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# LEARNING STRATEGIC SOPHISTICATION 

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#### Abstract

We experimentally investigate coordination games in which cognition plays an important role, i.e. where outcomes are affected by the agents' level of understanding of the game and the beliefs they form about each others' understanding. We ask whether and when repeated exposure permits agents to learn to improve cognition in a strategic setting. We find evidence for strategic sophistication being learned, generalized and promoted. Agents acquire strategic sophistication in simple settings. They may fail to do so in similar but more demanding settings. Given the opportunity, they transfer learning from the simple to the more demanding task. There is heterogeneity in sophistication. We find some evidence for sophisticated agents trying to spread sophistication early in the game, provided there is a long enough time horizon.


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# Learning Strategic Sophistication* 

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#### Abstract

We experimentally investigate coordination games in which cognition plays an important role, i.e. where outcomes are affected by the agents' level of understanding of the game and the beliefs they form about each others' understanding. We ask whether and when repeated exposure permits agents to learn to improve cognition in a strategic setting. We find evidence for strategic sophistication being learned, generalized and promoted. Agents acquire strategic sophistication in simple settings. They may fail to do so in similar but more demanding settings. Given the opportunity, they transfer learning from the simple to the more demanding task. There is heterogeneity in sophistication. We find some evidence for sophisticated agents trying to spread sophistication early in the game, provided there is a long enough time horizon.


[^0]
## 1 Introduction

Few would disagree that there are differences in cognition. Individuals differ in their spatial perception, their ability to perform arithmetic operations, to decide on logical truths, to verbalize emotions, to name just a few. One would suspect, and we think this is confirmed by casual observation, that differences in cognition matter for strategic interactions. For example, one's willingness to participate in a game of chess, an auction, or to be part of a contract, or other institution may be influenced by one's understanding of the working of the institution. And yet, the bulk of theory pays no attention to cognitive differences.

We experimentally study games with a spatial structure in which agents have common interests and lack of cognition is detrimental to coordination. In our setting a player's cognitive skill is essentially his ability to identify potential coordination opportunities. Others have studied differences in cognition in games where interests are not necessarily perfectly aligned. Prominent examples of experimental papers on this topic are Nagel's [1995] work on players' ability to reason through interative dominance in the guessing game, and Stahl and Wilson's [1995] and Costa-Gomes, Crawford and Broseta's [2001] work on players' varying cognitive abilities in dominance solvable games and games with unique equilibria. In a recent theoretical paper Crawford [2003] has studied deception in communication games in which fully rational players interact with boundedly rational ones. ${ }^{1}$

Focusing on common interest pure coordination games allows us to consider the possibility that players may be unaware of all possible coordination opportunities. Unawareness of this sort requires more than simple lack of knowledge. In addition to not knowing a specific coordination opportunity, the agent does not know that he does not know the opportunity exists, i.e. he must lack negative introspection. Unawareness seems commonplace in everyday life, and yet has only recently attracted attention in the literature. One likely reason is that unawareness does not easily fit into conventional models of information economics. Violations of negative introspection are not compatible with the standard partitional state space model of knowledge, Aumann [1976], as pointed out by Geanakoplos [1992]. More recently, Dekel, Lipman and Rustichini [1998] have demonstrated that any standard state space model precludes unawareness. They suggest that one way to avoid this conundrum is

[^1]to make a distinction between the agent's and the analyst's description of the state space, and to treat the state space as representing the agent's view of possibilities." Recently, there have been proposals of models of knowledge that permit unawareness, e.g. Li [2003] and Schipper [2002].

We accept unawareness as a simple empirical phenomenon and ask what happens when agents differ in their awareness in a simple strategic setting, i.e. when there is interactive unawareness. Common-interest games are attractive for this purpose because they help us focus on the central issue of how unawareness affects players strategic reasoning about others. We need not worry for example about how differential awareness interacts with players' ability to (iteratively) eliminate dominated strategies, signaling motives, bargaining motives, deception, or other-regarding preferences. Games with a spatial structure are appealing because agents may differ in how much of this structure and its possible uses they perceive.

Since improved cognition may improve coordination, agents may also benefit from experience. Therefore, in this paper we investigate the role of experience for cognition and coordination. Interestingly, the causal links are not straightforward. In a strategic setting experience is endogenous. Initial cognition, through its effect on initial coordination, may influence the scope for subsequent changes in cognition. Furthermore there may be cognitive learning, which involves altered perceptions about the structure of the game that manifest themselves in the form of an aha-experience. ${ }^{2}$ With a cognitively heterogeneous population, sophisticated players have to discover and adjust to the cognition of others, while less sophisticated players may observe behaviors they cannot rationalize from their frame of reference.

Our most important experimental findings are: There is evidence for strategic sophistication being learned, generalized and promoted. Agents acquire strategic sophistication in simple settings. They may fail to do so in similar but more demanding settings. Given the opportunity, they transfer learning from the simple to the more demanding task. There is heterogeneity in sophistication. Sophisticated agents try to spread sophistication early in the game, provided there is a long enough time horizon.

[^2]The paper is organized as follows. In the next section we describe the two-round stage game and the corresponding repeated game. In Section 3, we describe the design for our experiment. Section 4 presents our experimental results. Section 5 concludes.

## 2 The Game

We study repeated coordination games in which players are randomly paired each period. The stage game played in each period consists of two rounds as indicated in the time line of Figure 1.


## Figure 1

In the first round of the stage game two players simultaneously and independently choose one of $n$ identical sectors of a disk, where $n$ equals either three, as in Figure 2, or five, as in Figure 3. ${ }^{3}$ In the second round, after observing first-round choices, but without being able to distinguish one's own from one's partner's choice, both players choose again. In both rounds, payoffs are one if both players choose identical sectors and zero otherwise.

## Figure 2 and 3

A benchmark solution for the two-round stage game is provided by Crawford and Haller's [1990] approach to games in which players have common knowledge but lack a common language to describe their actions and positions. ${ }^{4}$ They formalize this lack of a common language by symmetry restrictions on players' strategies. According to them players must treat actions and player positions symmetrically, as long as they have not been distinguished by history. They propose to study optimal attainable strategies. An attainable strategy respects at every point in time all the symmetries that remain in the game. An optimal

[^3]attainable strategy is optimal within the class of attainable strategies. For any odd $n$, there is a unique optimal attainable strategy for the two-round stage game: By virtue of initial choices being identical, players are forced to randomize uniformly in the first round. In the second round, there is exactly one feasible strategy that if adopted by both players guarantees coordination: If players happen to coordinate in the first round, it is uniquely optimal to choose the coordinating sector in the second round. If players do not coordinate in the first round, note that each chosen sector will be separated from the other chosen sector by either no sectors or an even number of sectors in one direction, and by an odd number of sectors in the other direction, as in Figure 4 for the three-sector disk and Figure 5 and 6 for the five-sector disk where the previously chosen sectors are the shaded sectors on the disk. It is uniquely optimal to pick the middle sector among the odd number of sectors that separate the first-round choices.

Figure 4, 5 and 6

Cognitive issues may arise because the unique identified sector may be easier to spot for some individuals and some configurations than for others. Intuitively, it may seem obvious to return to coordination if by chance coordination was achieved in the first round. In contrast, if initial choices are not coordinated, the induced unique distinct sector may be less conspicuous.

