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IS LEVERAGE EFFECTIVE IN INCREASING PERFORMANCE UNDER MANAGERIAL MORAL HAZARD?

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Is leverage effective in increasing performance under managerial moral hazard?*

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Abstract

We consider a model in which the principal-agent relation between inside shareholders and the management affects the firm value. We study the effect of financing the project with risky debt in changing the incentive for a risk-neutral shareholder (the principal) to implement the project-value maximizing contract. We show the conditions under which leverage generates agency costs in terms of an ex-ante reduction of the firm value. The result also implies that the optimal remuneration structure includes "low-incentive" bonus when the firm is highly leveraged. This inefficiency does not arise when the the agent is paid with shares of the firm. We can then conclude that the use of debt is effective as a commitment device to implement higher operative performance only if it is accompanied with a compensation policy based on shares remuneration.

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1   Introduction

Since the work of Jensen and Meckling (1976), a vast literature has analyzed the role of ownership structure for the selection of projects. Various sources of agency costs have been characterized, where these costs are defined as the reduction of the value of the firm due to incentives to deviate from the optimal rule of selecting all projects with positive net-present-value.

In this paper we address the relevance of capital structure for efficiency, and more specifically, we study the conditions under which the use of high leverage is effective to improve ex-post efficiency. We focus our attention on the principal-agent relation between the owner of the firm (or a reference shareholder) and the management, who will be responsible for the operational activity of the firm.

The financial structure of the firm does not affect directly the payoffs of the investment in the Modigliani-Miller (1958) paradigm of irrelevance of capital structure; but it obviously changes the distribution of property rights over the firm. Shareholders and debtholders will share the future cash flows generated by the project in a way proportional to their claims. What happens if a big shareholder, or some of them, can negotiate contracts with the management in the presence of moral hazard? In this paper we show that changing the financial structure changes the "incentive to give incentive" by the shareholders-principals. This real effect influences the payoffs of the project, as well as their probability distribution. Therefore, these payoffs are not exogenous to the distribution of property rights, and the value of the firm changes with its capital structure.

We can show that financing the investment opportunities with risky debt reduces the ex-ante value of the firm, by inducing a future choice of contract that is suboptimal. If the cost of outside equity is considered negligible, and without corporate taxation, then the optimal capital structure involves not to issue risky debt at all. We also get some implications on the managerial incentives as a function of the capital structure: in highly leveraged firms, managerial compensation should be rather "flat", insuring the management against bankruptcy. The negative relation between leverage and compensation has been observed by Smith and Watts (1992) especially in firms with low ratio between growth opportunities and assets-in-place value, where managers are broadly remunerated with traditional monetary wages.

This inefficiency does not arise if the management is compensated with shares of the firm. We can then conclude that high leverage is effective in improving performance only if it is linked to a reorganization of managerial compensation scheme, centered on share-plans. Empirical evidence on LBOs (Denis (1994)) and Baker and Wruck (1989)) shows that performance increased after leveraged recapitalizations only when the executive compensation was restructured and based on shares or stock-options plans.

Up to now, the effects of human capital effort on the payoff structure of projects have been largely neglected in the literature studying the determinants of capital structure. Research has focused on asymmetric information between insiders and outsiders (Myers and Majluf, 1988, among many other signalling models), managerial discretion (Stulz, 1990, Grossman and Hart, (1983b)), or other forms of managerial inefficiency (Jensen, 1986). Only Innes (1990) has shown the optimality of debt contract for financing an entrepreneur who is wealth constrained and subject to limited liability. His result relies on the debt as a commitment device that the entrepreneur imposes on himself as an agent to implement a higher effort. Only in this paper we find the link between the shareholder payoff function, that is dependent on the capital structure, and the effort choice by individuals who are called to work on the project, which is underlying our results.

What is lacking, from our point of view, in the Innes' paper is the intrinsic moral hazard problem of agency relationships. Innes assumes that the entrepreneur is actually driving the project realization, and he assumes then the double role of the principal and the agent. In most of modern firms, and a fortiori in corporations, this is not the case. Even in small, project-based firms, the principal, the owner

of the firm, relies on the collaboration of (potentially) many agents in pursuing his production activity.

The model is a very stylized example of projects value and incentive structure. A principal proposes a complete contract to an agent, who chooses the utility-maximizing effort to implement according to it. Exogenous uncertainty is affected by this principal-agent relationship since higher effort increases the probability of getting higher results. The cost of the contract will also affect the final payments of the project, in a very simple way: higher payments to executives will reduce profits and hence the final value of the stock. At the end of the period the firm is liquidated. Dividends choice is then not strategical. This allows a substantial simplification with respect to the important issue of dividends as signals of higher valuable information about the firm (as in John and Williams (1985)). Moreover, it avoids any strategy of manipulation of the market via dividends policy by executives. Ownership and capital structure of the firm are exogenously given at the moment of contract negotiation: we analyze separately the case in which the firm is financed with riskless debt and the one where risky debt is issued. The capital structure is chosen at a precedent period, $t = -1$, and prior to this the decision about investing or not in the project is taken.

For simplicity we suppose that the all these choices are common knowledge between the investors in the firm: therefore, the principal has no informational advantage given his position (anyway, in many regulations of existing markets, trading on these advantages would be forbidden). Also, we do not consider any difference in the interests of the principal and other outside shareholders (who do not influence the choice of contracts) or the market. Under these assumptions, for very high debt over equity ratio, the contract which maximizes the market value of equities provides a lower incentive-bonus than the one which maximizes the total market value of the firm.

The main intuition of the result is simple. In the reference situation where the firm is entirely financed with equity, the principal, chooses the contract which maximizes the ex-ante value of the firm. When risky debt is issued, his payoff (as well as the payoff of all other shareholders) becomes convex in the future profit realizations. Then, he will not pay entirely the costs of the managerial remuneration: the fixed part, that is due in all the contingencies, is actually paid by the debtholders. As an opposite effect, he will have an incentive to increase the remuneration bonus in the good states in order to increase the likelihood of that states.

If the firm is highly leveraged, the first effect will prevail, as economic intuition suggests: why should the principal, who is going to enjoy positive profits only in a few states, give up a substantial part of them in the forms of high incentive bonuses to the management since most of the advantage of the higher effort will be cashed-in by the debtholders?

However, if the level of debt is lower, the second effect could prevail, according to the characteristics of the probability distribution over the states of future payoffs of the project as a function of managerial effort.

In any case, an inefficiency arises since the principal does not internalize all the benefits (in the first case) or all the costs (in the second) of an increased managerial bonus. This reduces the firm value. Risky debt does not provoke inefficiency if the managerial compensation scheme is based on shares (or share-options), a result

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analogous to Innes (1990).

We give then a rationale for implementing remuneration schemes based on managerial participation in firm capital when the firm is highly levered (ex. in LBOs).

The paper is organized as follows. Section 2 presents the general model. In section 3 we give the general result of irrelevance of financial structure . In section 4 we discuss our assumptions and give an alternative interpretation of the model as well as some possible extensions. Section 5 presents analytical example where the agency cost of debt is quantified and the new contract in the case of leveraged firm is characterized. Section 6 concludes. In the Appendix we collect the proofs of the basic lemmas.

2   The model

Consider a three-periods model of a firm whose value depends on independent projects. The value of the assets already in place is commonly known to insiders and investors, and does not affect the value of future projects: without loss of generality we normalize it to zero[1]. The firm is considering the investment decision in a new project, whose returns are independent of assets-in-place, and the related choice of project-financing[2].

The choice of investment is taken at date $t = -1$, and at the same time the financial structure is decided. We assume that the project earnings randomly realize at some future time $t = 1$: we denote this random variable with $\widetilde{y} = \{y_s; s \in \text{-}\}$ where - represents the space of possible gross profits level of the project, and it is assumed to be a closed and compact interval $[\underline{s}, \overline{s}]$; the project does not mature any earnings in time zero. After the realization of $\widetilde{y}$ it can be liquidated.

The main shareholder of the firm chooses whether to invest in the project, and, in the affirmative case, how to finance the investment. In he invests, he has to pay an amount $I > 0$ at date -1, and he has the choice of financing it with the issue of new equities or with debt. We are not interested in situations in which the principal is interested in carrying out investment projects for empire-building reasons (as in Stulz (1990)) if they are not profitable: hence, the choice of implementing or not the project will maximize the share value, and the principal can be seen as a representative shareholder who is endowed with decision-making power concerning the particular project[3].

If the project is implemented, he delegates the running of it to a manager: in terms of standard principal-agent models, the owner-(inside)shareholder is the principal and

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[1]Alternatively, and perhaps more easily, our firm is just to be established and it will be a one-project firm.

[2]We focus on a problem of project-financing, i.e. the project, if implemented, will be financed as a stand-alone entity. The assumption is required since we want to avoid the link between human capital effort devoted to the succes of a particular project and the one required for ruling the assets-in-place: considering a more complicated model of moral-hazard would complicate the analysis, introducing multi-dimensional effort choices.

[3]Alternatively, he can be considered as the unique owner of the firm, or as the CEO of it; in this case, it is crucial that he owns shares of the firm.

the manager is the agent. The principal is assumed not to be able to monitor the manager's actions, but he will be able to observe the outcome of these actions, and in particular the realization of the project's profit at $t = 1$.

The principal, if he decides to invest in the project, first chooses the financial structure, and then, at time $t = 0$, proposes a state-contingent, complete contract w_s to the agent specifying the payment the latter will get for any realized state of the world at time one. Notice that the financial decision anticipates the choice of the contract submitted to the agent.

The agent, after being told the contract, will choose the effort a maximizing his utility. The choice of effort lies in a continuous interval $a \in [0, \overline{a}]$. The agent suffers a cost for his effort that makes a potential conflict with the interest of the principal. Formally, his utility function $U(a, w_s)$ is a von Neumann-Morgenstern utility function which depends both on his action a and the remuneration earned (that we denote here as a state-dependent variable for sake of generality, but it could be a constant).

