CentER

Discussion paper

No. 2001-57

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August 2001

ISSN 0924-7815

A geometrical approach to computing expected cycle times for class-based storage layouts in AS/RS

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An exact, geometry-based analytical model is presented that can be used to compute the expected cycle time for a storage/retrieval (S/R) machine, executing single-commands, dual-commands, or both, in a rack structure that has been laid out in pre-specified storage zones for classes of goods. The rack may be either square-in-time or non-square-in-time. The approach is intuitively appealing, and it does not assume any certain layout shape, such as traditional "L-shaped" class layouts. The model has been coded in Turbo Pascal, and can be used by designers as a tool for quickly evaluating alternative layout configurations with respect to expected S/R cycle time in an AS/RS, and thereby the throughput of an automated warehouse over time. This model has been successfully applied in a major manufacturing plant in Europe to evaluate reconfigurations of their rack storage layouts over the past five years.

KEYWORDS: Automated storage and retrieval systems, AS/RS, class-based storage.

1. Introduction

In modern supply chains, suppliers, manufacturers, distributors, wholesalers and retailers are striving for increased profits in an economy that is highly charged, extremely competitive, customer service driven, and global. Accordingly, the material handling systems essential to support the dynamism of such supply chains must be flexible, agile and easily re-configurable.

Unfortunately, with a few notable exceptions, automated storage and retrieval systems (AS/RSs) have come to be viewed by many current and potential users as too inflexible to adequately function in the dynamic supply chain environment wherein the

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emphasis is on minimizing inventory, cross-docking, and other concepts designed to keep goods moving in the supply chain, rather than being stored.

AS/R systems have now been applied in manufacturing, warehousing and distribution facilities for about three decades. And, there have been many studies regarding the optimal policies for operating these systems in order to maximize throughput. To enhance the flexibility of AS/R systems, and to perhaps make them more useful components in the supply chain, attention should be directed toward finding an analytical approach to aid in easily evaluating the throughput resulting from frequent reconfigurations of storage assignments.

Prior studies of AS/RSs have defined three methods for assigning products to storage locations: (a) random storage; (b) class-based storage; and (c) dedicated storage. Although not widely analyzed in the literature, class-based storage assignment is effective when there are many products having different residence time requirements.

This paper presents a geometrical-based approach for determining the expected S/R machine cycle times, and therefore throughput, for class-based storage assignment layouts in an AS/RS that is either “square in time (SIT)” or “non-square in time (NSIT)”. It is believed that use of this approach can result in expedient evaluation of throughput resulting from re-layouts of the AS/R system racks, thus making these systems more appealing for use in integrated supply chain systems.

2. Literature review

There is a rich literature dealing with the operation of AS/RS systems. Researchers began with basic results such as computing the expected value and/or the distribution of single- and dual-command cycle times for storage/retrieval machines [see Hausman *et. al.* (1976), Graves *et. al.* (1977), Bozer and White (1984), Foley and Frazelle (1991) Chang *et. al.* (1995), Kouvelis and Papanicolaan (1995), Sarker and Babu (1995), among others]. Then, operational issues such as S/R machine dwell point strategies or storage/retrieval operation sequences received some attention [Egbelu and Wu (1993), Hwang and Lin (1993), Elsayed and Lee (1996), Lee and Schaefer (1996), Peters *et. al.* (1996), Chang and Egbelu (1997 a), Chang and Egbelu (1997 b),]. Later, twin-shuttle S/R machines [Keserla and Peters (1994), Sarker *et. al.* (1994)], multi-shuttle machines [Meller and Mungwattana (1997)], and storage and retrieval matching or AS/RS control/design strategies [Han *et. al.* (1987), Seidman (1988), Lim and Wysk (1990), Rosenblatt *et. al.* (1993), Wang and Yih (1997)] were considered. Simulation-based approaches have been employed to deal with random arrivals of storage and retrieval requests to an AS/RS [Lee (1997), Bozer and Cho (1998)]. Also, expected cycle time performance for AS/R systems having unequal sized cells, randomized storage assignments, and single- and dual-command cycles has been estimated [Lee *et. al.*(1999)]. One paper [Pan and Wang (1996)] takes a similar approach to that presented in this paper in developing a framework for the dual-command cycle, continuous travel, square-in-time model under class-based assignment.

Taken together, these studies have lead to a better understanding of the optimal operation of automated storage and retrieval systems. Furthermore, they have facilitated system design and performance evaluation of AS/R Systems.

In this paper, an exact, geometry-based analytical model is presented that can be used to compute the expected cycle time for a storage/retrieval (S/R) machine, executing single-commands, dual-commands, or both, in a rack structure that has been laid out in pre-specified storage zones. The rack may be either square-in-time or non-square-in-time.

3. Basic concepts

Veraart (1995) developed a model to calculate the expected cycle time of an S/R machine that is the basis of this paper. He makes the following assumptions that are common in the models of AS/RS operations cited in the preceding literature review section, i.e.:

1. A continuous approximation to the discrete rack face.
2. Each pallet holds only one part number or item type.
3. The system consists of a single S/R machine serving a single aisle, providing access to two, single-deep storage rack structures on either side.
4. Incoming and outgoing pallets are transferred at the same point, designated the I/O point, and this I/O point is situated at one corner (lower left-hand) of the rack face, in plan view oriented perpendicular to the aisle.
5. The S/R machine is capable of simultaneously moving both vertically and horizontally at constant speeds. Thus, the travel time required to reach any location in the rack is approximated by the Tchebyshev metric.
6. The S/R machine has a single shuttle and can operate only in single- or dual-command modes.
7. The actual time required for the S/R machine to load or unload a pallet at the I/O point or at a storage location is ignored, as is the time taken by the S/R machine

to travel from any external input/output device or hardware to the I/O point in the rack structure.

8. The fraction of single-command cycles, f, is known for a given planning horizon.

9. All pallets are randomly stored in empty locations within the appropriate zone assigned to the class of goods on the pallet.

10. The mean fraction of movements, p_i, for the items in each pre-determined storage zone for each class of goods is known for a specific time horizon.

11. Short-run dynamic considerations, like possible dependencies between successive retrieval and storage transactions, or seasonal demand distortions, are ignored.

