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# OVERCONFIDENCE AND DELEGATED PORTFOLIO MANAGEMENT

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## Overconfidence and delegated portfolio management\*

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#### Abstract

Following extensive empirical evidence about "market anomalies" and overconfidence, the analysis of financial markets with agents overconfident about the precision of their private information has received a lot of attention. However, all these models consider agents trading for their own account. In this article, we analyse a standard delegated portfolio management problem between a financial institution and a money manager who may be of two types: rational or overconfident. We consider several situations. In each case, we derive the optimal contract and results on the performance of financial institution hiring overconfident managers relative to institutions hiring rational agents, and results on the price impact of overconfidence.

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#### 1 Introduction

Following extensive empirical evidence about "market anomalies" <sup>1</sup> and overconfidence<sup>2</sup>, the analysis of financial markets with overconfident agents has received a lot of attention (see Kyle and Wang (1997), Odean (1998), Wang (1998), Daniel, Hirshleifer and Subrahmanyam (1998), Wang (2001)). <sup>3</sup>

The common results of all these studies is that overconfidence always leads to overly risky investment strategy but may also provide higher expected return.<sup>4</sup>

This last result led to conclusion about delegated portfolio management. For example, Kyle and Wang (1997) conclude that "for some parameter values a fund facing a major rival in an efficient market should hire an overconfident manager" and Wang (2001) shows that institutions hiring overconfident portfolio managers grow faster than those hiring rational managers. However, in order to reach these conclusions, these studies implicitly assume that (i) overconfidence is an observable characteristic, and (ii) managers' incentives are aligned with those of the employing institution. What if these two assumptions do not hold? What is then the compensation contract proposed by the financial institution? What are the consequences on the investment strategy of overconfident agents? In such a situation, what is the impact of overconfidence on prices?

To answer these questions, we analyse a standard delegated portfolio management problem in which a risk neutral financial institution (the principal) hires a money manager (the agent) with limited liability who may be of two types: rational or overconfident. If exerting effort, the agent acquires private information about the value of a risky asset. If the agent is rational, he updates his beliefs about the expected value of the risky asset in a Bayesian fashion. However, if overconfident, the agent over-estimates the precision of his private signal. Based on his updated beliefs, the agent then makes an investment decision. We consider two different cases. First, the agent is risk-averse and price taker (Case PT, hereafter). Second, the agent is risk-neutral and has market power. (Case MP, hereafter).

If hiring an overconfident agent, the principal faces a moral hazard problem on both effort and risk

<sup>&</sup>lt;sup>1</sup>See Daniel, Hirshleifer and Subrahmanyam (1998, Appendix I) for a review.

<sup>&</sup>lt;sup>2</sup>See, for example, Alpert and Raiffa (1982) and Heath and Tversky (1991). See Odean (1998, Section II) for a review of the literature.

<sup>&</sup>lt;sup>3</sup>Another way of modelling irrational behavior is misinterpretation of the expected value of the asset traded (Delong, Shleifer, Summers and Waldmann (1990), Palomino (1996)).

<sup>&</sup>lt;sup>4</sup>The exception is if agents are strategic (i.e., have market power) and do not trade simultaneously. In such a case, overconfidence yields lower expected returns.

(i.e., the amount invested in risky assets), in both cases PT and MP. If hiring a rational agent, the principal faces a moral hazard problem on both risk and effort in case PT but only a moral hazard problem on effort in case MP.

In Case PT, if overconfidence is an observable characteristic, we derive conditions under which the contract offered by the principal is *first-best*. A consequence of the first-best property is that both rational and overconfident agents choose the same investment strategy, i.e., that desired by the principal. Hence, overconfident and rational agents perform equally well and undertake the same amount of risk. This result implies that results about overconfidence obtained in the case of agents trading for own account may not hold in the case of delegated portfolio management since the contract offered by the principal modifies investment incentives.

If overconfidence is not observable, we derive conditions under which there exists a separating equilibrium such that the principal offers a menu of contracts, rational and overconfident agents choose different contracts, rational agents exert a low effort and overconfident agent exert a high effort. In this equilibrium, overconfident agents perform better (earn higher expected return) and undertake less risk (the variance of return is lower) than rational agents. This result is due to the fact that the contracts proposed by the principal align the risk taking incentives of the overconfident agent with his owns' while still giving incentives to overconfident agents to acquire a large amount of information (given their beliefs.)

In Case MP, we assume that trading takes place in a market similar to that described in Easley and O'Hara (1987). If overconfidence is observable, then, as in Case PT, we derive conditions under which the contract offered by the principal is first-best. Hence, rational and overconfident agents choose the same investment strategy. This implies that overconfidence does not have any impact on prices if informed agents trade on the behalf of a principal, while overconfidence would have some price impact if informed agents were trading for their own account. This results shows that results about the price impact of overconfidence depend on whether one considers trading for own account or delegated portfolio management. In the latter case, contracts offered by the principal influence investment strategies, hence the impact of overconfidence on prices.

If overconfidence is not observable, we show that overconfidence has an impact on prices in situations in which rational and overconfident agents do not acquire the same amount of information. In such situations, the market maker does not know whether informed trades come from rational of overconfident agents. As a consequence, he does not know the precision of the information of the informed agent. Therefore, if the market maker operates in a competitive environment (as is usually assumed in market microstructure models), then the quotes he posts take into account the fact that an informed order may come either from a rational agent or an overconfident agent.

Our results have several implications. First, in terms of accumulation of wealth. In this respect, they should be compared to those of Wang (2001). If overconfidence is observable, then a principal can choose what type of agent to hire and the contract proposed to an overconfident agent differs from that offered to a rational agent in two ways. First, given that an agent overconfident about the precision of his private information believes that he will realize a good performance with a higher probability than the correct one, he accepts "cheaper" contracts than a rational agent does. This information effect makes overconfident agents more attractive than rational agents for the principal. Second, if the overconfident agent is also overconfident about his outside opportunities, the minimum expected utility or revenue he accepts to derive from money management is higher than that accepted by a rational agent. This outside-option effect makes overconfident agents less attractive than rational agents for the principal. If the information effect dominates the outside-option effect, then our results are similar to those of Wang (2001): financial institution hiring overconfident agents grow faster than those hiring rational agents. However, in our model, the result is not due to strategic market interaction between rational and overconfidence acting as a commitment to trade aggressively (as in Kyle and Wang (1997) and Wang (2001)). This is due to their acceptance of "cheap" contracts that rational agents refuse. Conversely, if the outside-option effect dominates the information effect, then financial institution hiring rational agents grow faster than those hiring overconfident agents.

Second our results have implication for the debate on the origin of very high trading volume in financial markets. As already mentioned, the literature on overconfidence has established that overconfident agents trade too large quantities. In the case of delegated portfolio management, Dow and Gorton (1997) provide a second reason for excessive trading volume: agency problem between principals and agents who want to show that they are informed, hence are very active in the market. Our results suggest that a principal offering the appropriate contract can mitigate agents' incentives to trade due to overconfidence. Hence, excessive trading volume would have two sources: overconfidence of agents trading for own account and agency problem in the case of delegated portfolio management.

Last, our results also stress the difference between trading for proper account and delegated portfolio

management when comparing the performances of various categories of agents. If trading for their own account, overconfident agents perform better than rational ones (i.e., earn higher expected return and expected utility) if there is strategic market interaction. However, in the case of delegated portfolio management with observable overconfidence, overconfident agents always perform worse than rational ones. Principals (i.e., financial institutions) receive all the benefit from their overconfidence.

The organization of the paper is as follows. Section 2 reviews the related literature. Section 3 presents the model with a risk-averse, price-taking agent and consider trading for own account as a benchmark case. Section 4 presents the results in case of delegated portfolio management. Section 5 analyses the case with a risk-neutral agent with market power while Section 6 concludes. All the proofs are contained in the Appendix.

#### 2 Related literature

Our article bridges the literature on delegated portfolio management and that on overconfidence in financial markets.

Bhattacharya and Pfleiderer (1985) were the first to study delegated portfolio management in a principal-agent framework. However, their model is more one of hidden information rather than hidden action since the principal can verify the level of risk taken by the agent.<sup>5</sup>

Cohen and Starks(1988), Admati and Pfleiderer (1996), Diamond (1998), Palomino and Prat (2003) study delegated portfolio management with moral hazard on both effort and risk. Cohen and Stark (1988) derive conditions under which the manager exerts more effort but chooses a riskier portfolio than investors prefer. Admati and Pfleiderer (1996) look at the impact of benchmarking on behavior. They show that, in general, benchmarking is inconsistent with obtaining the optimal portfolio and tends to decrease incentives to exert effort. Diamond (1998) show that if the control space of the agent has full dimensionality, (i.e., the principal has fewer degrees of freedom in setting the incentives than the agent has degrees of freedom in responding), then as the cost of effort shrinks, the optimal contracts converges to a linear contract. Palomino and Prat (2003) consider the case in which the agent has limited liability. They show that there exists an optimal contract which takes the form of a bonus contract.

The influence of overconfidence on contracts has been studied by Gervais, Heaton and Odean (2002)

<sup>&</sup>lt;sup>5</sup>This type of problem is also analysed in Stoughton (1993).

and Hackbart (2002). Gervais, Heaton and Odean consider a capital budgeting problem faced by a risk averse manager who may be overoptimistic or overconfident. They find that a risk neutral principal may be better off hiring a moderately overconfident agent than rational one. The main difference between their study and ours is that we study the case in which the overconfidence level of the managers is not observable and the agent has the choice between several effort levels. In such a case, the principal cannot offer first-best contract. However, we show that the principal can use a menu of contracts to screen agents.

