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*Payment System Settlement and  
Bank Incentives*

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# Payment System Settlement and Bank Incentives <sup>1</sup>

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**Abstract:** In this paper we consider the relative merits of net versus gross settlement of interbank payments. Net settlement economizes on the costs of holding non-interest-bearing reserves, but increases moral hazard problems. The "put option" value of default under net settlement can also distort banks' investment incentives. Absent these distortions, net settlement dominates gross, although the optimal net settlement scheme may involve a positive probability of default. Net settlement becomes more attractive relative to gross settlement if bank assets have to be liquidated at less than book value.

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## Payment System Settlement and Bank Incentives

Various large-value interbank payment networks employ different rules for settling of interbank payments.<sup>1</sup> Some networks such as the Swiss Interbank Clearing (SIC) network operate under *real-time gross settlement* (RTGS). That is, payment messages entered into the payments network are continuously cleared and settled by transfer of reserve funds from the paying bank to the receiving bank. Other networks, such as the Clearing House Interbank Payment Network (CHIPS),<sup>2</sup> operate under *net settlement* rules. That is, at the close of each business day, the value of all payments due to and due from each bank in the network is calculated on a net basis. Banks ending the day in a net debit position (banks whose due-tos exceed their due-froms) transfer reserves to the network. The network, in turn, transfers these reserve funds to net creditor banks.<sup>3</sup>

The allocation of intraday credit in large-value net settlement networks is of policy concern, given the very large flows associated with these networks. The average gross daily volume of payments over CHIPS, for example, is easily over \$1 trillion, and Schoenmaker (1995, p. 21) puts average peak daily net debit positions on CHIPS at roughly \$50 billion. To give some perspective on these numbers, consider that annual GDP for the United States is roughly \$7.5 trillion, and that overnight reserve balances held by commercial banks at the Fed (i.e., the total of all non-currency reserves) averages roughly \$15 billion.

In policy circles, it has often been argued that net settlement of interbank payments can reduce the riskiness of interbank payment systems.<sup>4</sup> Formal analyses of this question have concluded that the real result is not a reduction but a reallocation of

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<sup>1</sup>For a survey of large-value payment systems in the G-10 countries, see Bank for International Settlements (1993) or Folkerts-Landau, Garber, and Schoenmaker (1996).

<sup>2</sup>CHIPS is operated by the New York Clearing House Association. Payments on CHIPS are most commonly associated with the dollar legs of foreign exchange transactions. See Federal Reserve Bank of New York (1991) or New York Clearing House Association (1995) on the operation of CHIPS.

<sup>3</sup>Other sets of rules for clearing and settlement are possible. For example, the Federal Reserve's Fedwire system is nominally a real-time gross settlement system, since all payment messages immediately become liabilities of the Federal Reserve System and therefore equivalent to reserve money. However, the Fedwire system resembles a net settlement system in the sense that participating banks are allowed to overdraw their accounts and have access to (subject to certain limits) free daylight credit. See Federal Reserve Bank of New York (1995) for details on the operation of Fedwire.

<sup>4</sup>See for example, the discussion on pp. 2-3 "Lamfalussy Report" of the Bank for International Settlements (1990).

risk, away from the banking system and towards either the government as guarantor of the payments system, deposit insurance facilities, or the public.<sup>5</sup> It follows that investigations of the relative merits of various settlement procedures should consider the effect that these procedures have on overall riskiness associated with a payments network, not on riskiness for a particular subset of the network participants.

In our analysis, we show that changes in the overall riskiness of interbank payment networks will be tied to changes in the banks' behavior induced by various rules for settlement. In particular, we examine the effects of settlement rules on banks' tendencies to honor interbank commitments ("net due-tos") rather than default. Small variations in settlement rules can result in significant changes in bank incentives and ultimately bank behavior.

At the most basic level, the tradeoff between net and real-time gross settlement can be characterized as a tradeoff between two distortions.<sup>6</sup> Net settlement increases default probability and thereby the costs associated with potential defaults, while gross settlement increases the costs associated with holding non-interest-bearing reserves. The relative merits of the two systems will depend on the relative size of these two costs. Different weighings of these costs as viewed by bank regulators and by banks can lead to different conclusions about optimality of particular settlement rules, and thus may explain the current lack of consensus on this issue.

We examine the tradeoff between these two distortions using the continuous-time inventory model described in Harrison (1985). This framework is particularly convenient for this problem because it provides tractable probability distributions for banks' net positions (under net settlement) and liquidity demands (under RTGS). This framework also allows us to analytically calculate the effects of placing upper limits ("caps") on banks' net debit positions. This is a relevant calculation because net debit caps are employed in all real-world payment networks that settle on a net basis.

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<sup>5</sup>See Emmons (1995a) and Freixas and Parigi (1996).

<sup>6</sup>This view of net versus gross settlement is in accord with policy-related discussions on this issue and the related literature (see below). There is another view of net settlement which emphasizes the effects of replacing gross contractual obligations with netted obligations (i.e., "netting by novation;" see e.g., Green 1996).

Our main conclusions are as follows. First, for the simplest case, where bank asset quality is fixed and bank assets can always be liquidated at book value, we find net settlement always dominates real-time gross settlement. However, the optimal net settlement scheme may be one which necessarily involves some probability of default. Second, we find that net settlement becomes even more attractive if bank assets have to be liquidated at less than book value. Third, when we examine the case where the quality of bank assets is a choice variable, we find that the potential costs of net settlement rise due to negative effects on bank asset quality.

## **I. Literature Review**

Angelini and Giannini (1994) present a systematic comparison of the tradeoffs between the costs of liquidity versus the costs associated with defaults under net settlement. Under the assumption that the risk of a bank failure rises monotonically with the interval between settlements, they derive an optimal settlement interval for a net settlement system. Using a similar approach, Schoenmaker (1994, 1995) compares the costs associated with real-time gross and net settlement of interbank payments, based on the costs associated with settlement failure (bank defaults), the opportunity costs of collateral holdings by the banks in the network, and “gridlock” or payment delay costs associated with gross settlement or higher collateral requirements. Using “macro” level data on payment flows through two large-value U.S. interbank payments networks (CHIPS and Fedwire), Schoenmaker concludes that for these networks, cost of real-time gross settlement (without free daylight overdrafts) would probably outweigh the reduction in the risks associated with bank failures. However, these analyses assume that the probability of default is exogenous to the structure and settlement rules of the network.

Furfine and Stehm (1996) analyze this issue from a different perspective. They consider the problem of a payments network which operates as a RTGS system, where intraday credit is extended by a central bank. The central bank’s optimal credit policy is based on balancing the costs of stricter credit policies (gridlock and collateral costs) and against the costs of more liberal credit policies (defaults), where these costs are

incorporated into nonstochastic functions. The optimal credit policy then depends on the relative weights associated with each type of cost.

Emmons (1995a) builds a microeconomic underpinning for interbank net settlement systems, by emphasizing cost savings resulting from minimization of the demand for non-interest-bearing reserves, and from the scale economies associated with costly state verification and delegated monitoring in the case of bank failures. He cautions, however, that the cost savings associated with each of these “natural monopolies” (i.e., in settling payments and in liquidation of failed banks) is unlikely to be realized under current institutional arrangements. Emmons (1995b) extends this framework to investigate the effects of net settlement in terms of risk shifting from other network participants to the deposit insurer and other bank creditors.

Freixas and Parigi (1996) analyze characteristics of net versus gross settlement, by considering a model in which gross settlement corresponds to an interbank settlement in reserves, and net settlement corresponds to settlement in debt claims. In the version of their model in where there is full information concerning the quality of bank assets, net settlement dominates. However, in the version of the model where there is private information concerning banks’ asset values, net settlement can lead to the risk of “contagion,” i.e., the risk that a failure of one bank can spread to another.<sup>7</sup> The issue of contagion is also taken up in a related paper by Rochet and Tirole (1995), who build a model of interbank lending in order to analyze the effects of government safety-net programs on bank’s incentives to monitor each other’s asset quality.

