# Macroeconomic Dynamics and Credit Risk: A Global Perspective*

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**Abstract**

This paper presents a new approach to modeling conditional credit loss distributions. Asset value changes of firms in a credit portfolio are linked to a dynamic global macroeconometric model, allowing macro effects to be isolated from idiosyncratic shocks from the perspective of default (and hence loss). Default probabilities are driven primarily by how firms are tied to business cycles, both domestic and foreign, and how business cycles are linked across countries. We allow for firm-specific business cycle effects and the heterogeneity of firm default thresholds using credit ratings. The model can be used, for example, to compute the effects of a hypothetical negative equity price shock in South East Asia on the loss distribution of a credit portfolio with global exposures over one or more quarters. We show that the effects of such shocks on losses are asymmetric and non-proportional, reflecting the highly non-linear nature of the credit risk model.

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# 1 Introduction

Risk management in general and credit risk analysis in particular has been the focus of extensive research in the past several years. Credit risk is the dominant source of risk for banks and the subject of strict regulatory oversight and policy debate. More recently, the proposal by the Bank for International Settlements (BIS) to reform the regulation of bank capital for credit risk (known as the New Basel Accord, or BIS 2) has initiated debates on a number of issues in the literature.[1] One of the issues under discussion centers on the effects of business cycles, especially of severe economic downturns, on bank risk and value-at-risk capital requirements (Carpenter, Whitesell and Zakrajšek 2001, Carey 2002, Allen and Saunders 2004). However, this discussion has been taking place largely without the benefit of an explicit model that links the loss distribution of a bank's credit portfolio to the evolution of directly observable macroeconomic factors at national and global levels. Given the increasing interdependencies in the global economy, risk managers of commercial and central banks alike may well be interested in questions like "What would be the impact on the credit loss distribution of a given bank (or banks) in a given region if there were large unfavorable shocks to equity prices, GDP or interest rates in that or other regions?"

The purpose of this paper is to show how global macroeconometric models can be linked to firm specific return processes which are an integral part of Merton-type credit risk models so that quantitative answers to such questions can be obtained. We propose a combined model of credit losses contingent on the macroeconomy that is able to distinguish between default (and loss) due to systematic versus idiosyncratic (or firm specific) shocks, providing an explicit channel for modeling default correlations. This enables us to conduct simulation experiments on the effect of changes in observable macroeconomic dynamics on credit risk.

In providing such a linkage, the main conceptual challenge is to allow for firm-specific business cycle effects and the heterogeneity of default probabilities across firms. Standard credit risk models pioneered by Vasicek (1987) and elaborated in Vasicek (1991, 2002) and Gordy (2003), adapt the option-based approach of Merton (1974) and allow for business cycle effects generally via one or more unobserved systematic risk factors. They assume that the processes generating asset values and the default thresholds are homogeneous across firms. The parameters of the loss distribution are then identified by fixing the cross-firm correlation of asset returns and the mean default rate of the credit portfolio. Operational versions of the Vasicek model, e.g. by KMV, allow for firm heterogeneity by making use of balance sheets, income statements and other similar reports issued by the firm. This process inevitably involves a certain degree of subjective evaluation, however, and the outcome is generally proprietary information.[2]

[1] For details of BIS 2 see BIS (2001, 2004), and for an account of the debates see, for example, Jones and Mingo (1998), and Altman, Bharath and Saunders (2002).

[2] Credit portfolio models also differ in the way they model changes to the firms' value. Some models operate on a mark-to-market basis by looking at the change of market value of credit assets based on credit migration and the term

Examples of credit risk portfolio models in the professional literature include Gupton, Finger and Bhatia's (1997) CreditMetrics, KMV's PortfolioManager, and the actuarial approach employed by CSFB's CreditRisk+ (Credit Suisse First Boston 1997) where the key risk driver is the variable mean default rate in the economy. Wilson's (1997a,b) model (CreditPortfolioView) is an exception. He allows for the macroeconomic variables to influence a firm's probability of default using a pooled logit specification. However, because the defaults are grouped, typically by industry, and modeled at the (single country) national level, any firm-specific heterogeneity is lost in the estimation process. For detailed comparisons, see Koyluoglu and Hickman (1998), Crouhy, Galai and Mark (2000), Gordy (2000) and Saunders and Allen (2002).

In this paper we depart from the literature in two important respects. First, we model individual firm returns (taken as proxies for changes in asset values) in terms of a number of directly observable contemporaneous risk factors, such as changes in equity indices, interest rates, inflation, real money balances, oil prices and output, both domestic and foreign. In this way we allow for the possible differential impacts of macroeconomic factors on the evolution of firm's asset values, and as such their default probabilities. Second, using historical observations on mean returns, volatility and default frequencies of firms for a particular credit rating, we compute firm-specific default threshold-equity ratios under the assumption that two firms with identical credit ratings are likely to have similar default threshold-equity ratios. Thus we are able to provide an empirical implementation of the Merton model using only two pieces of publicly available information for each firm, namely market returns and credit ratings, in a multi-country setting.