Assumption 1. The function $U(a, w_s)$ can be written as $\int_{-} u(w_s)g(s\,|a)ds - c(a)$ where $c(a)$ is twice differentiable and convex in a.

We suppose that the principal is risk-neutral, and hence he evaluates only the net profits of the project (gross profits y_s minus the payment to the manager w_s), while the agent is risk-averse. If the principal is able to fully diversify the risk of the project on the financial markets, then his neutrality towards the risk follows; since we will suppose he has free access on the credit market, (and also on the financial markets), and that markets are complete, this seems to us a quite reasonable assumption.

The compensation to the agent is assumed to be senior to any debt claim issued by the principal at time zero to finance the project; hence, all these claims are entitled to payment only if the agent has received his wage w_s.

We ignore problems of renegotiation of the agent contract, since the agent will be entitled to his compensation with priority to others claim-holders and since we assume completeness of the contract. Once it is decided whether to invest in the project, and the way of financing that investment, the principal will propose the contract that maximizes his claims value, i.e. the value of the shares on the project. The interests of other shareholders are perfectly aligned to the interests of our principal, and then there is no additional cost of outside equity financing[4]. Moreover, since the new project represents a growth opportunity for the firm it is reasonable that its value is accurately monitored by the market, and if the value of the assets-in-place has been released (in order to avoid the adverse selection problem pointed out by Myers and Majluf (1984)) the cost of outside equity financing can be considered exogenous. We suppose also that the information about the project returns and the contract is commonly known by market participants.

The net payoff of the project if state s realizes is then $y_s - w_s$. We assume no discount and market completeness and for simplicity all the claims are priced under the historical probability measure $g(s, a)$. The expected present value of the project, evaluated at time zero, is then $E_{g(s,a)}[y_s - w_s\,|\mathcal{F}_0]$ where $\mathcal{F}_0$ is the information set

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[4]Of course, this is restrictive in terms of design of optimal capital structure: as Jensen and Meckling (1976) point out, there are incentive problems associated with financement via outside equity (see also the discussion of this point in section 5).

at time zero that includes the contract stipulated and the financial structure, and $g(s, a)$ is the probability distribution on - . Let the states s ordered according to y_s: the worst state $\underline{s}$ is the one where y is minimum, and so on. The effort of the agent affects the probability distribution and the likelihood ratio is monotonically increasing (MLRP): the ratio $\frac{g_a(s|a)}{g(s|a)}$, where $g_a(s|a) = \frac{\partial g(s|a)}{\partial a}$, is monotonically increasing in s. In the following we impose a slightly more restrictive assumption on the likelihood ratio.

Assumption 2. $g_a(s|a)$ is increasing in s.

Hence, the project's future cash flows are dependent on the agency relationship existing between the shareholder-principal and the agent-manager: the latter's effort will affect the net profits and their probability distribution, together with exogenous uncertainty. If we refer to traditional theories of capital structure the main difference of our model is that here the cash flows depend on a contractual relation since human capital is involved in the process of production. The distribution, and the absolute value, of the project's cash flows is not exogenous to the distribution of property rights if these property rights affect the optimal contract for the principal: he is, in fact, the only one entitled to write contracts for the management.

The decision about taking or not the project relies only on its expected net present value, $E_{g(s|a)}[y_s - w_s | \mathcal{F}_0] - I$. If this latter is positive, the principal decides to invest in the project, otherwise he does not. If the principal invests, the total payoff to date-zero claim-holders will be $R = E_{g(s|a)}[y_s - w_s | \mathcal{F}_0] - I$.

If the principal is wealth constrained (the wealth he invests in the project $W < I$), he can alternatively raise the funds necessary for the project investment in the stock market, issuing new shares, or issuing debt claims. Once the necessary funds have been raised, he will negotiate the contract with the agent and then the production activity will start.

To begin with, suppose the principal can find on the stock market the funds necessary to take the project issuing new equities. If he invests I at period zero and proposes the agent contractual state-contingent payments of w_s, the project is an asset worth y_s at time $t = 1$, and the value of equity will be $y_s - w_s$ if state s realizes. If the investment is not made, no additional shares are issued, and the project is worthless (as well as the firm).

The value of the project at time zero is

$$V = \int_{-} (y_s - w_s) g(s|a) ds$$

In this case, the incentive of the principal coincides exactly with the incentive of outside equity-holders. In fact, the objective for both is to maximize the value of the project with the right choice of contract to propose to the manger-agent. In doing this, the principal faces the standard trade-off of moral hazard problems between incentive and insurance purposes. The problem for the principal, once investment has been paid and equities have been issued, reads as follows:

$$\max_{(w_s, a(w_s))_{s \in -}} \alpha E_{g(s|a)}[y_s - w_s | \mathcal{F}_0] = \alpha \left(\int_{-} (y_s - w_s) g(s|a) ds \right)$$

$$s.t. \quad a(w_s) \text{ is the effort implemented by } A \text{ given the contract } w_s \tag{P1}$$

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where α represents the fraction of principal's equities over the total amount of equities, i.e. $\alpha = \frac{W}{I}$ since the equities are priced under the martingale measure $g(s|a)$[5].

More precisely, we can express the constraint of (P1) describing the action-choice problem of the agent. If he has an outside option of obtaining an utility $\overline{U}$ working somewhere else, given assumption 1 his problem is

$$
\begin{aligned}
\max_{a} & \quad \int_{-} u(w_s)g(s|a)ds - c(a) \\
given & \quad \quad w_s
\end{aligned}
\tag{A}
$$

that generates the individual rationality and incentive compatibility constraints the principal must satisfy in (P1):

$$
\begin{aligned}
& \int_{-} u(w_s)g(s|a)ds - c(a) \geq \overline{U} \quad \quad (IR) \\
a \text{ solves } & \max_{\widehat{a}} \quad \int_{-} u(w_s)g(s|\widehat{a})ds - c(\widehat{a}) \quad (IC)
\end{aligned}
\tag{1}
$$

Notice that the agent's objective is not influenced by the capital structure of the project financement since his compensation is senior to the other claims on the payoff y_s. As immediate corollary of this observation, we can write the optimal choice of the agent, a, as a function of only the contract payments w_s. The principal will then take into account this in solving his maximization problem for the optimal contract.

Suppose now P decides to finance the investment issuing new debt[6]. Let $D < I$ be the face value of the debt issued, and assume that the proceeds of the debt issue are used to reduce the required initial equity investment. Then, consider the problem faced by the principal once the debt has been issued.

The optimal contract solves

$$
\begin{aligned}
\max_{(w_s, a(w_s))_{s \in -}} & \quad E_{g(s|a)}[\max\{y_s - w_s - D, 0\} | \mathcal{F}_0] = \int_{-} (\max\{y_s - w_s - D, 0\})g(s|a)ds \\
s.t. & \quad a(w_s) \text{ is the effort implemented by } A \text{ given the contract } w_s
\end{aligned}
\tag{P2}
$$

where the set of constraints is given by (1).

Two observations can be done at this point: firstly, due to the convexity of the payoff of his equity claims, the principal will have an incentive to increase the volatility of the project's payoffs (see Jensen and Meckling, (1976)). Secondly, these payoffs depend on the wage w_s in the optimal contract, that should create the right incentives to the agent. The trade-off between incentives and insurance purposes of the contract has not changed since the problem of the agent is independent of the financial structure. What has changed with the shape of the principal objective function is his cost to give incentives to the agent in order to make him working harder (and hence making the good states realization more probable (by MLRP)).

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[5] We assume that the new equity holders rationally take into account the determination of the optimal contract proposed by the principal to the agent, to evaluate the price of the new issued equities. In other words, we assume efficiency of the financial market.

[6] As we stressed in the introduction, we do not analyze the problem of debt-overhang illustrated by Myers (1977): in fact, financing the project with debt, in this setup, would reduce the value of the project due to agency costs of debt, since it induces a sub-optimal investment strategy. In fact, in our model as in Myers 1977 the debt matures after the firm investment decision is made. In the following, we will not consider these agency costs of debt.

Summarizing, the financing decision has an effect over the value of the project if[7] the solution of problem (P2) is different from the solution of (P1). In general, this will be the case. The main intuition is the following: if risky debt is issued, the principal is not entirely paying the compensation of the agent in the states with default; part of it is actually payed by the debtholders. This increases the willingness of the principal to offer an high compensation in the unfavorable states in exchange of a lower bonus, reducing the incentive structure for the agent. But doing this, the latter will probably choose to work less hard, reducing the probability of success of the project. On the other hand, this lower likelihood of good states, where P enjoys positive profits, is not beneficial for P. These two contrary effects make the capital structure relevant for the firm's value.

3 Relevance of capital structure and agency cost of debt

We characterize the solution of our model solving the problems backward in time, i.e. starting from the agent's choice of effort. We just recall here that capital markets are perfect and agent are rational[8]; information is distributed symmetrically in the market, in the sense that the structure of the model is commonly known among market participants: the optimal contract is then rationally anticipated by all the (potential) claimholders and their claims are correctly priced under the resulting probability measure $g(s|a)$. Given the contract choice (as a function of the capital structure), the principal chooses the optimal ratio of debt to equity to finance the project: this determines the expected value of the project. Finally, a decision about investing or not is taken according to the sign of the expected net present value of that project.

3.0.1 Some results of the moral-hazard problem

Unfortunately, the moral-hazard problem between P and A has very weak properties in a general framework, and then additional assumptions are necessary to give it some more tractability. As shown by Grossman and Hart (1983) the shape of the optimal contract in a principal-agent model can be extremely complex. Following the "first-order approach" (Holmstrom (1977)), the condition MLRP is sufficient to get monotonicity of the optimal contract. Our assumption 3 (that is stronger than MLRP) guarantees then the monotonicity of the optimal contract under the "first-order approach". In this framework, we can furthermore restrict the set of possible contracts to schemes of the following type:

Assumption 3. The function $y(s)$ is strictly monotone in s once the states have been appropriately ordinate, and $\underline{y} = y(\underline{s})$. The set of possible contracts is characterized by two positive parameters σ, b such that:

$$w_s = \sigma + b(y_s - \underline{y})$$

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[7] Under our conditions this should be also an "only if" statement.
[8] We include in the rational behaviour also the assumption of rational expectations.

