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On the Risk of Life Insurance Liabilities: Debunking Some Common Pitfalls

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The Working Paper Series is made possible by a generous grant from the Alfred P. Sloan Foundation

On the Risk of Life Insurance Liabilities: [1]
Debunking Some Common Pitfalls

November 1995

Abstract : The objective of this paper is to contribute to a better understanding of the driving forces of a life insurance company. More specifically, the issues of the duration and convexity of insurance liabilities and equity are addressed. These issues deserve a careful rethinking given the recent trends that have affected the insurance landscape. A correct assessment of these risk measures is critical as they constitute the primary ingredients of any sound asset-liability management approach. In addition, the effort toward a more detailed and more accurate risk picture of life insurance operations enables one to debunk some pitfalls that are commonly encountered in the insurance industry.

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ON THE RISK OF LIFE INSURANCE LIABILITIES : DEBUNKING SOME COMMON PITFALLS

Eric Briys
François de Varenne

November 1995

"How did a boring, straight-forward business become so interesting and so difficult to regulate?"

Salvatore R. Curiale
Superintendent of Insurance
New York State Insurance Department

The savings and loan debâcle - the soaring number of insolvencies of financial institutions - has been a traumatic event for the United States. During the "roaring eighties," a lot of changes affected the U.S. financial landscape. The rising interest rates in the early 1980s led to a significant flow of consumers dollar into mutual funds. The competition for the savings dollar became very fierce among financial institutions. This pressure combined with regulatory mistakes (see White [1991]) had a perverse consequence on many financial institutions. They reached for riskier assets offering higher yields and operated with less capital per dollar of assets. In that respect, the example of life insurance companies is very informative. Life insurance were forced to redesign their product lines and to migrate toward interest rate sensitive products (see Wright [1991]). This new environment induced life company investment officers to mismatch assets and liabilities and to lower quality standards by assuming higher credit risks. The result was, as we know now, disastrous. In 1987, nineteen companies went bankrupt; in 1989, a worrisome forty; in 1991, a new record of fifty-eight insolvent insurance companies. Canada went through the same kind of turmoil. In 1992, only a year after CompCorp, an industry-financed guarantee corporation, the insurance company, Les Coopérants, failed. It was followed by the collapse of Sovereign Life in 1993. In 1994, Confederation Life went into receivership.

Europe has not been spared by these costly events, either. Although less publicity has been given to the distresses of the major European insurance companies, concern is growing among European regulators and consumers. According to a recent article published in The Economist (July 17, 1993), insurance companies in the United Kingdom have started to cut bonuses on "with profits" policies. Most Scandinavian companies have been severely down graded.

The objective of this paper is to contribute to a better understanding of the driving forces of a life insurance company. More specifically, the issues of the duration and convexity of insurance liabilities and equity are addressed. As noted earlier, these issues deserve a careful rethinking given the recent trends that have affected the insurance landscape. A correct assessment of these risk measures is critical as they constitute the primary ingredients of any sound asset-liability management approach. In addition, the effort toward a more detailed and more accurate risk picture of life insurance operations enables one to debunk some pitfalls that are commonly encountered in the insurance industry, as shown in the next section.

DEBUNKING SOME COMMON PITFALLS IN LIFE INSURANCE

Most life insurers claim that their industry carries distinguishing features which makes it quite unique. In particular, they often reject the comparison with the banking industry by arguing that banks and life insurance companies are significantly different animals. To support their claim, life insurers usually put forward three basic arguments. These arguments, as the story goes, are sufficient motives for their industry to deserve a specific asset-liability management treatment.

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The first argument, which is also shared by property-casualty insurers, underlines the fact that the insurance production process is inverted: output prices (read insurance premiums) have to be established before input prices and costs (read claims) are known. The insured pays a known insurance premium and may incur in the future some damage which will be then reimbursed. This situation, according to insurers, is at odds with, say industry, where costs or input prices are known beforehand and used to set prices.

The second argument stressed by insurers is somehow a corollary of the first one. According to this argument, risk-taking initially occurs on the liability side of the balance sheet. Underwriters issue insurance policies which are transformed into liabilities (read technical reserves). Because of the time lag between the premium inflow and the indemnity outflow, the reserves are invested on the financial marketplace and generate the portfolio of assets of the company. Again, this is at odds with the banking industry which, according to insurers, initially take risks on the asset side (read loans, mortgages etc...).