The problem of obtaining accurate information about the health of a firm, while not new, is particularly relevant for modeling firms' bankruptcy or default. Our approach has the advantage that it does not rely on firm-specific accounting data which are at best noisy and at worst biased due to the information asymmetries between company managers (agents) and share/debt holders (principals). Rating agencies are likely to have access to private information about the firm's past performance and its current management, in addition to public information from balance sheets and company reports, in arriving at their firm-specific credit ratings. In the pursuit of better ratings, companies have more of an incentive to reveal (some of) their private information to the credit rating agencies than to their debt-holders, very much in the same spirit that second–hand car dealers have the incentive to reveal information about the cars they offer for sale by the duration of the guarantees and other after-sales services that they provide. Moreover, basing the analysis strictly on accounting data would make it difficult to harmonize information across different accounting standards and bankruptcy codes from different countries, a source of heterogeneity presumably addressed by rating agencies.

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structure of credit spreads (CreditMetrics). Others focus on predicting default losses (so-called default mode models such as CSFB's CreditRisk+). Yet there are other approaches that allow for both (e.g. KMV's PortfolioManager, Wilson's CreditPortfolioView).

In short, in our framework the *portfolio* loss distribution is driven by firms' credit ratings and how their returns are tied to business cycles, both domestic and foreign, and how business cycles are linked across countries. This is in contrast to credit risk analyses that explicitly focus on modeling of *individual* firm defaults, using panel probit or logit specifications (Altman and Saunders 1997, Lennox 1999). Since defaults are rare, to obtain sensible estimates these applications tend to impose strong homogeneity assumptions on the parameters, which could bias the estimates. For instance, it is impossible to allow for any firm-specific (e.g. fixed) effects. The probit/logit approach is also difficult to adapt for the analysis of multi-period credit loss distributions, whilst our approach can be readily extended for such purposes.[3]

To link the firm-specific returns to business cycle factors we shall make use of the global vector autoregressive (GVAR) macroeconometric model recently developed by Pesaran, Schuermann and Weiner (2004) – hereafter PSW. This model is composed of vector error-correcting models (VECM) estimated for individual countries (or regions), which are then combined into a global model that takes account of both intra- and inter-country/regional interactions. The model uses domestic macroeconomic variables such as GDP, inflation, the level of short term interest rates, exchange rate, equity prices (when applicable) and real money balances. These are related to corresponding foreign variables constructed exclusively to match the international trade pattern of the country under consideration. Because of the global nature of the model, we can analyze how a shock to one specific macroeconomic variable affects other macroeconomic variables, even (and especially) across countries, as well as shocks to risk factors, e.g. oil prices, affecting all regions.

We examine the credit risk of a fictitious corporate loan portfolio and its exposure to a wide range of observable risk factors in the global economy. We model a firm's probability of default as a function of those risk factors but assume for simplicity that loss given default is an exogenously given random variable whose specific parameterization can vary by country. Using the firm-specific return regressions and the GVAR model, single- and multi-period credit loss distributions of a given portfolio are then obtained through Monte Carlo simulations.

Our baseline expected losses are quite reasonable when compared with actual industry loan charge-offs. For example, expected loss over the course of four quarters is about 58bp (basis points) of exposure, compared with 89bp, the average net charge-offs (loans charged off less amount recovered over total loans) for the U.S. banking industry from 1987 to 2003. When compared with the actual industry charge-offs matched by our forecast horizon, namely 2000Q1, the difference is even smaller: those were 56bp (at an annual rate). Much of the fat-tailedness of our loss distribution is, however, due to the relatively small number of firms (119 in our portfolio) which entails a substantial degree of diversifiable idiosyncratic risk. Once this is controlled for (by including 'copies' of the existing firms within the portfolio), the EL to VaR multiples are in line with those obtained by others (e.g. Carey 2002 who has about 500 exposures). For instance, the tail values at 99% and

[3]A recent exception is Duffie and Wang (2003) who forecast default intensities over multiple periods.

99.5% are around three to four times expected losses. Moreover, when we impose extreme shocks such as those seen during the Great Depression, VaR is more than triple the baseline scenario, also consistent with Carey's results. We find further that symmetric shocks to the observable risk factors do not result in correspondingly symmetric loss outcomes reflecting the nonlinear nature of the credit risk model.

In attempting to provide a formal link between credit risk and the macro-economy we have been forced to make many difficult choices. First, we confined our analysis to publicly traded companies with a sufficiently long credit rating history. We assume that this credit rating is a sufficient summary statistic of unconditional default risk, meaning that we take credit ratings as the business cycle-neutral, 'common currency' of default risk across different geographies, legislations and accounting standards. But we allow for firm-specific conditional default probabilities over the course of the business cycle. To do this we need three different tools: (i) a model of the systematic (macroeconomic) risk factors, (ii) firm returns and how they are linked to those factors, and (iii) firm default thresholds. The GVAR satisfies the first requirement, and the link to firms is done through firm-level return regressions by allowing the loadings on the macro-variables to be firm specific. The default thresholds are identified by assuming that they are the same within a rating category.[4] Clearly, other modeling strategies and identification schemes can be adopted. The present paper demonstrates that such an approach to credit risk modeling is in fact feasible.

