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Abstract
Traditional resource economics has been criticised for assuming too high elasticities of substitution, not observing material balance principles and relying too much on planner solutions to obtain long-term growth. By analysing a multi-sector R&D-based endogenous growth model with exhaustible natural resources, labour, knowledge, and physical capital as inputs, the present paper addresses this critique. We study transitional dynamics and the long-term growth path and identify conditions under which firms keep spending on research and development. We demonstrate that long-run growth can be sustained under free market conditions even when elasticities of substitution between capital and resources are low and the supply of physical capital is limited, which seems to be crucial for today's sustainability debate.

Keywords : Growth, non-renewable resources, substitution, investment incentives, endogenous technological change, sustainability

JEL-Classification : Q20, Q30, O41, O33

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1. **Introduction**

Steady accumulation of man-made capital is the basic source of economic growth. Capital investments dropped substantially during the high-price period on markets for non-renewable resources in the 1970s. Since then, known stocks of resources have increased due to discoveries, prices have moderated, and investments have recovered. But given the finiteness of resources such as fossil fuels and precious minerals in the long run, will resource scarcity limit the future level of capital accumulation? This question is crucial in the context of sustainability, because, to maintain or increase consumption, capital has to substitute for resources in production. As soon as investments become critically low, future generations' welfare levels fall behind those of today's generations, which means that long-term development no longer satisfies the sustainability criterion.

Limited substitution of man-made capital for non-renewable natural resources may be the main obstacle to sustainable development. Most ecological economists argue that traditional economic theories are overly optimistic in this respect. Three major issues are under debate. First, to obtain unbounded growth in the standard neoclassical model, it has to be assumed that either the elasticity of substitution between natural resources and man-made capital is at least unity, or that exogenous resource-augmenting technological progress occurs at a constant rate, see the seminal papers of Dasgupta and Heal (1974) and Stiglitz (1974). Second, while many economic models rely on unlimited accumulation of man-made capital, ecological economists emphasise that material balances limit the use of physical capital in the long run, see Cleveland and Ruth (1997). Third, while sustained growth may be technically feasible, it is not necessarily reached under free market conditions. Low investment incentives and externalities may result in (too) little investment efforts in capital that substitutes for resources. Moreover, myopic behaviour may prevent the implementation by today's generations of policy measures that are needed to obtain sustainability for future generations.

The present paper reconsiders investment incentives and the limits to growth in the presence of non-renewable resource scarcity in a multi-sector endogenous model. We rule out exogenous technological change that offsets resource depletion as "manna from heaven", we bound the total supply of physical capital to take into account material balances, and we concentrate on market equilibria in which rates of return drive investment in physical capital and research and development. We show that growth can be sustained even if elasticities of substitution are small. The multi-sector structure of the model allows us to identify how substitution between sectors works as an additional mechanism for the substitution of natural resources. In particular, it will be shown that the effects on growth depend on which sector of the economy has poor or abundant substitution possibilities.

We focus on investment in knowledge capital through research and development (R&D) as the main engine of growth. R&D is often aimed at improving production techniques and thus at increasing capital productivity. For a large part of the economy, knowledge is only effective if it is embodied in certain types of physical capital. The decisive question, which is not answered yet in literature, reads: is it realistic to predict that knowledge accumulation is so powerful as to outweigh the physical limits of physical capital services and the limited substitution possibilities for natural resources? Only if the answer is positive the economy can provide a

constant or steadily increasing level of average individual utility in the long run, which is required for development to become sustainable. To eliminate hypothetical technical solutions for substitution, we focus on market outcomes to find out whether market incentives are strong enough to produce a sustainable level of investments in knowledge capital.

The present paper builds on various contributions in literature. The neoclassical literature on resource economics started with the seminal symposium issue of the Review of Economic Studies, 1974. This tradition stresses that the marginal returns on capital (and thus investment incentives) decrease when substitution possibilities between physical capital and exhaustible natural resources are low. In particular, Dasgupta and Heal (1974, 1979) show that, without technical progress and with an elasticity of substitution between natural resources and man-made capital being lower than unity, sustainable production is impossible. Indeed, if the elasticity of substitution is zero, which is implicitly assumed in the popular contribution of Meadows et al. (1972), the economic collapse is inevitable. Furthermore, Solow (1974) derives that sustainable production may be feasible without technical progress, whenever the elasticity equals unity. Provided that the production elasticity of capital exceeds the elasticity of natural resources, an appropriate constant share of income spent for savings leads to a constant flow of income, see Hartwick (1977). However, as soon as savings depend on interest rates, this result will not be obtained under free market conditions. Hence, an equilibrium with non-declining income is neither optimal nor does arise in a market equilibrium.

Taking into account technical change leads to less pessimistic results. In Stiglitz (1974), exogenous technical progress leads to sustained growth, which is feasible and optimal, provided that the elasticity is unity and the discount rate is not too high. Introducing exogenous resource-augmenting technical progress, sustained growth becomes feasible even when the elasticity is lower than unity; see Dasgupta and Heal (1979, p. 207). The optimistic view on technology in neoclassical models is criticised by Cleveland and Ruth (1997), who argue that in reality substitutability is low, that the continuity of technical progress is uncertain, and that the accumulation of physical capital is ultimately limited by biophysical constraints. Under these assumptions, sustainability indeed becomes a more demanding goal.

Long-run growth does not rely on exogenous technological change in the so-called new (or endogenous) growth theory developed in the 1990s. The endogenous accumulation of knowledge and human capital supplements the accumulation of physical capital as an engine of growth. The broadened view of man-made capital, the hypothesis of positive knowledge spillovers, and the assumed constant returns to aggregate capital inputs provide new perspectives on substitution of man-made capital for natural resources and sustained investment incentives.

For the case a unitary elasticity of substitution between capital and resource inputs, endogenous knowledge accumulation yields sustained optimal growth in the presence of non-renewable resources (Schou 1999, Scholz and Ziemes 1999, Aghion and Howitt 1998, Grimaud and Rougé 2003, Groth and Schou 2002). In the two-sector endogenous growth model with resource-augmenting knowledge production of Bovenberg and Smulders (1995), unbounded growth is feasible with elasticities of substitution between capital and resources that are smaller than unity. However, the model only applies to renewable resources. This is also true for Bretschger (1998),

who shows that lower resource use is compatible with sustained endogenous growth under free market conditions even when technical change is unbiased and the elasticity of substitution in production is smaller than unity.

The model of the present paper has the following special features.

First, we distinguish between two types of physical capital and knowledge capital. Physical capital stocks are a direct (but limited) substitute for natural resources whereas knowledge capital is directed at improving the efficiency of capital in one of the two consumer sectors. Accordingly, we call this consumer sector the "knowledge-using" sector. It is assumed that new ideas need to be embodied in a certain physical body before they can be used as a substitute for a natural resource. The heterogeneity of physical capital opens the possibilities of productivity gains in the knowledge-using sector by increasing division of labour. Applying the idea of expansion-in-varieties, this part of the model builds on Romer (1990) and Grossman and Helpman (1991). The increasing division of labour is assumed to have a similar effect on the efficiency of natural resource use, which is a scale effect that has to be distinguished from the basic substitution effect. In the other consumer sector, capital productivity depends on the input of skilled labour. As skilled labour is also an important input into R&D, which produces knowledge through positive spillovers, this sector is called "knowledge-competing" sector. Deviating from unitary elasticities, it will become evident that in the knowledge-using sector, a high elasticity of substitution fosters economic growth. In the knowledge-competing sector, however, a low elasticity is more favourable for long-term development.

Second, we introduce a non-renewable resource, which represents oil, precious metals, minerals etc. The resource is used in combination with physical capital to produce the two final outputs in the knowledge-using and the knowledge-competing sector. Natural resources are thus important for both sectors. Accordingly, a double-tracked input substitution process is modelled below.

Third, we emphasise the distinction between the market value and the production cost of additional designs. This alludes to Tobin's q-theory of investment. Profit expectations in research are a variable that explicitly depends on the price of physical capital and, indirectly, on the price of the natural resource. On the other hand, costs of inventions are determined in a separate R&D sector, where skilled labour and knowledge are used as inputs. As skilled labour is also used in combination with the natural resource to produce final output, production cost in the research lab depend on resource prices.

Fourth, we do not exclusively focus on balanced growth paths but also look at adjustment paths leading to long-run equilibria. During adjustment, goods and factor prices as well as relative sectoral outputs are allowed to vary. It is shown how the size of the sectors and the growth rate converge to long-run values according to the assumed parameters for substitution.

The remainder of the paper is organized as follows. In section 2, the theoretical five-sector model of the economy with two consumer goods is presented in detail. Section 3 shows how the model can be solved. Section 4 provides results for transitional dynamics and long-run growth for different types of parameter and substitution conditions. Section 5 concludes.

2. The model

2.1 Overview

We introduce three primary input factors: a non-renewable natural resource *R* as well as skilled labour *S* and unskilled labour *L*, see *figure 1*. Unskilled labour produces differentiated physical capital components, which are assembled to an aggregate physical capital stock K_Y. Skilled labour is employed in two activities. First, the development of designs for new capital components requires skills. Second, skilled labour produces the physical capital good K_T. The natural resource is combined with the two capital stocks and produces "standard" *T*-goods in the knowledge-competing sector and "high-tech" *Y*-goods in the knowledge-using sector. The substitution of capital for the natural resource takes place in both sectors: in the high-tech sector, which determines the reward to research investments, and in the standard sector, which affects wages of skilled labour and therefore production costs of new designs. Knowledge capital is accumulated through positive spillovers in research and is an input into subsequent R&D; it is the driving force for long-run development.

The essential elements of our model set-up are, first, substitution across sectors between goods that differ in their knowledge intensity and, second, (poor) substitution between man-made inputs and resources within sectors. The degree of within-sector substitution may differ across sectors. By combining these different mechanisms, the model is suited to capture the basic substitution process in modern economies.