Whether cognition is in fact an issue is an empirical matter that we address through our experiment. The cognitive issue in the three-sector game is recognizing that in the event of no first-round coordination, as in Figure 4, it is uniquely optimal, given the indistinguishability of the first round choices, to continue with the unchosen sector in the second round. With five sectors, three configurations are possible after the first round: Initial coordination, failure to coordinate with the initial choices separated, as in Figure 5, or failure to coordinate with two adjacent choices, as in Figure 6. In each of the three cases, there is a unique optimal way to continue in the second round provided that the structure of the game is common knowledge.

If there are differences in cognition, and therefore agents lack common knowledge of the game, they may either be attracted by the prominence of the chosen sectors, recognize the distinction between chosen and unchosen sectors, or fully appreciate the desymmetrizing
effect of first-round choices. These three perceptions correspond to a nested set of structures on an expanded universe of choices. Think of the choices presented to the players, the disk, as a subset of a larger set. A player, who does not appreciate that the choices at hand are singled out in the larger set, will look at a different complement of the set of choices made in the first round than a player who does focus on the set at hand. In the case of three sectors and no coordination in the first round, he will not view the complement as a single choice but as a potentially large set of unmade choices. He may then very well be attracted by the prominence of the sectors chosen in the first round. In contrast, a player who does focus on the disk will be able to distinguish between sectors that were chosen and sectors that were not chosen in the first round. In the case of three sectors and no coordination in the first round, he would be able to realize that there is a unique unchosen sector. In the five-sector case we have the additional issue that players may or may not make use of the circular structure of the disk. If there is no coordination in the first round, a player who only uses the fact that there are five sectors, may be attracted by the fact that there are fewer chosen sectors, two, than unchosen sectors, three. A more sophisticated player would also use the fact that the five sectors are arranged in a circular order to infer that there is a unique sector that does not belong to a symmetric pair. This sector is the midpoint of the odd number of sectors separating the two chosen sectors.

We refer to these structures as different languages in which agents describe the game to themselves, and in which they think about how others think about the game. We will say that players use plain, coarse or fine languages to describe the game. A plain-language player is attracted by chosen sectors, a coarse-language player distinguishes chosen and unchosen sectors, and a fine-language player uses the circular structure to identify in addition a unique sector that does not belong to a symmetric pair, Bacharach [1993] and Blume and Gneezy [2002]. ${ }^{5}$

We assume that when forming beliefs about others, players consider only languages no finer than their own. According to this no-refinement assumption of Bacharach [1993], a plain-language player assigns probability one to other players having a plain language, a coarse-language player only ascribes positive probabilities to his partner having either a

[^4]coarse or a plain language, and a fine-language player assigns positive probability to all three languages.

In the three-sector game, there is no distinction between coarse and fine-language players. In the case of no coordination, both recognize the two prior choices and the single unchosen sector. Therefore in the three-sector game we only refer to plain- and fine-language players. A plain-language player perceives all other players as having a plain language and being attracted to the prior choices, which only reinforces his prior attraction. Fine-language players assess the proportion of plain- and fine-language players in the population. If the proportion of fine-language players is sufficiently high, the unique optimal way of playing the three-sector game is for them to choose the unique distinct sector. However, if the proportion is sufficiently low, it is uniquely optimal for the fine-language players to conform with plain-language behavior.

In the five-sector game there is no distinction between plain- and coarse-language players. In the case of no coordination, both discriminate only between the two prior choices and the three unchosen sectors. For both it is always uniquely optimal to select one of the prior choices in round two. Therefore in the five-sector game we refer only to fine- and coarselanguage players. Fine-language players assess the proportion of coarse-language players in the population. If this proportion is low, it is uniquely optimal for them to select the unique distinct sector. Otherwise, they conform with coarse-language behavior and select one of the prior choices. In the five-sector game, we may also want to allow for the possibility that the configuration of prior choices, "separated" or "adjacent," may affect whether a given individual has a fine or a coarse language.

In the experiment, agents are repeatedly randomly matched in a fixed population to play the two-round game. If the structure of the game were common knowledge, the unique optimal strategy in the two-round game would induce a unique optimal strategy in the repeated game. If there are cognitive differences, and therefore players lack common knowledge of the game, behaviors depend on beliefs which may be affected by experience. To appreciate the intertemporal considerations, it suffices to consider a two-period version of our games.

Consider the three-sector game. Say that beliefs are such that fine-language players pick the unique distinct sector. Then, if there is miscoordination in round two of the first period, fine-language players revise their prior beliefs on the proportion of fine-language players
downward. Plain-language players on the other hand face an unanticipated event. They have no beliefs to update and not even a framework in which to rationalize their partner's choice. They may be prompted to reexamine their language.

In the five-sector game, coarse-language players may also find themselves surprised, which may prompt a reassessment of their language. Conversely, fine-language players update their beliefs in a conventional way and decide whether to pursue coordination on the unique distinct sector or to adjust to conform with coarse-language behavior.

## 3 Design

The experiment was conducted with twelve cohorts. A cohort consisted of twelve participants. All participants were recruited from undergraduate (sophomore and above) and graduate classes at the University of Iowa. None of the participants had previously taken part in or otherwise gained experience with this series of experiments. Upon arrival, participants were seated at separate computer terminals and given a copy of the instructions for the experiment. ${ }^{6}$ Since these instructions were read aloud, we assume that the information contained in them was mutual knowledge.

Each cohort played a repeated coordination game. Three each of these cohorts played ten periods of the three-sector game; ten periods of the five-sector game; twenty periods of the five-sector game; and, ten periods of the three-sector game followed by ten periods of the five-sector game.

Each period had the following structure. First, participants were paired using a randommatching procedure. Second, participants played a two-round game. In the first round participants chose a sector from a symmetric disk with $n$ identical sectors with $n$ odd. At the beginning of the first round the disks were randomly rotated, independently across participants to eliminate all possibilities for a priori coordination, before the disks appeared on participants computer screens. Participants then made their choices by using a mouse to click on their chosen sector. They were given an opportunity to either revise or confirm their choices. At the end of the round, when all participants had made and confirmed their choices, their respective disks disappeared from their computer screens. The disks re-

[^5]appeared, and participants were informed which sectors were chosen in their pair (without being told which was their own or their partner's choice). Participants were also informed whether they coordinated as a pair on a single sector or failed to coordinate. At the end of the first round the disks disappeared from participants computer screens.

At the beginning of the second round, the disks were randomly rotated before re-appearing on participants screens. Participants could then observe the previous round's choices, highlighted in gray, without being told which was their own or their partner's choice. See for example Figure 7 where the two players paired together did not coordinate and where the disks with the first-round choices have been randomly spun and presented to the two players, as in Figure 7A and Figure 7b respectively for the three-sector disk, at the beginning of round two. Participants then chose again. When all participants had made and confirmed their choices, their respective disks disappeared from their screens. The disks re-appeared, and participants were informed which sectors were chosen in their pair (without being told which was their own or their partner's choice). They saw the choices made in round two, marked by red dots, on the background of the choices made in the previous round, marked by the gray shaded sectors. Participants were also informed whether they coordinated as a pair on a single sector or failed to coordinate. At the end of the second round the disks disappeared from participants computer screens.

