CentER

Discussion Paper

No. 2005–116

MONETARY POLICY, DETERMINACY, AND LEARNABILITY IN THE OPEN ECONOMY

By James Bullard, Eric Schaling

May 2005

ISSN 0924-7815

Tilburg University

Monetary Policy, Determinacy, and Learnability in the Open Economy

James Bullard* **Eric Schaling**†

4 May 2005‡

Abstract

We study how determinacy and learnability of global rational expectations equilibrium may be affected by monetary policy in a simple, two country, New Keynesian framework. The two blocks may be viewed as the U.S. and Europe, or as regions within the euro zone. We seek to understand how monetary policy choices may interact across borders to help or hinder the creation of a unique rational expectations equilibrium worldwide which can be learned by market participants. We study cases in which optimal policies are being pursued country by country as well as some forms of cooperation. We find that open economy considerations may alter conditions for determinacy and learnability relative to closed economy analyses, and that new concerns can arise in the analysis of classic topics such as the desirability of exchange rate targeting and monetary policy cooperation.
Keywords: Indeterminacy, learning, monetary policy rules, new open economy macroeconomics, exchange rate regimes, second generation policy coordination.

*Research Department, Federal Reserve Bank of St. Louis, bullard@stls.frb.org. Any views expressed are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

†Department of Economics University of Johannesburg, and CentER for Economic Research, Tilburg University. Address: P.O. Box 524, 2006, Auckland Park, Johannesburg, Republic of South Africa. Telephone: + 27 (11) 489-2927. Email: esc@eb.rau.ac.za. This paper was written while Eric Schaling was a visiting scholar at the Research Department of the Federal Reserve Bank of St. Louis.

‡The authors thank Ed Nelson for helpful discussions concerning open economy New Keynesian models.

1 Introduction

1.1 Overview

New Keynesian macroeconomic models have become a workhorse for studying a variety of monetary policy issues in closed economy environments. An important component of this effort has been the development of the idea that equilibrium determinacy and learnability may be significantly influenced by monetary policy choices.[1] Recently, simple extensions of the New Keynesian model to open economy environments have been developed. Our primary concern in this paper is to provide an analysis of the extent to which the findings concerning determinacy and learnability for the closed economy New Keynesian framework may be altered when open economy considerations are brought to bear. Our learnability criterion is that of Evans and Honkapohja (2001).

Our approach to this question is to adopt a simple framework for a two-country world due to Clarida, Gali, and Gertler (2002). This framework provides one straightforward extension of the New Keynesian model to two countries and allows comparison to the more common single country and small open economy analyses as special cases. The model has a natural separation between countries that Clarida, Gali, and Gertler (2002) discuss in some detail. Roughly, after making certain adjustments to parameters accounting for the degree of openness of each economy, this version of the open economy New Keynesian model is qualitatively the same as the standard, Clarida, Gali, and Gertler (1999)-style closed economy New Keynesian model. We exploit this feature extensively in this paper. We show that the nature of monetary policy in each country can lead to more or less international feedback, and complicate or simplify the conditions for determinacy and learnability of worldwide equilibrium.

[1] See, for instance, Woodford (2003), Bullard and Mitra (2002), Evans and Honkapohja (2003a,b), and Preston (2003).

1.2 Main findings

We are able to make some progress analytically in showing how determinacy and learnability conditions depend on the nature of monetary policy in each country, the conditions under which one policymaker can or cannot mitigate the threats of indeterminacy and expectational instability posed by another country, and the degree to which international policy coordination may fail or succeed in delivering determinacy and learnability of worldwide equilibrium.

The main findings are as follows. Instrument rules which are focussed on domestic inflation and domestic output gaps lead to world determinacy and learnability conditions which must be met in each economy independently of whether they are met in the partner economy. Policymakers in even a large economy cannot take actions which will mitigate the threats of indeterminacy and expectational instability stemming from poor policy choices made by the partner.

For targeting rules, this result has a natural counterpart when policymakers in each country pursue non-cooperative optimal policy under discretion. The choice of how to implement the optimality condition stemming from the minimization problem faced by the monetary authorities can easily be made inappropriately, leading to indeterminacy and expectational instability. But the objectives of the two monetary authorities in this case involve only domestic variables and so under the most natural implementation strategies determinacy and learnability will again hinge on certain conditions being met in each economy independently of the conditions for the partner economy.

On the other hand, instrument rules which include responses to international economic conditions induce international feedback between the two economies even when there would otherwise be no such feedback. Policy choices in each economy will then influence all aspects of the conditions for determinacy and learnability of worldwide equilibrium. The separability of conditions between countries breaks down. Policymakers in a large country would then have some potential to render worldwide equilibrium determinate and learnable even if not all players worldwide are pursuing policies which meet minimal conditions for determinacy and learnability when viewed in isolation.

This second result again has a natural counterpart in the case of targeting rules, in the situation where the two countries agree to try to pursue the gains from cooperation which may exist in the model. In this case, the joint objective of the policymakers will involve weighted averages of inflation and output gaps in the two economies. Implementation strategy will again be an issue. Determinacy and learnability conditions will hinge on the nature of the joint policy of the two central banks, and one policy authority may be able to choose policy to deliver worldwide determinacy and learnability even if the partner authority adopts a policy which would be inconsistent with those objectives viewed in isolation.

We conclude that determinacy and learnability considerations can alter the evaluation of monetary policy options in an international context.

1.3 Recent related literature

The seminal work on the New Keynesian framework and the role of monetary policy is Woodford (2003). We have chosen to follow the extension to a two-country environment proposed by Clarida, Gali, and Gertler (2002). We stress that alternative extensions may produce substantively different findings concerning determinacy and learnability than the ones we report here, and for this reason we think it would be interesting to carry out the analysis below in other environments. Still, we think that Clarida, Gali, and Gertler (2002) provides a natural starting point.

Batini, Levine, and Pearlman (2004) study indeterminacy in a two-country New Keynesian model. Their focus is on the relationship between many-period forward-looking inflation forecast rules and indeterminacy conditions. We do not consider rules in this class in this paper. When forward-looking rules are considered here, they arise from the implementation of certain optimality considerations and do not involve forecasts more than one period into the future.

De Fiore and Liu (forthcoming) study indeterminacy in a small open New Keynesian economy. Their model is somewhat different from the one we study. They conclude that whether a given policy rule can deliver determinacy will depend on the degree of openness in the small economy, a result we also obtain.

Bencivenga, Huybens, and Smith (2001) study the relationship between dollarization and the scope for endogenous volatility (that is, indeterminacy implying the existence of sunspot equilibria). Their model emphasizes aspects of financial intermediation which are not part of the analysis here. They suggest that the degree of capital market integration plays an important role in judging whether certain policy options are consistent with determinacy or not. This type of capital market integration issue cannot be effectively addressed in the class of models we examine in this paper.

A number of papers study classic open economy issues in the New Keynesian framework. Pappa (2004) and Benigno and Benigno (2004), for example, study the gains from monetary policy coordination. Corsetti and Pesenti (2005) and Obstfeld and Rogoff (2002) analyze 'self-oriented' or 'inward-looking' national monetary policies in frameworks related to the one studied here. While touching on some related themes, these papers do not focus on the determinacy and learnability issues we emphasize.

Ellison, Sarno, and Vilmunen (2004) study central bank learning in the two-country world of Aghion, Becchetta, and Banerjee (2001). They allow fundamental parameters in the economy to follow Markov switching processes, and central banks update their inference concerning the current regime via Bayes rule. This optimal filtering conception of learning is quite interesting and, as the authors show, can have important implications for policy. But it is also conceptually distinct from the expectational stability analysis of Evans and Honkapohja (2001), in which small expectational errors may propagate and drive the economy away from the targeted equilibrium desired by policymakers. The present paper has both private sector and central bank learning in some circumstances where the central banks' implementation of policy involves a policy rule with forward-looking components.[2]

[2] For an analysis of how uncertainty about model parameters may drive central bank learning and alter our assessment of what would otherwise be optimal policy under full

Zanna (2004) studies determinacy and learnability in the small open economy case for a model due to Uribe (2003) which is again somewhat different from the one we study. We comment on the purchasing power parity rules of Zanna and relate them to our findings later in the paper. Zanna (2004) contains results on learnable sunspot equilibria under common factor representations, a topic we have not addressed here.

Working in parallel with us, Llosa and Tuesta (2005) study determinacy and learnability in a version of the Clarida, Gali, and Gertler (2002) model we use. Whereas we emphasize the two country model they analyze the model from the point of view of the small open economy. Llosa and Tuesta (2005) study instrument rules more extensively than we do, including different forms of Taylor-type rules as in Bullard and Mitra (2002). We discuss both instrument and targeting rules. The Llosa and Tuesta (2005) discussion of domestic inflation versus consumer price index inflation in the policy rule parallels some of our analysis, and we compare our results to theirs when appropriate.

1.4 Organization

We begin by presenting the basic model environment in the next section. We take up our analysis of the effects of policy on determinacy and learnability by first considering instrument rules, simple descriptions of policy that allow us to develop some basic results and intuition, especially concerning "country by country" determinacy and learnability conditions. Policymakers using rules in this class might break the natural separability of country analysis in the model should they decide to react in part to international variables when setting monetary policy, and we develop a version of this situation in some detail. We end this section by providing a brief discussion of purchasing power parity instrument rules.

We then turn to targeting rules, whereby the policy rule is inferred from an optimization exercise undertaken by each monetary authority. The na-

information, see Aoki and Nikolov (2004). They study the closed economy case.

ture of the optimization exercise will be important for our findings. Non-cooperative assumptions will preserve the natural separability of country analysis in the model and thus keep determinacy and learnability assessments unique to each country, while cooperative assumptions will not. The final portion of the paper takes up certain asymmetric situations which do not fit neatly into the above categories. One of these is the case of one country pegging its exchange rate to a second country which is following an independent monetary policy. We discuss our findings and directions for future research in the conclusion.

2 A two-country New Keynesian model

2.1 Overview

We employ the two-country model of Clarida, Gali, and Gertler (2002). This is one natural extension of the closed economy New Keynesian model to the open economy case in which two large economies are interacting, and so it provides a good starting point for the analysis of determinacy issues in the open economy. Our purpose is not to develop new aspects of this model *per se*, but to use the model to try to understand some of the main considerations that might arise concerning determinacy and learnability in the world economy, when determinacy and learnability can be importantly influenced by policymakers. Accordingly, we merely present the main equations of the Clarida, Gali, and Gertler (2002) framework here. Interested readers are referred to Clarida, Gali, and Gertler (2002) for details.

2.2 Environment

The model economy is log-linearized about a steady state and is given by

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \sigma_o^{-1} \left[r_t - E_t \pi_{t+1} - \overline{rr}_t \right], \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_o \tilde{y}_t + u_t \tag{2}$$

where $\kappa_o = \gamma(\sigma - 1)$, $\sigma_o = \sigma - \kappa_o$, $\kappa = \sigma - \kappa_o + \phi$, $\lambda_o = \delta\kappa$, and $\delta = [(1-\theta)(1-\beta\theta)]/\theta$. The variable $\tilde{y}_t$ represents the domestic output gap, π_t represents the deviation of domestic producer price inflation from a target value set by the domestic monetary authority, and r_t represents the deviation of the short term nominal interest rate from the value consistent with domestic inflation at target and a zero domestic output gap. Here $\overline{rr}_t$ is the domestic natural real interest rate (conditional on foreign output), given by

$$\overline{rr}_t = \sigma_o E_t \Delta\bar{y}_{t+1} + \kappa_o E_t \Delta y^{\star}_{t+1},$$

where $\Delta\bar{y}_{t+1}$ is the rate of growth of the domestic natural level of output and $\Delta y^{\star}_{t+1}$ is the rate of growth of the level of foreign output. The stochastic term u_t follows an $AR(1)$ process given by

$$u_t = \rho u_{t-1} + \epsilon_t,$$

with $0 \leq \rho < 1$, where $\epsilon_{u,t}$ is an *iid* stochastic term. The symbol E_t is the standard expectations operator. The equations (1) and (2) have five fundamental parameters: The household discount factor β, a parameter controlling the curvature in preferences over consumption σ, a parameter controlling the curvature in preferences over leisure ϕ, the mass of agents or degree of openness γ, and the probability that a firm will not be able to change its price today θ, which we sometimes refer to as the degree of price stickiness. The foreign economy is described analogously as

$$\tilde{y}^{\star}_t = E_t \tilde{y}^{\star}_{t+1} - \sigma_o^{\star,-1}\left[r^{\star}_t - E_t \pi^{\star}_{t+1} - \overline{rr}^{\star}_t\right], \qquad (3)$$

$$\pi^{\star}_t = \beta E_t \pi^{\star}_{t+1} + \lambda^{\star}_o \tilde{y}^{\star}_t + u^{\star}_t \qquad (4)$$

where $\kappa^{\star}_o = (1-\gamma)(\sigma - 1)$, $\sigma^{\star}_o = \sigma - \kappa^{\star}_o$, $\kappa^{\star} = \sigma - \kappa^{\star}_o + \phi$, $\lambda^{\star}_o = \delta\kappa^{\star}$, and $\delta = [(1-\theta)(1-\beta\theta)]/\theta$. In these equations $\tilde{y}^{\star}_t$ is the foreign output gap, $\pi^{\star}_t$ is the deviation of foreign producer price inflation from a target value set by the government, and $r^{\star}_t$ is the deviation of the foreign nominal interest rate from a value consistent with inflation at target and output at potential. The term $\overline{rr}_t^\star$ is the foreign natural real interest rate (conditional on domestic output), given by

$$\overline{rr}_t^\star = \sigma_o^\star E_t \Delta \bar{y}_{t+1}^\star + \kappa_o^\star E_t \Delta y_{t+1},$$

where $\Delta \bar{y}_{t+1}^\star$ is the rate of growth of the foreign natural level of output and Δy_{t+1} is the rate of growth of the level of domestic output. The stochastic term $u_t^\star$ is assumed to follow an $AR(1)$ process given by

$$u_t^\star = \rho u_{t-1}^\star + \epsilon_{u,t}^\star$$

with $0 \leq \rho < 1$, where $\epsilon_{u,t}^\star$ is an *iid* stochastic term. In equations (3) and (4), the fundamental parameters β, σ, ϕ, γ, and θ are all the same as in equations (1) and (2), reflecting a maintained assumption that the preferences and technologies in the two economies are the same. The only difference is that γ in (1) and (2) has been replaced by $1-\gamma$ in (3) and (4).