12. Storage and retrieval requests are triggered independently of teach other and are processed according to a first-come, first-served discipline.

These assumptions are rather typical in the papers cited previously. The most restrictive assumptions are the last five. These restrictions ensure that the times required for all retrievals and storages can be considered to be independent and identically distributed random variables.

The general model computes the expected cycle time per storage/retrieval operation, under the foregoing assumptions, as follows:

$$E(T) = 2\sum_i p_i E(t_i) + 2(1-f)\sum_i \sum_{j>i} p_i p_j E(t_{ij}) + (1-f)\sum_i p_i^2 E(t_{ii}) \tag{1}$$

where

$E(T)$ = expected cycle time per storage/retrieval operation

$E(t_i)$ = expected travel time between the I/O point and a random point in zone i

$E(t_{ij})$ = expected travel time between a random point in zone *i* and a random point in zone *j*

$E(t_{ii})$ = expected travel time between two random points within zone *i*

p_i = fraction of movements of the items stored in zone *i*

f = fraction of single-command transactions

As illustrated by Ashayeri *et. al.* (1997), the Tchebychev approximation of travel times by the S/R machine gives rise to the geometrical model of the time required to reach any point on a rack face, as shown in Figure 1.

In order to better comprehend Figure 1, imagine that the rack face is lying in the x-y plane. Then the cross-hatched figure above the (x,y)- plane represents the time required to reach each point in the rack face, assuming a Tchebyshev travel time metric.

Veraart (1995) and Ashayeri *et. al.* (1997) proved a lemma that the mean height of the geometrical surface, like that shown in figure 1, above the rack face is equal to the volume subtended by that surface divided by the projection of the surface onto the rack structure (domain). Therefore, the expected travel time, $E(t_1)$, between the I/O point, located at (0,0) and any rack face location (x_1,y_1) is:

[Insert figure 1 about here]

$$\text{Expected travel time, } E(t_1) = \frac{\int_0^{x_1}\int_0^{y_1} F(x,y)\,dy\,dx}{x_1 \cdot y_1}$$

where, $F(x_1, y_1) = \max\left(\frac{x_1}{s_h}, \frac{y_1}{s_v}\right)$, i.e., the Tchebyshev travel time (sec)

x_1 = horizontal location along the rack face (m)

y_1 = vertical location along the rack face (m)

s_h = horizontal speed of the S/R machine (m/sec)

s_v = vertical speed of the S/R machine (m/sec)

This implies that, if the rack structure is laid out in predetermined storage zones for classes of goods, the expected S/R travel time between the I/O point and a random location in zone i equals the volume of the geometrical surface that gives the travel time between the I/O point and any fixed location in zone i, divided by the surface area subtended by the projection of zone i onto the rack face.

This result may be used to compute each of the expected times in equation (1), i.e., $E(t_i)$, $E(t_{ij})$, and $E(t_{ii})$.

[Insert figure 2 about here]

3.1. *Computation of $E(t_i)$*

Consider a system of axes like that shown in figure 2, where the x-position of a location denotes the horizontal rack location, and the y-position the vertical rack location of a storage cell. The I/O point is defined as the point (0,0), and is situated in the lower left corner of the storage rack. Suppose that the function F(x,y) gives the Tchebyshev travel time between the I/O point and the fixed point (x,y) in zone i, and that the horizontal and vertical borders of zone i are given by x_1 and x_2, and y_1 and y_2, respectively. Then, according to the lemma, $E(t_i)$ can be computed as follows:

$$E(t_i) = \frac{\int_{x_1}^{x_2}\int_{y_1}^{y_2} F(x,y)\, dy\, dx}{(x_2 - x_1)(y_2 - y_1)}$$

The function F is the maximum of the horizontal travel time or vertical travel time to reach point (x,y).

Consider, for example, the situation shown in figure 3 below.

[Insert figure 3 about here]

The diagonal line starting at the I/O point is the line of movement when the S/R machine starts at the I/O point and travels at full horizontal and full vertical speed, simultaneously. This line corresponds to the fold (or crease) in the Tchebyshev time surface illustrated in figures 1 and 2. It has a slope equal to the quotient of the vertical travel speed of the S/R machine, s_v, and the horizontal travel speed, s_h. The points where this line crosses the boundaries of the projection of zone *i* in figure 3 are denoted (x_1, y_3) and (x_3, y_2). As can be seen, the travel time between the I/O point and points in area A of zone *i* equals the vertical travel time between those points. And the travel time between the I/O point and the points in the areas B is equal to the horizontal travel time between those points.

Figure 4 is a graphical illustration of the function F(x,y) over the domain of the surface of zone *i*.

[Insert figure 4 about here]

In fact, if we borrow the terminology from engineering mechanics, figure 4 could be considered a “free-body” diagram of the volume subtended by zone *i* on the Tchebyshev time surface (shown in figure 2) over the domain of its projection onto the rack face in the (x,y)- plane (as illustrated in figure 3).

In order to compute the mean height of this free-body diagram, which, according to the lemma proven by Veraart (1995), equals the expected travel time between the I/O point and a random location in zone *i*, the volume first needs to be computed.

This volume can be interpreted as the sums of the volumes above the parts of the projection of zone *i*, as defined in figure 4. The volumes above areas A and BI are of the same size (see Appendix for the proof), so only one of them must be computed. Since the travel time between the I/O point and any location in area A is determined only by the vertical travel time, y/s_v, the volume above area A and area BI is

$$\int_{x_1}^{x_3}\int_{y_3}^{y_2} F(x,y)dydx = 2\int_{x_1}^{x_3}\int_{\frac{s_v}{s_h}x}^{y_2} \frac{y}{s_v}\,dy\,dx \quad = \quad \frac{1}{s_v}y_2^2(x_3-x_1)-\frac{s_v}{3s_h^2}(x_3^3-x_1^3) \tag{2}$$

where, $x_3 \quad = \quad \dfrac{s_h}{s_v}y_2$.