Harckbart (2002) studies the impact of overconfidence on capital structure and also looks at contracts based on a cash salary, a bonus, an equity stake, and executive stock options. However, the optimal contract is not derived.

The consequences of overconfidence in financial markets has been studied both in the context of perfectly and imperfectly competitive markets. Under the assumption of perfect competition, Odean (1998, Section III.A) studies a market in which all informed agents are overconfident about the precision of their information. He shows that as overconfidence increases, trading volume and price volatility increase and overconfident agents' expected utility is lower than if their beliefs were properly calibrated.

Daniel et al. (1998) study price reactions to public and private information. They show that overconfidence increases price volatility around private signals, and that price moves resulting from the arrival of private information are on average partially reversed in the long run.

Wang (2001, Section III) studies population dynamics in the presence of rational and overconfident agents. He shows that if overconfident agents are moderately overconfident and their initial share of then population is above some treshold, then overconfident agents as a group will dominate the economy in the long run.

Finally, Daniel et al. (2001) derive an asset pricing model taking into account agents' overconfidence. In an economy in which agents are risk averse with negative exponential utility, and uncertainty is normally distributed, they show that price overreacts to private signals and true expected returns decompose additively into a risk premium and components arising from mispricing.

In imperfectly competitive markets, Odean (1998, Section III.A) shows that overconfidence can lead to market breakdowns, and that when a market equilibrium exists, expected volume, market depth, price volatility and the level of informational efficiency increase as the insider's overconfidence increase.

Kyle and Wang (1997) and Wang (2001, Section II) show that in market with two informed agents,

overconfidence acts as a commitment to trade aggressively. As a consequence, an overconfident informed agent may earn a higher expected utility than a rational one and overconfident agents may dominate the economy in the long run.

Finally, Caballe and Sakovics (2003) differentiate between private self-confidence (the self confidence of the speculators) and public self-confidence (the self-confidence they attribute the their competitors). They show that public self-confidence and private self-confidence have different effects (sometime opposite) on trading volume, price volatility, informational efficiency and expected profits.

#### 3 The model

We consider the following economy. There is one risky asset and a risk-free asset with return normalized to 1. The return V of the risky asset is  $V_H > 1$  with probability 1/2,  $V_L < 1$  with probability 1/2, and E(V) = 1.

If exerting effort at a cost c, the agent receives private information. The signal he receives is either  $s_H$  or  $s_L$ . Conditional on signals, the distribution of the return of the risky asset is

$$Prob(V_i|s_i) = (1+k)/2 \quad j = H, L$$
  
 $Prob(V_i|s_j) = (1-k)/2 \quad i = H, L, j = H, L, i \neq j$ 

with  $k \in (0, 1)$ .

We define overconfidence as in Gervais, Heaton and Odean (2002). That is, after receiving a signal  $s_i$  (i = H, L), an overconfidence agent believes that

$$Prob(V_i|s_i) = (1+K)/2 \quad j = H, L$$
  
 $Prob(V_i|s_i) = (1-K)/2 \quad i = H, L, j = H, L, i \neq j$ 

with  $K \in (k, 1)$ . Hence, overconfidence means that the agent perceives the information as more reliable than what it really is. The difference  $K - k \in (0, 1 - k)$  measures the degree of overconfidence of the agent.

The agent is risk averse with utility

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}$$

with  $\gamma \in (0,1)$ .

#### 3.1 Benchmark case: Trading for own account

As a Benchmark, we consider the case in which the agent first acquires information and then trades for his own account.

Proposition 1 Let

$$A_H(K) = \left(\frac{(1+K)(V_H - 1)}{(1-K)(1-V_L)}\right)^{1/\gamma} \quad A_L(K) = \left(\frac{(1-K)(V_H - 1)}{(1+K)(1-V_L)}\right)^{1/\gamma} \tag{1}$$

$$x(K,c,s_i) = \frac{(A_i(K)-1)(1-c)}{(V_H-1)+A_i(K)(1-V_L)} \qquad i = H,L$$
(2)

and

$$U_o(K, s_i) = (1 - c)^{1 - \gamma} \left\{ \frac{(1 + K)}{2(1 - \gamma)} \left[ 1 + \frac{x(K, c, s_i)}{(1 - c)} (V_i - 1) \right]^{1 - \gamma} + \frac{(1 - K)}{2(1 - \gamma)} \left[ 1 + \frac{x(K, c, s_i)}{(1 - c)} (V_j - 1) \right]^{1 - \gamma} \right\} \quad j \neq i$$

(i) If 
$$c < 1 - \left(\frac{2}{U_o(K, s_H) + U_o(K, s_L)}\right)^{1/(1-\gamma)}, \tag{4}$$

an agent with overconfidence level K acquires information and trades a quantity  $x(K, c, s_i)$  when receiving a signal  $s_i$ .

- (ii) The expected return of an overconfident agent is larger than that of a rational agent
- (iii) The variance of return of an overconfident agent is larger than that of a rational agent.

These results are similar to those of De Long, Shleifer, Summers and Waldmann (1990), Odean (1998) and Wang (2001) in the context of perfectly competitive markets. Overconfidence (or overoptimism) generates incentives to trade larger quantities than rational agents. As a consequence, overconfident agents earn a higher expected return but their investment is riskier.

## 4 Delegated portfolio management

The principal is risk-neutral and cannot acquire information at any cost. This implies that if he does not hire an agent, his expected revenue is zero. The amount invested in the risky asset x belongs to  $[-\bar{x}, \bar{x}]$ . This means that there is an upper limit to the amount the agent can borrow to invest in the risky asset or shortsell. To make the problem interesting, we assume  $\bar{x} > x(K, c, s_H)$  and  $-\bar{x} < x(K, c, s_L)$ .

We denote  $\bar{U}_K \geq 0$  the reservation expected utility of the agent. If  $\bar{U}_K$  is increasing in K, it means that the agent's overconfidence has two dimensions: precision of private information and the value of outside options. Conversely, if for all K,  $\bar{U}_K = \bar{U}$ , then overconfidence has one single dimension: the precision of the private information.

We also assume that the agent has limited liability, hence cannot receive a negative compensation.

In such a situation, if an agent acquires information and acts in the interest of the principal, he trades a quantity  $\bar{x}$  if he receives the signal  $s = s_H$  and he trades a quantity  $-\bar{x}$  if he receives the signal  $s = s_L$ .

If hired, the portfolio chosen by the agent is not verifiable. Therefore, it cannot be contracted upon. It follows that the moral hazard problem faced by the principal is twofold. He must provide the agent incentives to 1) exert effort and acquire information, and 2) take the appropriate level of risk.

We consider two cases. First, overconfidence is observable. That is the level of overconfidence of the agent is common knowledge to the principal and the agent. In the second case, we will assume that the principal does not know whether he is making an offer to a rational or an overconfident agent.

#### 4.1 Overconfidence is observable

Denote  $R[x(s_i), V_j]$  (i, j = H, L) the realized return of the agent if he trades a quantity x after having received a signal  $s_i$  and  $V_j$  is realized, i.e.,

$$R[x(s_i), V_i] = 1 + x(s_i)(V_i - 1)$$

with  $(j \neq i)$ 

As is standard in contract theory, the principal has all the bargaining power and makes a take-it-or-leave-it offer to the agent (see, e.g., Salanie (1997) and Laffont and Martimort (2001) for surveys on contract theory).

Denote  $E_K(.)$  and  $E_k(.)$  the expectation operators using overconfident or rational beliefs, respectively. The problem of the principal is to choose a contract  $h^*(R)$  which maximizes

$$\frac{1}{2}\left(E_k\{R(x^*(s_H), V) - h[R(x^*(s_H), V)]|s_H\} + E_k\{R(x^*(s_L), V) - h[R(x^*(s_L), V)]|s_L\}\right)$$
 (5)

subject to

$$x^*(s_i) \in \operatorname{argmax} E_K\{U[h^*(R(x, V) - c)]|s_i\} \qquad i = H, L$$
 (6)

$$x^*(\emptyset) \in \operatorname{argmax} E\{U[h^*(R(x,V))]|\emptyset\}$$
(7)

where "\mathbb{0}" means that the agent has not acquired information.

$$\frac{1}{2}E_K\{U[h^*(R(x^*(s_H),V))-c]|s_H\} + \frac{1}{2}E_K\{U[h^*(R(x^*(s_L),V))-c]|s_L\} \geq E\{U[h^*(R(x^*(\emptyset),V))]|\emptyset\} \ \ (8)$$

$$\frac{1}{2}E_K\{U[h^*(R(x^*(s_H), V)) - c]|s_H\} + \frac{1}{2}E_K\{U[h^*(R(x^*(s_L), V)) - c]|s_L\} \ge \bar{U}_K$$
(9)

$$h^*[R(x^*(s_i), V_j)] \ge c \quad i, j = H, L$$
 (10)

Equations (6) and (7) are the incentive compatibility constraints on risk if the agent has acquired information and has not acquired information, respectively. Equation (8) represents the incentive compatibility constraint on effort while Equations (9) and (10) represent the participation and limited liability constraints, respectively.

Lemma 2 For any contract h(R) of the shape

$$h(R|\alpha_0, \alpha_1) = \begin{cases} \alpha_0 & \text{if } R \le 1\\ \alpha_1 + \beta R & \text{if } R > 1 \end{cases}$$

$$\tag{11}$$

with  $\beta > 0$ , an informed agent trades quantities  $x(s_H) = \bar{x}$  and  $x(s_L) = -\bar{x}$ .