Under Assumption 3 we can write problem (A) as follows:

$$\max_{a} \quad \int_{\underline{s}}^{\overline{s}} u\left(\sigma + b(y_s - \underline{y})\right) g(s\,|a)ds - c(a)$$

and, given the first order approach, substitute the (IC) constraint with the first-order condition of the agent problem

$$\int_{\underline{s}}^{\overline{s}} u\left(w_s\right) g_a(s\,|a)ds - c'(a) = 0 \tag{2}$$

From equation (2) one gets the solution $a(\sigma, b)$. Given assumption 3 we can prove the following:

Lemma 1. *The optimal contract for the principal makes the (IR) constraint binding.*

Proof. If we restrict the set of contracts to the ones described in Assumption 3, the individual rationality constraint for the agent will not be binding if (i) the principal increases the flat wage σ keeping the bonus b constant; (ii) he increases b keeping σ constant; (iii) increase the pay in bad states reducing it in the good states.

The first deviation has no effect on the incentives for the manager to choose an higher effort, since the wage σ is paid in all future realizations. Hence, this deviation makes the contract more expensive for P without any benefits. The second can potentially increase the effort, but if this would be profitable, then the starting contract would not be optimal. The third deviation is analogous to reducing the bonus in terms of incentive for A, but is more costly for P. ∎

Since for the principal is always optimal to lower the remuneration until the constraint (IR) is binding, the bonus b and the flat wage σ will be functionally related by the equation (IR); we can write $\sigma = \sigma(b)$: once the bonus is fixed, σ is obtained giving the agent his reservation utility $\overline{U}$.

Lemma 2. $\frac{\partial \sigma(b)}{\partial b} < 0$.

(The proofs of this and the following lemmas are in the appendix).

Increasing the bonus b will also change the incentive for A to implement his effort, since it changes the relative remuneration in good and bad states; on the other hand, it makes more volatile the wage for the agent: the usual trade-off between insurance and incentives faced by the principal in moral-hazard models. In the following lemma we derive the bound on risk-aversion of the agent that makes the effort function $a(b)$ increasing in the incentive bonus b.

Lemma 3. *If* $-\frac{u''(w_s)}{u'(w_s)} \quad \frac{1}{b(\sigma'(b)+y_s-\underline{y})}$, *then* $a'(b) > 0$.

Given the properties on the functions $a(b)$ and $\sigma(b)$ we can determine a last result that will be useful in the following. In our setup, the total monetary cost of the contract for the principal reduces when b is reduced: one dollar less in terms of bonus remuneration requires an increase of less than one dollar in terms of σ to keep the agent at his reservation utility level.

Lemma 4. *Let* $C = \int_{\underline{s}}^{\overline{s}} w_s g(s\,|a)ds$; *if* $a'(b) > 0$ *then* $C'(b) > 0$.

Observation. Notice that Lemmas 1-2-3 and 4 are independent of the financing decision. Since the wage of A is assumed to be a senior claim with respect to debt,

the agent is completely indifferent to the capital structure of the firm he is working for. P will anyway reduce his utility to the reservation value $\overline{U}$. This property of our model is crucial to obtain the results, and needs a critical discussion (see section 4).

3.0.2 The principal's optimal contract as a function of the property rights distribution

Let V be the equilibrium market value of the project at time zero (when the contract is negotiated), and V_D, V_E the values of debt and equities respectively at the same time. From the balance sheet, we have that $V = V_E + V_D$ where:

$$V = \int_{\underline{s}}^{\overline{s}} (y_s - w_s) g(s\,|a\,) ds$$

$$V_E = \int_{\underline{s}}^{\overline{s}} \max\{y_s - w_s - D; 0\} g(s\,|a\,) ds$$

$$V_D = \int_{\underline{s}}^{\overline{s}} \min\{y_s - w_s; D\} g(s\,|a\,) ds$$

where $D \geq 0$ is the face value of the debt issued.

Suppose first the project is entirely financed by issuing equities, i. e. $D = 0$[9]. Then $V = V_E$. In this case the problem of the principal is (P1), that is equivalent to maximize V_E at time zero:

$$\max_{(w_s, a(w_s))_{s \in \cdot}} \int_{\underline{s}}^{\overline{s}} (y_s - w_s) g(s\,|a\,) ds$$

$$s.t. \quad \int_{\underline{s}}^{\overline{s}} u(w_s) g(s\,|a\,) ds - c(a) \geq \overline{U} \qquad (IR)$$

$$a \text{ solves } \max_{\widehat{a}} \int_{\underline{s}}^{\overline{s}} u(w_s) g(s\,|\widehat{a}\,) ds - c(\widehat{a}) \qquad (IC)$$

and given Lemma 1 and the "first-order approach" this is equivalent to:

$$\max_{b} V(b) = \int_{\underline{s}}^{\overline{s}} \left(y_s - \sigma(b) - b(y_s - \underline{y}) \right) g(s\,|a\,) ds$$

$$s.t. \quad a = a(b) : \int_{\underline{s}}^{\overline{s}} u(w_s) g_a(s\,|a\,) ds - c'(a) = 0$$

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[9] All what we say about all-equity financing is also true for the case that the debt is riskless, i.e. $D < y_{\underline{s}} - w_{\underline{s}}$. In fact, in this case

$$V_E = \int_{\underline{s}}^{\overline{s}} \max\{y_s - w_s - D; 0\} g(s\,|a\,) ds$$

$$= \int_{\underline{s}}^{\overline{s}} (y_s - w_s) g(s\,|a\,) ds - D$$

$$V_D = D$$

and no distortions are introduced since maximizing V_E is equivalent to maximize V. The capital structure is then irrelevant for the value of the project.

The F.O.C. of the above problem is:

$$\int_{\underline{s}}^{\overline{s}} -\sigma'(b)g(s\,|a)ds + a'(b) \int_{\underline{s}}^{\overline{s}} (y_s - w_s)\, g_a(s\,|a)ds = \int_{\underline{s}}^{\overline{s}} (y_s - \underline{y})g(s\,|a)ds \quad (1)$$

Equation (1) illustrates the marginal costs and benefit for P of an additional unit in bonus: on the left-hand side we have the marginal benefit due to the saving in $\sigma(b)$ plus the increased probability of good outcomes due to the higher effort induced; on the right-hand side the marginal cost of the bonus that has to be paid (equal to $E_{g(a)}[y_s] - \underline{y}$ in expected terms given the form of the contract).

This equality characterizes the optimal b, that we denote as b_E^*. b_E^* is a maximum only if the second derivative $\frac{\partial^2 V(b_E^*)}{\partial b^2} < 0$. Notice also that, if we impose global concavity of $V(b)$, any $b \neq b_E^*$ would reduce the value of the project. Sufficient conditions for the concavity of V in b are given in the following lemma (the proof is in the appendix).

Lemma 6. *The function $V(b)$ is concave in b if $\sigma''(b) \geq 0$, $a'(b) < 1$, $a''(b) \quad 0$.*

Financing with risky debt Suppose now the firm has borrowed funds to finance the project with the promised payment $D > 0$ where $D >> y_{\underline{s}}$. The debt issued is risky since in the contingencies with lowest project payoffs it will be impossible to payback D entirely. Moreover, the payment w_s has still to be subtracted from the total amount of resources the debtholders can claim: the credit structure we have imposed makes the debtholders paying the agent remuneration in the bad states.

This will change the principal's contract decision: the marginal costs and benefits for P, as a shareholder, are different from the one described in equation (1). P maximizes the value of his portfolio of shares, that is equivalent to solve

$$\max_b V_E = \int_{\underline{s}}^{\overline{s}} \max\{y_s - w_s - D; 0\}g(s\,|a)ds$$

$$s.t. \quad \begin{array}{c} \int_{\underline{s}}^{\overline{s}} u(w_s)g(s\,|a)ds - c(a) \geq \overline{U} \qquad (IR) \\ a \text{ solves } \max_{\widehat{a}} \quad \int_{\underline{s}}^{\overline{s}} u(w_s)g(s\,|\widehat{a})ds - c(\widehat{a}) \quad (IC) \end{array}$$

and letting $\widehat{s}(D, b)$ such that $y_{\widehat{s}} - w_{\widehat{s}} = D$ we can rewrite the previous problem as

$$\max_b \int_{\widehat{s}(D,b)}^{\overline{s}} \left(y_s - \sigma(b) - b(y_s - \underline{y}) - D \right) g(s\,|a)ds$$

$$s.t. \quad a = a(b) : \int_{\underline{s}}^{\overline{s}} u(w_s)g_a(s\,|a)ds - c'(a) = 0$$

The first-order condition gives, under the Assumptions of lemma 6, the necessary and sufficient conditions for the new maximum, $b^*(D)$, describing the marginal costs and benefits for P of an increase of b in the case of risky debt:

$$-\sigma'(b) \int_{\widehat{s}(D,b)}^{\overline{s}} g(s\,|a)ds + a'(b) \int_{\widehat{s}(D,b)}^{\overline{s}} (y_s - w_s - D)g_a(s\,|a)ds = \int_{\widehat{s}(D,b)}^{\overline{s}} (y_s - \underline{y})g(s\,|a)ds$$

$$(2)$$

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We can point out three differences from the F.O.C. (1):

(i) since P as a shareholder is not actually paying the wage of A in states $s < \widehat{s}(D, b)$ the marginal benefits from a reduction in $\sigma(b)$ reduces;

(ii) since the equity payoff is positive only in states $s > \widehat{s}(D, b)$ the marginal benefit from an increased bonus through an increased effort a is reduced to that states only;

(iii) the marginal cost of an increased b is also relevant only in the states where P is actually paying the agent.