## Figure 7

Before each game, we read the instructions aloud to each cohort, and participants individually filled out questionnaires confirming their knowledge and understanding of the instructions. We then went over the questionnaire orally and answered questions. Each replication lasted from one-half to one hour. Participants earnings ranged from $\$ 3$ to $\$ 21$ plus a "show up" payment of $\$ 5$.

## 4 Results and Analysis

We divide the description of the results and analysis in three parts. The first section presents the results on the proportion of distinct sector choices by treatment and their implications. The second section presents some insights into the dynamics of the choices made in each of the treatments. The final section addresses the issue of teaching by sophisticated players.

### 4.1 Proportion of Distinct-Sector Choices

Figure 8 presents the proportion of distinct-sector choices by treatment and over time. The treatments are the three-sector ten-period game, the five-sector ten-period game, the five-sector twenty-period game, and the treatment consisting of the three-sector ten-period game followed by the five-sector ten-period game. The results for the three-sector ten-period treatment are aggregated over six cohorts and other treatments' results are aggregated over three cohorts each. ${ }^{7}$

Denote the set of individuals who did not coordinate in the first round of period $t$ by $\mathcal{N}_{t}$. Denote the set of individuals who chose the distinct sector in the second round of period $t$ by $\mathcal{D}_{t}$. For any set $\mathcal{S}$ let $\#(\mathcal{S})$ stand for the cardinality of that set. Then the figure shows for each of the periods over which the treatment was run the ratio

$$
\sigma_{t}:=\frac{\#\left(\mathcal{N}_{t} \cap \mathcal{D}_{t}\right)}{\#\left(\mathcal{N}_{t}\right)}
$$

This is the fraction of individuals who chose the distinct sector in the second round conditional on not having been coordinated in the first round. It is a measure of the proportion of agents who engage in optimal learning in period $t$ when such learning is not obvious.

## Figure 8

For the three-sector ten-period treatment, four observations emerge from Figure 8. First, the initial value, $\sigma_{1}=0.55$, is close to the initial value in the one-period pencil-andpaper experiments reported in Blume and Gneezy [2000]. This confirms and strengthens those results and gives us some assurance that the computerized version of the experiment reproduces the natural features of the pencil-and-paper version. Second, there is learning, indicated by the fact that the last-period frequency $\sigma_{10}(=0.88)$ exceeds $\sigma_{1}(\mathrm{p}$-value $=.03)$. Third, the learning curve is quite flat. A simple regression of $\sigma_{t}$ on $t$ has a slope coefficient of 0.04 with a p-value $=0.00$. Fourth, learning is incomplete, indicated by the fact that the fraction of individuals engaging in optimal learning never reaches one.

The results from our five-sector ten-period treatment are considered next. As before, $\sigma_{t}$ measures the fraction of individuals who chose the distinct sector in the second round

[^6]conditional on not having been coordinated in the first round. We do not distinguish between adjacent and separated noncoordinated choices in the first round; it is clear from the graph that for this particular treatment this distinction is irrelevant.

Figure 8 shows that behavior in the five-sector ten-period treatment contrasts sharply with that in the three-sector ten-period treatment. The fraction of distinct sector choices in the first round is significantly lower $\left(\sigma_{1}=0.17\right)$. There is no tendency to learn the optimal strategy. The fraction of distinct sector choices remains consistently low ( $\sigma_{10}=0.09$ and the difference between $\sigma_{1}$ and $\sigma_{10}$ is not significant, $p=0.88$ ).

For the five-sector twenty-period treatment, the central message from Figure 8 is that increasing the number of periods promotes optimal play throughout the game. Remarkably, the fraction of optimal play is higher during the first ten periods of the five-sector twentyperiod treatment than during the ten periods of the five-sector ten-period treatment. Using a one-tailed Fisher Exact test, the hypothesis of no difference is rejected, $p=0.000$. This is the case both when the current configuration is "adjacent" and when it is "separated." ${ }^{8}$ The difference appears stable with no obvious signs of deterioration.

The second message is that the learning curve is quite flat. While the average frequency of optimal play is slightly higher over the last ten periods ( $\bar{\sigma}_{11-20}=0.41$ ) than over the first ten periods $\left(\bar{\sigma}_{1-10}=0.32\right)$, the frequency never exceeds $50 \%$. Any difference between the first ten periods of the five-sector twenty-period treatment and the last ten periods appears to be confined to the set of observations in which the current configuration was "separated." For the "separated" configuration we reject the hypothesis of no difference, $p=0.002$, whereas for the "adjacent" configuration we are unable to reject the hypothesis of no difference, $p=0.557$.

Finally, Figure 8 displays the distinct sector proportion for our last treatment, "fivesector ten-period preceded by three-sector ten-period." Here, the central message is that having the three-sector treatment precede the five-sector treatment is more effective in promoting optimal play than extending the length of the game. The level of optimal play in the last ten periods of this treatment dominates the level of optimal play in any period of the other two five-sector treatments. It appears to be easier for participants to generalize from

[^7]the cognitively simpler task (three sector) to the more demanding task (five sector) than to be exposed longer to the cognitively more demanding task. The benefit from learning in the three-sector game derives from the increase in optimal play under the "adjacent" sector condition. We can reject the hypothesis of no difference between the last ten periods of the five-sector twenty-period treatment and the last ten periods of "five-sector ten-period preceded by three-sector ten-period" treatment when the current configuration is "adjacent," $p=0.000$, but fail to reject when the current configuration is "separated," $p=0.224$.

To summarize, we observe significant cognitive issues and an absence of learning in the five-sector ten-period treatment. When the treatment is lengthened to twenty periods, coordination improves, particularly for the "separating" configuration, but learning is still relatively flat. Using the experience from the ten-period three-sector game, agents are able to generalize to the more complex five-sector game, particularly for the "adjacent" configuration, thereby raising the proportion of distinct-sector play. However, the proportion of distinct-sector play and learning are still less than in the three-sector ten-period game.

### 4.2 Dynamics

To gain some insight into the dynamics of the choices, we start with the three-sector tenperiod treatment. Table I reports, pooled across replications, the number of own choices of the distinct sector (D) and of a prior chosen sector $(\mathrm{P})$ in period $t$ as a function of period $t-1$ 's own choice and the choice of one's opponent.

We exclude all periods in which agents matched in the first round of a period. Thus, Period $t$ represents the period in which the agent did not match in the first round of that period; Period $t-1$ represents the last time prior to the present period that the agent did not match in the first round. DD indicates that an agent who was not matched in the first round of period $t-1$, chose the distinct sector and his opponent chose the distinct sector. DP indicates that an agent who was not matched in the first round of period $t-1$, chose the distinct sector and his opponent picked a prior choice. PD indicated the opposite and PP indicates both agents in the last period chose a prior choice and matched $(=1)$ or did not match $(=0).)^{9}$

To illustrate how to read the table, consider the two cells $\mathrm{D} / \mathrm{DD}$ and $\mathrm{P} / \mathrm{DD}$. There were

[^8]112 ( $=112 \mathrm{DD}$ ) prior period choices in which the participant chose D and the player $\mathrm{s} / \mathrm{he}$ was matched with chose D. In the current period, these same participants chose the distinct sector D 101 times (cell D/DD) and a prior chosen sector P 11 times (cell P/DD).

