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**GOING-PUBLIC AND THE INFLUENCE OF  
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# Going-Public and the Influence of Disclosure Environments

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## Abstract

This paper analyzes how differences in disclosure environments affect the firm's choice between private and public capital. Disclosure regulations prescribe to what extent the firm has to release confidential information that may lead to the firm incurring proprietary cost. We examine which firms go public in equilibrium, and how the equilibrium outcomes change with changes in the disclosure environments.

*Keywords:* Going-public decision, disclosure environments, proprietary cost.

*JEL codes:* G32, M49.

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# 1 Introduction

Information asymmetries hamper capital markets in allocating capital across the most productive investment opportunities. Firms in demand for capital need to inform potential capital market investors of the prospects of their investment opportunities, but in doing so they may simultaneously inform third parties with countervailing interests. The latter includes informing potential opponents like product market competitors, which may result in the opponent taking some adverse actions that harm the firm. Particularly, firms that consider an initial public offering are faced with this confidential or proprietary information problem.<sup>1</sup> Instead of a public placement, the firm can choose to issue privately which allows it to inform investors directly thereby reducing the incidence of leaking proprietary information. The extent to which private capital markets may mitigate proprietary information problems will typically depend on the difference in disclosure requirements with respect to the leakage of proprietary information between both capital market types. In addition to these disclosure considerations, other factors may influence the cost differential between both capital sources. In this paper we relate possible capital cost differences with the disclosure problem of leaking proprietary information and show its effect on the going-public decision.

Financing and disclosure are two important corporate activities, whose interaction appears to be strongest in the going-public decision. Central in our model are private firms that meet the requirements for listing and that are faced with a (positive net present value) investment opportunity. We assume that the internal financing means fall short or that the investment outlay is just too high so that the firm has to look for outside financing opportunities.

The problem of selecting the appropriate financing source is not limited to sorting out the cheapest form of capital. Other factors may also influence this choice. Amongst these are the costs related to the communication with capital suppliers. A natural difference in communication costs between an initial public or private offering are the higher direct costs to listing. Because these disclosure costs are too a large part fixed, their influence on the going-public decision is rather straightforward: the larger the capital need, the more attractive public capital becomes, *ceteris paribus*. In the present study, we are much more interested in the more intangible and important source of disclosure costs that stems from leaking confidential information to opposing parties. We posit that going-public has a large impact on the incidence of leaking confidential or proprietary information. First, a going-public firm has to deal with unknown investors that it can primarily reach via public disclosures. Second, exchange authorities make high demands upon public disclosures. They normally impose additional disclosure rules and watch more closely over the timeliness, completeness and precision of corporate disclosures. Third, public firms will attract more attention from financial analysts and the press, which further increases public scrutiny. Finally, being caught for fraudulent disclosure in public has far more serious consequences. Litigation and reputation costs, for instance, are likely to be higher because more investors will be harmed and the negative

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<sup>1</sup> The focus is on initial public offerings, for these firms will experience the largest change in disclosure environment. Firms considering a seasoned public offering may also consider proprietary information problems since each public offering itself comes with additional (disclosure) requirements. These firms, however, have already experienced the conversion to a more demanding disclosure environment and have already adapted to the longer-term public disclosure requirements. For this reason we consider the initial public offering firm as the most appropriate example for our study.

news will be disseminated more broadly. Summarizing, public firms will face more pressure on their disclosure activities and will have less flexibility in choosing their disclosure channels. As a result, disclosure costs stemming from the leakage of proprietary information, henceforth referred to as proprietary disclosure costs, are expected to be higher for public firms and, hence, are likely to affect a firm's going-public decision.

This paper analyzes the influence of disclosure environments on the going public decision of the firm. In our game-theoretical model, a firm can choose to finance its business on the private or on the public capital market, each of which having its own cost of finance and disclosure regulations. The going public decision is then a trade-off between the cost of finance and the disclosure related proprietary cost. Roughly speaking, we find that in equilibrium the relatively better firms prefer private financing to public financing.

One of the first papers that explicitly examines the question why firms go public is Pagano (1993). Pagano views the going public decision as a trade-off between portfolio diversification benefits and listing costs. In his model the propensity of a firm to go public within a particular economy depends on the going public decision of other firm's. The more firms are willing to bear the private listing costs the more efficient can be the general diversification opportunities. This externality, however, can create several equilibria, one of them featuring a stock market with very few companies listed. Zingales (1995) focuses solely on corporate control aspects associated with going public. In his model going-public is the result of a value-maximizing decision made by an initial owner who wants to sell his company. By first going public the initial owner can increase his gains from eventually selling the whole company to a large shareholder.<sup>2</sup> Pagano and Röell (1998) also view monitoring to be an important consideration on the side of an initial owner in deciding how to offer equity. To balance the benefits stemming from the firm's market value and future private benefits, the initial owner weighs against each other the cost of (over)monitoring<sup>3</sup> and the cost of providing a liquid market. The optimal solution contains some level of monitoring and some measure of dispersion. An interesting prediction of the model that relates to ours is that more stringent disclosure environments increase incentives to go public, because it offers more efficient monitoring.

In Chemmanur and Fulghieri (1999) the going public decision involves trading-off the bargaining power of private investors against information production costs. Large shareholders have more bargaining power which enhances the possibility of enforcing a higher return on their investments so as to compensate the idiosyncratic risk run on the relatively large shareholdings. By publicly selling equity to numerous small well-diversified investors the firm can mitigate the bargaining problem. Information production costs born by the issuer are higher in case of public placements though. In the model firms go public only when a sufficient amount of information about them has accumulated in the public domain.

More related to our study are Maksimovic and Pichler (1998b) and Yosha (1995). Instead of concentrating

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<sup>2</sup>Other papers that consider an IPO as part of an overall value-maximizing strategy of selling a firm are Mello and Parsons (1998) and Stoughton and Zechner (1998).

<sup>3</sup>In the paper the term overmonitoring is used from the perspective of the initial owner of the firm. Assuming that he wants to keep control over the firm, the initial owner will also be interested in future private benefits when selling part of the firm. In this case the optimal level of monitoring from the perspective of other investors, i.e. the level of monitoring that maximizes the market value of the firm, need not coincide with the level of monitoring that maximizes the utility of the initial owner.

on control issues, Maksimovic and Pichler (1998b) focus on the influence of leaking confidential information to product market competitors on the decision to go public or stay private. They too model the choice of outside financing as a trade off between a difference in cost of capital and indirect information disclosure costs. Their focus, however, is more on the timing of the going-public decision than on the decision itself. All firms in their analysis go public eventually. Early investing firms in an emerging or changing industry trade-off the higher cost of private capital against the higher likelihood of prematurely informing potential entrants by going public at an early stage. In contrast to the present study, they do not explicitly model the disclosure opportunities available to public and private issuers (i.e., they do not consider opportunities of strategic disclosure).

Yosha (1995) analyzes the effect of information disclosure cost on the decision between bilateral and multilateral financing, which can be related to private and public financing, respectively. Besides utilizing the somewhat uncommon view of public capital being more costly than private capital,<sup>4</sup> the effect of proprietary cost on firm value is rather limited in that model. This becomes particularly clear if one abandons Yosha's view and supposes that private capital is relatively costly compared to public capital. For in that case, the effect of proprietary cost on firm value is negligible and all firms prefer public to private financing.

In our paper the financing decision is the result of a trade-off between the cost of capital on the one hand and proprietary cost on the other hand. With respect to the cost of capital, we consider both scenarios: public capital may be cheaper or more costly than private capital. The first scenario is believed to be the most general one, however, adverse selection and agency costs can be so high for young and relatively unknown firms that it can make public capital more expensive than private capital. In addition, the cost of capital may be defined more broadly than just the price set by the capital market. One may, for instance, also include the fixed cost involved with preparing and disseminating the information that is subject to disclosure in the appropriate disclosure environment.

These two financing scenarios are studied under two different disclosure regimes. First, we consider the case in which firms must disclose their proprietary information when they go public and, second, we consider the case in which firms can disclose this information voluntarily. In both disclosure settings it is assumed that private firms do and can not publicly disclose their proprietary information in a credible way;<sup>5</sup> they disclose their private information exclusively to their investors.

The appropriate disclosure environment depends on the extant disclosure rules of a particular exchange applying to the informational item that is considered to be proprietary of nature. For example, if the proprietary information can be thought of as the firm's earnings figure,<sup>6</sup> a public firm in almost all developed countries is obliged to disclose it, whereas in most jurisdictions - most notably that of the US - a private firm can withhold

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<sup>4</sup>The explanation for this is that a bilateral financing arrangement involves communication with fewer agents than a multilateral financing arrangement. Therefore, private capital should be less costly. Herewith, Yosha (1995) disregards other factors that are generally believed to be of more importance in distinguishing public from private capital, like liquidity and diversification arguments. We refer to Maksimovic and Pichler (1998a) for a general explanation of the relatively lower cost of public capital. The cost of public capital needs to be distinguished from the cost of financing, which may differ due to other factors like proprietary disclosure costs.

<sup>5</sup>For example, because private markets lack disclosure standards that can be warranted by auditors or because private placements are usually not warranted by an investment banker.

<sup>6</sup>Other generally used examples of proprietary information are earnings expectations and segmented information, although a large array of corporate information has the potential of being proprietary in nature.

it. In EU-countries, where disclosure rules are predominantly code instead of listing based, differences in formal reporting requirements between public and potential public firms are smaller than in the US. One thus might conclude that our model is less appropriate in these instances. A firm's disclosure environment, however, is not solely determined by formal reporting requirements. Issues like public scrutiny, and the changes of being caught for breaking the rules as well as punishments adhered to it are as or even more important. It is clear that these informal disclosure requirements are higher for public as for private firms.<sup>7</sup>

The basic model that we present includes four risk neutral decision-makers: a privately informed firm, the public and private capital market, and an opponent.<sup>8</sup> At some stage of the game, the firm receives private information about its firm value, e.g. earnings. Private information ranges from relatively bad to relatively good with the interpretation that better private information results in a higher firm value. The kind of private information that a firm receives, depends on its type. Types can be ordered from good to bad on the basis of first order stochastic dominance, i.e. a better type receives valuable private information with higher probability. The game is then played as follows. Dependent on its type, the firm decides between public and private financing. Since we consider the financing decision to be the more fundamental decision with more long-term consequences relative to the disclosure decision, the financing decision is made before the firm receives its private information in more detail. Once the financing decision is made, the firm learns its private information about firm value that it discloses in the appropriate way. Subsequently, the capital markets and the competitor observe the firm's financing and disclosure decision and update their beliefs about firm value accordingly. Dependent on these beliefs, the competitor can decide to take an adverse action that imposes proprietary cost on the firm. It is assumed that the competitor benefits from taking the adverse action if and only if it believes that firm value exceeds a certain threshold value (cf. Wagenhofer (1990)). The goal of the firm is to maximize the resulting firm value, as perceived by its investors, including the cost of capital and the proprietary cost due to any adverse action by the competitor.

The second model introduces disclosure flexibility for the public firm in that disclosure of the private information is not longer mandated. This setting is applicable when the item that contains proprietary information is not subject to mandatory disclosure. Such a disclosure environment need not imply that the firm remains silent about its private information. Verrecchia (1983) and Wagenhofer (1990) show that firms may have an incentive to reveal their proprietary information. By introducing a voluntary disclosure environment for public financing the disclosure decision gets separated from the financing decision. The possibility of withholding the proprietary information is no longer directly attached to the choice of financing.

We show that in these settings several sequential equilibria may arise. The two extreme cases are a full private financing equilibrium and a full public financing equilibrium, in which all types choose private and public financing, respectively. In the intermediate case of a partial financing equilibrium, both privately and publicly

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<sup>7</sup> In the model we develop the main disclosure differences between going public and staying private can be represented by the emission prospectus that all public exchanges require newly listed firms to publish. This document can be seen as a device rendering credibility to public disclosures. Most of the information it contains is usually audited and additionally backed up by the sponsor(s) and other parties whose reputations are at stake. Although private firms have to inform their capital supplier(s) too, they do not have to follow as strict disclosure rules leading to an equally secured prospectus that can be publicly consulted.

<sup>8</sup> For ease of notation we restrict ourselves to one opponent. The results presented in this paper still hold true though, if we allow for more than one opponent.

financed types occur. The existence of either equilibrium depends mainly on the relative difference between the proprietary cost and the capital cost differential. Furthermore, in a partial financing equilibrium only the relatively better types opt for private financing. The latter result can be explained as follows. Suppose that the cost of private capital exceeds the cost of public capital. Then we can show that in equilibrium, the competitor refrains from taking the adverse action when observing private financing, implying that a privately financed firm incurs no proprietary cost. Since private capital is relatively costly, private financing is beneficial only for those firms that will most likely incur proprietary cost under the mandatory public disclosure rule of public financing, i.e. the relatively good types. Similar reasoning holds when public capital is relatively costly compared to private capital. Then we can show that in equilibrium the competitor will take its adverse action when observing private financing. Since public capital is relatively costly, public financing is beneficial only for those firms that will most likely avoid incurring proprietary cost in case of public financing, i.e. the relatively bad types. Hence, the relatively good types finance privately. Furthermore, the result that in a partial financing equilibrium the relatively good firms prefer private financing turns out to be robust to changes in the disclosure environments.

The paper proceeds as follows. Section 2 introduces and analyzes the basic model in which public financing comes with a mandatory public disclosure. Then Section 3 discusses the adjusted model in which public financing comes with a voluntary public disclosure. Section 4 presents some extensions and shows the robustness of our results while Section 5 discusses the implications of our study. Finally, Section 6 concludes.

## 2 The Model

Let us start with providing a mathematical description of the model. First, all parties participating in the game, i.e. the firm, the opponent, and the private and public capital market, are assumed to be risk neutral and rational decision makers. At some stage of the game the firm will receive private information about its firm value that is proprietary of nature. This private information will be denoted by  $y \in \mathbb{R}$  and belongs to the interval  $Y = [\underline{y}, \bar{y}]$ . Examples of what this private information can represent are profit figures, turnover, R&D expenses, production costs, or product quality. In fact, it can be given any meaning, as long as it can be represented by a one-dimensional compact interval. So, what the private information cannot contain is information about both quality *and* costs.

The private information  $y$  determines firm value  $v(y) \in \mathbb{R}_+$ . We assume that  $v(y)$  is strictly increasing and continuous in  $y$ . Hence, we can interpret  $\underline{y}$  as relatively bad and  $\bar{y}$  as relatively good information. Furthermore, since  $v$  is assumed to be strictly increasing, we may assume without loss of generality that  $v(y) = y$  for all  $y \in Y$ .<sup>9</sup> Since we assumed the firm to be risk neutral, the firm's objective of maximizing expected firm value is equivalent to maximizing the expected change in firm value, which, in fact, can be represented by the change in stock price. Hence, we can also state the firm's objective as maximizing the expected change in stock price.

The kind of private information that a firm can receive, depends on its type  $\theta$ . Given a firm of type  $\theta$ , also referred to as firm  $\theta$ , the private information that it receives is determined by a random variable  $\tilde{y}_\theta$  with probability distribution function  $F(y, \theta)$  and density function  $f(y, \theta)$ . A firm's type may be interpreted as a measure of the

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<sup>9</sup>For if  $v(y) \neq y$  for some  $y \in Y$  we can consider the information set  $Y' = \{v(y) | y \in Y\}$ . Since  $v$  is strictly increasing and continuous,  $Y'$  is compact and there is a one-to-one correspondence between  $Y$  and  $Y'$ . By defining  $v'(y) = y$  for all  $y \in Y'$  we obtain the desired result.