The three effects together imply both a reduction of marginal benefits and costs of an increase of b for P. It is not clear then which effect prevails, and it is not a priori sure if $b^*(D)$ is higher than b_E^*.

What are the basic effects that could induce P to propose a different bonus when risky debt has been issued? On one hand, his payoff is positive only in the favorable states, that would increase his willingness to increase b. On the other hand, he is actually paying b entirely as a shareholder (since b is paid just when shares are "in-the-money"), and the beneficial of an higher effort is not completely internalized by his payoff: higher effort is also increasing the debt value V_D. We see that these two forces do not trivially vanish in general.

Proposition 1. *Under Assumptions 1-2-3 the choice of optimal contract for the principal generically depends on the distribution of property rights over the project's payoff whenever risky debt has been issued. If $\int_{\widehat{s}}^{\overline{s}} g_a(s\,|a_E^*)ds > 0$, for high values of D the optimal contract bonus is lower than b_E^*.*

Proof. Rewriting the objective function for P, V_E in b_E^* we obtain

$$V_E(b_E^*) = \int_{\widehat{s}(D,b)}^{\overline{s}} \left(y_s - \sigma(b_E^*) - b_E^*(y_s - \underline{y}) - D\right) g(s\,|a_E^*)ds$$

and

$$
\begin{aligned}
\frac{\partial V_E(b_E^*)}{\partial b} &= \int_{\widehat{s}(D,b)}^{\overline{s}} \left((-y_s + \underline{y} - \sigma'(b_E^*))g(s\,|a_E^*) + (y_s - w_s(b_E^*) - D)g_a(s\,|a_E^*)a'(b_E^*)\right) ds + \\
&\quad + (y_{\widehat{s}} - w_{\widehat{s}}(b_E^*) - D)g_a(\widehat{s}\,|a_E^*)\frac{\partial \widehat{s}}{\partial b} \qquad\qquad (3) \\
&= -\sigma'(b_E^*) \int_{\widehat{s}}^{\overline{s}} g(s\,|a_E^*)ds + \\
&\quad - \int_{\widehat{s}}^{\overline{s}} (y_s - \underline{y})g(s\,|a_E^*)ds + \\
&\quad + a'(b_E^*) \int_{\widehat{s}}^{\overline{s}} (y_s - w_s(b_E^*) - D)g_a(s\,|a_E^*)ds
\end{aligned}
$$

But we know from f.o.c. (1) that

$$-\sigma'(b_E^*) - \int_{\widehat{s}}^{\overline{s}} (y_s - \underline{y})g(s\,|a_E^*) + a'(b_E^*) \int_{\widehat{s}}^{\overline{s}} (y_s - w_s(b_E^*))g_a(s\,|a_E^*)ds = 0 \qquad (4)$$

and then, $sgn[(3)] = sgn[(3) - (4)]$. The difference between (3) and(4) is

$$\sigma'(b_E^*)G(\widehat{s}\,|a_E^*) + \int_{\underline{s}}^{\widehat{s}} (y_s - \underline{y})g(s\,|a_E^*)ds - a'(b_E^*)\int_{\underline{s}}^{\widehat{s}} (y_s - w_s(b_E^*))g_a(s\,|a_E^*)ds +$$

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$$-Da'(b_E^*) \int_{\widehat{s}}^{\overline{s}} g_a(s \,|a_E^*)ds$$

that is decreasing in D if $\int_{\widehat{s}}^{\overline{s}} g_a(s \,|a_E^*)ds > 0$ since $a'(b_E^*) > 0$. Then two situations are possible:

1) $sgn[(3)] < 0$ for any D;

2) it exists a $\widetilde{D}: sgn[(3)] < 0$ for $D > \widetilde{D}$ and $sgn[(3)] > 0$ for $D < \widetilde{D}$.

To conclude, it is enough to observe that given the concavity of the function $V_E(b)$, whenever $\frac{\partial V_E(b_E^*)}{\partial b} < 0$ we can say that $b^*(D) < b_E^*$. ∎

The result has a very straightforward intuition, even if its statement seems to be a bit complicate. Take the case where D is very high. Increasing b would shift the probability distribution towards the right. Now, an increase of b is very costly for P, and is going to be beneficial for debtholders also. This situation can be described with the following words of P: "Why should I make my management work as crazy, that would cost me incredibly whenever we will have success in repaying our debts, if most of the advantage is probably going to the bankers?" The shareholders, in other words, do not internalize fully the benefits of an increased bonus: their "incentive to give incentives" is reduced.

If the leverage is much lower, things could change. Now, P has probably an interest in promising high incentives in case of good realizations since he enjoys most of the benefit of a shift towards the right of the probability distribution $g(s \,|a)$. One could ask why we observe an inefficiency in terms of ex-ante value loss of the project even when the bonus is increased, as the effort. This is due to the fact that bonus higher than b_E^* are too costly, and they reduce the overall value V: with respect to the optimal choice, b_E^* the firm is now paying the management "too much" in order to push them over-working with respect to the efficient level.

We can conclude stressing one main implications of proposition 1. An "over-exposure" in terms of debt of the firm produces a contract scheme for the management that is much more flat that in the case of full equity financement. A moderate leverage should be coupled with a very incentive-oriented compensation structure.

3.1 Optimal capital structure and the agency cost of debt

We can now solve the model backward analyzing the choice of capital structure. At time $t = -1$ P has to decide in which way to finance the project, if he did undertake it[10]. His purpose is to choose the ratio debt/equity that maximizes

$$V_E(D) = \int_{\underline{s}}^{\overline{s}} \max\{y_s - w_s - D; 0\}g(s \,|a)ds$$

Unfortunately, the solution of this problem is practically impossible, since we are not able to characterize completely the function $b^*(D)$[11]. The only possible observation we

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[10]Notice that we postpone the choice of capital structure to the one of investment in order to avoid the inefficiency due to "debt-overhang" illustrated by Myers (1977).

[11]See section 5 for a completely solved analytical example.

can make is that, whenever P chooses to issue risky debt, there will be an incentive for him to switch from the contract b_E^*; but then, the total value of the project, $\int_{\underline{s}}^{\overline{s}}(y_s - w_s)g(s\,|\,a)ds$, will be less than its all-equity value $V(b_E^*)$. This can be defined as an "agency cost" of risky debt (following Jensen and Meckling (1976)): if P is wealth-constrained but he has full access to credit markets, he will not be able to finance the projects whose cost I at time $t = -1$ is higher than $V(b^*(D))$, while without the obligation of issuing risky debt he could afford all the projects with $I \quad V(b_E^*)$. This is due to the fact that P will not have the incentive anymore to negotiate with its management the contract that is actually value-maximizing. The optimal policy of P as a shareholder, in choosing the capital structure, is then to issue no risky debt at all. Since issuing risky debt reduces the market value of the project, also $V_E(D)$, the market value of equities, will be maximum when D is riskless. This conclusion is opposite to the one of Innes (1990): this is due to the fact that in our model the principal is a different individual from the agent. Our contract is not a commitment to work done with ourselves, but is a standard employer-employee contract.

Naturally, we are aware that in reality our result about the optimal capital structure holds only if the agency costs of outside equities (in the terms of Jensen and Meckling (1976)) are smaller than the ones we have described here[12]. In the reality it is not true that the cost of outside equity is zero, as pointed out by many theories on agency costs between managers and shareholders (Stulz, 1990)), asymmetric information (Myers and Majluf (1984)), IPO's (Rock (1986)): but it could be that "Debt may be the lesser evil " (Myers, 1977). In that case, our model gives a rationale for project-owners to go public whenever they are wealth-constrained even if they have free and unlimited access to the credit market.

If P is actually forced to issue risky debt, then this will be costly. Writing the ex-ante value of the debt, we get

$$V_D = \int_{\underline{s}}^{\overline{s}} \min\{y_s - w_s; D\}g(s\,|\,a)ds$$

that is less than D. The cost of debt capital, $\frac{D}{V_D(b^*(D))} - 1$ is positive; notice also that it could be the case that increasing borrowing reduces the debt value $V_D(b^*(D))$, so that a rational credit rationing for P could arise: creditors could stop to borrow P money at some amount D^* such that $V_D(b^*(D^*))$ is maximum even if they would be offered an higher interest rate.

4   Discussion

In this section we critically analyze some of the assumptions of the model and draw some more implications of the results described up to now.

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[12] See also the discussion in section 5.

4.1 Shares plans for the agent

One important critic to our setup is that in reality is often observed that management, at least at a the highest levels, is rewarded with stocks of the firm (or with stock-options plans). The rationality of this compensation schemes is to align the incentives of the management with the ones of shareholders: in other words, compensation via stocks (or stock-options) should reduce the role of the incentive compatibility constraint, guaranteeing that the agent will always pick the effort that maximizes the value of equities, since his wealth will depend on it. Theoretical (Innes (1990), Jensen (1986), Hart and Moore (1995)) and empirical literature (Smith and Watts (1992), Denis (1994), Baker and Wruck (1989)) observes that leverage is in this case an useful device that commits the management to implement an efficient choice of effort, that produces an increase in operational performance of the firm. Solving our model when the principal pays the agent with shares of the project we reach the same conclusion: the critical assumption that guides the effect of debt over performance is then the seniority of the agent compensation.