In contrast to the last two papers, we will abstract from issues of contagion in the analysis below. We argue that while contagion is interesting and important, for most present-day payment networks, the high likelihood of regulatory intervention in the event of potential settlement failures insures that the probability of contagion will be virtually zero.<sup>8</sup> Thus it makes sense to concentrate first on the problem of the costs associated with

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<sup>7</sup>Kahn and Roberds (1996) present a related model, in which settlement via bank debt is only welfare-enhancing to the extent such debt can be collateralized.

<sup>8</sup>In the case of CHIPS and other privately operated payment networks, it should be emphasized that there are no explicit guarantees of settlement from the Fed or any other central bank. However, the perception that such private networks can be “too big to fail” is common among private sector observers. See for example, Eisenbeis (1987, p.48), Brimmer (1989, p. 15), or Bernanke (1990, p. 150). See also Bank for

individual defaults. Alternatively, one could interpret our analysis of “net-settlement” as applying to RTGS settlement systems that grant interest-free, uncollateralized daylight credit.

## II. A Model of Interbank Settlement

Many of the critical differences between net and gross settlement systems can be illustrated in the context of a simple example. In this setup, banks exist for a single “trading day.” A representative bank in an interbank payments network can hold three types of assets:  $A$  “earning assets,”  $M$  “reserves,” and payments due from other banks or “due-froms”  $DF$ ; it holds two types of liabilities, payments due to other banks or “due-tos”  $DT$ , and  $C$  “deposits.” By assumption, due-to positions cannot be collateralized.

A bank starts the day at time zero with only earning assets  $A$  and deposits  $C$ . During the course of the day, due-froms and due-tos will accumulate exogenously according to the demands of depositors. No delay is permitted: as soon as a bank receives instruction from a depositor to make a payment, the payment message must be entered into the payment network, or the bank will be in default.<sup>9</sup>

We will take as a legal restriction that net due-to positions must be settled by transfer of reserves. There is no legal reserve requirement. Instead, reserves will be purchased and/or accumulate as needed according to the settlement rules of the payment system. Under real-time gross settlement, for example, banks continually pay off net due-tos as they are realized. Initially we assume that bank assets have constant value over the trading day and that they can be exchanged for reserves at book value.<sup>10</sup> However, the market for reserves is imperfect in the sense that reserves accumulated during the day cannot be exchanged for earning assets during the day, but must be held overnight without receiving interest. We use the assumption that reserves cannot be reinvested during the day as a convenient approximation for the non-existence of a

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International Settlement (1996, pp. 6-8), which describes the resolution of four recent potential liquidity crises associated with foreign exchange markets, in each case without incidence of contagion.

<sup>9</sup>Some of the studies cited above have analyzed the costs associated with the delay of payments. In our model, allowing for delay of payments would only introduce an additional dimension of moral hazard and would be unlikely to change the qualitative nature of our results.



continuous auction market in reserves.<sup>11</sup> Note also that reserves accumulated during the day can be used to settle due-tos realized during the day. The “social” cost of holding reserves is the seignorage cost or implicit tax of  $r > 0$  per dollar of reserves held at the end of the day. We assume that these seignorage costs are passed on to depositors.<sup>12</sup>

The finality of reserves transfers will be the key factor in reducing the bank’s incentive to declare bankruptcy. In other words, settling interbank claims by transferring reserves to other banks irreversibly commits the bank to favor interbank claims on its assets over all others. We will take this “irreversibility” feature of reserves transfers as a given institutional feature of the model environment.<sup>13</sup>

Below we will calculate the optimal size of net debit caps for various environments. These calculations assume that the level of initial asset holdings are observable and known at the time decisions are made concerning the size of net debit caps, and that the size of the caps cannot subsequently be changed during the trading day. This assumption has some basis in real-world practice. Net debit caps on CHIPS, for example, can be in most circumstances be changed only during a twenty-minute interval before the opening of business.<sup>14</sup> It would be easy to extend this model to situations in which the level of asset holdings by a bank is a random variable unknown to other parties at the instant in which debit caps are set.

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<sup>10</sup>These assumptions are relaxed in Section IV.

<sup>11</sup>This assumption is also consistent with Stigum’s (1990, pp. 537-74) description of the operation of the market for reserves (fed funds) in the U.S. In particular, Stigum notes (p. 550) that “...if a bank that is normally a net buyer of funds accumulates a big surplus position, it may have difficulty working off that surplus because it has insufficient lines to sell it.”

<sup>12</sup>Here we use the nominal rate times reserve holdings as a first-order approximation for the welfare costs of inflation that could be derived in a more complete model, e.g., as in Freeman (1996). The assumption that the inflation tax can be passed on to depositors simplifies the analysis by allowing us to do separate calculations of banks’ reserve holdings and their net worth. This assumption is consistent with a legal restriction against paying interest on demand deposits, coupled with a restriction that funds received during the day must be held overnight as demand deposits.

<sup>13</sup>We note that this assumption is consistent with current U.S. institutions. Transfers of reserves via Fedwire immediately become liabilities of the Federal Reserve System; see Federal Reserve Bank of New York (1995, p. 11). In the words of Corrigan (1990, p. 131), “...the money in question is ‘good money’ even if at the next instant the sending institution goes bust.”

<sup>14</sup>See New York Clearing House Association (1995, p.26).

A bank's decision to default is made on the basis of profit maximization. The net worth of a bank if it does not declare bankruptcy is the value of its assets minus its liabilities, i.e.,

$$NW = A + DF + M - C - DT$$

Note that net worth at time zero is simply  $NW_0 \equiv A - C$ , which we assume to be positive.

If a bank declares bankruptcy, its net worth is given by  $\alpha$  times its assets, minus  $\alpha$  times its deposit liabilities, minus  $\beta$  times its interbank liabilities or net due-tos  $ND \equiv DT - DF$ , i.e.,

$$NW = \alpha(\text{assets}) - \alpha(\text{deposits}) - \beta(\text{net due - tos}) = \alpha(A - C) - \beta(ND)$$

where  $1 > \alpha > \beta > 0$ . In other words, the cost of bankruptcy procedures diminishes the value of a bank's assets, but it also allows the bank to partially shift priority away from other banks participating in the payments network. Under this assumption, bankruptcy disproportionately punishes holders of interbank claims, implying that bankruptcy is a tempting option for banks with a large net debit position relative to their capital.<sup>15</sup>

End-of-day default under net settlement occurs if net worth under bankruptcy exceeds net worth under normal settlement, which is equivalent to

$$\gamma ND(\text{end - of - day}) > NW_0, \text{ where } \gamma \equiv \frac{\alpha - \beta}{1 - \alpha}$$

The social cost of default is  $\Xi$ , where  $\Xi \geq (1 - \alpha)A$ . That is, the cost of bankruptcy is at least as great as the value of assets lost from the defaulting bank. However, the total cost of default may also include additional costs, such as the costs of payment system disruptions.

We now consider a discrete-time example in which the day is divided into three periods, 0 (morning), 1 (noon) and 2 (close of business). The intraday evolution of net due-tos is stochastic. We assume that there is an equal probability of depositors receiving

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<sup>15</sup>Note that banks need not literally have positive net worth in the bankruptcy state in order for this incentive to exist. A similar incentive could exist if bankruptcy favored a select group of bank creditors. Such inequities in bankruptcy priority could result from political considerations, e.g., if the failing bank is based in one country while the payment network is based in another, or from statutory provisions favoring certain depositors over other creditors.

$C/2$  in funds or wishing to send  $C/2$  in funds in the morning and again in the afternoon.

