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The Design of Private Reinsurance Contracts

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The Design of Private Reinsurance Contracts [1]

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Abstract: This paper examines the role of reinsurance relationships in the trading of underwriting risk when this trade takes place in an environment that is characterized by asymmetric information and in which information is revealed only over time. It begins by explaining how information problems affect the efficiency of the allocation of risk between insurer and reinsurer, and how long-term implicit contracts between insurers and reinsurers allow the inclusion of new information in the pricing of both future and past reinsurance coverage. Because of these features, the ceding company purchases a more efficient quantity of reinsurance. Specifically, such arrangements lead to more reinsurance coverage, higher insurer profits, and lower expected distress in the industry. It is, in short, Pareto improving.

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1 Introduction

The capacity of the insurance industry to absorb large losses resulting from natural disasters has been severely tested in the 1990's and has caused concern among practitioners and policymakers. Although partly due to the growth in insured property values, the unprecedented scale of the losses caused by catastrophic events such as hurricanes and earthquakes in this decade has led to a re-assessment of commonly held beliefs about the frequency and magnitude of these phenomena.[1] It has also sparked interest in a better understanding of current risk-sharing arrangements within the industry as well as the relative effectiveness of markets for sharing catastrophic risks with the wider community of investors. Traditionally, private reinsurance contracts whereby an insurer cedes part of the original risk to a reinsurer, have been used to share risk within the insurance industry. This study analyzes the nature of these risk sharing contracts.

Most analyses of these contracts up to this point have emphasized their ability to pool capacity: reinsurance effectively transforms the insurance industry into one big insurer whose capacity is the sum of the capacities of individual insurers. This literature pioneered by Borch (1962) and developed by many others[2], predicts that in equilibrium, all insurers' portfolios are perfectly correlated after reinsurance is taken into account. While this literature has significantly contributed to our understanding of reinsurance, it does have limiting assumptions which warrant consideration.

First, in these models both insurers and reinsurers have the same information set. In reality, this may not be true. As insurance underwriters have first hand information concerning the nature and extent of the vulnerability one would suspect that they are better informed than their reinsurance counterparts. In the case of natural catastrophes, it may seem reasonable to assume that both insurer and reinsurer are equally informed about the frequency of losses. However, even in this case, insurers control the relationship with the insured and are likely to have private information about the magnitude of potential losses, which depends on factors that are more easily observed by the insurer such as the adequacy of mitigation measures. Even in this case, reinsurance may reduce the insurer's incentives to expend resources to identify policyholders with low expected losses. In effect, risk sharing may result in a deterioration of the quality of underwriting standards. If this deterioration is severe enough, it may actually increase the industry's vulnerability to catastrophic events.

Second, most analyses are developed in a static, one-period framework. In contrast, reinsurance is characterized by an intimate long-term relationship between insurer and reinsurer.[3] Some elements of this relationship are explicitly stated in contractual provisions but others are tacit agreements. The value of reinsurance may lie in the nature of this intertemporal relationship and its resulting effect on the market's ability to deal with monitoring and verification issues. These features cannot be captured in one-period models that essentially treat reinsurance as a onetime arm's length transaction.

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[1] See Cummins, Lewis and Phillips (1996)
[2] For instance Cummins and Doherty (1998)
[3] See Ferguson (1980)

This paper examines the role of reinsurance relationships in the trading of underwriting risk when this trade takes place in an environment that is characterized by asymmetric information and in which information is revealed only over time. It starts with the observation that each layer of the transaction introduces asymmetric information. First, at the origination level, the insured is likely to be better informed about the risk of loss from catastrophic events for which it is purchasing insurance from the primary insurer. Second, the insurer itself has more information about and control of the risk being ceded than the reinsurer. Finally, the reinsurer may have an information advantage over capital markets because of its ability to audit and monitor the insurer. This pyramid of asymmetric information limits the ability of reinsurance to yield the optimal allocation of risk.

The model considers a risk-neutral world in which the demand for reinsurance is created by the existence of bankruptcy costs. Although both insurer and reinsurer face the same capital markets and incur substantial costs in financial distress, the reinsurer usually has better diversification opportunities, which may lower its expected bankruptcy costs. This implies that the cost of taking on an additional risk is higher for the primary insurer than for the reinsurer. However, because reinsurers are less informed about the quality of the risks they are assuming, their evaluation of their risk of insolvency is not accurate; this distorts their pricing of reinsurance. In response to this information asymmetry, the quantity of reinsurance that is demanded by insurers is not first-best. However, because the reinsurer learns the insurer's expected losses in the course of the relationship, the pricing accuracy improves over time. This has one clear and interesting effect, insurers purchase more reinsurance as the expected length of the relationship increases. This is the case because as information arrives, the reinsurance premium is likely to decline, because it will reflect expected losses more accurately.

Although reinsurance is primarily used to share the risk from the liabilities of the insurer, there are other factors that affect the demand for reinsurance and these will not be considered here. These omitted factors can broadly be classified into three categories. One advantage of reinsurance is that it allows the primary insurer to tap the reinsurer's expertise while keeping control of the valuable relationship with the insured. For instance, although the primary insurer may specialize in a particular line, it may use reinsurance to offer its customers a wider diversity of products.[4] The demand for reinsurance is also affected by regulatory accounting constraints that determine the insurer's underwriting capacity. In this context, reinsurance allows the primary insurer to strengthen its financial structure, stabilize its earnings and alter its regulatory capital.[5] Finally, the demand for reinsurance can be dictated by tax considerations. Reinsurance can be used as a mechanism to transfer tax shields benefits to those insurers that have the greatest capacity for utilizing them.[6] None of these ancillary characteristics of reinsurance are modeled here. Rather, its role in risk pooling and the difficulty associated with the

[4] Reinsurance can be tied to other services; this is the "real service" purpose of reinsurance. See Baker (1980).
[5] See Strain (1980)
[6] See Garven and Loubergé (1996), Cummins and Grace (1994), Mayers and Smith (1990)

asymmetric information environment that is an unavoidable part of the market are the focal point of the current analysis.