Lemma 2 means that a contract of the shape of h solves the moral hazard problem on risk.

Definition 3 A contract  $h^*$  is first best if it is a solution of the program (5)-(10) such that the agent chooses (i)  $x(s_H) = \bar{x}$ ,  $x(s_L) = -\bar{x}$  and (ii)  $E_K[U(h^*)] = \bar{U}_K$ .

Hence, a contract is said to be first-best if the agent chooses the same trading strategy as that the principal would chooses were trading for his own account (Condition (i)) and the contract leaves no rent to the agent (Condition (ii)). We derive now conditions under which such a contract exist.

Proposition 4 There exist  $\bar{U}$  and  $\bar{c}(\bar{U}_K)$  such that if  $\bar{U}_K \in (0, \bar{U})$  and  $c < \bar{c}(\bar{U}_K)$ , then there exists a first-best contract

$$h_K^*(R|\alpha_0^*(K), \alpha_1^*(K)) = \begin{cases} \alpha_0^*(K) & \text{if } R \le 1\\ \alpha_1^*(K) + R & \text{if } R > 1 \end{cases}$$

where  $\alpha_1^*(K)$  and  $\alpha_0^*(K)$  are given by Equations (15) and (16), respectively, in Appendix.

From Lemma 2 and Proposition 4, we deduce that rational and overconfident agents make the same investment decisions. In other words, the optimal contracts align overconfident and rational agents' investment incentives. This result highlights the differences between trading for own account and delegated portfolio management when dealing with the impact of overconfidence on investment strategies.

Corollary 5 Assume that for all  $K \in (k,1)$ , a first best contract  $h_K^*$  exists and  $\bar{U}_K = \bar{U}$ . Then,

- (i)  $\alpha_0^*(K)$  and  $\alpha_1^*(K)$  are decreasing functions of K.
- (ii)  $\alpha_0^*(K)$  and  $\alpha_1^*(K)$  are increasing functions of  $\bar{U}$ .

The corollary implies that if overconfidence has one single dimension (i.e., the precision of the private information), then principals prefer to hire overconfident agents. However, this is not due to better performance by overconfident agents. Here, rational and overconfident agents perform equally well. Principals prefer to hire overconfident agents because they accept "cheaper" contracts than rational agents (i.e., the expected compensation paid to an overconfident agent is lower than that to a rational agent.)

Conversely, if overconfidence has two dimensions (i.e., the precision of the private information and the value of outside options), then a principal who has the choice between hiring a rational and an overconfident agent faces a trade-off. The information effect of overconfidence (part (i) of the corollary) makes overconfident agent accept cheap contract. However, the outside-option effect (part (ii) of the corollary) makes the contract accepted by overconfident agents more expensive than that accepted by rational agent. Hence, if the information effect dominates the outside-option effect, the principal is better off hiring an overconfident agent. If the outside-option effect dominates, then the principal is better off hiring a rational agent.

These results should be compared to those of Wang (2001) on the comparison of performances between rational and overconfident money managers. If the only dimension of overconfidence is the precision of the private information, then the results of Wang (2001) hold when the compensation contract of the agent is taken into account. However, financial institution hiring overconfident agents do not accumulate more wealth than those hiring rational agents because overconfident agents perform better than rational agents but because, overconfident agents accept cheaper contracts than rational agent.

Conversely, if overconfidence has two dimensions and the outside-option effect dominates the information effect, then financial institution hiring rational agents accumulate more wealth that those hiring overconfident agent. In this case, rational agents are those accepting cheaper contracts.

#### 4.2 Overconfidence is not observable

We assume now that the principal cannot observe whether the agent is rational or overconfident. The principal correctly believes that the agent is overconfident with probability  $\theta$  and rational with probability  $1 - \theta$ .

With respect to the case of observable types, the principal faces an additional incentive compatibility constraint. If the first-best contract  $h_k^*$  is offered (such that the participation constraint of a rational agent is binding), then the participation constraint of an overconfident agent is not binding. In other words, this contract leaves some rent to an overconfident agent. As a consequence, an overconfident agent prefers the contract  $h_k^*$  relative to the contract  $h_k^*$ . This implies that if overconfidence is not observable, then the principal faces a trade-off. Either he proposes the contract  $h_k^*$  which is accepted by both types of agents (but leaves some rent to an overconfident agent), or the principal proposes the contract  $h_K^*$  which will be rejected by the agent with probability  $(1 - \theta)$ , i.e., if the agent happens to be rational. We derive the following result.

Proposition 6 Assume that first-best contracts  $h_k^*$  and  $h_K^*$  exist and for all  $K \in (k,1)$ ,  $\bar{U}_K = \bar{U}$ . There exists  $\bar{\theta}$  such that if  $\theta > \bar{\theta}$ , the principal only offers the contract  $h_K^*$ . The contract is accepted by an overconfident agent and rejected by a rational one. If  $\theta < \bar{\theta}$ , then the principal proposes the contract  $h_k^*$  and it is accepted by both types of agents.

The Proposition states that if overconfidence has one single dimension, the agent's trade does not influence prices (hence, returns), and the probability that the agent is overconfident is large (i.e., larger than  $\bar{\theta}$ ), then the principal is willing to screen agents and only hires overconfident agents. If the probability than the agent is overconfident is small (i.e., smaller than  $\bar{\theta}$ ), then both types of agents are hired.

#### 4.3 Two levels of information.

So far, we have assumed that there is only one level of private information. As a consequence, the only possible separating equilibrium is such that only overconfident agent are hired. Here, we extend the previous analysis by assuming that there are two levels (1 and 2) of private information. If a rational manager pays a cost  $c_i$  (i = 1, 2 and  $c_1 < c_2$ ), he receives a signal  $s_{i,j}$  (j = H, L) such that

$$Prob(V_j|s_{ij}) = (1 + k_i)/2 \quad j = H, L$$
  
 $Prob(V_j|s_{ij'}) = (1 - k_i)/2 \quad j = H, L, j' = H, L, j' \neq j$ 

with  $k_i \in (0,1)$  and  $k_1 < k_2$ .

Hence, if exerting a high effort and paying a high cost  $(c_2)$ , the manager receives a more precise information than if exerting a low effort and paying a low cost  $(c_1)$ .

After paying a cost  $c_i$  (i = 1, 2) and receiving a signal  $s_{ij}$  (j = H, L), an overconfident agent believes that

$$Prob(V_j|s_{ij}) = (1 + K_i)/2 \quad j = H, L$$
  
 $Prob(V_j|s_{ij'}) = (1 - K_i)/2 \quad j = H, L, j' = H, L, j \neq j'$ 

with  $K_i \in (k_i, 1)$  and  $K_1 < K_2$ .

For the sake of simplicity, we assume that  $\bar{U}_K = 0$  and restrict our attention to contracts of the shape of (11). From Lemma 2, we know that these contracts solve the moral hazard problem on risk.

Proposition 7 Assume that

$$\frac{1+K_2}{1+K_1} \ge Max\left(1+k_1, \frac{1+k_2}{1+k_1}\right)$$

then there exists  $\bar{c} > 0$ ,  $\bar{\delta} > 0$  and  $\bar{\theta} < 1$  such that if  $c_1 < \bar{c}$ ,  $c_2 - c_1 < \bar{\delta}$  and  $\theta > \bar{\theta}$ , then there exists a separating equilibrium such that

- (i) the principal offers a menu of contracts  $(h(R|c_1, \alpha_1^1), h(R|c_2, \alpha_1^2))$ ,
- (ii) a rational agent chooses the contract  $h(R|c_1, \alpha_1^1)$  and exerts a low effort,
- (iii) an overconfident agent chooses  $(h(R|c_2,\alpha_1^2))$  and exert a high effort.

The proposition states that for some sets of parameters, types are revealed in equilibrium, overconfident agents exert a high effort and rational agents exert a low effort. The contract offered by the principal aligns rational and overconfident agents' investment incentives but leaves overconfident agent with more incentives to acquire information given their beliefs. To complete the Proposition, it should also be mentioned that there is no separating equilibrium such that rational agent exerts a high effort and overconfident agent exerts low effort, the reason being that any contract which provides a rational agent with incentives to exert a high effort, also provides an overconfident agent to exert a high effort.

One can wonder whether agents have incentives to signal their type before the contract is proposed. This is not the case. The contract accepted by overconfident agents is the same as that they would get if types were observable. As a consequence, they are indifferent between revealing their type or not. A rational agent is better off if types are not observable. The reason is that if types are observable, the principal offers one contract to each type of agent. In such a case, the optimal contract is such that a rational agent is better off exerting low effort rather than no effort. When types are not observable, the principal faces an additional constraint. The contract  $h(R|c_1, \alpha_1^1)$  must provide an expected compensation high enough so that a rational agent does not deviate to choose  $h(R|c_2, \alpha_1^2)$  and still exert a low effort. As a consequence, a rational agent is better off when types are not observable, hence has no incentives to reveal his type.

The separating equilibrium described in Proposition 7 has implication for the comparison of performances and risk undertaken by rational and overconfident agent.

#### Corollary 8 Assume that the separating equilibrium of Proposition 7 holds. Then,

- (i) Overconfident agents perform better than rational agents (i.e., their expected return is higher than that of rational agents).
- (ii) Overconfident agents take less risk than rational agents (i.e., the variance of return of overconfident agents is lower than that of rational agents)

Part (ii) of the corollary contrasts with previously established results showing that overconfidence leads to investment strategies riskier than those of rational agents. However, these results were obtained in the context of agents trading for their own account. Corollary 8 shows how, in the context of delegated portfolio management, the contract offered by the principal influences investment incentives and the risk taking behaviour of overconfident agents relative to rational agents.