TABLE I
History's Effect on Choices
Three Sector - Ten Period Treatment

| Period $t-1:$ | DD | DP | PD | PP1 | PP0 | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Period $t:$ |  |  |  |  |  |  |
| D | 101 | 40 | 28 | 2 | 0 | 171 |
| P | 11 | 7 | 21 | 4 | 4 | 47 |
| Total | 112 | 47 | 49 | 6 | 4 | 218 |

Table I shows that the predominant choice was $D$, and $D$ choices were persistent even if there was no positive payoff reinforcement. Forty participants out of 47 ( $=47 \mathrm{DP}$ ) who unsuccessfully chose $D(85 \%)$ chose $D$ again at the next opportunity. $P$ choices were rarely successful, only six times. In the overwhelming number of cases $P$ choices were met with $D$ choices and this led to zero payoffs. Nevertheless, 21 participants out of 49 who chose $P$ and met $D(43 \%)$ continued to choose $P$ in the next period. Thus, there was a fair amount of persistence in $P$ choices, regardless of payoff experience, which accounts for the fact that the convergence to distinct-sector play remains incomplete in the three-sector ten-period treatment.

To better understand the dynamics of choices in the five-sector ten-period treatment, we report in TABLE II the number of current period choices in a similar way to that in Table I, the distinct sector $(\mathrm{D})$, the prior sector $(\mathrm{P})$ and new to the five-sector game, the sector that is neither prior nor distinct (N). Since the current period's configuration can be either adjacent or separated in the five-sector game, we augment the current period's choice by whether the current configuration is "adjacent" (A) or "separated" (S).

Similar to TABLE I, last period's choices are organized by last period's own choice (D, $\mathrm{P}, \mathrm{N})$, the choice of the matched opponent ( $\mathrm{D}, \mathrm{P}, \mathrm{N}$ ), and whether it was a success (1) or
failure (0). New to the five-sector game, we present last period's choices as a function of last period's configuration (Separated ( $\mathrm{t}-1$ ) or Adjacent ( $\mathrm{t}-1$ )). For example the entry in the cell PA/PP0 under the heading "Separated "(21) indicates the number of instances in which a participant chose a prior choice ( P ) when the current configuration was adjacent (A), last period's configuration was "Separated," last period's choices by both the participant and his match in that period were prior (PP), which did not lead to a match (0).

TABLE II
History's Effect on Choices
Five Sector - Ten Period Treatment

Period $t-1: \quad$ DD $\quad$ DP $\quad$ PD PP1 PP0 $\quad$ DN $\quad$ ND $\quad$ PN $\quad$ NP NN1 NN0 Total

Period $t$ :
Separated $(t-1)$

| DS | 1 | 3 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 4 | 5 | 15 | 31 | 0 | 0 | 1 | 0 | 0 | 0 | 56 |
| NS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| DA | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| PA | 0 | 8 | 10 | 13 | 21 | 0 | 0 | 1 | 2 | 0 | 0 | 55 |
| NA | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 4 |
| Total | 2 | 17 | 17 | 28 | 52 | 0 | 0 | 2 | 2 | 0 | 2 | 122 |
|  |  |  |  | Adjacent $(t-1)$ |  |  |  |  |  |  |  |  |
| DS | 0 | 1 | 0 | 6 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 13 |
| PS | 0 | 1 | 0 | 22 | 15 | 0 | 0 | 1 | 1 | 0 | 0 | 40 |
| NS | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 2 | 0 | 0 | 4 |
| DA | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| PA | 0 | 1 | 1 | 29 | 22 | 0 | 0 | 5 | 5 | 0 | 0 | 63 |
| NA | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 4 |
| Total | 0 | 3 | 3 | 59 | 39 | 2 | 2 | 9 | 9 | 0 | 0 | 126 |

Table II starkly shows that in this treatment P choices were strongly persistent (PS and $\mathrm{PA})$ for both configurations. For example, the PP condition in period $t-1$ almost invariably led to a choice of P in period $t$. Remarkably, it was irrelevant whether the PP condition in the prior period led to a success (PP1) or not (PP0). Out of a total 91 PP0 pairs only 2 prompted a participant to switch to a choice other than P at the next opportunity. In contrast D choices were not as persistent and more susceptible to whether or not they led to a success. Out of a total of 22 unsuccessful D choices (20DP and 2DN), 14 prompted the corresponding participant to switch to $P$ at the next opportunity. Interestingly, there is a small number, cell Adjacent $\mathrm{DS} \backslash \mathrm{PP} 1=6$, of switches from P to D when the configuration in the prior period was "adjacent", the participant was successful with a prior choice and at the next opportunity faced a "(S)eparated" configuration. One possible explanation would be that in the "separated" configuration the distinct sector is more prominent and that participants who did not recognize the distinct sector in the prior "adjacent" configuration recognized it in the current "separated" configuration.

In the five-sector twenty-period treatment, Table III, both P and D choices tended to persist. Notably there was persistence even when the prior choice had not been successful. For example, in the "Adjacent"-DP column, there were 33 instances in which last period's configuration had been "adjacent," last period's choice had been $D$ and it had been matched by a P choice. In 28 ( $=18 \mathrm{DS}+10 \mathrm{DA})$ of these cases the current period's choice remained D despite the lack of success with D in the prior period. A similar situation holds for P choices in the "adjacent" - PD column.

Contrary to the five-sector ten-period treatment however, a second observation that emerges from the current treatment, Table III, is that the frequency of D versus P choices varies considerably with the current period's configuration, "S" versus "A." There where $141 \mathrm{D}(=71 \mathrm{DS}+70 \mathrm{DS})$ choices under the " S " configuration compared to only 59 D (= $35 \mathrm{DA}+24 \mathrm{DA}$ ) choices under the "A" configuration. It also seems that the persistence of D choices is independent of the prior period's configuration. Focusing on the DD choices, if the participants saw the "separated" configuration in the prior period and the "S" configuration in the current period, they chose D 43 out of 44 times $(=43 \mathrm{DS}+0 \mathrm{PS}+1 \mathrm{NS}$ under DD in "separated"). If the participants saw the "adjacent" configuration in the prior period and
the "S" configuration in the current period, they chose D 4 out of 4 times $(=4 \mathrm{DS}+0 \mathrm{PS}+$ 0DS under DD "adjacent"). Similar results hold for the DP condition in period t-1.

Recall that overall the proportion of distinct-sector choices was higher in the five-sector twenty-period treatment than in the five-sector ten-period treatment. This increase is statistically significant both when the present configuration is "Adjacent" and when it is "Separated." Table III shows however that the main contributor to the increased frequency of distinct-sector choices came from those instances when the present configuration was "Separated."