The travel times between the I/O point and locations in areas BII and BIII are defined only by the horizontal travel time, x/s_h. Therefore, the volume above area BII in figure 4 is:

$$\int_{x_3}^{x_2}\int_{y_3}^{y_2} \frac{x}{s_h}\,dy\,dx \quad = \quad \frac{1}{2s_h}(y_2-y_3)(x_2^2-x_3^2) \tag{3}$$

where, $y_3 \quad = \quad \dfrac{s_v}{s_h}x_1$.

Finally, the volume above area BIII in Figure 4 is:

$$\int_{x_1}^{x_2}\int_{y_1}^{y_3} \frac{x}{s_h}\,dy\,dx \quad = \quad \frac{1}{2s_h}(y_3-y_1)(x_2^2-x_1^2) \tag{4}$$

Dividing the sum of expressions (2), (3), and (4) by the domain surface area of zone i, i.e. $(x_2-x_1)(y_2-y_1)$, gives the expected travel time between the I/O point and a random location in zone i, $E(t_i)$.

Using the foregoing model, the expected travel time between the I/O point and a random location in a certain storage zone i can be computed for any such zone. However, it should be clear that the equation for the volume of the geometrical figure that gives the travel time between the I/O point and a random location in a certain zone depends upon the location of the zone with respect to the fold (or crease) in the Tchebyshev travel time surface shown in figures 1 and 2. In fact, there are six possible orientations, as shown in figure 5.

[Insert figure 5 about here]

Case 2 in figure 5 corresponds to the zone *i* orientation of the example solved in equations (2) through (4) above. For the other orientations, the expected travel time between the I/O point and a random location in the zone can be calculated in a similar way. The zones can be divided into areas, where the travel time from the I/O point is determined by the horizontal travel time, designated B areas, and areas where the travel time is determined by the vertical travel time, designated A areas. Then the sums of the volumes over each zone of the geometrical figure like in figure 4 can easily be computed. All that is left to do is divide the total volume by the area of the projection of the zone onto the surface of the rack face. In fact, the last three orientations can be changed into the first three by appropriate axis translations. The volume equations for all six orientations depicted in figure 5 are provided in figure 6 for ease of reference.

With this information, it is possible to compute the expected cycle time of single-command cycles (i.e. $f = 1$), the first term in equation (1):

$$E(T_{SC}) = 2 \cdot \sum_i p_i E(t_i)$$

where,

$E(T_{sc})$ = expected cycle time of a single-command cycle

$E(t_i)$ = expected travel time between the I/O point and a random location in zone i

p_i = fraction of movements to zone i (known *a priori*)

[Insert figure 6 about here]

3.2. *Computation of $E(t_{ij})$*

In executing a dual-command cycle, assume that the S/R machine has completed a storage operation by going from the I/O to a location (a,b) in zone i. From here, the S/R machine is directed to another random location (p,q) in another storage zone j to retrieve a load and take it to the I/O point.

In order to compute the expected travel time between these two random points in different zones, $E(t_{ij})$, the same approach used in the previous section for computing the expected travel time from the I/O to a random point in zone i, $E(t_i)$, is used.

Without loss of generality, assume that zones i and j are situated with respect to each other in the rack face as shown in figure 7 wherein one zone, called the source zone i, lies beneath the other zone, the destination zone j. This situation can be created for any two zones in the rack face by a suitable translation of axes, if necessary. Let x_1 and x_2, and y_1 and y_2 be the horizontal and vertical coordinate bounds, respectively, of zone i. And, let d_1 and d_2, and y_3 and y_4 be the similarly defined coordinate bounds of the destination zone j. For the purpose of assisting in the calculation of the volume integrals, the source zone can be divided into thirteen regions, as is shown in figure 7. Depending upon which side of the destination zone is crossed, the diagonals connecting the points in the two zones have slopes equal to the vertical travel speed of the S/R machine divided by the horizontal speed, or the negative of this ratio.

[Insert figure 7 about here]

To compute the expected travel time between two random points located in different storage zones, the expected travel time from a certain location (a,b) in the source zone to a location (p,q) in the destination zone is computed using an approach similar to that discussed in the computation of $E(t_i)$.

For example, take a random point (a,b) in the source zone of region III in figure 7. The projection of the destination zone onto the rack face can then be divided into a number of areas, four in this case, as illustrated in figure 8. Here, areas designated A correspond to those where the expected travel time from (a,b) is determined by the vertical travel time to (p,q), and areas designated B correspond to parts where the travel time from (a,b) is determined by the horizontal travel time to (p,q).

[Insert figure 8 about here]

The volume of the geometrical figure subtended by the Tchebychev time surface projected onto the rack face in figure 8 is given by:

$$\int_{d_1}^{d_2}\int_{y_3}^{y_4} \max\left[\frac{|p-a|}{s_h}, \frac{|q-b|}{s_v}\right] dq\,dp$$

We assume that the S/R machine travels along the appropriate "crease" in the time surface in going from (a,b) in zone i to (p,q) in zone j. Depending upon the position of (p,q) relative to the crease, the travel time is determined by s_h if (p,q) is to the right of the crease, and s_v if (p,q) is to the left of the crease. The volume subtended by the travel time surface in going between location (a,b) in zone i and a location (p,q) in area AI of zone j is:

$$\int_{d_1}^{d_3}\int_{y_3}^{y_4} \frac{q-b}{s_v} dq\,dp = \left(\frac{y_4 + y_3 - 2b}{2s_v}\right)(y_4 - y_3)(d_3 - d_1) \qquad (5)$$

where, $d_3 = a + \dfrac{s_h}{s_v}(y_3 - b)$

From the proof in the Appendix, the volume subtended over triangular area AII equals the volume subtended over triangular area BI. Then, the total volume over both areas AII and BI in zone j from point (a,b) in zone i is:

$$2\int_{d_3}^{d_4}\int_{y_3+\frac{s_v}{s_h}(p-d_3)}^{y_4}\frac{q-b}{s_v}\,dq\,dp=\frac{s_h}{s_v}\left(\frac{y_3+2y_4-3b}{3s_v}\right)(y_4-y_3)^2 \qquad (6)$$

where, $d_4=a+\frac{s_h}{s_v}(y_4-b)$

And, the volume subtended by the travel time between location (a,b) and a location in area BII of zone j is:

$$\int_{d_4}^{d_2}\int_{y_3}^{y_4}\frac{p-a}{s_h}\,dq\,dp=\left(\frac{d_2-a}{2s_h}+\frac{y_4-b}{2s_v}\right)(y_4-y_3)(d_2-d_4) \qquad (7)$$

Equations (5), (6) and (7) are summed, then divided by the domain surface area of zone j , i.e. $(d_2-d_1)(y_4-y_3)$, on the rack face in order to get the expected travel time from a specific location (a,b) in region III of zone i to a random location (p,q) in zone j.