### 5 Extension: A risk-neutral strategic agent.

In this section, we consider a situation in which the agent is risk neutral and has market power when trading the risky asset. As a consequence, overconfidence will have an impact on prices.

As in the previous sections, we assume that there is one risky asset. Its value V is  $V_H > 1$  with probability 1/2,  $V_L < 1$  with probability 1/2 and E(V) = 1. This asset is traded in a market similar to that described in Easley and O'Hara (1987). That is, three types of agents participate in the market: a market maker, noise traders and the informed agent. With equal probabilities, noise traders buy a small quantity  $(Z_1)$ , buy a large quantity  $(Z_2)$ , sell a low quantity  $(-Z_1)$ , or sell a large quantity  $(-Z_2)$ . The timing of the trading game is the following. The market maker posts bid and ask prices for the various quantities submitted. With probability 1/2, an order is sent to the market maker by the informed agent, and with probability 1/2, it is sent by a noise trader. The market maker operates in a competitive environment. This implies that for any order (X) that he receives, the market maker expects zero profit from trade. After trading takes place, the value V of the asset is realized.

If the agent acquires information, then the precision of his signal and his beliefs are as in the previous sections.

#### 5.1 Benchmark case: Trading for own account.

Assume that the informed agent's beliefs are common knowledge. When trading for his own account, the informed agent acts so as to maximize his expected profit. Denote P(X) the price posted by the market for a trading order X. We have the following results about strategies and market prices.

Proposition 9 Let  $\rho = Z_2/Z_1$ .

(i) If  $\rho \geq \frac{3K}{3K-2k}$ , then there exists a unique separating equilibrium: an agent who observes a signal  $s_H$   $(s_L)$  always trades a quantity  $Z_2$   $(-Z_2)$ . For all X different from  $Z_2$  and  $-Z_2$ , P(X) = 1.

$$P(Z_2) = \frac{1}{3} \left( (1+k)V_H + (1-k)V_L + 1 \right)$$

and

$$P(-Z_2) = \frac{1}{3} \left( (1-k)V_H + (1+k)V_L + 1 \right)$$

(ii) If  $\rho < \frac{3K}{3K-2k} < 3$  then there exists a unique pooling equilibrium: an agent receiving a signal  $s_H$  ( $s_L$ ) trades a quantity  $Z_2$  (- $Z_2$ ) with probability  $\mu_K^*$  and trades a quantity  $Z_1$  (- $Z_1$ ) with a probability  $(1-\mu_K^*)$ 

where  $\mu_K^*$  is the unique positive solution of Equation (45) in Appendix. For all X different from  $Z_1$ ,  $Z_2$ ,  $-Z_1$  and  $-Z_2$ , P(X) = 1.

$$P(Z_2) = \frac{\mu_K^*[(1+k)V_H + (1-k)V_L] + 1}{2\mu_K^* + 1}$$

$$P(Z_1) = \frac{(1-\mu_K^*)[(1+k)V_H + (1-k)V_L] + 1}{2(1-\mu_K^*) + 1}$$

$$P(-Z_1) = \frac{(1-\mu_K^*)[(1+k)V_L + (1-k)V_H] + 1}{2(1-\mu_K^*) + 1}$$

$$P(-Z_2) = \frac{\mu_K^*[(1+k)V_L + (1-k)V_H] + 1}{2\mu_K^* + 1}$$

The proposition states that overconfidence has an impact on the bid-ask spread when agents trade for their own account. First, if  $\rho \in [3K/(3K-2k),3]$  and the insider is rational then there is a strictly positive bid-ask spread for both small and large quantities. Conversely, if the insider is overconfident, he always trade large quantities, hence  $P(Z_1) - P(-Z_1) = 0$ . Second, if  $\rho < 3K/(3K-2k)$ , then there are positive bid-ask spreads for both small and large quantities. However, these spreads are different if the insider is rational or overconfident. The reason is that, due to their difference in beliefs, different types of traders use different probabilities for randomization between the small and the large quantity in equilibrium, i.e., the unique positive solution of Equation (45) is different if K = k (rational insider) and if K > k (overconfident insider).

#### 5.2 Delegated portfolio management

As in Section 4, we assume that the principal does not have access to private information (hence, if he does not hire an agent his expected profit from trading is zero) and the agent has limited liability, and a reservation utility  $\bar{U}_K \geq 0$ .

First, we assume that overconfidence is observable. Two cases have to be distinguished. If  $\rho > 3$ , then there is no moral hazard on risk. Provided that the agent acquires information, the principal and the agent's incentives are aligned. This implies that the principal only faces moral hazard on effort when hiring an agent. Conversely, if  $\rho \in [3K/(3K-2k), 3]$ , then the principal also faces moral hazard on risk: the expected trading volume of the agent is too large (i.e., if maximizing his expected return, the agent trades a quantity  $|Z_2|$  with probability 1.)

Note that, with respect to the Section 4, the moral hazard problem on risk is different. Here, the objective of the principal is to reduce the trading intensity of the agent while in Section 4, the principal was aiming at increasing trading quantities.

The following proposition derives conditions under which contracts align overconfident agents' incentives with those of rational agents.

Proposition 10 (i) Assume that  $\rho > 3$ . If  $0 < \bar{U}_K < \frac{Z_2}{2(1+k)}[(1+k)V_H + (1-k)V_L - 2]$  and  $c < K\bar{U}_K$  then there exists a first-best contract

$$h_K^o(\pi) = \begin{cases} 0 & \text{if } \pi \le 0\\ \frac{2(\bar{U}_K + c)}{1 + K} - Z_2(V_H - P(Z_2)) + \pi & \text{if } \pi > 0 \end{cases}$$
 (12)

The agent acquires information and trades a quantity  $Z_2$   $(-Z_2)$  when observing  $s_H$   $(s_L)$ .

(ii) Assume that  $\rho < 3$ . There exists  $\bar{\bar{U}} > 0$  such that if  $0 < \bar{U}_K < \bar{\bar{U}}$ , and  $c < K\bar{U}_K$  then there exists a first best contract

$$g_K(\pi) = \begin{cases} 0 & \text{if } \pi \le 0\\ \frac{2(\bar{U}_K + c)}{1 + K} & \text{if } \pi > 0 \end{cases}$$
 (13)

An agent receiving a signal  $s_H$  ( $s_L$ ) trades a quantity  $Z_2$  (- $Z_2$ ) with probability  $\mu_k^*$  and trades a quantity  $Z_1$  (- $Z_1$ ) with a probability  $(1-\mu_k^*)$  where  $\mu_k^*$  is the unique positive solution of Equation (45) with K=k.

The proposition states than when the principal only faces moral hazard on effort (i.e.,  $\rho \geq 3$ ), then there is a contract of the shape described in Lemma 2 which is optimal.

Conversely, when the principal also faces moral hazard on risk (i.e.,  $\rho < 3$ ), such contracts are not optimal. In order to limit the agent's risk taking incentive, there must be a cap to the compensation he receives. This result is along the lines of Palomino and Prat (2003).

We deduce that for parameters such that Proposition 10 holds then rational and overconfident agents choose the same investment strategy. This implies that overconfidence does not influence bid-ask spreads (hence prices) in the case of delegated portfolio management, although overconfident agents have market power.

#### 5.3 Overconfidence is not observable

We assume that overconfidence is not observable and the principal and the market maker correctly believe (ex-ante) that the agent is overconfident with probability  $\theta$  and rational with probability  $1 - \theta$ .

From Proposition 10, we deduce that if for all  $K \in (k, 1)$ ,  $\bar{U}_K = \bar{U}$ , then the optimal contracts for overconfident agents are cheaper than those for rational agents. Therefore, if overconfidence is not observable, the principal faces the same trade-off as in the case of a price-taking agent: if  $\rho > 3$  ( $\rho < 3$ ) either the principal proposes the contract  $h_k^o$  ( $g_k$ ) which is accepted by both types of agents (but leaves some rent to an overconfident agent), or the principal proposes the contract  $h_K^o$  ( $g_K$ ) which will be rejected by the agent with probability  $(1 - \theta)$ , i.e., if the agent happens to be rational. Hence, we have the same type of result as in the previous section.

Proposition 11 Assume that overconfidence is not observable and for all  $K \in (k,1)$ ,  $\bar{U}_K = \bar{U}$  and first-best contracts  $h_K^o$  and  $g_K$  exist. Then, there exists  $\bar{\theta}$  such that if  $\theta > \bar{\theta}$ , if  $\rho > 3$  then the principal only offers the contract  $h_K^o$  and if  $\rho < 3$  the principal only offers the contract  $g_K$ . The contracts are accepted by an overconfident agent and rejected by a rational one.

Here, again, if overconfidence has one single dimension (the interpretation of private information), the principal can screen agents with contracts and only hire overconfident agents. However, since optimal contracts align overconfident agents' investment incentives with those of rational agents, overconfidence does not have any impact on bid-ask spreads.