The paper is organized as follows. The next section describes the contracting environment when risk is traded repeatedly. With repeated transactions long-term relationships can develop; because of this feature the present framework is suitable to the analysis of the effect of contractual arrangements on the demand for reinsurance. This section also formalizes the reasons for the reinsurer's lower bankruptcy cost and analyzes the effect of learning in the pricing of reinsurance. Section 3 analyzes risk-trading and the optimal quantity of reinsurance under different contractual possibilities in a one-shot model. Sections 4 and 5 extend this analysis to a dynamic setting, first with infinitely lived agents, then within a multi period finite setting. Section 6 summarizes our findings, indicating the importance of our informational setting to understanding the reinsurance market and pointing to areas of fruitful future research.

2 A Dynamic reinsurance model

2.1 The primary insurer's problem

Consider an economy in which risk-sharing takes place over several (and possibly an infinity of) discrete time periods. At the beginning of period *t* (date *t-1*) the insurer collects revenue P_{t-1} for the coverage that it provides its clients. Total losses for each period are realized at the end of the period and can be thought of as independent draws from the distribution of the random variable

$$\tilde{X} = \mu + \tilde{\varepsilon} \qquad \tilde{\varepsilon} \equiv N(0,\sigma^2)\,.$$

In other words, losses at date *t* (noted X_t) are independent realizations of $\tilde{X}$ and can be thought of as random variables $\tilde{X}_t$ such that

$$\tilde{X}_t = \mu + \tilde{\varepsilon}_t \qquad \tilde{\varepsilon}_t \equiv N(0,\sigma^2)$$

where $\tilde{\varepsilon}_t$ are independently and identically distributed, i.i.d., random variables. In this expression $\tilde{\varepsilon}$ is a random variable that is beyond the insurer's control; noise variables in different periods are independent of each other. The parameter μ is characteristic of the insurer's exposure to catastrophic losses. μ can be interpreted as the result of the insurer's previous decisions relating to selecting the risks that it wants to insure. For instance, the insurer may investigate the adequacy of building codes as well as the degree of enforcement of these codes in the area where it operates. Such screening activities are costly and higher levels of expenditure on screening result in better quality of underwriting and lower average losses from subsequent loss events. This screening process is not specifically modeled here; it is simply assumed that at the beginning of the analysis screening related costs have been sunk once and for all. The intrinsic quality of the insurer's operations is represented by the parameter μ.

In any period, in addition to the losses themselves, the insurer incurs an additional cost that can be interpreted as the cost of a higher likelihood of insolvency. The existence of these non-linear costs across the range of loss states essentially transform insurers into risk-averse agents that are interested in both the expected magnitude of losses and their variability. In this case, the volatility of losses is a variable that the insurer can choose by adjusting the level of reinsurance. This justification for reinsurance is well-known in the literature on financial risk management.[7] The specification of the insurer's objective function that is adopted in this article includes a term related to actual losses as a proxy for the cost of insolvency and the reinsurer's distaste for risk. This approach is well established in the literature.[8] More specifically if L_t represents the share of date t losses that the insurer is responsible for ($L_t \leq X_t$ since the insurer can reinsure) the insurer's cost of bankruptcy for the period that extends between dates $t-1$ and t is given by:

$$\frac{R}{2} L_t^2 \quad \text{with} \quad R>0$$

where R converts the square of the loss rate to the cost of such losses to the insurer. As such, it includes such things as the actual increasing firm level costs to such losses, the increased capital market scrutiny and the likely regulatory pressure associated with such outcomes. It is the insurers concern over such distress costs that leads it to consider risk transfer through reinsurance.

2.2 The reinsurer's problem

If the insurer reinsures its portfolio each period, it pays a reinsurance premium π_t, $t \in \{0,1\}$ at the beginning of each period. At the outset, the reinsurer cannot directly observe the parameter μ. However, at date 0, the reinsurer can inspect the insurer's operations and learn the realization of a signal, $\tilde{\alpha}$, of the quality of the insurer's operations before determining the premium structure:

$$\tilde{\alpha} = \mu + \tilde{\varepsilon}_0 \qquad\qquad \tilde{\varepsilon}_0 \equiv N(0, \sigma_0^2) \, .$$

[7] See Santomero (1995) and Allen and Santomero (1997)

[8] See Doherty 1991 and Niehaus and Mann (1992)

Note that the observation of $\tilde{\alpha}$ is a noisy signal of μ and is presumed to differ by an unbiased error term.

It will be further assumed that reinsurance markets are competitive and insurers have all the bargaining power. While this assumption is made for convenience, it insures that the reinsurance premium is always equal to the expected cost to the reinsurer. The sequence of events is then summarized in Figure 1.

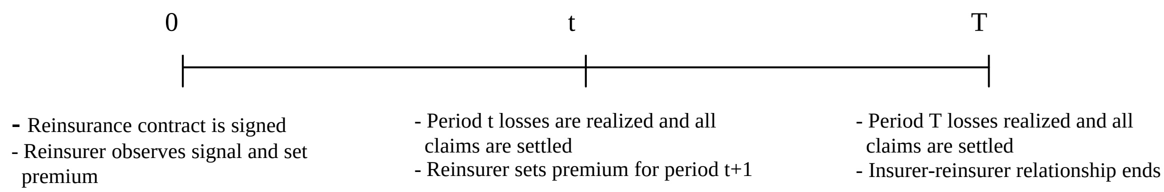

Figure 1. Sequence of events

2.3 The reinsurer's cost function

Similar to the primary insurer, the reinsurer is susceptible to bankruptcy and the form of its value function is the same as that of an insurer. However, the reinsurer deals with several primary insurers, and to the extent that the losses of these insurers are not perfectly correlated the reinsurer's operations will in general be less risky and its cost of bankruptcy lower than that of any single insurer.