Suppose some risky debt D has been issued in the first period and assume for simplicity that it exists a state $\underline{s} < \widehat{s} < \overline{s}$ such that $D = y_{\widehat{s}}$[13]. Suppose also for simplicity that the set of feasible contracts is composed just by shares remuneration[14], i.e.

$$w_s = \max\{q(y_s - D); 0\}$$

where the quota $q < 1$ of shares is transferred from the principal portfolio to the agent[15]. If we now write the problem of the agent we realize that this is not anymore independent of the chosen capital structure:

$$\max_a \int_{\underline{s}}^{\overline{s}} u(w_s)g(s\,|a\,)ds - c(a)$$

$$\Rightarrow \max_a \int_{\widehat{s}}^{\overline{s}} u(q(y_s - D))g(s\,|a\,)ds - c(a)$$

$$given \quad q.$$

By first order approach this can be replaced with its first-order condition, and hence by a function $a(q)$. The (IR) constraint implicitly fixes a lower bound for the quota q of shares to allocate to the management:

$$\max_a \int_{\widehat{s}}^{\overline{s}} u(q(y_s - D))g(s\,|a\,)ds - c(a) \quad \geq \overline{U}$$

$$\Rightarrow q \geq q^{IR}$$

The problem of the principal for a given capital structure is then to choose the stock sharing $(q, 1-q)$ that is equity-value maximizing, under the constraint of being

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[13] If the payoffs y_s are not continuous in s we can only be sure that D belongs to an interval $[y_{s_j}, y_{s_{j+1}}]$ where the states have been ordered increasingly in y_s. With a continuum of states the assumption seems rather innocuous.

[14] Any "mixed regime" where compensation is partly paid in money and partly in shares will probably combine the two effects.

[15] We will assume here for simplicity that P is the only shareholder, then he owns the remaining $1-q$ shares.

individually rational for the agent; P solves

$$\max_{q} \quad (1-q)\int_{\hat{s}}^{\overline{s}}(y_s - D)g(s\,|a(q))ds$$
$$given \qquad \begin{array}{c} a(q) \\ q \geq q^{IR} \end{array}$$

The solution of this problem is made complicate by the fact that we should be able to explicitly check if the (IR) is binding or not: we present in the last section of the paper an analytical example where this can be done. In general terms we can obtain the following result (the proof is in the appendix):

Proposition 2. *If the function y_s is differentiable and if $-\dfrac{u''(w_s)}{u'(w_s)} \quad \dfrac{dy_s/ds}{y_s}$ whenever the agent is remunerated with shares of the firm the value of the firm increases with leverage.*

Hence, our model is able to generate the prediction of Innes (1990) when the agent is made residual claimant, being paid by shares, exactly as in his model. As predicted by the literature, with share-remuneration the principal indeed aligns the interests of the management to his own ones. In this case, the trade-off that creates the agency cost of debt illustrated before disappears, since now the agent is paid only in the good states, and the principal does not give up some wealth to increase the payoff of debtholders: then leverage works as a commitment device (exactly as in Innes (1990)) to maximize the value of the project, that is now also in the interest of A given his remuneration via shares. Notice also that the fact that (IR) is binding finally depends on reservation utility $\overline{U}$, but in a complex way that we are not able to point out in this general setup.

A second implication of the proposition is also interesting: with share-remuneration, the agent is constrained again to his reservation utility level, but higher leverage will also imply harder work. Then, giving shares to the management seems to be effective in implementing an high effort. Whenever problems of monitoring are serious at a management level, a share-remuneration scheme should be definitely preferable for equityholders. As Smith and Watts (1992) argue, monitoring and evaluation problems are especially serious in firms with high growth opportunities and low values of assets-in-place, where they indeed find empirical evidence for a wider use of share plans for management (independently of the capital structure).

4.1.1   LBOs: when debt is an effective tool to implement efficiency

Case studies on leverage buyouts (Denis (1994), Baker and Wruck (1989)) have shown that this financial reorganization often improved performance of the firm. Since in LBOs the leverage is dramatically increased, this seems contradictory to the result of our proposition 1. One usual interpretation of this evidence is that debt is indeed a good device to increase the value of the firm. But, as the literature quoted above observes, in LBOs the management is usually heavily rewarded with shares, and proposition 2 can therefore explain the phenomenon. A closer look to the empirical evidence is therefore necessary if we want to understand when debt is really effective as a value-maximizing instrument. Denis (1994) finds in a comparative study between

similar firms that have been purchased via a LBO that the one where management compensation plan was focus on participation increased its post-LBO performance much more than another firm where management compensation structure was still based on traditional monetary-bonus scheme. Moreover, he observes that the incentive structure in this second firm was not changed even after the very dramatic increase in leverage due to the LBO. Then, Denis concludes that "lacking the financial incentive of significant managerial equity ownership or the monitoring of a large shareholder, [...] using leverage to increase effectiveness of internal control systems [...] may be risky." Palepu and Wruck (1992) also provides evidence for poor ex-post performance for defensive leveraged recapitalizations.

4.2   Some other critical points

4.2.1   Seniority of wage

What has been promised on the basis of a legally enforceable contract as remuneration for the management usually has to be paid in priority to all the debt, and not many objections should arise on this. Unfortunately[16] a relevant part of managerial remuneration is usually paid in terms of fringe-benefits, other kinds of monetary and non-monetary benefits difficult to evaluate quantitatively, stocks of the company, stock options, or in other forms. For some of these instruments, that typically form the major part of incentives to management, our assumption on seniority of claim could be questioned with good reasons. Even if we recognize that the critic has some valuable background, we think that the monetary component of incentive remuneration is definitely predominant, at least to the pure non-monetary aspects (keeping aside the stock-options that we discuss later).

4.2.2   Cost of outside equities

There are many theoretical studies that prove that the cost of outside equity is not equal to zero. We certainly do not claim the opposite. We just consider the cost of outside equity as exogenous to the economic trade-offs we illustrate here, and we "parameterized" it to zero for simplicity. The point that should be discussed is then if it is reasonable to consider the cost of outside equity independent of the management incentive structure. If the outside shareholders are not risk-neutral, they will price the risk of their investment in shares of the project, risk that can be measured with the volatility of the returns. When P proposes a contract to A this affect this volatility: different contracts induce a different payoff structure.

This problem is avoided under the assumption that the pricing rule is risk-neutral, i.e. the equilibrium pricing measure is $g(s\,|a)$. Using a more sophisticated pricing kernel would invalidate our assumption, and it would lead to a much more complex model where both the costs of outside equities and debt are endogenous: we leave this problem as a further research topic.

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[16]For us, not for the management.

4.2.3 The perfection of capital market.

Who is actually crucial to produce, the principal or the agent? Of course, the agent: his effort is going to change the project results. This observation leads to a very sensitive objection: why A cannot borrow and finance himself the project? The imperfection on the credit market that we implicitly assume is definitely driving all our results, especially if we compare them with the one in Innes (1990). Innes shows that the optimal contract from the point of view of a risk-neutral entrepreneur under limited liability is actually a debt contract, since it commits him to work hard. We would find the same result if our agent A could borrow himself the money necessary to invest in the project. This is assumed to be not possible for institutional constraints and then it is necessary the presence of an individual, P, who actually plays no role in the process of production, but has the necessary access to the credit market. The inefficiency we derive is finally due to this sub-optimality of the credit market. Any policy which stimulates the credit for potentially profitable new projects will increase the efficiency of the final allocation.

4.2.4 P as a CEO endowed with stock options

With a simple mathematical observation, we could say that the agency cost of risky debt is due to the convexity of P's payoff, that makes him not to internalize completely the costs (or the benefits) of an increased bonus.

This convexity can be actually the result of a remuneration based on stock-options plan for CEO's, who are usually called to design also the managerial compensation schemes. Viewing P as a CEO who is going to negotiate contracts with lower management A is an alternative interpretation of our model. The result is then not encouraging for stock-options plans. Aside this, with this alternative interpretation, we can derive a possible testable implication of the model: if the managerial stock-options are prohibitive ex-ante (i.e. the strike price is relatively high given the current stock price) this should encourage the optionholders (CEOs) to design relatively "flat-wage" contracts for the management; the reverse should be true when the strike price is actually closed to the stock price (as it is often observed). This conclusion would then reverse one of the arguments usually in favor of using highly challenging stock-options plans for the top management.

4.2.5 Solutions to the problem: bankers in the board of firms.

An easy solution to the problem we have illustrated would be to appoint some representative of the debtholders in the Board of the directors, or in any institution called to discuss the contracts for employees. Notice that this should be true even when the leverage is not very high, because in that case P could not internalize all the costs of the contract and offer then "too expensive" remuneration schemes. For high leveraged firms, our proposition 1 predicts that P will not push its management to work hard enough in order to be able to repay the debt, or at least not as hard as it would be optimal from the debtholders' point of view.

The presence of banks' representative is common when firms are close to situations of financial distress, but it is not, at least in some countries, in firms during normal, financially healthy periods. Our research suggests that debtholders should instead monitor constantly the remuneration policy whenever this latter is decided by individuals who are also shareholders.

5 An analytical example

The analysis done in section 3 is aimed to be as general as possible[17], but this has the drawback that its implications are descriptively weak. The aim of the following example is then complementary to section 3, i.e. to show the magnitude of the effects described up to now in a special functional framework.

Suppose the state of the world is composed by two states, $\{\underline{s}, \overline{s}\}$; the effort choice a be unidimensional in the interval $[0, 1]$, and the probability distribution

$$g(s \mid a) = \left\{ \begin{array}{l} a = \Pr(\overline{s} \mid a) \\ 1 - a = \Pr(\underline{s} \mid a) \end{array} \right. \tag{5}$$

Let the expected utility function of the agent be

$$E[u(w_s, a)] = E[w_s - c(a)] = a w_{\overline{s}} + (1 - a) w_{\underline{s}} - \gamma a^2 \tag{6}$$

while the set of contract is given by the couples $\{w_{\underline{s}}, w_{\overline{s}}\} = \{\sigma, \sigma + b\}$.

Since the agent's problem is invariant to the capital structure we start from its solution.

$$\max_{a \in [0,1]} \ \sigma + ab - \gamma a^2$$
$$given \quad \sigma, b$$

that has a solution, whenever $\gamma > b/2$:

$$a(b) = \frac{b}{2\gamma} \tag{7}$$

and using the fact that the (IR) must be binding:

$$\sigma(b) = \frac{b^2}{4\gamma} \tag{8}$$

The value of the project is then

$$V(b) = a \left(y_{\overline{s}} - (\sigma(b) + b) \right) + (1 - a)(y_{\underline{s}} - \sigma(b)) \tag{9}$$

and substituting for (7) and (8)

$$V(b) = -\frac{b^2}{4\gamma} + \frac{b}{2\gamma}(y_{\overline{s}} - y_{\underline{s}}) + y_{\underline{s}} - \overline{U}$$

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[17] Given the constraint of the author's ability to solve the model.