## TABLE III

History's Effect on Choices
Five Sector - Twenty Period Treatment

Period $t-1: \quad$ DD $\quad$ DP $\quad$ PD PP1 PP0 DN ND PN NP NN1 NN0 Total

Period $t$ :

|  | Separated $(t-1)$ |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| DS | 43 | 14 | 1 | 0 | 2 | 2 | 1 | 1 | 4 | 0 | 3 | 71 |
| PS | 0 | 2 | 14 | 3 | 7 | 1 | 2 | 6 | 0 | 0 | 0 | 35 |
| NS | 1 | 6 | 1 | 0 | 1 | 2 | 2 | 0 | 4 | 0 | 2 | 19 |
| DA | 15 | 9 | 2 | 0 | 0 | 2 | 3 | 0 | 4 | 0 | 0 | 35 |
| PA | 8 | 8 | 30 | 11 | 11 | 0 | 2 | 6 | 1 | 0 | 0 | 77 |
| NA | 12 | 10 | 1 | 0 | 1 | 3 | 1 | 0 | 0 | 0 | 1 | 29 |
| Total | 79 | 49 | 49 | 14 | 22 | 10 | 11 | 13 | 13 | 0 | 6 | 266 |
|  |  |  |  |  | Adjacent $(t-1)$ |  |  |  |  |  |  |  |
| DS | 4 | 18 | 5 | 8 | 4 | 6 | 6 | 7 | 7 | 1 | 4 | 70 |
| PS | 0 | 0 | 10 | 19 | 21 | 1 | 0 | 8 | 1 | 0 | 1 | 61 |
| NS | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 4 | 1 | 0 | 9 |
| DA | 3 | 10 | 0 | 2 | 1 | 2 | 1 | 0 | 4 | 0 | 1 | 24 |
| PA | 2 | 2 | 18 | 22 | 19 | 1 | 2 | 12 | 3 | 0 | 0 | 81 |
| NA | 1 | 2 | 0 | 2 | 0 | 1 | 3 | 2 | 11 | 1 | 2 | 25 |
| Total | 10 | 33 | 33 | 54 | 46 | 12 | 12 | 29 | 30 | 3 | 8 | 270 |

Finally, Table IV presents the dynamics for the five-sector ten-period game that is preceded by the three-sector ten-period game. According to Table IV, N choices are rare relative to both P and D choices. There is strong persistence in D choices regardless of whether there was a success with a D choice in the prior period or not. There were 68 ( $=68 \mathrm{DD}$ ) successful D choices in the prior period; 55 chose D in the current period. There
were $70(=54 \mathrm{DP}+16 \mathrm{DN})$ unsuccessful D choices in the prior period; 54 out of the 70 chose D in the current period.

Observe that P choices were fairly persistent as well, despite the fact that they were rarely successful. For example out of 19 instances ( $=1$ from cell DS/PD +18 from cell PS/PD under separated) in which someone transited from a "Separated" configuration to an S configuration and chose P against $\mathrm{D}(\mathrm{PD})$ in the prior period, 18 times $\mathrm{s} /$ he chose P again in the current period. This suggests that there may have been coarse language players who just "did not get it."

A key observation here concerns the choice of the D sector in period $t$ when the current configuration is "Adjacent," DA. Comparing the relative proportion of DA choices to PA and NA choices in period $t$ (for both the "separated" and "adjacent" categories of period $t-1$ ) in the current table to the relative proportions from the five-sector twenty-period treatment, Table III, the proportion of DA choices is higher in the current table. ${ }^{10}$ This confirms our previous result that the proportion of distinct-sector choices under the current "Adjacent" configuration improves more with experience with a simple game than with more periods of the same game.

[^9]TABLE IV
History's Effect on Choices
Five Sector-Ten Period preceded by Three Sector-Ten Period Treatment

Period $t-1: \quad$ DD $\quad$ DP $\quad$ PD PP1 PP0 $\quad$ DN $\quad$ ND $\quad$ PN $\quad$ NP NN1 NN0 Total

Period $t$ :

|  | Separated $(t-1)$ |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| DS | 30 | 18 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 52 |
| PS | 0 | 1 | 18 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 21 |
| NS | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 3 |
| DA | 11 | 6 | 0 | 0 | 0 | 4 | 3 | 0 | 1 | 0 | 0 | 25 |
| PA | 7 | 4 | 7 | 2 | 1 | 1 | 2 | 0 | 0 | 0 | 0 | 24 |
| NA | 3 | 1 | 3 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 8 |
| Total | 52 | 31 | 29 | 2 | 2 | 7 | 6 | 2 | 2 | 0 | 0 | 133 |
|  |  |  |  |  | Adjacent $(t-1)$ |  |  |  |  |  |  |  |
| DS | 9 | 12 | 5 | 4 | 0 | 4 | 3 | 1 | 0 | 2 | 0 | 40 |
| PS | 1 | 0 | 9 | 2 | 2 | 0 | 0 | 1 | 0 | 0 | 0 | 15 |
| NS | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 5 |
| DA | 5 | 7 | 3 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 19 |
| PA | 0 | 2 | 4 | 3 | 0 | 1 | 0 | 3 | 0 | 0 | 0 | 13 |
| NA | 0 | 1 | 1 | 1 | 0 | 1 | 2 | 0 | 6 | 1 | 0 | 13 |
| Total | 16 | 23 | 23 | 10 | 2 | 9 | 8 | 5 | 6 | 3 | 0 | 105 |

### 4.3 Teaching

In this section, we address the possibility of teaching by sophisticated players in the fivesector twenty-period treatment. Overall, the picture that emerged from Table III and the previous section's discussion is that of a tug of war between P and D , with D being the modal choice when the current configuration is "Separated" and P being the modal choice when the current configuration is "Adjacent." Players tend to adhere to one language
or the other and this attitude tends to dominate the effect of past successes and failure with the use of their language. It is conceivable that players adhere to a language despite adverse payoff experiences in the hope of convincing others to adopt the same language, which would constitute a form of strategic teaching of concepts. This view is supported by our observation that there are significantly more $D$ choices in the first ten periods of the twenty-period treatment than in the five-sector ten-period treatment. The longer time horizon in the twenty-period treatment would increase the expected rewards from teaching as well as actual rewards from teaching if teaching were successful. ${ }^{11}$

If indeed there is teaching, be it successful or otherwise, we would expect to see relatively more persistent D choices in the early periods of the twenty-period treatment than in the early periods of the ten-period treatment. Figures 9 and 10 show the persistence of D choices by players in the two treatments for the "Separated" configuration. Each figure presents the cumulative proportion of players in the "Separated" configuration who by period $t$ have chosen their first distinct sector that resulted in $m$ consecutive choices of the distinct sector, $\mathrm{m}=1,2,3,4$. Comparing Figure 9 and Figure 10, we see that while no one ever uses D four consecutive times in the "Separated" configuration with ten-periods, there is a significant proportion who by the third period start using $D$ four consecutive times with twenty-periods. The same is true for distinct sector choices made one, two and three consecutive times. Although not as strong, a similar result holds for the "Adjacent" configuration as well.

## Figure 9

## Figure 10

This result is similar to the sacrifice of current payoffs with the intent of changing future behavior of adaptive players that Camerer, Ho and Chong [2002] refer to as strategic teaching. Strategic teaching of concepts involves the same intertemporal payoff considerations and adds the idea that sophisticated players may hope to change the view of the world of unsophisticated players. Strategic teaching of concepts combines the idea of strategic teaching with what Weber [2003] refers to as a-ha learning, Bühler [1907-1908] and Köhler[1925].