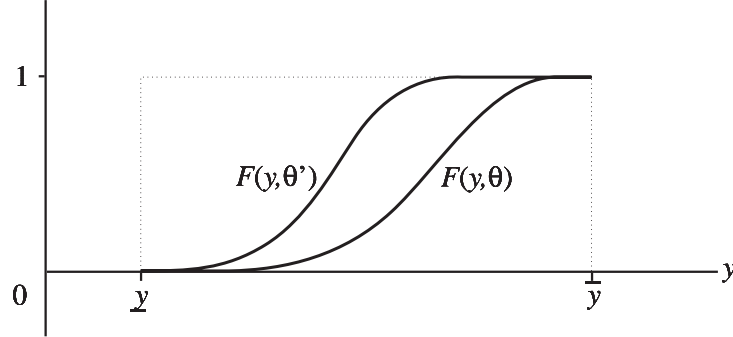


Figure 1: Stochastic dominance of firm's types  $\theta, \theta' \in \Theta$  with  $\theta > \theta'$ .

firm's potential performance or long-term profitability, for it describes, albeit indirectly, the probability distribution of the firm's future value. Note that there are many factors that determine a firm's potential performance. Some of these factors like the product market in which it operates or the state of the economy are publicly observable. Many other factors, however, like technology used, capacity, know how, and experience, are not. Since there is no direct, verifiable evidence of how much each of these factors contribute to the firm's potential performance, we assume that it is impossible for a firm to make a credible revelation of its type to either the capital markets, the opponent, or both. If the firm wants to communicate any information about its type, it can only do so by strategically making its publicly observable financing and disclosure decision.

The type space  $\Theta \subset \mathbb{R}$  equals  $[\underline{\theta}, \bar{\theta}]$  by assumption. A firm's type is determined by a random variable  $\tilde{\theta}$  with probability distribution function  $G$  and density function  $g$ . We assume that  $F, f, G,$  and  $g$  are common knowledge, that  $G(\theta)$  is continuous in  $\theta$ , and that  $F(y, \theta)$  is continuous in  $y$  and decreasing in  $\theta$ . The latter assumption implies that  $F(y, \theta) \leq F(y, \theta')$  for all  $y \in Y$  if  $\theta > \theta'$ , so that  $\tilde{y}_\theta$  stochastically dominates  $\tilde{y}_{\theta'}$  (see also Figure 1). Consequently, we can order firm types on the basis of first order stochastic dominance from the relatively bad type  $\underline{\theta}$  to the relatively good type  $\bar{\theta}$ .

A description of the order in which the game is played is depicted in Figure 2. First, nature determines the firm's type. Subsequently, the firm makes its financing decision while taking into account that each type of financing comes with its own particular cost and disclosure environment. Private capital comes with a cost  $C_b \geq 0$ . Furthermore, a privately financed firm can disclose its private information exclusively to its investor(s), but cannot make a credible public disclosure. Public capital, on the other hand, comes with a cost  $C_m \geq 0$  and a mandatory public disclosure of its private information. Since we focus on the relation between the choice of financing and the disclosure environment, we assume that the firm can always acquire the necessary capital on the market that it desires. Once the firm has made its financing decision, the firm receives private information about its firm value and discloses this information in the appropriate way. We assume that due to some anti-fraud rule the firm is not able to misrepresent its information so that any public disclosure is truthful. Subsequently, the



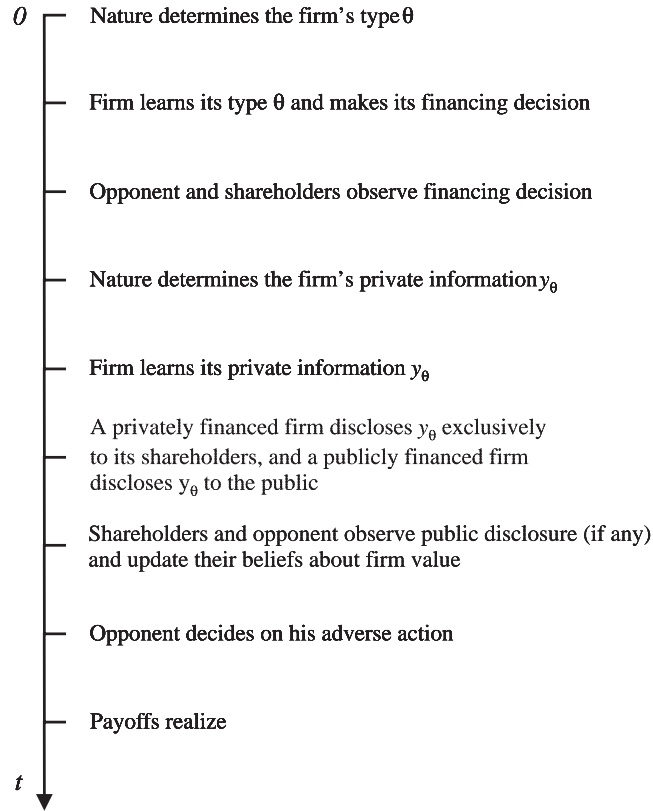


Figure 2: Time schedule of the game

opponent and the shareholders update their beliefs about firm value and the opponent makes a decision regarding his adverse action. For modeling the opponent's behavior we take the same approach as Wagenhofer (1990). This means that it is beneficial for the opponent to take the adverse action, and thereby impose proprietary cost  $C_p > 0$  on the firm, if and only if he believes that the expected firm value exceeds a certain threshold value  $K \in Y$ .

Since the disclosure environment completely determines a firm's disclosure decision, the only decision that remains to the firm is the financing decision. A financing strategy is described by a pair  $(\Theta_b, \Theta_m)$  with the interpretation that a firm of type  $\theta \in \Theta_b$  chooses for private financing while a firm of type  $\theta \in \Theta_m$  chooses for public financing. Note that in the forthcoming analysis we confine ourselves to pure financing strategies only, so that randomization between private and public financing is excluded.

In our model, we abstract from any agency problems, and assume that the manager of the firm strives to obtain the goal of its investor(s), which is to maximize expected firm value including the cost of capital and possibly the proprietary cost. With regard to the latter cost, recall that the opponent takes the adverse action, if his beliefs regarding the expected firm value exceed a certain threshold value  $K \in Y$ . Thus, we can model the opponents action by

$$a(\beta) = \begin{cases} 1, & \text{if } \beta \geq K, \\ 0, & \text{if } \beta < K, \end{cases} \quad (1)$$

where  $\beta \in Y$  denotes the opponent's beliefs about expected firm value. Note that since a publicly financed firm  $\theta$  makes a mandatory public disclosure of its private information  $y_\theta$ , the opponent learns the firm's private information  $y_\theta$ . Hence, we only need to specify the opponent's beliefs if he observes private financing.

Let  $\beta_b(\Theta_b, \Theta_m)$  denote the opponent's beliefs about the expected firm value of a privately financed firm when the financing strategy is  $(\Theta_b, \Theta_m)$ . Recall that the probability distribution functions  $F$  and  $G$  of  $\tilde{y}_\theta$  and  $\tilde{\theta}$ , respectively, are common knowledge. Thus, the opponent's prior beliefs about expected firm value equal  $E(\tilde{y}_{\tilde{\theta}})$ . Next, suppose that private financing occurs with strictly positive probability, that is  $\mathbb{P}(\tilde{\theta} \in \Theta_b) > 0$ , then we can update the opponent's prior beliefs as follows. Given that only firms  $\theta \in \Theta_b$  are privately financed, his beliefs concerning the expected firm value conditional upon observing private financing equal  $E(\tilde{y}_{\tilde{\theta}} | \tilde{\theta} \in \Theta_b)$ . If private financing occurs with zero probability, that is  $\mathbb{P}(\tilde{\theta} \in \Theta_b) = 0$ , then the conditional expectation does not exist. In this case, the beliefs about expected firm value upon observing private financing may be any firm value  $y_b \in Y$ . The beliefs  $y_b$  may be considered as the so-called out-of-equilibrium beliefs.

Summarizing, the beliefs of the opponent upon observing private financing equal

$$\beta_b(\Theta_b, \Theta_m) = \begin{cases} E(\tilde{y}_{\tilde{\theta}} | \tilde{\theta} \in \Theta_b), & \text{if } \mathbb{P}(\tilde{\theta} \in \Theta_b) > 0, \\ y_b, & \text{if } \mathbb{P}(\tilde{\theta} \in \Theta_b) = 0, \end{cases} \quad (2)$$

where  $y_b \in Y$ .

Now, we can determine the expected payoff of the firm. Since a privately financed firm of type  $\theta$  can disclose its private information exclusively to its investors, firm value equals  $y_\theta$  minus the costs  $C_b$  of private capital and, in case the opponent takes the adverse action, minus the proprietary costs  $C_p$ . The expected payoff of private financing thus equals

$$\begin{aligned} V_b(\theta, (\Theta_b, \Theta_m)) &= E(\tilde{y}_{\tilde{\theta}} - C_b - C_p a(\beta_b(\Theta_b, \Theta_m))) \\ &= E(\tilde{y}_\theta) - C_b - C_p a(\beta_b(\Theta_b, \Theta_m)). \end{aligned} \quad (3)$$

Note that the beliefs of the opponent only affect a privately financed firm's payoff through the proprietary cost.

Since a publicly financed firm of type  $\theta$  makes a mandatory, truthful public disclosure of its private information  $y_\theta$ , the opponent's beliefs about the firm's private information equal  $y_\theta$ . The expected payoff of public financing thus equals

$$\begin{aligned} V_m(\theta, (\Theta_b, \Theta_m)) &= E(\tilde{y}_{\tilde{\theta}} - C_m - C_p a(\tilde{y}_\theta)) \\ &= E(\tilde{y}_\theta) - C_m - \mathbb{P}(\tilde{y}_\theta \geq K)C_p \\ &= E(\tilde{y}_\theta) - C_m - (1 - F(K, \theta))C_p \end{aligned} \quad (4)$$

A sequential equilibrium (cf. Kreps and Wilson (1982)) consists of a financing strategy  $(\Theta_b^*, \Theta_m^*)$  and beliefs  $\beta_b(\Theta_b^*, \Theta_m^*)$  concerning the expected firm value upon observing private financing, such that

- (a) private financing is the optimal choice for each firm  $\theta \in \Theta_b^*$  with respect to the beliefs  $\beta_b(\Theta_b^*, \Theta_m^*)$ , that is  $V_b(\theta, (\Theta_b^*, \Theta_m^*)) \geq V_m(\theta, (\Theta_b^*, \Theta_m^*))$  for all  $\theta \in \Theta_b^*$ ,

- (b) public financing is the optimal choice for each firm  $\theta \in \Theta_m^*$  with respect to the beliefs  $\beta_b(\Theta_b^*, \Theta_m^*)$ , that is  $V_m(\theta, (\Theta_b^*, \Theta_m^*)) \geq V_b(\theta, (\Theta_b^*, \Theta_m^*))$  for all  $\theta \in \Theta_m^*$ ,
- (c) the beliefs  $\beta_b(\Theta_b^*, \Theta_m^*)$  are as defined in (2).

Condition (3) states that the beliefs  $\beta_b(\Theta_b^*, \Theta_m^*)$  are sequentially rational with respect to the financing strategy  $(\Theta_b^*, \Theta_m^*)$ . We will not go into the formal details of sequentially rational beliefs. For this, the interested reader is referred to Kreps and Wilson (1982). If for some type the firm is indifferent between public and private financing, it may arbitrarily choose one of them. Since we consider a continuum of types, the choice of the indifferent type is irrelevant.

We call a sequential equilibrium a full private financing equilibrium if  $\mathbb{P}(\tilde{\theta} \in \Theta_b) = 1$  and a full public financing equilibrium if  $\mathbb{P}(\tilde{\theta} \in \Theta_m) = 1$ . If in equilibrium both  $\mathbb{P}(\tilde{\theta} \in \Theta_b) > 0$  and  $\mathbb{P}(\tilde{\theta} \in \Theta_m) > 0$ , then we speak of a partial financing equilibrium. The following theorem concerns the existence of full private and full public financing equilibria. The proof of this and all forthcoming theorems are provided in the appendix.

**Theorem 2.1** A full public financing equilibrium exists if and only if  $C_p F(K, \bar{\theta}) \geq C_m - C_b$ . A full private financing equilibrium exists if and only if

- (a)  $E(\tilde{y}_{\tilde{\theta}} | \tilde{\theta} \in \Theta) < K$  and  $C_p(1 - F(K, \underline{\theta})) \geq C_b - C_m$ , or
- (b)  $C_p F(K, \underline{\theta}) \leq C_m - C_b$ .

A full public financing equilibrium arises from skeptical beliefs of the opponent, which imply that the opponent takes the adverse action whenever he observes private financing. Skeptical beliefs, however, cannot always sustain a full public financing equilibrium. Reason for this is that a privately financed firm can exclusively disclose its private information to its shareholders. Consequently, the exact firm value as perceived by the opponent is not that important to the firm, it is only the action that results from these beliefs that counts. Then the worst that can happen from the firm's point of view is, that the opponent believes that the firm's private information is valuable enough to take the adverse action, for such beliefs result in proprietary cost for the firm. Thus, a full public financing equilibrium always exists if private capital is relatively costly, i.e.  $C_m < C_b$ . For in that case, the proprietary cost resulting from the opponent's skeptical beliefs make private financing even more costly, so that each type prefers public financing to private financing. A full public financing equilibrium, on the other hand, need not always exist if public capital is relatively costly, i.e.  $C_m > C_b$ . In that case, the publicly financed firm's expected advantage  $C_p F(K, \theta)$  of incurring no proprietary cost should exceed the publicly financed firm's disadvantage  $C_m - C_b$  in capital cost. Thus, a full public financing equilibrium only exists if the proprietary cost  $C_p$  is sufficiently large compared to the difference in capital cost  $C_m - C_b$ , so that private financing becomes more costly than public financing for all possible firm types.

In a full private financing equilibrium, skeptical beliefs are absent. What is important are the opponent's prior beliefs  $E(\tilde{y}_{\tilde{\theta}})$  about the firm's private information. These prior beliefs and the value of  $K$  determine whether or not the firm incurs proprietary cost in a full private financing equilibrium. If public capital is relatively costly, i.e.  $C_m > C_b$ , and the opponent's threshold value  $K$  is sufficiently large, i.e.  $K \geq E(\tilde{y}_{\tilde{\theta}})$ , then a privately financed firm can always avoid incurring proprietary cost. Consequently, private financing is preferred to public financing by all types of firms. If public capital is still relatively costly, i.e.  $C_m > C_b$ , but the opponent's threshold value  $K$  is sufficiently small, i.e.  $K < E(\tilde{y}_{\tilde{\theta}})$ , then a privately financed firm can not avoid incurring proprietary

cost in a full private financing equilibrium. Consequently, private financing is preferred to public financing by all types, only if the capital cost advantage  $C_m - C_b$  exceeds the proprietary cost disadvantage  $C_p F(K, \theta)$  related to private financing. Thus, a full private financing equilibrium exists if the proprietary cost are sufficiently small compared to the difference in capital cost  $C_m - C_b$ . For in that case, the relatively low cost of private capital still outweighs the proprietary cost.

If private capital is relatively costly, i.e.  $C_b > C_m$ , and the opponent's threshold value  $K$  is sufficiently large, i.e.  $K \geq E(\tilde{y}_{\tilde{\theta}})$ , then a privately financed firm can again avoid proprietary cost in a full private financing equilibrium. Private financing is then preferred to public financing if the proprietary cost advantage  $C_p(1 - F(K, \theta))$  related to private financing exceeds the capital cost disadvantage  $C_b - C_m$ . Thus, a full private financing equilibrium exists if the proprietary cost are sufficiently large compared to the difference in capital cost  $C_b - C_m$ . For in that case the lower cost of public capital does no longer outweigh the expected proprietary cost of public financing.

**Theorem 2.2** Let  $(\Theta_b^*, \Theta_m^*)$  be a partial financing equilibrium. Then the set of privately financed firms equals  $\Theta_b^* = [\theta_1^*, \bar{\theta}]$  where

$$C_p(1 - F(K, \theta_1^*)) - C_p a(E(\tilde{y}_{\tilde{\theta}} | \tilde{\theta} \in \Theta_b^*)) = C_b - C_m. \quad (5)$$

Furthermore,

- (a) if public capital is relatively costly, i.e.  $C_m > C_b$ , then a partial financing equilibrium exists if and only if  $C_p F(K, \underline{\theta}) > C_m - C_b > C_p F(K, \bar{\theta})$  and  $E(\tilde{y}_{\tilde{\theta}} | \tilde{\theta} \in \Theta_b^*) \geq K$ ;
- (b) if private capital is relatively costly, i.e.  $C_m < C_b$ , then a partial financing equilibrium exists if and only if  $C_p(1 - F(K, \underline{\theta})) < C_b - C_m < C_p(1 - F(K, \bar{\theta}))$  and  $E(\tilde{y}_{\tilde{\theta}} | \tilde{\theta} \in \Theta_b^*) < K$ .