Thus

$$ND(1) = \begin{cases} C/2 \text{ with probability } 1/2 \\ -C/2 \text{ with probability } 1/2 \end{cases}$$

$$ND(2) = \begin{cases} C \text{ with probability } 1/4 \\ 0 \text{ with probability } 1/2 \\ -C \text{ with probability } 1/4 \end{cases}$$

We choose parameter values so that a bank with  $C$  in net due-tos finds it advantageous to default, but a bank with  $C/2$  in net due-tos does not. This means that

$$\gamma C > NW_0 = (A - C) > \gamma(C/2) \quad (1)$$

Under RTGS banks pay off net due-tos as they are realized. They do so by selling earning assets in return for reserves. Hence if the bank incurs a net due to the position of  $C/2$  in period 1, it must immediately liquidate  $C/2$  worth of earning assets. This implies that the bank's maximum net due-to position in period 2 will be  $C/2$ , which in turn implies that the bank will have no incentive to default in either period (the evolution of the bank's net position under RTGS is shown in Figure 1).

To verify this claim, consider the bank's situation if it arrives in period 2 holding  $A - C/2$  in net assets and facing a due-to position of  $C/2$ . If it defaults its net worth is

$$\alpha(A - C/2) - \beta C/2$$

which under (1) is less than  $A - C$ , which is its net worth if it settles. Since its net worth is therefore  $A - C$  in period 2, regardless of the due-tos that arrive in period 2, it has no interest in defaulting in period 1 if its due-to position is  $C/2$ .

Since there is no possibility of bankruptcy under real-time gross settlement, the expected social costs under RTGS are simply the seignorage costs times the expected reserve holdings at the end of period 2. To evaluate this cost, consider the four equally likely possibilities. If the bank pays out to other banks in each of the two periods it holds no reserves in the final period. If the bank receives funds in each of the two periods it holds reserves in an amount equal to  $C$  in the final period. If it receives funds in the first period and pays in the second, it holds zero reserves. Finally, if it pays in the first period

but receives funds in the second period it holds reserves equal to  $C/2$ . Thus the expected level of reserve holdings is  $(3/8)C$ , and social costs are  $(3/8)Cr$ .

Under net settlement without net debit caps, the bank ends up with a period 2 net debit position of  $C$  with probability  $1/4$  (see Figure 2). In this case the bank will default under assumption (1). With probability  $1/4$  the bank ends up with a period 2 net credit position of  $C$ , and with probability  $1/2$  the bank ends up with a zero net position. **The social costs of this settlement system are therefore given by the social cost of default  $\Xi$ , times the probability of default ( $1/4$ ), plus the cost  $Cr$  of holding reserves at the end of the day times the probability of ending the day in a net credit position, i.e.,  $1/4$ . The total social cost of net settlement is therefore given by the sum of these two costs, i.e.,  $rC/4 + \Xi/4$ .**

In the last calculation we have assumed that the failure of the representative bank does not result in the failure of other banks participating in the payments network. In other words, failure of a bank does not lead to a problem of systemic risk. This could happen if either the failing bank's due-to position were spread over a large number of creditor banks, or if in the case of a default, the failing bank's liabilities were assumed by a private or governmental guarantor.

There are several points that can be made from this simple example. First, changing the rules from real-time gross to net settlement necessarily (though perhaps, weakly) increases the risk of a settlement failure. Second, the social costs of a net settlement system may be less than under gross settlement, even though net settlement can increase the risk of a default. This can happen because net settlement reduces the seignorage costs associated with settlement. Obviously if strategic default were unavailable, then net settlement would always dominate gross settlement, since net settlement would lessen seignorage costs without increasing default risk. We do not regard this as a telling rejoinder to the relevance of the model: strategic default as modeled here is merely the simplest form of moral hazard problem. More complicated forms of moral hazard, such as choice of riskier investment, offering depositors early payment, and the like, will generate similar costs, as long as interbank liability is limited in cases of default.

Finally, we note that a system of net settlement with a net debit cap  $D > 0$  offers the possibility of some economization on reserve balances with reduced default

probability. For this example it is easy to show that a net debit cap of  $D^* = C - \gamma^{-1}NW_0$  decreases expected reserve holdings while eliminating bankruptcy. Therefore net settlement with a net debit cap of  $D^*$  dominates gross settlement, and indeed if bankruptcy costs are greater than some critical level, this is the optimal net debit cap. (Because of the discrete nature of this example, the optimal cap is constant for a wide range of parameter values. In the continuous models of subsequent sections, we will examine the effects of changing social cost parameters on optimal net debit caps.)

### III. The Model with Continuous Payment Flows

We can better model the incentive problems faced by interbank payments networks if we modify the setup of the previous section to allow for smooth evolution of payment flows over the trading day. In this section, we will index time as a continuous parameter  $t$  on the unit interval, where  $t = 0$  indicates the start of the trading day and  $t = 1$  indicates the close of business. Net payment flows  $X(t)$  for a representative bank will evolve as a driftless standard Brownian motion (see e.g., Harrison 1985, p. 1) over the unit interval. This assumption is natural, if we regard each bank as small relative to the overall size of the network, and if we regard individual payment orders as small relative to the size of the bank. If either of these presumptions is grossly violated, a more complicated process will be needed to model net payments.<sup>16</sup>

We assume that initial net payments  $X(0)$  are zero. At any time  $t \in [0,1]$ ,  $X(t) \sim N(0,t)$ . In the case of net settlement with no caps, the bank's net debit position at any time  $t$  is equal to  $X(t)$ . In the case of net settlement with net debit cap  $D$ , the net debit position  $Z(t)$  at any time  $t$  is a *regulated* Brownian motion with upper control barrier at  $D$  (Harrison, p. 14). The process  $Z(t)$  ranges over  $(-\infty, D]$  and may be represented as

$$z(t) = X(t) - L(t)$$

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<sup>16</sup>Also note that it would be feasible to bound the net payment process within finite "reflecting barriers," but that such a restriction would entail a considerable increase in the model's mathematical complexity. See, e.g., the computations in Appendix B of Bertola and Caballero (1994).

where  $L(t)$  is defined as

$$L(t) = \sup_{0 \leq s \leq t} [\max\{X(s) - D, 0\}]$$

The interpretation of  $L(t)$  is the total cumulated quantity of earning assets that must be sold in order to keep the bank's net debit position  $Z(s)$ , below the cap  $D$  at all times  $s \in [0, t]$ . Note that  $L$  is a nondecreasing and continuous process and only increases when the cap is binding. Representative realizations of  $X(t)$ ,  $L(t)$ , and  $Z(t)$  are shown in Figure 3 for the case where  $D = 0$ .

We begin our analysis by considering the social costs of real-time gross settlement, which corresponds to a net debit cap  $D = 0$ . Under a net debit cap of zero it is never in the interest of the bank to default as long as its net worth is positive. Since the net worth of the bank remains constant as long as there is no cost to liquidating assets, no default occurs under gross settlement. (This will change once we consider a cost to liquidation of assets; see the following section.) Thus social costs of gross settlement are proportional to the seignorage costs associated with the level of reserves the bank holds at the end of the day. To calculate the level of reserve holdings at time 1, note that this level equals  $-Z(t)$ . Since  $D = 0$ ,  $-Z(t)$  is a regulated Brownian motion with lower reflecting barrier at zero. The random variable  $-Z(1)$  has a distribution

$$F_1(x) = \Phi(x) - \Phi(-x) = 2\Phi(x) - 1, \quad x \geq 0$$

where  $\Phi$  is the standard Normal distribution. Thus the expected social cost of gross settlement (per bank) is

$$SC_{GS} = r \int_0^{\infty} x dF_1(x)$$

To evaluate the social costs of net settlement, we again note that in this framework, there is never a reason to default before the end of the trading day. Since liquidation of assets imposes no penalty, there is no gain from early default. Early default is also costly in the sense that it eliminates the option value of a firm waiting until the end to discover if there is an influx of net due to increase the firm's holdings. Thus for net settlement the social costs can be calculated by observing the costs associated with the terminal position of the bank. For simplicity, we begin by considering net settlement

without debit caps. The social cost consists of two components--one proportional to the probability of default, and the other proportional to the expected holdings of reserves at the end of the day. The expected holding of reserves is  $\max\{-Z(1),0\}$ . As in the previous section, default occurs if  $Z(1) > \gamma^{-1}NW_0$ . Since there is no net debit cap,  $Z(t) = X(t)$ , implying that  $Z(1)$ , and therefore  $-Z(1)$  is a standard Normal variable. Thus

$$SC_{NS} = r \int_0^{\infty} x d\Phi(x) + \Xi \Phi(-\gamma^{-1}NW_0)$$

For general values of  $D$ , the social costs of net settlement involve identical considerations; however the distribution of  $-Z(1)$  is more complicated. The distribution is given by

$$\begin{aligned} F_2(x; D) &= \Phi(x) - \Phi(-x - 2D), \quad x \geq -D \\ &= 0 \quad \text{otherwise} \end{aligned}$$