Before turning to the third argument, the first and the second argument can be shown to be quite dubious. They are easily debunked. Indeed, the first argument is rather astonishing when the insurance contract is framed into an option setting. An insurance contract is a put option: like the equity put option, its primary function is to provide his holder with an asset value guarantee. The writer of any option has to fix a premium before he knows whether or not the option is going to be exercised. Bankers are familiar with options: they embed prepayment options in mortgages (which is a way of insuring the borrower against interest rate risk) without knowing whether or not the borrower is going to use them. Bankers, option markets players do not however complain about a so-called inverted production process!

The second argument is also quite easy to dismantle. Bank deposits can be viewed as providing liquidity insurance. A depositholder knows that, in case of unexpected liquidity needs, he can totally or partially cash out his deposit. The analogy with insurance is even stronger: because of the law of large numbers, not everybody is expected to run and require from his bank his total deposit back. The notion of risk mutualization is precisely at the heart of the insurance industry. To make the counterargument even more convincing, the case of indexed CDs issued by banks can be referred to. Banks offer deposits whose return is composed of a fixed component and a variable component pegged to some index. By issuing such deposits banks are indeed taking risks: they guarantee a floor and add up a potential bonus.

The third argument is by far the most compelling one. Life insurers frequently insist upon the long maturity of their liabilities. This long term view stems from the actuarial dimension of insurance liabilities. A typical example is the life annuity which is paid until death which may occur very late. Until recently, as stressed by Wright [1991], “portfolio philosophy in the life insurance business was centered on the matching of assets and liabilities... The traditional practices of buying long-term bonds and mortgages and holding them to maturity were based on the long duration of liabilities”. Wright hastens to add that today “a rethinking of the duration of these (insurance) products is essential”. This rethinking is not only urged by the redesign of life insurance policies but also by the pressure of competition. Insurance agents bring considerable pressure on companies headquarters to set initial rates high enough to match competition and keep them high in the future even though interest rates might have fallen down. These interwoven effects challenge the long view of insurance liabilities. The duration, in other words the interest rate risk exposure of insurance liabilities, is not only a matter of mortality tables and proper discounting but is also significantly affected by the “geometry” of the contractual liabilities cash-flows. Because insurance liabilities are not traded and accounting practices tend to distort the cash-flow picture, this last point is not clearly understood. A lot of insurance companies still manage their liabilities using a long term horizon while they should be shooting at a much shorter time frame. The outcome of such practices is clear: an outsized mismatch between the durations of assets and liabilities jeopardize the value of equity when interest rates increase.

MODELLING STAKEHOLDERS CASH-FLOWS

In the previous section, three common pitfalls have been debunked using rather qualitative arguments. The purpose of the current section is to set up a simple quantitative model of a life insurance company. Indeed, the use of fairly standard financial tools such as options, duration and convexity measures yields a more accurate, not to say a more convincing, risk picture of life insurance liabilities. More specifically, the outputs of this simple quantitative model reinforce the views expressed in the previous section by giving some order of magnitude.

Our model is based on Merton’s approach to financial intermediaries [1977, 1990]. Only one type of life insurance policy is considered here. The closest example of such a policy on the US life insurance market is the Universal Life insurance contract (UL) that was introduced in 1979. The cash value of a UL contract typically earns a minimum guaranteed rate of return. Some insurers then tie an extra return on UL insurance accumulations to the portfolio rate of return earned by the insurer. Some other US life insurers leave it to the Board of Directors to determine the rate credited on the UL contract. It is however fairly clear that this decision cannot really depart from the performance of the insurer's portfolio. It also corresponds to the UK "with profits" policies. These British policies insure customers for a lowish basic sum, which is then topped up with discretionary bonuses (the profits), depending on how the insurer's investments perform. This type of policy is also quite common in France. The first component of the rate to be served on these French policies is fixed and guaranteed. However, regulation imposes a profit-sharing mechanism. Indeed, by law, French life insurance companies have to pay policyholders at least 85 percent of their net financial revenues - namely, dividends, coupons, rents, realized capital gains, and so on. This profit-sharing mechanism is known in France under the name of *Participation aux Bénéfices.* To sum up, policyholders benefit from a guaranteed interest rate and a percentage of the performance of the company's asset portfolio. As a result, two key inputs characterize such policies - the guaranteed interest rate and the participation level. The model enables one to determine the fair interest rate or the fair participation level policyholders should require to fully compensate them for the risks they face.[1] In other words, the model yields the fair price of the insurance liabilities given the current structure of the company balance sheet.[2] Based on this valuation, interest rate risk measures can be computed. These computations show, as already pointed out in the first section, that conventional wisdom needs to be "turned upside down."