When the similar calculations are performed for all of the different regions of the source zone illustrated in figure 7, the entire time volume can be calculated. This geometrical volume connecting zone i and zone j is constructed by considering all points (a,b) in zone i in the (x,y) –plane with the height $E(t_{ij}|(a,b))$ as function of (a,b). This total volume is then divided by the domain (rack) surface of the source zone i, i.e., $(x_2-x_1)(y_2-y_1)$, to get the expected travel time from a random location in the source zone to a random location in the destination zone, i.e. $E(t_{ij})$.

Of course some other shapes of zones than those illustrated in figure 7 exist. However, it can be shown that all such layouts can be reduced to the geometrical outline corresponding to the original layout illustrated in figure 7.

3.3. *Computation of* $E(t_{ii})$

The expected travel time between two random locations within the same storage zone *i* can be determined rather easily. For any random location (a,b), the storage zone can be divided into four parts where the destination random location (p,q) can be located. The expected travel times between the random location (a,b) and the destination random location (p,q) in one of regions 1, 2, 3, or 4 of figure 9 are similar for all areas. Therefore, it is sufficient to illustrate the computation of the expected travel time for region 1.

[Insert figure 9 about here]

To calculate the expected travel time from (a,b) to a destination (p,q) in region 1 of figure 9, the volume subtended over areas AI, AII and BI on the rack face (domain)surface must be computed, i.e.

$$\int_{x_1}^{a}\int_{b}^{y_2} \max\left[\frac{a-p}{s_h}, \frac{q-b}{s_v}\right]$$

The volume above part AI, where the travel time from (a,b) is equal to the horizontal travel time to location (p,q) in part AI, is:

$$\int_{x_1}^{x_3}\int_{y_1}^{y_2} \frac{a-p}{s_h}\, dq\, dp \quad = \quad \left[\frac{2a-x_3-x_1}{2s_h}\right](y_2-y_1)(x_3-x_1)$$

where,

$$x_3 \quad = \quad a-\frac{s_h}{s_v}(y_2-b)$$

Since the volumes above areas AII and BI are equal to each other (see Appendix), and because the travel time between location (a,b) and location (p,q) in area BI is determined by the vertical travel time between these locations, the volume of these two parts is determined from:

$$2\int_{x_3}^{a}\int_{b+\frac{s_v}{s_h}(p-a)}^{y_2}\frac{q-b}{s_v}dqdp = \frac{2(a-x_3)(y_2-b)^2}{3s_v}$$

The volumes above the three areas are summed and divided by the surface area of region 1, i.e. $(y_2\text{-}b)*(a\text{-}x_1)$, to get the expected travel time from location (a,b) to a random location in region 1 of Figure 9. Similar calculations will provide equations for the expected travel times between (a,b) and random locations in the remaining three zones. Similar calculations can be used to construct a geometrical figure for zone *i* located in the the (x,y)-plane and a height is $E(t_{ii})$ as function of (a,b) with (a,b) a point in zone *i*. This total volume of this figure is then divided by the domain (rack) surface of the source zone *i*, i.e. $(x_2\text{-}x_1)(y_2\text{-}y_1)$, to get the expected travel time from a random location in the source zone to a random location in same zone, i.e. $E(t_{ii})$.

Thus, for any given layout of storage zones in a rack face, the calculations illustrated in the foregoing sections may be used to compute the values of $E(t_i)$, $E(t_{ij})$, and $E(t_{ii})$ in the general S/R machine cycle time model of equation (1). Specifically, these values of $E(t_i)$, $E(t_{ij})$, and $E(t_{ii})$, the x and y coordinates of the corners of all storage zones, the percentages of storage and retrievals for each storage zone, p_i, and the fraction of single cycle commands, *f*, provide all information necessary to compute the expected S/R machine cycle time per operation.

Due to the tedious mathematical evaluations required by this methodology, it has been coded in Turbo Pascal [Veraart (1995) and Valkenburg (1997)]. The resulting code requires only a few seconds to compute the expected S/R machine cycle time for a given storage layout design.

4. Model validation and results

In order to validate this modeling approach, optimal class boundaries for a single command (SC) square-in-time (SIT) AS/R system from Hausman *et. al.* (1976) and the class boundaries in Graves *et. al.* (1977) for a dual command (DC) SIT system, were used. The rack layout shown in figure 10 was specified for this model where zone 1 is the first class, zones 2 and 3 together constitute the second class, and zones 4 and 5 together constitute the third class. For a two-class rack layout, only the first three zones are needed.

[Insert figure 10 about here]

The fraction of movements to the second class storage is divided among zones 2 and 3 in proportion to the surfaces of each zone. The same applies to the allocation among zones 4 and 5 comprising class three.

In evaluating system performance for single command cycles only, the fraction of single command cycles in equation (1), f, is set to one. And, for evaluating system performance for dual command cycles only, this fraction is set to zero.

Table 1 shows the results of both the geometry-based analytical model of this paper (Model) along with the corresponding results of Hausman *et. al.* (1976) and the results of Graves *et. al.* (1977). For single command cycles, the table shows the expected travel time from the I/O point to a random location, i.e. half the expected cycle time, $E(t_i)$. Further, in the first column of table 1, the ratios such as 20/60 means that 20% of the items in inventory represents 60% of total demand, and so forth.

Table 1 shows that the results from the model proposed herein are very similar to the results of Hausman *et. al.* (1976) and Graves *et. al.* (1977). This is not surprising because all of the methods are analytical and should therefore generate the same results. The small differences between the tabular column entries can be attributed to rounding errors. So, it appears that the model proposed herein provides valid results for both single- and dual-command SIT systems.