If there are two levels of information, the situation is different. In equilibria in which rational and overconfident agents pool on the same effort level, then overconfidence will not have an impact on the bid-ask spread since the principal will offer contracts of the shape of  $h_j^o$  and  $g_j$  (j = k, k) in these equilibria. Conversely, in equilibria in which rational and overconfident agents do not pool on the same effort level, overconfidence always has an impact on bid-ask spreads. To see this, consider a situation in  $Z_2/Z_1$  is large and conditions similar to those of Proposition 7 hold (i.e.,  $c_1$  and  $c_2 - c_1$  are small and  $(1 + K_2)/(1 + K_1)$  and  $\theta$  are large). In such a case, a rational agent chooses the low effort level, an overconfident agents chooses a high effort level and both types of agents trade large quantities with probability 1. If overconfidence is not observable, then the expected value of the asset conditional on receiving an order  $Z_2$  is

$$E(V|Z_2) = \frac{2}{3} [\theta E_{k_2}(V|s_H) + (1-\theta)E_{k_1}(V|s_H) + 1]$$

Since market makers operate in a competitive environment,  $P(Z_2) = E(V|Z_2)$ . This implies that overconfidence has an impact on price. However, this is not due to trading intensity. This is due to over-acquisition of information by overconfident agents.

#### 6 Conclusion

We have studied models of delegated portfolio management in which a risk neutral principal hires an agent who is either rational or overconfident.

When overconfidence is observable, we have derived conditions under which the contract proposed to the agents is first best. This implies that the optimal contract fully aligns overconfident agents' incentives to invest with those of rational agents. As a consequence, overconfident and rational agents perform equally well. This implies that if agents have market power, then overconfidence does not have any impact on prices.

When overconfidence is not observable, we have derived conditions under which there exists a separating equilibrium such that the principal offers a menu of contracts, rational and overconfident agents choose different contracts, rational agents exert a low effort while overconfident agent exert a high effort. In this situation, overconfident managers may perform better than rational managers and take less risk and if they have market power, they will have an impact on prices

These results have consequences for the analysis of the price impact of overconfidence. If overconfidence can be detected before proposing compensation contracts (i.e., overconfidence is observable), then contracts can align overconfident agents' risk taking incentives with those of rational agents. This implies that the price impact of overconfidence should be small. If overconfidence is not observable, it will have an impact on price if it leads to over-acquisition of information by overconfident money managers.

Our article extends the current literature on the performance and risk taking behaviour of overconfident agents in financial markets. In particular, it shows the difference between comparing the performance of agents trading for their own account and comparing performances in the context of delegated portfolio management.

## **Appendix**

#### Proof of Proposition 1

Proof of (i) Assume that after paying a cost c, the agent receives a signal  $s_H$ .<sup>6</sup> He maximizes

$$U(W(x)) = \frac{1}{2(1-\gamma)} \left( (1+K)[1-c+x(V_H-1)]^{1-\gamma} + (1-K)[1-c+x(V_L-1)]^{1-\gamma} \right)$$

The first-order condition of utility maximization yields

$$(1+K)(V_H-1)[1-c+x(V_H-1)]^{-\gamma}+(1-K)(V_L-1)[1-c+x(V_L-1)]^{-\gamma}=0$$

This is equivalent to

$$x = x(K, c, s_H) = \frac{(A_H(K) - 1)(1 - c)}{(V_H - 1) + A_H(K)(1 - V_L)}$$

It follows that the expected utility of the agent before receiving his signal is

$$U = \frac{1}{2(1-\gamma)} (1-c)^{1-\gamma} [U_o(K, s_H) + U_o(K, s_L)]$$

where  $U_o(K, s_H)$  and  $U_o(K, s_L)$  are given by Equation (3). The agent acquires information if U > 1. This is equivalent to

$$c < 1 - \left(\frac{2}{U_o(K, s_H) + U_o(K, s_L)}\right)^{1/(1-\gamma)},$$

*Proof of (ii)* The expected return of the agent is

$$R(K) = 1 + \frac{(1+k)}{4} [x(K,c,s_H)(V_H - 1) + x(K,c,s_L)(V_L - 1)] + \frac{(1-k)}{4} [x(K,c,s_H)(V_L - 1) + x(K,c,s_L)(V_H - 1)]$$

Since  $A_H$  and  $A_L$  are increasing and decreasing in K, respectively. This implies that  $x(K, c, s_H)$  and  $x(K, c, S_L)$  are increasing and decreasing in K, respectively. Hence, R(K) is increasing in K.

Proof of (iii) Given that  $(V_H + V_L)/2 = 1$ , we have  $1 - V_L = V_H - 1$ , and  $x(K, c, s_L) = -x(K, c, s_H)$ . The variance of the return is then

$$Var_K = x_H^2 (1 - k^2)(V_H - 1)^2$$

where  $x_H$  stands for  $x(K, c, s_H)$ . Given that  $A_H(K)$  is increasing in K,  $x_H$  is increasing in K. Hence,  $Var_K$  in increasing in K.

<sup>&</sup>lt;sup>6</sup>The proof for the case  $s = s_L$  is identical.

Proof of Lemma 2: Given that h(R) is constant if  $R \leq 1$  and increasing in R if R > 1, it is straightforward that the agent will trade a positive quantity when receiving a signal  $s_H$  and will trade a negative quantity when receiving a signal  $s_L$ . Therefore, if observing  $s_H$ , the expected utility of the agent is

$$E_K[U(h^*)|s_H] = \frac{1 - K}{2} \frac{\alpha_0^{1 - \gamma}}{(1 - \gamma)} + \frac{1 + K}{2} \frac{(\alpha_1 + \beta x(V_H - 1))^{1 - \gamma}}{(1 - \gamma)}$$

and if observing  $s_L$ , the expected utility of the agent is

$$E_K[U(h^*)|s_L] = \frac{1-K}{2} \frac{\alpha_0^{1-\gamma}}{(1-\gamma)} + \frac{1+K}{2} \frac{(\alpha_1 + \beta x(V_L - 1))^{1-\gamma}}{(1-\gamma)}$$

 $E_K[U(h^*)|s_H]$  and  $E_K[U(h^*)|s_L]$  are increasing and decreasing in x, respectively. Hence, the agent chooses  $x(s_H) = \bar{x}$  and  $x(s_L) = -\bar{x}$ .

#### Proof of Proposition 4:

*Proof:* The proof is divided in three steps.

Step 1: Assume that the agent exerts effort, then the contract

$$h(R) = \begin{cases} \alpha_0^* & \text{if } R \le 1\\ \alpha_1^* + R & \text{if } R > 1 \end{cases}$$

with

$$M(k,K) = \left(\frac{(1-k)(1+K)}{(1+k)(1-K)}\right)^{1/\gamma} \tag{14}$$

$$\alpha_1^* = c - 1 - \bar{x}(V_H - 1) + \left(\frac{2(1 - \gamma)\bar{U}M(k, K)^{1 - \gamma}}{1 - K + (1 + K)M(k, K)^{1 - \gamma}}\right)^{1/(1 - \gamma)}$$
(15)

$$\alpha_0^* = c - 1 + \frac{\alpha_1^* + \bar{x}(V_H - 1) - c}{M(k, K)} \tag{16}$$

maximizes the principal's expected revenue and leaves no rent to the agent.

Proof: From the assumption that the unconditional distribution of the risk asset is  $V_H$  with probability 1/2,  $V_L$  with probability 1/2 and E(V) = 1, we deduce that  $V_H - 1 = 1 - V_L$ . Let  $\bar{r} = \bar{x}(V_H - 1) = \bar{x}(1 - V_L)$ . From Step 1, we deduce that the objective of the principal is to maximize

$$\frac{1-k}{2}[-\alpha_0 + 1 - \bar{r}] - \frac{1+k}{2}\alpha_1 \tag{17}$$

subject to

$$\frac{1-k}{2}(\alpha_0 - c)^{1-\gamma} + \frac{1+k}{2}(\alpha_1 + 1 + \bar{r} - c)^{1-\gamma} \ge \bar{U}_K$$
 (18)

The first order condition of revenue maximization for the principal yields

$$\frac{(1-k)(\alpha_0-c)^{\gamma}}{(1-K)} = \frac{(1+k)(\alpha_1+1+\bar{r}-c)^{\gamma}}{(1+K)}$$
(19)

and the constraint (18) is binding. The system of equations (19) and constraint (18) binding has a unique solution  $(\alpha_0^*, \alpha_1^*)$ .

Step 2: Assume that the contract  $h^*$  is proposed. Then, there exists  $\bar{c} > 0$  such that if  $c < \bar{c}$ , then the agent exerts effort.

*Proof:* The agent exerts effort if

$$\frac{1}{2}\left[U\left(\alpha_{0}^{*}\right) + U(\alpha_{1}^{*} + 1 + \bar{r})\right] < \frac{(1 - K)}{2}U\left(\alpha_{0}^{*} - c\right) + \frac{1 + K}{2}U(\alpha_{1}^{*} + 1 + \bar{r} - c) \tag{20}$$

Let

$$Z = \left(\frac{2(1-\gamma)\bar{U}_K M(k,K)^{1-\gamma}}{1-K+(1+K)M(k,K)^{1-\gamma}}\right)^{1/(1-\gamma)}$$
(21)

Using Equation (21), Inequality (20) can be rewritten as

$$\frac{1}{2}\left[U\left(\frac{Z}{M(k,K)}+c\right)+U(Z+c)\right]<\frac{(1-K)}{2}U\left(\frac{Z}{M(k,K)}\right)+\frac{(1+K)}{2}U(Z) \tag{22}$$

If  $\bar{U}_K > 0$  then Z > 0. Given, that K > k > 0, M(k, K) > 1. Therefore, given that U is concave,

$$\frac{1}{2}\left[U\left(\frac{Z}{M(k,K)}\right) + U(Z)\right] < \frac{(1-K)}{2}U\left(\frac{Z}{M(k,K)}\right) + \frac{(1+K)}{2}U(Z) \tag{23}$$

Given that Z is increasing in  $\bar{U}_K$ , We deduce that there exists  $\bar{c}_1(\bar{U}_K) > 0$  such that if  $c < \bar{c}_1(\bar{U}_K)$ , we have the desired result.

