1+AM-M)^\maltese:$$

The first two terms reduce to:

$$1+\mu-\sigma^2{}^\complement\,A^{2-m'}(A(1-m')-2+m')+A^{1-M}(1+AM-M)^i:$$

For $m'=M+1$ this equals zero, for $m'=1$ this equals $-A+A^{1-M}(1+AM-M)$ which is always less than zero, it is straightforward to verify this is true for values between these extremes too, so that expression is less than or equal to zero for all values of m':

The final term is $-\sigma^2{}^\pounds A-A^{1-M}(1+AM-M)^\maltese$; which, for the same reason, is also less than or equal to zero.

Thus, since the overall sign of the square bracket is negative, the whole expression is positive. An increase in m' increases the RHS. Since the LHS is unchanged, growth unambiguously rises as m' increases. Since the effect of increasing $\circledR$ is for m' to fall, g falls. $\maltese$

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