To fix these ideas, consider the following example: the reinsurer insures a fraction *s* of the portfolios of each of $n \geq 2$ insurers whose losses are i.i.d. $N(\mu,\sigma)$. Further, assume that there is zero correlation across losses for ease of analysis. Then, expected losses for the reinsurer, are $ns\mu$ and expected losses for each insurer are $(1-s)\mu$. Suppose further that the world lasts just one period and that insurers and reinsurer have the same bankruptcy cost function, $\frac{R}{2}L^2$. Here, L represents the part of the losses for which the firm is responsible, i.e., $L=(1-s)X_i$ for insurer i and $L=s\sum X_i$ for the reinsurer, and the coefficient of the quadratic term is the same for every firm in the market. This is equivalent to arguing that both insurers and reinsurers are evaluated by the same capital market and have similar distress costs, conditional on a proportional loss value.

In this example, the expected cost of bankruptcy for the reinsurer is:

$$\frac{R}{2}E\left[\left(\sum_1^n s\tilde{X}_i\right)^2\right] = \frac{R}{2}s^2E\left[\left(\sum_1^n \tilde{X}_i\right)^2\right] = \frac{R}{2}(ns)^2\left[\frac{\sigma^2}{n}+\mu^2\right].$$

Using this notation for the losses experienced by the insurer, the expected cost of bankruptcy of each insurer indicated above can be written in equivalent form as:

$$\frac{R(1-s)^2}{2}E\left(X_i^{\ 2}\right)=\frac{R(1-s)^2}{2}\left(\sigma^2+\mu^2\right).$$

Notice that this illustrates both the similarity between insurers and reinsurers in this setup and the unique value of the reinsurer spotlighted by Cummins and Doherty (1998). In addition to the distress cost factor *R/2*, the generalized cost of bankruptcy for all insurers can be decomposed into two other factors. The first factor is due to size and is equal to $(ns)^2$ for the reinsurer and to $(1-s)^2$ for the insurers. The second factor is due to risk and is equal to $\left(\frac{\sigma^2}{n}+\mu^2\right)$ for the reinsurer and to $(\sigma^2+\mu^2)$ for the insurers. Obviously, the latter factor will be smaller for the reinsurer than for individual insurers. Given equal size, i.e. $ns=(1-s)$, the total cost of the reinsurer is smaller than that of each insurer.

Therefore, the cost factor of the reinsurer can be denoted as

$$\overline{R}=rR \qquad \text{with} \qquad r=\frac{\frac{\sigma^2}{n}+\mu^2}{\sigma^2+\mu^2}<1\,.$$

Note that when $\mu=0$ or when the cost function is changed to $\frac{R}{2}[L-E(L)]^2$ we have $r=1/n$. This result is simply an application of the law of large numbers. In the rest of the paper it will be assumed that the value function of the reinsurer is identical to that of an insurer, except for the coefficient of the quadratic term which will be equal to $\overline{R}<R$.

2.4 Learning in the reinsurance relationship

Because the reinsurer does not observe either the initial cost incurred by the insurer to screen its customers or establish its underwriting standards, the realized losses in every period contain information about the value μ. Note that from the point of view of the reinsurer μ is a random variable, noted $\tilde{\mu}$, such that

$$\tilde{X}=\tilde{\mu}+\tilde{\varepsilon}\,.$$

If total losses of X_t are observed at date *t*, the reinsurer's belief about μ are updated using Bayes' rule. The following lemma describes the reinsurer's beliefs about the insurer's quality after *T* periods.

LEMMA 1: *At date t the reinsurer's beliefs about the distribution of the insurer's losses are characterized by*

$$E_T(\tilde{X}) = a_T \alpha + b_T \sum_{t=1}^{T} X_t \qquad \textit{and} \qquad \operatorname{var}_T(\tilde{X}) = \sigma^2(1+b_T)$$

$$\textit{where} \quad a_T = \frac{\sigma^2}{\sigma^2 + T\sigma_0^2} \quad b_T = \frac{\sigma_0^2}{\sigma^2 + T\sigma_0^2}.$$

Proof: See appendix.

Date T beliefs about the quality of the insurer's portfolio are simply a weighted sum of the realization, α, of the initial signal and the history of losses, X_t. Higher realized losses signal poorer underwriting quality (higher values of μ). As long as the reinsurer's liability is a non-decreasing function of losses, higher past losses signal higher future losses for the reinsurer. Therefore, as time passes the ceding company's past experience allows the reinsurer to form a more precise opinion of the insurer's quality.

3 The optimal quantity of reinsurance in spot markets

Before analyzing repeated reinsurance purchases, it is useful to consider the optimal reinsurance arrangement in a single period setting. Suppose that the insurer operates only for one period and is liquidated at date 1 after all claims have been settled. In this setting the Pareto optimal (first-best) quantity of reinsurance s minimizes the total expected cost of bankruptcy for both the insurer and reinsurer:

$$\underset{s}{Min} \qquad \frac{E(\tilde{X}^2)}{2}[R(1-s)^2 + \bar{R}s^2].$$

The solution to this problem is

$$\bar{s} = \frac{R}{R+\bar{R}}.$$

The simplest risk-sharing contract shifts an ex-ante agreed-upon part of the insurer's losses at date 1 to the reinsurer in exchange for a premium paid at date 0, a fixed pricing scheme. However, the market will not achieve the first best because of the information asymmetry that is an integral part of the reinsurer contract ex ante. In fact in this setting because bankruptcy costs are non-linear, reinsurers are effectively risk-averse and charge an additional premium for the noise contained in their information. This can be seen from the expression for π. In this case, given that the reinsurance market is priced at the zero profit point, the premium is set so that the reinsurer just breaks even, i.e.,

$$\pi = sE_0\left[\tilde{X} + \bar{R}s\frac{\tilde{X}^2}{2}\right]$$

$$= s\left[\alpha + \frac{\bar{R}s}{2}(\alpha^2 + \sigma^2 + \sigma_0^2)\right].$$

The reinsurance premium is proportional to both the riskiness of the primary insurer's policies and the noisiness of the reinsurer's signal.