In order to illustrate one of the real advantages of the geometric modeling approach, consider the three different layouts of the rack face shown in figure 11. In addition, assume that the S/R speeds are such that the racks are non-square in time. Specifically, assume that T = 1.25 and b = 0.64, where T is the maximum time required by the S/R machine to reach the most distant location in the rack from the I/O point and b is the shape factor, as defined in Bozer and White (1984). The only restriction on the layout zones to make this model applicable in analyzing an AS/R system is that the rack face be divided into a number of rectangular zones.

[Insert figure 11 about here]

The results from the application of the geometric model to these three layouts are presented in table 2. For layout III in figure 11, only the results for the three-class scenario are presented because if the third class is not present, layouts II and III are equivalent, and the results for the two-class layouts are identical.

For comparison purposes, the model was also executed for each of the three layouts assuming that the racks are square-in-time, i.e., T = 1 and b = 1. The percentage differences with regard to the SIT results, assuming L-shaped classes, compared to the NSIT results are given in parentheses beneath each entry in table 2 for comparison purposes.

From the results in table 2 it is clear that it makes a great difference how the storage zones in an AS/R system are laid out. The expected cycle times for NSIT systems designed as shown in figure 11 are significantly larger than the expected cycle times of a SIT systems, as defined by Hausman *et. al.* (1976) and Graves *et. al.* (1977). Therfore, whenever feasible, AS/RS designers should choose equipment or rack configureations that result in SIT systems in order to optimize cycle time. But, as shown in table 2, estimating the expected cycle times of layouts I, II and III using a model assuming only a SIT situation would result in erroneous throughput estimates, especially for systems of layout types I or II in figure 11.

5. Conclusions

In practice, the operation of many automated storage and retrieval systems do not satisfy all the assumptions of the analytical models found in the literature. The geometrically based model presented in this paper can be used by designers as a tool for quickly evaluating alternative layout configurations with respect to expected S/R cycle time in an AS/RS, and thereby the throughput of the warehouse over time. The approach is intuitively appealing, and it does not assume any certain layout shape, such as traditional "L-shaped" class layouts. It can be used for any distribution of demand, can handle rack layouts that are either square-in-time or non-square-in-time, and considers both single-command and dual-command transactions.

This model has been successfully applied in a major manufacturing plant in Europe to evaluate reconfigurations of their rack storage layouts over the past five years.

Further work has also been done in applying the approach to evaluate systems with multiple input/output locations. Work is also directed toward streamlining the difficult practical problem of assigning goods to classes and the layout of storage zones in the rack structure.

6. Appendix: Equality of volumes

To prove: The volumes above areas A and BI in figure 12 are equal.

[Insert figure 12 about here]

Divide the volumes above both parts in two volume parts for each part A and BI: a part under height d, and a part above height d. The parts under height d are two halves of the same block. So it is obvious that the volumes of these are equal. Then there are two pyramids above height d.

For the pyramids above height d, we apply: $volume = \frac{1}{3} \times surface\ base \times height$

The pyramid above part A in figure 12 has:

$surface\ base = (x_3-x_1)*(h-d)$, and $height = y_2-y_3$. Thus, this pyramid has

$volume = (x_3-x_1)*(y_2-y_3)*(h-d)/3$.

Similarly, the pyramid above part BI has: $surface\ base = (y_2-y_3)*(h-d)$, and $height = x_3-x_1$. Therefore, this pyramid also has $volume = (x_3-x_1)*(y_2-y_3)*(h-d)/3$. This is a general result for all volumes that are defined in a similar manner.

7. References

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Figures and Tables

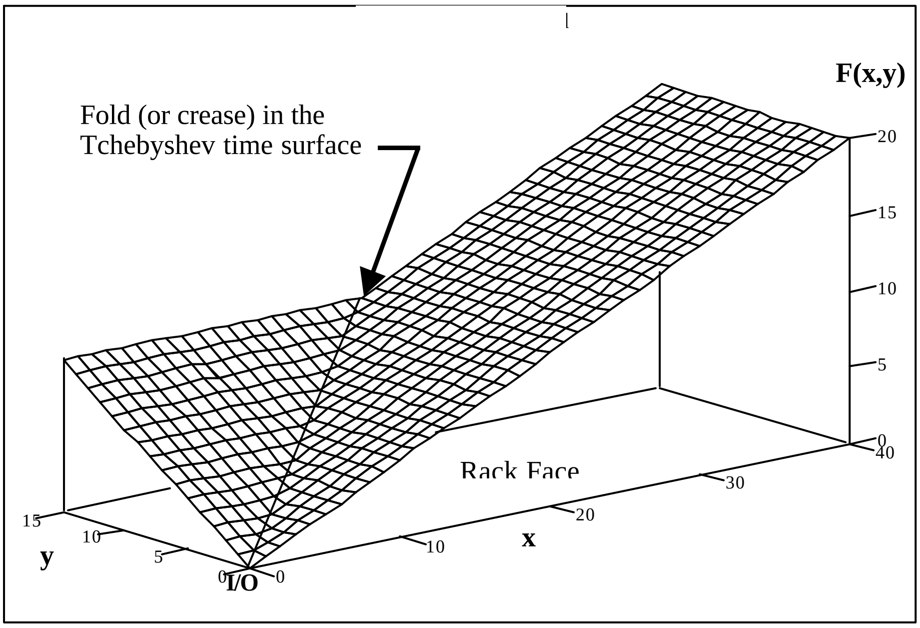


Figure 1. Plot of Tchebyshev time surface, F(x,y), showing the time required by the S/R machine to reach any point on a rack face in the (x,y)-plane from the I/O point (0,0)