The domestic and foreign natural rates of interest, $\overline{rr}_t$ and $\overline{rr}_t^\star$, in part reflect expected changes in potential output in the foreign and the domestic economies. In order to remain consistent with the closed economy New Keynesian literature, such as Woodford (1999), in this section we will view the natural rates of interest as exogenous stochastic terms following $AR(1)$ processes, again with serial correlation $0 \leq \rho < 1$, and *iid* disturbances $\epsilon_{\overline{rr},t}$ and $\epsilon_{\overline{rr},t}^\star$, respectively. Later in the paper we will use the Clarida, Gali, and Gertler (2002) formulation in which these terms are observed directly by the economy's participants.

The nominal exchange rate e_t obeys consumer price index-based, or "aggregate" purchasing power parity, and is given by

$$\begin{aligned} e_t &= (p_{C,t} - p_{C,t}^\star) \\ &= (p_t + \gamma s_t) - (p_t^\star - \{(1-\gamma) s_t\}) \\ &= p_t - p_t^\star + s_t \end{aligned}$$

where p_t is shorthand for the domestic producer price level $p_{H,t}$, $p_t^\star$ is shorthand for the foreign producer price level $p_{F,t}^\star$, and where $p_{C,t}$ and $p_{C,t}^\star$ represent the home and foreign consumer price index, respectively. Finally, a simple expression links the terms of trade to movements in the output gap:

$$\begin{aligned} s_t &= (\tilde{y}_t - \tilde{y}_t^\star) + (\bar{y}_t - \bar{y}_t^\star) \\ &= (\tilde{y}_t - \tilde{y}_t^\star) + \bar{s}_t \end{aligned}$$

where $\bar{s}_t$ is the natural level of the terms of trade.

The open economy effects in this model come through the composite parameters κ_o and $\kappa_o^\star$. In the special case where $\sigma = 1$, the effects vanish as $\kappa_o = \kappa_o^\star = 0$. In this special case, each economy evolves as if it were an isolated, closed economy. In addition, the special cases where either $\gamma \to 0$ or $\gamma \to 1$ respectively place all the mass of agents in the home or the foreign economy. In these cases, the home or foreign economy behaves as if it were an isolated, closed economy, while the partner behaves as if it were a small open economy.[3]

The two-country model involves the short-term nominal interest rates r_t and $r_t^\star$. In the remainder of the paper we will analyze the model under different scenarios for how these interest rates are determined by policymakers. We will begin with a simple specification that produces simple intuition, and later move to more complicated optimal policy specifications under a variety of assumptions on the nature of the optimization policymakers undertake.

3 Instrument rules

3.1 Simple Taylor-type rules

3.1.1 The dynamic system

In this section we simply assume that the policymakers in each country follow Taylor-type policy rules given by

$$r_t = \varphi_\pi \pi_t + \varphi_y \tilde{y}_t \tag{5}$$

[3] Clarida, Gali and Gertler (2001) and Gali and Monacelli (2002) analyze the case of a small open economy using a similar framework.

for the domestic economy, and by

$$r_t^\star = \varphi_\pi^\star \pi_t^\star + \varphi_y^\star \tilde{y}_t^\star \tag{6}$$

for the foreign economy. By substituting (5) and (6) into equations (1) and (3), we can write the four equation system as follows. First, define $\mathcal{Z}_t = [\tilde{y}_t, \pi_t, \tilde{y}_t^\star, \pi_t^\star]'$ along with $\mathcal{V}_t = [\overline{rr}_t, u_t, \overline{rr}_t^\star, u_t^\star]'$. Then write the system as

$$\mathcal{Z}_t = \mathcal{A}_0 + \mathcal{B} E_t \mathcal{Z}_{t+1} + \mathcal{X} \mathcal{V}_t \tag{7}$$

where $\mathcal{A}_0 = 0$,[4]

$$\mathcal{B} = \begin{bmatrix} B_{11} & \mathbf{0} \\ \mathbf{0} & B_{22} \end{bmatrix},$$

$$B_{11} = \frac{1}{\sigma_o + \varphi_y + \lambda_o \varphi_\pi} \begin{bmatrix} \sigma_o & 1 - \beta\varphi_\pi \\ \lambda_o \sigma_o & \lambda_o + \beta\left(\sigma_o + \varphi_y\right) \end{bmatrix},$$

$$B_{22} = \frac{1}{\sigma_o^\star + \varphi_y^\star + \lambda_o^\star \varphi_\pi^\star} \begin{bmatrix} \sigma_o^\star & 1 - \beta\varphi_\pi^\star \\ \lambda_o^\star \sigma_o^\star & \lambda_o^\star + \beta\left(\sigma_o^\star + \varphi_y^\star\right) \end{bmatrix},$$

where $\mathbf{0}$ is a 2×2 matrix of zeroes, and

$$\mathcal{X} = \begin{bmatrix} X_{11} & \mathbf{0} \\ \mathbf{0} & X_{22} \end{bmatrix},$$

with

$$X_{11} = \begin{bmatrix} \sigma_o^{-1} & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$X_{22} = \begin{bmatrix} \sigma_o^{\star,-1} & 0 \\ 0 & 1 \end{bmatrix},$$

and where $\mathcal{V}_t$ follows a vector $AR(1)$ process with serial correlation given by the scalar ρ.

[4] We stay consistent with Bullard and Mitra (2002) in allowing for constant terms.

3.1.2 Determinacy

Because the four variables in this system are free in the terminology of Blanchard and Kahn (1980), we require all eigenvalues of $\mathcal{B}$ to be inside the unit circle for determinacy. Since $\mathcal{B}$ is block diagonal, this requirement means that the eigenvalues of B_{11} and B_{22} must be inside the unit circle. From a version of Proposition 1 in Bullard and Mitra (2002), this implies that the following two conditions must hold for determinacy in this system:

$$\lambda_o \left(\varphi_\pi - 1\right) + \left(1 - \beta\right) \varphi_y > 0 \tag{8}$$

and

$$\lambda_o^\star \left(\varphi_\pi^\star - 1\right) + \left(1 - \beta\right) \varphi_y^\star > 0. \tag{9}$$

These conditions are versions of the Taylor principle for each country and depend on the household discount factor β, which is assumed to be the same in the two countries, on the policy parameters in the Taylor-type rules in the two countries, and on the composite parameters λ_o and $\lambda_o^\star$. We can write the composite parameters as

$$\begin{aligned}
\lambda_o &= \delta \left[\sigma + \phi - \gamma \left(\sigma - 1\right)\right], \\
\lambda_o^\star &= \delta \left[\sigma + \phi - \left(1 - \gamma\right) \left(\sigma - 1\right)\right].
\end{aligned}$$

Thus the conditions (8) and (9) can be written as

$$\delta \left[\sigma + \phi - \gamma \left(\sigma - 1\right)\right] \left(\varphi_\pi - 1\right) + \left(1 - \beta\right) \varphi_y > 0 \tag{10}$$

and

$$\delta \left[\sigma + \phi - \left(1 - \gamma\right) \left(\sigma - 1\right)\right] \left(\varphi_\pi^\star - 1\right) + \left(1 - \beta\right) \varphi_y^\star > 0. \tag{11}$$

The term in brackets is positive, so that if $\varphi_y = \varphi_y^\star = 0$, the conditions state that each central bank has to move nominal interest rates more than one-for-one in response to deviations of inflation from target. We have several remarks on conditions (10) and (11).

First, the conditions are not the same except in the special case where policies are identical (in the sense that $\varphi_\pi = \varphi_\pi^\star$ and $\varphi_y = \varphi_y^\star$) and $\gamma = 1/2$, which would be interpreted as the case that the two economies are equally open.[5] Otherwise, the degree of openness differs and this translates into a difference in the two conditions. This means in particular that identical policy in the two countries, in the sense of identical values for the Taylor-type policy rule coefficients, may be enough to meet one determinacy condition but not the other.

Second, the policy parameters from a single country can only influence one of the two conditions. Thus policymakers from each country must separately meet conditions for determinacy: Determinacy conditions are met, in some sense, "country by country."[6] If one country fails, then worldwide equilibrium is indeterminate, and one country cannot "make up for" the failure of a second country to meet appropriate conditions.

3.1.3 Learnability

We now turn to the learnability of rational expectations equilibrium for cases where that equilibrium is unique. We allow the expectations in equation (7) to initially be different from rational expectations.[7] The MSV solution of equation (7) is given by

$$\mathcal{Z}_t = \bar{\mathcal{A}} + \bar{\mathcal{C}}\mathcal{V}_t$$

where the conformable matrix $\bar{\mathcal{A}}$ is null and

$$\bar{\mathcal{C}} = (I - \rho\mathcal{B})^{-1}\mathcal{X}.$$

[5] If $\gamma = \frac{1}{2}$, there is no home bias in consumption. More generally, for $\gamma < \frac{1}{2}$, both country H and country F consumers will demand relatively more H goods than F goods, while for $\gamma > \frac{1}{2}$, both sets of consumers will demand relatively more F goods than H goods. Likewise, $1-\gamma$ corresponds to the share of country F consumption allocated to goods imported from country H, and so is a measure of openness for that country. This has the important implication that as γ increases country H becomes more open and F moves towards more autarky ($1-\gamma$ declines).

[6] This is due to the combination of flexible exchange rates and purchasing power partity which keeps countries perfectly insulated from foreign instabilities transmitted via the terms of trade. With a fixed exchange rate this will no longer be the case.

[7] Preston (2003) considers deriving the fundamental equations of models in this class assuming agents are learning. Under his interpretation of the microfoundations, the equations are altered and long-horizon forecasts matter. We think it would be interesting to carry out an analysis of this type for the open economy case.

We endow agents with a *perceived law of motion*

$$\mathcal{Z}_t = \mathcal{A} + \mathcal{C}\mathcal{V}_t \tag{12}$$

where $\mathcal{A}$ and $\mathcal{C}$ are conformable. Using this perceived law of motion and assuming time t information $(1, \overline{rr}_t, u_t, \overline{rr}_t^\star, u_t^\star)'$ we can calculate

$$E_t\mathcal{Z}_{t+1} = \mathcal{A} + \mathcal{C}\rho\mathcal{V}_t.$$

Substituting this into equation (7) yields the *actual law of motion*

$$\begin{aligned} \mathcal{Z}_t &= \mathcal{B}\left(\mathcal{A} + \mathcal{C}\rho\mathcal{V}_t\right) + \mathcal{X}\mathcal{V}_t \\ &= \mathcal{B}\mathcal{A} + \left(\mathcal{B}\mathcal{C}\rho + \mathcal{X}\right)\mathcal{V}_t. \end{aligned}$$

We then define a map T from the perceived law of motion to the actual law of motion as

$$T\left(\mathcal{A}, \mathcal{C}\right) = \left(\mathcal{B}\mathcal{A}, \mathcal{B}\mathcal{C}\rho + \mathcal{X}\right).$$

Expectational stability is attained if the differential equation

$$\frac{d}{d\tau}\left(\mathcal{A}, \mathcal{C}\right) = T\left(\mathcal{A}, \mathcal{C}\right) - \left(\mathcal{A}, \mathcal{C}\right)$$

is locally asymptotically stable at $\left(\bar{\mathcal{A}}, \bar{\mathcal{C}}\right)$. Results in Evans and Honkapohja (2001) establish that under weak conditions, expectational stability governs stability in the real-time learning dynamics.