[^10]One simple fact that has a bearing on whether there is teaching is that the incidence of N choices relative to D choices is relatively low. For players for whom the D choice is not cognitively available one would expect approximately twice as many N choices as D choices, whereas in the data there are considerably more D than N choices. This is true across replications and, most interestingly, also in the replication with the lowest frequency of sophisticated play (Replication 3 in Table A.III.3). The conclusion is that the distinct sector is cognitively available to and systematically used by some players in all replications of the twenty-period treatment.

## 5 Related Literature and Conclusion

In Sir Arthur Conan Doyle's "Silver Blaze," Colonel Ross and Sherlock Holmes engage in the following conversation:
"Is there any point to which you would wish to draw my attention?"
"To the curious incident of the dog in the night-time."
"The dog did nothing in the night-time."
"That was the curious incident," remarked Sherlock Holmes.
Going back to Geanakoplos [1992], the story is frequently used to illustrate that decision makers may be unaware of certain possibilities. There are two states of the world, state $a$ in which the dog barked and Colonel Ross realizes that there was an intruder, and state $b$ in which the dog did nothing and Colonel Ross fails to infer that there cannot have been an intruder.

The second round of our five-sector game, in the absence of coordination in the first round, has a similar knowledge structure. Recall that in this game a player faces three possible choices: to pick one of the two sectors chosen in the first round ("grey"), to pick the distinct sector ("distinct white"), or to pick one of the two other unchosen sectors ("other white"). Consider for a moment the decision problem derived from this game in which a single player plays against himself. The single player can solve this decision problem perfectly provided he understands it sufficiently well to recognizes the unique distinct sector. However, like Colonel Ross, the player may find himself in one of two states. In state $a$ he realizes that he has three choices, "grey," "distinct white" and "other white." In state $b$ he fails to
realize that the presence of the two grey sectors desymmetrizes the three white sectors, and accordingly thinks that he has only two choices, "grey" and "white." We as analysts know that in this decision problem the agent always has three choices, and also know that there are versions of the problem in which the agent has only two choices, i.e. if we eliminate the circular structure of the choices and leave only the partitional structure of chosen and unchosen sectors. The agent on the other hand may be convinced that there is only one possibility, namely that there are just two choices.

Translated into a nondegenerate game, with multiple agents, it is then natural to have different state spaces for different agents. Bacharach [1993] calls for a model of games in which "one specifies the way players conceive the situation and how this varies." He provides details of such a model of variable universe games for the case where players have identical sets of actions and payoffs. In Bacharach's model, a player's perception is essentially given by a partition of the set of actions. Blume and Gneezy [2002] extend Bacharach's approach to permit more general structures on the sets of actions than partitions, or collections of partitions. This permits us to accommodate the circular structure in our games.

Our experiments create an environment in which agents with differential awareness interact repeatedly. In this environment agents may learn. Given our setup, learning can occur at two levels. At one level, in each period, agents can learn by labeling actions in the first round and using these labels in the second round. This type of optimal learning has been analyzed by Crawford and Haller [1990] and Blume [2000]. ${ }^{12}$ At the other level, agents can learn across periods how to learn within a period. This type of learning, which we call cognitive learning has to the best of our knowledge not yet been addressed in the literature. Initially, there may be agents who are unaware of the fact that the labels introduced by first-round choices can always be used to identify a unique distinct sector. Other agents may be aware of this possibility. In the course of the multi-period interaction, agents may become aware of this possibility, and in any given period, agents who are aware may want to make aware those who are not.

We find evidence for differential awareness and cognitive learning. That is, in simple environments agents learn across periods to make better use within a period of labels created

[^11]in that period. We observe transfer of cognitive learning from simple environments to more complicated environments. That is, following a repeated three-sector game, optimal play is more likely in the five-sector game than otherwise. Our evidence is consistent with attempts at strategic teaching of concepts. That is, in the repeated twenty-period five-sector game players who are aware of the fine language appear to sacrifice payoffs early in the game in order to make aware other players who are currently unaware of the fine language. Our data suggest that despite these attempts there remain coarse language players who just "don't get it." While the incidence of distinct-sector play increases in the five-sector treatment when lengthening the time horizon or permitting players to gain experience in a simpler environment, it is always less than in the three-sector treatment.

## A Appendix

## TABLE A.I

History's Effect on Choices
Three Sector - Ten Period
Period $t-1$ : DD DP PD PP1 PP0 Total

Period $t$ :

| Replication 1 |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: |
| D | 26 | 12 | 10 | 0 | 0 | 48 |  |  |  |
| P | 3 | 3 | 6 | 0 | 2 | 14 |  |  |  |
| Total | 29 | 15 | 16 | 0 | 2 | 52 |  |  |  |
|  | Replication 2 |  |  |  |  |  |  |  |  |
| D | 34 | 15 | 9 | 0 | 0 | 58 |  |  |  |
| P | 4 | 3 | 9 | 0 | 0 | 16 |  |  |  |
| Total | 38 | 18 | 18 | 0 | 0 | 74 |  |  |  |
|  | Replication 3 |  |  |  |  |  |  |  |  |
| D | 41 | 13 | 9 | 2 | 0 | 65 |  |  |  |
| P | 4 | 1 | 6 | 4 | 2 | 17 |  |  |  |
| Total | 45 | 14 | 15 | 6 | 2 | 82 |  |  |  |

TABLE A.II. 1
History's Effect on Choices
Five Sector - Ten Period
Replication 1

Period $t-1: \quad$ DD $\quad$ DP $\quad$ PD PP1 PP0 $\quad$ DN $\quad$ ND $\quad$ PN $\quad$ NP NN1 NN0 Total

Period $t$ :
Separated $(t-1)$

| DS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 1 | 0 | 10 | 20 | 0 | 0 | 0 | 0 | 0 | 0 | 31 |
| NS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| DA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| PA | 0 | 0 | 1 | 8 | 12 | 0 | 0 | 0 | 0 | 0 | 0 | 21 |
| NA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Total | 0 | 1 | 1 | 18 | 32 | 0 | 0 | 0 | 0 | 0 | 0 | 52 |
|  |  | Adjacent $(t-1)$ |  |  |  |  |  |  |  |  |  |  |


| DS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 0 | 0 | 9 | 10 | 0 | 0 | 0 | 0 | 0 | 0 | 19 |
| NS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| DA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| PA | 0 | 0 | 0 | 4 | 9 | 0 | 0 | 0 | 0 | 0 | 0 | 13 |
| NA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Total | 0 | 0 | 0 | 13 | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 32 |