The insurer's problem is to select the optimal quantity of reinsurance given the reinsurance premium given above. Mathematically,

$$\underset{s}{Max}\, E(\Phi) = P - (1-s)E(\tilde{X}) - \frac{R(1-s)^2}{2}E(\tilde{X}^2) - E(\pi)\,.$$

This objective function can be rewritten as

$$E(\Phi) = P - \mu - \frac{R(1-s)^2}{2}(\sigma^2 + \mu^2) - \frac{\bar{R}s^2}{2}(\sigma^2 + \mu^2 + 2\sigma_0^2)$$

because

$$E(\pi) = s\left[\mu + \bar{R}s\frac{2\sigma_0^2 + \sigma^2 + \mu^2}{2}\right].$$

The optimal quantity of reinsurance that results from this optimization can be written as

$$s^f = \frac{R(\sigma^2 + \mu^2)}{R(\sigma^2 + \mu^2) + \bar{R}(\sigma^2 + \mu^2 + 2\sigma_0^2)}\,.$$

At this level the insurer's total expected profit can be expressed as

$$E(\Phi^f) = P - \mu - \frac{R\bar{R}}{2}\frac{(\sigma^2 + \mu^2)(\sigma^2 + \mu^2 + 2\sigma_0^2)}{R(\sigma^2 + \mu^2) + \bar{R}(\sigma^2 + \mu^2 + 2\sigma_0^2)}\,.$$

Comparative static analysis of these results has some interesting implications. First, simple algebra shows that the insurer's expected cash flow increases with the accuracy of the reinsurer's information (i.e. $\frac{dE(\Phi^f)}{d\sigma_0^2} < 0$). The more accurate the reinsurer's signal, the lower the reinsurance premium and the higher the ceding company's expected profit. Second, another point worth mentioning is that the quantity of reinsurance is lower than the first-best level. The reason for this under-purchase of reinsurance is that the insurer must trade-off two risks in determining the optimal quantity of coverage: the original risk of insolvency and the excess price that results from the reinsurer's noisy information. Although the insurer is made better off by reinsurance, the asymmetry of information between insurers and reinsurer limits risk-sharing opportunities between the two agents.

This situation can be improved upon, even in a spot market. Imagine for instance that the two parties can contract on ex-post adjustments to the original agreement. More specifically, assume that the reinsurer is allowed to adjust the original premium ex-post by incorporating the new information contained in the realized losses at date 1. This is equivalent to offering rebates or commissions as a function of losses, a feature common to the institutional structure of the reinsurance market.[1] If this ex-post repricing of the reinsurance is unconstrained, then this contingent pricing mechanism allows the reinsurer to charge the insurer an additional premium of

$$\Delta\pi = s\left\{E_1\left[\tilde{X} + \frac{\bar{R}s}{2}\tilde{X}^2\right] - E_0\left[\tilde{X} + \frac{\bar{R}s}{2}\tilde{X}^2\right]\right\}.$$

In this case the combination of both ex ante and ex-post pricing makes the contract equivalent to a contingent pricing mechanism which results in the following lemma.

LEMMA 2: With the ex post adjustment in ex ante fixed cost pricing $E_0(\Delta\pi) = 0$.

Proof: See appendix.

This premium structure effectively insures that reinsurance pricing incorporates the information obtained from the current period insurance coverage, and with a competitive reinsurance market, zero expected profit is achieved.

Assume that the latter pricing mechanism is in place. In this case, the adjustment in the period one premium can be deducted from (or added to depending on the case) the reinsurer's part of the total losses at date 1. In this case at date 1, after observing the realized losses, the reinsurer's evaluation of the insurer's riskiness becomes

$$E_1\left[\tilde{X} + \frac{\bar{R}s}{2}\tilde{X}^2\right] = a_1\alpha + b_1 X_1 + \frac{\bar{R}s}{2}[\sigma^2(1+b_1) + (a_1\alpha + b_1 X_1)^2].$$

Knowing this, the insurer's problem is altered to the following maximization:

$$\underset{s}{Max}\, E(\Phi) = P - (1-s)E(X_1) - \frac{R(1-s)^2}{2}E(X_1^2) - E(\pi + \Delta\pi).$$

[1] See Ferguson (1980)

However, notice that the last term can be rewritten as

$$E(\pi+\Delta\pi) = sE_1\left[\tilde{X} + \frac{\bar{R}s}{2}\tilde{X}^2\right] = s\left[\mu + \frac{\bar{R}s}{2}(\sigma^2+\mu^2+2a_1\sigma_0^2)\right].$$

In this case, the insurer's objective function can be rewritten as

$$E(\Phi) = P - \mu - \frac{R(1-s)^2}{2}(\sigma^2+\mu^2) - \frac{\bar{R}s^2}{2}(\sigma^2+\mu^2+2a_1\sigma_0^2)\,.$$

Then the optimal quantity of reinsurance and the insurer's cash flow become respectively

$$s^c = \frac{R[\sigma^2+\mu^2]}{R(\sigma^2+\mu^2)+\bar{R}(\sigma^2+\mu^2+2a_1\sigma_0^2)}$$

$$E(\Phi^c) = P - \mu - \frac{R\bar{R}}{2}\frac{(\sigma^2+\mu^2)(\sigma^2+\mu^2+2a_1\sigma_0^2)}{R(\sigma^2+\mu^2)+\bar{R}(\sigma^2+\mu^2+2a_1\sigma_0^2)}\,.$$

It easy to see that contingent pricing has improved the insurer's total cash flow compared to the ex ante fixed price contract. This improvement obtains because, in effect, contingent pricing makes the effective premium contingent upon more accurate information: the total premium is based not only on the reinsurer's initial assessment of the insurer's exposure to catastrophic events, but also on the information conveyed by the actual realized losses. The concomitant decrease in the noisiness of the reinsurer's information results in a lower total premium. A by-product of more accurate pricing is that the quantity of reinsurance purchased is higher with contingent than with fixed pricing. However, this quantity is still lower than the first-best. The proposition below summarizes the analysis.