The function $V(b)$ is concave in b, hence the f.o.c. gives us the value of b that maximizes the value of the project, b^*

$$b^* = y_{\overline{s}} - y_{\underline{s}} \tag{10}$$

For notational simplicity we call $\overline{y} = y_{\overline{s}}$ and $\underline{y} = y_{\underline{s}}$. The maximum value of the project at time zero is hence

$$V^* = \frac{(\overline{y} - \underline{y})^2}{4\gamma} + \underline{y} - \overline{U}$$

and P will invest in all the projects for which the date $t = -1$ payment I is lower or equal to V^*.

If the project has been financed entirely with equities all the results before go through, since P will implement the same contract b^* at time zero, maximizing the value of the project.

As proved in section 3, if P has decided to issue risky debt at time $t = -1$ with a promised repayment equal to $\overline{y} >> D > \underline{y}$ the solution will be different. The value of the project at $t = 0$ is just the sum of $V_E(b) + V_D(b)$ with

$$V_E(b, D) = a(b)(\overline{y} - (\sigma(b) + b) - D)$$
$$V_D(b, D) = a(b)D + (1 - a(b))(\underline{y} - \sigma(b))$$

and at $t = 0$ P maximizes $V_E(b)$ given the functions (7) and (8). The function $V_E(b)$ is not anymore concave for all values of b, and hence the f.o.c and the s.o.c. give the solution b_D^*

$$b_D^* = \frac{4}{3}\gamma - \frac{1}{6}\sqrt{64\gamma^2 - 48\gamma(\overline{y} - D - \overline{U})} \tag{11}$$

that puts a restriction on the cost of effort for the agent, $\gamma \geq \frac{3}{4}(\overline{y} - D - \overline{U})$. Notice that the optimal bonus is a function of the promised payment D, i.e. of the leverage at time $t = -1$. We have then verified in this simple example our proposition 1, but we now can tell more precise answer on the comparative static of the model: is b_D^* higher or lower of b^*? Is the optimal bonus decreasing with D? Can we quantify the agency cost of debt financing?

First, it is not immediately verified, but it is very intuitive, that $b_D^* < b^*$. It is intuitive because in this example the bonus is extremely high in the case of 100% equity financement, since it results to give to A all the surplus $y_{\overline{s}} - y_{\underline{s}}$ in case of good realization. Obviously, P cannot promise an higher premium.

Secondly, it is easily shown deriving (b_D^* w. r. to D) that the optimal bonus decrease with leverage, according to our proposition 1. The more the claims of debtholders, the less P gets in any case, and hence the less he is ready to pay to A in the good state, since most probably the payoff will not benefit himself. In the particular example here illustrated this trades off the incentive for P to push A working more in order to make his shares "in-the-money".

Third, we can proceed backward characterizing the optimal capital structure. Substituting (11) in $V_E(D)$ we find that

$$\frac{dV_E}{d D} < 0$$

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and hence the debt financement is more costly than the equity financement: since the P's objective is to maximize V_E, the optimal policy is to issue no risky debt at all. Also, writing $V_D(D)$ we get

$$V_D(D) = -\frac{b^3}{8\gamma^2} + \frac{b^2}{4\gamma} + \frac{b}{2\gamma}(D + \overline{U} - \underline{y}) + \underline{y} - \overline{U} \qquad (12)$$

and we can show three results:

1) $V_D(D) < D$: rational debtholders anticipate the choice of optimal contract for P given the capital structure and ask for an higher return to protect themselves against the induced risk of default; the return on debt, $\frac{D}{V_D(D)} - 1 > 0$.

2) $V_E(b_D^*, D) + V_D(b_D^*, D) < V(b^*)$: changing the payoff function for P causes an inefficiency in terms of project-financing possibilities. All the projects whose cost $I \in [V_E(b_D^*, D) + V_D(b_D^*, D), V(b^*)]$ are not implementable with risky debt, while they have a positive expected NPV[18].

3) $\frac{\partial V_D(b_D^*)}{\partial b} > 0$: for debtholders it would be optimal to design contracts with higher incentives.

Apart from the disclaim applicable to all the paper, about the agency cost of outside equity (see section 4) this example is somewhat special because implies that whenever risky debt is issued the optimal contract from the point of view of shareholders induce less effort. Proposition 1 tells us that this is not always the case. But we can quantify the effects over the return on risky debt and its agency cost, that is necessary to proceed into an empirical investigation.

5.1 Remuneration with shares

We solve here the proposed example considering the case where the agent remuneration consists of shares of the firm, verifying our Proposition 2. The example also allows to see when the (IR) constraint is the binding constraint, a feature that is extremely complex to be analyzed in the more general case.

We start assuming that the project is entirely financed with equities, and the contract specifies the following remuneration to A:

$$w_s = \left\{ \begin{array}{l} q\underline{y} \text{ in } \underline{s} \\ q\overline{y} \text{ in } \overline{s} \end{array} \right. \qquad (13)$$

where q is the quota of shares paid to A. His choice of effort solves

$$\max_a aq\overline{y} + (1-a)q\underline{y} - \gamma a^2$$

and the solution is $a_E(q) = \dfrac{q(\overline{y} - \underline{y})}{2\gamma}$. This represents the (IC) constraint for the optimal choice of q in the principal problem given the first order approach.

But from (IR) we get:

$$aw_{\overline{s}} + (1-a)w_{\underline{s}} - \gamma a^2 \geq \overline{U}$$

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[18] For simplicity we assume here no discount between $t = -1$ and $t = 0$.

and solving for q:

$$q \geq q_E^{IR} = 2\left(\frac{\sqrt{\gamma\left(\gamma\underline{y}^2 + \overline{U}(\overline{y} - \underline{y})^2\right)} - \gamma\underline{y}}{(\overline{y} - \underline{y})^2}\right) \tag{14}$$

Now solve the problem for P in choosing the optimal q[19]:

$$\max_q \quad (1-q)\left(a\overline{y} + (1-a)\underline{y}\right)$$
$$given \quad a_E(q), q \geq q_E^{IR}$$

and from its first order condition:

$$q_E^* = \frac{1}{2} + \frac{\gamma\underline{y}}{(\overline{y} - \underline{y})^2}$$

Whenever $q \geq q^{IR}$ is binding, i.e. $\frac{1}{2} + \frac{\gamma\underline{y}}{(\overline{y}-\underline{y})^2} - 2\left(\frac{\sqrt{\gamma(\gamma\underline{y}^2 + \overline{U}(\overline{y}-\underline{y})^2)} - \gamma\underline{y}}{(\overline{y}-\underline{y})^2}\right) < 0$[20] the optimal choice for P is given by q_E^{IR}. In that case the value of the firm entirely financed by equities is equal to

$$V_E = \sqrt{\frac{\overline{U}(\overline{y} - \underline{y})^2}{\gamma} + \underline{y}^2} \tag{15}$$

We analyze now the impact of a different capital structure on the value of the firm. Suppose $D > \underline{y}$ has been issued at time zero: given the shares-remuneration plan proposed to A, his problem becomes

$$\max_a aq\overline{y} - \gamma a^2$$

and the effort solution is then $a_D(q) = \dfrac{q\overline{y}}{2\gamma}$ that is higher than $a_E(q)$. It is intuitive that now the agent puts an higher effort since he enjoys a positive payment only in the good state. But (IR) must also be satisfied in the choice of q, hence

$$aw_{\overline{s}} - \gamma a^2 \geq \overline{U}$$

that is equivalent to

$$q \geq q_D^{IR} = \frac{2}{\overline{y}}\sqrt{\gamma\overline{U}}$$

---

[19] Observe that the utility of the agent increases with q, then we obtain the second constraint from the (IR) relation (14).

[20] This is equivalent to

$$\frac{(a-b)^2}{4\sqrt{\gamma}} + \frac{3b\sqrt{\gamma}}{2} < \sqrt{\left(\gamma + u(a-b)^2\right)}$$

Again we can solve the principal problem

$$\max_{q} \quad (1-q)a_D(q)(\overline{y}-D)$$
$$given \quad a_D(q), q \geq q_D^{IR}$$

and the solution is now

$$q_D^* = \frac{1}{2}$$

without considering the second constraint. The solution will then be $q = \max\{\frac{1}{2}, \frac{2}{\overline{y}}\sqrt{\gamma\overline{\overline{U}}}\}$. This gives a value for the leveraged firm

$$V_D = \max\{\frac{\overline{y}}{4\gamma}, \sqrt{\frac{\overline{\overline{U}}}{\gamma}}\}(\overline{y}-\underline{y})+\underline{y}$$

and it is easy to verify that $V_D \geq V_E$ since $V_D \geq \sqrt{\frac{\overline{\overline{U}}}{\gamma}}(\overline{y}-\underline{y})+\underline{y}$ and $V_E < \sqrt{\left(\sqrt{\frac{\overline{\overline{U}}}{\gamma}}(\overline{y}-\underline{y})\right)^2 + \underline{y}^2}$.

Then, we have verified the general principle illustrated before: leverage is indeed a value-maximizing device whenever the individual who is putting the human capital (and the effort) necessary to produce payoffs is endowed with shares of the project (or is remunerated by them). With a remuneration scheme where the agent is not paid with claims on the project payoffs, risky debt creates an agency cost.

6   Conclusion

We have shown that when the firm value depends on the result of some human capital effort that is not perfectly observable and it is then exposed to moral hazard behaviour, the way of financing the firm will affect its value.