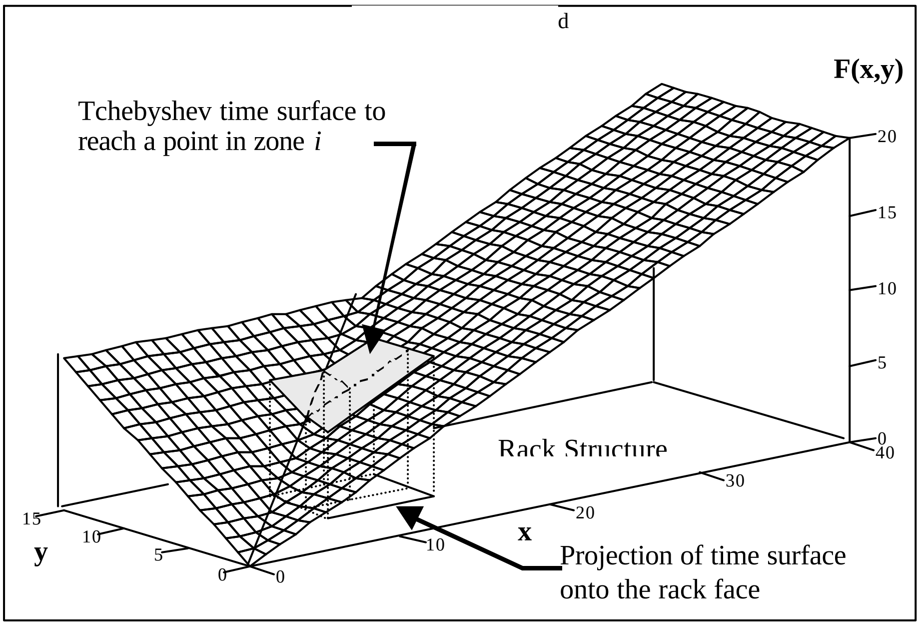


Figure 2. Plot of Tchebyshev time, F(x,y), required by the S/R machine to reach any point in storage zone *i* in the (x,y)-plane from the I/O point at (0,0)

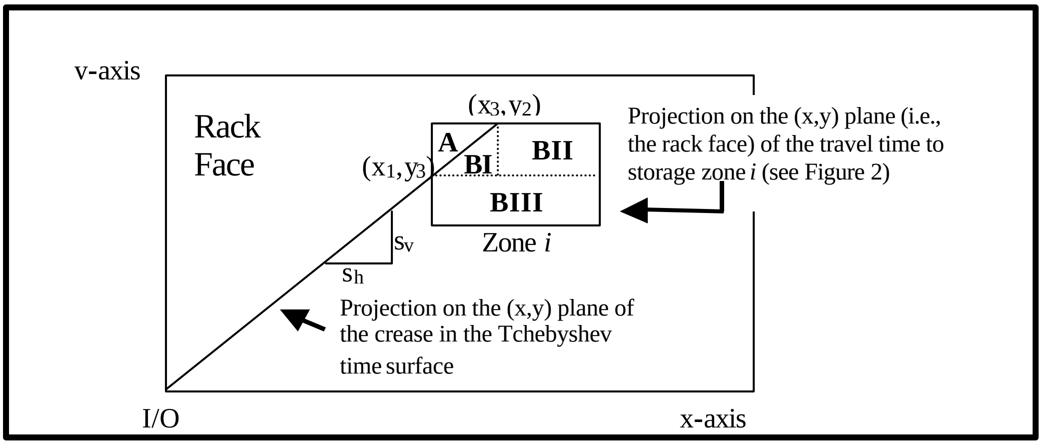


Figure 3: Projection onto the rack face of the time surface to reach a storage location in zone *i*

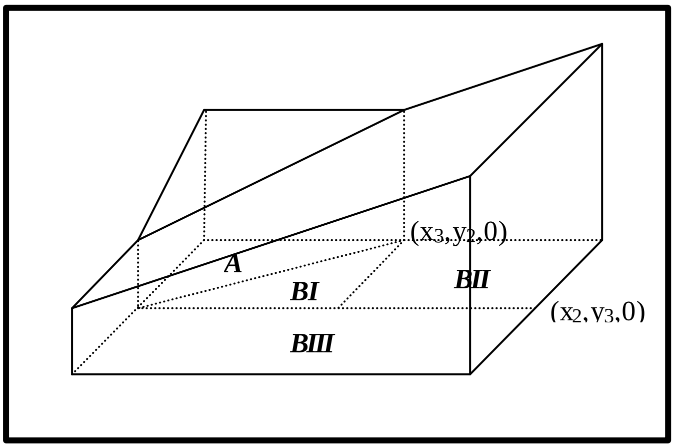


Figure 4. "Free-body" diagram of zone *i*, its projection onto the rack face, and the subtended volume from Figure 2

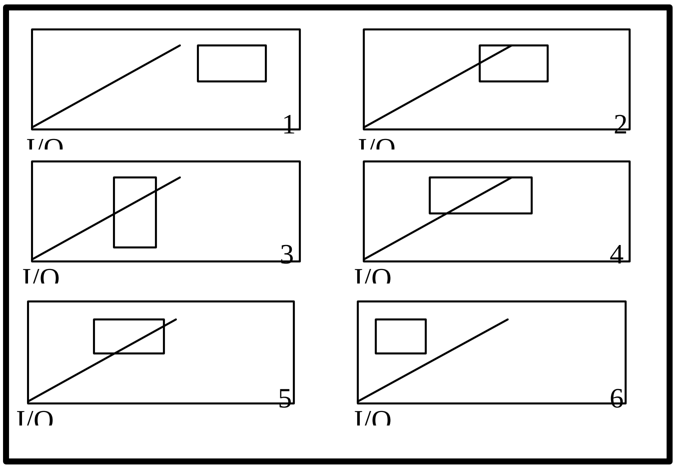
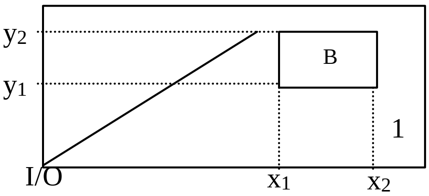
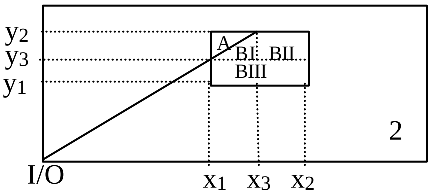


Figure 5. Possible orientations of storage zones with respect to the crease in the Tchebyshev time travel surface