Fig. 1
(about here)

2.2 Production sector

Let us discuss the different productive sectors of the model in turn, beginning with high-tech goods, followed by standard goods, the two types of capital goods, and, finally, R&D.

High-tech goods Y are produced with different physical capital services *k* and natural resources *R* as inputs. We adopt a nested CES-function. The constant elasticity of substitution between aggregate capital and natural resources σ in the *Y*-sector take any positive value. Aggregate capital input is a CES-index of a continuum of differentiated components of mass *n*; the constant elasticity of substitution between them equals $1/(1-\beta)>0$. Each producer in the high-tech sector uses all types of components as well as natural resources, according to:

$$Y = \left[\bar{\theta} \left(\int_0^n k_j^{\beta} dj \right)^{\frac{\sigma-1}{\beta\sigma}} + (1-\bar{\theta}) \left(n^{\delta} \cdot R_Y \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \qquad (1)$$

where $0<\beta,\bar{\theta}<1$, and $\delta,\sigma>0$ are given parameters. In a symmetrical equilibrium, the quantities of capital services *k* are equal for the different components, i.e. $k_1=k_2=...=k_n=k$. With *n* different components at a certain point of time, aggregate input of capital services is denoted by

$$K_Y = n \cdot k \,. \tag{2}$$

Due to gains from specialization, an expansion-in-varieties of capital components leads to productivity gains of *Y*-producers. According to (1), holding aggregate capital input K_Y constant, the production of the high-tech goods increases with the number of capital components, *n*. In a similar way, it is reasonable to argue that *Y*-producers are able to use natural resources the more efficiently, the higher is the specialization of capital components a given amount of resources is combined with. We will come back to this point when discussing the general results at the end of section 4. The gains from specialization for both types of inputs can be seen more clearly when reformulating (1) by using (2) to get:

$$Y=\left[\bar{\theta}\left(n^{\frac{1-\beta}{\beta}}\cdot K_Y\right)^{\frac{\sigma-1}{\sigma}}+(1-\bar{\theta})\left(n^{\delta}\cdot R_Y\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

The specialization effect of an additional capital variety is given by $(1-\beta)/\beta$ whereas the efficiency gain of resources *R* used with additional varieties is expressed by the factor δ. If $\delta=0$, there is no efficiency gain from resource use. Rearranging the above equation, we find:

$$Y = n^{(1-\beta)/\beta}\cdot Q = n^{(1-\beta)/\beta}\cdot\left[\bar{\theta}\,K_Y{}^{(\sigma-1)/\sigma}+(1-\bar{\theta})(n^{-\nu}R_Y)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \tag{3}$$

with $\nu=(1-\beta)/\beta-\delta$. The expression in brackets on the r.h.s. of (3) corresponds to the familiar CES-approach of resource economics, see Dasgupta/Heal (1979, p. 199). It aggregates capital inputs K_Y and effective resource inputs $n^{-\nu}R_Y$ into a composite input *Q*. The total factor productivity term preceding brackets is the result of the expansion-in-varieties approach according to Romer (1990) and Grossman/Helpman (1991). Note that the degree of specialization (measured by *n*) affects productivity in two ways. First, it raises total factor productivity. Second, it introduces a bias in technological change: if $(1-\sigma)\nu < 0$ technological change is capital-using; if $(1-\sigma)\nu > 0$, it is resource-using.

The market for Y-goods is fully competitive. Producers take prices of output, resource inputs and capital components (denoted by p_Y, p_R and p_{kj}, respectively) as given. They maximize total profits $p_Y Y - p_R R_Y - \int_0^n p_{kj}k_j\,dj$, subject to the production function (1). Under symmetry ($p_{kj}=p_{kY}$), see (2), relative demand for capital and energy is given by:

$$\frac{K_Y}{R_Y} = \left(\frac{\bar{\theta}}{1-\bar{\theta}} \right)^{\sigma} \left(\frac{p_{kY}}{p_R} \right)^{-\sigma} n^{-(1-\sigma)\nu} . \tag{4}$$

The production of *standard goods T* requires homogenous capital K_T and resources R_T as inputs. We use again a CES-formulation:

$$T = \left[\bar{\eta} \cdot K_T^{\frac{\omega-1}{\omega}} + \left(1-\bar{\eta}\right) R_T^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \tag{5}$$

with ω being the elasticity of substitution in the *T*-sector and $0 < \bar{\eta} < 1$. Producers take prices as given and maximize profits $p_T T - p_{KT} K_T - p_R R_T$ subject to (5). This gives relative factor demand:

$$\frac{K_T}{R_T} = \left(\frac{\bar{\eta}}{1-\bar{\eta}} \right)^{\omega} \left(\frac{p_{KT}}{p_R} \right)^{-\omega} \tag{6}$$

The *differentiated capital services k* are produced by monopolists. The production of one unit requires one unit of unskilled labour *L*. The profit maximizing monopolistic supplier faces a price elasticity of demand equal to –1/(1 – β). As in the standard Dixit-Stiglitz approach, this follows from the Y-producers demand for *k*. Thus, the monopolist optimally sets a rental rate that is a mark-up 1/β times the labour costs w_L. All monopolistic suppliers set this same price:

$$p_{kY} = w_L / \beta \tag{7}$$

Associated profits π for each supplier of a capital components can be calculated as:

$$\pi = (1-\beta) \cdot p_{kY} \cdot K_Y / n \ . \tag{8}$$

Profits are used to cover the expenses for fixed costs in the production of *k*-goods, which consist of payments for the blueprint of the capital component. Each design contains the know-how for the production of one capital component *k*. Thus, each *k*-firm has to acquire one design as an up-front investment before it can start production.

New blueprints $\dot{n}$ are produced in the *R&D sector*. Per blueprint, *a/n* units of skilled labour are required. Here it is assumed that an increase in variety also increases the stock of public knowledge on which R&D builds so that research cost decline with *n*. Thus, the cost of a blueprint, c_n, equals:

$$c_n = (a/n) \cdot w_S \tag{9}$$

The production of one unit of homogenous capital K_T used in the *T*-sector requires skilled labour S_T and materials inputs *M* according to a Cobb-Douglas production

function $K_T = S_T^{\upsilon} M^{\mu}$. We assume that materials are fully recyclable, such that each moment in time a fixed stock, of which we normalize the size to $M = 1$, is available. For simplicity, we disregard recycling costs, set $\upsilon = 1$, and assume perfect competition. The price of capital then equals the wage of skilled labour w_S:

$$p_{KT} = w_S . \tag{10}$$

2.3 Capital markets

There are three assets i.e. investment possibilities in the economy: riskless bonds, patents for the production of capital varieties *k*, and natural resources *R*. Let p_n denote the market price of a patent in time *t* and consider the brief time interval between *t* and *t*+*dt*. In this time, the return for an investment of size p_n in a bond is $r \cdot p_n \cdot dt$. A firm holding a patent for the production of an intermediate capital input earns an infinite stream of profits. Per-period profit π is given in (8). So in the same time interval, the total return on a patent is $\pi \cdot dt + \dot{p}_n \cdot dt$ which yields the following no-arbitrage condition:

$$\pi + \dot{p}_n = r \cdot p_n \tag{11}$$

Perfect capital markets affect the timing of R&D. Over a small period *dt* with positive investment in R&D, inventors should be indifferent between incurring the R&D cost c_n at the beginning or the end of the period. That is, the return to postponing investment, which amounts to interest payments on postponed costs $rc_n dt$, should equal the cost of postponing, which amount to forgone dividends and change $(\pi + \dot{c}_n) \cdot dt$. However, if over a small period the net benefits of postponing investment are positive, no investment will take place in this period. Hence, we may write the following no-arbitrage condition:

$$\pi + \dot{c}_n \leq r \cdot c_n \qquad \text{with equality (inequality) if } \dot{n} > 0 \ (\dot{n} = 0). \tag{12}$$

The final no-arbitrage condition concerns the comparison of returns between bonds and stocks of natural resources. Resource owners extract resources without costs and supply them at spot prices, which are denoted by p_R per unit of *R*. The returns for investments of size p_R in bonds during the brief time interval between *t* and *t*+*dt* are $r \cdot p_R$. In analogy to above, the return per unit of the stock of natural resource *R* can be expressed as $\pi_R \cdot dt + \dot{p}_R \cdot dt$. However, it is a basic characteristic of natural resources to have no direct return like capital goods, i.e. it is that $\pi_R = 0$. So in equilibrium we are left with:

$$\dot{p}_R = r \cdot p_R \tag{13}$$

The Hotelling-rule in (13) implies that resource owners are exactly indifferent between selling resources (and investing the profit with interest rate r) and preserving the stock of resources. The compensation for keeping the stock is the price rise of the resource.

2.4 Factor markets

The total stock of resource *R* at time *t* is denoted by W_R. It is depleted according to:

$$\dot{W}_R = -R_Y - R_T, \qquad W_R(0) \text{ given}, \; W_R(t) \geq 0, \tag{14}$$

$$\int_0^\infty \left(R_Y(t) + R_T(t)\right)dt = W_R(0), \tag{15}$$

which says that any flow of resource use depletes the total resource stock proportionally, that the resource stock is predetermined, and that the stock can never become negative. Profit maximization by resource owners implies that the price of natural resources increases at a rate equal to the interest rate, see (13). Resource owners do not have an incentive to conserve a part of the stock so that total extraction must be equal to the total resource stock in equilibrium, see (15). Total extraction can be ensured by setting the optimum price at the beginning of optimisation. Initial price level and price increase of natural resources do not deviate from the optimum path, provided that agents form rational expectations. This will be assumed in the following.