Thus the social cost can in general be written as a function of  $D$ :

$$SC(D) = r \int_{-D}^{\infty} x dF_2(x; D) + \Xi F_2(-\gamma^{-1}NW_0; D) \quad (2)$$

Note that when  $D=0$  this formula reduces to  $SC_{GS}$  and when  $D=\infty$  it reduces to  $SC_{NS}$ .

In short, equation (2) says that the total social cost of a settlement system is equal to the sum of expected seignorage costs and expected costs associated with default. Note that (2) presumes that seignorage costs do not vary depending on the fact of a default. There are two equivalent ways of interpreting this assumption: first we measure seignorage costs as the long-run costs of diverting funding from return-bearing assets in order to make the payments system work. Second we imagine that in the event of a default the bank is immediately taken over by the network and depositors are made whole but continue to bear the costs of the reserves used for the settling payments. This is consistent with our abstraction from concerns about systemic risk.

As long as the debit cap is below the critical level  $\gamma^{-1}NW_0$ , there is zero probability of default. Moreover, increases in the debit cap reduce the expected seignorage cost. To see this note that the family of distributions  $F_2(\cdot; D)$  is ordered in the sense of first order stochastic dominance as  $D$  increases, i.e.,  $\partial F_2(x; D) / \partial D > 0$  for  $x > -D$ . Thus gross settlement is always dominated by net settlement for small levels of the net debit cap, and

the social costs of the payment network unambiguously fall as  $D$  increases to  $\gamma^{-1}NW_0$ . Desirability of further increases in the net debit cap depends on the parameters of the model. The following results describe this dependence.

**Proposition 1.** *As  $\Xi/r$  increases, i.e., as the cost of bankruptcy increases relative to seignorage charges, the optimal value of  $D$  decreases.*

*Proof:* From the discussion above  $SC(D)$  is minimized on  $[\gamma^{-1}NW_0, \infty)$ . Clearly the proposition holds (weakly) for either of the endpoints of this interval, so we restrict our attention to interior minima. Defining  $SC^*(D) \equiv SC(D)/r$ , first- and second-order conditions for the minimization of  $SC^*$  are given by

$$\begin{aligned} -2\Phi(-2D) + (\Xi/r)\phi(\gamma^{-1}NW_0 + 2D) &= 0 \\ \phi(2D) - (\Xi/r)(\gamma^{-1}NW_0 + 2D)\phi(\gamma^{-1}NW_0 + 2D) &> 0 \end{aligned} \quad (3)$$

where  $\phi$  is the standard Normal density. Straightforward comparative statics manipulations yield

$$\frac{\partial D}{\partial(\Xi/r)} = -\frac{\phi(\gamma^{-1}NW_0 + 2D)}{4[\phi(2D) + (\Xi/r)(\gamma^{-1}NW_0 + 2D)\phi(\gamma^{-1}NW_0 + 2D)]} < 0 \quad \text{Q.E.D.}$$

**Proposition 2.** *As  $\gamma^{-1}NW_0$  increases, i.e., as we increase the critical level of net debit position for bankruptcy to be tempting, the optimal value of  $D$  increases.*

*Proof.* Again restricting our attention to interior minima, standard comparative statics calculations using conditions (3) yield

$$\frac{\partial D}{\partial(\gamma^{-1}NW_0)} = \frac{2(\Xi/r)(\gamma^{-1}NW_0 + 2D)\phi(\gamma^{-1}NW_0 + 2D)}{4[\phi(2D) + (\Xi/r)(\gamma^{-1}NW_0 + 2D)\phi(\gamma^{-1}NW_0 + 2D)]} > 0 \quad \text{Q.E.D.}$$

**Corollary.** *If  $(\Xi/r) > 0$  is sufficiently small, the optimal net debit cap implies a positive probability of default.*

*Proof.* The first derivative of  $SC(D)$  is given by



$$SC'(D) = -2r\Phi(-2D) + \Xi\phi(\gamma^{-1}NW_0 + 2D)$$

For  $\Xi/r$  sufficiently close to zero, it follows that  $SC'(\gamma^{-1}NW_0) < 0$ . Q.E.D.

Propositions 1 & 2 characterize the socially optimal choice of net debit cap  $D$ , where the socially optimal cap minimizes the sum of seignorage costs (borne by the bank's customers) and bankruptcy costs (which are borne by unspecified parties). If some or all of the costs of default can be shifted to parties outside of the network, conflicts may arise concerning the proper level of net debit cap.

To see this last point, suppose that all bankruptcy costs are borne by outside parties. It is then easy to show that the true expected net worth of the representative bank in the network is increasing in the net debit cap  $D$ . Denoting the “true” (as opposed to notional) net worth of the bank as  $\hat{W}$ , at time  $t=1$  this quantity is given by

$$\hat{W}(NW_0, Z(1)) = \begin{cases} NW_0, & \text{if } Z(1) \leq (\gamma^{-1}NW_0), \text{ i.e., if no default} \\ \alpha NW_0 + (\alpha - \beta)Z(1), & \text{if } Z(1) > (\gamma^{-1}NW_0), \text{ i.e., if a default occurs} \end{cases}$$

Hence if  $D$  is large enough so that defaults can occur, the “true” time  $t = 0$  expected net worth of the bank can be calculated as

$$E_0\hat{W} = NW_0(1 - F_2(-\gamma^{-1}NW_0)) + \alpha NW_0 F_2(-\gamma^{-1}NW_0) - (\alpha - \beta) \int_{-D}^{-\gamma^{-1}NW_0} x dF_2(x)$$

which simplifies to

$$E_0\hat{W} = NW_0 + (\alpha - \beta) \int_{-D}^{-\gamma^{-1}NW_0} F_2(x; D) dx \quad (4)$$

Equation (4) says the true expected net worth of the bank is given by its notional net worth  $NW_0$  plus the “option value” associated with defaults. We can now show:

**Proposition 3.** *The true expected net worth of the bank  $E_0\hat{W}$  is increasing in the net debit cap  $D$ .*

*Proof:* Restricting our attention to the nontrivial case where  $D$  is large enough to allow defaults, differentiate (4) to obtain

$$\frac{\partial E_0 \hat{W}}{\partial D} = (\alpha - \beta) \int_{-D}^{-\gamma^{-1}NW_0} \left( \frac{\partial}{\partial D} \right) dF_2(x; D) = 2(\alpha - \beta)(\Phi(\gamma^{-1}NW_0 - 2D) - \Phi(-D)) > 0$$

Q.E.D.

## IV. Extensions

### Changes in the Payments Process

We can extend this model to consider the effects of changes in the volume of payments flows or changes in the intervals between settlement. Formally, we do this by changing the variance parameter of the net payment process  $X(t)$  to be  $\sigma^2$ , and by allowing the settlement time to be some arbitrary time  $\tau > 0$ . This implies that  $X(\tau) \sim N(0, \sigma^2 \tau)$ , and that the distribution of net credit positions at settlement time  $-Z(\tau)$  will be given by

$$F_3(x; D) = \begin{cases} \Phi\left(\frac{x}{\sigma\tau^{1/2}}\right) - \Phi\left(\frac{-x-2D}{\sigma\tau^{1/2}}\right), & x \geq -D \\ 0 & \text{otherwise} \end{cases}$$

We interpret higher values of  $\sigma$  as an increase in payments volume through the network, and higher values of  $\tau$  as an increase in the interval between settlements. The effects of increasing payments volume and/or settlement intervals on the optimal net debit cap depend critically on the relative costs of the inflation tax and bankruptcy. The following result describes this dependence.