A firm prefers private to public financing if, from a private financing point of view, the expected proprietary cost advantage  $C_p(1 - F(K, \theta)) - C_p a(E(\tilde{y}_{\tilde{\theta}} | \tilde{\theta} \in \Theta_b))$  exceeds the cost of capital disadvantage  $C_b - C_m$ . Since the proprietary cost advantage increases with the firm's type and the cost of capital does not, the relatively better firms prefer private financing to public financing in a partial financing equilibrium. Furthermore, if private capital is relatively costly, then a privately financed firm does not incur proprietary cost, for otherwise it would have gone public. This means that firms prefer private financing to public financing, if the additional cost of private capital outweighs the expected proprietary cost in case of public financing. Since proprietary cost are more likely to be incurred by the better firms, the better firms choose private financing. Moreover, even though the opponent knows that only the better firms finance privately, his threshold value  $K$  is that high that it does not pay to take the adverse action.

If public capital is relatively costly, then the opponent takes the adverse action when observing private financing. Hence, a firm cannot avoid proprietary cost by choosing private financing. This means that private financing is preferred to public financing if the cost advantage of private capital outweighs the possibility of no proprietary cost in case of public financing. Since the better firms have relatively little chance of avoiding proprietary cost when publicly financed, they prefer the cheaper option of private capital.

If there is no cost difference between private and public capital, i.e.  $C_b = C_m$ , then only full financing equilibria exist. The explanation is straightforward. If there is no difference in capital cost, then the financing

decision is completely determined by the opponent's action when observing private financing. Suppose that the opponent does not take the adverse action so that a privately financed firm avoids proprietary cost. If this is the case, a full private financing equilibrium results because public financing yields proprietary cost with positive probability. Similarly, suppose that the opponent does take the adverse action when observing private financing, so that a privately financed firm incurs proprietary cost. Then a full public financing equilibrium results, because a publicly financed firm incurs no proprietary cost with positive probability,

Summarizing, in a partial financing equilibrium, the relatively better firms choose private financing, whatever type of capital is more costly. Furthermore, private financing is a means to avoid incurring proprietary cost only if private capital is more costly than public capital. Also note that in the absence of proprietary cost, i.e.  $C_p = 0$ , partial equilibria cease to exist. Depending on which of the two types of capital is least costly, either a full private or a full public financing equilibrium arises.

### 3 Public Financing in a Voluntary Disclosure Environment

In this section we change the disclosure environment of a publicly financed firm. Instead of a mandatory public disclosure, a publicly financed firm may now decide by itself, whether or not to disclose its private information to the public. More specifically, we implement Wagenhofer's voluntary disclosure model in our model so as to introduce a less stringent disclosure environment for publicly financed firms. The more flexible disclosure environment should make public financing more attractive to the relatively good firms, for it offers publicly financed firms with additional possibilities to avoid proprietary cost. A voluntary disclosure environment is applicable when the proprietary information is not subject to mandatory disclosure.

In this setting, a strategy of the firm comprises the financing decision and, in case of public financing, the disclosure decision. Hence, it is described by the tuple  $(\Theta_b, \Theta_m, \{N_\theta\}_{\theta \in \Theta_m})$ , where  $\Theta_b \subset \Theta$  represents the privately financed firms,  $\Theta_m \subset \Theta$  the publicly financed firms, and  $N_\theta$  describes the nondisclosure set for each publicly financed firm  $\theta \in \Theta_m$ . The latter means that firm  $\theta \in \Theta_m$  discloses its private information  $y_\theta$  if and only if  $y_\theta \notin N_\theta$ . We maintain the assumption that a public disclosure is truthful and completely reveals the firm's private information.

Since the disclosure environment of a privately financed firm has not changed, the expected firm value for a privately financed firm  $\theta \in \Theta_b$  equals (3), i.e.

$$V_b(\theta, (\Theta_b, \Theta_m, \{N_\theta\}_{\theta \in \Theta_m})) = E(\tilde{y}_\theta) - C_b - C_p a(\beta_b(\Theta_b, \Theta_m, \{N_\theta\}_{\theta \in \Theta_m})),$$

where

$$\beta_b(\Theta_b, \Theta_m, \{N_\theta\}_{\theta \in \Theta_m}) = \begin{cases} E(\tilde{y}_\theta | \tilde{\theta} \in \Theta_b), & \text{if } \mathbb{P}(\tilde{\theta} \in \Theta_b) > 0, \\ y_b, & \text{if } \mathbb{P}(\tilde{\theta} \in \Theta_b) = 0. \end{cases}$$

are the opponent's beliefs about the expected firm value of a privately financed firm (cf. (2)).

In order to determine the expected firm value for a publicly financed firm, we first have to determine the opponent's and the public capital market's beliefs about firm value when they observe nondisclosure of a publicly

financed firm. Similar to  $\beta_b$ , we distinguish two cases. If nondisclosure by a publicly financed firm occurs with positive probability, i.e.  $\mathbf{P}(\tilde{y}_{\tilde{\theta}} \in \cup_{\theta \in \Theta_m} N_{\theta} | \tilde{\theta} \in \Theta_m) > 0$ , then the updated beliefs about expected firm value equal  $E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}, \tilde{\theta} \in \Theta_m)$ . If nondisclosure by a publicly financed firm occurs with zero probability, i.e.  $\mathbf{P}(\tilde{y}_{\tilde{\theta}} \in \cup_{\tilde{\theta} \in \Theta_m} N_{\tilde{\theta}} | \tilde{\theta} \in \Theta_m) = 0$ , then the conditional expectation does not exist. In this case, the beliefs about expected firm value upon observing public financing may be any firm value  $y_m \in Y$ . Thus, we obtain that

$$\beta_m(\Theta_b, \Theta_m, \{N_{\theta}\}_{\theta \in \Theta_m}) = \begin{cases} E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}, \tilde{\theta} \in \Theta_m), & \text{if } \mathbf{P}(\tilde{y}_{\tilde{\theta}} \in \cup_{\theta \in \Theta_m} N_{\theta} | \tilde{\theta} \in \Theta_m) > 0, \\ y_m, & \text{if } \mathbf{P}(\tilde{y}_{\tilde{\theta}} \in \cup_{\theta \in \Theta_m} N_{\theta} | \tilde{\theta} \in \Theta_m) = 0. \end{cases} \quad (6)$$

where  $y_m \in Y$ .

Since the goal of firm  $\theta$  is to maximize the expected firm value as perceived by its investors, firm value in case of public financing equals

$$\beta_m(\Theta_b, \Theta_m, \{N_{\theta'}\}_{\theta' \in \Theta_m}) - C_m - C_p a(\beta_m(\Theta_b, \Theta_m, \{N_{\theta'}\}_{\theta' \in \Theta_m}))$$

if firm  $\theta$  withholds its private information  $y_{\theta}$  from the public, and it equals  $y_{\theta} - C_m - C_p a(y_{\theta})$  if firm  $\theta$  discloses  $y_{\theta}$ . The expected firm value then equals<sup>10</sup>

$$\begin{aligned} V_m(\theta, (\Theta_b, \Theta_m, \{N_{\theta'}\}_{\theta' \in \Theta_m})) = & \\ (1 - F(N_{\theta}, \theta)) (E(\tilde{y}_{\theta} | \tilde{y}_{\theta} \notin N_{\theta}) - C_m - C_p \mathbf{P}(\tilde{y}_{\theta} \geq K | \tilde{y}_{\theta} \notin N_{\theta})) & \\ + F(N_{\theta}, \theta) (\beta_m(\Theta_b, \Theta_m, \{N_{\theta'}\}_{\theta' \in \Theta_m}) - C_m - C_p a(\beta_m(\Theta_b, \Theta_m, \{N_{\theta'}\}_{\theta' \in \Theta_m}))) & \end{aligned} \quad (7)$$

In a sequential equilibrium  $(\Theta_b^*, \Theta_m^*, \{N_{\theta}^*\}_{\theta \in \Theta_m^*})$ , the financing and disclosure decision of the firm is optimal with respect to the sequentially rational beliefs  $\beta_b(\Theta_b^*, \Theta_m^*, \{N_{\theta}^*\}_{\theta \in \Theta_m^*})$  and  $\beta_m(\Theta_b^*, \Theta_m^*, \{N_{\theta}^*\}_{\theta \in \Theta_m^*})$ .

Since sequential equilibria are subgame perfect, the (non)disclosure strategy  $N_{\theta}^*$  must be the optimal strategy with respect to the beliefs  $\beta_m(\Theta_b^*, \Theta_m^*, \{N_{\theta}^*\}_{\theta \in \Theta_m^*})$ . This subgame shows great resemblance to the voluntary disclosure model discussed in Wagenhofer (1990). In equilibrium, private information  $y_{\theta}$  is disclosed to the public if and only if disclosure results in a higher firm value than nondisclosure, that is

$$y_{\theta} - C_m - C_p a(y_{\theta}) \geq \beta_m(\Theta_b, \Theta_m, \{N_{\theta'}\}_{\theta' \in \Theta_m}) - C_m - C_p a(\beta_m(\Theta_b, \Theta_m, \{N_{\theta'}\}_{\theta' \in \Theta_m})).$$

Hence, in a sequential equilibrium  $(\Theta_b^*, \Theta_m^*, \{N_{\theta}^*\}_{\theta \in \Theta_m^*})$  it holds for the nondisclosure set  $N_{\theta}^*$  that

$$N_{\theta}^* = \left\{ y \in Y \left| \begin{array}{l} y - C_m - C_p a(y) \leq \\ E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*, \tilde{\theta} \in \Theta_m^*) - C_m - C_p a \left( E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*, \tilde{\theta} \in \Theta_m^*) \right) \end{array} \right. \right\} \quad (8)$$

for all  $\theta \in \Theta_m^*$ .

We distinguish two types of disclosure equilibria. In a full disclosure equilibrium, a publicly financed firm  $\theta \in \Theta_m$  always discloses its private information. Thus,  $N_{\theta}^* = \emptyset$ . This equilibrium is supported by skeptical beliefs. When the opponent and the public capital market observe nondisclosure of a publicly financed firm, they believe the worst possible, i.e.  $y_m = \min\{\underline{y}, K - C_p\}$ .

In a partial disclosure equilibrium, a publicly financed firm  $\theta$  withholds some of its private information from the public. The nondisclosure set  $N_{\theta}^*$  is characterized by two intervals, one containing relatively bad information

<sup>10</sup>For ease of notation we denote  $\mathbf{P}(\tilde{y}_{\theta} \in N_{\theta})$  by  $F(N_{\theta}, \theta)$  in the remainder of this paper.

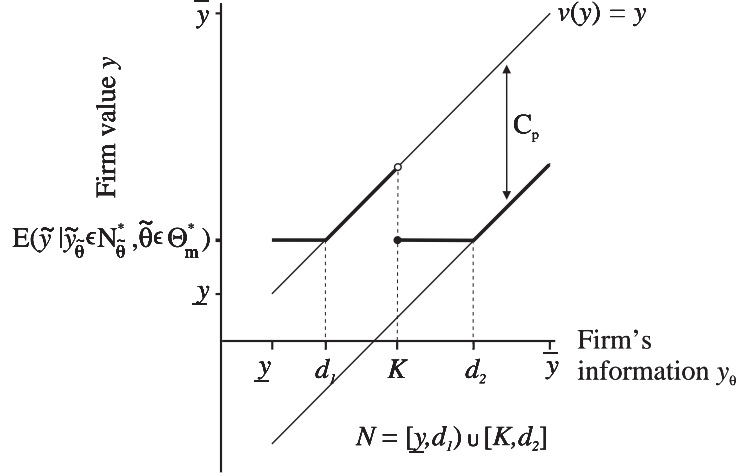


Figure 3: A partial disclosure equilibrium.

and the other containing relatively good information (see Figure 3).<sup>11</sup> When the opponent observes nondisclosure, he cannot find out whether this is because the private information is good and the firm wants to avoid proprietary cost, or just because the private information is bad. Since the latter thought always dominates in equilibrium, the opponent refrains from taking the adverse action when he observes nondisclosure.

The next proposition makes a statement about how the equilibrium nondisclosure sets of publicly financed firms relate to each other.

**Proposition 3.1** Let  $(\Theta_b^*, \Theta_m^*, \{N_\theta^*\}_{\theta \in \Theta_m^*})$  be a sequential equilibrium with  $\mathbb{P}(\tilde{y}_{\tilde{\theta}} \in \cup_{\theta \in \Theta_m^*} N_\theta^* | \theta \in \Theta_m^*) > 0$ , and define  $N^* = [y, d_1^*] \cup [K, d_2^*]$ , with  $d_1^* = E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_\theta^*, \tilde{\theta} \in \Theta_m^*)$  and  $d_2^* = \min\{\bar{y}, d_1^* + C_p\}$ . Then  $d_1^* < K$  and  $\mathbb{P}(\tilde{y}_\theta \in N^* - N_\theta^*) = \mathbb{P}(\tilde{y}_\theta \in N_\theta^* - N^*) = 0$  for all  $\theta \in \Theta_m^*$ .

Proposition 3.1 states that all publicly financed firms essentially use the same disclosure strategy. The intuition behind this proposition is clear. Since the opponent cannot distinguish between the various types of firms that choose public financing, he cannot have skeptical beliefs for one type  $\theta \in \Theta_m^*$  and other beliefs for another type  $\theta' \in \Theta_m^*$ . As a result, either all publicly financed firms play a full disclosure strategy, or all publicly financed firms play the same partial disclosure strategy.

Since full disclosure yields the same payoff as mandatory disclosure, Theorem 2.1 and Theorem 2.2 also apply if a full disclosure equilibrium occurs.

### Theorem 3.2

(a) A full public financing equilibrium that features a partial disclosure equilibrium exists if and only if there exists  $N^* = [y, d_1^*] \cup [K, d_2^*]$  such that

$$N^* = \left\{ y \in Y \mid E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N^*, \tilde{\theta} \in \Theta) - C_p a(E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N^*, \tilde{\theta} \in \Theta)) \geq y - C_p a(y) \right\} \quad (9)$$

<sup>11</sup> Figure 3 illustrates one of the two general forms of partial disclosure equilibria. The alternative is characterized by  $d_2 = \bar{y}$  and implies only one disclosure interval.

and

$$F(N^*, \bar{\theta}) (d_1^* - E(\tilde{y}_{\bar{\theta}} | \tilde{y}_{\bar{\theta}} \in N^*)) + C_p a(E(\tilde{y}_{\bar{\theta}} | \tilde{\theta} \in \Theta_b^*)) - C_p(1 - F(d_2^*, \bar{\theta})) \geq C_m - C_b. \quad (10)$$

(b) A full private financing equilibrium exists if and only if

$$C_p(1 - F(K, \underline{\theta})) - C_p a(E(\tilde{y}_{\hat{\theta}})) \geq C_b - C_m. \quad (11)$$

(c) In a partial financing equilibrium with partial disclosure strategy  $N^* = [\underline{y}, d_1^*] \cup [K, d_2^*] \neq \emptyset$ , the set of privately financed firms equals  $\Theta_b^* = [\theta_2^*, \bar{\theta}]$ , where  $\theta_2^* \in \Theta$  is such that

$$F(N^*, \theta_2^*)(d_1^* - E(\tilde{y}_{\theta_2^*} | \tilde{y}_{\theta_2^*} \in N^*)) + C_p a(E(\tilde{y}_{\hat{\theta}} | \tilde{\theta} \in \Theta_b^*)) - C_p(1 - F(d_2^*, \theta_2^*)) = C_m - C_b.$$

Furthermore,  $\theta_2^* \geq \theta_1^*$ , and if private capital is relatively costly, i.e.  $C_b > C_m$ , then it holds that  $E(\tilde{y}_{\hat{\theta}} | \tilde{\theta} \in \Theta_b^*) < K$ .