The market for skilled labour is in equilibrium if the fixed supply *S* equals demand for production of capital and for research labour:

$$S = K_T + (a/n)\cdot \dot{n} \tag{16}$$

The market for unskilled labour clears, which requires that demand for unskilled labour from the differentiated capital goods sector equals the fixed supply *L*:

$$L = K_Y. \tag{17}$$

2.5 Consumer sector

The representative household maximizes a lifetime utility function subject to the usual intertemporal budget constraint; the function is additively separable in time and contains logarithmic intratemporal utility of the Cobb-Douglas type:

$$U(t) = \int_t^\infty e^{-\rho(\tau - t)} \log C(\tau) d\tau \qquad \rightarrow \max \tag{18}$$

$$\text{with } C = Y^{\phi} \cdot T^{1-\phi} \tag{19}$$

The Cobb-Douglas specification in (19) implies constant expenditure shares for *T*- and *Y*-goods. Accordingly, relative demand for final goods is given by:

$$\frac{p_Y \cdot Y}{p_T \cdot T} = \frac{\phi}{1-\phi}. \tag{20}$$

where $p_T(, p_Y)$ is the price of T-goods (Y-goods). Intertemporal optimisation gives the well-known Keynes-Ramsey rule stating that the growth rate of consumer expenditures is equal to the difference between the nominal interest rate *r* and the discount rate ρ, that is:

$$\hat{p}_T + \hat{T} = r - \rho \,. \tag{21}$$

where hats denore growth rates.

Finally, intertemporal utility maximization requires that the value of household wealth, np_n, discounted by the interest rate, *r*, approaches zero if time goes to infinity. This transversality condition can be written as:

$$\lim_{t\to\infty} \hat{n}(t) + \hat{p}_n(t) - r(t) < 0 \,. \tag{22}$$

3. Solving the model

To facilitate the analysis, we reduce the model to three differential equations, which characterize the dynamics of the system. Our three key variables will be the capital shares in the knowledge-using and knowledge competing sectors, to be denoted by θ and η respectively, and the growth rate of product variety, also to be interpreted as the rate of innovation or the growth rate of the (public) knowledge stock, to be denoted by *g*. Hence, we define:

$$\theta = \frac{p_{kY} K_Y}{p_Y Y} \tag{23}$$

$$\eta = \frac{p_{KT} K_T}{p_T T} \tag{24}$$

$$g = \hat{n} \,. \tag{25}$$

Note that the initial knowledge stock, $n(0)$, is given. Because total revenue equals total cost for energy and capital inputs, the energy shares in the two sectors are given by $1-\eta = p_R R_T / p_T T$ and $1-\theta = p_R R_Y / p_Y Y$, respectively. Inserting the definitions (23) and (24) into the firms demand functions (4) and (6), differentiating (4) and (7) with respect to time and using (7), (10) and (13) to eliminate capital and resource prices, we find:

$$\hat{\theta} = -(1-\theta)(1-\sigma)(r - \hat{w}_L + \nu g) \,, \tag{26}$$

$$\hat{\eta} = -(1-\eta)(1-\omega)(r - \hat{w}_S) \,. \tag{27}$$

The equations show how interest cost, wage changes, and technical change (only in the *Y*-sector) drive factor substitution.

Substituting (23) and (24) into (19) to eliminate $p_Y Y$ and $p_T T$ and inserting (7) and (17), we find:

$$\frac{w_L}{w_S} = \frac{\theta}{\eta} \cdot \frac{\beta\phi}{1-\phi} \frac{K_T}{L} . \tag{28}$$

This equation reflects that in final-goods market equilibrium the relative wage of unskilled labour increases with the share of capital in the unskilled-labour-intensive *Y*-sector (θ), increases with the share of the *Y*-good in total demand for final goods (ϕ), and falls with the relative abundance of unskilled labour.

Substituting $\hat{p}_T + \hat{T} = \hat{w}_S + \hat{K}_T - \hat{\eta}$, which follows from (24) and (10), into (21), we find:

$$r - \hat{w}_S = \rho + \hat{K}_T - \hat{\eta} . \tag{29}$$

Substituting (25) into (16), we express K_T in terms of the innovation rate:

$$K_T = S - ag . \tag{30}$$

From (28), (29) and (26) and from (29), (30) and (27), we derive the following equations of motion for the cost shares, respectively:

$$\dot{\theta} = -\theta \left(\frac{(1-\sigma)(1-\theta)}{1-(1-\sigma)(1-\theta)} \right) (\rho + v g) \tag{31}$$

$$\dot{\eta} = -\eta \left(\frac{(1-\omega)(1-\eta)}{1-(1-\omega)(1-\eta)} \right) \left(\rho - \frac{\dot{g}}{S/a - g} \right) \tag{32}$$

Dividing the capital market equilibrium (12) by the R&D cost c_n, inserting (7), (8), (9), (17) and (25), we find:

$$r - \hat{w}_S \geq \frac{1-\beta}{\beta} \frac{L}{a} \frac{w_L}{w_S} - g . \qquad \text{with equality for } g > 0 \tag{33}$$

This right-hand side of (33) represents the rate of return to R&D. It increases with the mark-up rate $1/\beta$ and with the size of the market as captured by *L*, which is the labour supply that produces the stock of physical capital in which innovations are embodied. The rate of return falls with the cost of R&D, which is proportional to aw_S.

Substituting (30), (28), (27) and (32) into (33), we find:

$$\dot{g} = \left(\frac{S}{a} - g\right)\left\{\left[1-(1-\omega)(1-\eta)\right]\left[g - \Phi\frac{\theta}{\eta}\left(\frac{S}{a} - g\right)\right] + \rho\right\}, \qquad \text{for } g>0, \qquad (34)$$

$$\frac{\rho}{1-(1-\omega)(1-\eta)} > \Phi\frac{\theta}{\eta}\frac{S}{a}, \qquad \text{for } g = \dot{g} = 0, \qquad (35)$$

with $\Phi = \frac{(1-\beta)\phi}{1-\phi}$.

Equations (34), (32) and (31) now form a dynamic system in three variables, which are the rate of innovation g, the share of capital in Y-goods production θ, and the share of capital in T-goods production η. None of these three variables is predetermined, so we need to formulate the conditions that restrict initial values and end values of them. Crucial is that cumulative extraction cannot exceed the available resource stock, see (13) and (14). We therefore need to focus on extraction and express it in terms of our key variables g, η and θ. Eliminating prices between (4) and (23) and between (6) and (24), and combining (23), (24) and (20), we find, respectively:

$$R_Y = \left(\frac{\theta}{1-\theta}\Big/\frac{\bar{\theta}}{1-\bar{\theta}}\right)^{\sigma/(1-\sigma)} Ln^{\nu} \qquad (36)$$

$$R_T = \left(\frac{\eta}{1-\eta}\Big/\frac{\bar{\eta}}{1-\bar{\eta}}\right)^{\omega/(1-\omega)} (S - ag) \qquad (37)$$

$$\frac{R_Y}{R_T} = \frac{1-\theta}{1-\eta}\frac{\Phi}{1-\beta} \qquad (38)$$

After choosing initial values for θ, η, and g, equations (36), (37) and (38), together with the given initial value $n(0)$ and equations (25), (32), (31), (34), and (35), allow us to calculate the extraction path. The initial values for θ, η, and g must be chosen such that the no-depletion condition (15) holds and the transversality condition (22) holds. In a steady state without innovation, $\lim_{t\to\infty} \hat{n}(t) = 0$, discounted stock prices change at rate $\hat{p}_n - r = -\pi / p_n < 0$, see (11). The transversality condition (22) then always holds. In a steady state with innovation ($\lim_{t\to\infty} g(t) > 0$), stock prices equal the cost of innovation, $p_n = aw_S / n$, see (9), (11) and (12). Then, the transversality condition (22) boils down to:

$$\lim_{t\to\infty} r(t) - \hat{w}_S(t) > 0 \qquad \text{for } \lim_{t\to\infty} g(t) > 0 \qquad (39)$$

For future use, we express sectoral depletion rates in both final goods sectors in terms of the three key variables. Differentiating (36) and (37) with respect to time and substituting (31) and (32), we find, respectively:

$$-\hat{R}_Y = \frac{1}{1-(1-\sigma)(1-\theta)}\left[\sigma\rho + v\,g(1-\sigma)\theta\right] \tag{40}$$

$$-\hat{R}_T = \frac{1}{1-(1-\omega)(1-\eta)}\left[\omega\rho + \frac{\dot{g}}{S/a-g}(1-\omega)\eta\right] \tag{41}$$

4. Solutions for different substitution conditions

To see the different mechanisms in the model most clearly, it is useful to first consider three specific cases for parameter values which are obtained by setting either elasticity of substitution or both elasticities equal to one. After that, the general case is evaluated in section 4.4.