We use Proposition 10.3 in Evans and Honkapohja (2001) to calculate the condition for expectational stability. According to the proposition, the condition for expectational stability is that the real parts of the eigenvalues of the matrices $\mathcal{B}$ and $\rho\mathcal{B}$ are less than unity. Because $0 \leq \rho < 1$, this means that the we need only check the real parts of the eigenvalues of $\mathcal{B}$. Also, because of the block diagonality of $\mathcal{B}$, the expectational stability condition can be calculated country by country, that is, via B_{11} and B_{22}, and by a version of Proposition 2 in Bullard and Mitra (2002) yields conditions (10) and (11). This means that both countries must meet the open economy version of the Taylor principle in order for the world equilibrium to be learnable. It also means that the conditions for determinacy are the same as the conditions for learnability in the special case where both countries follow simple Taylor-type instrument rules. This is known not to be true in general in models in this class with alternative instrument rules, but it provides a good benchmark.[8]

In this calculation we have proceeded as if all actors in the world economy possessed the entire information set $(1, \overline{rr}_t, u_t, \overline{rr}_t^\star, u_t^\star)'$. This is a natural assumption in a domestic economy setting, but it may not be as natural in a multi-country setting. It means that all actors in all countries are keeping track of all state variables worldwide. Instead, one might assume that domestic residents have the information set $(1, \overline{rr}_t, u_t)'$ and that foreign residents have the information set $(1, \overline{rr}_t^\star, u_t^\star)'$. One would then postulate perceived laws of motion for each set of agents as

$$\begin{aligned} Z_{11,t} &= A_{11} + C_{11} V_{11,t}, \\ Z_{22,t} &= A_{22} + C_{22} V_{22,t}, \end{aligned}$$

where $Z_{11} = [\tilde{y}_t, \pi_t]'$, $Z_{22} = [\tilde{y}_t^\star, \pi_t^\star]'$, $V_{11} = [\overline{rr}_t, u_t]'$, and $V_{22} = [\overline{rr}_t^\star, u_t^\star]'$ and where A_{11}, A_{22}, C_{11}, and C_{22} are conformable matrices. Because of the block diagonality in the system, proceeding in this way will yield the same conditions for expectational stability. However, this may not be the case for other systems, as we discuss below.

3.1.4 Quantitative effects

As stressed by Clarida, Gali, and Gertler (2001, 2002), the nature of the policy problem faced by each country in this open economy framework is isomorphic to the closed economy case, but there are nevertheless quantitative consequences. Figure 1 illustrates. Here the calibration has been chosen so that the domestic economy collapses to the one studied by Woodford (1999) when the openness parameter $\gamma \to 0$. Woodford's (1999) values have been

[8] An example of a case in which determinacy and learnability conditions do not coincide is when the policy authorities use a Taylor-type policy rule but react to lagged information on inflation and the output gap. See Bullard and Mitra (2002). For a wider variety of Taylor-type instrument rules in a similar model, see Llosa and Tuesta (2005).

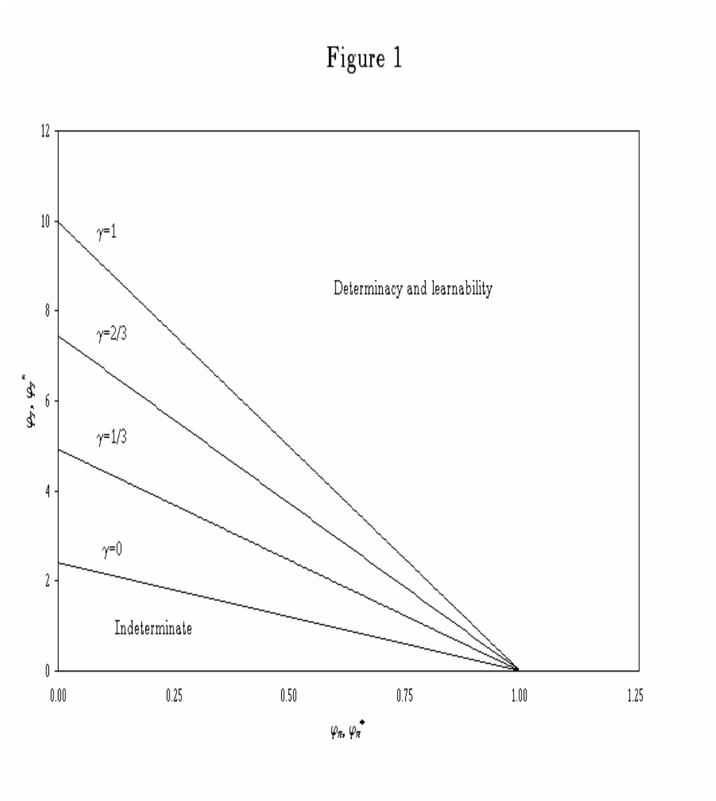

Figure 1: The conditions for determinacy and learnability when each monetary authority uses a simple contemporaneous data Taylor rule. The more open economy will have a steeper tradeoff in the Figure.

widely used and provide a simple benchmark. The discount factor $\beta = 0.99$. When $\gamma \to 0$, $\sigma_o \to \sigma$ and we set this to Woodford's value of $\sigma = 0.157$. The coefficient λ_o would correspond to a value of $\lambda_o = 0.024$ in the Woodford calibration. When $\gamma \to 0$, $\kappa_o \to 0$ so that $\kappa \to \sigma + \phi$, and $\lambda_o = \delta\left(\sigma + \phi\right)$, with $\delta = \left[\left(1-\theta\right)\left(1-\beta\theta\right)\right]/\theta$. We follow Woodford (2003) and set ϕ to the nearly linear value 0.11. Given other parameters, this means that $\theta = 0.745$ to obtain $\lambda_o = 0.024$.

The figure plots (10) as a function of φ_π and φ_y using this calibration for values of γ between zero and unity. Since (11) is the same condition for the foreign country with γ replaced by $1-\gamma$, one can view the lines in Figure 1 as representing this condition as well. The line labelled $\gamma = 0$ represents the case when the home country is closed and corresponds to the condition from Bullard and Mitra (2002, their Figure 1). The determinacy and learnability condition for the foreign country would then correspond to the line labelled $\gamma = 1$ (that is, $1-\gamma$ would equal one if $\gamma = 0$).[9] Thus the small open foreign economy would have to choose its Taylor rule coefficients to the northeast of this line in the figure, while the large closed home economy would only have to choose its Taylor rule coefficients to the northeast of the line labelled $\gamma = 0$. Failure of either country to abide by its condition would produce indeterminacy and the possibility of sunspot fluctuations in the world equilibrium. These lines are closer together if the degree of openness γ is intermediate between zero and one, as illustrated by the lines labelled $\gamma = 1/3$ and $\gamma = 2/3$. For $\gamma = 1/2$, the conditions for determinacy and learnability in the two countries would be identical.

One of the main implications of Figure 1 is that the open economy lines with γ greater than zero all lie above the closed economy line, so that conditions for determinacy and learnability become more stringent when open economy considerations are introduced. A central bank analyzing its economy as if it were closed might mistakenly chose Taylor rule coefficients that are too small to deliver determinacy and learnability of equilibrium.

Another stark implication of this simple Taylor rule case is that the determinacy and learnability conditions must be met country by country. The coefficients in the Taylor-type rule chosen by policymakers in one country do not influence the conditions that apply to the partner economy. If the policymakers in the foreign economy chose values which are inappropriate, then domestic policymakers, even if they are very aggressive, cannot alter this facet of world equilibrium. This complete dichotomy can easily break down however, and we shall now turn to a simple case where this occurs.

[9] Llosa and Tuesta (2005) focus on the small open economy and depict the same line in their Figure 1 for their domestic inflation rule, which would correspond to this case. Their model is similar to the one used here and has a similar calibration.

3.2 Instrument rules with international variables

3.2.1 Consumer versus producer price inflation

In this section we illustrate how these results are altered when each country pays attention to an international variable in its policy rule. We do this by positing simple variants on the ad hoc rule of the previous section. In this case, the block diagonality property of the system can be lost. In some cases, one country can take a policy stance that will mitigate the threats of indeterminacy and expectational instability caused by another country's poor policy. The purpose of this section is to establish this fact in a simple environment.

We begin by supposing that each country pursues a Taylor-type rule featuring consumer price index, or CPI, inflation instead of domestic producer price inflation.[10] This is intuitively plausible as in an open economy CPI inflation, not domestic producer price inflation, is often the variable of interest for the monetary authority. We show that targeting CPI inflation is equivalent to having a conventional Taylor-type rule augmented by a third term which is the *terms of trade*. The monetary policy rule in the home country is given by

$$r_t = \varphi_\pi \pi_{C,t} + \varphi_y \tilde{y}_t, \tag{13}$$

and the monetary authority in the foreign economy pursues

$$r_t^\star = \varphi_\pi^\star \pi_{C,t}^\star + \varphi_y^\star \tilde{y}_t^\star \tag{14}$$

where

$$\pi_{C,t} = \pi_t + \gamma s_t \tag{15}$$

is home CPI inflation, and

$$\pi_{C,t}^\star = \pi_t^\star - (1-\gamma) s_t \tag{16}$$

is foreign CPI inflation. The inflation targets of the monetary authorities implicit in these specifications would then be in terms of CPI inflation. Using

[10] This is also the second rule analyzed by Llosa and Tuesta (2005) for their small open economy analysis.

(15) and (16) in (13) and (14) implies that the home and foreign country policymakers effectively respond to both their domestic inflation rates and to the terms of trade

$$r_t = \varphi_\pi \pi_t + \varphi_y \tilde{y}_t + \varphi_s s_t, \tag{17}$$

$$r_t^\star = \varphi_\pi^\star \pi_t^\star + \varphi_y^\star \tilde{y}_t^\star - \varphi_s^\star s_t, \tag{18}$$

where $\varphi_s = \varphi_\pi \gamma$ and $\varphi_s^\star = \varphi_\pi^\star (1-\gamma)$.

From (17) and (18) it can be seen that the home (foreign) central bank raises (lowers) interest rates when the terms of trade $s_t = p_{F,t} - p_{H,t} = p_t^\star - p_t$ increases, that is, deteriorates from the perspective of the home country. This is because a worsening of the terms of trade—a higher s_t—raises (lowers) the home (foreign) CPI inflation rate. The effect on domestic (foreign) interest rates is higher the more open the economy, that is, the larger (smaller) the parameter γ. In the special case where the home (foreign) economy is closed, CPI and domestic inflation are equivalent and the domestic (foreign) central bank simply responds to domestic (foreign) inflation.

Rules (17) and (18) can be further simplified by taking into account that in equilibrium the terms of trade (in logs) depends on the difference between the home and foreign output levels, or

$$s_t = y_t - y_t^\star = \tilde{y}_t - \tilde{y}_t^\star + \bar{s}_t,$$

where $\bar{s}_t$ is the level of the real exchange rate consistent with output at potential in each economy, the natural level of the terms of trade. Substituting into (17) and (18) yields

$$r_t = \varphi_\pi \pi_t + [\varphi_y + \varphi_s]\tilde{y}_t - \varphi_s \tilde{y}_t^\star + \varphi_s \bar{s}_t, \tag{19}$$

$$r_t^\star = \varphi_\pi^\star \pi_t^\star + [\varphi_y^\star + \varphi_s^\star]\tilde{y}_t^\star - \varphi_s^\star \tilde{y}_t - \varphi_s^\star \bar{s}_t. \tag{20}$$

Now both central banks can be viewed as responding not only to domestic economic events, but also to *foreign* conditions in the form of foreign output gaps. In addition, they now respond more aggressively to domestic output gaps as is evident from the higher coefficients, $\varphi_y + \varphi_s > \varphi_y$ and $\varphi_y^\star + \varphi_s^\star > \varphi_y^\star$, than one finds in the simple Taylor rule of the previous section. The reason for this is that for the home country a higher foreign output gap (higher $\tilde{y}_t^\star$) *ceteris paribus* implies a more favorable terms of trade, which lowers home CPI inflation and thus allows the home central bank to lower interest rates. The opposite is true for a higher home output gap, hence the higher response coefficient on $\tilde{y}_t$. Similarly, from the perspective of the foreign country a higher home output gap (higher $\tilde{y}_t$) gives rise to a more favorable terms of trade, which lowers the foreign CPI and thereby makes room for interest rate cuts abroad. The opposite is true for a higher foreign output gap, hence the higher response coefficient on $\tilde{y}_t^\star$.