A life insurance company whose planning horizon extends over a given time interval [0, *T]* is considered. Time $t = T$ can be considered as the time to maturity of a single cohort of life insurance policies issued at time $t = 0$. Insurance and financial markets are assumed to be competitive. At time $t = 0$, the insurance company acquires an asset portfolio A_0 and finances this portfolio with paid-in capital E_0 and the premiums of the life insurance cohort. The life insurance policy is structured as follows. The policyholder is guaranteed a fixed interest rate r^*. On top of this fixed rate, the policyholder is entitled to a share δ of the net financial revenues (dividends, net capital gains, coupons, rents...) of the life insurance company. This participation coefficient δ is sometimes bounded from below by regulation. In France, for instance, it cannot go below 85 percent. The guaranteed rate r^* is usually less than the market rate for a risk-free asset of the same maturity as the policy. The participation coefficient δ can be viewed as making up for the difference between the two rates and embodying the required risk premium by policyholders holding risky life insurance policies. Indeed, shareholders have a limited liability, and if the company is declared insolvent at time $t = T$, they simply walk away.

At time $t = 0$, the company's balance sheet is given by Table 1. For the sake of simplicity, we normalize A_0 to the value $1. The proportion of the initial assets financed by equity is then given by $(1-\alpha)$. The portfolio of assets is assumed to be totally invested in risky assets (equity, bonds, real estate).

Table 1 :The company's balance sheet at time t = 0

Assets		Liabilities and equity	
Assets	A_0	Liabilities	$L_0 = \alpha \cdot A_0$
		Equity	$E_0 = (1-\alpha) \cdot A_0$
Total	A_0	Total	A_0

The first risk element of the balance sheet is interest rate risk. To capture the uncertainty in the term structure of interest rates, we use the Heath, Jarrow and Morton [1992] process[3]. The second element of risk is asset risk - that is, all risk affecting assets other than interest rate risk[4]. To give a complete picture of the riskiness of the insurance company, the portfolio of assets is assumed to be affected by both the interest rate risk and the asset risk. As a result, the value of the portfolio of assets, A_t, as of time *t*, is governed by a stochastic process whose path is affected both at the same time by asset risk and interest-rate risks.[5]

Under this simple setting and using the contractual definition of stakeholders' cash-flows, one can write the policyholders payoffs as follows. Three states of nature have to be clearly distinguished.

- In the first scenario (the worst case scenario), the insurance company is totally insolvent : the value of assets as of time T is less than the guaranteed payment to policyholders. The company is declared bankrupt, and the policyholders receive what is left:

$$L_T = A_T \quad (1)$$

- In the second case, the insurance company is able to fulfill its guaranteed commitment but unable to serve the bonus. Assuming that the policyholders assets are earmarked, we define B_T, the financial bonus to policyholders once guaranteed payments have been cashed out as of time t = *T:*

$$B_T = \delta\left[\frac{L_0}{A_0}(A_T - A_0) - (\dot{L}_T - L_0)\right] = \delta\left[\alpha A_T - \dot{L}_T\right]$$

where δ denotes the level of the bonus and $L_T^* = \alpha A_0 \exp(r^* T)$ is the guaranteed payoff to policyholders. If the bonus B_T is negative, it amounts to $A_T < L_T^*/\alpha$.

- In the third scenario, B_T *is* positive. Assets generate enough value to match the guaranteed payment and the policyholders bonus. In such a case the liabilities as of time T are equal to :

$$\begin{aligned} L_T &= L_T^* + B_T \\ &= (1-\delta) L_T^* + \delta\alpha\, A_T \end{aligned} \qquad (2)$$

To sum up, the first case corresponds to a case of total insolvency; the second case is a partial insolvency in the sense that only guaranteed commitments are fulfilled; the third case is the best case scenario where the insurance company is performing well.

Because of the limited liability feature of equity, the shareholders' stake is a residual stake. Their final payoffs E_T are depicted across the three previous scenarios:

Ž First case :

$$E_T = 0 \text{ if } A_T < L_T^*$$

Ž Second case:

$$E_T = A_T - L_T^* \text{ if } L_T^* \le A_T < \frac{L_T^*}{\alpha}$$

Ž Third case:

$$E_T = (1-\delta\alpha) A_T - (1-\delta) L_T^* \text{ if } \frac{L_T^*}{\alpha} \le A_T$$

These payoffs suggest that equity and liabilities have the features of contingent claims. This result is obviously not a surprise. Indeed, since the seminal works of Black and Scholes [1973] and Merton [1973], it is well-known that the limited liability equity of a levered firm can be valued as a contingent claim on the firm's underlying assets. The only difference here is that the contractual structure is more involved and includes features that are not present in the simplified picture à la Black-Scholes-Merton.