4.1 Cobb-Douglas Case

We first study unitary elasticities both in the knowledge-using and the knowledge-competing sector, i.e. $\sigma = \omega = 1$. From (32) and (31), we see that θ and η are constant; (36) and (37), or equivalently (4) and (7), reveal that they equal the parameters $\bar{\theta}$ and $\bar{\eta}$, respectively. Then the dynamics is represented by a single differential equation for *g*, given by (34), which can now be simplified as:

$$\dot{g} = (\frac{S}{a} - g)\left\{g\left[1+\Phi\frac{\bar{\theta}}{\bar{\eta}}\right] - \left[\Phi\frac{\bar{\theta}}{\bar{\eta}}\frac{S}{a} - \rho\right]\right\} \qquad \text{if } \Phi\frac{\bar{\theta}}{\bar{\eta}}\frac{S}{a} \geq \rho$$

$$g = 0 \qquad \text{if } \Phi\frac{\bar{\theta}}{\bar{\eta}}\frac{S}{a} < \rho$$

The corresponding phase diagram is drawn in figure 2 for the case $\Phi(\bar{\theta}/\bar{\eta})(S/a) > \rho$. The path converging to a negative growth rate must be ruled out. The same holds for the path converging to $g = S/a$, since it violates the transversality condition (39) (first substituting $\hat{\eta} = 0$, (30) and (34) into (29), and then setting $g = S/a$, we find $r - \hat{w}_S = -S/a < 0$). Hence, the equilibrium growth rate jumps to the value for which $\dot{g} = 0$ and remains there. Thus, the equilibrium growth equals:

$$\bar{g} = \max\{0, \frac{(\Phi\bar{\theta}/\bar{\eta})(S/a) - \rho}{1+(\Phi\bar{\theta}/\bar{\eta})}\} \tag{42}$$

The rate of innovation is stimulated by a higher supply of skilled labour *S*, a lower unit input coefficient research *a*, and a lower discount rate ρ. This corresponds to the findings in other R&D-models. Our multi-sector model reveals ho innovation incentives depend on the expenditures shares and factor shares. In particular, the rate of innovation increases with $\Phi\bar{\theta}/\bar{\eta}$, which captures three effects. First, since innovation takes place in the *Y*-sector only, a higher expenditure share on *Y*-goods (Φ) boosts innovation. Second, since innovation is embodied in physical capital

goods in the *Y*-sector, a greater role for capital, as measured by a larger capital share in the *Y*-sector $\bar{\theta}$, increases the market for innovations and boosts research. Alternatively, a high value for $\bar{\theta}$ implies a low share of non-renewable resources in *Y*-production: the sector is less dependent on non-man-made inputs and this stimulates innovation. Finally, and most important, innovation is high when the share of non-renewable resources in the *T*-sector is high (low $\bar{\eta}$). If the *T*-sector relies heavily on resources rather than skilled labour input, less skilled labour is allocated in this sector, and more becomes available for the research sector. Hence, greater natural-resource dependence in the knowledge-competing sector reduces output in this sector, but raises innovation.

Fig. 2
(about here)

To study how resource dependence is related to growth of consumption rather than innovation, we need to calculate output growth in both final goods sectors. Besides innovation, only depletion of resource inputs drives growth, since labour and materials inputs are constant. The rate of depletion equals the discount rate ($-\hat{R}_Y = -\hat{R}_T = \rho$), see (40)-(41). Differentiating consumption and production functions (19), (3) and (5) with respect to time and substituting (40)-(41), we obtain the consumption growth rate according to:

$$\hat{C} = \left[\bar{\theta}(1-\beta)/\beta + (1-\bar{\theta})\delta\right]\phi g - \left[\phi(1-\bar{\theta}) + (1-\phi)(1-\bar{\eta})\right]\rho . \qquad (43)$$

Consumption grows at a positive rate only if the right-hand side of (43) is positive: innovation (at rate *g*, see first term at right-hand side) has to be sufficient to offset the decline in resource inputs (at rate ρ, see second term) and to overcome the constancy of physical capital inputs. For a *given* rate of innovation (*g*), consumption growth is the bigger, the higher are the gains from specialisation (low β), the larger are productivity spillovers (δ), the lower is the discount rate, and the higher are the factor shares $\bar{\theta}$ and $\bar{\eta}$. A lower discount rate (ρ) reduces resource depletion and implies a smaller drag on growth from the scarcity of non-renewable resources. For a *given* rate of innovation *g*, a higher capital share in both sectors ($\bar{\theta}$ and $\bar{\eta}$) imply smaller dependence of production on non-man-made scarce resource inputs, which is good for growth.

Overall, resource dependence in the knowledge-competing sector (as measured by 1-$\bar{\eta}$) has an ambiguous impact on growth. First, higher resource dependence makes *T*-goods more expensive and shifts skilled labour to innovation activities, which increases growth through innovation (see (42)). However, higher resource dependence implies that the decline in necessary resource inputs in production weighs more heavily, which reduces consumption growth (see (43)). Substituting (42) into (43) and differentiating growth with respect to $\bar{\eta}$, we find:

$$\frac{\partial \hat{C}}{\partial \bar{\eta}} < 0 \quad \Leftrightarrow \quad \frac{1}{\bar{\theta}(1-\beta)}\left(\bar{\eta}\frac{1-\phi}{\phi}\right)^2 + \left(\bar{\eta}\frac{1-\phi}{\phi}\right) - \bar{\tau}\left(\frac{S}{a\rho}+1\right) < 0 ,$$

where $\bar{\tau}$ stands for the expression in the first brackets in (43) and represents the effect of innovation on output growth in the Y-sector. The inequality reveals that for $\bar{\eta}(1-\phi)/\phi$ sufficiently small, higher resource dependence in the knowledge-competing sector goes together with higher growth rates.

We finally need to solve for the initial levels. Initial resource use is calculated by using the fact that the rate of extraction $\hat{R}$ decreases with the discount rate (see 40 and 41) and cumulative extraction corresponds to total resource stock. This gives $R_T(0) + R_Y(0) = \rho \cdot W_R(0)$. For a given initial knowledge stock $n(0)$, we can calculate the initial consumption and income levels. Note that a change in the initial knowledge stock, $n(0)$, has no effect on initial factor shares and the innovation rate. The reason is that with a Cobb-Douglas production function, technological change is neutral with respect to production factors and affects levels of output without changing relative prices.

4.2 Poor substitution in the knowledge-using sector

The assumption of a unitary elasticity of substitution in the knowledge-competing sector, i.e. setting $\omega = 1$, reduces the model to a two-dimensional system in g and θ, given by the differential equations:

$$\dot{g} = (\frac{S}{a} - g)\left\{g\left[1 + \Phi\frac{\theta}{\bar{\eta}}\right] - \left[\Phi\frac{\theta}{\bar{\eta}}\frac{S}{a} - \rho\right]\right\},$$

$$\dot{\theta} = -(\rho + \nu g)\cdot\theta\cdot\frac{(1-\sigma)(1-\theta)}{1-(1-\sigma)(1-\theta)}.$$

The corresponding phase diagram is depicted in figure 3. We have drawn the diagram assuming that ν is positive, so that the $\dot{\theta} = 0$ locus appears in the negative quadrant. If μ is positive, but sufficiently small to prevent the $\dot{\theta} = 0$ locus to intersect the $\dot{g} = 0$ locus, the dynamics will be qualitatively the same. Any path converging to $g=S/a$ violates the transversality condition and must be ruled out. Any path that hits the $g=0$ line at $\theta > \bar{\eta}a\rho/\Phi S$ must also be ruled out since it violates (35). It can be seen that there is a unique trajectory that neither does violate the transversality condition (39) nor condition (35). This saddle path lies below the $\dot{g} = 0$ locus. The economy jumps on the saddle path and asymptotically approaches the equilibrium with $\theta = g = 0$.

Which point on the saddle path is the equilibrium for a given initial resource and knowledge stock, is determined by (15), (36) and (38). The saddle path defines θ as a function of g, say $\theta(t) = f(g(t))$. Substituting this and $\eta = \bar{\eta}$ into (37) and (38), we find aggregate resource use, $R_T(t)+R_Y(t)$, as a function of $g(t)$. Since we know the equation of motion for g, this solves for the entire extraction path. In equilibrium, $g(0)$ must be such that cumulative extraction over the entire horizon exactly equals the initial resource stock $W_R(0)$. It now follows that a higher initial resource stock implies an initial point further to the right on the saddle path. So a (sufficiently large)

resource boom boosts short-run growth. A higher initial knowledge stock increases depletion for given θ and g, see (36). Hence to prevent running out of resources, a higher knowledge stock implies higher resource prices, and a lower initial growth rate in equilibrium.

During the adjustment, the rate of innovation g gradually falls to zero and then remains zero; the share of capital θ steadily falls. With rising resource prices and poor substitution in the *Y*-sector, compensation for R&D-investments is steadily falling. Skilled labour moves from the R&D to the *T*-sector. From the phase diagram it is clear that innovation stops when θ reaches the level that is implied by the intersection between the $\dot{g} = 0$ locus and the g = 0 axis, $\theta = \bar{\eta} a \rho / \Phi S$. This means that in finite time, resource prices reach such a high level that R&D becomes unprofitable. From then on, all skilled labour is in the knowledge-competing sector, knowledge growth is zero, and the growth rate of consumption is negative because of resource depletion.

Fig. 3
(about here)

The main conclusion from this case is that poor substitution in the knowledge-using sector is unambiguously unfavourable for innovation and growth.

4.3 Poor substitution in the knowledge-competing sector

Assuming a unitary elasticity for the knowledge-using sector, i.e. setting $\sigma = 1$ so that $\theta = \bar{\theta}$, see (31) and (36), the model reduces to a two-dimensional system in g and η, that reads, according to (34) and (32):

$$\dot{g} = (\frac{S}{a} - g)\left\{ g\left[1 + \Phi\frac{\bar{\theta}}{\eta}\right] - \left[\Phi\frac{\bar{\theta}}{\eta}\frac{S}{a} - \frac{\rho}{1-(1-\omega)(1-\eta)}\right]\right\}\left[1-(1-\omega)(1-\eta)\right],$$

$$\dot{\eta} = \eta(1-\omega)(1-\eta)\left\{ g\left[1 + \Phi\frac{\bar{\theta}}{\eta}\right] - \Phi\frac{\bar{\theta}}{\eta}\frac{S}{a}\right\}.$$

The corresponding phase diagram is depicted in figure 4. Any path converging to η=1 must be ruled out since it violates the transversality condition (it implies $\hat{\eta} > 0$, so that $r - \hat{w}_S < 0$, see (27), which violates (39)). Any path converging to g=0 and η=0 must also be ruled out since it violates (35). Hence, the economy jumps on the saddle path, which lies between the $\dot{\eta} = 0$ and $\dot{g} = 0$ loci, and asymptotically approaches the equilibrium with $\eta = 0$ and g = *S/a*.