3.2.2 The dynamic system

Substituting (19) and (20) into (1) and (3) yields a dynamic system in the same form as equation (7). We will use the fact that $\varphi_s = \gamma\varphi_\pi$ and $\varphi_s^\star = (1-\gamma)\,\varphi_\pi^\star$ to facilitate comparison with the findings of the previous section. The system can be written as

$$\mathcal{Z}_t = \mathcal{A}_0 + \mathcal{B}E_t\mathcal{Z}_{t+1} + \mathcal{X}\mathcal{V}_t, \tag{21}$$

where we have included the terms involving $\bar{s}_t$ in the vector

$$\mathcal{V}_t = \left[\overline{rr}_t - \gamma\varphi_\pi\bar{s}_t, u_t, \overline{rr}_t^\star - (1-\gamma)\,\varphi_\pi^\star\bar{s}_t, u_t^\star\right]'.$$

The terms in this vector are again assumed to follow independent $AR\,(1)$ processes with serial correlation coefficient ρ and iid disturbances. The terms $\mathcal{Z}$, $\mathcal{A}_0$, and $\mathcal{X}$ are defined as in equation (7), but the key matrix $\mathcal{B}$ is no longer block diagonal. Instead, it is defined as

$$\mathcal{B} = \frac{1}{b_0}\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22}\end{bmatrix},$$

where

$$\begin{aligned} b_0 = {} & \left(1+\sigma_o^{-1}\left[(\gamma+\lambda_o)\,\varphi_\pi+\varphi_y\right]\right)\times \\ & \qquad \left(1+\sigma_o^{\star,-1}\left[(1-\gamma+\lambda_o^\star)\,\varphi_\pi^\star+\varphi_y^\star\right]\right) \\ & \qquad\qquad -\sigma_o^{-1}\sigma_o^{\star,-1}\,(1-\gamma)\,\gamma\varphi_\pi\varphi_\pi^\star, \end{aligned}$$

$$B_{11} = \left\{1 + \sigma_o^{\star,-1}\left(\left[1-\gamma+\lambda_o^\star\right]\varphi_\pi^\star + \varphi_y^\star\right)\right\} \begin{bmatrix} 1 & \sigma_o^{-1}\left(1-\beta\varphi_\pi\right) \\ \lambda_o & b_{11,22} \end{bmatrix},$$

with

$$b_{11,22} = \beta\left[1 + \sigma_o^{-1}\left(\gamma\varphi_\pi + \varphi_y + \lambda_o\beta^{-1}\right)\right] - \frac{\beta\sigma_o^{-1}\sigma_o^{\star,-1}\varphi_\pi\varphi_\pi^\star\gamma\left(1-\gamma\right)}{\left\{1 + \sigma_o^{\star,-1}\left(\left[1-\gamma+\lambda_o^\star\right]\varphi_\pi^\star + \varphi_y^\star\right)\right\}},$$

$$B_{12} = \sigma_o^{-1}\gamma\varphi_\pi \begin{bmatrix} 1 & \sigma_o^{\star,-1}\left(1-\beta\varphi_\pi^\star\right) \\ \lambda_o & \lambda_o\sigma_o^{\star,-1}\left(1-\beta\varphi_\pi^\star\right) \end{bmatrix},$$

$$B_{21} = \sigma_o^{\star,-1}\left(1-\gamma\right)\varphi_\pi^\star \begin{bmatrix} 1 & \sigma_o^{-1}\left(1-\beta\varphi_\pi\right) \\ \lambda_o^\star & \lambda_o^\star\sigma_o^{-1}\left(1-\beta\varphi_\pi\right) \end{bmatrix},$$

and

$$B_{22} = \left\{1 + \sigma_o^{-1}\left(\left[\gamma+\lambda_o\right]\varphi_\pi + \varphi_y\right)\right\} \begin{bmatrix} 1 & \sigma_o^{\star,-1}\left(1-\beta\varphi_\pi^\star\right) \\ \lambda_o^\star & b_{22,22} \end{bmatrix},$$

with

$$b_{22,22} = \beta\left\{1 + \sigma_o^{\star,-1}\left[\left(1-\gamma\right)\varphi_\pi^\star + \varphi_y^\star + \lambda_o^\star\beta^{-1}\right]\right\} - \frac{\beta\sigma_o^{-1}\sigma_o^{\star,-1}\varphi_\pi\varphi_\pi^\star\gamma\left(1-\gamma\right)}{\left\{1 + \sigma_o^{-1}\left(\left[\gamma+\lambda_o\right]\varphi_\pi + \varphi_y\right)\right\}}.$$

The main difference is that B_{12} and B_{21} are no longer null. We stress that the loss of the block diagonality of this matrix is induced by policy alone. Policymakers are reacting to consumer rather than producer prices and this is creating international linkages that would otherwise not exist. This means, in principle, that policy parameters in one country will influence all aspects[11] of worldwide conditions for determinacy and learnability. The separability of these conditions across borders breaks down because the policymakers are reacting to variables that have foreign components.

[11] That is, all four eigenvalues.

3.2.3 Determinacy and learnability

One economy is large Determinacy again requires that all eigenvalues of $\mathcal{B}$ are inside the unit circle. We begin with the special case in which the home country is closed so that $\gamma \to 0$. From equation (17) it is as if the home country is using the simple Taylor rule analyzed in the previous section. In this case B_{12} becomes null, B_{11} is the submatrix associated with the simple Taylor rule, B_{21} does not become null, and B_{22} is altered from the simple Taylor rule case. The eigenvalues of $\mathcal{B}$ are then the eigenvalues of $(1/b_0)\, B_{11}$ and $(1/b_0)\, B_{22}$ since $\mathcal{B}$ is lower block triangular. The opposite pattern occurs if the foreign country is very large so that $(1-\gamma) \to 0$.

The eigenvalues of $(1/b_0)\, B_{11}$ will be inside the unit circle if the condition

$$\delta\left[\sigma+\phi\right]\left(\varphi_{\pi}-1\right)+\left(1-\beta\right)\varphi_{y}>0$$

is met, that is, the closed economy condition for a simple Taylor rule. For $(1/b_0)\, B_{22}$ in the case $\gamma = 0$, we have

$$(1/b_0)\, B_{22} = \frac{1}{\sigma_o^{\star}+\varphi_y^{\star}+\left(1+\lambda_o^{\star}\right)\varphi_{\pi}^{\star}} \times \left[\begin{array}{cc} \sigma_o^{\star} & 1-\beta\varphi_{\pi}^{\star} \\ \lambda_o^{\star}\sigma_o^{\star} & \lambda_o^{\star}+\beta\left(\sigma_o^{\star}+\varphi_y^{\star}+\varphi_{\pi}^{\star}\right) \end{array}\right].$$

This is the same matrix that would apply in the simple Taylor rule economy except that an extra term $\varphi_{\pi}^{\star}$ enters in the denominator of the factor multiplying the matrix, and an extra term $\beta\varphi_{\pi}^{\star}$ enters the lower right position in the matrix. The characteristic polynomial of $(1/b_0)\, B_{22}$ is $v^2 + a_1 v + a_0 = 0$, with

$$a_0 = \frac{\beta\sigma_o^{\star}}{\sigma_o^{\star}+\varphi_y^{\star}+\left(1+\lambda_o^{\star}\right)\varphi_{\pi}^{\star}},$$

and

$$a_1 = \frac{-\left(\sigma_o^{\star}+\lambda_o^{\star}+\beta\left[\sigma_o^{\star}+\varphi_y^{\star}+\varphi_{\pi}^{\star}\right]\right)}{\sigma_o^{\star}+\varphi_y^{\star}+\left(1+\lambda_o^{\star}\right)\varphi_{\pi}^{\star}}.$$

The conditions for both eigenvalues to be inside the unit circle are that $|a_0| < 1$ and $|a_1| < 1+a_0$. The first condition is always met under maintained

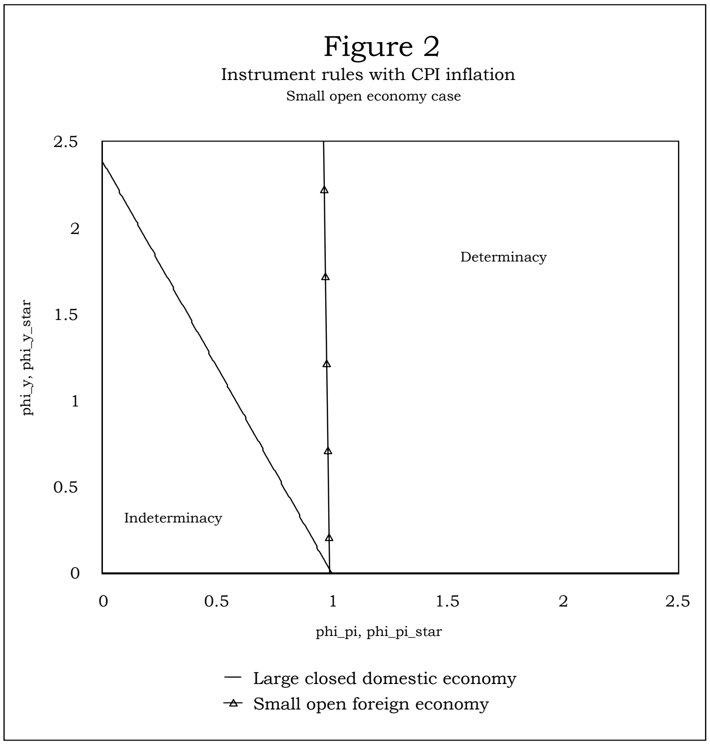

Figure 2: The small open economy faces a much steeper tradeoff between $\varphi_y^\star$ and $\varphi_\pi^\star$ when both countries react to CPI inflation.

assumptions, and the second condition implies

$$\lambda_o^\star (\varphi_\pi^\star - 1) + (1 - \beta)\, \varphi_\pi^\star + (1 - \beta)\, \varphi_y^\star > 0. \tag{22}$$

This differs from the Taylor rule case by the term $(1 - \beta)\, \varphi_\pi^\star$. We have several remarks on condition (22).

First, condition (22) depends only on foreign economy parameters. Thus, this condition must be met by the foreign authorities independently of the policy rule parameters adopted domestically.[12] The condition essentially still represents the Taylor principle. In particular, for a value of $\varphi_y^\star = 0$, the

[12] We stress that we have achieved this dichotomy only by assuming $\gamma \to 0$ here, whereas the Taylor rules used in the previous section allowed this dichotomy regardless of the value of γ.

authorities still must choose a value of $\varphi_\pi^\star > \beta \approx 1$ to meet the determinacy condition. The main quantitative effect of condition (22) is to greatly steepen the tradeoff between values of $\varphi_y^\star$ and $\varphi_\pi^\star$ that are consistent with determinacy. The $\varphi_y^\star$-intercept increases to $\lambda_o^\star (1-\beta)^{-1} \approx 100$ at the baseline calibration. These facts are illustrated in Figure 2.

Both economies have positive mass When $0 < \gamma < 1$ the analysis of the previous subsection no longer holds. Instead, the policy parameters from both countries will enter into consideration for all four eigenvalues of the system. One way to get some idea of this tradeoff is to consider the special case where the two countries do not respond directly to output gaps in their policy rules. This means that the values $\varphi_y = \varphi_y^\star = 0$, and that the two central banks only respond to consumer price index inflation in each country. We set $\gamma = 1/3$ for the purposes of this calculation, meaning that the home economy is larger and less open, and we set the remaining parameters at their baseline values, other than the two remaining policy parameters φ_π and $\varphi_\pi^\star$.

The analysis of learnability of determinate equilibria in this case follows Section 3.1.3. As in that discussion, the expectational stability condition will be that real parts of the eigenvalues of the matrix $\mathcal{B}$ are less than unity.[13] Quantitatively, the conditions for determinacy and learnability coincide again in this case. Although the system has been altered relative to the system with the simple Taylor rule, each monetary authority is still reacting to contemporaneous information in its policy rule, and this is keeping the two conditions consistent.[14]

Figure 3 shows how the values of these two parameters can trade off against one another in order to generate determinacy and learnability of the world equilibrium. The larger home country needs to have the larger

[13]The question of learnability of sunspot equilibria is more complicated and is left to future work.

[14]Learnability would fail if the agents were only allowed to use information from their own countries. Agents would be given a perceived law of motion that would not include foreign variables, and this would be insufficient to learn the REE.