Valuing Equity and Insurance Liabilities

By applying the option pricing framework (see Briys and de Varenne [1994]), the market value of both equity and insurance liabilities can be computed. As far as equity is concerned, Table 2 indicates that the value of equity as of time T is given by :

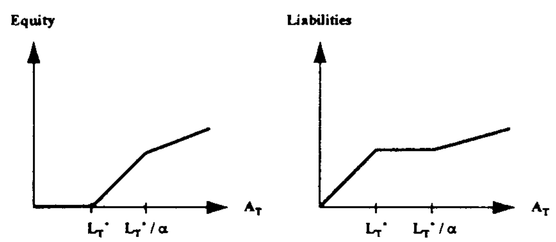
$$E_T = \max\left[0, A_T - L_T^*\right] - \delta\alpha \max\left[0, A_T - \frac{L_T^*}{\alpha}\right] \qquad (3)$$

The two terms on the right hand side are nothing but the final payoffs on two call options maturing at time T and with exercise price L_T^*. The value of equity at any time $t < T$ can thus be rewritten as:

$$E_t = C_E(A_t, L_T^*) - \delta\alpha\, C_E\left(A_t, \frac{L_T^*}{\alpha}\right) \qquad (4)$$

where $C_E(A_t, L_T^*)$ and $C_E(A_t, L_T^*/\alpha)$ are both European calls[6] maturing at time $(T-t)$ and with exercise price L_T^* and L_T^*/α respectively.

Table 2 : Policyholders and shareholders final payoffs



The equity of the life insurance company is an hybrid entity. The first term on the right hand side is easy to understand. Indeed, if one let δ equal to zero, this first term is the familiar limited liability call option (see Black and Scholes [1973] and Merton [1973]). Shareholders have the option to walk away if things go wrong. The second term appears because of the potential bonus. This term is also a call option. A bonus is equivalent for the policyholders to having a call option on the performance of the insurance company assets. Equity is thus made of a long (limited liability) call position and a short call position, the latter being weighted by the bonus level δ.

As far as liabilities are concerned, the final cash-flows depicted in Table 2 indicate that they can be priced as follows:

$$L_t = L_T^* \cdot P(t, T) - P_E(A_t, L_T^*) + \delta\alpha\, C_E\left(A_t, \frac{L_T^*}{\alpha}\right) \qquad (5)$$

where $P_E(A_t, L_T^*)$ denotes the price of the shareholders' put to default - that is to walk away from their guaranteed commitments.[7] The intuition underlying the pricing of insurance liabilities is obvious.

Indeed, insurance liabilities can be disentangled into three basic components as shown in Table 3. The first term values the insurance liabilities of a default-free no-bonus insurance company. The second term reflects the fact that the insurance company may default: policyholders write an option to default to shareholders. The third term is related to the bonus which the company may pay if it does well.

Table 3 : Liabilities as an hybrid portfolio

Treasury zero-coupon bond	−	Shareholders' option to default	+	Bonus option
Long position		Short position		Long position
Risky zero-coupon bond				

Policyholders have two ways to be rewarded for the risk of non performance they face. For a given bonus δ, they will require a guaranteed rate r^* so that they do get a fair rate of return on their policy. Or, for a given r^*, they will make sure that the bonus level is such that the insurance policy offers an ex-ante fair rate of return. Viewed from the shareholders' side, the pricing of the insurance policy should be such that they are fairly compensated for owning the stock of the insurance company. This in turn means that the guaranteed rate and the bonus should be such that the initial cash outlay by shareholders is equal to the present value of their claim on future equity cash-flows. The present value of this claim is already known: it is given by expression (4). To put it even more simply, shareholders will not "subsidize" policyholders, nor will policyholders "subsidize" shareholders. For a given guaranteed rate r^*, the matching bonus can be computed. For a given bonus level, the guaranteed rate is obtained. It is worthwhile to observe that while r^* cannot be computed explicitly, an analytical expression for the bonus can be derived:

$$\delta = \frac{C_E(A_0, L_T) - (1-\alpha)A_0}{\alpha C_E(A_0, L_T^*/\alpha)} \tag{6}$$

The model can thus be used to draw some pricing implications. In particular, the relationships linking the pricing parameters to the various inputs (volatility, leverage, etc...) can be thoroughly examined (see Briys and de Varenne [1994] for more details). The objective of the current paper is not so much in terms of pricing but primarily in terms of risk assessment. Indeed, the knowledge of the valuation of insurance liabilities as given in expression (5) is most helpful: the interest rate sensitivity of these liabilities can be directly measured by simply computing the derivative of expression (5) with respect to the short-term interest rate[8]. In the same vein convexity measures (second-order derivative) can be established.