Step 3: There exists  $\bar{\bar{U}}$  such that if  $\bar{U}_K < \bar{\bar{U}}$ , then, the expected revenue of the principal from hiring an agent and proposing the contract  $h_K^*$  is strictly positive.

*Proof:* The expected revenue of the principal is

$$Rev = \frac{(1-k)}{2} \left( -c - \frac{Z}{M(k,K)} + 1 - \bar{r} \right) + \frac{1+k}{2} \left( 1 + \bar{r} - c - Z \right)$$

The principal hires an agent if Rev > 0. This is equivalent

$$c < 1 + k\bar{r} - \frac{Z}{2} \left( 1 + k + \frac{1 - k}{M(k, K)} \right)$$
 (24)

If Z is small then the RHS of this inequality is strictly positive. Denote  $\bar{c}(\bar{U}_K)$  the RHS of (24) as a function of  $\bar{U}_K$ . Given that Z is increasing in  $\bar{U}_K$ , there exists  $\bar{U}^o > 0$ , then the such that if  $\bar{U}_K < \bar{U}^o$  then  $\bar{c}(\bar{U}_K) > 0$  and if  $c < \bar{c}(\bar{U}_K)$  then Rev > 0.

Hence, if 
$$\bar{U}_K < \bar{U}^o$$
 and taking  $\bar{c}(\bar{U}_K) = Min(\bar{c}_1(\bar{U}_K), \bar{c}(\bar{U}_K))$ , we have the desired result.

#### Proof of Corollary 5:

(i) Assume that for all  $K \in (k, 1)$ ,  $\bar{U}_K = \bar{U}$ .  $\alpha_0^*$  and  $\alpha_1^*$  are given by Equations (16) and (15), respectively. M(k, K) is increasing in K and

$$\left(\frac{2(1-\gamma)\bar{U}_K M(k,K)^{1-\gamma}}{1-K+(1+K)M(k,K)^{1-\gamma}}\right)^{1/(1-\gamma)}$$

is decreasing in K. As a consequence, we have the desired result.

(ii) From Equations (16) and (15), it is straightforward that 
$$\alpha_0^*$$
 and  $\alpha_1^*$  are increasing in  $\bar{U}$  2.

Proof of Proposition 6: Denote  $Rev_j$  (j=k,K) the revenue of the principal is higher when hiring an agent with the contract  $h_j^*$ . From Corollary 5, we know that  $Rev_K > Rev_k$ . Let  $D = Rev_K - Rev_k$ . If the principal offers the contract  $h_K^*$ , it is only accepted by an overconfident agent. Therefore, the expected revenue of the principal is  $\theta Rev_K$ . If the principal offers the contract  $h_k^*$ , the contract is accepted by both types of agents, hence the revenue of the principal is  $Rev_k$ . Let  $\bar{\theta} = Rev_k/Rev_K$ . If  $\theta \geq \bar{\theta}$ , then the principal proposes the contract  $h_k^*$  while if  $\theta < \leq \bar{\theta}$ , the principal proposes the contract  $h_K^*$ .

Proof of Proposition 7: Given that  $\bar{U} = 0$ , it is straightforward that for any contract of the shape of (11) with parameters  $(\alpha_0, \alpha_1, \beta)$  such that an agent pays an effort cost c, there exists a contract with parameters  $(c, \alpha'_1, 1)$  which provides the same expected utility to an agent. In what follows, we focus on this type of contracts.

Let  $\bar{R} = 1 + \bar{r}$ . A separating equilibrium must satisfy the following conditions:

1) An overconfident agent choosing the contract  $(c_2, \alpha_2, 1)$  is better off exerting a high effort (i.e., paying

 $c_2$ ) than exerting no effort, i.e.,

$$\frac{1+K_2}{2}(\alpha_2+\bar{R}-c_2)^{1-\gamma} \ge \frac{1}{2}\left(c_2^{1-\gamma}+(\alpha_2+\bar{R})^{1-\gamma}\right)$$
 (25)

2) An overconfident agent choosing the contract  $(c_2, \alpha_2, 1)$  is better off exerting a high effort (i.e., paying  $c_2$ ) than exerting a low effort (i.e. paying a cost  $c_1$ ), i.e.,

$$\frac{1+K_2}{2}(\alpha_2+\bar{R}-c_2)^{1-\gamma} \ge \frac{1-K_1}{2}(c_2-c_1)^{1-\gamma} + \frac{1+K_1}{2}(\alpha_2+\bar{R}-c_1)^{1-\gamma}$$
 (26)

3) An overconfident agent choosing the contract  $(c_2, \alpha_2, 1)$  and exerting a high effort is better off than choosing the contract  $(c_1, \alpha_1, 1)$  and exerting a low effort, i.e.,

$$\frac{1+K_2}{2}(\alpha_2+\bar{R}-c_2)^{1-\gamma} \ge \frac{1+K_1}{2}(\alpha_1+\bar{R}-c_1)^{1-\gamma}$$
 (27)

4) An overconfident agent choosing the contract  $(c_2, \alpha_2, 1)$  and exerting a high effort is better off than choosing the contract  $(c_1, \alpha_1, 1)$  and exerting no effort, i.e.,

$$\frac{1+K_2}{2}(\alpha_2+\bar{R}-c_2)^{1-\gamma} \ge \frac{1}{2}\left(c_1^{1-\gamma}+(\alpha_1+\bar{R}-c_1)^{1-\gamma}\right)$$
(28)

5) A rational agent choosing the contract  $(c_1, \alpha_1, 1)$  is better off exerting a low effort than exerting no effort, i.e.,

$$\frac{1+k_1}{2}(\alpha_1+\bar{R}-c_1)^{1-\gamma} \ge \frac{1}{2}\left(c_1^{1-\gamma}+(\alpha_1+\bar{R})^{1-\gamma}\right) \tag{29}$$

6) A rational agent choosing the contract  $(c_1, \alpha_1, 1)$  and exerting low effort is better off than choosing the contract  $(c_2, \alpha_2, 1)$  and exerting no effort, i.e.,

$$\frac{1+k_1}{2}(\alpha_1+\bar{R}-c_1)^{1-\gamma} \ge \frac{1}{2}\left(c_2^{1-\gamma}+(\alpha_2+\bar{R})^{1-\gamma}\right)$$
(30)

7) A rational agent choosing the contract  $(c_1, \alpha_1, 1)$  and exerting low effort is better off than choosing the contract  $(c_2, \alpha_2, 1)$  and exerting a low effort, i.e.,

$$\frac{1+k_1}{2}(\alpha_1+\bar{R}-c_1)^{1-\gamma} \ge \frac{1-k_1}{2}(c_2-c_1)^{1-\gamma} + \frac{1+k_1}{2}(\alpha_2+\bar{R}-c_1)^{1-\gamma}$$
(31)

8) A rational agent choosing the contract  $(c_1, \alpha_1, 1)$  and exerting low effort is better off than choosing the contract  $(c_2, \alpha_2, 1)$  and exerting a high effort (i.e., paying  $c_2$ ), i.e.,

$$\frac{1+k_1}{2}(\alpha_1+\bar{R}-c_1)^{1-\gamma} \ge \frac{1+k_2}{2}(\alpha_2+\bar{R}-c_2)^{1-\gamma}$$
(32)

A separating equilibrium is then a pair of contracts  $(c_i, \alpha_i, 1)$  (i = 1, 2) that maximizes the revenue of the principal and satisfies constraints (25)-(32).

Step 1: there exist contracts such that (27) and (32) are simultaneously satisfied if and only if

$$\frac{1+K_2}{1+K_1} \ge \frac{1+k_2}{1+k_1}$$

*Proof:* Constraints (27) and (32) can be rewritten as

$$\left(\frac{1+K_2}{1+K_1}\right)^{1/(1-\gamma)} (\alpha_2 + \bar{R} - c_2) > \alpha_1 + \bar{R} - c_1$$

and

$$\left(\frac{1+k_2}{1+k_1}\right)^{1/(1-\gamma)}(\alpha_2+\bar{R}-c_2)<\alpha_1+\bar{R}-c_1,$$

respectively. These two inequalities hold simultaneously if and only if

$$\frac{1+K_2}{1+K_1} \ge \frac{1+k_2}{1+k_1}$$

Step 2: For any contract  $(c, \alpha, 1)$  with  $c \ge c_1$ , if  $\alpha > -\bar{R}$ , then there exists  $\bar{c}_1(\alpha)$  such that if  $c_1 < \bar{c}_1(\alpha)$ , then low effort is always preferred to no effort.

*Proof:* If exerting low effort, the profit from the contract  $(-c, \alpha, 1)$  is

$$\frac{1 - L_1}{2} (c - c_1)^{1 - \gamma} + \frac{1 + L_1}{2} (\alpha + \bar{R} - c_1)^{1 - \gamma}$$
(33)

with L = k, K. If exerting no effort, the expected profit is

$$\frac{1}{2}\left(c^{1-\gamma} + (\alpha + \bar{R})^{1-\gamma}\right) \tag{34}$$

If  $c_1 = 0$ , (33) is strictly larger than (34). Therefore, there exists  $\bar{c}_1(\alpha)$  such that if  $c_1 < \bar{c}_1(\alpha)$ , then low effort is always preferred to no effort.