When the principal-shareholder can finance the investment with equities or with riskless debt, he will implement a contract which maximizes the firm value. This does not occur if the firm is financed partially with risky debt. Then, contracts are an additional sources of agency cost for debt financing. The basic intuition of the result is that, whenever risky debt is issued, the principal does not completely internalize the benefits (resp. the costs) of an higher incentive bonus (with respect to the bonus he would propose with 100 % equity financement) since they are shared with (resp. paid by) debtholders. This implies that in highly leveraged firms we should observe relatively "flat" compensation schemes for the management: in that case inside-shareholders have no incentive to pay a very high remuneration in the case of success since they suffer most of its cost, enjoying the benefits only in relatively few contingencies. The shape of the optimal incentive scheme depends in general on the characteristics of the probability distribution of the future payoffs as a function of the human effort.

The result seems to contradict previous papers (Innes (1990)) in which issuing debt serves as commitment for the entrepreneur to work hard and maximize the value of

the firm. We show that the key difference is due to the kind of managerial compensation: when management is rewarded with shares of the firm, leverage is actually an useful device for implementing value-maximizing choices. We can then reconcile some empirical evidence (Smith and Watts (1992)) that shows a negative association between leverage and compensation when this is done via monetary payments, and other characteristics from case studies in LBOs (Denis (1994)) in which this form of financial reorganization proves much more successful in increasing operational performance when it is linked with a reorganization of managerial compensation that focus on share-plans.

We also present an example where the loss in ex-ante value due to risky debt financement can be quantified.

7 Appendix

Proof of Lemma 2. Write again the (IR) constraint, that for Lemma 1 holds with equality:

$$\int_{\underline{s}}^{\overline{s}} u\left(\sigma + b(y_s - \underline{y})\right) g(s\,|a)ds - c(a) = \overline{U}$$

and differentiate it w.r.to b, obtaining

$$0 = \int_{\underline{s}}^{\overline{s}} \left(u'(w_s)\left(\frac{\partial w_s}{\partial \sigma}\frac{\partial \sigma}{\partial b} + \frac{\partial w_s}{\partial b}\right)g(s\,|a) + u(w_s)g_a(s\,|a)\frac{\partial a}{\partial b}\right)ds - c'(a)\frac{\partial a}{\partial b}$$

$$0 = \frac{\partial a}{\partial b}\left[\int_{\underline{s}}^{\overline{s}} u(w_s)g_a(s\,|a)ds - c'(a)\right] + \int_{\underline{s}}^{\overline{s}} u'(w_s)\left(\frac{\partial w_s}{\partial \sigma}\frac{\partial \sigma}{\partial b} + \frac{\partial w_s}{\partial b}\right)g(s\,|a)ds$$

and given equation (2)

$$0 = \int_{\underline{s}}^{\overline{s}} u'(w_s)\left(\frac{\partial w_s}{\partial \sigma}\frac{\partial \sigma}{\partial b} + \frac{\partial w_s}{\partial b}\right)g(s\,|a)ds$$

$$\frac{\partial \sigma}{\partial b}\int_{\underline{s}}^{\overline{s}} u'(w_s)\frac{\partial w_s}{\partial \sigma}g(s\,|a)ds + \int_{\underline{s}}^{\overline{s}} u'(w_s)\frac{\partial w_s}{\partial b}g(s\,|a)ds = 0$$

and substituting for $\frac{\partial w_s}{\partial b} = (y_s - \underline{y})$, $\frac{\partial w_s}{\partial \sigma} = 1$ we obtain:

$$\frac{\partial \sigma}{\partial b} = -\frac{\int_{\underline{s}}^{\overline{s}} u'(w_s)(y_s - \underline{y})g(s\,|a)ds}{\int_{\underline{s}}^{\overline{s}} u'(w_s)g(s\,|a)ds} \tag{3}$$

that proves the result given that $u(\sigma)$ is monotone increasing. ■

Proof of Lemma 3. Assuming that the "first-order" approach holds, the solution of the agent's problem is characterized by the first order condition (2). Rewriting it observing that $\sigma(b)$ for (IR) constraint:

$$\int_{\underline{s}}^{\overline{s}} u(\sigma(b) + b(y_s - \underline{y}))g_a(s\,|a)ds - c'(a) = 0$$

and differentiating w.r.to b:

$$\int_{\underline{s}}^{\overline{s}} \left(u'(w_s)\left(\frac{\partial w_s}{\partial \sigma}\frac{\partial \sigma}{\partial b} + \frac{\partial w_s}{\partial b}\right)g_a(s\,|a) + u(w_s)g_{aa}(s\,|a)a'(b)\right)ds - c''(a)a'(b) = 0$$

$$\int_{\underline{s}}^{\overline{s}} u'(w_s)\left(\frac{\partial w_s}{\partial \sigma}\frac{\partial \sigma}{\partial b} + \frac{\partial w_s}{\partial b}\right)g_a(s\,|a)ds + a'(b)\left[\int_{\underline{s}}^{\overline{s}} u(w_s)g_{aa}(s\,|a)ds - c''(a)\right] = 0$$

Since $\int_{\underline{s}}^{\overline{s}} u(w_s)g_{aa}(s\,|a)ds - c''(a) < 0$ for the second order condition of problem (A)

$$sgn(a'(b)) = sgn\left[\int_{\underline{s}}^{\overline{s}} u'(w_s)\left(\frac{\partial \sigma}{\partial b} + (y_s - \underline{y})\right)g_a(s\,|a)ds\right]$$

Rewriting the second function one gets

$$\int_{\underline{s}}^{\overline{s}} u'(w_s)\frac{\partial \sigma}{\partial b}g_a(s\,|a)ds + \int_{\underline{s}}^{\overline{s}} u'(w_s)(y_s - \underline{y})g_a(s\,|a)ds$$

For MLRP $\frac{g_a(s|a)}{g(s|a)}$ is increasing in s, and since $\int_{\underline{s}}^{\overline{s}} g(s\,|a)ds = 1$, deriving with respect to a: $\int_{\underline{s}}^{\overline{s}} g_a(s\,|a)ds = 0$. Then there exists a state $\widehat{s}$ such that

$$\int_{\underline{s}}^{\widehat{s}} g_a(s\,|a)ds \; < \; 0$$

$$\int_{\widehat{s}}^{\overline{s}} g_a(s\,|a)ds \; > \; 0$$

Notice now that, form standard results in principal-agent literature (Holmstrom, 1977) if the agent is risk-neutral the principal is not asked to offer him a remuneration scheme that insures him from bad realizations, and hence increasing the bonus will increase the effort: $a'(b) > 0$ if $u(w_s) = w_s$. Then, $\int_{\underline{s}}^{\overline{s}} w_s(\frac{\partial \sigma}{\partial b} + y_s - \underline{y})g_a(s\,|a)ds > 0$. Since $g_a(s\,|a)$ is increasing in s we can say that the states with negative weights $g_a(s\,|a)$ have a lower value w_s. The idea of the proof is that the effort function will still be increasing if the same happens when $u'(w_s)\left(\frac{\partial \sigma}{\partial b} + y_s - \underline{y}\right)$ is substituted to w_s. A sufficient condition for $a'(b) > 0$ is then

$$\frac{\partial}{\partial s}u'(w_s)\left(\frac{\partial \sigma}{\partial b} + y_s - \underline{y}\right) \geq 0$$

that is

$$u''(w_s)\frac{\partial w_s}{\partial s}\left(\sigma'(b) + y_s - \underline{y}\right) + u'(w_s)\frac{\partial y_s}{\partial s} \geq 0$$

and since we have ordinate the states $\frac{\partial y_s}{\partial s} \geq 0$, and the above condition is then equivalent to

$$-\frac{u''(w_s)}{u'(w_s)} \quad \frac{\frac{\partial y_s}{\partial s}}{b(\sigma'(b) + y_s - \underline{y})}$$

∎

Proof of Lemma 4. Rewrite $C = \int_{\underline{s}}^{\overline{s}} (\sigma(b) + b(y_s - \underline{y}))g(s\,|a)ds$ and compute

$$
\begin{aligned}
C'(b) &= \int_{\underline{s}}^{\overline{s}} \left((\sigma'(b) + (y_s - \underline{y}))g(s\,|a) + w_s g_a(s\,|a)a'(b)\right) ds \\
&= \int_{\underline{s}}^{\overline{s}} (\sigma'(b) + (y_s - \underline{y}))g(s\,|a)ds + a'(b)\int_{\underline{s}}^{\overline{s}} w_s g_a(s\,|a)ds \\
&= \sigma'(b)\int_{\underline{s}}^{\overline{s}} g(s\,|a)ds + \int_{\underline{s}}^{\overline{s}} (y_s - \underline{y})g(s\,|a)ds + a'(b)\int_{\underline{s}}^{\overline{s}} w_s g_a(s\,|a)ds \\
&= \sigma'(b) + E_g\left[y_s - \underline{y}\right] + a'(b)\int_{\underline{s}}^{\overline{s}} w_s g_a(s\,|a)ds \\
&= -\frac{\int_{\underline{s}}^{\overline{s}} u'(w_s)(y_s - \underline{y})g(s\,|a)ds}{\int_{\underline{s}}^{\overline{s}} u'(w_s)g(s\,|a)ds} + E_g\left[y_s - \underline{y}\right] + a'(b)\int_{\underline{s}}^{\overline{s}} w_s g_a(s\,|a)ds \\
&= -\frac{E_g\left[u'(w_s)(y_s - \underline{y})\right]}{E_g\left[u'(w_s)\right]} + E_g\left[y_s - \underline{y}\right] + a'(b)\int_{\underline{s}}^{\overline{s}} w_s g_a(s\,|a)ds
\end{aligned}
$$

and since $a'(b) > 0$ and $\int_{\underline{s}}^{\overline{s}} w_s g_a(s\,|a)ds > 0$ because both w_s and $g_a(s\,|a)$ are increasing in s, and $\int_{\underline{s}}^{\overline{s}} g_a(s\,|a)ds = 0$, a sufficient condition for $C'(b) > 0$ is

$$-\frac{E_g\left[u'(w_s)(y_s - \underline{y})\right]}{E_g\left[u'(w_s)\right]} + E_g\left[y_s - \underline{y}\right] > 0$$

or

$$
\begin{aligned}
-\frac{Cov\left[u'(w_s), (y_s - \underline{y})\right] + E_g\left[u'(w_s)\right]E_g\left[y_s - \underline{y}\right]}{E_g\left[u'(w_s)\right]} + E_g\left[y_s - \underline{y}\right] &> 0 \\
-\frac{Cov\left[u'(w_s), (y_s - \underline{y})\right]}{E_g\left[u'(w_s)\right]} &> 0 \\
-Cov\left[u'(w_s), (y_s - \underline{y})\right] &> 0
\end{aligned}
$$

that is proven in the following lemma. ∎

Lemma 5. If $-\frac{u''(w_s)}{u'''(w_s)} > b(1 + \overline{y})$ then $Cov\left[u'(w_s), (y_s - \underline{y})\right] < 0$.