Measuring the Duration of Insurance Liabilities

As indicated in the previous section our valuation model has interesting implications for assessing the riskiness of a life insurance company. Indeed, the assets and liabilities of a life insurance company are interest rate sensitive. To implement a sound Asset-Liability Management (ALM) approach, the insurance manager needs to accurately evaluate his risk exposure. Interest rate elasticity, duration and convexity measures are now commonplace. Nevertheless, most of them are quite restrictive and only apply under a specific set of assumptions. Corporate default, bonus schemes are for instance rarely taken into account. This is unfortunate and produces biased estimates of the true elasticity of insurance liabilities. In that respect, our model corrects these pitfalls.

After some computations,[9] the following expression for the duration of insurance liabilities obtain as of time t :

$$\begin{aligned} D_L &= -\frac{1}{L_t}\cdot\frac{\partial L_t}{\partial r_t} \\ &= (T-t) - G_t\cdot\frac{A_t}{L_t}\cdot\left(N(-d_1) + \delta\alpha N(d_3)\right) \end{aligned} \tag{7}$$

$$\text{where } G_t = \rho\frac{\sigma_A}{\sigma} + (T-t)$$

From expression (7) several polar cases can be recovered. The first one deals with the situation where there is no bonus and the volatility of assets is nil. Under these assumptions, the duration of insurance liabilities is exactly equal to maturity. This makes sense because insurance liabilities become just a default-free zero-coupon bond.

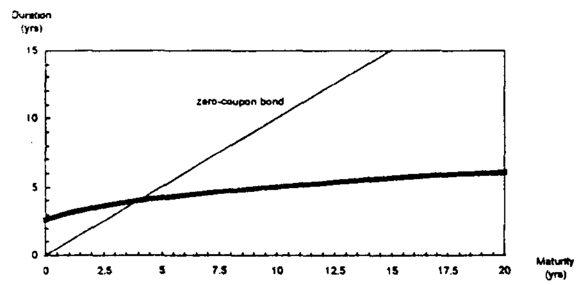
In the second case, no bonus is attached to the insurance policy. Assets are, however, risky. As a result, expression (7) collapses to:

$$D_L = (T-t) - G_t\cdot\frac{A_t}{L_t}\cdot N(-d_1) \tag{8}$$

A careful inspection of expression (8) is illuminating for getting a better insight into the more complex expression (7). Indeed, expression (8) is basically composed of three terms which can be disentangled as follows. As already mentioned above, the first term corresponds to the time to maturity *(T – t)* of a default free zero-coupon bond. The second term G, is equivalent to the interest rate elasticity gap between the insurance company's assets and the default-flee zero-coupon bond. This is so because the ratio $(-\rho\sigma_A/\sigma)$ measures the interest rate elasticity of the insurance firm's assets. The third term in (8) can be viewed as a probability adjusted leverage ratio.

Expression (7) is obviously more complex than expression (8). It reflects not only the above mentioned effects but also the influence of the bonus scheme. For policyholders, this bonus is equivalent to a long position on a call option whose duration affects the total duration of the insurance policy.

Table 4 : The duration of liabilities of as function of time-to-maturity

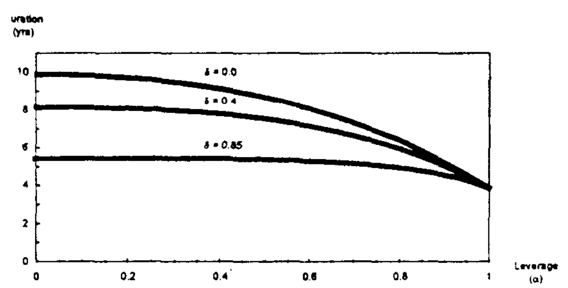


$\alpha = 0.8, \delta = 0.85, r_0 = 0.1, \sigma_A = 0.2, \sigma = 0.01, \rho = -0.2$