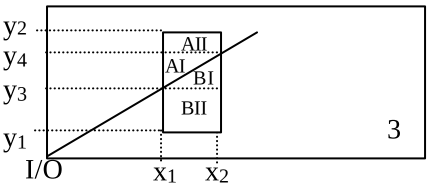
$$\text{Vol. B} = \frac{1}{2s_h}(y_2 - y_1)(x_2^2 - x_1^2)$$



$$\text{Vol. A \& BI} = \frac{1}{s_v} y_2^2 (x_3 - x_1) - \frac{s_v}{3s_h^2}(x_3^3 - x_1^3) : y_2 = \frac{s_v}{s_h} x_3$$

$$\text{Vol. BII} = \frac{1}{2s_h}(y_2 - y_3)(x_2^2 - x_3^2) : y_3 = \frac{s_v}{s_h} x_1$$

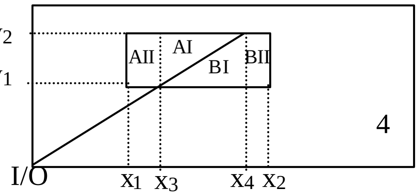
$$\text{Vol. BIII} = \frac{1}{2s_h}(y_3 - y_1)(x_2^2 - x_1^2)$$



$$\text{Vol. AI \& BI} = \frac{1}{s_v} y_4^2 (x_2 - x_1) - \frac{s_v}{3s_h^2}(x_2^3 - x_1^3) : y_4 = \frac{s_v}{s_h} x_3$$

$$\text{Vol. AII} = \frac{1}{2s_v}(x_2 - x_1)(y_2^2 - y_4^2)$$

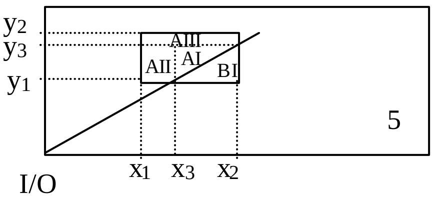
$$\text{Vol. BII} = \frac{1}{2s_h}(y_3 - y_1)(x_2^2 - x_1^2) : y_3 = \frac{s_v}{s_h} x_1$$



$$\text{Vol. AI \& BI} = \frac{1}{s_v} y_2^2 (x_4 - x_3) - \frac{s_v}{3s_h^2}(x_4^3 - x_3^3) : y_2 = \frac{s_v}{s_h} x_4$$

$$\text{Vol. BII} = \frac{1}{2s_h}(y_2 - y_1)(x_2^2 - x_4^2) : y_1 = \frac{s_v}{s_h} x_3$$

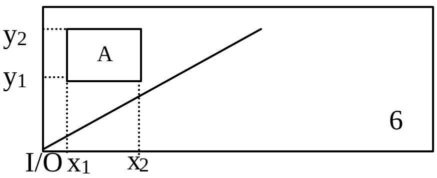
$$\text{Vol. AII} = \frac{1}{2s_v}(x_3 - x_1)(y_2^2 - y_1^2)$$



$$\text{Vol. AI \& BI} = \frac{1}{s_v} y_3^2 (x_2 - x_3) - \frac{s_v}{3s_h^2}(x_2^3 - x_3^3) : y_3 = \frac{s_v}{s_h} x_2$$

$$\text{Vol. AII} = \frac{1}{2s_v}(x_3 - x_1)(y_3^2 - y_1^2) : y_1 = \frac{s_v}{s_h} x_3$$

$$\text{Vol. AIII} = \frac{1}{2s_v}(x_2 - x_1)(y_2^2 - y_3^2)$$



$$\text{Vol. A} = \frac{1}{2s_v}(x_2 - x_1)(y_2^2 - y_1^2)$$

Figure 6. Equations for the volumes for each of the six orientations in Figure 5.

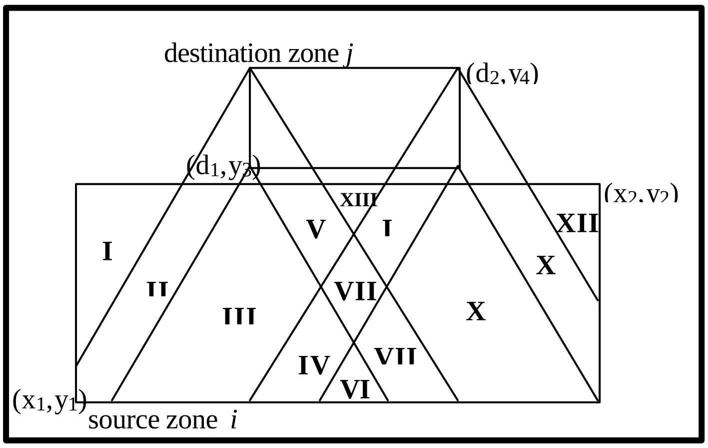


Figure 7. Relation of a source storage zone i to a destination storage zone j in the rack face

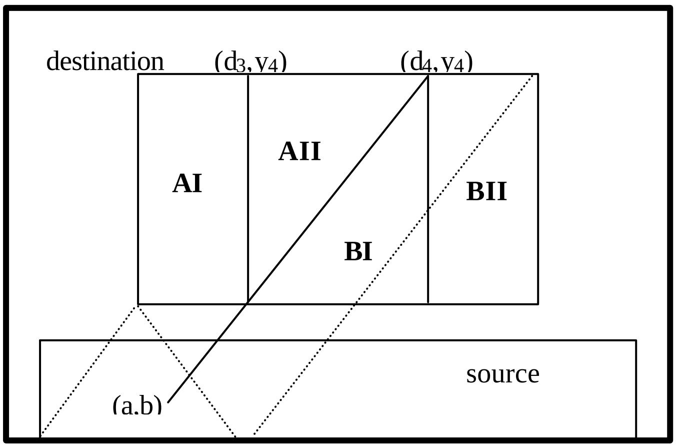


Figure 8. Example computation of a component of the expected time to travel between locations in two storage zones