This implies that if  $c_1$  is small enough, then constraints (25), (28), (29) and (30) are satisfied.

Step 3: Constraints (31) and (27) can be rewritten as

$$\alpha_2 < \left\{ (\alpha_1 + \bar{R} - c_1)^{1-\gamma} - \frac{1 - k_1}{1 + k_1} (c_2 - c_1)^{1-\gamma} \right\}^{1/(1-\gamma)} + c_1 - \bar{R}$$
 (35)

and

$$\alpha_2 > \left(\frac{1+K_1}{1+K_2}\right)^{1/(1-g)} (\alpha_1 + \bar{R} - c_1) + c_2 - \bar{R}$$
 (36)

respectively. Denote  $F(\alpha_1)$  and  $G(\alpha_1)$ , the right-hand sides of (35) and (36), respectively. A contract must satisfy  $\alpha_i < 0$ , otherwise this contract generates a negative revenue for the principal. If F(0) > G(0), then there exists contracts such that (27) and (31) are satisfied simultaneously. F(0) > G(0) is equivalent to

$$\left\{ (\bar{R} - c_1)^{1-\gamma} - \frac{1 - k_1}{1 + k_1} (c_2 - c_1)^{1-\gamma} \right\}^{1/(1-\gamma)} > \left( \frac{1 + K_1}{1 + K_2} \right)^{1/(1-g)} (\bar{R} - c_1) + c_2 - c_1$$

Given that  $K_2 > K_1$ , we deduce that there exists  $\delta$  such that if  $c_2 - c_1 < \delta$ , then F(0) > G(0), i.e., Constraints (27) and (31) are satisfied simultaneously.

Step 4: Denote  $\alpha_2^*$  the unique solution of

$$\frac{1+K_2}{2}(\alpha_2+\bar{R}-c_2)^{1-\gamma} = \frac{1-K_1}{2}(c_2-c_1)^{1-\gamma} + \frac{1+K_1}{2}(\alpha_2+\bar{R}-c_1)^{1-\gamma}$$
(37)

(That is,  $\alpha_2^*$  is such that Constraint (26) holds with equality.)

A sufficient condition for Constraint (26) and (27) to hold simultaneously, is that  $F(0) > \alpha_2^*$ . Rewriting Constraint (26),  $\alpha_2^*$  is the solution of

$$\alpha_2 + \bar{R} - c_2 = \left(\frac{1 - K_1}{1 + K_2}(c_2 - c_1)^{1 - \gamma} + \frac{1 + K_1}{1 + K_2}(\alpha_2 + \bar{R} - c_1)^{1 - \gamma}\right)^{1/(1 - \gamma)} = H(\alpha_2)$$
 (38)

Given that  $H(\alpha_2)$  is an increasing function of  $\alpha_2$ ,  $F(0) > \alpha_2^*$  is equivalent to

$$F(0) + \bar{R} - c_2 > H(F(0))$$

This last inequality can be rewritten as

$$\left(\frac{1+K_1}{1+K_2}\right)^{1/(1-\gamma)} (\bar{R}-c_1) > \left(\frac{1-K_1}{1+K_1}(c_2-c_1)^{1-\gamma} + \frac{1+K_1}{1+K_2} \left[ \left(\frac{1+K_1}{1+K_2}\right)^{1/(1-\gamma)} (\bar{R}-c_1) + (c_2-c_1) \right]^{1-\gamma} \right)^{1/(1-\gamma)}$$
(39)

If  $c_2 = c_1$ , then the LHS of (39) is strictly larger than its RHS. Therefore, by continuity, there exists  $\delta_1$  such that if  $c_2 - c_1 < \delta_1$ , then  $F(0) > \alpha_2^*$ .

Step 5: Steps 1 to 4 ensure that the set of contracts such that constraints (25)-(32) are satisfied is non-empty. In this set, the principal chooses the contract that maximizes his expected revenue.

Rewrite (32) as

$$\alpha_2 \le \left(\frac{1+k_1}{1+k_2}\right)^{1/(1-\gamma)} (\alpha_1 + \bar{R} - c_1) + c_2 - \bar{R}$$
 (40)

and denote  $\mathcal{G}(\alpha_1)$ , the RHS of this equation as a function of  $\alpha_1$ . Let  $\hat{\alpha}$  be the unique solution of  $F(\alpha) = G(\alpha)$ .<sup>7</sup> If  $\alpha_2^* \leq F(\hat{\alpha})$ , then the principal chooses  $\alpha_1 = \hat{\alpha}$  and  $\alpha_2 = F(\hat{\alpha}) = G(\hat{\alpha})$ . If  $\alpha_2^* > F(\hat{\alpha})$ , the principal chooses  $\alpha_2 = \alpha_2^*$  and  $\alpha_1 = \mathcal{G}^{-1}(\alpha_2^*)$ .

Claim: There exists  $\delta_2$  such that if  $c_2 - c_1 < \delta_2$  then  $\alpha_2^* > F(\hat{\alpha})$ .

*Proof:* Let  $\alpha_1^* = G^{-1}(\alpha_2^*)$ , i.e.,

$$\alpha_1^* = \left(\frac{1+K_2}{1+K_1}\right)^{1/(1-\gamma)} (\alpha_2^* + \bar{R} - c_2) + c_1 - \bar{R}$$

A sufficient condition for  $\alpha_2^* > F(\hat{\alpha})$  is that  $F(\alpha_1^*) > \alpha_2^*$ . This last inequality is equivalent to

$$\left\{ \left[ \left( \frac{1+K_2}{1+K_1} \right)^{1/(1-\gamma)} (\alpha_2^* + \bar{R} - c_2) + c_2 - c_1 \right]^{1-\gamma} + \frac{1-k_1}{1+k_1} (c_2 - c_1) \right\}^{1-\gamma} > \alpha_2^* + \bar{R} - c_1$$
(41)

Since  $K_2 > K_1$ , the LHS of (41) is strictly larger than its RHS at  $c_2 = c_1$ . By continuity, there exists  $\delta_2$  such that if  $c_2 - c_1 < \delta_2$  then  $\alpha_2^* > F(\hat{\alpha})$ .

Step 6: For this menu of contracts to be an equilibrium, it must also be the case that the principal does not have incentives to deviate from this equilibrium. First, From Step 2, we know that if  $c_1 < \min(c_1(\alpha_2^*), c_1(\mathcal{G}^{-1}(\alpha_2^*)))$ , an agent always exert at least a low effort. As a consequence, whatever the contracts proposed by the principal, there cannot be a situation such that one type of agent exerts effort while the other does not. Second, the principal never deviates and proposes another menu of contracts that satisfies constraints (25)-(32) since the menu derived in Step 5 maximizes his revenue in the set of contracts such that an overconfident agent exerts high effort and a rational agent exerts a low effort.

We deduce that the only potentially profitable deviations for the principal are such that agents pool on the same effort level.

The most profitable deviation such that agents pool on the high effort level is a contract  $(c_2, \hat{\alpha}_2, 1)$  such that Constraint (26) is satisfied (i.e.,  $\hat{\alpha}_2 \geq \alpha_2^*$ ) and

$$\frac{1+k_2}{2}(\hat{\alpha}_2+\bar{R}-c_2)^{1-\gamma} \ge \frac{1-k_1}{2}(c_2-c_1)^{1-\gamma} + \frac{1+k_1}{2}(\hat{\alpha}_2+\bar{R}-c_1)^{1-\gamma}$$
(42)

Let  $\hat{\alpha}$  be such that (42) holds with equality.

Claim: There exists  $\delta_3$  such that if  $c_2 - c_1 < \delta_3$  then  $\alpha_2^* < \hat{\hat{\alpha}}$ .

*Proof:* If  $c_2 = c_1$ , then the LHS of (42) is strictly larger than its RHS. Hence, by continuity, There exists

<sup>&</sup>lt;sup>7</sup>Since  $F(\alpha)$  and  $G(\alpha)$  are increasing in  $\alpha$ , F(0) > G(0), and  $F(c_1 - \bar{R}) < G(c_1 - \bar{R})$ ,  $\hat{\alpha}$  is unique.

 $\delta_3$  such that if  $c_2 - c_1 < \delta_3$  then  $\alpha_2^* < \hat{\hat{\alpha}}$ .

It follows that the cheapest contract such that agents pool on the high effort level is  $(c_2, \hat{\alpha}, 1)$ . The expected revenue of the principal from this contract is

$$Rev(Pool_h) = \frac{1 - k_2}{2}(-c_2 + 1 - \bar{r}) - \frac{1 + k_2}{2}\hat{\hat{\alpha}}$$

The expected revenue from the separating equilibrium is

$$Rev[Sep, \theta] = \theta\left(\frac{1-k_2}{2}(-c_2+1-\bar{r}) - \frac{1+k_2}{2}\alpha_2^*\right) + (1-\theta)\left(\frac{1-k_1}{2}(-c_1+1-\bar{r}) - \frac{1+k_1}{2}\mathcal{G}(\alpha_2^*)\right)$$

Let  $\bar{\delta} = min(\delta, \delta_1, \delta_2, \delta_3)$ . If  $\delta < \bar{\delta}$  then  $\alpha_2^* < \hat{\alpha} < 0$ . We deduce that  $Rev[Sep, 1] > Rev(Pool_h)$ . As a consequence, there exists  $\bar{\theta}_h < 1$  such that if  $\theta > \bar{\theta}_h$ , then there is no profitable deviation such that agent pool on the high effort level.