To confirm the qualitative arguments put forward in section 2, we now turn to a numerical implementation of our duration model. Table 4 depicts the relationship between the duration of insurance liabilities and the maturity of the life insurance policy. It is based on the following basic parameters values: $\alpha = 0.8$, $\delta = 0.85$, $\sigma_A = 0.2$, $\sigma = 0.01$, $\rho = -0.2$ and $r_0 = 0.1$. In Table 4, the 45° line represents the locus of points where duration is exactly equal to maturity. Several comments are in order. One can first observe that the effective duration of insurance liabilities is generally smaller than the maturity, i.e. the Macaulay duration. Insurance liabilities are composed of three terms. The default put reduces the duration of insurance liabilities, so does the long position on the bonus call option. Indeed, the call is an increasing function of the interest rate. For instance, life insurance liabilities with a maturity of 20 years have an effective duration of 6.1 years. But this result does not carry over to short term insurance liabilities. For maturity below 4 years, the effective duration is greater than Macaulay duration. This result may seem quite counterintuitive. This lengthening of the effective duration finds its main roots in the bonus scheme and the interest rate elasticity gap between the insurance company's assets and the default-free zero-coupon bond. In the case of a negative correlation[10] (which we have here) and short maturities the gap is negative. Because the bonus is indeed a bonus ($\delta > 0$), the total impact on the effective duration is a positive one. The risk of life insurance liabilities is a more complex issue than conventional wisdom suggests. As recalled in section 2, life insurers frequently insist upon the long maturity (read duration) of their liabilities. The message conveyed by our contingent-claim approach is somewhat different and boils down to[11]: "short means long, long means short"!

A last point concerning Table 4 deserves some explanation. When the maturity of the life insurance liability is zero, it appears that the effective duration is not nil. In Table 4, it is roughly equal to three. Such a result is strongly at odds with what intuition would suggest. However, a careful investigation of expression (7) gives the clue to the puzzle. The effective duration of insurance liabilities is driven by the duration of assets through the bonus mechanism. When the bonus is nil, intuition is recovered: effective duration is equal to zero. Again, the message conveyed by our model is clear; "real short means quite long!"

Table 5 : The duration of liabilities as a function of leverage



$T = 10, r_0 = 0.1, \sigma_A = 0.2, \sigma = 0.01, \rho = -0.2$

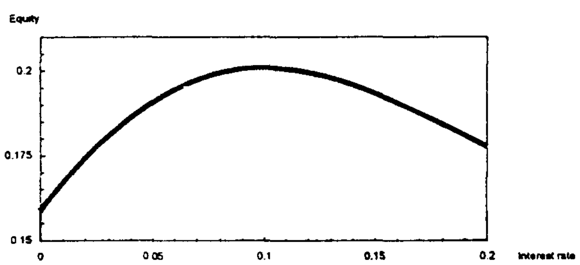
Table 4 yields interesting results. One could however argue that it does not clearly disentangle the respective effects of the shareholders' option to default and the bonus scheme. To cope with this, some more numerical simulations are implemented. More specifically, in Table 5, a life insurance liability with a maturity of ten years is considered. In this table, the effective duration of the insurance liability is related to the insurance company leverage ratio. For low leverage ratios[12], the effect of the put to default is significantly neutralized. The bulk of the impact on the effective duration of the insurance liability stream stems primarily from the bonus scheme. Indeed, in the comer case where the leverage ratio is close to zero, the effective duration of the insurance policy with a typical bonus of 85% is roughly 55% of that of an insurance policy with no bonus at all.

Some Implications for Equity Immunization

A relevant objective of any asset-liability management policy is to ensure that equity is insensitive to unexpected shifts in interest rates. Indeed, shareholders do not need a cumbersome vehicle such as a life insurance company to bet on the future course of interest rates. They can do it themselves by buying or selling bonds or interest rate futures contracts !

In what follows we look at the implications of our previous results for the asset-liability posture of the life insurance company. The market value of equity is obviously equal to the difference between the market value of assets and the market value of liabilities. As such, its duration is directly affected by both the duration of assets, the duration of liabilities and the leverage effect. Table 6 portrays the behavior of equity value as a function of the level of the interest rate.