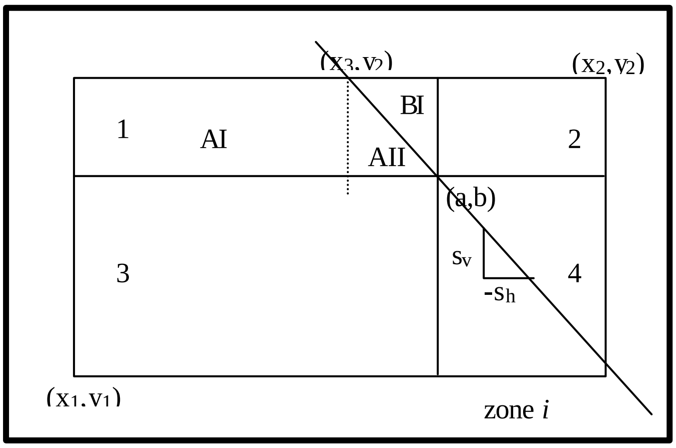


Figure 9. Example computation of the expected time to travel between random locations in the same storage zone *i*

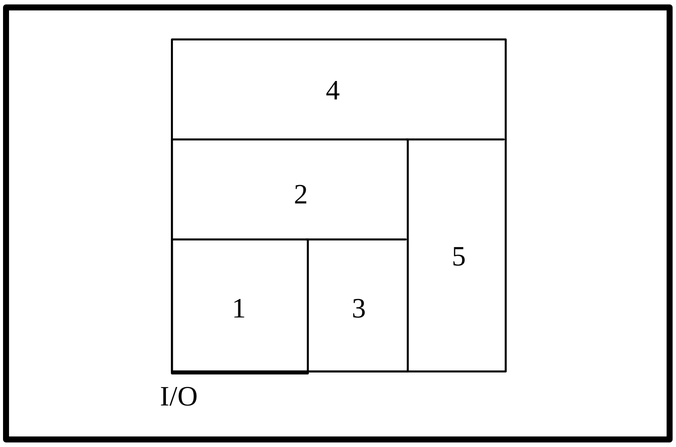


Figure 10. Class layout structure comprised of rectangular storage zones used for model validation

Table 1. Results of the model validation study

Class Dist.	Model (SC)	Hausman (SC)	Model (DC)	Graves (DC)
20/60				
2 classes	0.5459	0.546	1.5375	1.537
3 classes	0.5176	0.518	1.4811	1.481
20/70				
2 classes	0.4966	0.497	1.4255	1.425
3 classes	0.4573	0.457	1.3436	1.343
20/80				
2 classes	0.4273	0.427	1.2614	1.261
3 classes	0.3750	0.375	1.1455	1.145
20/90				
2 classes	0.3143	0.314	0.9757	0.976
3 classes	0.2500	0.250	0.8160	0.816

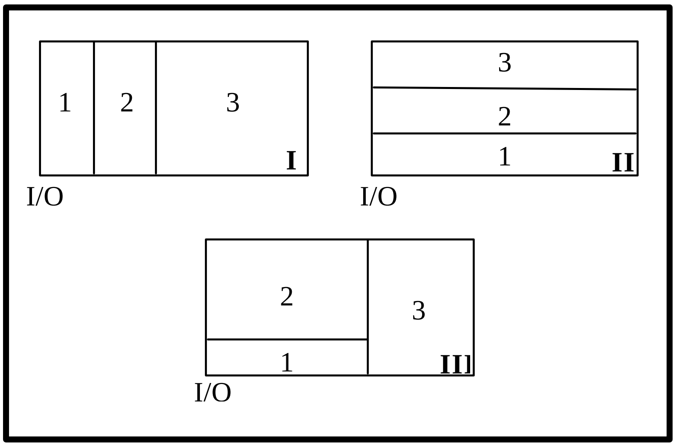


Figure 11. Three different non-square in time rack layouts

Table 2. Geometric model results

	I, SC	II, SC	III, SC	I, DC	II, DC	III, DC
20/60						
2 classes	1.2358 (+13.2%)	1.3698 (+25.5%)	xxxxx	1.7218 (+12.0%)	1.8615 (+21.1%)	xxxxx
3 classes	1.1989 (+15.8%)	1.3595 (+31.3%)	1.1314 (+9.3%)	1.6882 (+14.0%)	1.8521 (+25.0%)	1.6074 (+8.5%)
20/70						
2 classes	1.1810 (+18.9%)	1.3547 (+36.4%)	xxxxx	1.6501 (+15.8%)	1.8419 (+29.2%)	xxxxx
3 classes	1.1310 (+23.6)	1.3408 (+46.6%)	1.0207 (+11.6%)	1.6094 (+19.8%)	1.8305 (+36.2)	1.4823 (+10.3%)
20/80						
2 classes	1.1132 (+30.3%)	1.3361 (+56.3%)	xxxxx	1.5682 (+24.3%)	1.8197 (+44.3%)	xxxxx
3 classes	1.0513 (+40.2%)	1.3190 (+75.8%)	0.8716 (+16.2%)	1.5033 (+31.2%)	1.8003 (+57.2%)	1.3014 (+13.6%)
20/90						
2 classes	1.0210 (+62.4%)	1.3108 (+108.5%)	xxxxx	1.4464 (+48.2%)	1.7850 (+82.9%)	xxxxx
3 classes	0.9568 (+91.4%)	1.2931 (+158.6%)	0.6411 (+28.2%)	1.3578 (+66.4%)	1.7575 (+115.4%)	1.0152 (+24.4%)

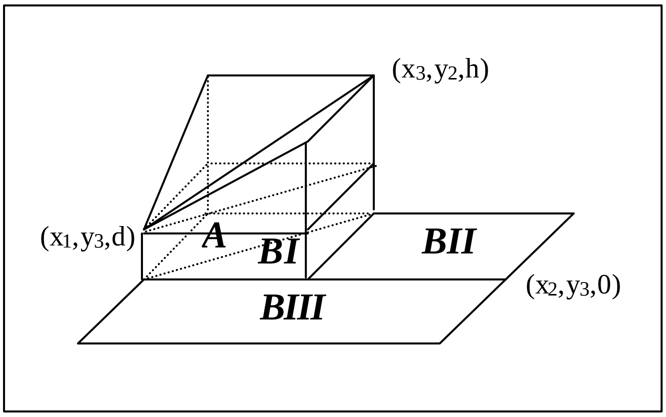


Figure 12. Equal volumes above A and BI