Which point on the saddle path is the equilibrium for a given initial resource and knowledge stock, is determined by (15) and (37)-(38). The saddle path defines η as a function of g, say $\eta(t) = F(g(t))$. Substituting this and $\theta = \bar{\theta}$ into (37) and (38), we

find aggregate resource use, $R_T(t)+R_Y(t)$, as a function of $g(t)$. Since we know the equation of motion for g, this solves for the entire extraction path. In equilibrium, $g(0)$ must be such that cumulative extraction over the entire horizon exactly equals the initial resource stock $W_R(0)$. As in the Cobb-Douglas case in section 4.1, the initial condition $n(0)$ has no effect on the growth rate. However, a higher initial resource stock implies an equilibrium with a higher cost share $\eta(0)$, and a lower growth rate $g(0)$.

During the adjustment, the growth rate increases. This happens because, with rising resource prices and poor substitution in the T-sector, T-production becomes relatively more expensive. Skilled labour moves from the T-sector to R&D. In the long run, all skilled labour is in research so that that the asymptotic growth rate is S/a irrespective of further model parameters.

Fig. 4
(about here)

The main conclusion from this case is that poor substitution is in the knowledge-competing sector is not a problem for growth and investment in man-made (knowledge) capital. To the contrary: the rate of innovation in the steady state is higher than in the case with unitary elasticities in both sectors, the case considered in the previous section.

To draw conclusions about consumption growth rather than innovation, we need to calculate again the rate of consumption growth. In the long run, the rate of depletion is again equal to the discount rate. However, depletion has a greater weight in production in the case of poor substitution, since its share tends to one in the long run, $(1-\eta)\to 1$. Therefore, on the one hand the poor substitution case yields higher growth because of faster innovation, but on the other hand it yields a bigger drag on growth through depletion. We can show that the former effect dominates the latter, so that *less substitution in the knowledge-competing sector implies higher long-run growth*. Equation (43) still holds, provided $\bar{\eta}$ is replaced by $\eta = 0$. After substituting $g = S/a$ and $\bar{\eta} = 0$, we find the long-run growth rate of consumption for the case with $\sigma = 1 > \omega$:

$$\hat{C}_{|\sigma=1,\omega<1} = \left[\bar{\theta}(1-\beta)/\beta + (1-\bar{\theta})\delta\right]\phi\frac{S}{a} - \left[\phi(1-\bar{\theta}) + (1-\phi)\right]\rho\,. \qquad (44)$$

This growth rate exceeds the growth rate of consumption with unitary elasticities in both sectors, cf. (42)-(43), by the following positive amount:

$$\hat{C}_{|\sigma=1,\omega<1} - \hat{C}_{|\sigma=1,\omega=1} = \frac{1}{1+\Phi\bar{\theta}/\bar{\eta}}\left\{\frac{S}{a} + \rho\left[(1-\phi)(1-\bar{\eta}) + \phi(1-(1-\beta)\bar{\theta})\right]\right\}.$$

4.4 Poor substitution in both consumer sectors

We now turn to the general – and most interesting – case with elasticities unequal unity in both sectors. To be on the conservative side with respect to technological opportunities, we assume poor substitution, $0<\sigma<1$, $0<\omega<1$, and small spillovers

to resource augmenting so that technological change is resource-using, $\nu = (1-\beta)/\beta - \delta > 0$. We have to examine the full system of three differential equations (32), (31), and (34), the latter to be replaced by (35) in a corner solution. Solving for $\dot{g} = \dot{\eta} = \dot{\theta} = 0$ with $g \geq 0$, we can identify the different steady states.

From equation of motion (31), we see that θ always falls, so that in the steady state θ must approach zero. Furthermore, from (32) we see that constancy of η in the steady state requires either $\eta = 0$ or $\eta = 1$. Since η=1 can only be reached if $\hat{\eta} > 0$ at time infinity, and since $\hat{\eta} > 0$ violates the transversality condition (see (27) and (39)), any path converging to $\eta = 1$ must be ruled out. Hence, in the long run both θ and η approach zero:

$$\eta(\infty) \to 0, \quad \theta(\infty) \to 0. \tag{45}$$

According to (34), the dynamics of g depend on the ratio θ/η. The growth rates of θ and η approach asymptotically a (strictly negative) constant. Subtracting (32) from (31) and substituting (45), we find how the steady state ratio θ/η evolves over time:

$$\hat{\theta}(\infty) - \hat{\eta}(\infty) = \frac{\sigma - \omega}{\omega\sigma}\rho - \frac{1-\sigma}{\sigma}\nu g(\infty). \tag{46}$$

Depending on parameters, three types of steady states arise: an interior solution, a corner solution with zero innovation, or a corner solution with maximal innovation. First consider for which value of $\theta(\infty)/\eta(\infty)$ an interior steady state, $0 < g(\infty) < S/a$, can arise. The inequality in (35) rules out $\theta(\infty)/\eta(\infty) \to 0$, since this would imply $g = 0$. Equation (34) rules out $\theta(\infty)/\eta(\infty) \to \infty$ since this would imply $\dot{g} < 0$. Hence $\theta(\infty)/\eta(\infty)$ must be a constant in an interior steady state. This requires the both sides of equation (46) to be zero, so that $g(\infty) = \rho(\sigma - \omega)/\nu(1-\sigma)\omega$. This solution is an interior solution, $0 < g(\infty) < S/a$, only if $g(\infty) = \rho(\sigma - \omega)/\nu(1-\sigma)\omega < S/a$, which can be reformulated as $0 < (\sigma - \omega)\rho < (1-\sigma)\omega\nu S/a$. Second, consider the case $\sigma < \omega$. We see from (46) that then $\hat{\theta}(\infty) - \hat{\eta}(\infty) < 0$, so that $\theta(\infty)/\eta(\infty) \to 0$, which implies, by (35), the corner solution $g = 0$. Third, if $0 < (1-\sigma)\omega\nu S/a < (\sigma - \omega)\rho$ and $g(\infty) \to S/a$, we see from (46) that $\hat{\theta}(\infty) - \hat{\eta}(\infty) > 0$ so that $\theta(\infty)/\eta(\infty) \to \infty$. Equation (34) reveals that this is a steady state ($\dot{g}$ =0) provided $(\theta/\eta)\cdot(S/a - g)$ approaches a bounded constant that is smaller than $\rho/\omega + S/a$. This is a rational expectations equilibrium as we show in the appendix, where we use (34) to solve for the steady state value of $(\theta/\eta)\cdot(S/a - g)$ in each equilibrium.

Collecting these results we have:

$$g(\infty) = 0 \qquad \text{if } \sigma \leq \omega. \tag{47a}$$

$$g(\infty) = \frac{\rho(\sigma - \omega)}{\nu(1-\sigma)\omega} \qquad \text{if } 0 < \frac{\sigma - \omega}{(1-\sigma)\omega} < \frac{\nu}{\rho}\frac{S}{a} \tag{47b}$$

$$g(\infty) \to \frac{S}{a} \qquad \text{if } \frac{\sigma - \omega}{(1-\sigma)\omega} \geq \frac{\nu}{\rho}\frac{S}{a} \tag{47c}$$

The equations in (48) show that for given parameters, there is a unique steady state. In the appendix we show the existence and stability of these steady states as well as the transition paths to the steady states, which - together with the initial stocks $n(0)$ and $W_R(0)$ - define the initial conditions for all endogenous variables. The remainder of the section discusses the results and the implications for consumption growth.

Equation (47b) reveals that innovation incentives keep intact and a constant innovation rate can be maintained in the long run even with poor substitution, provided substitution in the knowledge-using Y-sector is better than in the R&D-competing T-sector ($0 < \omega < \sigma < 1$). To understand this result, we have to sort out why there is no incentive for skilled labour to move out of or into R&D in the long run. Two opposing – but inseparable – forces, from depletion and technological change respectively, determine labour allocation. On the one hand, as the resource stock is depleted and the amount of resource input per unit of labour falls, the wage falls, especially for the type of labour that is the poorest substitute for the resource. Thus, if $\omega < \sigma$, the T-sector is hurt most by depletion and the relative wage of skilled labour, which is employed in this sector, falls. This lowers innovation costs and tends to raise innovation. On the other hand, any shift into R&D speeds up the pace of innovation, which makes capital goods relatively more abundant, lowers their reward (provided $(1-\sigma)\nu > 0$, see (4)), and lowers the profits from innovation. On balance, in the interior steady state (47b), innovation takes place at a rate that makes profits from innovation fall at the same rate as costs of innovation (which happens because of depletion). The steady state with maximal R&D (47c) arises if the supply of skilled labour is small since then the supply of skilled labour constrains the rate of innovation such that the depletion effect dominates: the relative wage paid by the T-sector falls even if asymptotically all skilled labour has moved out of the T-sector into R&D. The zero innovation steady state (47a) arises if substitution is poorest in the Y-sector since then both depletion and innovation reduce the returns to innovation.

The interaction between depletion and innovation implies that the innovation rate becomes bounded by substitution elasticities when the supply of skilled labour grows large. With $\omega < \sigma < 1$, and S sufficiently large that (47b) applies, a rise in skilled labour supply does not affect the innovation rate. Hence, the so-called scale effect, for which endogenous growth model were criticised (notably by Jones 1995, 1999), is not present. The reason is that the growth rate is determined by the equality of depletion and innovation bias effect, so that g is solely governed by technical and preference parameters, notably the elasticities (note that the scale effect is present in the case with $\sigma = 1$, see section 4.1 and 4.3).

Another remarkable feature of the interior long-run innovation rate in (47b) is that it rises with the discount rate. In the Cobb-Douglas case (section 4.1), and in most endogenous growth models, the opposite happens. Usually, discounting disfavours investment in general and investment in R&D in particular. However, in the present model there are two types of investment, resource conservation and innovation. Higher discounting reduces investment in the resources by speeding up depletion, see (40)-(41). Thus the wage of skilled labour in the T-sector falls relatively faster, which reduces the cost of R&D and speeds up innovation.