**Proposition 4.** *For small values of  $(\Xi/r) > 0$ , the optimal net debit cap increases with increases in payments volume ( $\sigma$ ) or with increases in the settlement interval ( $\tau$ ). For sufficiently large  $\Xi/r$ , the optimal net debit cap decreases with increasing  $\sigma$  or  $\tau$ .*

*Proof.* Concentrating on interior minima, first- and second-order conditions for social cost minimization are given by

$$\begin{aligned}
-2\Phi\left(\frac{-2D}{\sigma\tau^{1/2}}\right) + (\Xi/r)\phi\left(\frac{\gamma^{-1}NW_0 + 2D}{\sigma\tau^{1/2}}\right) &= 0 \\
\phi\left(\frac{2D}{\sigma\tau^{1/2}}\right) - (\Xi/r)\left(\frac{\gamma^{-1}NW_0 + 2D}{\sigma\tau^{1/2}}\right)\phi\left(\frac{\gamma^{-1}NW_0 + 2D}{\sigma\tau^{1/2}}\right) &> 0
\end{aligned} \tag{5}$$

For the purpose of comparative statics calculation, it is convenient to define payments “intensity”  $\iota = \sigma\tau^{1/2}$ . Applying standard comparative static methods to (5) yields

$$\frac{\partial \mathcal{D}}{\partial \iota} \propto \frac{\left(-\frac{D}{\iota^2}\right)\left[\phi\left(\frac{2D}{\iota}\right) - \left(\frac{\Xi}{r}\right)\left(\frac{\gamma^{-1}NW_0 + 2D}{\iota}\right)\phi\left(\frac{\gamma^{-1}NW_0 + 2D}{\iota}\right)\right]}{\phi\left(\frac{2D}{\iota}\right) - \left(\frac{\Xi}{r}\right)\left(\frac{\gamma^{-1}NW_0 + 2D}{\iota}\right)\phi\left(\frac{\gamma^{-1}NW_0 + 2D}{\iota}\right)} \tag{6}$$

Since the denominator of (6) is positive, the sign of (6) varies with its numerator, which is clearly negative for small values of  $\Xi/r$ , and positive for  $\Xi/r$  sufficiently large.

Q.E.D.

In words, Proposition 4 says that increased payments volume and/or longer settlement intervals imply lower optimal net debit caps only when bankruptcy costs are small relative to seignorage costs.

### Liquidation Costs

So far we have assumed that a bank in need of reserves can instantaneously liquidate its assets at par in order to obtain reserves. This assumption has simplified the analysis in several ways. The most important simplification is that there is no cost to the bank from settling due-to positions during the day, so that default only was an issue at the end of the trading period. A second implication was that there was no need to distinguish between the liquidity of various earning assets. We now drop the assumption of costless liquidation. Instead we assume that banks start with two categories of earning assets in their initial holding: bonds in an amount  $A_1$  and loans in an amount  $A_2$ , where  $A = A_1 + A_2$ . Bonds are “liquid” in the sense that liquidating them is costless to the bank. Loans are “illiquid:” to turn a loan into reserves costs a liquidity penalty of  $\lambda$  per dollar of loans sold, where  $\lambda \in (0,1)$ . Since this extra wrinkle to the model considerably increases

its mathematical complexity, we will confine our analysis to the two extreme cases: gross settlement and net settlement without caps.

Under gross settlement, the net worth of the bank at any point during the day is the difference between original net worth and the level of liquidity penalty paid for reserves so far during the day. Recall that the total amount of reserves purchased as of time  $t$  is given by  $L(t)$ . Hence the total liquidation penalty paid as of time  $t$  is given by

$$\pi(t) \equiv \lambda \max\{L(t) - A_1, 0\}$$

Thus if asset liquidations exceed  $A_1$ , loans must be liquidated at a loss and the bank's net worth is diminished. Net worth as of time  $t$  will be

$$NW(t) = A - C - \pi(t)$$

Bankruptcy can occur if  $NW(t)$  is driven to zero, which will occur if

$$L(t) = L^* = \lambda^{-1}(A - C) + A_1$$

Note that under our assumptions  $NW(t)$  is nonincreasing so there is no chance that a zero net-worth bank can be bailed out of bankruptcy. If asset value were stochastic, then attempting to continue would have option value, so the analysis would be considerably more complicated. Despite the fact that default can occur during the trading day, the nonincreasing property of  $NW(t)$  implies that the probability of a default at some time during the day can be calculated using the distribution of  $NW(1)$ . To do this we adopt the convention that a negative value of  $NW(1)$  implies that default has already occurred by time  $t = 1$ . Thus, the probability of a default at some time during the day is given by

$$\Pr\{NW(1) \leq 0\} = \Pr\{L(1) \geq L^*\} = 1 - F_1(L^*)$$

The last equality follows from the so-called "reflection principle," which implies that for  $Z(0) = 0$  and a reflecting barrier of zero for  $Z(t)$ , the processes  $L(t)$  and  $Z(t)$  have identical distributions, even though their sample paths are almost surely different (Harrison, p.14). Thus the expected costs associated with default in this environment are  $\Xi[1 - F_1(L^*)]$ . Note that a more complete analysis might adjust for the timing of default during the day; however such adjustments are likely to be swamped by the costs of the default occurring at all.

The total social costs associated with the settlement regime have three components, a seignorage cost, a liquidation cost, and a default cost. The total is

$$SC_{GS} = r \int_0^{\infty} x dF_1(x) + \lambda \int_{A_1}^{L^*} (L - A_1) dF_1(L) + \Xi[1 - F_1(L^*)] \quad (7)$$

Note that in (7) we assume that liquidation penalties are not borne after the bank defaults. In the case of net settlement, default occurs only at time 1; until then it is not necessary to bear any liquidation costs. Default occurs when the bank's net worth under default exceeds its net worth under normal settlement. The latter is

$$A - C - \lambda\pi(1)$$

The former is

$$\alpha(A - C) - \beta X(1) - \lambda\pi(1)$$

and so the critical value for default is identical to the value used in the previous section:  $\gamma^{-1}NW_0$ .<sup>17</sup> Thus social costs under a net settlement system are

$$SC_{NS} = r \int_0^{\infty} x d\Phi(x) + \lambda \int_{A_1}^{\gamma^{-1}NW_0} (x - A_1) d\Phi(x) + \Xi[1 - \Phi(\gamma^{-1}NW_0)] \quad (8)$$

where we have assumed that the bank's holding of bonds  $A_1$  are less than the critical net debit position  $\gamma^{-1}NW_0$ . If this condition were violated, the middle term in (8) would simply drop out. In other words, the expressions (7) and (8) are the analogous, with  $\Phi$  substituting for  $F_1$  and  $\gamma^{-1}NW_0$  for  $L^*$ . Although the middle terms of the two expressions correspond in this analogy, they are actually based on different quantities: under net settlement, the liquidation decision depends on the bank's net position at the end of the period, whereas under gross settlement, liquidation costs depend on the history of accumulated requirements for reserves.

We now consider the situations under which one or the other of these two are more desirable. To do so we will assume the following parametric restriction:

$$\gamma = 1 > \lambda \quad (9)$$

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<sup>17</sup>Here we have chosen assumptions on cost of liquidating in default to make this hold. Alternative assumptions are possible and easily analyzed. Formally this could be done by simply incorporating additional liquidity costs as adjustments in the parameter  $\beta$ .