A full public financing equilibrium involves skeptical beliefs about privately financed firms. The advantage of public financing consists of two parts, the proprietary cost advantage  $C_p a(E(\tilde{y}_{\hat{\theta}} | \tilde{\theta} \in \Theta_b^*)) - C_p(1 - F(d_2^*, \bar{\theta}))$  and the nondisclosure advantage  $F(N^*, \bar{\theta}) (d_1^* - E(\tilde{y}_{\bar{\theta}} | \tilde{y}_{\bar{\theta}} \in N^*))$ . The latter advantage represents the misvaluation of the firm by the opponent and the public capital market when they observe nondisclosure. Note that if there is an undervaluation of the firm, there is actually a nondisclosure disadvantage for the publicly financed firm. Thus, in a full public financing equilibrium, the proprietary cost advantage and the nondisclosure advantage exceed the capital cost disadvantage  $C_m - C_b$  for all types.

Skeptical beliefs about firm value of a publicly financed firm is not necessary to sustain a full private financing equilibrium. If, however, such an equilibrium exists with other than skeptical beliefs, then also such an equilibrium exists with skeptical beliefs. In other words, if skeptical beliefs cannot sustain a full private financing equilibrium, then no other sequentially rational beliefs can sustain such an equilibrium. Furthermore, if private capital is more costly than public capital, i.e.  $C_b > C_m$ , then expression (11) can only be satisfied if  $E(\tilde{y}_{\hat{\theta}}) < K$ , which implies that the opponent refrains from taking the adverse action. So, although private capital is more expensive, the benefits from no proprietary cost is sufficiently high to make all firms prefer private financing to public financing.

In a partial financing equilibrium the relatively better firms still prefer private financing. In this regard, nothing has changed compared to a mandatory disclosure environment. In a voluntary disclosure environment though, public financing occurs more often than in a mandatory disclosure environment, because the opportunity to disclose voluntarily reduces the expected proprietary disclosure cost. This finding reflects the value of disclosure flexibility that arises when the disclosure of proprietary information is not mandated.

## 4 Extensions and Related Issues

For both disclosure environments discussed thus far it holds that in a partial financing equilibrium the better firms prefer private financing. Moreover, it can happen that privately financed firms incur proprietary costs in a partial financing equilibrium. When this is the case, a privately financed firm might want to publicly disclose its private information when this information turns out to be relatively bad, for a disclosure of bad information keeps the opponent from taking the adverse action. Privately financed firms, however, are assumed not to be able to make credible public disclosures. Next, we examine how the results of Theorem 2.1 and Theorem 2.2 change if we



allow for such public disclosures. It turns out that relaxing the disclosure rules in this way will not radically change the equilibria: the relatively better firms still prefer private financing to public financing.

#### 4.1 Credible Public Disclosures by Privately Financed Firms

Let us return to our basic model with mandatory public disclosures for publicly financed firms, and suppose that privately financed firms are able to make a credible public disclosure about their private information. The incentive to make such a disclosure arises when the opponent imposes proprietary cost on privately financed firms. For if this is the case, a privately financed firm with bad private information could still avoid proprietary cost by publicly disclosing this information. Since such a disclosure environment makes private financing more attractive, we should expect to see more firms choose private financing.

In such disclosure environments, a strategy is described by the tuple  $(\Theta_b, \{N_\theta\}_{\theta \in \Theta_b}, \Theta_m)$ , where  $\Theta_b \subset \Theta$  represents the set of privately financed firms,  $N_\theta \subset Y$  the disclosure strategy of a privately financed firm  $\theta \in \Theta_b$ , and  $\Theta_m \subset \Theta$  the set of publicly financed firms. Note that a privately financed firm  $\theta \in \Theta_b$  publicly discloses the information if and only if  $y \notin N_\theta$ , and that such a disclosure is truthful by assumption.

In order to determine the expected firm value for a privately financed firm, we first need to specify the beliefs of the opponent when observing nondisclosure. We distinguish two cases. If nondisclosure by a privately financed firm occurs with positive probability, i.e.  $\mathbf{P}(\tilde{y}_{\tilde{\theta}} \in \cup_{\theta \in \Theta_b} N_\theta | \tilde{\theta} \in \Theta_b) > 0$ , then the updated beliefs concerning the expected firm value equal  $E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}, \tilde{\theta} \in \Theta_b)$ . If nondisclosure occurs with zero probability, i.e.  $\mathbf{P}(\tilde{y}_{\tilde{\theta}} \in \cup_{\theta \in \Theta_b} N_\theta | \tilde{\theta} \in \Theta_b) = 0$ , then the conditional expectation does not exist, so that the beliefs are allowed to be any value  $y_b \in Y$ . Hence, the beliefs  $\beta_b(\Theta_b, \{N_\theta\}_{\theta \in \Theta_b}, \Theta_m)$  of the opponent when observing nondisclosure by a privately financed firm with strategy  $(\Theta_b, \{N_\theta\}_{\theta \in \Theta_b}, \Theta_m)$  equal

$$\beta_b(\Theta_b, \{N_\theta\}_{\theta \in \Theta_b}, \Theta_m) = \begin{cases} E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}, \tilde{\theta} \in \Theta_b) & \text{if } \mathbf{P}(\tilde{y}_{\tilde{\theta}} \in \cup_{\theta \in \Theta_b} N_\theta | \tilde{\theta} \in \Theta_b) > 0, \\ y_b, & \text{if } \mathbf{P}(\tilde{y}_{\tilde{\theta}} \in \cup_{\theta \in \Theta_b} N_\theta | \tilde{\theta} \in \Theta_b) = 0. \end{cases} \quad (12)$$

Since the opponent learns  $y_\theta$  if a privately financed firm makes a public disclosure, the expected firm value for private financing equals

$$V_b(\theta, (\Theta_b, \{N_{\theta'}\}_{\theta' \in \Theta_b}, \Theta_m)) = E(\tilde{y}_\theta) - C_b - C_p \mathbf{P}(\tilde{y}_\theta \in N_\theta) a(\beta_b(\Theta_b, \{N_\theta\}_{\theta \in \Theta_b}, \Theta_m)) - C_p \mathbf{P}(\tilde{y}_{\tilde{\theta}} \notin N_\theta) \mathbf{P}(\tilde{y}_{\tilde{\theta}} \geq K | \tilde{y}_{\tilde{\theta}} \notin N_\theta). \quad (13)$$

For a publicly financed firm  $\theta \in \Theta_m$ , the disclosure regulations are the same as in our basic model. Thus, the expected firm value  $V_m(\theta, (\Theta_b, \{N_{\theta'}\}_{\theta' \in \Theta_b}, \Theta_m))$  equals (4), i.e.

$$V_m(\theta, (\Theta_b, \{N_{\theta'}\}_{\theta' \in \Theta_b}, \Theta_m)) = E(\tilde{y}_\theta) - C_m - C_p(1 - F(K, \theta)).$$

In a sequential equilibrium  $(\Theta_b^*, \{N_\theta^*\}_{\theta \in \Theta_b^*}, \Theta_m^*)$ , the financing and disclosure decision are optimal with respect to the beliefs  $\beta_b(\Theta_b^*, \{N_\theta^*\}_{\theta \in \Theta_b^*}, \Theta_m^*)$ . Subgame perfection implies that the equilibrium disclosure strategy  $N_\theta^*$  is optimal against the beliefs  $\beta_b(\Theta_b^*, \{N_\theta^*\}_{\theta \in \Theta_b^*}, \Theta_m^*)$ . Since the opponent's beliefs only affect expected firm value of a privately financed firm through the proprietary cost  $C_p$ , the firm is indifferent between disclosure and nondisclosure for many kinds of private information. For instance, if the opponent's beliefs are such that a

nondisclosing privately financed firm incurs no proprietary cost, this firm is indifferent between disclosing and nondisclosing any information  $y < K$ .

The next proposition makes a statement about the equilibrium disclosure strategy of privately financed firms.

**Proposition 4.1** Let  $(\Theta_b^*, \{N_\theta^*\}_{\theta \in \Theta_b^*}, \Theta_m^*)$  be a sequential equilibrium. Then for all  $\theta \in \Theta_b^*$  it holds that  $[K, \bar{y}] \subset N_\theta^*$  if  $\beta_b(\Theta_b^*, \{N_\theta^*\}_{\theta \in \Theta_b^*}, \Theta_m^*) < K$ , and  $N_\theta^* \subset [K, \bar{y}]$  if  $\beta_b(\Theta_b^*, \{N_\theta^*\}_{\theta \in \Theta_b^*}, \Theta_m^*) \geq K$ .

If in equilibrium the opponent does not take the adverse action when observing a nondisclosing privately financed firm, then a privately financed firm does not disclose relatively good information, i.e.  $y \geq K$ . Note that also some bad information should not be disclosed so as to keep the opponent from taking the adverse action. For if it would only conceal information  $y \geq K$ , then the opponent would know that the privately financed firm possesses relatively good information  $y \geq K$  when he observes nondisclosure. Consequently, he would impose proprietary cost on the firm. So, a privately financed firm should be careful not to release so much bad information that it will change the opponents beliefs in such a way that he will take the adverse action when observing nondisclosure.

If in equilibrium the opponent does take the adverse action when observing a nondisclosing privately financed firm, then a privately financed firm discloses all its relatively bad information, i.e.  $y < K$ . Note that in this case the firm is indifferent between disclosure and nondisclosure of good information, that is  $y \geq K$ . Furthermore, note that if bad information  $y < K$  is disclosed to the public, then the opponent's beliefs when observing nondisclosure always exceed the threshold value  $K$ . Hence, no inconsistency arises.

#### Theorem 4.2

- (a) A full public financing equilibrium exists if and only if private capital is more costly than public capital, i.e.  $C_b \geq C_m$ .
- (b) If  $C_m > C_b$ , then a full private financing equilibrium always exists. If  $C_b > C_m$  then a full private financing equilibrium exists if and only if  $C_p(1 - F(K, \underline{\theta})) \geq C_b - C_m$  and  $E(\tilde{y}_\theta | \tilde{y}_\theta \in N_\theta^*) < K$ .
- (c) If a partial financing equilibrium exists, then private capital is relatively costly, i.e.  $C_b > C_m$ , and the relatively good firms finance privately, i.e.  $\Theta_b = [\theta_3^*, \bar{\theta}]$ , with  $\theta_3^*$  such that  $C_p(1 - F(K, \theta_3^*)) = C_b - C_m$ . Furthermore,  $\theta_3^* = \theta_1^*$  (cf. Theorem 2.2), and the opponent refrains from taking the adverse action when observing nondisclosure by a privately financed firm, i.e.  $E(\tilde{y}_\theta | \tilde{y}_\theta \in N_\theta^*, \tilde{\theta} \in \Theta_b^*) < K$ .

A full public financing equilibrium is driven by skeptical beliefs, which means that the opponent takes the adverse action whenever he observes nondisclosure by a privately financed firm. The existence of a full private financing equilibrium when private capital is less costly than public capital is obvious. For in that case, private financing with full disclosure of information dominates public financing.

The opportunity to credibly disclose private information to the public, does not change the preferences of the firms between private and public financing. Compared to the situation where privately financed firms cannot make any credible public disclosures, exactly the same firms opt for private financing. The only difference is that partial financing equilibria can only exist when private capital is relatively costly. That a partial financing equilibrium does not exist in the opposite case, is due to the fact that a privately financed firm can choose to fully

disclose its private information to the opponent. By choosing this disclosure strategy, the firm mimics the behavior of a publicly financed firm. Thus, the difference between private and public financing is just the difference in the cost of capital. Since private capital is less costly than public capital, all types prefer private financing and a full private financing equilibrium arises.

## 4.2 Credible Voluntary Disclosures for Private and Public Financing

In the second relaxation, we allow voluntary credible public disclosures by both publicly and privately financed firms. Hence, as in the former subsection, a private firm can make a credible public disclosure in addition to informing its private investors exclusively. A publicly financed firm is no longer compelled to reveal its private information publicly. So, we can describe a strategy by  $(\Theta_b, \Theta_m, \{N_\theta\}_{\theta \in \Theta})$ , where  $\Theta_b$  describes the privately financed firms,  $\Theta_m$  the publicly financed firms, and  $N_\theta$  the disclosure strategy of firm  $\theta \in \Theta$ .

For determining the expected payoff for both types of financing, recall that the beliefs of the opponent when he observes nondisclosure by a privately and a publicly financed firm, are given by (12) and (6), respectively. Hence, the expected firm value for private and public financing equals (13) and (7), respectively.

In a sequential equilibrium  $(\Theta_b^*, \Theta_m^*, \{N_\theta^*\}_{\theta \in \Theta})$ , the financing and disclosure decision are optimal with respect to the beliefs  $\beta_b(\Theta_b^*, \Theta_m^*, \{N_\theta^*\}_{\theta \in \Theta})$  and  $\beta_m(\Theta_b^*, \Theta_m^*, \{N_\theta^*\}_{\theta \in \Theta})$ . Subgame perfection implies that Proposition 3.1 and Proposition 4.1 hold true. Furthermore, if a full disclosure equilibrium arises for publicly financed firms, Theorem 3.2 also applies.

### Theorem 4.3

- (a) A full public financing equilibrium featuring partial disclosure always exists if (9) holds and if public capital is relatively cheap compared to private capital, that is if  $C_m \leq C_b$ .
- (b) If  $C_m > C_b$ , then a full private financing equilibrium always exists. If  $C_b > C_m$  then a full private financing equilibrium exists if and only if  $C_p(1 - F(K, \underline{\theta})) \geq C_b - C_m$  and  $E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*) < K$ .
- (c) In a partial financing equilibrium with  $E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*, \tilde{\theta} \in \Theta_b^*) < K$ , the relatively better firms choose private financing, i.e.  $\Theta_b = [\theta_4^*, \bar{\theta}]$ . In particular, it holds that  $\theta_4^* = \theta_2^*$  (cf. Theorem 3.2). Furthermore, in a partial financing equilibrium with  $E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*, \tilde{\theta} \in \Theta_b^*) \geq K$ , public capital must be more costly than private capital, i.e.  $C_m \geq C_b$ .

Again, a full public financing equilibrium involves skeptical beliefs by the opponent when he observes nondisclosure by a privately financed firm. Skeptical beliefs by the opponent impose proprietary cost on the firm so that it wants to disclose all its bad information. Such a disclosure strategy, however, elicits the same behavior of the opponent as a full disclosure strategy for a publicly financed firm. And since public capital is relatively cheap, all firms prefer public financing to private financing. Note that in contrast with the previous models, a full public financing equilibrium may also exist if public capital is relatively costly. This equilibrium will only arise, of course, if the benefit from nondisclosure by publicly financed types is sufficiently large.

Since a full private financing equilibrium features skeptical beliefs, the optimal disclosure strategy of a publicly financed firm is full disclosure. Hence, the conditions for such an equilibrium are equivalent to that of Theorem 4.2.

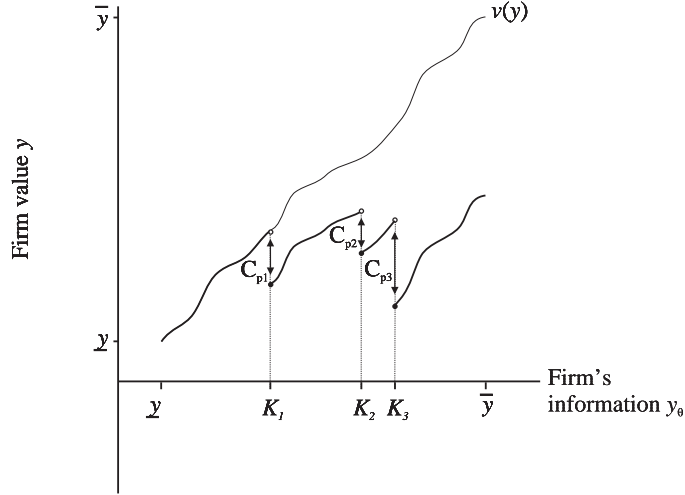


Figure 4: Firm value and proprietary cost with multiple opponents

If in a partial financing equilibrium privately financed firms can avoid proprietary cost, then the possibility for privately financed firms to credibly disclose their information to the public does not make private financing more attractive. Compared to the environment that excludes such disclosures, the same type of firms prefer private to public financing. If, however, in a partial financing equilibrium the opponent imposes proprietary costs on a nondisclosing privately financed firm, we cannot draw any conclusions but that public capital must be more costly than private capital.