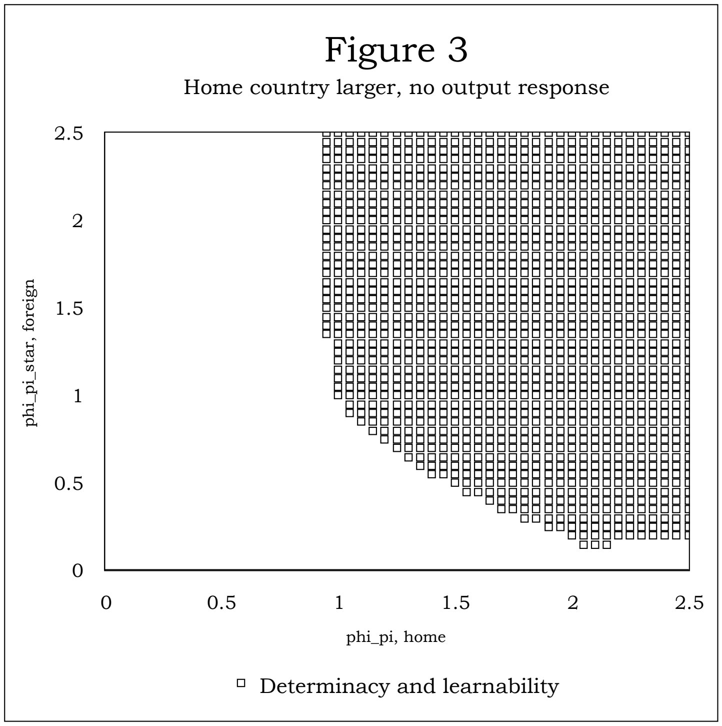

Figure 3: The region consistent with determinacy and learnability involves a trade-off between the policy responses, φ_π and $\varphi_\pi^\star$, in the two economies. The output responses have been set to zero. The blank region is associated with indeterminacy. The home country has the larger, relatively closed economy.

coefficient on inflation in the policy rule, while the foreign policy authorities can choose something less than unity. If foreign policymakers choose a low value of $\varphi_\pi^\star$, the home policymakers may be able to react more aggressively to inflation and therefore generate a determinate world equilibrium.

International sunspot transmission Is it possible for poor monetary policy in one country to create worldwide endogenous fluctuations in the form of sunspot equilibria? Figure 2 suggests that the answer is yes and in this subsection we provide a simple example. Here we follow Woodford's (1999) discussion of sunspot equilibria in the domestic economy case. We again set $\gamma = 1/3$ and $\varphi_y = \varphi_y^\star = 0$, with remaining parameters other than φ_π and $\varphi_\pi^\star$ set to baseline values. We then suppose that the monetary authorities in the home country adopt a policy of $\varphi_\pi = 1.2$ while the foreign policy authorities adopt a policy of $\varphi_\pi^\star = 0.25$. Figure 2 suggests that this combination is insufficient to generate determinacy worldwide. In some sense the problem is the foreign policymakers since $\varphi_\pi^\star = 0.25$ is "too low."

Write equation (21) as

$$E_t \mathcal{Z}_{t+1} = \mathcal{B}^{-1}\mathcal{A}_0 + \mathcal{B}^{-1}\mathcal{Z}_t + \mathcal{B}^{-1}\mathcal{X}\mathcal{V}_t. \tag{23}$$

Three of the eigenvalues of $\mathcal{B}^{-1}$ are outside the unit circle, but one is inside indicating indeterminacy for this system written in terms of $\mathcal{B}^{-1}$. This eigenvalue is $v \approx 0.94$ and the associated eigenvector is $b_v \approx [0.46, 0.35, -0.02, 0.82]'$. If $\{\mathcal{Z}_t\}$ satisfies (23), then another bounded solution is given by

$$\mathcal{Z}_t' = \mathcal{Z}_t + b_v \zeta_t$$

where

$$\zeta_{t+1} = v\zeta_t + \eta_{t+1}.$$

The stochastic term η_{t+1} can be any mean zero bounded random variable unforecastable at time t. The stochastic process η_{t+1} can have any correlation with the fundamental disturbances in the two economies, and can be highly variable so long as the linear approximations on which (23) is based remain valid. Therefore, as stressed by Woodford (1999) for the closed economy, the fluctuations induced in $[\tilde{y}_t, \pi_t, \tilde{y}_t^\star, \pi_t^\star]'$ can be large in magnitude and therefore some of the equilibrium sequences would be quite bad from a welfare perspective. Furthermore, although we do not stress it in this paper, recent research has isolated conditions under which sunspot equilibria can be learnable in the sense defined by Evans and Honkapohja (2001). An example related to the current context is provided by Zanna (2004). We conclude that there is considerable policy risk in the indeterminate region.

The potential for expectations to fluctuate considerably in excess of fundamentals and yet remain consistent with rational expectations equilibrium is arguably the primary policy problem in contexts like this one. When choosing among policies that induce a unique rational expectations equilibrium, policymakers are comparing outcomes with at most the volatility induced by the fundamental driving stochastic processes of the system. But policies that allow indeterminacy can be associated with equilibrium outcomes with a volatility many times that of the fundamental shocks. The welfare gain in avoiding the high volatility outcomes associated with indeterminacy is first order, while the welfare gain in choosing among alternative policies all of which induce determinacy is second- or even third-order.[15]

3.3 Two PPP instrument rules

Zanna (2004, p. 2) points out that the *real exchange rate* is perhaps the most popular real target in developing economies. Policymakers are often concerned about avoiding losses in competitiveness in foreign markets, or similarly, about maintaining purchasing power parity (PPP). According to Zanna, in order to achieve the real exchange rate target policymakers often follow PPP rules. Such rules link the nominal rate of devaluation of the domestic currency to the deviation of the real exchange rate from its long run level or to the difference between the domestic inflation rate and the foreign inflation rate. For instance, Calvo, *et al.* (1995) argued that Brazil, Chile and Columbia followed such rules in the past.[16]

An important contribution to the theoretical literature on this topic is Uribe (2003), who analyzes a PPP rule whereby the government increases the devaluation rate when the real exchange rate is below its steady-state value. Uribe (2003) argues that PPP rules may lead to indeterminacy, potentially inducing aggregate instability in the economy via endogenous fluctuations due to self-fulfilling expectations.

[15] This latter argument has recently been reiterated by Lucas (2003).

[16] Calvo, *et al.* (1995) mention that starting in 1968, Brazil's government implemented a rule by which the exchange rate was adjusted as a function of the difference between domestic and U.S. inflation. In addition, between 1985 and 1992, Chile used an exchange rate band whose trend was determined by the difference between the domestic inflation rate and a measure of the average inflation in the rest of the world.

Zanna (2004) studies the conditions under which a Uribe-type PPP rule may be associated with indeterminacy in a small open economy with traded and non-traded goods. Zanna (2004) finds that the lower the degree of openness of the economy, the more likely it is that the rule will induce aggregate instability in the economy by generating multiple equilibria. Further, he studies the learnability properties of equilibria induced by PPP rules. He investigates both a Uribe and a Dornbusch (1982)-type rule—which we here call the Zanna rule—whereby the nominal devaluation rate is positively linked to the difference between the domestic and foreign CPI inflation rates.

In this section we investigate the properties of stylized Uribe (2003) and Zanna (2004) instrument rules in our environment. We start with the former. Suppose the home country follows a Uribe-type PPP rule whereby the home central bank sets the nominal devaluation rate as a function of the deviation of the (log of the) current CPI-based real exchange rate q_t with respect to its steady state level $\bar{q}$

$$\Delta e_t = \rho_q (q_t - \bar{q}) \tag{24}$$

where

$$q_t = p^{\star}_{C,t} + e_t - p_{C,t} \tag{25}$$

and $\rho_q < 0$.[17] The nominal exchange rate in our two country world is given by the law of one price—that is, the (log) CPI-based real exchange rate is zero, so that we have $p_{C,t} = p^{\star}_{C,t} + e_t$, or $q_t = 0$. Using this result in the PPP rule (24) and normalizing the initial $(t-1)$ level of the nominal exchange rate at zero, the Uribe rule reduces to

$$e_t = 0. \tag{26}$$

Thus, in our environment we have the result that if the home country follows the Uribe PPP rule (24)—if it targets the real exchange rate—under conditions where the nominal exchange rate already obeys PPP, the home country effectively pegs its *nominal* exchange rate to the foreign country.

[17] Using the notation of this paper Uribe assumes that $\rho_q = \frac{d\rho}{de_t} < 0$ where $\rho(.)$ is a continous function that in steady state satisfies $\Delta \bar{e}_t = \rho(0)$.

Next, suppose the home country follows a Zanna (2004)-type PPP rule whereby it links the nominal depreciation of the exchange rate to the difference between home CPI inflation, $\pi_{C,t}$, and foreign CPI inflation $\pi^{\star}_{C,t}$,

$$\Delta e_t = \rho_\pi(\pi_{C,t} - \pi^{\star}_{C,t}) \tag{27}$$

where $0 < \rho_\pi < 1$.[18] This specification tries to capture the previously mentioned stylized facts about PPP rules in Brazil, Columbia and Chile.

The intuition behind this rule is that by increasing the nominal depreciation rate in response to the international inflation differential the home country stabilizes its *real exchange rate*. Again, we have to deal with the phenomenon that the nominal exchange rate in our two country world is already given by the law of one price: the (log) CPI-based real exchange rate is zero. That is, we have $\pi_{C,t} = \pi^{\star}_{C,t} + \Delta e_t$. Using this result in the Zanna PPP rule (27) we have $(1 - \rho_\pi)\Delta e_t = 0$, which after the usual normalization implies (26).

Altogether, this section provides a simple general result for PPP rules, namely that if the home country follows a PPP rule (that is, if it targets the real exchange rate)—either in the form of the Zanna rule (27) or in the form of the Uribe rule (24)—under conditions *where the nominal exchange rate already obeys PPP*, the home country effectively pegs its *nominal* exchange rate to the foreign country. The learnability and determinacy properties of the two PPP rules above are therefore identical to those of a system in which one country follows a nominal exchange rate peg. This provides some motivation for investigating the desirability of such an exchange rate regime. We consider this situation in Section 5.1 below under targeting rules.

[18] In Zanna's analysis foreign variables are considered exogenous and constant, therefore (using the notation of this paper) Zanna's PPP rule is $\Delta e_t = \rho_\pi(\pi_{C,t})$ with $\frac{d\rho_\pi(\pi_{C,t})}{d\pi_{C,t}} > 0$.

4 Targeting rules

4.1 Overview

In this section we assume that the central bank sets policy optimally. This means that the nominal interest rate is set according to a rule inferred from an explicit optimization exercise.[19] We investigate the benchmark case of discretion[20] and consider two implementation strategies of the first-order condition along the lines of Evans and Honkapohja (2003). The various implementation strategies may or may not provide determinacy and learnability of rational expectations equilibrium. In the next sub-section we focus on the non-cooperative case in which each policymaker sets monetary policy autonomously. We will turn to the cooperative case in Section 4.3.

4.2 Non-cooperative discretionary policy

4.2.1 The policy problem

Importantly, as Clarida, Gali, and Gertler (2002) mention, the correct inflation variable for the policymaker following a non-cooperative discretionary policy is domestic producer price inflation. This means π_t will enter into the objective for the domestic policymaker. Under discretion the monetary authority will choose a sequence of current and future short-term nominal interest rates to minimize loss defined by

$$L = (1-\gamma)\,\Lambda E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{2} [\left(\pi_\tau - \pi^T\right)^2 + \alpha_o \left(\tilde{y} - \tilde{y}^T\right)^2] \tag{28}$$

with $\Lambda \equiv \frac{\xi}{\delta}$ and $\alpha_o \equiv \frac{\delta\kappa}{\xi} = \frac{\lambda_o}{\xi}$, and where π^T and $\tilde{y}^T$ are target values which we will often view as being zero. The parameter ξ represents the price elasticity of demand for intermediate goods in Clarida, Gali, and Gertler

[19] For a recent discussion about targeting versus instrument rules see Svensson (2003), McCallum and Nelson (2004), as well as Svensson (2004).

[20] For a discussion of determinacy issues for optimal rules in a closed economy where the timing protocol is commitment, see Giannoni and Woodford (2002a,b).

(2002). The minimization is subject to

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \sigma_o^{-1}(r_t - E_t\pi_{t+1} - \overline{rr}_t), \tag{29}$$

and

$$\pi_t = \beta E_t\pi_{t+1} + \lambda_o\tilde{y}_t + u_t, \tag{30}$$

where[21]

$$\overline{rr}_t = \sigma_o E_t\Delta\bar{y}_{t+1} + \kappa_o E_t\Delta y^\star_{t+1},$$

and

$$u_t = \rho u_{t-1} + \epsilon_t.$$

Optimal policy under discretion reduces to a sequence of static problems in which the nominal interest rate is chosen to deliver the values $\tilde{y}_t$ and π_t which minimize the loss. We can reformulate the problem above as choosing the indirect control variable $\{\tilde{y}_\tau\}_{\tau=t}^{\infty}$ to minimize (28) where the central bank treats $E_t\pi_{t+1}$ as given.

This problem can be viewed as a Stackelberg game in which the private sector (the leader) sets $E_t\pi_{t+1}$ and $E_t\tilde{y}_{t+1}$ before the central bank determines r_t. This means that the private sector commits—the natural counterpart to what the literature calls the 'commitment case' where it is the central bank which commits. So, here we are looking at a Stackelberg equilibrium where the private sector is the dominant player. This would be the case where the private sector can 'commit' to inflation and output forecasts before the central bank's optimal level of the short term nominal interest rate is set. In this environment, two conditions define the equilibrium: (i) For any private sector forecast, the central bank's policy response is optimal given that forecast, and (ii) Given the reaction function of the central bank as defined in (i), the private sector forecast is optimal.