This assumption sets the “leverage factor”  $\gamma$  equal to unity, so that default decisions are undertaken on the basis of one-for-one comparisons of a bank’s net debit position to its capital.<sup>18</sup> Under assumption (9), difference between the costs of gross and net settlement is given by

$$SC_{GS} - SC_{NS} = r \int_0^\infty x d\Phi(x) + 2\lambda \int_{NW_0}^{L^*} (L - A_1) d\Phi(L) + \lambda \int_{A_1}^{NW_0} (L - A_1) d\Phi(L) + \Xi(1 + \Phi(NW_0) - 2\Phi(L^*)) \quad (10)$$

While the first three terms on the RHS of (10) are nonnegative, the sign of the last term is ambiguous. Thus net settlement (without caps) does not always dominate gross settlement. However, it is possible to show that imposing liquidation costs favors net settlement in the following sense.

**Proposition 5.** *Under restriction (9), for liquidation costs  $\lambda$  sufficiently small, the difference between the cost of gross and net settlement is increasing in  $\lambda$ .*

*Proof:* Differentiating (10) with respect to  $\lambda$  we obtain

$$\frac{\partial(SC_{GS} - SC_{NS})}{\partial\lambda} = 2 \int_{NW_0}^{L^*} (L - A_1) d\Phi(L) - \frac{(L^* - A_1)\phi(L^*)NW_0}{\lambda} + \int_{A_1}^{NW_0} (L - A_1) d\Phi(L) + 2\Xi \left( \frac{\phi(L^*)NW_0}{\lambda^2} \right)$$

As we drive  $\lambda \downarrow 0$ ,  $L^* \rightarrow \infty$  and  $\phi(L^*) \downarrow 0$  at a faster-than-quadratic rate, implying

$$\lim_{\lambda \downarrow 0} \frac{\partial(SC_{GS} - SC_{NS})}{\partial\lambda} = 2 \int_{NW_0}^\infty (L - A_1) d\Phi(L) + \int_{A_1}^{NW_0} (L - A_1) d\Phi(L) > 0.$$

Q.E.D.

Proposition 5 implies that if net settlement (without caps) is preferred to gross settlement without a liquidation penalty, a slight increase in this penalty increases the attractiveness of net settlement.

## Settlement Rules and Bank Portfolio Choice

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<sup>18</sup>In policy discussions it is common to directly compare a bank’s net debit cap with its capital position. For example, the Fed’s caps on daylight credit over the Fedwire system are typically set at a fixed percentage of an institution’s risk-based capital. See Federal Reserve Bank of New York (1995, pp. 30-31).

We extend the model to consider the interaction of various settlement rules and bank portfolio decisions. We again divide portfolios between loans and bonds; and we return to the assumption that all assets can be liquidated costlessly. Instead, we consider the effect of differences in the riskiness of the two sorts of assets.

In this section, bonds are riskless and yield zero net return. That is, a portfolio of  $A_1$  in bonds turns out to be worth  $A_1$  with certainty. A loan portfolio of size  $A_2$  will turn out to have one of two realizations. With probability  $1-p$  the portfolio will turn out to be worthless. With probability  $p$  the portfolio will turn out to be worth  $R(A_2)$ , where the gross return function  $R(\cdot)$  is strictly increasing and concave. In other words, the expected marginal value of an additional unit of the loan portfolio is  $pR'(A_2) > 0$  and the portfolio is subject to diminishing marginal returns.

We assume that on the night before (at time  $t = -1$ ) the bank allocates its total assets  $A$  between the loan and bond portfolios. It learns the realization of the return on the loan portfolio in the morning before trading begins. The managers of the payments network must set the rules of the settlement scheme without knowing either the bank's portfolio decision or the realization of the loan value.

In the absence of other considerations the optimal size of the loan portfolio would be determined by the marginal condition  $pR'(A_2) = 1$ , which we assume is satisfied for  $A_2^* \in (0, A)$ . Given limited liability there is the possibility of the bank preferring to overinvest in the risky portfolio. The main result of this section is that the use of net settlement *increases* the temptation of firms to overinvest in risky portfolios.<sup>19</sup>

To see this, let us set parametric restrictions on the function  $R$  so that in the absence of any participation in the payments network, the bank would have an incentive to choose the efficient level of  $A_2$ . The following condition is necessary and sufficient for this to be the case:

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<sup>19</sup>The idea that the “put option” feature of equity creates a conflict between equity-holders and creditors of a firm is hardly new. For a general discussion of this idea and its relevance in banking environments, see Flannery (1994). In this section, we show that to the extent that net settlement creates a new class of unsecured creditors, it also has the potential to create a new set of conflicts between equity holders and these creditors.

$$p(R(A_2^*) - A_2^*) + A - C \geq p(R(\tilde{A}_2) - \tilde{A}_2 + A - C) \quad (11)$$

where  $\tilde{A}_2$  is defined by the condition  $R'(\tilde{A}_2) = 1$ , and by assumption  $\tilde{A}_2 \in (0, A)$ .

Condition (11) says that, absent settlement considerations, the bank would choose a level of investment in loans  $A_2^*$  that would leave the bank with positive net worth under the bad investment outcome. By assumption, this level of investment is more profitable than the most profitable level of investment in loans, assuming that the bank has negative net worth under the bad outcome.

Propositions 6 and 7 describe the effect of net and gross settlement on the bank's investment decision.

**Proposition 6.** *Given condition (11), under real-time gross settlement the bank chooses a loan portfolio of efficient size  $A_2^*$ .*

*Proof:* Since there is no incentive to default under RTGS (see the discussion of Section III), participation in the payments network cannot change a bank's net worth. Hence, under condition (11), the bank will choose a loan portfolio of efficient size. Q.E.D.

**Proposition 7.** *Given condition (11), and given a net debit cap  $D$  large enough to allow for default, under net settlement the bank chooses a loan portfolio of greater than efficient size, and the size of the bank's loan portfolio increases as the net debit cap increases.*

*Proof:* To show the first part of the proposition, we first show that the bank's time  $t = 0$  true expected net worth  $E_0 \hat{W}$  grows less than proportionately with its notional net worth  $NW_0$ . Totally differentiating the expression for true net worth (4), we obtain

$$\frac{dE_0 \hat{W}}{d(NW_0)} = 1 - (1 - \alpha) F_2(-\gamma^{-1} NW_0) \in (0, 1) \quad (12)$$

Now consider the bank's time-zero notional net worth under successful and unsuccessful investment outcomes



$$NW_0 = \begin{cases} NW_0^s \equiv R(A_2) - A_2 + A - C, & \text{if successful} \\ NW_0^u \equiv -A_2 + A - C, & \text{if unsuccessful} \end{cases}$$

and define true time-zero expected net worth  $E_0\hat{W}^s$  and  $E_0\hat{W}^u$  (i.e., under success and failure) analogously. The bank's problem is to choose  $A_2$  at time  $t = -1$  to maximize true expected net worth  $E_{-1}\hat{W}$ . If the resulting value of  $A_2$  is so large as to cause  $NW_0^u < 0$ , then the proposition holds trivially. So suppose that the optimal choice of  $A_2$  implies  $NW_0^u \geq 0$ . Then the first-order condition for maximization of  $E_{-1}\hat{W}$  will be given by

$$\frac{d}{dA_2} (pE_0\hat{W}^s + (1-p)E_0\hat{W}^u) = p \frac{dE_0\hat{W}^s}{d(NW_0^s)} \frac{d(NW_0^s)}{dA_2} + (1-p) \frac{dE_0\hat{W}^u}{d(NW_0^u)} \frac{d(NW_0^u)}{dA_2} = 0$$

Which is equivalent to

$$pR'(A_2) = p \frac{dE_0W^s}{d(NW_0^s)} + (1-p) \frac{dE_0W^u}{d(NW_0^u)} \quad (13)$$

Note that (12) implies that the RHS of (13) is in  $(0,1)$ , implying that if  $A_2$  satisfies (13), it must be the case that  $A_2 > A_2^*$ . To verify that a solution to (13) represents a unique maximum, consider the second-order condition

$$pR''(A_2) + p\gamma(1-\alpha)F_2'(-\gamma^{-1}NW_0^s)(R'(A_2)-1) - (1-p)\gamma F_2'(-\gamma^{-1}NW_0^u) < 0 \quad (14)$$

The first and last terms of the LHS of (14) are clearly negative, while the second term must be negative under the assumption that  $NW_0^u \geq 0$ .