PROPOSITION 1: *In the spot market for reinsurance:*

a) *the quantity of reinsurance traded with contingent pricing is larger than the quantity traded with fixed pricing but smaller than the efficient quantity of reinsurance* ($\bar{s} > s^c > s^f$);
b) *the quantity of reinsurance traded with any type of contract increases as the noisiness of the reinsurer's information declines;*
c) *the insurer prefers contingent pricing to fixed pricing and its expected profit increases with the accuracy of the reinsurer's signal.*

However, contingent pricing is not always feasible in spot markets for three reasons. First, in many tax jurisdictions such ex post state contingent pricing would be viewed as not satisfying the requirements of real insurance. This would have devastating effects on the net cost of such contract premia, and make the arrangement non-economic. Second, in spot markets the insurer's wealth or available surplus (which is represented by P in the model) effectively bounds the magnitude of ex-post adjustments to the reinsurance premium. Thus, in order to be feasible contingent pricing schemes may need to be accompanied by some complementary intertemporal smoothing mechanism. Third, even when wealth is not a binding constraint, contingent pricing is possible in spot markets only to the extent that explicit contracts can be written. In practice, this is not always the case because contrary to what is assumed in this stylized model, losses for a period are not always observed immediately at the end of the period. Claims are usually filed and settled gradually. In fact, both insurers and reinsurers may not really know the extent of their liabilities for past periods before several years elapse. In this context the insurer may have an incentive to behave dishonestly or to take advantage of the reinsurer. However, all of these issues can be dealt with in the context of the long-term relationships that are part of the reinsurance markets operating structure. It is to this structure that we now turn.

4 Infinitely repeated risk-trading and the optimal quantity of reinsurance

The fact that contingent pricing is more efficient than fixed pricing is not surprising because contingent pricing allows the price to reflect information as it arrives. In short, it improves the quality of the decisions that agents make. The efficiency of ex-post adjustments to the reinsurance premium also suggests that if these adjustments are repeated over time, the insurer might be made even better off without any adverse consequences for the reinsurer. In other words, in a repeated setting, the first-best Pareto optimal allocation of risk may be attainable. The purpose of this section is to investigate this possibility.

Suppose for instance that a primary insurer enters into a long-term relationship with a reinsurer. Here, long-term means an infinite number of periods. Suppose that when the ceding company buys reinsurance at date t (which cover losses for period $t+1$) the insurer is charged an original premium of

$$\pi_t = s_t E_t \left\{ \tilde{X} + \frac{\overline{R} s_t}{2} \tilde{X}^2 \right\}$$

Then, at each successive date thereafter the insurer is charged an adjustment in addition to the premium for new periods equal to

$$\Delta_{t+j}(\pi_t) = s_t E_{t+j}\left\{\tilde{X} + \frac{\overline{R}s_t}{2}\tilde{X}^2\right\}$$
$$- s_t E_{t+j-1}\left\{\tilde{X} + \frac{\overline{R}s_t}{2}\tilde{X}^2\right\}$$

j=1,2,... Furthermore,

$$E_t[\Delta_{t+j}(\pi_t)] = 0 .$$

This equality results from the well-known fact that beliefs follow a martingale, that is any expected change in tomorrow's beliefs is reflected in today's beliefs (see the proof of Lemma 2). Notice that at any date $t+j$ the total premium, including all successive adjustments, paid for the reinsurance that was purchased at date t is

$$\pi_t + \sum_{i=1}^{j}\Delta_{t+i}(\pi_t) = s_t E_{t+j}\left(\tilde{X} + \frac{\overline{R}s_t}{2}\tilde{X}^2\right).$$

This result is important. It indicates that the price of past transactions is constantly being revised to include new information as it arrives. In fact, at any date $t+j$ the total cost of the reinsurance that was purchased at date t is equal to the expected cost to the reinsurer taking into account the information that arrived between dates t and $t+j$. In effect, this scheme progressively reduces the noisiness of the reinsurer's information.

As time passes $(j\to\infty)$ this expression tends to

$$\overline{\pi}_t = \lim_{j\to\infty}\left(\pi_t + \sum_{i=1}^{j}\Delta_{t+i}(\pi_t)\right)$$
$$= s_t E\left(\tilde{X} + \frac{\overline{R}s_t}{2}\tilde{X}^2\right)$$
$$= s_t\mu + \frac{\overline{R}s_t}{2}(\sigma^2 + \mu^2) .$$

In the limit, the total reinsurance premium paid by the insurer is equal to the full information premium that would have been paid if there were no information asymmetry between insurer and reinsurer. The insurer's total expected cash flows for insurance and reinsurance activities for period $t+1$ can be expressed as

$$E(\Phi_{t+1}) = P_t - (1-s_t)\mu - \frac{R(1-s_t)^2}{2}E(\tilde{X}^2) - \overline{\pi}_t$$
$$= P_t - \mu - \frac{R(1-s_t)^2}{2}(\sigma^2+\mu^2) - \frac{\overline{R}s_t^2}{2}(\sigma^2+\mu^2)$$

The resulting optimal quantity of reinsurance purchased at date t solves the following problem

$$\underset{s}{Max}\, E(\Phi_{t+1});$$

The solution to that problem is obviously $\bar{s}$, the efficient quantity of reinsurance. This discussion is summarized in the following proposition.

PROPOSITION 2: *In an infinitely lived reinsurance relationship even with asymmetric information, contingent pricing achieves the Pareto optimal allocation of risk.*

5 Finitely repeated risk-trading

The previous section assumes that firms live forever and can enter into infinitely lived relationships. This assumption is unrealistic for several reasons. First, firms do not last forever and under some conditions are liquidated. Liquidation of one of the parties effectively terminates the reinsurance relationship. Second, an insurer may have an incentive to terminate the relationship after large unexpected losses because of the resulting future loss of ceding commission if it stays in the relationship. This is particularly relevant if contracts of infinite duration cannot be written or enforced. Moreover, reputation mechanisms may not fully mitigate the effects of the incompleteness of contracts. The reason is that when an insurer experiences extreme losses as in the case of catastrophic events, the value of maintaining a reputation decreases.