The equilibrium can be computed by working backwards. First, we compute the optimal level of the nominal interest rate given private sector output

[21] In this section we follow Clarida, Gali, and Gertler (2002) and we do not specify a stochastic process for $\overline{rr}_t$.

and inflation expectations. To do this we write the central bank's Lagrangian as[22]

$$\mathcal{L} = E_t \sum_{\tau=t}^{\infty} -\frac{\beta^{\tau-t}}{2} \Lambda (1-\gamma) \times$$
$$\left[\left(\pi_\tau - \pi^T\right)^2 + \alpha_o \left(\tilde{y}_\tau - \tilde{y}^T\right)^2 - \beta^{\tau-t} \mu_\tau \left(\pi_\tau - \beta E_\tau \pi_{\tau+1} - \lambda_o \tilde{y}_\tau - u_\tau\right) \right]$$

where π_t and $E_t \pi_{t+1}$ are state variables. The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial \tilde{y}_t} = -\alpha_o \Lambda (1-\gamma) \left(\tilde{y}_t - \tilde{y}^T\right) + \mu_t \lambda_o = 0, \tag{31}$$

$$\frac{\partial \mathcal{L}}{\partial \pi_t} = -\Lambda (1-\gamma) \left(\pi_t - \pi^T\right) - \mu_t = 0. \tag{32}$$

From equation (31) we have $\mu_t = \frac{\alpha_o \Lambda (1-\gamma)}{\lambda_o} \left(\tilde{y}_t - \tilde{y}^T\right)$. Using this result in (32) yields

$$\tilde{y}_t - \tilde{y}^T = -\frac{\lambda_o}{\alpha_o} \left(\pi_t - \pi^T\right). \tag{33}$$

This equation gives us the optimal level of the indirect control which can be mapped into the direct control (the nominal interest rate) by combining this expression with equation (29). An explicit form will be given below. Under strict inflation targeting we simply have $\pi_t = \pi^T$, while under strict output stabilization we obtain $\tilde{y}_t = \tilde{y}^T$. The intuition behind (33) is that if the central bank cares about output, inflation will be allowed to over (under) shoot the inflation target if output is below (above) target. The extent to which this is allowed to happen is increasing in the central bank's weight on output stabilization.[23]

It is well-known in the closed economy literature that there are a variety of strategies for implementing conditions like (33), and that these strategies can have differing implications for determinacy and learnability. We now

[22]For a discussion of the relative merits of dynamic programming and the Lagrange method see Schaling (2001). For applications of the latter to a non-linear optimization problem, and a regime switching model see Schaling (2004) and Bullard and Schaling (2001), respectively.

[23]Similar intuition is provided in Clarida, Gali, and Gertler (1999, p. 1672). If $\tilde{y}^T = \pi^T = 0$ we obtain exactly Clarida *et al* (2002) equation (57).

turn to two implementations for the open economy model in order to see how these results may or may not be altered.

4.2.2 Two implementations

An expectations-based optimal rule

The rule Combining the first-order condition (33) with equation (29) we obtain

$$E_t\tilde{y}_{t+1} - \sigma_o^{-1}(rr_t - \overline{rr}_t) - \tilde{y}^T = -\frac{\lambda_o}{\alpha_o}\left(\pi_t - \pi^T\right)$$

(where rr_t is the ex ante real interest rate $r_t - E_t\pi_{t+1}$). This can be written as

$$rr_t - \overline{rr}_t = \frac{\lambda_o\sigma_o}{\alpha_o}\left(\pi_t - \pi^T\right) + \sigma_o\left(E_t\tilde{y}_{t+1} - \tilde{y}^T\right).$$

Substituting for π_t from equation (30) we obtain

$$rr_t - \overline{rr}_t = \frac{\lambda_o\sigma_o}{\alpha_o}\left(\beta E_t\pi_{t+1} + \lambda_o\tilde{y}_t + u_t - \pi^T\right) + \sigma_o\left(E_t\tilde{y}_{t+1} - \tilde{y}^T\right).$$

Eliminating output via (29) yields

$$r_t - \overline{rr}_t = \delta_{0,0} + \delta_{\pi,0}E_t\pi_{t+1} + \delta_{y,0}E_t\tilde{y}_{t+1} + \delta_{u,0}u_t \tag{34}$$

where the coefficients are given by

$$\delta_{0,0} = -\frac{\sigma_o\left(\lambda_o\pi^T + \alpha_o\tilde{y}^T\right)}{\alpha_o + \lambda_o^2}, \tag{35}$$

$$\delta_{\pi,0} = \frac{\alpha_o + \lambda_o^2 + \sigma_o\lambda_o\beta}{\alpha_o + \lambda_o^2}, \tag{36}$$

$$\delta_{y,0} = \sigma_o, \tag{37}$$

$$\delta_{u,0} = \frac{\lambda_o\sigma_o}{\alpha_o + \lambda_o^2}. \tag{38}$$

Equation (34) is an example of a targeting rule, as discussed for example in Svensson (1999) and Woodford (2003, pp. 290-295). This rule is the Evans and Honkapohja (2003) expectations-based optimal rule. By construction, it implements what Evans and Honkapohja label ‘optimal discretionary policy’ in every period and for all values of private expectations.

The dynamic system, determinacy and learnability Setting the targets π^T and y^T to zero, the world economy can be written as

$$\mathcal{Z}_t = \mathcal{A}_0 + \mathcal{B}E_t\mathcal{Z}_{t+1} + \mathcal{X}\mathcal{V}_t,$$

where $\mathcal{Z}_t = [\tilde{y}_t, \pi_t, \tilde{y}_t^{\star}, \pi_t^{\star}]'$, and the key matrix $\mathcal{B}$ is given by

$$\mathcal{B} = \begin{bmatrix} B_{11} & \mathbf{0} \\ \mathbf{0} & B_{22} \end{bmatrix},$$

where

$$B_{11} = \begin{bmatrix} 0 & \frac{-\beta\lambda_o}{\alpha_o+\lambda_o^2} \\ 0 & \frac{\alpha_o\beta}{\alpha_o+\lambda_o^2} \end{bmatrix}$$

and

$$B_{22} = \begin{bmatrix} 0 & \frac{-\beta\lambda_o^{\star}}{\alpha_o^{\star}+(\lambda_o^{\star})^2} \\ 0 & \frac{\alpha_o^{\star}\beta}{\alpha_o^{\star}+(\lambda_o^{\star})^2} \end{bmatrix}.$$

Because $\mathcal{B}$ is block diagonal, determinacy conditions will be have to be met country by country. A unique rational expectations equilibrium exists since

$$0 < \frac{\alpha_o\beta}{\alpha_o + \lambda_o^2} < 1$$

for the domestic economy, and

$$0 < \frac{\alpha_o^{\star}\beta}{\alpha_o^{\star} + (\lambda_o^{\star})^2} < 1$$

for the foreign economy. Expectational stability will again depend on the real parts of the eigenvalues of this same matrix, and can be shown to hold as well.

We conclude that this implementation of (33) generates determinacy and learnability of worldwide rational expectations equilibrium with an optimal, non-cooperative discretionary policy being followed in both countries. This occurs because this implementation is known to work well in the closed economy case, the good performance does not break down with the extension to the open economy model, and the policy does not introduce spillover effects that would otherwise not exist. Other implementations of (33) are known to have poor properties, however, and we now turn to this case.

A fundamentals-based policy rule

The rule A fundamentals-based policy rule implementing (33) generates a different reduced form. To obtain an optimal interest rate rule under rational expectations conjecture a solution of the form

$$\begin{aligned} \tilde{y}_t &= a_1 + d_1 u_t, \\ \pi_t &= a_2 + d_2 u_t, \end{aligned}$$

for the domestic economy, with an analogous conjectured solution for the foreign economy. The MSV solution has

$$\begin{aligned} \bar{a}_1 &= \frac{-\delta_{0,0} - (\delta_{\pi,0} - 1)\,\bar{a}_2}{\sigma_o}, \\ \bar{d}_1 &= \frac{-\rho\bar{d}_2\,(\delta_{\pi,0} - 1) - \delta_{u,0}}{\sigma_o}, \\ \bar{a}_2 &= \frac{-\delta_{0,0}\lambda_o}{\sigma_o\,(1 - \beta\rho) + \rho\lambda_o\,(\delta_{\pi,0} - 1)}, \\ \bar{d}_2 &= \frac{\sigma_o - \lambda_o\delta_{u,0}}{\sigma_o\,(1 - \beta\rho) + \rho\lambda_o\,(\delta_{\pi,0} - 1)}. \end{aligned}$$

where $\delta_{0,0},\ \delta_{\pi,0},\ \delta_{y,0}, \delta_{u,0}$ are given by (35) through (38) respectively. The policy feedback rule is then

$$r_t = \psi_0 + \psi_u u_t + \overline{rr}_t, \tag{39}$$

with

$$\psi_0 = \delta_{0,0} + \delta_{\pi,0}\bar{a}_2 + \delta_{y,0}\bar{a}_1$$

and

$$\psi_u = \rho\left(\delta_{\pi,0}\bar{d}_2 + \delta_{y,0}\bar{d}_1\right) + \delta_{u,0}.$$

This is sometimes called the fundamentals form of the RE-optimal policy rule.

The dynamic system, determinacy, and learnability It is known that this interest rate rule is associated with indeterminacy in the closed economy case.[24] The world economy can be written as

$$\mathcal{Z}_t = \mathcal{A}_0 + \mathcal{B} E_t \mathcal{Z}_{t+1} + \mathcal{X} \mathcal{V}_t,$$

with $\mathcal{B}$ block diagonal,

$$B_{11} = \begin{bmatrix} 1 & \sigma_o^{-1} \\ \lambda_o & \beta + \lambda_o \sigma_o^{-1} \end{bmatrix},$$

and

$$B_{22} = \begin{bmatrix} 1 & \sigma_o^{\star,-1} \\ \lambda_o^\star & \beta + \lambda_o^\star \sigma_o^{\star,-1} \end{bmatrix}.$$

Determinacy requires $|a_0| < 1$ and $|a_1| < 1 + a_0$ in $v^2 + a_1 v + a_0 = 0$, the characteristic equation for B_{11} and B_{22}, respectively. For the domestic economy (and analogously for the foreign economy),

$$a_1 = \frac{-\left(\lambda_o + \sigma_o + \beta \sigma_o\right)}{\sigma_o}$$

and $a_0 = \beta$. The condition $|a_1| < 1 + a_0$ is never met under maintained assumptions and so worldwide equilibrium is indeterminate, as in the domestic economy case discussed by Evans and Honkapohja (2003). The MSV solution will also be unstable in the learning dynamics. We conclude that the method of implementing (33) will matter in the open economy case just as it does in the closed economy.

4.2.3 Summary for discretionary non-cooperative policy

There are other possible implementations of the first order condition under discretionary non-cooperative policy. All implementations we have seen have the same feature, namely that they produce world systems which are block diagonal. The conditions for determinacy and learnability will then have to be met economy by economy. This is driven by the fact that under a non-cooperative approach to optimal policy, each policymaker can approximate

[24]See for instance Woodford (1999, 2003) and Svensson and Woodford (2003).

the utility of a typical household by including only its own, "inward-looking" variables in its objective function.

As we have seen, block diagonality breaks down if policymakers put weight on international variables in their policy rules, or, in a targeting approach, in their objective function. That is exactly what happens should policymakers in each country attempt to pursue the gains to cooperation which normally exist in this model. We now turn to this issue.

4.3 Cooperative discretionary policy

4.3.1 Overview

Clarida, Gali and Gertler (2002) study cooperation in the context of their New Keynesian model and are thus part of what Canzoneri, Cumby and Diba (2004) call second generation models of policy coordination. Canzoneri, *et al.*, state that the gains from coordination are larger in second generation models than in first generation models.[25]

Clarida, Gali, and Gertler (2002) show in their Proposition 3 that gains to international policy cooperation will accrue to both countries when $\sigma > 1$ and each country follows a rule dictated by the solution to a joint optimization problem. We now follow Clarida, Gali, and Gertler (2002) and discuss the prospects for determinacy and learnability if each country attempts to pursue the gains from cooperation.