Proof. Suppose A is risk neutral, i.e. $u(w_s) = w_s$; then $u'(w_s) = 1$ and $Cov\left[u'(w_s), (y_s - \underline{y})\right] = 0$. If A is risk-averse $u'(w_s)$ is decreasing and convex in y_s. Call $f(y_s) = u'(w_s)$. We have

$$
\begin{aligned}
Cov(u'(w_s), (y_s - \underline{y})) &= Cov(f(y_s), y_s) \\
&= E_g\left[f(y_s) * y_s\right] - E_g\left[f(y_s)\right]E_g\left[y_s\right]
\end{aligned}
$$

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and for convexity of $f(y_s)$ we can use Jensen's inequality $E_g\left[f(y_s)\right] \geq f(E_g[y_s])$. We have a sufficient condition for $Cov(f(y_s), y_s) < 0$ that is

$$E_g\left[f(y_s) * y_s\right] - f(E_g[y_s])E_g\left[y_s\right] < 0$$

Call $h(y_s) = f(y_s) * y_s$; the above condition becomes

$$
\begin{aligned}
E_g\left[h(y_s)\right] - h(E_g[y_s]) &< 0 \\
E_g\left[h(y_s)\right] &< h(E_g[y_s])
\end{aligned}
$$

that is equivalent to $h(y_s)$ being strictly concave.

$$
\begin{aligned}
\frac{\partial h(y_s)}{\partial y_s} &= f'(y_s)y_s + f(y_s) \\
\frac{\partial^2 h(y_s)}{\partial y_s^2} &= f''(y_s)y_s + f'(y_s) + f''(y_s) < 0
\end{aligned}
$$

and substituting for the definition of f

$$
\begin{aligned}
(1 + y_s)b^2 u'''(w_s) + bu''(w_s) &< 0 \\
b(1 + y_s) &< -\frac{u''(w_s)}{u'''(w_s)}
\end{aligned}
$$

and then the sufficient condition of the text is easily derived. ∎

Proof of Lemma 6. Deriving (1) w.r.to b :

$$
\begin{aligned}
\frac{\partial^2 V}{\partial b^2} &= -\sigma''(b) + a''(b)\int_{\underline{s}}^{\overline{s}} (y_s - w_s)g_a(s\,|a)ds + \\
&\quad + a'(b)\int_{\underline{s}}^{\overline{s}} \left(-\frac{\partial w_s}{\partial b}g_a(s\,|a) + a'(b)(y_s - w_s)g_{aa}(s\,|a)\right)ds + \\
&\quad - a'(b)\int_{\underline{s}}^{\overline{s}} y_s g_{aa}(s\,|a)ds \\
&= -\sigma''(b) + a''(b)\int_{\underline{s}}^{\overline{s}} (y_s - w_s)g_a(s\,|a)ds + \\
&\quad + a'(b)\int_{\underline{s}}^{\overline{s}} (-\sigma'(b) - y_s + \underline{y})g_a(s\,|a)ds + \\
&\quad + a'(b)^2\int_{\underline{s}}^{\overline{s}} (y_s - w_s)g_{aa}(s\,|a)ds - a'(b)\int_{\underline{s}}^{\overline{s}} y_s g_{aa}(s\,|a)ds \\
&= -\sigma''(b) + a''(b)\int_{\underline{s}}^{\overline{s}} (y_s - w_s)g_a(s\,|a)ds + \\
&\quad + a'(b)(a'(b) - 1)\int_{\underline{s}}^{\overline{s}} y_s g_{aa}(s\,|a)ds + \\
&\quad - a'(b)^2\int_{\underline{s}}^{\overline{s}} w_s g_{aa}(s\,|a)ds - a'(b)\int_{\underline{s}}^{\overline{s}} (\sigma'(b) + y_s - \underline{y})g_a(s\,|a)ds
\end{aligned}
$$

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and the sufficient conditions follow restricting all the terms to be negative where $\int_{\underline{s}}^{\overline{s}} w_s g_{aa}(s\,|a)ds > 0$ and $\int_{\underline{s}}^{\overline{s}} (\sigma'(b) + y_s - \underline{y}) g_a(s\,|a)ds > 0$ for Assumption 3. ∎

Proof of proposition 2. The proof is divided in three steps: first, we show that the effort of the agent a increases in his quota, q of shares, under some bounds on his risk-aversion; secondly, we prove that the principal has to increase that quota when leverage increases in order to satisfy the (IR) constraint of the agent; third, we show that the value of the firm V for a given capital structure D/E increases in the quota earned by the agent.

First part: $a'(q) \geq 0$.

Write the first-order condition for the effort choice by the agent

$$\int_{\underline{s}}^{\overline{s}} u(qy_s)g_a(s\,|a)ds - c'(a) = 0$$

and differentiate it w.r.to q:

$$\int_{\underline{s}}^{\overline{s}} \left(u'(w_s)y_s g_a(s\,|a) + u(w_s)g_{aa}(s\,|a)a'(q) \right) ds - c''(a)a'(q) = 0$$

$$\int_{\underline{s}}^{\overline{s}} u'(w_s)y_s g_a(s\,|a)ds + a'(q)\left[\int_{\underline{s}}^{\overline{s}} u(w_s)g_{aa}(s\,|a)ds - c''(a)\right] = 0$$

and given that the coefficient of $a'(q)$ is negative for S.O.C we have

$$sgn[a'(q)] = sgn[\int_{\underline{s}}^{\overline{s}} u'(w_s)y_s g_a(s\,|a)ds]$$

Now, following the reasoning explained in the proof of Lemma 3 and applying the same consideration, we get that a sufficient condition for $a'(q) \geq 0$ is

$$\frac{\partial}{\partial s}(u'(w_s)y_s) \geq 0$$

and hence

$$-\frac{u''(w_s)}{u'(w_s)} \quad \frac{dy_s/ds}{y_s} \tag{16}$$

If risky debt D has been issued, now the problem, of the agent looks different, and the F.O.C. becomes

$$\int_{\widehat{s}}^{\overline{s}} u(q(y_s - D))g_a(s\,|a)ds - c'(a) = 0$$

and proceeding as before we have

$$\int_{\widehat{s}}^{\overline{s}} \left(u'(w_s)(y_s - D)g_a(s\,|a) + u(w_s)g_{aa}(s\,|a)a'(q) \right) ds - c''(a)a'(q) = 0$$

and hence

$$sgn[a'(q)] = sgn[\int_{\widehat{s}}^{\overline{s}} u'(w_s)(y_s - D)g_a(s\,|a)ds]$$

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and then we get a new sufficient condition for $a'(q) \geq 0$:

$$-\frac{u''(w_s)}{u'(w_s)} \quad \frac{dy_s/ds}{y_s - D}$$

that is a less restrictive bound than (16).

Second part: $q'(D) \geq 0$.

Take the (IR) constraint for the agent when risky debt has been issued:

$$\int_{\widehat{s}}^{\overline{s}} u(q(y_s - D))g(s\,|a)ds - c(a) \geq \overline{U}$$

and suppose it was binding. Then differentiate w.r.to D

$$\int_{\widehat{s}}^{\overline{s}} (u'(w_s)(-q(D) + q'(D)(y_s - D))g(s\,|a) + u(w_s)g_a(s\,|a)a'(q)q'(D))\,ds +$$
$$u(q(y_{\widehat{s}} - D))g(\widehat{s}\,|a)\frac{d\widehat{s}}{dD} - c'(a)a'(q)q'(D) = 0$$

and assuming that $u(0) = 0$ we get

$$\int_{\widehat{s}}^{\overline{s}} u'(w_s)(-q(D) + q'(D)(y_s - D))g(s\,|a)ds +$$
$$a'(q)q'(D)[\int_{\widehat{s}}^{\overline{s}} u(w_s)g_a(s\,|a)ds - c'(a)] = 0$$

that reduces to (by F.O.C. of agent problem)

$$q'(D) \int_{\widehat{s}}^{\overline{s}} u'(w_s)(y_s - D)g(s\,|a)ds - q(D) \int_{\widehat{s}}^{\overline{s}} u'(w_s)g(s\,|a)ds = 0$$
$$q'(D) = \frac{q(D) \int_{\widehat{s}}^{\overline{s}} u'(w_s)g(s|a)ds}{\int_{\widehat{s}}^{\overline{s}} u'(w_s)(y_s - D)g(s|a)ds}$$

that is positive since $u'(w_s) > 0$, $y_s - D > 0$ for $s > \widehat{s}$ and $q(D) > 0$.

Third part: $q'(D) \geq 0$.

If we write the ex-ante value of the firm, for any ratio of debt over equity we have

$$V = \int_{\underline{s}}^{\overline{s}} y_s g(s\,|a(q))ds$$

and differentiating w.r.to q

$$\frac{dV}{dq} = a'(q) \int_{\underline{s}}^{\overline{s}} y_s g_a(s\,|a(q))ds$$

and since $a'(q) \geq 0$, $\frac{dV}{dq} \geq 0$ whenever $\int_{\underline{s}}^{\overline{s}} y_s g_a(s\,|a(q))ds > 0$, that is true under assumption 2.

Putting together the three results, we have that the value of the firm increases with leverage if the (IR) constraint of the agent is binding. ∎

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