### 4.3 Multiple Opponents

In modeling the opponent's behavior, we took the same approach as Wagenhofer (1990). This means that the motives behind the opponent's actions are not explicitly modeled. Instead, it is assumed that of all the decisions that the opponent may make to obtain his goal, only one imposes a fixed proprietary cost on the firm. In addition, it is assumed that the opponent takes this decision if and only if he believes that firm value is sufficiently high. Agents that may act as an opponent include for instance product market competitors and governmental authorities. The present model, however, only takes into account one opponent. The results of our study remain valid though, if we allow for more than one opponent. In the case of  $n$  opponents, each opponent  $i$  imposes a proprietary cost  $C_{p_i} > 0$  if and only if he believes that the expected firm value exceeds the threshold  $K_i \in Y$ , where  $K_1 < K_2 < \dots < K_n$ . This means that if the opponents<sup>12</sup> believe that firm value equals  $y \in [K_i, K_{i+1})$ , then the firm incurs proprietary cost  $C_{p_1} + C_{p_2} + \dots + C_{p_i}$  (see Figure 4).

Note that this generalized setup can also be used to vary the height of the proprietary cost with the opponent's beliefs. So instead of yes or no proprietary cost, the proprietary cost may, for instance, be absent for low firm values, low for average firm values, and high for high firm values.

<sup>12</sup>Since all opponents behave rationally and possess the same information, they form identical beliefs about firm value.

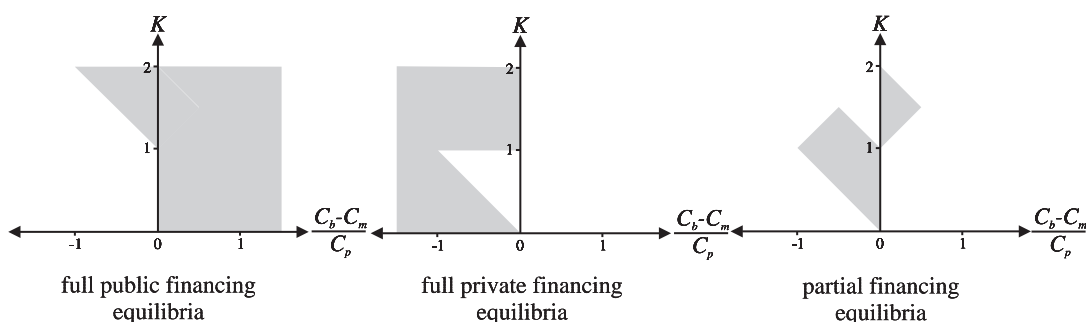


Figure 5: The existence of the different financing equilibria in relation to the exogenous variables.

## 5 Implications of the Model

Our analysis shows that in equilibrium a relatively good firm prefers private financing to public financing. In this regard, relatively good refers to the probability distribution of the private information that such a firm can receive. That a firm possesses valuable private information need not necessarily imply that the firm is of a relatively good type, for even a relatively bad firm may possess valuable information from time to time. As a result, a single profit figure is no unambiguous indicator of a firm's type. Indicators of a firm's type should provide information about the probability distribution of firm value like a (time) series of profit figures does<sup>13</sup> or a firm's long term profitability, i.e. a firm's permanent earnings. Regarding the latter as a reasonable indicator of a firm's type and assuming that a positive relation exists between the firm's proprietary information and its profitability, our results state that private firms are more profitable in the long term than public firms. This inference is in line with the empirical results of Brav and Gompers (1997), Loughran and Ritter (1995), and Ritter (1991), who observe a long-run underperformance by IPO's.

In our model, the exogenous variables  $C_p$ ,  $C_b - C_m$ , and  $K$  partly determine the existence of the several financing equilibria. To illustrate the relation between the exogenous variables and a partial financing equilibrium consider the following example. Let the private information be described by  $\tilde{y}_{\tilde{\theta}} = \tilde{y} + \tilde{\theta}$  where  $\tilde{y}$  and  $\tilde{\theta}$  are uniformly distributed on the interval  $[0, 1]$ . Hence,  $Y = [0, 2]$  and  $\Theta = [0, 1]$ . Figure 5 shows the existence of the different financing equilibria in relation to  $K$  and  $\frac{C_b - C_m}{C_p}$ , the ratio between the difference in capital cost and the proprietary cost.

Since a full public financing equilibrium features skeptical beliefs of the opponent when observing private financing, private financing yields proprietary cost with certainty. In accordance with Theorem 2.1, we see that a public financing equilibrium always exists if public capital is relatively cheap compared to private capital.

<sup>13</sup>To have a proper view of the profit figures over time, the time series should, of course, be appropriately adjusted for factors like economic growth and inflation.

Reason for this is that a publicly financed firm may avoid proprietary cost with positive probability. Hence, public financing has a capital cost advantage and a proprietary cost advantage. In addition, if the proprietary cost advantage of public financing is sufficiently large compared to the capital cost disadvantage, a full public financing equilibrium may also exist in case that public capital is relatively costly. That such an equilibrium can only arise for  $K > 1$  has the following explanation. If  $K \leq 1$  then a firm of type  $\theta \geq K$  will always incur proprietary cost when it opts for public financing. Since in that case there is no proprietary cost advantage for public capital, this firm prefers the cheaper option of private capital.

As Figure 5 shows, full private financing equilibria can only exist if private capital is less costly than public capital. If  $K > E(\tilde{y}_{\hat{\theta}})$ , then the opponent will not impose proprietary cost on a privately financed firm. Since public capital yields proprietary cost with positive probability, private financing comes with a capital cost advantage and a proprietary cost advantage, so that all firms prefer private to public financing. Note, however, that even if the proprietary cost advantage of private financing is sufficiently large, a full private financing equilibrium cannot arise in case private capital is relatively costly. To see this, observe that a firm of type  $\theta < K - 1$  will not incur proprietary cost when it opts for public financing. Since in that case there is no proprietary cost advantage for private capital, this firm prefers the cheaper option of public capital. If  $K \leq E(\tilde{y}_{\hat{\theta}})$ , then the opponent will impose proprietary cost on a privately financed firm. Since public financing yields no proprietary cost with positive probability, there is a proprietary cost disadvantage for private financing. Then a full private financing equilibrium only arises if private capital is sufficiently cheap compared to public capital.

For the partial financing equilibria, we know from Theorem 2.2 that the relatively good firms choose private financing, i.e.  $\Theta_b^* = [\theta^*, \bar{\theta}]$ . Figure 6 pictures the  $\Theta_b^*$  as a function of the threshold value  $K$  and  $\frac{C_b - C_m}{C_p}$ . It follows that partial financing equilibria only exist if the proprietary cost exceeds the difference in capital cost, that is  $C_p \geq |C_b - C_m|$ . Furthermore, one can derive that more firms prefer private financing when the threshold value  $K$  decreases. Similarly, one can derive that  $\theta^*$  decreases as  $\frac{C_b - C_m}{C_p}$  decreases. Note, however, that the effect of proprietary cost depends on which type of capital is relatively costly. If private capital is relatively costly, that is  $C_b > C_m$ , an increase in proprietary cost makes private financing more attractive. The intuition is that when  $C_b > C_m$ , privately financed firms avoid incurring proprietary cost in a partial financing equilibrium otherwise it would not exist. If the proprietary cost increases, then the expected proprietary cost of public financing increases so that private financing becomes beneficial for more firms. The opposite holds if public capital is relatively costly, that is  $C_m > C_b$ . In that case, privately financed firms incur proprietary cost in a partial financing equilibrium. If the proprietary cost increase, then the cost benefit of private financing does no longer outweigh the proprietary cost for the relatively bad privately financed firm. Hence, they prefer public financing.

It is a straightforward exercise to show that similar interdependencies can be derived for the general case in which public financing is subject to mandatory disclosure. Since Theorem 3.2 states that under a voluntary disclosure rule more firms go public in a partial financing equilibrium than under a mandatory disclosure rule, the interdependencies should follow similar trends in a voluntary disclosure environment. So, we pose that public financing occurs more often as the threshold value  $K$  increases or  $\frac{C_b - C_m}{C_p}$  increases. This assertion leads to the following implications.

When proprietary cost is positively related to product market competition, the cost of leaking proprietary

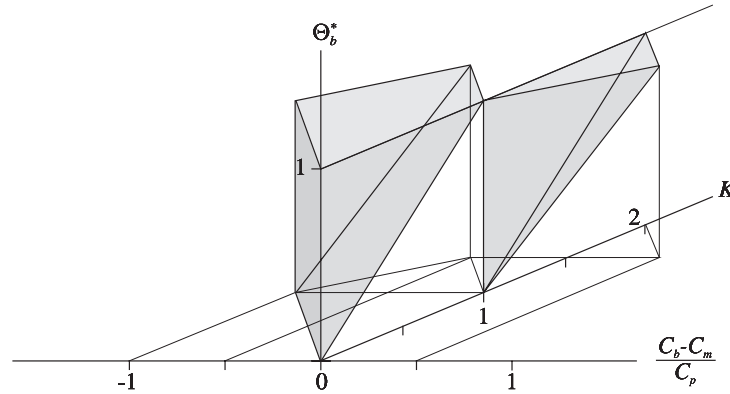


Figure 6: The attractiveness of private financing in relation to the exogenous variables  $K$  and  $\frac{C_b - C_m}{C_p}$ .

information is relatively high for competitive markets.<sup>14</sup> Thus, competitive markets are characterized by high proprietary cost, a low threshold value, or both. As a result, one should observe more private firms in competitive markets where private capital is more costly than public capital. Two remarks are in place here. First, our model does not take into account differences in the risk of returns of different firms. Firms in highly competitive markets may have more risky returns, making the premium for bearing idiosyncratic risk in case of private financing higher. Instead, public capital markets allow better risk sharing opportunities. Hence, the capital cost difference for risky firms is larger and therefore it is difficult to say what overall effect an increase in product market competition has on the going-public decision. Second, if the ability to hide proprietary information in highly competitive markets is more difficult,<sup>15</sup> the influence of differences in the disclosure environments attached to the alternative financing opportunities reduces.

When proprietary cost is related to the entry of a new competitor on the product market, a market with high entry barriers may be represented by a high threshold value. Assuming that the proprietary cost resulting from entry is fixed, public financing should become more attractive as the entry barrier increases. This result links up with Chemmanur and Fulghieri (1999). They suggest that firms from more capital intensive industries go public earlier.

As in Yosha (1995) our model supports findings of positive stock price responses to the announcement of private equity or debt placements. Firms that are on to something good might choose to attract private capital to prevent having to disclose publicly about their investment plans. This consideration is also applicable in case of seasoned offerings.

<sup>14</sup> Harris (1998), however, finds evidence that might point to a negative association between proprietary cost and competition. Her finding suggests that a reluctance towards disclosure is highest for firms enjoying abnormal returns. Assuming that abnormal returns are more likely in less competitive industries yields a negative association.

<sup>15</sup> Competitive markets are typically characterized by less information asymmetry, e.g. because the competition for proprietary information is also higher in such markets.

Finally, our model may also have implications concerning the discussion about the unification of accounting rules across jurisdictions. To the extent that proprietary disclosure cost considerations influence a firm's financing decision, the decision to list on a domestic or foreign public capital market can be driven by differences in disclosure requirements. For instance, American public security markets are generally believed to be the most liquid markets. These markets, however, are also known to have the most stringent disclosure regimes.<sup>16</sup> Hence, a firm considering an (initial) public offering of securities might forego the liquidity advantage offered by the NYSE, ASE, or NASDAQ because proprietary cost considerations makes a listing to a less demanding disclosure environment more attractive. This result is in contrast with Huddart, Hughes and Brunnermeier (1999) who find that when several public markets compete in trading volume disclosure requirements increase. Their model considerably differs from ours, particularly in the modeling of the proprietary cost and how disclosure resolves the information asymmetry.

To return to our model, the proprietary cost argument might help to explain why we do not see all IPO's to be executed on American public capital markets. Moreover, our model predicts that it are the relatively worse (foreign) firms that will enter American stock exchanges. With regard to the ongoing efforts of harmonizing accounting rules worldwide, the former argument might explain why stock market authorities are reluctant in changing their disclosure requirements for listing to meet GAAP. Uniform disclosure regulations across capital markets favors those markets that offer the least capital cost. Hence, representatives of less liquid capital markets, i.e. non-US capital markets, may oppose GAAP proposals that reduce disclosure flexibility in fear of losing their competitive disclosure advantage. On the other hand, US market officials might oppose obscure accounting proposals in an attempt to protect their liquidity lead.

## 6 Conclusions

This paper analyzes how differences in disclosure regulations between private and public capital markets may affect the firm's going-public decision. Disclosure regulations prescribe which of the firm's private information is subject to disclosure. Particularly, any confidential information that is subject to disclosure may lead to the firm incurring proprietary cost. In our study, the going-public decision is a trade-off between the difference in capital cost and the difference in proprietary disclosure cost. The main result of our analysis is that the relatively better firms remain private and that the relatively bad firms go public. The latter result might explain the minimum requirements for going-public that currently exist at public capital markets.

Our model implies that firms for which proprietary disclosure cost considerations are important, are more likely to stay private as the private information that is proprietary in nature becomes more valuable. If this latter property can be associated with the value of growth opportunities, our model shows that the more valuable a firm's growth options, the more attractive private financing becomes. This implies that the recent tendency for young fast-growing firms to enter public markets across jurisdictions may say more about differences in capital cost, particularly differences in the efficiency of private financing opportunities, than that it says something about the relevance of confidential information in IPO-decisions. Furthermore, our analysis confirms the general notion

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<sup>16</sup> Moel (1999) documents a difference in the rigor of disclosure rules across different American security markets.



that more stringent disclosure requirements for public firms decrease the likelihood of an IPO due to an increase in expected proprietary disclosure cost.

There are of course several other factors influencing the public/private financing decision that may confuse proprietary disclosure cost considerations. A prominent candidate for additional public financing cost that are not considered in the present study, are costs stemming from agency problems between managers and investors. For as long as these costs are fixed, they can be captured in our model by broadening the definition of the cost of capital. It may well be, however, that these costs vary with the quality of the firm. Underinvestment problems like those introduced by Myers (1977) become more serious for lower quality firms. Chances that a firm's management creates or discovers profitable investments increases with the quality of the firm and its management. In instances where the firm's management wants to forego positive NPV-projects, a few private capital investors has better opportunities to redirect the management's investment decision than a large group of public investors has. The introduction of such an agency cost may change our results in such a way that a disjoint set of privately financed firms arise in a partial financing equilibrium: besides the relatively better firms, also the relatively bad firm prefer staying private. The latter do so to benefit from the private investors influence on management.

## 7 Proofs

PROOF OF THEOREM 2.1: Let us start with proving the existence of a full public financing equilibrium, i.e.  $(\Theta_b^*, \Theta_m^*) = (\emptyset, \Theta)$ . Since  $\Theta_b^* = \emptyset$  it holds that  $\beta_b(\emptyset, \Theta) = y_b \in Y$ , so that the expected firm value for a privately financed firm equals  $E(\tilde{y}_\theta) - C_b - a(\beta_b(\emptyset, \Theta))C_p = E(\tilde{y}_\theta) - C_b - a(y_b)C_p$ . Note that a privately financed firm can exclusively disclose its information to its investor(s), so that the beliefs of the opponent only affect the presence of the proprietary cost. Since the expected payoff for a publicly financed firm equals  $V_m(\theta, (\emptyset, \Theta)) = E(\tilde{y}_\theta) - C_m - C_p(1 - F(K, \theta))$ , a full public financing equilibrium exists if and only if there exists  $y_b \in Y$  such that  $V_m(\theta, (\emptyset, \Theta)) \geq V_b(\theta, (\emptyset, \Theta))$  for all  $\theta \in \Theta$ , i.e.

$$E(\tilde{y}_\theta) - C_m - C_p(1 - F(K, \theta)) \geq E(\tilde{y}_\theta) - C_b - C_p a(y_b)$$

for all  $\theta \in \Theta$ . Since the right hand side of this inequality is minimal for  $a(y_b) = 1$ , such an equilibrium exists if and only if

$$E(\tilde{y}_\theta) - C_m - C_p(1 - F(K, \theta)) \geq E(\tilde{y}_\theta) - C_b - C_p,$$

for all  $\theta \in \Theta$ . Rearranging terms gives  $C_p F(K, \theta) \geq C_m - C_b$  for all  $\theta \in \Theta$ . From  $F(K, \theta)$  decreasing in  $\theta$  we then obtain that  $C_p F(K, \bar{\theta}) \geq C_m - C_b$ .