In the long run, with $\eta = \theta = 0$ growth of consumption is (equation (43) still holds, provided $\bar{\eta}$ and $\bar{\theta}$ are replaced by $\eta = 0$ and $\theta = 0$):

$$\hat{C} = \phi\delta g - \rho$$

From this equation it becomes clear that growing consumption requires $\delta > 0$, that is, endogenous knowledge has to affect the productivity of resource use in *Y*-production positively, or, in other words, technological change is resource-augmenting. In this case, long-run consumption growth is (technically) feasible in principle. However, our analysis shows that in the market equilibrium, consumption grows in the long run only if in addition to $\delta > 0$, substitution is higher in the knowledge-using sector than in the knowledge-competing sector ($\sigma > \omega$), and the discount rate is low enough.

5. Conclusions

This paper shows that unbounded economic growth can be sustained if non-renewable resources are an essential input in production, even without exogenous technological change and with elasticities of substitution between man-made capital and resources which lie below unity. We have used a multi-sector framework in which differences in substitution opportunities across sectors cause labour to move from production to R&D when the resource stock becomes depleted. Poor substitutability in the sector that competes for skilled labour input with the R&D sector turns out to be favourable for growth. Resource depletion makes final goods production activities that heavily rely on resources more expensive. Thus, increased resource scarcity lowers the opportunity costs of innovation and shifts labour from final goods production to innovation effort. The sectoral shift supplements input changes as a substitution mechanism. As a consequence, growth is higher with this kind of poor substitutability compared to the case of unitary elasticities. In contrast, strong dependence on resources in the sector that implements the innovations is bad for growth: with a poor substitutability in this case, resource depletion increases production costs and lowers the demand for innovations. We conclude that the *relative* resource dependence of the knowledge-using and knowledge-competing sectors (measured by cost shares and elasticities of substitution) determine whether incentives for investment and innovation are sustained and growth is unbounded in the presence of poor substitution possibilities. We also find that in the case of poor substitution, the size of the elasticities of substitution, rather than resource and labour endowments, bound the rate of growth. Hence in the interior solution, the scale of the economy has no effect on long-run growth.

We have made some simplifying assumptions that may be relaxed in future research. First, we have stressed that (in contrast to knowledge capital) physical capital inputs are bounded because material use is bounded. Instead of completely abstracting from increases in the physical capital stock, physical capital accumulation can be modelled subject to explicit material balances constraints. Second, we have modelled technological change embodied in capital goods and we have found that if research spillovers are large, technological change may become resource-saving.

Alternatively, we may model two types of innovation, one directed at improving capital productivity and the other at resource productivity. Third, we have abstracted from resource extraction costs and polluting resource use, which may be taxed by the government. These features may change the price profile of the resource but they hit both consumer sectors in the same way. As the effects of price changes in the two sectors work in opposite direction, as seen in sections 4.2 and 4.3, the quality of our results is not expected to change substantially when enlarging the general model set-up in this way. Fourth, to keep the set-up tractable we have assumed that there are two specific labour factors and that no innovation is possible in one of the sectors. The use of a single type of labour does not qualitatively change the results. The difference of the two sectors concerning knowledge use is an extreme form of input intensity which is not decisive for the outcome either. Finally, as the paper focuses on market solutions, the issue of optimal policies has not been treated. Resource use produces no negative externalities in this model, only R&D generates positive spillovers which leads, as in the original "Romer-type" approach to R&D, to positive subsidies for innovations in the social optimum.