TABLE A.II. 2
History's Effect on Choices
Five Sector - Ten Period
Replication 2

Period $t-1: \quad$ DD $\quad$ DP PD PP1 PP0 DN ND PN NP NN1 NN0 Total

Period $t$ :
Separated $(t-1)$

| DS | 1 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 3 | 3 | 2 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 12 |
| NS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| DA | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| PA | 0 | 4 | 5 | 2 | 4 | 0 | 0 | 1 | 1 | 0 | 0 | 17 |
| NA | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| Total | 2 | 10 | 10 | 4 | 8 | 0 | 0 | 1 | 1 | 0 | 0 | 36 |
|  |  |  | Adjacent $(t-1)$ |  |  |  |  |  |  |  |  |  |


| DS | 0 | 1 | 0 | 4 | 2 | 1 | 1 | 0 | 0 | 0 | 0 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 1 | 0 | 4 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 8 |
| NS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| DA | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| PA | 0 | 1 | 1 | 14 | 7 | 0 | 0 | 1 | 1 | 0 | 0 | 25 |
| NA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Total | 0 | 3 | 3 | 22 | 12 | 1 | 1 | 1 | 1 | 0 | 0 | 44 |

TABLE A.II. 3
History's Effect on Choices
Five Sector - Ten Period
Replication 3

Period $t-1$ : DD DP PD PP1 PP0 DN ND PN NP NN1 NN0 Total

Period $t$ :
Separated $(t-1)$

| DS | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 0 | 2 | 3 | 7 | 0 | 0 | 1 | 0 | 0 | 0 | 13 |
| NS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| DA | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| PA | 0 | 4 | 4 | 3 | 5 | 0 | 0 | 0 | 1 | 0 | 0 | 17 |
| NA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 |
| Total | 0 | 6 | 6 | 6 | 12 | 0 | 0 | 1 | 1 | 0 | 2 | 34 |
|  |  |  | Adjacent $(t-1)$ |  |  |  |  |  |  |  |  |  |


| DS | 0 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 0 | 0 | 9 | 2 | 0 | 0 | 1 | 1 | 0 | 0 | 13 |
| NS | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 2 | 0 | 0 | 4 |
| DA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| PA | 0 | 0 | 0 | 11 | 6 | 0 | 0 | 4 | 4 | 0 | 0 | 25 |
| NA | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 4 |
| Total | 0 | 0 | 0 | 24 | 8 | 1 | 1 | 8 | 8 | 0 | 0 | 50 |

TABLE A.III. 1
History's Effect on Choices
Five Sector - Twenty Period
Replication 1
Period $t-1$ : DD DP PD PP1 PP0 DN ND PN NP NN1 NN0 Total

Period $t$ :
Separated $(t-1)$

| DS | 32 | 5 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 39 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 |
| NS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| DA | 11 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 13 |
| PA | 2 | 2 | 10 | 2 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 17 |
| NA | 10 | 2 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 14 |
| Total | 55 | 13 | 13 | 2 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 87 |
|  |  |  | Adjacent $(t-1)$ |  |  |  |  |  |  |  |  |  |


| DS | 1 | 9 | 1 | 4 | 3 | 3 | 2 | 5 | 4 | 1 | 4 | 37 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 0 | 5 | 1 | 3 | 0 | 0 | 0 | 1 | 0 | 0 | 10 |
| NS | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 2 |
| DA | 1 | 4 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 11 |
| PA | 2 | 0 | 8 | 2 | 7 | 1 | 0 | 4 | 1 | 0 | 0 | 25 |
| NA | 0 | 1 | 0 | 1 | 0 | 1 | 3 | 1 | 3 | 1 | 1 | 12 |
| Total | 4 | 14 | 14 | 10 | 14 | 6 | 6 | 10 | 10 | 3 | 6 | 97 |

TABLE A.III. 2
History's Effect on Choices
Five Sector - Twenty Period
Replication 2
Period $t-1: \quad$ DD $\quad$ DP $\quad$ PD PP1 PP0 $\quad$ DN $\quad$ ND $\quad$ PN $\quad$ NP NN1 NN0 Total

Period $t$ :
Separated $(t-1)$

| DS | 8 | 2 | 1 | 0 | 2 | 1 | 0 | 1 | 2 | 0 | 2 | 19 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 0 | 6 | 1 | 0 | 1 | 2 | 5 | 0 | 0 | 0 | 15 |
| NS | 1 | 3 | 1 | 0 | 1 | 2 | 2 | 0 | 4 | 0 | 2 | 16 |
| DA | 3 | 3 | 1 | 0 | 0 | 2 | 3 | 0 | 3 | 0 | 0 | 15 |
| PA | 3 | 3 | 7 | 1 | 2 | 0 | 1 | 4 | 1 | 0 | 0 | 22 |
| NA | 1 | 5 | 0 | 0 | 1 | 2 | 1 | 0 | 0 | 0 | 0 | 10 |
| Total | 16 | 16 | 16 | 2 | 6 | 8 | 9 | 10 | 10 | 0 | 4 | 97 |
|  |  |  |  | Adjacent $(t-1)$ |  |  |  |  |  |  |  |  |


| DS | 3 | 3 | 3 | 2 | 1 | 3 | 4 | 1 | 2 | 0 | 0 | 22 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 0 | 3 | 3 | 2 | 1 | 0 | 7 | 0 | 0 | 1 | 17 |
| NS | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 3 | 0 | 0 | 6 |
| DA | 2 | 3 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 0 | 0 | 8 |
| PA | 0 | 1 | 3 | 2 | 2 | 0 | 2 | 4 | 1 | 0 | 0 | 15 |
| NA | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 6 | 0 | 1 | 11 |
| Total | 6 | 9 | 9 | 8 | 6 | 6 | 6 | 13 | 14 | 0 | 2 | 79 |

TABLE A.III. 3
History's Effect on Choices
Five Sector - Twenty Period
Replication 3
Period $t-1: \quad$ DD $\quad$ DP $\quad$ PD PP1 PP0 $\quad$ DN $\quad$ ND $\quad$ PN $\quad$ NP NN1 NN0 Total

Period $t$ :
Separated $(t-1)$

| DS | 3 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 1 | 13 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 0 | 6 | 2 | 7 | 0 | 0 | 1 | 0 | 0 | 0 | 16 |
| NS | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 |
| DA | 1 | 4 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 7 |
| PA | 3 | 3 | 13 | 8 | 9 | 0 | 0 | 2 | 0 | 0 | 0 | 38 |
| NA | 1 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 5 |
| Total | 8 | 20 | 20 | 10 | 16 | 0 | 0 | 3 | 3 | 0 | 2 | 82 |
|  |  |  | Adjacent $(t-1)$ |  |  |  |  |  |  |  |  |  |


| DS | 0 | 6 | 1 | 2 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 11 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 0 | 2 | 15 | 16 | 0 | 0 | 1 | 0 | 0 | 0 | 34 |
| NS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| DA | 0 | 3 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 5 |
| PA | 0 | 1 | 7 | 18 | 10 | 0 | 0 | 4 | 1 | 0 | 0 | 41 |
| NA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 2 |
| Total | 0 | 10 | 10 | 36 | 26 | 0 | 0 | 6 | 6 | 0 | 0 | 94 |

TABLE A.IV. 1
History's Effect on Choices
Five Sector-Ten Period preceded by Three Sector-Ten Period
Replication 1