In a full private financing equilibrium it holds that  $(\Theta_b^*, \Theta_m^*) = (\Theta, \emptyset)$ . This is equivalent to  $V_b(\theta, (\Theta, \emptyset)) \geq V_m(\theta, (\Theta, \emptyset))$  for all  $\theta \in \Theta$ , i.e.

$$E(\tilde{y}_\theta) - C_b - C_p a(\beta(\Theta, \emptyset)) \geq E(\tilde{y}_\theta) - C_m - C_p(1 - F(K, \theta)),$$

for all  $\theta \in \Theta$ . Using that  $\beta_b(\Theta, \emptyset) = E(\tilde{y}_{\hat{\theta}})$  and rearranging terms yields

$$C_p(1 - F(K, \theta)) - C_p a(E(\tilde{y}_{\hat{\theta}} | \tilde{\theta} \in \Theta)) \geq C_b - C_m, \quad (14)$$

for all  $\theta \in \Theta$ . Since  $F(K, \theta)$  decreases with  $\theta$ , it follows that (14) is satisfied for all  $\theta \in \Theta$  if and only if  $C_p(1 - F(K, \underline{\theta})) - C_p a(E(\tilde{y}_{\tilde{\theta}}|\tilde{\theta} \in \Theta)) \geq C_b - C_m$ . Claim (a) then follows from  $a(E(\tilde{y}_{\tilde{\theta}})) = 0$  if  $E(\tilde{y}_{\tilde{\theta}}) < K$ , while (b) follows from  $a(E(\tilde{y}_{\tilde{\theta}})) = 1$  if  $E(\tilde{y}_{\tilde{\theta}}) \geq K$ .  $\square$

PROOF OF THEOREM 2.2: Recall that in equilibrium  $\theta \in \Theta_b^*$  if

$$E(\tilde{y}_{\theta}) - C_b - C_p a(E(\tilde{y}_{\tilde{\theta}}|\tilde{\theta} \in \Theta_b^*)) \geq E(\tilde{y}_{\theta}) - C_m - C_p(1 - F(K, \theta)).$$

Rearranging terms yields that  $\theta \in \Theta_b^*$  if

$$C_m - C_b - C_p a(E(\tilde{y}_{\tilde{\theta}}|\tilde{\theta} \in \Theta_b^*)) \geq -C_p(1 - F(K, \theta)). \quad (15)$$

Since the right hand side is decreasing in  $\theta$ , it follows that  $\theta' \in \Theta_b^*$  whenever  $\theta \in \Theta_b^*$  and  $\theta' > \theta$ . Hence,  $\Theta_b^* = [\theta_1^*, \bar{\theta}]$  where  $\theta_1^* \in \Theta$  is such that

$$C_m - C_b - C_p a(E(\tilde{y}_{\tilde{\theta}}|\tilde{\theta} \in \Theta_b^*)) = -C_p(1 - F(K, \theta_1^*)), \quad (16)$$

or, equivalently,

$$C_p(1 - F(K, \theta_1^*)) - C_p a(E(\tilde{y}_{\tilde{\theta}}|\tilde{\theta} \in \Theta_b^*)) = C_b - C_m.$$

Next, we prove part (a). Therefore, let  $C_m > C_b$ . From  $1 - F(K, \theta) \geq 0$  for all  $\theta \in \Theta$  and  $C_b - C_m \leq 0$  it follows that expression (15) is satisfied for all  $\theta \in \Theta$  if  $a(E(\tilde{y}_{\tilde{\theta}}|\tilde{\theta} \in \Theta_b^*)) = 0$ , i.e.  $E(\tilde{y}_{\tilde{\theta}}|\tilde{\theta} \in \Theta_b^*) < K$ . Hence, in a partial financing equilibrium we must have that  $E(\tilde{y}_{\tilde{\theta}}|\tilde{\theta} \in \Theta_b^*) \geq K$  and  $a(E(\tilde{y}_{\tilde{\theta}}|\tilde{\theta} \in \Theta_b^*)) = 1$ , so that  $\theta_1^* \in \Theta$  is such that  $C_p F(K, \theta_1^*) = C_m - C_b$ . Furthermore,  $\theta_1^* \in (\underline{\theta}, \bar{\theta})$  if and only if  $C_p F(K, \underline{\theta}) > C_m - C_b > C_p F(K, \bar{\theta})$ .

For the proof of part (b), let  $C_m < C_b$ . From  $1 - F(K, \theta) \geq 0$  for all  $\theta \in \Theta$  and  $C_b - C_m \geq 0$  it follows that expression (15) is satisfied for all  $\theta \in \Theta$  if  $a(E(\tilde{y}_{\tilde{\theta}}|\tilde{\theta} \in \Theta_b^*)) = 1$ , i.e.  $E(\tilde{y}_{\tilde{\theta}}|\tilde{\theta} \in \Theta_b^*) \geq K$ . Hence, in a partial financing equilibrium we have that  $E(\tilde{y}_{\tilde{\theta}}|\tilde{\theta} \in \Theta_b^*) < K$  and  $a(E(\tilde{y}_{\tilde{\theta}}|\tilde{\theta} \in \Theta_b^*)) = 0$ , so that  $\theta_1^* \in \Theta$  is such that  $C_p(1 - F(K, \theta_1^*)) = C_b - C_m$ . Furthermore,  $\theta_1^* \in (\underline{\theta}, \bar{\theta})$  if and only if  $C_p(1 - F(K, \underline{\theta})) < C_b - C_m < C_p(1 - F(K, \bar{\theta}))$ .  $\square$

PROOF OF PROPOSITION 3.1: let  $(\Theta_b^*, \Theta_m^*, \{N_{\tilde{\theta}}^*\}_{\tilde{\theta} \in \Theta_m^*})$  be a sequential equilibrium. In equilibrium it holds that

$$N_{\tilde{\theta}}^* = \left\{ y \in Y \left| \begin{array}{l} y - C_m - C_p a(y) \leq \\ E(\tilde{y}_{\tilde{\theta}}|\tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*, \tilde{\theta} \in \Theta_m^*) - C_m - C_p a(E(\tilde{y}_{\tilde{\theta}}|\tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*, \tilde{\theta} \in \Theta_m^*)) \end{array} \right. \right\},$$

for all  $\tilde{\theta} \in \Theta_m^*$ . Now, suppose that  $E(\tilde{y}_{\tilde{\theta}}|\tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*, \tilde{\theta} \in \Theta_m^*) \geq K$ . Then  $a(E(\tilde{y}_{\tilde{\theta}}|\tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*, \tilde{\theta} \in \Theta_m^*)) = 1$  and

$$\begin{aligned} N_{\tilde{\theta}}^* &= \left\{ y \in Y \left| y - C_p a(y) \leq E(\tilde{y}_{\tilde{\theta}}|\tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*, \tilde{\theta} \in \Theta_m^*) - C_p \right. \right\} \\ &= \left\{ y \in Y \left| C_p - C_p a(y) \leq E(\tilde{y}_{\tilde{\theta}}|\tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*, \tilde{\theta} \in \Theta_m^*) - y \right. \right\}. \end{aligned}$$

Since  $C_p - C_p a(y) \geq 0$  we obtain for all  $\tilde{\theta} \in \Theta_m^*$  and all  $y \in N_{\tilde{\theta}}^*$  that  $E(\tilde{y}_{\tilde{\theta}}|\tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*, \tilde{\theta} \in \Theta_m^*) \geq y$ . Obviously, this is a contradiction. Hence,  $E(\tilde{y}_{\tilde{\theta}}|\tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*, \tilde{\theta} \in \Theta_m^*) < K$ .

Next, define

$$N^* = [\underline{y}, d_1^*) \cup [K, d_2^*),$$

where  $d_1^* = E(\tilde{y}_\theta | \tilde{y}_\theta \in N_\theta^*, \tilde{\theta} \in \Theta_m^*) < K$  and  $d_2^* = \min\{\bar{y}, d_1^* + C_p\}$ . If  $N_\theta^*$  is firm  $\theta$ 's nondisclosure set, the expected payoff equals

$$\int_{Y-N_\theta^*} y - C_p a(y) dF(y, \theta) + \int_{N_\theta^*} d_1^* - a(d_1^*) dF(y, \theta). \quad (17)$$

Similarly, if  $N^*$  is firm  $\theta$ 's nondisclosure set, the expected payoff equals

$$\int_{Y-N^*} y - C_p a(y) dF(y, \theta) + \int_{N^*} d_1^* - a(d_1^*) dF(y, \theta). \quad (18)$$

Subtracting expression (17) from (18) and using that  $a(d_1^*) = 0$  yields

$$\begin{aligned} & \int_{Y-N^*} y - C_p a(y) dF(y, \theta) + \int_{N^*} d_1^* dF(y, \theta) \\ & - \int_{Y-N_\theta^*} y - C_p a(y) dF(y, \theta) - \int_{N_\theta^*} d_1^* dF(y, \theta) \\ & = \int_{N_\theta^*-N^*} y - C_p a(y) dF(y, \theta) - \int_{N_\theta^*-N^*} d_1^* dF(y, \theta) \\ & - \int_{N^*-N_\theta^*} y - C_p a(y) dF(y, \theta) - \int_{N^*-N_\theta^*} d_1^* dF(y, \theta) \\ & = \int_{N_\theta^*-N^*} y - C_p a(y) - d_1^* dF(y, \theta) - \int_{N^*-N_\theta^*} y - C_p a(y) - d_1^* dF(y, \theta) \\ & \geq 0, \end{aligned}$$

where the inequality follows from the following two observations. First, if  $y \in N^* - N_\theta^*$  then  $y \in N^*$  which implies that  $y - C_p a(y) \leq d_1^*$ . Second, if  $y \in N_\theta^* - N^*$  then  $y \notin N^*$  which implies that  $y - C_p a(y) \geq d_1^*$ . Since at least one of the inequalities is strict if either  $\mathbb{P}(\tilde{y}_\theta \in N^* - N_\theta^*) > 0$  or  $\mathbb{P}(\tilde{y}_\theta \in N_\theta^* - N^*) > 0$ , it follows that in equilibrium  $\mathbb{P}(\tilde{y}_\theta \in N^* - N_\theta^*) = 0$  and  $\mathbb{P}(\tilde{y}_\theta \in N_\theta^* - N^*) = 0$ . For if this is not the case, a firm of type  $\theta$  can improve its expected payoff by playing a disclosure strategy with nondisclosure set  $N^*$  instead of  $N_\theta^*$ .  $\square$

**Lemma 7.1** Let  $(\Theta_b^*, \Theta_m^*, \{N_\theta^*\}_{\theta \in \Theta_m^*})$  be a partial financing equilibrium with  $\mathbb{P}(\tilde{y}_\theta \in \cup_{\theta \in \Theta_m^*} N_\theta^* | \tilde{\theta} \in \Theta_m^*) > 0$  and let  $N^* = [\underline{y}, d_1^*) \cup [K, d_2^*)$  be such that  $d_1^* = E(\tilde{y}_\theta | \tilde{y}_\theta \in N_\theta^*, \tilde{\theta} \in \Theta_m^*)$  and  $d_2^* = \min\{\bar{y}, d_1^* + C_p\}$ . Then  $F(N^*, \theta)(d_1^* - E(\tilde{y}_\theta | \tilde{y}_\theta \in N^*)) - C_p(1 - F(d_2^*, \theta))$  is decreasing in  $\theta$ .

PROOF: Since

$$\begin{aligned} \int_{N^*} y f(y, \theta) dy &= \int_{\underline{y}}^{d_1^*} y f(y, \theta) dy + \int_K^{d_2^*} y f(y, \theta) dy \\ &= \int_{\underline{y}}^{d_1^*} \int_0^y f(y, \theta) dx dy + \int_K^{d_2^*} \int_0^y f(y, \theta) dx dy \\ &= \int_0^{\underline{y}} \int_{\underline{y}}^{d_1^*} f(y, \theta) dy dx + \int_{\underline{y}}^{d_1^*} \int_x^{d_1^*} f(y, \theta) dy dx + \end{aligned}$$

$$\begin{aligned}
& \int_0^K \int_K^{d_2^*} f(y, \theta) dy dx + \int_K^{d_2^*} \int_x^{d_2^*} f(y, \theta) dy dx \\
&= \int_0^{\underline{y}} F(d_1^*, \theta) - F(\underline{y}, \theta) dx + \int_{\underline{y}}^{d_1^*} F(d_1^*, \theta) - F(x, \theta) dx + \\
& \quad \int_0^K F(d_2^*, \theta) - F(K, \theta) dx + \int_K^{d_2^*} F(d_2^*, \theta) - F(x, \theta) dx \\
&= \underline{y} F(d_1^*, \theta) + \int_{\underline{y}}^{d_1^*} F(d_1^*, \theta) - F(x, \theta) dx + \\
& \quad K (F(d_2^*, \theta) - F(K, \theta)) + \int_K^{d_2^*} F(d_2^*, \theta) - F(x, \theta) dx, \\
&= F(d_1^*, \theta) d_1^* - \int_{\underline{y}}^{d_1^*} F(x, \theta) dx + F(d_2^*, \theta) d_2^* - F(K, \theta) K - \\
& \quad \int_K^{d_2^*} F(x, \theta) dx,
\end{aligned}$$

it follows that

$$\begin{aligned}
& F(N^*, \theta) (d_1^* - E(\tilde{y}_\theta | \tilde{y}_\theta \in N^*)) - C_p (1 - F(d_2^*, \theta)) = F(N^*, \theta) d_1^* - \int_{N^*} y f(y, \theta) dy \\
&= (F(d_1^*, \theta) + F(d_2^*, \theta) - F(K, \theta)) d_1^* - F(d_1^*, \theta) d_1^* + \int_{\underline{y}}^{d_1^*} F(y, \theta) dy \\
& \quad - F(d_2^*, \theta) d_2^* + F(K, \theta) K + \int_K^{d_2^*} F(y, \theta) dy - C_p (1 - F(d_2^*, \theta)) \\
&= \int_{\underline{y}}^{d_1^*} F(y, \theta) dy + F(d_2^*, \theta) (C_p - (d_2^* - d_1^*)) + F(K, \theta) (K - d_1^*) \\
& \quad + \int_K^{d_2^*} F(y, \theta) dy - C_p. \tag{19}
\end{aligned}$$

Since  $d_2^* = \min\{\bar{y}, d_1^* + C_p\}$ , we obtain that  $C_p - (d_2^* - d_1^*) \geq 0$ . Then  $d_1^* < K$  and the fact that  $F(y, \theta)$  decreases in  $\theta$  prove the result.  $\square$