The only thing left to be done is to show that there is no profitable deviation such that agent pool on the low effort level.

The most profitable deviation such that agents pool on the low effort level is a contract  $(c_1, \hat{\alpha}_1, 1)$  such that

$$\frac{1+k_1}{2}(\hat{\alpha}_1+\bar{R}-c_1)^{1-\gamma} = \frac{c_1^{1-\gamma}}{2} + \frac{1}{2}(\hat{\alpha}_1+\bar{R})^{1-\gamma}$$
(43)

That is, a rational agent is indifferent between paying a cost  $c_1$  and not. The expected revenue of the principal from this contract is

$$Rev(Pool_l) = \frac{1 - k_1}{2}(-c_1 + 1 - \bar{r}) - \frac{1 + k_1}{2}\hat{\alpha}_1$$

Claim: Assume that  $\frac{1+K_2}{1+K_1} > 1+k_1$ . There exists  $\hat{c}$  and  $\delta_4$  such that if  $c_1 < \hat{c}$  and  $\delta < \delta_4$ , then  $\alpha_2^* < \hat{\alpha}_1$ . Proof: Inequality (43) can be rewritten as

$$\hat{\alpha}_1 = \left[ \frac{1}{1+k_1} \left( c_1^{1-\gamma} + (\hat{\alpha}_1 + \bar{R})^{1-\gamma} \right) \right]^{1/(1-\gamma)} + c_1 - \bar{R} = H_1(\hat{\alpha}_1)$$

Let  $H_2(\alpha) = H(\alpha) + c_2 - \bar{R}$  where  $H(\alpha)$  is given by the RHS of (38). Then  $\alpha_2^*$  is the solution of  $\alpha = H_2(\alpha)$ . Given that  $K_2 > k_1$ , we have  $\frac{1-K_1}{1+K_2} < \frac{1}{1+k_1}$ . Furthermore, we have assumed that  $\frac{1+K_1}{1+K_2} < \frac{1}{1+k_1}$ . As a consequence, there exist  $\bar{c}_2$  and  $\delta_4$  such that if  $c_1 < \bar{c}_2$  and  $\delta < \delta_4$ , then for any  $\alpha \in (c_1 - \bar{R}, 0)$ ,  $H_2(\alpha) < H_1(\alpha)$ . This implies that  $\alpha_2^* < \hat{\alpha}_1$ .

Now, assume that  $\alpha_2^* < \hat{\alpha}_1$ .  $Rev[Sep, 1] > Rev(Pool_l)$  is equivalent to

$$(k_2 - k_1)(1 - \bar{r}) - [(1 - k_2)c_2 - (1 - k_1)c_1] > (1 + k_2)\alpha_2^* - (1 + k_1)\hat{\alpha}_1 \tag{44}$$

Given that  $\alpha_2^* < \hat{\alpha}_1 < 0$ , the RHS of (44) is negative. Therefore, there exists  $\delta_5 \leq \delta_4$  such that if  $c_2 - c_1 < \delta_5$ , then the RHS of (44) is positive, hence Inequality (44) holds. As a consequence, there exists  $\bar{\theta}_l < 1$  such that if  $c_2 - c_1 < \delta_5$  and  $\theta > \bar{\theta}_l$  then  $Rev[Sep, \theta] > Rev(Pool_l)$  (i.e., there is no profitable deviation such that agent pool on the low effort level.)

Taking  $\bar{\theta} = \max(\bar{\theta}_h, \bar{\theta}_l)$ ,  $\bar{\delta} = \min(\delta, \delta_1, \dots, \delta_5)$  and  $\bar{c} = \min(c_1(\alpha_2^*), c_1(\mathcal{G}^{-1}(\alpha_2^*), \bar{c}_2))$ , we have the desired result.

Proof of Corollary 8: The expected return of an agent paying an effort cost  $c_i$  (i = 1, 2) is

$$E(R|k_i) = 1 + \frac{\bar{x}}{2}k_i(V_H - V_L)$$

which is an increasing function of  $k_i$ .

The variance of the return of an agent paying an effort cost  $c_i$  (i = 1, 2) is

$$Var(R|k_i) = \frac{\bar{x}^2}{2} \left( (1+k_i)(V_H - 1)^2 + (1-k_i)(V_L - 1)^2 - \frac{1}{2}k_i^2(V_H - V_L)^2 \right)$$

Given that  $(V_H + V_L)/2 = 1$ ,  $\frac{Var(R|k_i)}{dk_i} < 0$  is equivalent to  $k_i > 0$ . Hence,  $Var(R|k_2) < Var(R|k_1)$ .

Proof of Proposition 9: Assume that the agent observes  $s_H$ . (The proof for the case  $s_L$  is similar). First, it is straightforward for any quantity different from  $Z_1$ ,  $Z_2$  the market makers knows that he is facing an insider. Hence, for any X different from  $Z_1$ ,  $Z_2$ , he sets  $P(X) = V_H$ . Second, if the market maker anticipates the insider to always trade a quantity  $Z_2$ , then he sets  $P(Z_1) = 1$  and

$$P(Z_2) = \frac{\mu_K[(1+k)V_H + (1-k)V_L] + 1}{2\mu_K + 1}$$

Therefore, for a separating equilibrium to exist, it must be the case that

$$(E_K(V|S_H) - P(Z_2))Z_2 > (E_K(V|S_H) - P(Z_1))Z_1$$

Using the assumption that  $(V_H + V_L)/2 = 1$ , this last inequality is equivalent to

$$\frac{Z_2}{Z_1} > \frac{3K}{3K - 2k}$$

If this inequality does not hold then the only equilibria are of pooling type. Assume that the insider when observing  $s_H$  chooses trades a quantity  $Z_2$  with probability  $\mu$  and  $Z_1$  with probability  $(1 - \mu)$ . A pooling equilibrium must satisfy the following three conditions:

$$P(Z_2, \mu) = \frac{\mu_K[(1+k)V_H + (1-k)V_L] + 1}{2\mu_K + 1}$$
 
$$P(Z_1, \mu) = \frac{(1-\mu_K)[(1+k)V_H + (1-k)V_L] + 1}{2(1-\mu_K) + 1}$$
 
$$Z_2(E_K(V|s_H) - P(Z_2, \mu)) = Z_1(E_K(V|s_H) - P(Z_1, \mu))$$

This last equation is then equivalent to

$$4\mu^{2}(K-k)(\rho-1) + 2\mu[(K-k)(2-3\rho) + K(1+\rho)] + K(1-3\rho) = 0$$
(45)

where  $\rho = Z_2/Z_1$ .

This completes the proof.

Proof of Proposition 10: Assume that  $\rho \geq 3$ . The expected compensation of the agent is increasing in his profit. Therefore, we deduce from Proposition 9 that if the market maker anticipates that an informed agent trades large quantities with probability 1, then an informed agents always trades large quantities.

It is straightforward that the participation constraint of the agent is binding, i.e., his expected compensation is  $\bar{U}_K$ . Therefore, we only need to check that the incentive compatibility constraint on effort holds and that the expected revenue of the principal is positive. Given the distribution of uncertainty in the economy,  $V_H - P(Z_2) = P(-Z_2) - V_L$ . This implies that the incentive compatibility constraint on effort holds if  $\bar{U}_K > (\bar{U}_K + c)/(1 + K)$ . This is equivalent to  $c < K\bar{U}_K$ .

The principal proposes the contract  $h^{o}(\pi)$  if

$$\frac{1-k}{2}Z_2(V_L - P(Z_2)) - \frac{1+k}{2}\left(\frac{2(\bar{U}_K + c)}{1+K} - Z_2(V_H - P(Z_2))\right) \ge 0 \tag{46}$$

Let  $E_k(V) = \frac{1}{2}[(1+k)V_H + (1-k)V_L]$ . Then, Inequality (46) is equivalent to

$$\bar{U}_K + c < \frac{1+K}{1+k} Z_2 [E_k(V) - 1]$$

Since  $c < K\bar{U}_K$ , if  $\bar{U}_K < Z_2[E_k(V) - 1]/(1 + k)$ , we have the desired result.

Assume now that  $\rho < 3$ . The contract proposed by the principal makes the agent indifferent between trading small and large quantities. As a consequence, he is willing to randomize and trade a large quantity with probability  $\mu_k^*$  and a small quantity with probability  $1 - \mu_k^*$ . In such a case, prices are as given in Proposition 9 (ii).

The incentive compatibility constraint on effort is the same as in the case  $\rho \geq 3$ , we only need to check that the expected revenue of the principal is positive.

The mixed strategy equilibrium requires  $(E_k(V) - P(Z_1))Z_1 = (E_k(V) - P(Z_2))Z_2 = \Pi$ . The principal proposes the contract  $g_K$  if

$$\frac{1-k}{2}Z_2(V_L - P(Z_2)) + \frac{1+k}{2}\left(-\frac{2(\bar{U}_K + c)}{1+K} + \Pi\right) \ge 0 \tag{47}$$

Given that  $c < K\bar{U}_K$  (for the incentive compatibility constraint on effort to hold), Inequality (47) is always satisfied if

$$\bar{U}_K < \frac{1-k}{1+k}Z_2(V_L - P(Z_2)) + \Pi = \frac{2}{1+k}(E_k(V) - P(Z_2))Z_2$$

Taking  $\bar{\bar{U}}$  as the RHS of this last inequality, we have the desired result. This completes the proof. 2

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