In a more realistic environment, firms expect the relationship to last a finite number of periods. Therefore, this section analyses risk-sharing in a simple T-period framework. In this case it is assumed that the ceding company decides on how much reinsurance to buy at date 0 for the entire period 0 to T. This assumption is meant to capture the fact that in the short run, the primary insurer cannot always change its implicit agreement with the reinsurer or switch reinsurers after a poor loss experience without incurring a reputation penalty.[1] The respective total cash flows of the insurer and the reinsurer over the finite number of periods are (assuming P_t=P for all t):

$$\Phi = TP - \pi_T - (1-s)\sum_{t=1}^{T} X_t - \frac{R(1-s)^2}{2}\sum_{t=1}^{T} X_t^2$$

$$\Psi = \pi_T - s\sum_{t=1}^{T} X_t - \frac{\bar{R}s^2}{2}\sum_{t=1}^{T} X_t^2 .$$

In these expressions π_T represents total premia and adjustments paid over the T periods.

[1] See Gilliam (1980)

5.1 Contingent pricing

In a finite period relationship, the total price that is paid by the primary insurer is the expected cost to the reinsurer given the reinsurer's total information set gathered over the entire period i = 1…T. More specifically, the reinsurer's revised expected cost of bankruptcy at date *T* is

$$\frac{\bar{R}s^2}{2} E_T\left(\sum_{t=1}^{T} \tilde{X}_t^2\right) = \frac{\bar{R}s^2 T}{2} E_T(\tilde{X}^2) .$$

The term in the expectation bracket corresponds to total bankruptcy costs over the *T* periods. Using the Bayesian updating formulas of section 2.4 above, we have

$$E_T(\tilde{X}) = b_T \sum X_t + a_T \alpha$$

$$\text{var}_T(\tilde{X}) = \sigma^2 (1 + b_T) .$$

Therefore at date *T*, the reinsurer's revised total expected premium over the *T* periods is

$$\pi_T = sT\left(b_T \sum X_t + a_T \alpha\right) + \frac{\bar{R}s^2 T}{2}\left[(b_T + 1)\sigma^2 + \left(b_T \sum X_t + a_T \alpha\right)^2\right].$$

With contingent pricing, the ceding company expects to pay only this revised cost estimate. At date 0 the insurer expectation of the revised total premium can be shown to be equal to

$$E(\pi_T) = sT\left[\mu + \frac{\bar{R}s}{2}(\sigma^2 + \mu^2 + 2a_T \sigma_0^2)\right].$$

Notice that this cost is higher than the real (full-information) expected cost of bankruptcy. The difference is due to the noise in the reinsurer's information set as represented by a_T. The ceding company's total expected cash flow at date 0 is:

$$E(\Phi) = TP - T\mu - \frac{R(1-s)^2 T}{2}(\sigma^2 + \mu^2) - \frac{\bar{R}s^2 T}{2}[\sigma^2 + \mu^2 + 2a_T \sigma_0^2] .$$

The optimal quantity of reinsurance, *s*, solves:

$$\max_s E(\Phi) .$$

The first and second order conditions for this problem are

$$\frac{\partial E(\Phi)}{\partial s} = T\left[R(1-s)(\sigma^2+\mu^2) - \overline{R}s(\sigma^2+\mu^2+2a_T\sigma_0^2)\right] = 0$$

$$\frac{\partial^2 E(\Phi)}{\partial s^2} = -T\left[(\sigma^2+\mu^2) + \overline{R}(\sigma^2+\mu^2+2a_T\sigma_0^2)\right] < 0\,.$$

Therefore the optimal value of s noted s_T^* exists and is unique:

$$s_T^* = \frac{R(\sigma^2+\mu^2)}{R(\sigma^2+\mu^2)+\overline{R}(\sigma^2+\mu^2+2a_T\sigma_0^2)}\,.$$

Straightforward algebra shows that $s_T^* < \overline{s}$, and simple derivation shows that

$$\frac{\partial s_T^*}{\partial a} < 0 \quad \text{and} \quad \frac{\partial s_T^*}{\partial T} > 0\,.$$

This simply means that as the longer the insurer-reinsurer relationship is expected to last, the larger the quantity of reinsurance traded.

The reinsurer's maximal expected profit is given by

$$E(\Phi_T^*) = T\left\{P - \mu - \frac{R\overline{R}(\sigma^2+\mu^2)(\sigma^2+\mu^2+2a_T\sigma_0^2)}{2\left[R(\sigma^2+\mu^2)+\overline{R}(\sigma^2+\mu^2+2a_T\sigma_0^2)\right]}\right\}.$$

A straightforward application of the envelope theorem shows that:

$$\frac{\partial E(\Phi_T^*)}{\partial R} = -(1-s_T^*)^2(\sigma^2+\mu^2) < 0$$

$$\frac{\partial E(\Phi_T^*)}{\partial \overline{R}} = -s_T^{*\,2}(\sigma^2+\mu^2+2a_T\sigma_0^2)] < 0\,.$$

This confirms the intuition that the equilibrium profit of the ceding company is inversely related to both its own cost of bankruptcy and the reinsurer's. Similarly, it simple to verify that

$$\frac{\partial E(\Phi_T^*)}{\partial T} > 0 .$$

This analysis is summarized in the following proposition.

PROPOSITION 3: *In a finitely lived reinsurance relationship, under contingent pricing the quantity of reinsurance is lower than the Pareto optimal level (* $s_T^* < \bar{s}$ *).*
As the length of the relationship increases:

a) the quantity of reinsurance traded increases, that is $\frac{\partial s_T^*}{\partial T} > 0$ *;*

b) the insurer's expected profit increases, that is $\frac{\partial E(\Phi_T^*)}{\partial T} > 0$;

c) in the limit the quantity of reinsurance traded is efficient, that is $\lim_{T \to \infty} s_T^* = \bar{s}$.

5.2 Fixed Pricing

An alternative to contingent pricing is for the insurer and the reinsurer to sign a long-term contract for *T* periods in which the price of reinsurance is fixed once and for all at date 0. Let π_T be the total premium under that contract. Because competition is assumed in the reinsurance market, the ceding company has all the bargaining power so that π_T can be written as:

$$\pi_T = sE_0\left[\sum X_t + \frac{\bar{R}s}{2}\sum X_t^2\right]$$
$$= sT\left[\alpha + \frac{\bar{R}s}{2}\left(\sigma^2 + \alpha^2 + \sigma_0^2\right)\right].$$

In this case by construction, the reinsurer cannot include the new information it learns at subsequent dates in the pricing of reinsurance. From the ceding company's perspective, total expected premia at date 0 are:

$$E(\pi_T) = sT\left[\mu + \frac{\bar{R}s}{2}\left(\sigma^2 + \mu^2 + 2\sigma_0^2\right)\right].$$

Comparison with the expressions obtained in the previous section shows that the total expected premium is higher under fixed pricing than under contingent pricing. The two expressions are almost identical except for the last term. The difference is due to the lower noise in the contingent pricing scheme which decreases the last term.