Period $t-1: \quad$ DD $\quad$ DP PD PP1 PP0 DN ND PN NP NN1 NN0 Total

Period $t$ :
Separated $(t-1)$

| DS | 8 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 |
| NS | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| DA | 2 | 4 | 0 | 0 | 0 | 3 | 2 | 0 | 0 | 0 | 0 | 11 |
| PA | 4 | 0 | 2 | 0 | 0 | 1 | 2 | 0 | 0 | 0 | 0 | 9 |
| NA | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 3 |
| Total | 15 | 7 | 6 | 0 | 0 | 5 | 4 | 0 | 0 | 0 | 0 | 37 |
|  |  |  | Adjacent $(t-1)$ |  |  |  |  |  |  |  |  |  |


| DS | 4 | 3 | 3 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 0 | 14 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 3 |
| NS | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 3 |
| DA | 4 | 3 | 3 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 14 |
| PA | 0 | 2 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 5 |
| NA | 0 | 1 | 1 | 1 | 0 | 1 | 2 | 0 | 2 | 0 | 0 | 8 |
| Total | 10 | 9 | 9 | 2 | 0 | 7 | 6 | 2 | 2 | 0 | 0 | 47 |

TABLE A.IV. 2
History's Effect on Choices
Five Sector-Ten Period preceded by Three Sector-Ten Period
Replication 2

Period $t-1: \quad$ DD $\quad$ DP $\quad$ PD PP1 PP0 DN ND PN NP NN1 NN0 Total

Period $t$ :
Separated $(t-1)$

| DS | 16 | 13 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 32 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 1 | 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 12 |
| NS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| DA | 6 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8 |
| PA | 2 | 1 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 |
| NA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Total | 24 | 17 | 16 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 59 |
|  |  |  | Adjacent $(t-1)$ |  |  |  |  |  |  |  |  |  |


| DS | 4 | 7 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 14 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 0 | 6 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 |
| NS | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| DA | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 |
| PA | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| NA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Total | 4 | 10 | 9 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 27 |

TABLE A.IV. 3
History's Effect on Choices
Five Sector-Ten Period preceded by Three Sector-Ten Period
Replication 3

Period $t-1: \quad$ DD $\quad$ DP $\quad$ PD PP1 PP0 DN ND PN NP NN1 NN0 Total

Period $t$ :
Separated $(t-1)$

| DS | 6 | 3 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 6 |
| NS | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 2 |
| DA | 3 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 6 |
| PA | 1 | 3 | 1 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 8 |
| NA | 3 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 |
| Total | 13 | 7 | 7 | 2 | 2 | 1 | 1 | 2 | 2 | 0 | 0 | 37 |
|  |  |  | Adjacent $(t-1)$ |  |  |  |  |  |  |  |  |  |


| DS | 1 | 2 | 1 | 2 | 0 | 2 | 2 | 0 | 0 | 2 | 0 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| PS | 0 | 0 | 2 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 5 |
| NS | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| DA | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| PA | 0 | 0 | 2 | 1 | 0 | 0 | 0 | 3 | 0 | 0 | 0 | 6 |
| NA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 1 | 0 | 5 |
| Total | 2 | 4 | 5 | 4 | 2 | 2 | 2 | 3 | 4 | 3 | 0 | 31 |

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Three-Sector Disk
Figure 2


Five-Sector Disk
Figure 3


Chosen Sectors
Three-Sector Disk
Figure 4


Separated Chosen Sectors
Five-Sector Disk
Figure 5


Adjacent Chosen Sectors
Five-Sector Disk
Figure 6


Player 1
Three-Sector Disk
Figure 7A


Player 2
Three-Sector Disk
Figure 7B

Distinct-Sector Choices
Proportion by Treatment
Figure 8


## Consecutive Number of Distinct Sector Choices in Separated Configuration

$\square$ First of Four in a Row ( $\mathrm{n}=0$ )
$\square$ First of Three in a Row $(n=3)$
$\square$ First of Two in a Row ( $n=5$ )
$\square$ First Distinct Choice ( $\mathrm{n}=12$ )


Five-Sector Ten-Period Treatment
Figure 9
Cumulative Proportion: proportion of players who by period $t$ have taken their first distinct choice that resulted in $m$ consecutive choices of the distinct sector, $\mathrm{m}=1,2,3,4$.
n : total number of players out of 36 who made m consecutive distinct choices.


Five-Sector Twenty-Period Treatment
Figure 10
Cumulative Proportion: proportion of players who by period thave taken their first distinct choice that resulted in m consecutive choices of the distinct sector, $m=1,2,3,4$.
n : total number of players out of 36 who made m consecutive distinct choices.


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[^1]:    ${ }^{1}$ Experimental evidence for deception and credulity has been reported in Forsythe, Lundholm and Rietz [1999] and Blume, DeJong, Kim and Sprinkle [2001].

[^2]:    ${ }^{2}$ The Aha-Erlebnis was first introduced in the literature and studied by the cognitive and language psychologist Karl Bühler [1907 and 1908]. In his words: "Man trifft auf einen etwas schwierigen fremden Gedanken, stutzt in wenig und plötzlich geht einem das Verständnis auf wie durch eine Erleuchtung." (One encounters a somewhat difficult foreign thought, hesitates for a moment, and suddenly one attains the insight as if through enlightenment) Bühler [1908, p. 14]. Subsequently, the aha-experience was studied by Köhler in his experiments with chimpanzees, Köhler [1925].

[^3]:    ${ }^{3}$ For the sectors to be identical, it is important that the orientation of the disk is not common to both players. This can be achieved by spinning the disk before presenting it to each player. Furthermore, we wish to eliminate asymmetries arising from a directional structure on the disk (clockwise vs counter-clockwise). This can be achieved by having agents in a match choose from opposite sides of the disk, or by randomizing over the side which is presented to each player before each choice. The randomizing scheme is more powerful in preserving symmetries, but for our purposes the opposite-side scheme suffices.
    ${ }^{4}$ Blume [2000] extends this approach to models with partially common languages.

[^4]:    ${ }^{5}$ Bacharach [1993] develops a formal model of interactive unawareness via variable universe games. Blume and Gneezy [2002] extend Bacharach's approach to permit a more general structure on the sets of actions than partitions or collections of partitions, i.e., the spatial or circular structure used in this paper.

[^5]:    ${ }^{6}$ Instructions are available from the authors upon request.

[^6]:    ${ }^{7}$ We find no difference between the three-sector ten-period treatment and the first ten periods of "fivesector ten-period preceded by three-sector ten-period" treatment, $p=0.101$.

[^7]:    ${ }^{8}$ Similarly, the hypothesis of no difference between the ten periods of the five-sector ten-period treatment and the last ten periods of five-sector twenty-period treatment is rejected, $p=0.000$.

[^8]:    ${ }^{9}$ For data by replication, consult the appendix.

[^9]:    ${ }^{10}$ Recall that the tables are constructed by only considering second-round choices if there was no match in the first-round. Further, $t-1$ is the last time prior to the present period, an individual was not coordinated in the first round. Thus, the relative comparison across the two treatments is the correct comparison.

[^10]:    ${ }^{11}$ In the "Separated" configuration, the frequency of the distinct sector and profits are significantly higher in the last ten periods than in the first ten periods. There are no significant differences in the "Adjacent" configuration.

[^11]:    ${ }^{12}$ Other applications of this idea can be found in Alpern and Reyniers [2002], Bhaskar [2000] and Kramarz [1996].