**PROOF OF THEOREM 3.2:** Let  $(\Theta_b^*, \Theta_m^*, \{N_{\theta'}^*\}_{\theta' \in \Theta_m^*})$  be a sequential equilibrium with  $\mathbf{P}(\tilde{y}_{\tilde{\theta}} \in \cup_{\theta \in \Theta_m^*} N_\theta^* | \tilde{\theta} \in \Theta_m^*) > 0$ . First, we make some preliminary calculations. Since for all publicly financed firms  $\theta \in \Theta_m^*$  we have that  $N_\theta^* = N^* = [\underline{y}, d_1^*) \cup [K, d_2^*)$  with  $d_1^* = E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_\theta^*, \tilde{\theta} \in \Theta_m^*) < K$  and  $d_2^* = \min\{\bar{y}, d_1^* + C_p\}$ , it follows that

$$\begin{aligned}
& V_m(\theta, (\Theta_b^*, \Theta_m^*, \{N_{\theta'}^*\}_{\theta' \in \Theta_m^*})) \\
&= (1 - F(N_\theta^*, \theta)) (E(\tilde{y}_\theta | \tilde{y}_\theta \notin N_\theta^*) - C_m - C_p \mathbf{P}(\tilde{y}_\theta \geq K | \tilde{y}_\theta \notin N_\theta^*)) + \\
& \quad F(N_\theta^*, \theta) \left( E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_\theta^*, \tilde{\theta} \in \Theta_m^*) - C_m - C_p a \left( E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_\theta^*, \tilde{\theta} \in \Theta_m^*) \right) \right) \\
&= (1 - F(N^*, \theta)) (E(\tilde{y}_\theta | \tilde{y}_\theta \notin N^*) - C_m - C_p \mathbf{P}(\tilde{y}_\theta \geq K | \tilde{y}_\theta \notin N^*)) + \\
& \quad F(N^*, \theta) \left( E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N^*, \tilde{\theta} \in \Theta_m^*) - C_m - C_p a \left( E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N^*, \tilde{\theta} \in \Theta_m^*) \right) \right) \\
&= (1 - F(N^*, \theta)) \left( \frac{\int_{Y-N^*} y dF(y, \theta)}{1 - F(N^*, \theta)} - C_m - \frac{C_p \mathbf{P}(\tilde{y}_\theta \in [K, \bar{y}] \cap (Y - N^*))}{1 - F(N^*, \theta)} \right) +
\end{aligned}$$

$$\begin{aligned}
& F(N^*, \theta) (d_1^* - C_m - C_p a(d_1^*)) \\
= & \int_{Y-N^*} y dF(y, \theta) - (1 - F(N^*, \theta)) C_m - C_p \mathbf{P}(\tilde{y}_\theta > d_2^*) + F(N^*, \theta) (d_1^* - C_m) \\
= & \int_Y y dF(y, \theta) - \int_{N^*} y dF(y, \theta) - C_m - C_p (1 - F(d_2^*, \theta)) + F(N^*, \theta) d_1^* \\
= & E(\tilde{y}_\theta) - F(N^*, \theta) \frac{\int_{N^*} y dF(y, \theta)}{F(N^*, \theta)} - C_m - C_p (1 - F(d_2^*, \theta)) + F(N^*, \theta) d_1^* \\
= & E(\tilde{y}_\theta) - F(N^*, \theta) E(\tilde{y}_\theta | \tilde{y}_\theta \in N^*) - C_m - C_p (1 - F(d_2^*, \theta)) + F(N^*, \theta) d_1^* \\
= & E(\tilde{y}_\theta) - C_m - F(N^*, \theta) (d_1^* - E(\tilde{y}_\theta | \tilde{y}_\theta \in N^*)) - C_p (1 - F(d_2^*, \theta)), \tag{20}
\end{aligned}$$

where the second equality follows from Proposition 3.1. For all  $\theta \in \Theta_b^*$  it holds that  $V_b(\theta, (\Theta_b^*, \Theta_m^*, \{N_{\theta'}^*\}_{\theta' \in \Theta_m^*})) \geq V_m(\theta, (\Theta_b^*, \Theta_m^*, \{N_{\theta'}\}_{\theta' \in \Theta_m^*}))$ . By using (20), this is equivalent to

$$\begin{aligned}
& E(\tilde{y}_\theta) - C_b - C_p a(\beta_b(\Theta_b^*, \Theta_m^*, \{N_{\theta'}^*\}_{\theta' \in \Theta_m^*})) \geq \\
& E(\tilde{y}_\theta) - C_m + F(N^*, \theta) (d_1^* - E(\tilde{y}_\theta | \tilde{y}_\theta \in N^*)) - C_p (1 - F(d_2^*, \theta))
\end{aligned}$$

for all  $\theta \in \Theta_b^*$ . Rearranging terms yields

$$C_m - C_b - C_p a(\beta_b(\Theta_b^*, \Theta_m^*, \{N_{\theta'}^*\}_{\theta' \in \Theta_m^*})) \geq F(N^*, \theta) (d_1^* - E(\tilde{y}_\theta | \tilde{y}_\theta \in N^*)) - C_p (1 - F(d_2^*, \theta)) \tag{21}$$

for all  $\theta \in \Theta_b$ .

Now, let us start with proving claim (a). Recall that in a full public financing equilibrium, it holds that  $\mathbf{P}(\tilde{\theta} \in \Theta_b^*) > 0$ , so that the opponent's beliefs about firm value when observing private financing equal  $y_b \in Y$ . Expected firm value then equals  $E(\tilde{y}_\theta) - C_b - C_p a(y_b)$  for a firm of type  $\theta$ . From (21), it then follows that a full public financing equilibrium exists if and only if there exists  $y_b \in Y$  such that

$$F(N^*, \theta) (d_1^* - E(\tilde{y}_\theta | \tilde{y}_\theta \in N^*)) - C_p (1 - F(d_2^*, \theta)) \geq C_m - C_b - C_p a(y_b) \tag{22}$$

for all  $\theta \in \Theta$ . From Lemma 7.1 we know that the left hand side of the inequality is decreasing in  $\theta$ . Hence, expression (22) is satisfied for all  $\theta \in \Theta_m^*$  if

$$F(N^*, \bar{\theta}) (d_1^* - E(\tilde{y}_{\bar{\theta}} | \tilde{y}_{\bar{\theta}} \in N^*)) - C_p (1 - F(d_2^*, \bar{\theta})) \geq C_m - C_b - C_p a(y_b).$$

Since the right hand side is minimal for  $a(y_b) = 1$ , it follows that a full public financing equilibrium exists if and only if (9) and

$$F(N^*, \bar{\theta}) (d_1^* - E(\tilde{y}_{\bar{\theta}} | \tilde{y}_{\bar{\theta}} \in N^*)) + C_p F(d_2^*, \bar{\theta}) \geq C_m - C_b$$

are satisfied.

Next, we prove part (c). Since  $\mathbf{P}(\tilde{\theta} \in \Theta_b^*) > 0$  in a partial financing equilibrium, the opponent's beliefs about the expected firm value when observing private financing, equal  $E(\tilde{y}_{\tilde{\theta}} | \tilde{\theta} \in \Theta_b^*)$ . From (21) it then follows that  $\theta \in \Theta_b^*$  if

$$C_m - C_b - C_p a(E(\tilde{y}_{\tilde{\theta}} | \tilde{\theta} \in \Theta_b^*)) \geq F(N^*, \theta) (d_1^* - E(\tilde{y}_\theta | \tilde{y}_\theta \in N^*)) - C_p (1 - F(d_2^*, \theta)) \tag{23}$$

for all  $\theta \in \Theta_b$ . By Lemma 7.1 the right hand side of (23) decreases in  $\theta$ . Hence, private financing is preferred by all firms  $\theta \geq \theta_2^*$ , where  $\theta_2^* \in \Theta$  is such that

$$-C_p a(E(\tilde{y}_\theta | \tilde{\theta} \in \Theta_b^*)) + C_m - C_b = F(N^*, \theta_2^*) (d_1^* - E(\tilde{y}_{\theta_2^*} | \tilde{\theta}_{\theta_2^*} \in N^*)) - C_p (1 - F(d_2^*, \theta_2^*)).$$

Rearranging terms then gives that

$$F(N^*, \theta_2^*) (d_1^* - E(\tilde{y}_{\theta_2^*} | \tilde{\theta}_{\theta_2^*} \in N^*)) + C_p a(E(\tilde{y}_\theta | \tilde{\theta} \in \Theta_b^*)) - C_p (1 - F(d_2^*, \theta_2^*)) = C_m - C_b,$$

which completes the first part of the proof. For the second part, recall that in a mandatory disclosure environment,  $\theta \in \Theta_b^*$  if (15) is satisfied. Subgame perfection implies that in a partial disclosure equilibrium, the equilibrium disclosure strategy yields a higher payoff than full disclosure. Hence, the right hand side of (15) is less than or equal to the right hand side of (23). From this, it follows that  $\theta_2^* \geq \theta_1^*$ .

Next, let  $C_m - C_b < 0$ . By subgame perfection, the payoff from private financing exceeds the payoff from public financing with full disclosure for all privately financed firms. Hence, for all  $\theta \in \Theta_b^*$  it holds that

$$E(\tilde{y}_\theta) - C_b - C_p a(E(\tilde{y}_\theta | \tilde{\theta} \in \Theta_b^*)) \geq E(\tilde{y}_\theta) - C_m - C_p (1 - F(K, \theta)).$$

Rewriting yields

$$C_b - C_m \leq C_p (1 - F(K, \theta)) - C_p a(E(\tilde{y}_\theta | \tilde{\theta} \in \Theta_b^*)).$$

Since  $C_b - C_m > 0$ , this inequality can only be satisfied if  $a(E(\tilde{y}_\theta | \tilde{\theta} \in \Theta_b)) = 0$ , that is, if  $E(\tilde{y}_\theta | \tilde{\theta} \in \Theta_b^*) < K$ .

Finally, we prove part (b). In a full private financing it holds that  $\Theta_b^* = \Theta$ . This implies that private financing is preferred to public financing irrespective of the disclosure strategy. The expected firm value for a privately financed firm equals  $E(\tilde{y}_\theta) - C_b - C_p a(E(\tilde{y}_\theta | \tilde{\theta} \in \Theta))$  for all  $\theta \in \Theta$ . For determining the expected firm value of a publicly financed firm, we must first specify the opponent's and shareholder's beliefs when they observe nondisclosure of a publicly financed firm. Since  $\Theta_m^* = \emptyset$  these beliefs equal  $y_m \in Y$ . Given these beliefs, the optimal disclosure strategy for a publicly financed firm is to disclose its private information if and only if this disclosure yields a firm value higher than  $y_m$ , i.e.  $N = \{y \in Y | y - C_m - C_p a(y) \leq y_m - C_m - C_p a(y_m)\}$ . From Figure 7 it follows that  $N = [\underline{y}, d_1) \cup [K, d_2)$ , with

$$d_1 = \begin{cases} \underline{y} & \text{if } y_m - C_p a(y_m) \leq \underline{y}, \\ y_m - C_p a(y_m) & \text{if } \underline{y} < y_m - C_p a(y_m) \leq K, \\ K & \text{if } K < y_m - C_p a(y_m), \end{cases}$$

and

$$d_2 = \begin{cases} K & \text{if } y_m - C_p a(y_m) \leq K - C_p, \\ y_m - C_p a(y_m) + C_p & \text{if } K - C_p < y_m - C_p a(y_m) \leq \bar{y} - C_p, \\ \bar{y} & \text{if } \bar{y} - C_p < y_m - C_p a(y_m). \end{cases}$$

Then the expected firm value for a publicly financed firm  $\theta$  equals

$$(1 - F(N, \theta)) (E(\tilde{y}_\theta | \theta \notin N) - C_m - C_p \mathbf{P}(\tilde{y}_\theta \geq K | \tilde{y}_\theta \notin N)) + F(N, \theta) (y_m - C_m - C_p a(y_m)),$$

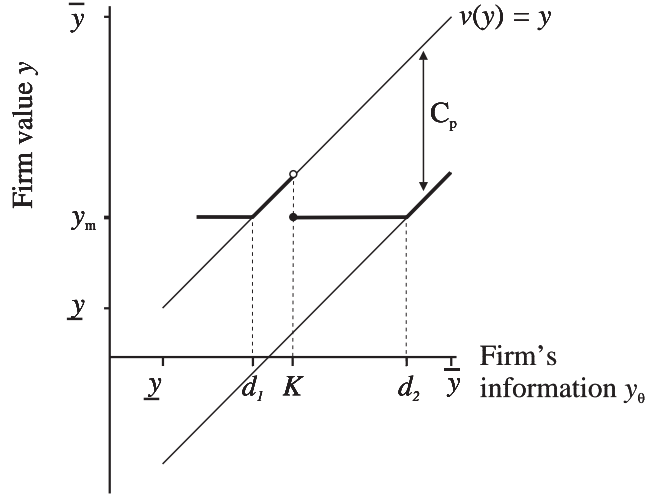


Figure 7: The optimal disclosure strategy at beliefs  $y_m \in Y$ .

which is minimal for  $y_m = \min\{\underline{y}, K - C_p\}$ . To see this, note that  $y_m = \min\{\underline{y}, K - C_p\}$  yields  $N = \emptyset$  and an expected firm value of  $E(\tilde{y}_\theta) - C_m - C_p(1 - F(K, \theta))$ . Then

$$\begin{aligned}
& 1 - F(N, \theta) (E(\tilde{y}_\theta | \theta \notin N) - C_m - C_p \mathbf{P}(\tilde{y}_\theta \geq K | \tilde{y}_\theta \notin N)) + F(N, \theta)(y_m - C_m - C_p a(y_m)) \\
& \quad - (E(\tilde{y}_\theta) - C_m - C_p(1 - F(K, \theta))) \\
& = \int_{Y-N} y dF(y, \theta) - C_m - C_p \mathbf{P}(\tilde{y}_\theta \geq d_2^*) + F(N, \theta)(y_m - C_p a(y_m)) \\
& \quad - E(\tilde{y}_\theta) + C_m + C_p(1 - F(K, \theta)) \\
& = - \int_N y dF(y, \theta) - C_p(1 - F(d_2^*, \theta)) + \int_N y_m - C_p a(y_m) dF(y, \theta) + C_p(1 - F(K, \theta)) \\
& = \int_N y_m - C_p a(y_m) - y dF(y, \theta) + C_p(F(d_2^*, \theta) - F(K, \theta)) \geq 0,
\end{aligned}$$

where the inequality follows from  $y_m - C_p a(y_m) \geq y$  for all  $y \in N$ . Thus, a full private financing equilibrium exists if and only if there exist beliefs  $y_m$  such that

$$\begin{aligned}
& E(\tilde{y}_\theta) - C_b - C_p a(E(\tilde{y}_{\tilde{\theta}} | \tilde{\theta} \in \Theta)) \geq (1 - F(N, \theta))E(\tilde{y}_\theta | \theta \notin N) - \\
& \quad (1 - F(N, \theta)) (C_m - C_p \mathbf{P}(\tilde{y}_\theta \geq K | \tilde{y}_\theta \notin N)) + F(N, \theta)(y_m - C_m - C_p a(y_m))
\end{aligned}$$

holds true for all  $\theta \in \Theta$ . Since the right-hand side of this inequality is minimal for  $y_m = \min\{\underline{y}, K - C_p\}$ , the expression above is equivalent to

$$E(\tilde{y}_\theta) - C_b - C_p a(E(\tilde{y}_{\tilde{\theta}} | \tilde{\theta} \in \Theta)) \geq E(\tilde{y}_\theta) - C_m - C_p(1 - F(K, \theta))$$

for all  $\theta \in \Theta$ . Rearranging terms yields

$$C_b - C_m \leq C_p(1 - F(K, \theta)) - C_p a(E(\tilde{y}_{\tilde{\theta}} | \tilde{\theta} \in \Theta))$$

for all  $\theta \in \Theta$ . The result then follows from  $F(K, \theta)$  is decreasing in  $\theta$ .  $\square$

PROOF OF PROPOSITION 4.1: Let  $(\Theta_b^*, \{N_\theta^*\}_{\theta \in \Theta_b^*}, \Theta_m^*)$  be a sequential equilibrium. Note that by subgame perfection we have for all  $\theta \in \Theta_b^*$  that

$$\begin{aligned} & \left\{ y \in Y \mid y - C_b - C_p a(y) < y - C_b - C_p a(\beta_b(\Theta_b^*, \{N_\theta^*\}_{\theta \in \Theta_b^*}, \Theta_m^*)) \right\} \subset N_\theta^* \\ & \subset \left\{ y \in Y \mid y - C_b - C_p a(y) \leq y - C_b - C_p a(\beta_b(\Theta_b^*, \{N_\theta^*\}_{\theta \in \Theta_b^*}, \Theta_m^*)) \right\}. \end{aligned}$$

If  $\beta_b(\Theta_b^*, \{N_\theta^*\}_{\theta \in \Theta_b^*}, \Theta_m^*) < K$ , then

$$\{y \in Y \mid a(y) > 0\} \subset N_\theta^* \subset \{y \in Y \mid a(y) \geq 0\}.$$

Hence,  $[K, \bar{y}] = \{y \in Y \mid a(y) = 1\} \subset N_\theta^*$ . If  $\beta_b(\Theta_b^*, \{N_\theta^*\}_{\theta \in \Theta_b^*}, \Theta_m^*) \geq K$  then

$$\{y \in Y \mid a(y) > 1\} \subset N_\theta^* \subset \{y \in Y \mid a(y) \geq 1\}.$$

Hence,  $N_\theta^* \subset \{y \in Y \mid a(y) \geq 1\} = [K, \bar{y}]$ . □

PROOF OF THEOREM 4.2: Let us start with some preliminary calculations. Given a strategy  $(\Theta_b, \{N_\theta\}_{\theta \in \Theta_b}, \Theta_m)$ , let  $N_\theta \subset Y$  be the optimal disclosure strategy with respect to the beliefs  $\beta_b(\Theta_b, \{N_\theta\}_{\theta \in \Theta_b}, \Theta_m)$ . Then firm  $\theta \in \Theta$  prefers private financing if

$$\begin{aligned} & E(\tilde{y}_\theta) - C_b - C_p (\mathbf{P}(\tilde{y}_\theta \in N^\beta) a(\beta) + \mathbf{P}(\tilde{y}_\theta \notin N^\beta) \mathbf{P}(\tilde{y}_\theta \geq K \mid \tilde{y}_\theta \notin N^\beta)) \geq \\ & E(\tilde{y}_\theta) - C_m - C_p (1 - F(K, \theta)). \end{aligned}$$

Rearranging terms then yields that a firm  $\theta \in \Theta$  prefers private financing if

$$C_b - C_m \leq C_p (1 - F(K, \theta)) - C_p (\mathbf{P}(\tilde{y}_\theta \in N^\beta) a(\beta) + \mathbf{P}(\tilde{y}_\theta \notin N^\beta) \mathbf{P}(\tilde{y}_\theta \geq K \mid \tilde{y}_\theta \notin N^\beta)).$$

If  $\beta_b(\Theta_b, \{N_\theta\}_{\theta \in \Theta_b}, \Theta_m) < K$ , i.e.  $a(\beta_b(\Theta_b, \{N_\theta\}_{\theta \in \Theta_b}, \Theta_m)) = 0$ , Proposition 4.1 implies that  $[K, \bar{y}] \subset N$ .