The ceding company's total expected cash flows are:

$$E(\Phi) = TP - T\mu - \frac{R(1-s)^2 T}{2}(\sigma^2 + \mu^2) - \frac{\overline{R}s^2 T}{2}(\sigma^2 + \mu^2 + 2\sigma_0^2) .$$

Following a procedure similar to the one outlined in the previous section yields the unique optimal quantity of reinsurance which is given by the expression

$$s_T = \frac{R(\sigma^2 + \mu^2)}{R(\sigma^2 + \mu^2) + \overline{R}(\sigma^2 + \mu^2 + 2\sigma_0^2)} .$$

As could be expected $s_T < s_T^*$, that is insurers purchase more reinsurance under the contingent pricing scheme than under the fixed-price contract. Furthermore, the quantity of reinsurance is independent of the length of the contract that is:

$$\frac{ds_T}{dT} = 0 .$$

Finally, the maximal expected profit for the insurer is

$$E(\Phi_T) = T\left\{P - \mu - \frac{R\overline{R}(\sigma^2 + \mu^2)(\sigma^2 + \mu^2 + 2\sigma_0^2)}{2\left[R(\sigma^2 + \mu^2) + \overline{R}(\sigma^2 + \mu^2 + 2\sigma_0^2)\right]}\right\} .$$

It is straightforward to show that this is lower than with contingent pricing, that is $E(\Phi_T) < E(\Phi_T^*)$. The results of this section are summarized in the following proposition.

PROPOSITION 4: *Under fixed pricing in a finitely lived reinsurance relationship, both the quantity of reinsurance and the ceding company's total expected profit are lower than the corresponding contingent pricing levels (* $s_T < s_T^* < \overline{s}$ *and* $E(\Phi_T) < E(\Phi_T^*)$*). The quantity of reinsurance is independent of the length of the relationship.*

In a fixed price contract the price depends only on the initial information of the reinsurer. This is noisier than the information that is accumulated over *T* periods which determines the price under a contingent pricing contract.

5 Conclusion

This paper has developed a model of the role of private reinsurance contracts in allocating the losses from random property and casualty losses among insurers. First, it generally explains how information problems affect the efficiency of the allocation of risk between insurer and reinsurer. Here is demonstrated that the traditional results obtained from standard models of reinsurance omit relevant, indeed important, aspects of the market. Specifically, traditional models from Borch to the present omit the information asymmetry prevalent in this market and the intertemporal nature of these contracts. Adding these characteristics substantially alters both the nature of the problem and the design of optimal contracts. Second, the paper explains the rationale for the commission structure that prevails in the industry in which the ceding commission is usually adjusted according to realized losses. Long-term implicit contracts between insurers and reinsurers allow the inclusion of new information in the pricing of both future and past reinsurance coverage. When the effective price of reinsurance for one specific period is contingent upon the outcome in all subsequent periods, the price of reinsurance is likely to reflect more accurately the underlying risk. Because of this, the ceding company purchases a more efficient quantity of reinsurance. Specifically, such arrangements lead to more reinsurance coverage, higher insurer profits, and lower expected distress in the industry. The conclusion is that in an environment characterized by imperfect information and in which learning occurs over time, adjustments of the reinsurance premium allows insurers to diversify informational risk intertemporally, and improve social welfare. It is, in short, Pareto improving.

Finally, the paper also shows the limitations of the rebate structure. Most of the analysis rests on the implicit assumption that the pooled capacity of insurer and reinsurer is large enough to allow for full settlement every period. To the extent that the realized outcome falls within reasonable bounds this scheme works. However, in extreme cases the scheme falls apart and crises occur. In particular, when both insurer and reinsurer are susceptible to the same aggregate shocks, they may be unable to fulfill their commitments to each other in case of extremely high losses. This suggests that risk-sharing mechanisms that pool capacity within the insurance industry have their limitations and that there is a role for the broader capital markets in the effective mitigation of catastrophic risks. Whether markets can improve the intertemporal smoothing ability of private reinsurance contracts is left for further research.

Appendix

A- PROOF OF LEMMA 1

Given the normality assumption, Bayes' formula can be written as

$$E_t(\tilde{X}) = E_{t-1}(\tilde{X}) + \frac{\text{cov}_{t-1}(\tilde{X}_t, \tilde{X}_{t+1})}{\text{var}_{t-1}(\tilde{X})}[X_t - E_{t-1}(\tilde{X})]$$

$$\text{var}_t(\tilde{X}) = \text{var}_{t-1}(\tilde{X}) - \frac{\left[\text{cov}_{t-1}(\tilde{X}_t, \tilde{X}_{t+1})\right]^2}{\text{var}_{t-1}(\tilde{X})}.$$

The proof consists of two parts: first we show that the lemma holds for *T=1*; second we show that if the lemma is true for any *T*, it must also hold for *T+1*.

a) The lemma is true for *T=1*.
At date 0 the reinsurer's beliefs can be expressed as:

$$\tilde{\mu} = \alpha - \tilde{\varepsilon}_0 \qquad \tilde{X} \equiv \alpha - \tilde{\varepsilon}_0 + \tilde{\varepsilon} \quad \tilde{X} \equiv N(\alpha, \sigma^2 + \sigma_0^2)$$

where α is the realization of $\tilde{\alpha}$. The reinsurer's beliefs at date 1, can be computed with Bayes' formula above as:

$$E_1(\tilde{X}) = \lambda\alpha + (1-\lambda)X_1 \qquad \text{var}_1(\tilde{X}) = \lambda(\sigma^2 + 2\sigma_0^2) \qquad \lambda = \frac{\sigma^2}{\sigma^2 + \sigma_0^2}.$$