To show the second part of the proposition, denote the solution to (13) as  $\hat{A}_2$ . Straightforward manipulation of conditions (13) and (14) implies

$$\frac{\partial \hat{A}_2}{\partial D} = \frac{-2(p(1-\alpha)\Phi(\gamma^{-1}NW_0^s - 2D) + (1-p)(1-\alpha)\Phi(\gamma^{-1}NW_0^u - 2D))}{pR''(A_2) + p\gamma(1-\alpha)F_2'(-\gamma^{-1}NW_0^s)(R'(A_2)-1) - (1-p)\gamma F_2'(-\gamma^{-1}NW_0^u)} > 0$$

Q.E.D.

**Corollary.** *The possibility of asset substitution reduces the optimal debit cap.*

*Proof:* If banks can invest in risky loans, then social cost function (2) becomes

$$SC(D) = r \int_0^\infty x dF_2(x; D) + \Xi \left[ p(F_2(-\gamma^{-1}NW_0^s; D)) + (1-p)(F_2(-\gamma^{-1}NW_0^u; D)) \right]$$

and first- and second-order conditions (3) become

$$\begin{aligned}
& -2\Phi(-2D) + (\Xi/r) \left[ p\phi(\gamma^{-1}NW_0^s + 2D) + (1-p)\phi(\gamma^{-1}NW_0^u + 2D) \right] = 0 \\
& \phi(2D) - \\
& (\Xi/r) \left[ p(\gamma^{-1}NW_0^s + 2D)\phi(\gamma^{-1}NW_0^s + 2D) + (1-p)(\gamma^{-1}NW_0^u + 2D)\phi(\gamma^{-1}NW_0^u + 2D) \right] > 0
\end{aligned} \tag{15}$$

Comparative statics using conditions (15) yield

$$\frac{\partial D}{\partial p} = \frac{(\Xi/r)(\phi(\gamma^{-1}NW_0^s + 2D) - \phi(\gamma^{-1}NW_0^u + 2D))}{4 \left[ \phi(2D) + (\Xi/r) \left[ p(\gamma^{-1}NW_0^s + 2D)\phi(\gamma^{-1}NW_0^s + 2D) + (1-p)(\gamma^{-1}NW_0^u + 2D)\phi(\gamma^{-1}NW_0^u + 2D) \right] \right]}$$

which is negative for  $p \in [0,1)$  and approaches zero as  $p \uparrow 1$ . Q.E.D.

In words, Proposition 7 says that under net settlement, the option value of default can cause banks to overinvest in risky assets. From the individual bank's point of view, a poor investment outcome can sometimes be mitigated by forcing other banks (or their guarantors) to share in this loss. Proposition 6 says that under gross settlement, no such incentive exists. Finally, the Corollary says that the optimal net debit cap must fall as a result of these considerations.

## V. Summary and Conclusion

Those whose primary concern is the leeway given to participants in interbank payments networks favor real-time gross settlement. However RTGS is expensive for banks, who must retain large holdings of non-interest-bearing reserves and potentially face liquidation costs in order to satisfy the liquidity demands of a RTGS system. This paper outlines the tradeoffs between the costs and benefits associated with net settlement relative to RTGS. The costs of net settlement result from allowing banks the "put option" of default on their interbank obligations, and from the resulting distortions in asset holding decisions, while the benefits of net settlement result from saving on seignorage and liquidation costs.

In practice, default considerations represent a very small aspect of the decisions made by banks in a payment network during the course of their day-to-day operations. There are three justifications for concentrating on default as the decision that a bank controls. First, when defaults do occur, they are likely to be expensive. Second, the default decision is representative of many other decisions which a bank makes -- for example more general portfolio decisions, the decision to allow depositors access to uncleared funds and similar extensions of intraday credit, or the decision by a bank to delay the sending of payment messages to other banks -- in which the actions are at best imperfectly controlled by the managers of the payments network or governmental regulators, but which have an impact on the overall risk associated with the network. For all of these decisions, the conflict between the interest of an individual bank and the “social” interest of the payments network are exacerbated by the bank’s holding a large due-to position. Third, the default decision is very simple to analyze. The framework we have developed can be usefully extended to consider the more complex decisions which are of concern to both operators of private payments networks and to governmental regulators.

Our results demonstrate the complexity of welfare comparisons between different settlement rules, even in a very restrictive model environment. In the simplest case, with no asset substitution and no liquidity costs, some form of net settlement dominates (Proposition 1). Increases in banks’ net worth increase the optimal net debit cap in a non-linear fashion (Proposition 2). Increases in payments flows and/or settlement times can either increase or decrease the optimal net debit cap (Proposition 4). The presence of liquidity costs increases the costs associated with RTGS (Proposition 5), while the possibility of asset substitution increases the likelihood of default, and therefore increases the costs associated with net settlement (Propositions 6 and 7).

Despite these ambiguities, our analysis does have some clear policy implications for the regulation of payment networks that settle on a net basis. First, Proposition 3 says that to the extent that default costs can be shifted to outside parties, payment network participants have an incentive to maximize this subsidy by setting as large a net debit as is feasible. This result would therefore be consistent with the “Lamfalussy standards” set

forth in Bank for International Settlements (1990), which require payment networks to share in the costs of potential defaults by posting collateral sufficient to cover the maximum net debit of any single payment network member. Second, our analysis clearly shows that in comparing various settlement rules, the risk of a default is an endogenous variable and should not be taken parametrically. Finally, Propositions 6 and 7 show that the welfare costs associated with suboptimal settlement schemes are not limited to liquidity costs and/or the costs of default. When settlement rules distort banks' investment decisions, losses in allocational efficiency can also result.

## Model Notation

### Basic Model and Discrete-Time Example:

$A$	Initial level of representative bank's earning assets
$C$	Initial level of deposits
$M$	Initial reserve holdings
$DF$	Payments due from other banks
$DT$	Payments due to other banks
$NW$	Bank's (notional) net worth
$ND$	Net debit position (due-tos less due-froms)
$\alpha$	Share of assets (deposits) recoverable by failed bank (depositors)
$\beta$	Share of due-tos of failed bank recoverable by network
$\gamma$	Leverage factor for comparison of net debit position to capital
$r$	Unit tax on overnight reserve holdings
$\Xi$	Social cost of default
$D$	Net debit cap