4.3.2 The policy problem

Clarida, Gali, and Gertler (2002) define cooperation to mean that the two central banks in the model agree to maximize a weighted average of the utility of the home and foreign households under discretion. The weights are naturally γ and $1-\gamma$. Both governments refrain from creating a surprise appreciation, and hand out employment subsidies that just offset the monopolistic competition distortion. The monetary authorities jointly maximize an

[25] For a survey of the lessons from the first generation literature see Nolan and Schaling (1996).

approximation to weighted household utility given by

$$L=-\frac{1}{2}\Lambda E_0\sum_{t=0}^{\infty}\beta^t\times$$
$$\left[(1-\gamma)\left(\pi_t^2+\alpha\left(\tilde{y}_t^C\right)^2\right)+\gamma\left(\left(\pi_t^\star\right)^2+\alpha^\star\left(\tilde{y}_t^{\star,C}\right)^2\right)-2\Phi\tilde{y}_t^C\tilde{y}_t^{\star,C}\right],$$

where $\Lambda=\xi/\delta$, $\alpha_o=\lambda_o/\xi$, $\alpha_o^\star=\lambda_o^\star/\xi$,

$$\Phi\equiv\frac{\delta\left(1-\sigma\right)\gamma\left(1-\gamma\right)}{\xi},$$

and $\tilde{y}_t^C$ and $\tilde{y}_t^{\star,C}$ are the output gaps defined under cooperation as the deviation, in percent, of output from the cooperative steady state level for the domestic and foreign economy, respectively. The cooperative steady state level of output is associated with flexible prices in both countries and the absence of shocks. These gaps are related to the standard output gaps in the model via

$$\begin{aligned}\tilde{y}_t^C &= \tilde{y}_t-\frac{\kappa_o}{\kappa}\tilde{y}_t^{\star,C},\\ \tilde{y}_t^{\star,C} &= \tilde{y}_t^\star-\frac{\kappa_o^\star}{\kappa^\star}\tilde{y}_t^C.\end{aligned}$$

The first order conditions for this problem can then be written in terms of standard output gaps as

$$\begin{aligned}\tilde{y}_t &= -\xi\left(\pi_t+\frac{\kappa_o}{\kappa}\pi_t^\star\right),\\ \tilde{y}_t^\star &= -\xi\left(\pi_t^\star+\frac{\kappa_o^\star}{\kappa^\star}\pi_t\right).\end{aligned}$$

4.3.3 One implementation

The rule Combining these conditions with (1) and (3) gives optimal cooperative policy rules

$$\begin{aligned}r_t &= \vartheta E_t\pi_{t+1}+\frac{\kappa_o}{\kappa}\left(\vartheta-1\right)E_t\pi_{t+1}^\star+\overline{rr}_t,\\ r_t^\star &= \vartheta^\star E_t\pi_{t+1}^\star+\frac{\kappa_o^\star}{\kappa^\star}\left(\vartheta^\star-1\right)E_t\pi_{t+1}^\star+\overline{rr}_t^\star,\end{aligned}$$

where

$$\vartheta = 1+\frac{\xi\sigma_o(1-\rho)}{\rho},$$
$$\vartheta^\star = 1+\frac{\xi\sigma_o^\star(1-\rho)}{\rho}.$$

The dynamic system, determinacy, and learnability The world economy can be written as

$$\mathcal{Z}_t = \mathcal{A}_0 + \mathcal{B}E_t\mathcal{Z}_{t+1} + \mathcal{X}\mathcal{V}_t,$$

where the key matrix $\mathcal{B}$ is

$$\mathcal{B} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

and

$$B_{11} = \begin{bmatrix} 1 & -\sigma_o^{-1}(\vartheta-1) \\ \lambda_o & \beta-\sigma_o^{-1}\lambda_o(\vartheta-1) \end{bmatrix},$$
$$B_{12} = \begin{bmatrix} 0 & -\sigma_o^{-1}\kappa_o\kappa^{-1}(\vartheta-1) \\ 0 & -\sigma_o^{-1}\lambda_o\kappa_o\kappa^{-1}(\vartheta-1) \end{bmatrix},$$
$$B_{22} = \begin{bmatrix} 1 & -\sigma_o^{\star,-1}(\vartheta^\star-1) \\ \lambda_o^\star & \beta-\sigma_o^{\star,-1}\lambda_o^\star(\vartheta^\star-1) \end{bmatrix},$$

and

$$B_{21} = \begin{bmatrix} 0 & -\sigma_o^{\star,-1}\kappa_o^\star\kappa^{\star,-1}(\vartheta^\star-1) \\ 0 & -\sigma_o^{\star,-1}\lambda_o^\star\kappa_o^\star\kappa^{\star,-1}(\vartheta^\star-1) \end{bmatrix}.$$

Determinacy properties will again depend on the eigenvalues of the matrix $\mathcal{B}$. The lack of block diagonality indicates that policy in each country will influence determinacy properties. The four eigenvalues of $\mathcal{B}$ are given by

$$v_{1,\pm} = \frac{(1+\beta)\rho+\delta(1+\phi)(\rho-1)\xi}{2\rho} \pm \frac{\left[(\delta[1+\phi]\xi-\rho[1+\beta+\delta(1+\phi)\xi])^2-4\beta\rho^2\right]^{1/2}}{2\rho}$$

and

$$v_{2,\pm} = \frac{(1+\beta)\,\rho + \delta\,(\sigma+\phi)\,(\rho-1)\,\xi}{2\rho} \pm \frac{\left[(\delta\,[\sigma+\phi]\,\xi - \rho\,[1+\beta+\delta\,(\sigma+\phi)\,\xi])^2 - 4\beta\rho^2\right]^{1/2}}{2\rho}.$$

These eigenvalues are independent of γ, the degree of openness. This is because the two economies are following a cooperative policy which takes the size of each economy into account. Determinacy does not always hold. In particular,

$$\begin{aligned} \lim_{\rho\to 0} v_{1,-} &= -\infty, \\ \lim_{\rho\to 0} v_{2,-} &= -\infty. \end{aligned}$$

Unless the serial correlation in the shock is sufficiently large, this cooperative policy will generate indeterminacy.[26]

We use the baseline calibration with the addition of $\xi = 7.88$ implying a markup of about 15 percent, and we report results for values of ρ. The cutoff value for the serial correlation parameter is $\rho_c \approx 0.165$.[27] Values less than this will create indeterminacy given the baseline calibration. Should the shock process become something more like white noise, optimal policy cooperation implemented in this way will be associated with indeterminacy. For determinate cases, we verified numerically at baseline parameter values that expectational stability holds.

4.3.4 Summary for cooperative policy

Cooperation means that the two central banks have agreed to jointly pursue an objective defined over weighted world averages of key variables. The implied policy rule will have each monetary authority responding to world economic events. Policies are no longer inward-looking. Determinacy and

[26] This is a version of a similar result for the closed economy in Evans and Honkapohja (2003).

[27] For $\sigma = 2$, $\rho_c \approx 0.28$.

learnability conditions will be met, not country by country, but by a type of weighted average of world monetary policy. This weighted average takes on a specific value, because the objectives weigh each economy appropriately by γ.

We stress that there may be other implementations of optimal cooperative policy which may or may not generate either determinacy or learnability.

One might wonder if full cooperation is really a good positive model for world monetary policy. In the international policy arena, we seem to observe a variety of strategies in play. So far in the paper we have only considered certain types of symmetry in policy, but there are also interesting asymmetric situations. We now turn to one of these.

5 Asymmetry in monetary policy

5.1 An exchange rate peg

5.1.1 Overview

In this section we suppose the home country targets its nominal exchange rate e *vis-a-vis* the foreign country. We assume the foreign economy sets its monetary policy based on its own domestic considerations. The home country gives up its domestic monetary autonomy in return for "importing monetary stability" from the foreign, anchor country.

This is a leading example of an *asymmetric* exchange rate regime, as only the anchor country's variables matter for its interest rate (depending on the nature of the policy adopted there), and the home country simply sets its interest rate to ensure it realizes a fixed exchange rate. The home country in setting policy takes foreign monetary conditions into account, but the foreign country need not incorporate the home country's conditions in its own monetary policy stance. This arrangement is similar to the regimes adopted by some European countries prior to economic and monetary union and to the present peg of the Chinese renminbi to the U.S. dollar.

5.1.2 The policy problem

The home country minimizes

$$(1-\gamma)\,\Lambda E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t}\frac{1}{2}[(e_\tau - e^T)]^2. \tag{40}$$

The minimization is subject to

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \sigma_o^{-1}\left[r_t - E_t\pi_{t+1} - \overline{rr}_t\right], \tag{41}$$

$$\pi_t = \beta E_t\pi_{t+1} + \lambda_o\tilde{y}_t + u_t, \tag{42}$$

$$\bar{r}r_t = \sigma_o E_t\Delta\bar{y}_{t+1} + \kappa_o E_t\Delta y^{\star}_{t+1},$$

$$u_t = \rho u_{t-1} + \epsilon_t,$$

$$e_t = e_{t-1} + s_t - s_{t-1} + \pi_t - \pi^{\star}_t,$$

and

$$s_t = (\tilde{y}_t - \tilde{y}^*_t) + \bar{s}_t.$$

For ease of exposition we normalize the initial levels of the nominal exchange rate and terms of trade at zero ($e_{t-1} = s_{t-1} = 0$), so that

$$e_t = s_t + \pi_t - \pi^{\star}_t. \tag{43}$$

In what follows we normalize the exchange rate target at zero ($e^T = 0$). From (40) the first-order condition then becomes $e_t = 0$, which combined with (43) implies

$$s_t = -(\pi_t - \pi^{\star}_t). \tag{44}$$

The intuition behind (44) is the following. The nominal exchange rate obeys CPI-based purchasing power parity and, after appropriate normalization, is given by $e_t = \pi_t - \pi^{\star}_t + s_t$. In order to prevent fluctuations in e_t, the home central bank should manipulate the terms of trade s_t, which it can affect via the domestic output gap, in such a way as to offset the GDP deflator-based inflation differential. Thus we have (44).

Since the terms of trade can be affected by the domestic output gap, which in turn is affected by the home nominal interest rate, the home central bank should try to achieve a level of the home output gap given by

$$\tilde{y}_t = -(\pi_t - \pi_t^\star) + \tilde{y}_t^\star - \bar{s}_t. \tag{45}$$

Equation (45) is obtained by substituting the expression for the terms of trade into the first-order condition and rearranging.

5.1.3 The policy rule

Substituting (42) into (45), we obtain the home country's optimal monetary policy rule in terms of its indirect control $\tilde{y}_t$

$$\tilde{y}_t = -\frac{\beta}{1+\lambda_o} E_t \pi_{t+1} + \frac{1}{1+\lambda_o}(\pi_t^\star + \tilde{y}_t^\star) - \frac{1}{1+\lambda_o}(\bar{s}_t + u_t). \tag{46}$$

The home interest rate reaction function can be obtained by combining (46) with (41) to obtain

$$r_t - \overline{rr}_t = \delta'_{\pi,0} E_t \pi_{t+1} + \delta'_{y,0} E_t \tilde{y}_{t+1} + \delta'_{\star,0}\pi_t^\star + \delta'_{\star,0}\tilde{y}_t^\star + \delta'_{u,0}\left(u_t + \bar{s}_t\right), \tag{47}$$

where the coefficients are given by

$$\delta'_{\pi,0} = \frac{(1+\lambda_o) + \sigma_o \beta}{1+\lambda_o}, \tag{48}$$

$$\delta'_{y,0} = \sigma_o, \tag{49}$$

and

$$\delta'_{\star,0} = -\delta'_{u,0} = -\frac{\sigma_o}{1+\lambda_o}. \tag{50}$$

The rule (47) describes the optimal home monetary reaction function that implements its monetary policy of pegging the exchange rate to the foreign anchor country.[28]

We substitute the home country's policy rule (47) into (41). This implies

$$\tilde{y}_t = \sigma_o^{-1}(1 - \delta'_{\pi,0}) E_t \pi_{t+1} - \sigma_o^{-1}\delta'_{\star,0}(\pi_t^\star + \tilde{y}_t^\star) - \sigma_o^{-1}\delta'_{u,0}\left(u_t + \bar{s}_t\right), \tag{51}$$

Here the dependence of home's economic outcomes on the foreign macroeconomy is evident from the presence of the terms $\pi_t^\star$ and $\tilde{y}_t^\star$.

[28] We stress that there may be other ways to implement the first order condition for the fixed exchange rate.

5.1.4 The dynamic system, determinacy, and learnability

Whether or not a fixed exchange rate regime is compatible with determinacy of worldwide rational expectations equilibrium depends on how the foreign, anchor country implements monetary policy, and on any international spillover effects on the home country. We make the assumption that the foreign, anchor country is inward-looking, and concerned only about reacting to developments in its own economy. We proceed with the most straightforward assumption, namely that the foreign inflation country follows a simple Taylor-type policy rule. This allows us to easily study cases where the foreign, anchor monetary authorities are pursuing policies either consistent or inconsistent with determinacy and learnability of worldwide rational expectations equilibrium.