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Figures

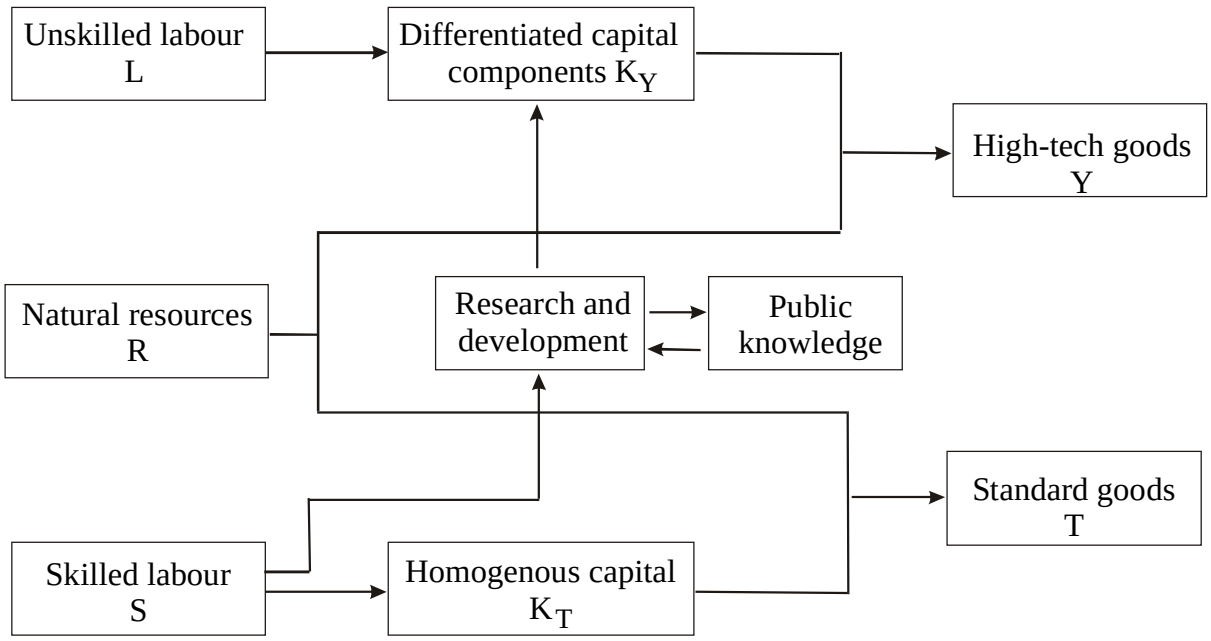

Fig. 1

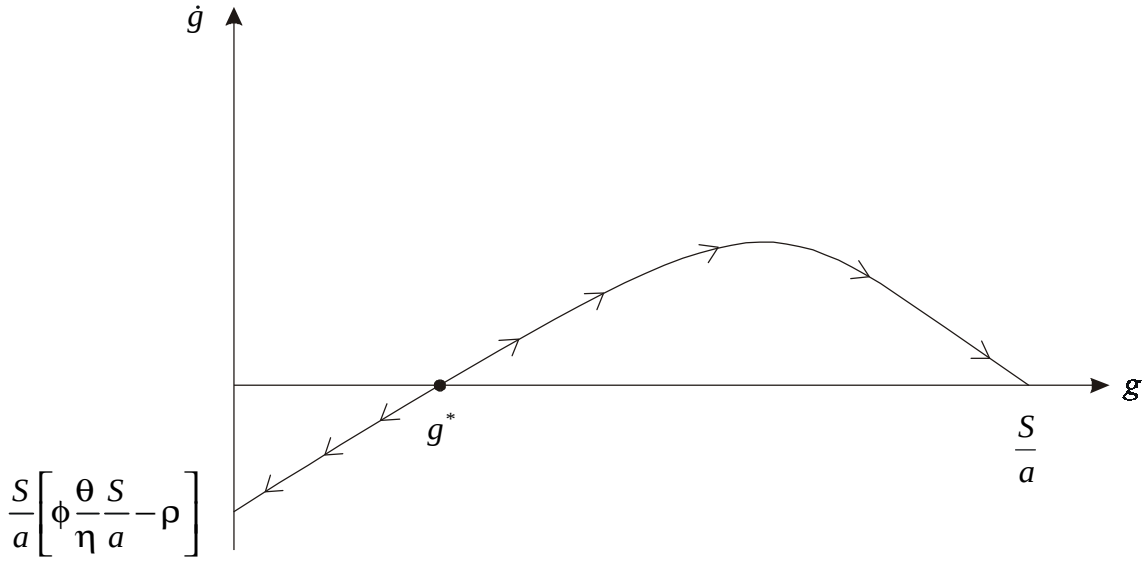

Fig. 2

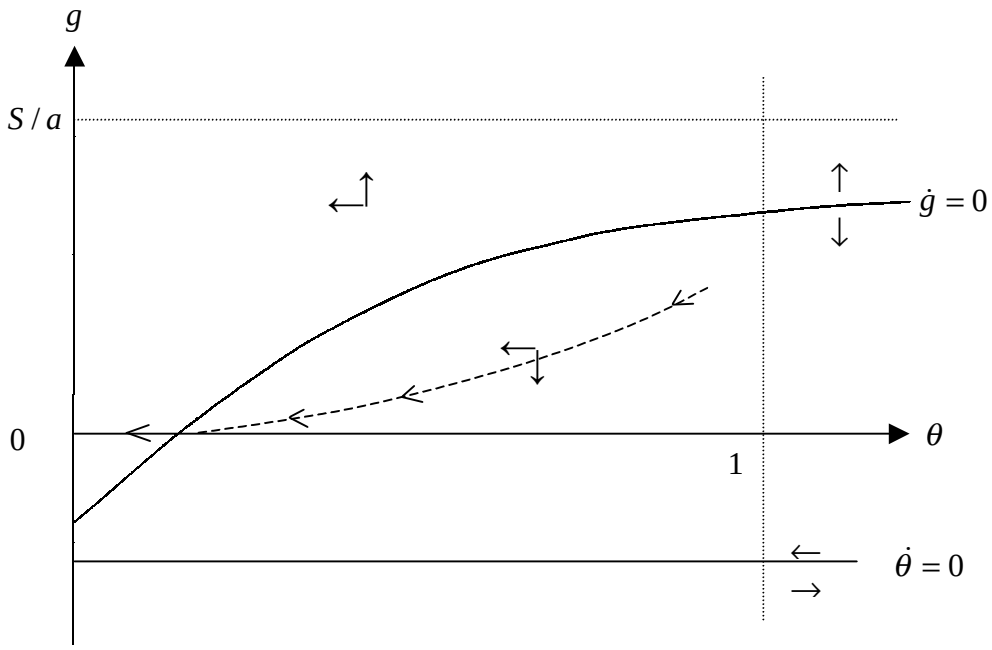

Fig. 3

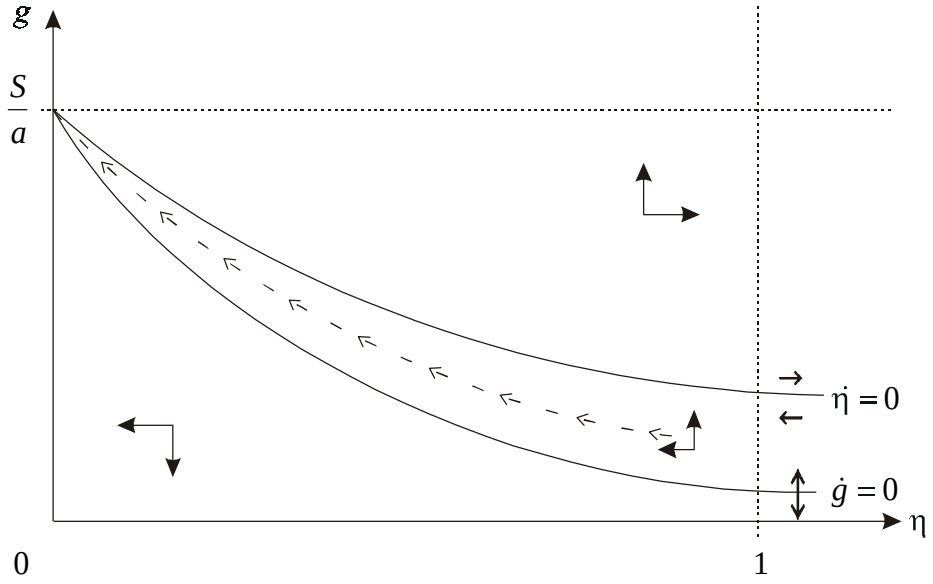

Fig. 4

Appendix to section 4.4

This appendix studies existence and stability of the steady state with poor substitution in both sectors ($0 < \sigma < 1$, $0 < \omega < 1$, and $\nu > 0$). Because $\dot{g}$ depends on θ/η, see (34), whereas $\eta = 0$ in the steady state, see (45), we cannot directly differentiate the system in the steady state. Instead of studying the dynamics in terms of θ, η, and g, we therefore rewrite the system in terms of the three endogenous variables g, η, and h, where h is defined as

$$h = \Phi \frac{\theta}{\eta}\left(\frac{S}{a} - g\right). \tag{A.1}$$

Existence of steady state with positive growth
We first examine a steady state with positive growth. Substituting $\theta = h\eta/\Phi(S/a - g)$ into from (34), (32) and (31), and subsequently setting $\theta = \eta = 0$, see (45), we find that the following must hold in such a steady state:

$$\dot{g} = \left(\frac{S}{a} - g\right)\omega\left(\frac{\rho}{\omega} + g - h\right) = 0 \tag{A.2}$$

$$\hat{h} = h - \frac{\rho}{\sigma} - \left[1 + \nu\left(\frac{1-\sigma}{\sigma}\right)\right]g \tag{A.3}$$

Moreover, if $g > 0$, $\eta \to 0$ and $\theta \to 0$, the transversality condition boils down to (see (28), (33) and (39)):

$$r - \hat{w}_S = h - g > 0 \tag{A.4}$$

From these three equations, it follows immediately that a positive growth rate requires $h \leq g + \rho/\omega$, which rules out that h goes to infinity, which requires by (A.3) that $h \leq g[1 - \mu(1-\sigma)/\sigma] + \rho/\sigma$. The transversality condition rules out that h goes to zero, which requires by (A.3) that $h \geq g[1 - \mu(1-\sigma)/\sigma] + \rho/\sigma$. Hence, a positive growth rate requires constant h and from setting (A.3) equal zero we get

$$h(\infty) = g(\infty) \cdot [1 + \nu(1-\sigma)/\sigma] + \rho/\sigma.$$

Substituting this solution into (A.2), and setting $\dot{g} = 0$, we find two solutions for the innovation rate, corresponding to (47b) and (47c).

Existence of steady state without innovation.
See main text.

Local stability
We prove that the steady state in the case of poor substitution in both sectors ($\omega < 1, \sigma < 1$) has two negative eigenvalues and one positive eigenvalue. We save on

notation by defining $s = S/a$. Provided that g is not at its corner $g = 0$, we may write from (32), (31), and (34):

$$\begin{aligned}
\dot{h} &= h\left\{h - g - \rho - (\rho + \nu g)\frac{(1-\sigma)\left(1 - \dfrac{\eta h}{\Phi(s-g)}\right)}{1-(1-\sigma)\left(1 - \dfrac{\eta h}{\Phi(s-g)}\right)}\right\} \\
\dot{\eta} &= -\eta(1-\eta)(1-\omega)(h-g) \qquad \text{(A.5)} \\
\dot{g} &= (s-g)\{\rho - [\omega + (1-\omega)\eta](h-g)\}
\end{aligned}$$

The Jacobian of this system is:

$$J(h,\eta,g) \equiv \begin{bmatrix} \partial\dot{h}/\partial h & \partial\dot{h}/\partial\eta & \partial\dot{h}/\partial g \\ \partial\dot{\eta}/\partial h & \partial\dot{\eta}/\partial\eta & \partial\dot{\eta}/\partial g \\ \partial\dot{g}/\partial h & \partial\dot{g}/\partial\eta & \partial\dot{g}/\partial g \end{bmatrix} =$$

$$\begin{bmatrix} J_{11} & J_{12} & J_{13} \\ -\eta(1-\eta)(1-\omega) & -(1-2\eta)(1-\omega)(h-g) & \eta(1-\eta)(1-\omega) \\ -(s-g)[\omega+(1-\omega)\eta] & -(s-g)(1-\omega)(h-g) & (h-g+s-g)[\omega+(1-\omega)\eta]-\rho \end{bmatrix}$$

where

$$\begin{aligned}
J_{11} &= h + h - g - \rho - (\rho + \nu g)(1-\sigma)\left(\frac{1-\theta}{\sigma+(1-\sigma)\theta} - \frac{\theta}{[\sigma+(1-\sigma)\theta]^2}\right) \\
J_{12} &= \frac{h^2}{\Phi(s-g)}(\rho+\nu g)(1-\sigma)\left(\frac{1}{[\sigma+(1-\sigma)\theta]^2}\right) \\
J_{13} &= -h\left[1 + \nu\frac{(1-\sigma)(1-\theta)}{\sigma+(1-\sigma)\theta} - (\rho+\nu g)\frac{\theta}{s-g}\left(\frac{1}{[\sigma+(1-\sigma)\theta]^2}\right)\right]
\end{aligned}$$

Interior growth rate, IG

First, consider the steady state with $0 < g < s$. In this case, we have $h = (\rho/\omega)[\sigma - \omega + \nu(1-\sigma)]/\nu(1-\sigma) \equiv h_{IG}$, $\eta = 0$, $g = (\rho/\omega)(\sigma-\omega)/\nu(1-\sigma) \equiv g_{IG}$. To facilitate calculations, note also that $h = g[1+\nu(1-\sigma)/\sigma] + \rho/\sigma = g + \rho/\omega$. For this equilibrium, all elements of the Jacobian J turn out to be finite, and the Jacobian can be evaluated as:

$$J(h_{IG}, 0, g_{IG}) = \begin{bmatrix} h_{IG} & J_{12} & -h_{IG}[1+\nu(1-\sigma)/\sigma] \\ 0 & -\rho(1-\omega)/\omega & 0 \\ -(s-g_{IG})\omega & -(s-g_{IG})\rho(1-\omega)/\omega & (s-g_{IG})\omega \end{bmatrix}$$

We find that the determinant is positive:

$$\mathrm{Det}J(h_{IG}, 0, g_{IG}) = h_{IG}\nu\frac{1-\sigma}{\sigma}(1-\omega)\rho(s-g_{IG}) > 0$$

Because the second row has zero elements only, except for the diagonal element, the diagonal element is an eigenvalue. Hence it follows immediately that one eigenvalue is negative:

$$\lambda_{IG,1} = -\left(\frac{1-\omega}{\omega}\right)\rho .$$

Since the determinant of the Jacobian, which equals the product of the three eigenvalues, is positive, we must have $\mathrm{Det}J = \lambda_1\lambda_2\lambda_3 > 0$, so there are two negative and one positive eigenvalue.

$$\lambda_{IG,2} = \frac{1}{2}[h_{IG} + \omega(s - g_{IG})] - \frac{1}{2}\sqrt{[h_{IG} + \omega(s - g_{IG})]^2 + 4\omega(s - g_{IG})h_{IG}\nu(1-\sigma)/\sigma} < 0$$
$$\lambda_{IG,3} = \frac{1}{2}[h_{IG} + \omega(s - g_{IG})] + \frac{1}{2}\sqrt{[h_{IG} + \omega(s - g_{IG})]^2 + 4\omega(s - g_{IG})h_{IG}\nu(1-\sigma)/\sigma} > 0$$

Maximum growth rate, MG

Next consider the steady state equilibrium with $g \to s$. For this case, we evaluate the Jacobian at $h = s[1+\nu(1-\sigma)/\sigma] + \rho/\sigma \equiv h_{MG}$, $\eta = 0$, $g = s$. To facilitate calculations, note that it implies $\theta = 0$, $h = g[1+\nu(1-\sigma)/\sigma] + \rho/\sigma < g + \rho/\omega$. For this equilibrium, J_{12} and J_{13} cannot be evaluated because they involve a division by zero. However, the determinant and the characteristic equation can be determined since multiplying elements from the first row with elements from the third row always gives finite expressions. In particular, the determinant and characteristic equation can be written as:

$$\mathrm{Det}J(h_{MG}, 0, s) = h_{MG}(1-\omega)(h_{MG} - s)[\rho - \omega(h_{MG} - s)] > 0$$
$$0 = [(1-\omega)(h_{MG} - s) + \lambda]\cdot[\rho - \omega(h_{MG} - s) + \lambda]\cdot(h_{MG} - \lambda)$$