Table 6 : The equity as a function of interest rates

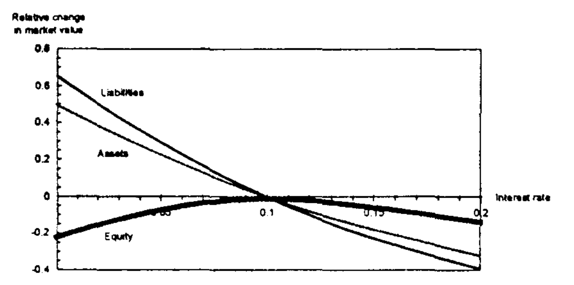


$\alpha = 0.8, \delta = 0.85, T = 10, \sigma_A = 0.2, \sigma = 0.01, \rho = -0.204$

To draw such a picture, expression (4) is used. In that equation, the change in asset value in response to the change in interest rates is approximated by using the asset duration and the asset convexity.[13] The correlation coefficient is set such that the equity duration)[14] is equal to zero for a 10% interest rate. The striking feature in Table 6 is that equity behaves like a short straddle. In options markets, short straddle are achieved by simultaneously selling a put and a call on the same underlying asset. The rationale for choosing such a profile is betting on small movements in the price of the underlying asset whatever the direction. In such a case, the options are not exercised by the long side and the short straddler is ensured to retain the option premiums. In case of large movements, this short straddling position is obviously a very risky one.

From Table 6 and its set of parameters, one can observe that the duration of equity is nil at a 10% interest rate. If rates go below that level, the slope becomes positive: when rates decrease, equity goes down. Beyond that level, the slope is negative: when rates increase, the value of equity decreases. The fact that the equity of a life insurance company may resemble a short straddle is not new. It has been described by Babbel and Stricker [1989]. Babbel and Stricker's framework however is quite different and their short straddling equity is a direct result of their choices for assets and liabilities. In their model insureds hold a surrender option, namely the equivalent of an American put option on a zero-coupon bond. On the asset side, the company, in its quest for extra yields, is assumed to invest in callable bonds. By buying such bonds, the insurance company implicitly sells an American call bond option to the issuer. The short straddle outcome is then quite straightforward : the insurance company ends up selling options not only on the liability side but also on the asset side. In our setting, this explanation is not valid. For instance, no option is assumed to be sold on the asset side. No surrender option is considered. The only options at work are the put to default and the bonus call option. Both of them are on the liability side. The concavity of equity ("negative gamma" to use an option pricing terminology) is the by-product of the interest rate risk exposure of assets and liabilities and the leverage effect. As a result, even in a very simple framework, the insurance company may turn out to "bet the bank on interest rates". To clarify why a short straddle is obtained, it is useful to disentangle the various effects at play on the asset side and the liability side.

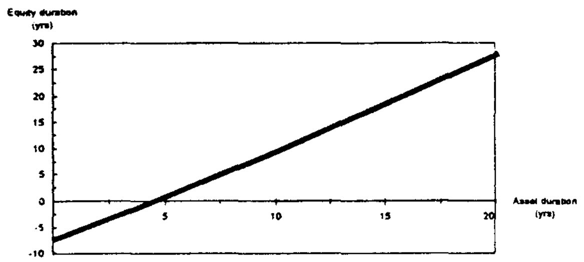
Table 7 : The relative impact of changes in interest rates



$\alpha = 0.8, \delta = 0.85, T = 10, \sigma_A = 0.2, \sigma = 0.01, \rho = -0.204$

Table 7 depicts these various effects by relating the relative change in assets, liabilities and equity values to the level of the interest rate. Given the choice of (reasonable) parameters, liabilities and assets are shown to exhibit a classical convex behavior. The convexity effect is however stronger on the liability side than on the asset side. When rates go down, assets tend to go up because of the negative correlation: the total effect on the bonus call option is a positive one. Liabilities increase more than they would in the absence of a bonus. When rates go up, assets tend to decrease: the dominant effect is that of the put to default which goes deeper into the money. Liabilities lose more value than they would if no default were possible. As a result, liabilities turn out to be more convex than assets. The consequence for equity is immediate. Equity exhibits a negative convexity. In Table 7, equity is interest rate risk immune at a 10% interest rate. Any change from that level entails a destruction of shareholder value.

Table 8 : Equity duration as a function of asset duration



$\alpha = 0.8, \delta = 0.85, r_0 = 0.1, \sigma_A = 0.2, \sigma = 0.01, T = 10$

To avoid such a prospect, that is gambling on interest rates on behalf of shareholders, assets must be allocated such that their duration results in a zero-duration for equity. To show that this is possible, equity duration is related to asset duration in Table 8. Because the interest rate elasticity of assets is given by $(-\rho\sigma_A/\sigma)$, asset allocation policies can be mimicked by tuning the correlation coefficient for a given level of asset volatility. Indeed, a lower coefficient means that the asset portfolio is more heavily invested in bonds. On the contrary, a higher coefficient implies that the portfolio is geared toward equity investment. In Table 8, an asset duration of 4.1 years, namely a coefficient equal to -0.204, yields a zero equity duration. A greater asset duration would imply a company betting on a decrease in interest rates.