### Continuous-Time Model:

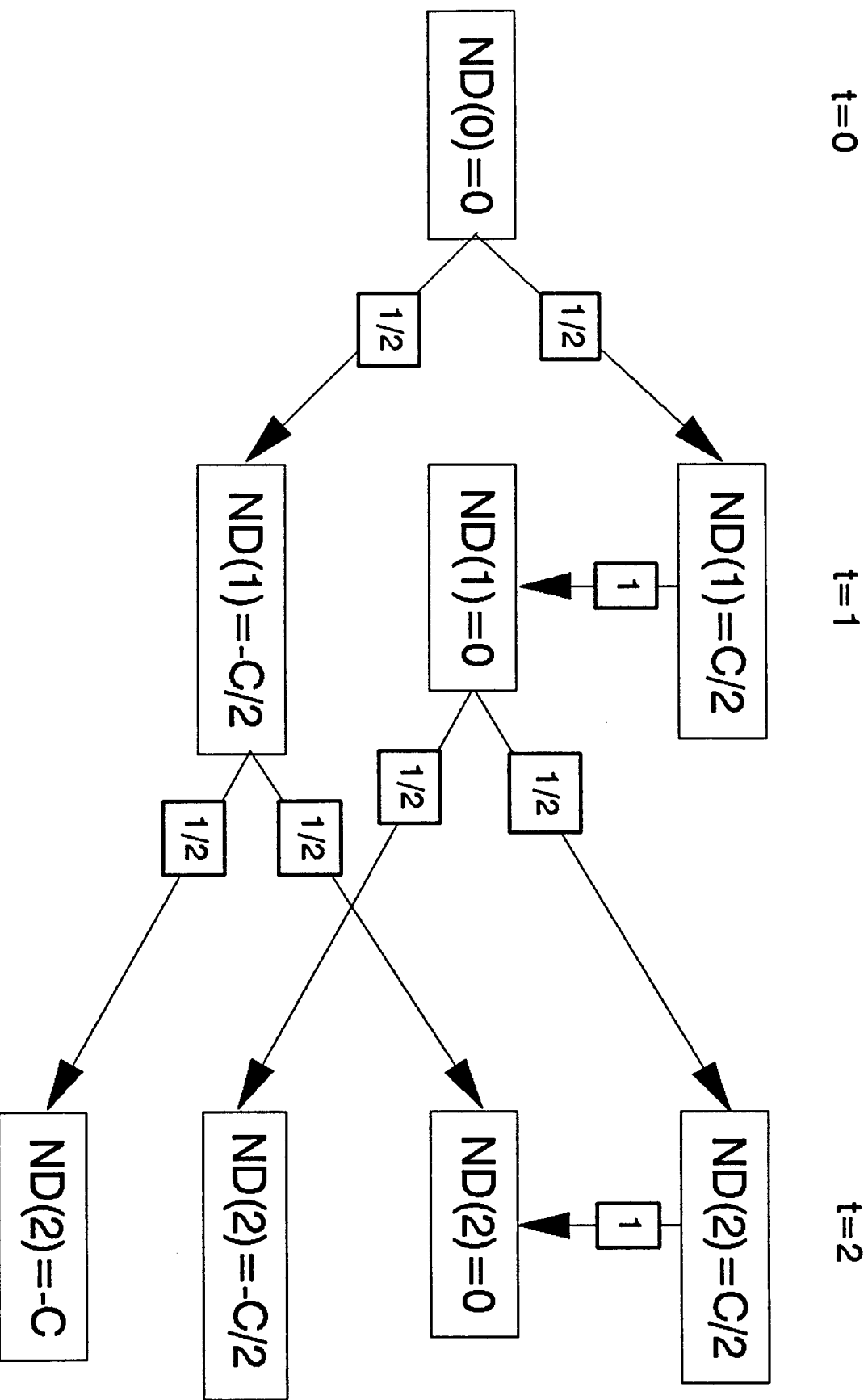
$X(t)$	Bank's net payments, i.e., net debit position with no caps
$Z(t)$	Bank's adjusted net debit position, subject to a cap $D$
$L(t)$	Total asset liquidations necessary to stay under net debit cap
$SC$	Social cost of running a payments network
$F_1$	Distribution of $-Z(1), L(1)$ when $D=0$
$F_2$	Distribution of $-Z(1)$ when $D>0$
$\hat{W}$	Bank's true net worth
$\sigma$	Standard deviation of $X(1)$
$\tau$	Time to settlement
$A_1$	Portion of $A$ held as bonds
$A_2$	Portion of $A$ held as loans
$\lambda$	Unit liquidation penalty for loans
$\pi(t)$	Cumulated liquidation penalty
$L^*$	Level of liquidations that exhausts bank capital
$R(\cdot)$	Return on successful loan portfolio
$p$	Probability of a successful loan portfolio
$A_2^*$	Size of efficient loan portfolio

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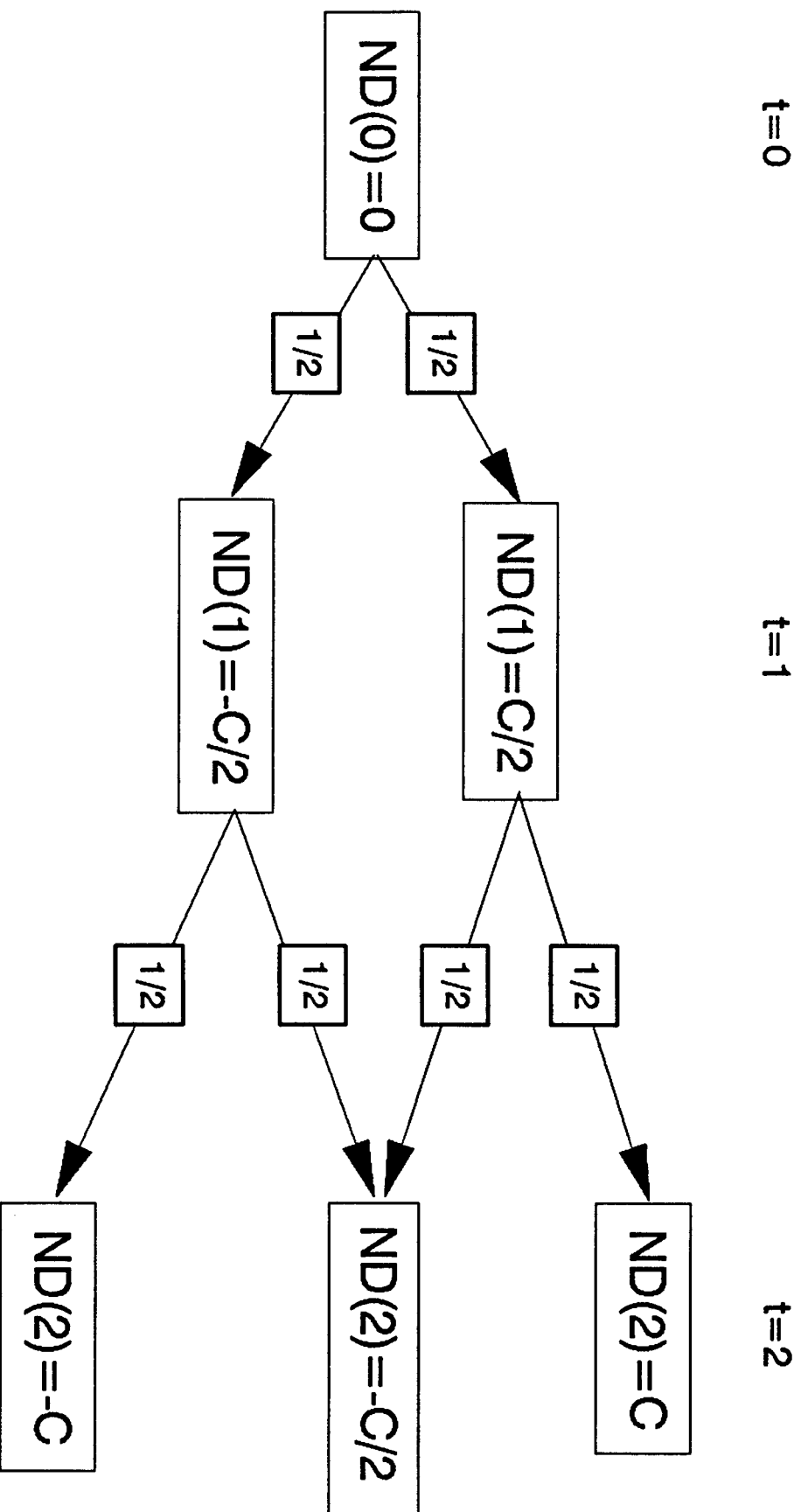
Figure 1: Evolution of Bank's Net Debit Position under Real-Time Gross Settlement



Numbers in boxes represent conditional probabilities



Figure 2: Evolution of Bank's Net Debit Position under Net Settlement



Numbers in boxes represent conditional probabilities

# Figure 3: Evolution of Net Debits

## Continuous-time Model