Hence, the eigenvalues are

$$\lambda_{MG,1} = -(1-\omega)(h_{MG} - s) < 0,$$
$$\lambda_{MG,2} = -[\rho - \omega(h_{MG} - s)] < 0$$
$$\lambda_{MG,3} = h_{MG} > 0$$

Zero growth rate, ZG

Finally, consider the steady state with $g = \dot{g} = 0$. The dynamics are now governed by (32) and (31) (note that we cannot use the system in (A.5), which is valid for $g > 0$ only). The Jacobian, evaluated at the steady state with $\eta = \theta = g = 0$ reads:

$$\begin{bmatrix} \partial\dot{\eta}/\partial\eta & \partial\dot{\eta}/\partial\theta \\ \partial\dot{\theta}/\partial\eta & \partial\dot{\theta}/\partial\theta \end{bmatrix} = \begin{bmatrix} -\rho(1-\omega)/\omega & 0 \\ 0 & -\rho(1-\sigma)/\sigma \end{bmatrix}$$

So the two eigenvalues are negative:

$$\lambda_{ZG,1} = -\left(\frac{1-\omega}{\omega}\right)\rho$$

$$\lambda_{ZG,2} = -\left(\frac{1-\sigma}{\sigma}\right)\rho$$

Adjustment and initial conditions

Two negative eigenvalues are associated with each steady state. This implies that the initial values of (g, η, h) have to be located on the two-dimensional stable manifold, spanned by the eigenvectors associated with the two negative eigenvalues. In the linearised version, we can identify the initial condition as a the intersection between the manifold and two planes to be defined as follows. First, the marginal product of resource use has to be equalized across the two sectors, as described in equations (36)-(38). By eliminating R_Y and R_T between these equations, we find a relation between g, η, θ and n, say $J(g, \eta, \theta, n)=0$, which – after substitution of the definition of h (A.1) – defines a plane in the (g, η, h) space. Second, the resource stock has to be asymptotically depleted, as described in equation (15). Substituting (36)-(37) into (15) and integrating, we find a relation between g, η, θ, n, and W_R, say $J(g, \eta, \theta, n, W_R)=0$, which – after substitution of the definition of h – defines a plane in the (g, η, h) space. The intersection between the two planes, defined for initial values $n(0)$ and $W_R(0)$, and the stable manifold determines the initial equilibrium.

Some notes on the derivations of the equations in the main text.

Derivation of (4) and (6), relative factor demand.
The producer of Y-goods maximizes

$$\pi_Y = p_Y \left[\overline{\theta} \left(\int_0^n k_j^{\beta} dj \right)^{\frac{\sigma-1}{\beta\sigma}} + (1-\overline{\theta})(n^{\delta} R_Y)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - p_R R_Y - \int_0^n p_{kj} k_j dj \qquad \text{(B.1)}$$

The first order conditions are:

$$\frac{\partial \pi_Y}{\partial k_j} = 0 \quad \Leftrightarrow \quad p_Y Y^{1/\sigma} \overline{\theta} \left(\int_0^n k_j^{\beta} dj \right)^{\frac{\sigma-1}{\beta\sigma}-1} k_j^{\beta-1} = p_{kj} \qquad \text{(B.2)}$$

$$\frac{\partial \pi_Y}{\partial R_Y} = 0 \quad \Leftrightarrow \quad p_Y Y^{1/\sigma} (1-\overline{\theta}) n^{\delta(\sigma-1)/\sigma} R_Y^{-1/\sigma} = p_R \qquad \text{(B.3)}$$

Inserting the symmetry result, $k_j = K_Y / n$, we can rewrite (B.2) as:

$$p_Y Y^{1/\sigma} \overline{\theta} n^{(1-\beta)(\sigma-1)/\beta\sigma} K_Y^{-1/\sigma} = p_{kY} . \qquad \text{(B.4)}$$

Dividing (B.3) by (B.4), we find (4).
Equation (6) is derived analogously.

Derivation of (7), monopoly price of capital.
The monopolistic supplier of k_j maximizes $\pi = p(k)k - w_L k$, where we omit the subscript j and where $p(k)$ is the inverse demand function for capital, defined by the second equality in (4.2). The first order condition with respect to k reads: $p(1 + p'(k)k / p) - w_L = 0$. From (4.2) we see that $p'(k)k / p = \beta - 1$ if we make the standard assumption that the producer ignores its own influence on aggregate demand (Dixit and Stiglitz ,1977). Substituting this result, we find (7).

Derivation of (20),(21),(22), consumer demand
The representative household maximizes (18)-(19), subject to the budget constraint $\dot{N} = rN + w_L L + w_S S - p_T T - p_Y Y$, where $N = np_n$ is asset holdings. The Hamiltonian reads:

$$\text{(13.1)} \quad H = \phi \log Y + (1-\phi) \log T + \lambda [rN + w_L L + w_S S - p_T T - p_Y Y] \qquad \text{(B.5)}$$

where λ is the co-state variable. The conditions for a maximum are:

$$\frac{\partial H}{\partial Y} = 0 \Leftrightarrow \phi / Y - \lambda p_Y = 0 , \qquad \text{(B.6)}$$

$$\frac{\partial H}{\partial Y} = 0 \Leftrightarrow (1-\phi)/T - \lambda p_T = 0, \tag{B.7}$$

$$\frac{\partial H}{\partial N} = \rho\lambda - \dot{\lambda} \Leftrightarrow \lambda r = \rho\lambda - \dot{\lambda}. \tag{B.8}$$

$$\lim_{t\to\infty} \lambda N e^{-\rho t} = 0. \tag{B.9}$$

Eliminating λ between (B.6) and (B.7), we find (20). Solving for λ in (B.7), we find $\lambda = (1-\phi)/(Tp_T) \Leftrightarrow \dot{\lambda} = \lambda \cdot (\hat{T} + \hat{p}_T)$. Substituting these results in (B.8), we find (21). Substituting (B.8) and $N = np_n$ into (B.9), we find (22).

Derivation of (26) and (27), factor shares in terms of prices.
By the definitions in (23) and (24), the capital and resource share in the Y-sector are $\theta = (p_{kY} \cdot K_Y)/(p_Y \cdot Y)$ and $1-\theta = (p_R \cdot R_Y)/(p_Y \cdot Y)$, respectively, the capital and resource share in the T-sector are $\eta = (p_{KT} K_T)/(p_T T)$ and $1-\eta = (p_R R_T)/(p_T T)$, respectively. Hence, (4) and (6) can be written as:

$$\frac{\theta}{1-\theta} = \left(\frac{p_{kY}}{n^{\nu} p_R}\right)^{1-\sigma} \left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^{\sigma}, \tag{B.10}$$

$$\frac{\eta}{1-\eta} = \left(\frac{p_{KT}}{p_R}\right)^{1-\omega} \cdot \left(\frac{\bar{\eta}}{1-\bar{\eta}}\right)^{\omega}, \tag{B.11}$$

or, expressed in percentage changes:

$$\hat{\theta}(1-\theta)^{-1} = (1-\sigma)(\hat{p}_{kY} - \nu\hat{n} - \hat{p}_R), \tag{B.12}$$

$$\hat{\eta}(1-\eta)^{-1} = (1-\omega)(\hat{p}_{KT} - \hat{p}_R). \tag{B.13}$$

where hats denote growth rates. Substituting (7), (10), (13) and (25) to eliminate $\hat{w}_L, \hat{w}_S, \hat{p}_R, \hat{n}$ respectively, we find (26) and (27).

Derivation of (31) and (32), factor shares in terms of innovation rate.
Differentiating (28) with respect to time, we find:

$$\hat{w}_L - \hat{w}_S = \hat{\theta} - \hat{\eta} + \hat{K}_T. \tag{B.14}$$

Substituting (29) to eliminate $\hat{K}_T$, we find:

$$r - \hat{w}_L = \rho - \hat{\theta}. \tag{B.15}$$

Substituting this result into (26), we find (31).
Differentiating (30) with respect to time, we find:

$$\hat{K}_T = -\frac{\dot{g}}{S/a - g}. \tag{B.16}$$

Substituting (29) into (27) to eliminate $r - \hat{w}_S$, we find:

$$\hat{\eta} = -(1-\omega)(1-\eta)(\rho + \hat{K}_T - \hat{\eta}). \tag{B.17}$$

Substituting (B.16) into (B.17), and rewriting, we find (32).

Deriving (36) and (37), rates of depletion.

Multiplying both sides of (4) by $(K_Y / R_Y)^{-\sigma}$, we find:

$$\left(\frac{K_Y}{R_Y}\right)^{1-\sigma} = \left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^{\sigma} \left(\frac{p_{kY}}{p_R}\frac{K_Y}{R_Y}\right)^{-\sigma} n^{-(1-\sigma)\nu} \tag{B.18}$$

Inserting the definition of θ from (23), we find:

$$\left(\frac{K_Y}{R_Y}\right)^{1-\sigma} = \left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^{\sigma} \left(\frac{\theta}{1-\theta}\right)^{-\sigma} n^{-(1-\sigma)\nu} \tag{B.18}$$

Using (17) to eliminate K_Y, and rewriting, we find (36).

Multiplying both sides of (6) by $(K_T / R_T)^{-\omega}$ and inserting the definition of η from (24), we find:

$$\left(\frac{K_T}{R_T}\right)^{1-\omega} = \left(\frac{\bar{\eta}}{1-\bar{\eta}}\right)^{\omega} \left(\frac{\eta}{1-\eta}\right)^{-\omega} \tag{B.19}$$

Using (30) to eliminate K_T, and rewriting, we find (37).