A last implication of our results is that derivatives like options or structured products are certainly nice candidates for fine-tuning equity immunization strategies. Indeed, to compensate negative convexity effects, buying options (i.e. buying positive convexity) on the asset side is a decision that deserves careful consideration.

CONCLUSION

In this paper, we have developed a life insurance liabilities valuation model which debunks some pitfalls commonly encountered in the insurance industry. Using a contingent claim based methodology, a fairly simple closed-form solution for the pricing of life insurance liabilities is obtained. Because it accounts for stochastic interest rates, default risk and bonus schemes, this model produces more accurate duration and convexity measures. The effective duration of life insurance liabilities has been shown to be significantly different from the traditional Macaulay duration measure. For reasonable sets of parameters, equity has been shown to resemble a short straddle even though we did not rely on the traditional explanations to achieve such a result. Immunization implications have also been considered.

Some additional work can usefully complement these results. Extensions to more complex life insurance policies is a natural candidate. Indeed, surrender options have not been taken into account. Because they amount to selling options to policyholders, introducing them would amplify our picture. Liabilities would be even more sensitive to increases in interest rates.

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Footnotes

1. For a detailed analysis of these pricing issues and some regulatory implications, the interested reader is invited to refer to Briys and de Varenne [1994].
2. We do not consider the mortality issue. Indeed, introducing a mortality table would not change our results.
3. Under this representation, the price dynamics of a default-free zero-coupon maturing at time T is given by: $dP(t,T)/P(t,T) = r_t\,dt + \sigma(T-t)\,dW_t$ where r_t is the instantaneous risk-free rate at time t and W_t a standard Wiener process. Other representations could have been used. The important point here is that interest rates are stochastic.
4. That is, all risk affecting assets (equity, real estate...) other than the interest rate risk.
5. More precisely, the stochastic process is given by $dA_t/A_t = \mu dt + \sigma_A\left[\rho dW_t + \sqrt{1-\rho^2}\,dZ_t\right]$ where μ and σ_A denote respectively the instantaneous expected return on assets and their instantaneous volatility. Z_t is a standard Wiener process independent of W_t capturing the asset risk other than the interest rate risk. The coefficient p represents the correlation between the total value of assets and the interest rate.
6. See Appendix for the detailed formula.
7. See Appendix
8. The term structure model that we use here is a one factor model. Extensions can obviously be made to multi-factor model. In that case, multiple durations (with respect to each factor) can be computed.
9. Detailed computations are available from the authors upon request.
10. This assumption is obviously a reasonable one.
11. Under our set of assumptions.
12. Reasonable interest rate and asset volatility levels are also retained.
13. More precisely, the asset duration is equal to $D_A = -\rho\sigma_A/\sigma$ and the convexity to $C_A = D_A^2$.
14. The equity duration is defined along the same lines as equation (7): $D_E = T - [\rho\sigma_A/\sigma + T]\cdot[N(d_1) - \delta\alpha N(d_3)]\cdot A_0/E_0$

Appendix

From Heath Jarrow Morton [1992], we have the following expressions.

$$C_E(A_t, \overset{\bullet}{L}_T) = A_t\,N(d_1) - P(t,T)\,\overset{\bullet}{L}_T\,N(d_2)$$

$$P_E(A_t, \overset{\bullet}{L}_T) = -A_t\,N(-d_1) + P(t,T)\,\overset{\bullet}{L}_T\,N(-d_2)$$

$$C_E\left(A_t, \frac{\overset{\bullet}{L}_T}{\alpha}\right) = A_t\,N(d_3) - P(t,T)\,\frac{\overset{\bullet}{L}_T}{\alpha}\,N(d_4)$$

where :

$$\begin{cases} d_1 = \dfrac{\ln A_t/\overset{\bullet}{L}_T P(t,T) + \bar{\sigma}^2(T-t)/2}{\bar{\sigma}\sqrt{T-t}} \\ d_2 = d_1 - \bar{\sigma}\sqrt{T-t} \\ d_3 = \dfrac{\ln \alpha A_t/\overset{\bullet}{L}_T P(t,T) + \bar{\sigma}^2(T-t)/2}{\bar{\sigma}\sqrt{T-t}} \\ d_4 = d_3 - \bar{\sigma}\sqrt{T-t} \\ \bar{\sigma}^2 = \sigma_A^2 + \rho\sigma\sigma_A(T-t) + \sigma^2\dfrac{(T-t)^2}{3} \\ N(\,) = \text{the cumulative normal distribution} \end{cases}$$