The world economy can again be written in standard form. The matrix $\mathcal{B}$ is given by

$$\mathcal{B} = \begin{bmatrix} B_{11} & B_{12} \\ \mathbf{0} & B_{22} \end{bmatrix}$$

where B_{22} is the matrix associated with a simple Taylor rule in use in the foreign country. The eigenvalues there will depend on whether the foreign country is following the open economy version of the Taylor principle or not, as discussed earlier in the paper. The eigenvalues of B_{11} will also have to be less than unity for determinacy. This matrix is given by

$$B_{11} = \begin{bmatrix} 0 & \sigma_o^{-1}\left(1-\delta'_{\pi,0}\right) \\ 0 & \beta + \sigma_o^{-1}\lambda_o\left(1-\delta'_{\pi,0}\right) \end{bmatrix}.$$

The eigenvalues are zero and

$$v = \frac{\beta}{1+\lambda_o} < 1.$$

We conclude that determinacy holds under maintained assumptions provided the foreign, anchor monetary authorities are following the Taylor principle. Learnability holds under the same conditions.

One may be able to imagine scenarios under which this result would break down, if the foreign, anchor economy had some other policy. But this result suggests there need not be anything intrinsically unstable in the use of an exchange rate peg.

6 Conclusion

We have developed results on determinacy and expectational stability for a simple open economy New Keynesian model due to Clarida, Gali, and Gertler (2002). We used this model with an eye toward comparing the open economy findings to known results for closed economies under similar assumptions.

We have shown that even for simple Taylor-type policy rules, open economy considerations will have quantitative effects on determinacy and learnability conditions. Closed economy analyses tend to understate the degree of aggressiveness the policymaker must adopt to avoid indeterminacy and expectational instability. Quantitative differences of this type are alluded to by Clarida, Gali, and Gertler (2002) and are in accord with the findings of Llosa and Tuesta (2005).

When central banks are inward-looking—reacting to domestic variables in their policy rules—our results indicate that determinacy and learnability conditions for worldwide equilibrium must be met country by country. This is true whether we are considering inward-looking instrument rules or targeting rules which are implied by non-cooperative policy objectives. Optimal policy will require an implementation, but the natural implementations suggested in the closed economy literature imply the separability of determinacy and learnability conditions across economies. We interpret this finding as follows. If one country out of many adopts an instrument rule that is inconsistent with determinacy and learnability, or one country out of many adopts an implementation of an optimal policy which is inconsistent with determinacy and learnability, then worldwide equilibrium will be indeterminate and expectationally unstable. The remaining countries, even if they attempt to be very aggressive in promoting determinacy and learnability, will not have an impact on this facet of the world equilibrium. This might be viewed as an undesirable aspect of inward-looking policies, even if they are judged 'optimal' on other grounds.

When monetary authorities are actively responding to international variables, our results indicate that determinacy and learnability conditions for worldwide equilibrium are met by something akin to an average of world monetary policy. This occurs for simple instrument rules which include international elements, or for targeting rules where monetary authorities are attempting to pursue cooperative policies to achieve the available gains. Optimal cooperative policy will also require an implementation, and the baseline implementation from the literature may not be consistent with determinacy and learnability. Still, inclusion of reactions to international variables allows the monetary authorities from a sufficiently large economy to mitigate the threats of indeterminacy and expectational instability posed by a partner country that is pursuing a poor policy, either through an ad hoc policy or through an inadvertently bad implementation of an optimal policy. The ability to influence these conditions may be viewed as a desirable aspect of monetary policy in an open economy context.

References

[1] Aghion, P., P. Bacchetta, and A. Banerjee. 2001. "A Simple Model of Monetary Policy and Currency Crises." *European Economic Review* 44: 728-738.

[2] Aoki, M. 1981. *Dynamic Analysis of Open Economies.* New York, Academic Press.

[3] Aoki, K., and K. Nikolov. 2004. "Rule-Based Monetary Policy Under Central Bank Learning." Working Paper, London School of Economics.

[4] Benhabib, J., and R. Farmer. 1999. "Indeterminacy and Sunspots in Macroeconomics." Chapter 6: 387-448 in Taylor, J. B., and M. Woodford (Eds.). *Handbook of Macroeconomics, Volume 1 A.* Elsevier.

[5] Batini, N., P. Levine, J. Pearlman. 2004. "Indeterminacy with Inflation-Forecast-Based Rules in a Two-Bloc Model." International Finance Discussion Paper #797, Board of Governors of the Federal Reserve System.

[6] Benigno, G., and P. Benigno. 2004. "Designing Targeting Rules for International Monetary Policy Cooperation." European Central Bank, Working Paper #279.

[7] Betts, C., and M. Devereux. 2000. "Exchange Rate Dynamics in a Model of Pricing-to-Market." *Journal of International Economics* 50: 215-244.

[8] Blanchard, O., and C. Kahn. 1980. "The Solution of Linear Difference Models under Rational Expectations," *Econometrica* 48(5): 1305-1312.

[9] Bullard, J., and E. Schaling. 2001. "New Economy-New Policy Rules?" *Federal Reserve Bank of St. Louis Review* 83(5): 57-66.

[10] Bullard, J., and K. Mitra. 2002. "Learning About Monetary Policy Rules." *Journal of Monetary Economics* 49: 1105-1129.

[11] Calvo, G. 1983. "Staggered Prices in a Utility Maximizing Framework." *Journal of Monetary Economics* 12: 383-398.

[12] Calvo, G., Reinhart, C. and C. Vegh 1995. "Targeting the Real Exchange Rate: Theory and Evidence." *Journal of Development Economics* 47(1): 97-134.

[13] Campa, J., and L. Goldberg. 2002. "Exchange Rate Pass-Through into Import Prices: A Macro or Micro Phenomenon." Mimeo, Federal Reserve Bank of New York.

[14] Canzoneri., M., R. Cumby, and B. Diba 2004. "The Need for International Policy Coordination: What's Old, What's New, What's Yet to Come." *Journal of International Economics,* forthcoming.

[15] Chari, V., P. Kehoe and E. McGrattan. 2002. “Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?” *Review of Economic Studies* 69: 533-563.

[16] Chaippori, P.A., and R. Guesnerie. 1991. “Sunspot Equilibria in Sequential Market Models.” Chapter 32: 1683-1762 in Hildenbrand, W., and H. Sonneschein (Eds.). *Handbook of Mathematical Economics, Volume 4A.* Elsevier.

[17] Clarida, R., J. Gali, and M. Gertler. 1999. “The Science of Monetary Policy: A New Keynesian Perspective.” *Journal of Economic Literature* 37(4): 1661-1707.

[18] Clarida, R., J. Gali, and M. Gertler. 2001. “Optimal Monetary Policy in Open versus Closed Economies: An Integrated Approach.” *American Economic Review Papers and Proceedings* 91: 253-257.

[19] Clarida, R., J. Gali, and M. Gertler. 2002. “A Simple Framework for International Monetary Policy Analysis.” *Journal of Monetary Economics* 49: 879-904.

[20] Corsetti, G., and P. Pesenti. 2001. “Welfare and Macroeconomic Interdependence.” *Quarterly Journal of Economics* 116: 421-445.

[21] Corsetti, G., and P. Pesenti. 2005. “International Dimensions of Optimal Monetary Policy.” *Journal of Monetary Economics* 52(2): 281-306.

[22] De Fiore, F., and Z. Lui. “Does Trade Openness Matter for Aggregate Instability?” *Journal of Economic Dynamics and Control*, forthcoming.

[23] Devereux, M. B., and C. Engel. 1998. “Fixed vs. Floating Exchange Rates: How Price Setting Affects the Optimal Choice of Exchange-Rate Regime.” NBER Working Paper #6867, December.

[24] Devereux, M. B., and C. Engel. 2001. "Endogenous Currency of Price Setting in a Dynamic Open Economy Model." NBER Working Paper #8559.

[25] Devereux, M. B., and C. Engel. 2002. "Exchange-Rate Pass-Through, Exchange Rate Volatility, and Exchange Rate Disconnect." *Journal of Monetary Economics*, June, 913-940.

[26] Dornbusch, R. 1982 "PPP Exchange Rate Rules and Macroeconomic Stability." *Journal of Political Economy* 90(3): 158-165.

[27] Ellison, M., L. Sarno, and J. Vilmunen. 2004. "Caution or Activism? Monetary Policy Strategies in an Open Economy." Manuscript, University of Warwick.

[28] Evans, G., and S. Honkapohja. 2001. *Learning and Expectations in Macroeconomics*. Princeton University Press.

[29] Evans, G., and S. Honkapohja. 2003a. "Adaptive Learning and Monetary Policy Design." *Journal of Money, Credit, and Banking* 35(6): 1045-1073.

[30] Evans, G., and S. Honkapohja. 2003b. "Expectations and the Stability Problem for Optimal Monetary Policies." *Review of Economic Studies* 70: 807-824.

[31] Gali and Monacelli. 2002. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy." NBER Working Paper #8905.

[32] Giannoni, M., and M. Woodford. 2002a. "Optimal Interest Rate Rules: I. General Theory." NBER Working Paper #9419.

[33] Giannoni, M., and M. Woodford. 2002b. "Optimal Interest Rate Rules: II. Applications." NBER Working Paper #9420.

[34] Golub, G., and C. Van Loan. 1996. *Matrix Computations*. John Hopkins University Press.

[35] Llosa, G., and V. Tuesta. 2005. "Learning About Monetary Policy Rules in Small Open Economies." Manuscript, Central Reserve Bank of Peru.

[36] Lucas, R.E., 2003. "Macroeconomic Priorities." *American Economic Review* 93(1): 1-14.

[37] Marcet, A., and T. Sargent. 1989. "Convergence of Least Squares Learning to Rational Expectations in Self-Referential Linear Stochastic Models." *Journal of Economic Theory* 48: 337-368.

[38] McCallum, B, and E. Nelson. 2004. "Targeting vs. Instrument Rules for Monetary Policy." Federal Reserve Bank of St. Louis *Review*, forthcoming.

[39] Monacelli, T. 2005. "Monetary Policy in a Low Pass-Through Environment." *Journal of Money, Credit and Banking*, forthcoming.

[40] Nolan, C. and E. Schaling. 1996. "International Monetary Policy Coordination: Some Lessons from the Literature." *Bank of England Quarterly Bulletin* 36 (4): 412-417.

[41] Obsfeld, M. and K. Rogoff. 1995. "Exchange Rate Dynamics Redux." *Journal of Political Economy*, Vol. 103, No. 3, 624-660.

[42] Obsfeld, M. and K. Rogoff. 1996. *Foundations of International Macroeconomics*. MIT Press, Cambridge, MA.

[43] Obstfeld, M. and K. Rogoff. 2000. "New Directions for Stochastic Open Economy Models." *Journal of International Economics* 50(1), 117-153.

[44] Otani, A. 2002. "Pricing to Market (PTM) and the International Transmission Effect of Monetary Policy: The New Open-Economy Macroeconomics Approach." Institute for Monetary and Economic Studies Bank of Japan, Discussion Paper No. 2002-E-5, May.

[45] Pappa, E. 2004. "Do the ECB and the Fed Really Need to Cooperate? Optimal Monetary Policy in a Two-Country World." *Journal of Monetary Economics* 51: 753-779.

[46] Preston, B. 2003. "Learning About Monetary Policy Rules When Long-Horizon Forecasts Matter." Working paper, Columbia University.

[47] Schaling, E.2001. "Dynamic Programming versus Lagrange: Is there a Horse Race in Dynamic Optimization?" in Klok, H., T. van Schaik and S. Smulders (eds), *Economologues - Liber Amicorum for Theo van de Klundert*, Tilburg University Press, pp. 334-349.

[48] Schaling, E. 2004. "The Non-Linear Phillips Curve and Inflation Forecast Targeting: Symmetric vs. Asymmetric Monetary Policy Rules." *Journal of Money, Credit and Banking* 36(3): 361-386.

[49] Schmitt-Grohe, S. and M. Uribe 2002. "Closing Small Open Economy Models." *Journal of International Economics* 61: 163-185.

[50] Svensson, L. 1999. "Inflation Targeting as a Monetary Policy Rule." *Journal of Monetary Economics* 43: 607-654.

[51] Svensson, L. 2003. "What's Wrong with Taylor Rules? Using Judgment in Monetary Policy Through Targeting Rules." *Journal of Economic Literature* 41: 426-477.

[52] Svensson, L. 2004. "Targeting Rules Versus Instrument Rules for Monetary Policy: What is Wrong With McCallum and Nelson?" Federal Reserve Bank of St. Louis *Review*, forthcoming.

[53] Svensson, L and M. Woodford. 2003. "Implementing Optimal Policy Through Inflation-Forecast Targeting," in B. Bernanke and M. Woodford (eds.), *Inflation Targeting*, Chicago: University of Chicago Press.

[54] Taylor, J. 1993. "Discretion versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy* 39: 195-214.

[55] Uribe, M. 2003. "Real Exchange Rate Targeting and Macroeconomic Instability." *Journal of International Economics*, forthcoming.

[56] Woodford, M. 1999. "Optimal Monetary Policy Inertia." NBER Working Paper #7261.

[57] Woodford, M. 2003. *Interest and Prices.* Princeton University Press.

[58] Zanna, L. 2004. "PPP Rules, Macroeconomic (In) stability and Learning." Board of Governors of the Federal Reserve System, International Finance Discussion Papers, #814, August.