Hence,  $\mathbf{P}(\tilde{y}_\theta \geq K \mid \tilde{y}_\theta \notin N) = 0$ , so that firm  $\theta \in \Theta$  prefers private financing if

$$C_b - C_m \leq C_p (1 - F(K, \theta)). \tag{24}$$

If  $\beta_b(\Theta_b, \{N_\theta\}_{\theta \in \Theta_b}, \Theta_m) \geq K$ , i.e.  $a(\beta_b(\Theta_b, \{N_\theta\}_{\theta \in \Theta_b}, \Theta_m)) = 1$ , then Proposition 4.1 implies that  $N \subset [K, \bar{y}]$ , so that firm  $\theta \in \Theta$  prefers private financing if

$$\begin{aligned} C_b - C_m & \leq C_p (1 - F(K, \theta)) - C_p (\mathbf{P}(\tilde{y}_\theta \in N) + \mathbf{P}(\tilde{y}_\theta \geq K, \tilde{y}_\theta \notin N)) \\ & = C_p (1 - F(K, \theta)) - C_p (\mathbf{P}(\tilde{y}_\theta \in N, \tilde{y}_\theta \geq K) + \mathbf{P}(\tilde{y}_\theta \geq K, \tilde{y}_\theta \notin N)) \\ & = C_p (1 - F(K, \theta)) - C_p (\mathbf{P}(\tilde{y}_\theta \geq K)) \\ & = C_p (1 - F(K, \theta)) - C_p (1 - F(K, \theta)) = 0. \end{aligned} \tag{25}$$

Now, let us prove part (a). In a full public financing equilibrium it holds that  $\beta_b(\Theta_b^*, \{N_\theta^*\}_{\theta \in \Theta_b^*}, \Theta_m^*) = y_m \in Y$ . If  $y_m < K$ , a full public financing equilibrium exists, if (24) is violated for all  $\theta \in \Theta$ , i.e.  $C_b - C_m \geq C_p (1 - F(K, \theta))$  for all  $\theta \in \Theta$ . Since  $F(K, \theta)$  is decreasing in  $\theta$ , this is equivalent to

$$C_p (1 - F(K, \bar{\theta})) \geq \frac{C_b - C_m}{C_p}.$$



If  $y_m \geq K$ , a full public financing equilibrium exists, if (25) is violated for all  $\theta \in \Theta$ , i.e.  $C_b - C_m \geq 0$  for all  $\theta \in \Theta$ . Summarizing, a full public financing equilibrium exists if and only if  $C_b - C_m \geq \min\{C_p(1 - F(K, \bar{\theta})), 0\}$ , that is if and only if  $C_b \geq C_m$ .

Next, we prove part (b). In a full private financing equilibrium it holds that  $\beta_b(\Theta_b^*, \{N_\theta^*\}_{\theta \in \Theta_b^*}, \Theta_m^*) = E(\tilde{y}_\theta | \tilde{y}_\theta \in N_\theta^*, \tilde{\theta} \in \Theta_b^*)$ . If  $E(\tilde{y}_\theta | \tilde{y}_\theta \in N_\theta^*, \tilde{\theta} \in \Theta_b^*) < K$ , it follows from (24) that  $C_b - C_m \leq C_p(1 - F(K, \theta))$  for all  $\theta \in \Theta$ . Since  $F(K, \theta)$  is decreasing in  $\theta$ , this is equivalent to  $C_b - C_m \leq C_p(1 - F(K, \underline{\theta}))$ . Note that this condition is satisfied if  $C_b < C_m$ . If  $E(\tilde{y}_\theta | \tilde{y}_\theta \in N_\theta^*, \tilde{\theta} \in \Theta_b^*) \geq K$ , it follows from (25) that  $C_b - C_m \leq 0$  for all  $\theta \in \Theta$ . Hence, a full private financing equilibrium exists if  $C_b < C_m$ .

Finally, we prove part (c). Let  $(\Theta_b^*, \{N_\theta^*\}_{\theta \in \Theta_b^*}, \Theta_m^*)$  be a partial financing equilibrium such that  $\mathbf{P}(\tilde{y}_\theta \in \cup_{\theta \in \Theta_b^*} N_\theta^* | \tilde{\theta} \in \Theta_b^*) > 0$ . Since  $\beta_b(\Theta_b^*, \{N_\theta^*\}_{\theta \in \Theta_b^*}, \Theta_m^*) = E(\tilde{y}_\theta | \tilde{y}_\theta \in N_\theta^*, \tilde{\theta} \in \Theta_b^*)$ , we have for all  $\theta \in \Theta_b^*$  that

$$E(\tilde{y}_\theta) - C_b - C_p \left( \mathbf{P}(\tilde{y}_\theta \in N_\theta^*) a(E(\tilde{y}_\theta | \tilde{y}_\theta \in N_\theta^*, \tilde{\theta} \in \Theta_b^*)) + \mathbf{P}(\tilde{y}_\theta \notin N_\theta^*) \mathbf{P}(\tilde{y}_\theta \geq K | \tilde{y}_\theta \notin N_\theta^*) \right) \geq E(\tilde{y}_\theta) - C_m - C_p(1 - F(K, \theta)).$$

Rearranging terms yields

$$C_p(1 - F(K, \theta)) - C_p \left( \mathbf{P}(\tilde{y}_\theta \in N_\theta^*) a \left( E(\tilde{y}_\theta | \tilde{y}_\theta \in N_\theta^*, \tilde{\theta} \in \Theta_b^*) \right) + \mathbf{P}(\tilde{y}_\theta \geq K, \tilde{y}_\theta \notin N_\theta^*) \right) \geq C_b - C_m$$

for all  $\theta \in \Theta_b^*$ .

If  $E(\tilde{y}_\theta | \tilde{y}_\theta \in N_\theta^*, \tilde{\theta} \in \Theta_b^*) < K$  then (24) implies that  $\theta \in \Theta_b^*$  if and only if  $C_b - C_m \leq C_p(1 - F(K, \theta))$ . From the fact that  $F(K, \theta)$  is decreasing in  $\theta$ , it follows that  $\theta \in \Theta_b^*$  if and only if  $\theta \geq \theta_3^*$ , where  $\theta_3^* \in \Theta$  is such that  $C_p(1 - F(K, \theta_3^*)) = C_b - C_m$ . Furthermore, from (16) with  $E(\tilde{y}_\theta | \tilde{\theta} \in \Theta_b^*) < K$  it follows that  $\theta_3^* = \theta_1^*$ .

If  $E(\tilde{y}_\theta | \tilde{y}_\theta \in N_\theta^*, \tilde{\theta} \in \Theta_b^*) \geq K$  then (25) implies that  $\theta \in \Theta_b^*$  if  $C_b - C_m \leq 0$ . Since this inequality does not depend on  $\theta$ , we obtain that  $\Theta_b^* = \Theta$  if  $C_b - C_m < 0$  and  $\Theta_b^* = \emptyset$  if  $C_b - C_m > 0$ . So, no partial financing equilibrium exists.  $\square$

**PROOF OF THEOREM 4.3:** Let us start with proving part (a). In a full public financing equilibrium we have that  $\beta_b(\Theta_b^*, \Theta_m^*, \{N_\theta^*\}_{\theta \in \Theta}) = y_b \in Y$ . Let  $N \subset Y$  denote the optimal disclosure strategy given the beliefs  $y_b$ . Since  $\Theta_m = \Theta$  it holds that  $V_m(\theta, \Theta_m, \{N_{\theta'}\}_{\theta' \in \Theta_m}) \geq V_b(\theta, \Theta_b, \{N_{\theta'}\}_{\theta' \in \Theta_b})$  for all  $\theta \in \Theta$ , that is,

$$E(\tilde{y}_\theta) - C_m + F(N^*, \theta)(d_1^* - E(\tilde{y}_\theta | \tilde{y}_\theta \in N^*)) - C_p(1 - F(d_2^*, \theta)) \geq E(\tilde{y}_\theta) - C_b - C_p \left( \mathbf{P}(\tilde{y}_\theta \in N^\beta) a(\beta) + \mathbf{P}(\tilde{y}_\theta \notin N^\beta) \mathbf{P}(\tilde{y}_\theta \geq K | \tilde{y}_\theta \notin N^\beta) \right),$$

for all  $\theta \in \Theta$ . Since the expected payoff for a privately financed firm is minimal when  $\beta \geq K$ , that is,  $a(\beta) = 1$ , Proposition 4.1 implies that  $N^\beta \subset [K, \bar{y}]$ . Similar to (24) we derive for all  $\theta \in \Theta$  that

$$F(N^*, \theta)(d_1^* - E(\tilde{y}_\theta | \tilde{y}_\theta \in N^*)) - C_p(1 - F(d_2^*, \theta)) + C_p(1 - F(K, \theta)) \geq C_m - C_b. \quad (26)$$

Subgame perfection implies that the equilibrium disclosure strategy  $N^*$  yields a higher payoff than full disclosure, i.e.

$$F(N^*, \theta)(d_1^* - E(\tilde{y}_\theta | \tilde{y}_\theta \in N^*)) - C_p(1 - F(d_2^*, \theta)) \geq -C_p(1 - F(K, \theta)),$$

for all  $\theta \in \Theta$ . Hence,

$$F(N^*, \theta)(d_1^* - E(\tilde{y}_\theta | \tilde{y}_\theta \in N^*)) - C_p(1 - F(d_2^*, \theta)) + C_p(1 - F(K, \theta)) \geq 0, \quad (27)$$

so that a full public financing equilibrium always exists if  $C_m - C_b \leq 0$  (cf. (26)).

Next, we prove part (b). In a full private financing equilibrium we have that  $\beta_m(\Theta_b^*, \Theta_m^*, \{N_\theta^*\}_{\theta \in \Theta}) = y_m \in Y$ . Let  $N = [\underline{y}, d_1] \cup [K, d_2] \subset Y$  denote the optimal disclosure strategy given the beliefs  $y_m$ . Since  $\Theta_b = \Theta$  it holds that  $V_b(\theta, \Theta_b, \{N_{\theta'}\}_{\theta' \in \Theta_b}) \geq V_m(\theta, \Theta_m, \{N_{\theta'}\}_{\theta' \in \Theta_m})$  for all  $\theta \in \Theta$ , that is,

$$\begin{aligned} E(\tilde{y}_\theta) - C_b - C_p \left( \mathbf{P}(\tilde{y}_\theta \in N_\theta) a \left( E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}, \tilde{\theta} \in \Theta_b) \right) + \mathbf{P}(\tilde{y}_\theta \notin N_\theta) \mathbf{P}(\tilde{y}_\theta \geq K | \tilde{y}_\theta \notin N_\theta) \right) \\ \geq E(\tilde{y}_\theta) - C_m + F(N, \theta)(d_1 - E(\tilde{y}_\theta | \tilde{y}_\theta \in N)) - C_p(1 - F(d_2, \theta)). \end{aligned}$$

The expected payoff for a publicly financed firm is minimal when the beliefs  $y_m$  induce full disclosure, which is the case when beliefs are skeptical, that is  $y_m = \min\{\underline{y}, K - C_p\}$ . Since publicly financed firms fully disclose their private information, the proof continues in the same way as the proof of Theorem 4.2(b).

In order to prove part (c), let  $(\Theta_b^*, \Theta_m^*, \{N_\theta^*\}_{\theta \in \Theta})$  be a partial financing equilibrium such that  $\mathbf{P}(\tilde{y}_{\tilde{\theta}} \in \cup_{\theta \in \Theta_b^*} N_\theta^* | \tilde{\theta} \in \Theta_b^*) > 0$  and  $\mathbf{P}(\tilde{y}_{\tilde{\theta}} \in \cup_{\theta \in \Theta_m^*} N_\theta^* | \tilde{\theta} \in \Theta_m^*) > 0$ . Then  $\theta \in \Theta_b^*$  if

$$\begin{aligned} E(\tilde{y}_\theta) - C_b - C_p \left( \mathbf{P}(\tilde{y}_\theta \in N_\theta^*) a \left( E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*, \tilde{\theta} \in \Theta_b^*) \right) + \mathbf{P}(\tilde{y}_\theta \notin N_\theta^*) \mathbf{P}(\tilde{y}_\theta \geq K | \tilde{y}_\theta \notin N_\theta^*) \right) \\ \geq E(\tilde{y}_\theta) - C_m + F(N^*, \theta)(d_1^* - E(\tilde{y}_\theta | \tilde{y}_\theta \in N^*)) - C_p(1 - F(d_2^*, \theta)). \end{aligned}$$

Rearranging terms yields

$$\begin{aligned} F(N^*, \theta)(d_1^* - E(\tilde{y}_\theta | \tilde{y}_\theta \in N^*)) - C_p(1 - F(d_2^*, \theta)) \\ - C_p \left( \mathbf{P}(\tilde{y}_\theta \in N_\theta^*) a \left( E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*, \tilde{\theta} \in \Theta_b^*) \right) + \mathbf{P}(\tilde{y}_\theta \notin N_\theta^*) \mathbf{P}(\tilde{y}_\theta \geq K | \tilde{y}_\theta \notin N_\theta^*) \right) \leq C_m - C_b, \end{aligned}$$

for all  $\theta \in \Theta_b^*$ .

If  $E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*, \tilde{\theta} \in \Theta_b^*) < K$  then Proposition 4.1 states that  $[K, \bar{y}] \subset N_\theta^*$ . Hence,  $\theta \in \Theta_b^*$  if

$$F(N^*, \theta)(d_1^* - E(\tilde{y}_\theta | \tilde{y}_\theta \in N^*)) - C_p(1 - F(d_2^*, \theta)) \leq C_m - C_b.$$

Since the left hand side is decreasing in  $\theta$  by Lemma 7.1, it follows that  $\Theta_b^* = [\theta_4^*, \bar{\theta}]$ , where  $\theta_4^* \in \Theta$  is such that

$$F(N^*, \theta_4^*)(d_1^* - E(\tilde{y}_{\theta_4^*} | \tilde{y}_{\theta_4^*} \in N^*)) - C_p(1 - F(d_2^*, \theta_4^*)) = C_m - C_b.$$

If  $E(\tilde{y}_{\tilde{\theta}} | \tilde{y}_{\tilde{\theta}} \in N_{\tilde{\theta}}^*, \tilde{\theta} \in \Theta_b^*) \geq K$  then Proposition 4.1 states that  $N_\theta^* \subset [K, \bar{y}]$ . Hence,  $\theta \in \Theta_b^*$  if

$$F(N^*, \theta)(d_1^* - E(\tilde{y}_\theta | \tilde{y}_\theta \in N^*)) - C_p(1 - F(d_2^*, \theta)) + C_p(1 - F(K, \theta)) \leq C_m - C_b,$$

Since the left hand side is greater than or equal to zero by (27) it must hold that  $C_m - C_b > 0$ .  $\square$

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