The same result is obtained by replacing *T* by 1 in the lemma an recognizing that

$$a_1 = \lambda \qquad b_1 = 1 - \lambda.$$

b) If the lemma is true for any *T*, then it must also be true for *T+1*.
Assume that the lemma is true for *T* periods. Therefore we have:

$$E_T(\tilde{X}) = a_T\alpha + b_T\sum_{t=1}^{T} X_t \qquad \text{var}_T(\tilde{X}) = \frac{\sigma^2[\sigma^2 + (T+1)\sigma_0^2]}{\sigma^2 + T\sigma_0^2}$$

$$a_T = \frac{\sigma^2}{\sigma^2 + T\sigma_0^2} \qquad b_T = \frac{\sigma_0^2}{\sigma^2 + T\sigma_0^2}.$$

Again using Bayes' formula we can compute the reinsurer's beliefs at date *T+1*:

$$E_{T+1}(\tilde{X}) = E_T(\tilde{X}) + \frac{\text{cov}_T(\tilde{X}_{T+1}, \tilde{X}_{T+2})}{\text{var}_T(\tilde{X})}[X_{T+1} - E_T(\tilde{X})]$$

$$E_{T+1}(\tilde{X}) = \frac{\sigma^2}{\sigma^2 + (T+1)\sigma_0^2}\alpha + \frac{\sigma_0^2}{\sigma^2 + (T+1)\sigma_0^2}\sum_1^{T+1} X_t$$

$$\text{var}_{T+1}(\tilde{X}) = \frac{\sigma^2[\sigma^2 + (T+2)\sigma_0^2]}{\sigma^2 + (T+1)\sigma_0^2}.$$

This is the same as replacing *T* by *T+1* in the lemma.

B- <u>PROOF OF LEMMA 2</u>

$$\Delta\pi = s\left\{E_1\left[\tilde{X} + \frac{\overline{R}s}{2}\tilde{X}^2\right] - E_0\left[\tilde{X} + \frac{\overline{R}s}{2}\tilde{X}^2\right]\right\}$$

$$E_0[E_1(\tilde{X})] = E_0(a_1\alpha + b_1 X_1) = \alpha$$

$$E_0[E_1(\tilde{X}^2)] = E_0[\sigma^2(1+b_1) + (a_1\alpha + b_1 X_1)^2] = \alpha^2 + \sigma^2 + \sigma_0^2 = E_0(\tilde{X}^2).$$

The lemma follows.

More generally:

$$E_T[E_{T+J}(\tilde{X})] = E_T(a_{T+J}\alpha + b_{T+J}\sum_1^{T+J} X_t) = a_{T+J}\alpha + b_{T+J}\sum_1^{T} X_t + b_{T+J}\, jE_T(\tilde{X})$$

$$= \frac{b_{T+J}}{b_T}E_T(\tilde{X}) + b_{T+J}\, jE_T(\tilde{X}) = (1 + Tb_{T+J} + jb_{T+J})E_T(\tilde{X}) = E_T(\tilde{X}).$$

$$E_T[E_{T+J}(\tilde{X}^2)] = E_T\left[\sigma^2(1+b_{T+J}) + \left(a_{T+J}\alpha + b_{T+J}\sum_1^{T+j} X_t\right)^2\right]$$

$$= E_T\left[\sigma^2(1+b_{T+J}) + b_{T+j}^2\left[\left(\frac{\sigma^2}{\sigma_0^2}\alpha + \sum_1^T X_t\right)^2 + 2\left(\frac{\sigma^2}{\sigma_0^2}\alpha + \sum_1^T X_t\right)\sum_{T+1}^{T+j} X_t + \left(\sum_{T+1}^{T+j} X_t\right)^2\right]\right]$$

$$= E_T\left[\sigma^2(1+b_{T+J}) + b_{T+j}^2\left[\left(\frac{E_T(\tilde{X})}{b_T}\right)^2 + 2\left(\frac{E_T(\tilde{X})}{b_T}\right)\sum_{T+1}^{T+j} X_t + \left(\sum_{T+1}^{T+j} X_t\right)^2\right]\right]$$

$$= \sigma^2(1+b_{T+J}) + (a_{T+J} + Tb_{T+J})^2[E_T(\tilde{X})]^2 + 2j(a_{T+J} + Tb_{T+J})b_{T+J}[E_T(\tilde{X})]^2 + b_{T+j}^2 E_T\left(\sum_{T+1}^{T+j} X_t\right)^2$$

$$= \sigma^2(1+b_{T+J}) + (1-jb_{T+J})^2[E_T(\tilde{X})]^2 + 2j(1-jb_{T+J})b_{T+J}[E_T(\tilde{X})]^2 + b_{T+j}^2 E_T\left(\sum_{T+1}^{T+j} X_t\right)^2$$

$$= \sigma^2(1+b_{T+J}) + (1-j^2 b_{T+j}^2)[E_T(\tilde{X})]^2 + b_{T+j}^2 E_T\left(\sum_{T+1}^{T+j} X_t\right)^2$$

$$= \sigma^2(1+b_{T+J}) + (1-j^2 b_{T+j}^2)[E_T(\tilde{X}^2) - \sigma^2(1+b_T)] + b_{T+j}^2\{jE_T(\tilde{X}^2) + 2\tbinom{j}{2}[E_T(\tilde{X}^2) + \sigma^2 b_T]\}$$

$$= \sigma^2(1+b_{T+J}) + (1-j^2 b_{T+j}^2)[E_T(\tilde{X}^2) - \sigma^2(1+b_T)] + b_{T+j}^2[j^2 E_T(\tilde{X}^2) + j(j-1)\sigma^2 b_T]$$

$$= E_T(\tilde{X}^2) + \sigma^2[(1+b_{T+j}) - (1+b_T)(1-b_{T+j}^2 j^2) + j(j-1)b_T b_{T+j}^2]$$

$$= E_T(\tilde{X}^2) + \sigma^2(1+jb_{T+j})[b_{T+j} - b_T(1-jb_{T+j})]$$

$$= E_T(\tilde{X}^2)\,.$$

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