Our model is particularly suited for an international and multi-factor interpretation of the standard corporate finance view of firm risk: total risk is the sum of systematic and idiosyncratic (i.e. firm-specific) risk. The GVAR is ostensibly a global model of systematic risk and its dynamics. Having a model of those factor dynamics can go a long way to understanding firm risk (and return) characteristics and to address specific risk management related questions. One which we find particularly valuable is the ability to rank-order possible shock scenarios. Given a particular portfolio of credit exposures, is a $1\sigma$ shock (one standard error shock) to Japanese money supply more damaging (or beneficial, depending on the sign of the shock) than a $1\sigma$ shock to South East Asian or U.S. equity markets? What will the portfolio loss distribution look like one year from now? What if the portfolio changes? Such counterfactual questions are central to policy analysis, be it by commercial or central bankers who might wish to investigate the impact on a representative bank portfolio in their country of various economic shocks in other countries. If the model is not compact enough, it cannot be practically used in this repetitive fashion.

The remainder of the paper is as follows: Section 2 provides an overview of the alternative

[4] The bankruptcy models of Altman (1968), Lennox (1999) and Shumway (2001) generate firm specific default forecasts, as does the industry model by KMV (Kealhofer and Kurbat (2002)). However, all of these studies impose more significant parameter homogeneity than we do, and they focus on just one country at a time (the U.S. and U.K in this list), and thus do not address the formidable challenges of point in time bankruptcy forecasting with a multi-country portfolio.

approaches to credit portfolio modeling and puts forward our proposed approach. Section 3 briefly discusses the GVAR model and shows how it is linked to the credit risk model. Mathematical expressions for the conditional one-period and multi-period loss distributions of a given credit portfolio under various shock scenarios are also obtained. Section 4 provides an empirical analysis of the impact of the different types of shocks (to output, money supply, equity and oil prices) on the loss distribution. Section 5 offers some concluding remarks.

## 2 Credit Portfolio Modeling

Credit risk modeling is concerned with the tail properties of the loss distribution for a given portfolio of credit assets such as loans or bonds, and attempts to provide quantitative analysis of the extent to which the loss distribution varies with changes to firm/industry-specific, national and global risk factors. It can be approached from the perspective of the individual loans that make up the portfolio, or it could be addressed by considering the return on the loan portfolio directly. In this paper we follow the former approach and simulate the portfolio loss distribution from the bottom up by considering how individual firms default.

Broadly speaking, there are two important variables describing asset/firm level credit risk: the probability of default ($PD$) and the loss given default ($LGD$).[5] Most of the work on $PD$ and $LGD$ has been done without explicit conditioning on business cycle variables; exceptions include Carey (1998), Frye (2000) and Altman, Brady, Resti and Sironi (2002). These studies find, perhaps not surprisingly, that losses are indeed worse in recessions. Tapping into information contained in equity returns (as opposed to credit spreads from debt instruments), Vassalou and Xing (2004) show that default risk varies with the business cycle.[6] Carey (2002), using re-sampling techniques, shows that mean losses during a recession such as 1990/91 in the U.S. are about the same as losses in the 0.5% tail during an expansion. Bangia et al. (2002), using a regime switching approach, find that capital held by banks over a one-year horizon needs to be 25-30% higher in a recession that in an expansion.

In this paper we shall consider the loss distribution of the credit portfolio of a financial institution such as a bank by conditioning on observable macroeconomic variables or factors. The conditional loss distribution allows for the effect of business cycle variations and captures such effects at a global level by explicitly taking account of the heterogeneous interconnections and interdependencies that exist between national and international factors.

[5] The New Basel Accord explicitly mentions two additional variables: exposure at default and maturity. As these affect credit risk only moderately (and are often taken to be non-stochastic), our discussion will focus on the $PD$ and $LGD$ which are the two dominant determinants of the credit loss distribution.

[6] See also the survey by Allen and Saunders (2004).

## 2.1 A Merton-Based Model of Default

Following Merton (1974), a firm is expected to default when the value of its assets falls below a threshold value determined by its callable liabilities. The lender is effectively writing a put option on the assets of the borrowing firm. If the value of the firm falls below a certain threshold, the owners will put the firm to the debt-holders.[7] Default, as considered by the rating agencies and banks, typically constitutes non-payment of interest or a coupon.[8]

Thus there are three aspects which require modeling: (i) the evolution of firm value, (ii) the default threshold, and in a portfolio context, (iii) return correlations across firms in the portfolio. We discuss the first two aspects in this section, while modeling of return correlations is treated in Section 3.2. In Merton-type portfolio models, such as KMV, asset value and asset volatility are typically derived from balance sheet data as well as observable equity returns and (estimated) return volatility (see Kealhofer and Kurbat 2002). The default threshold in these models is typically taken to be short term debt plus a proportion of long term debt. Asset value, asset volatility and the default threshold are then used to determine the distance from default. In what follows we advance an alternative approach where instead of using balance sheet data we make use of firm credit ratings.

Consider a firm $j$ in country or region $i$ having asset values $V_{ji,t}$ at time $t$, and an outstanding stock of debt, $D_{ji,t}$. Under the Merton (1974) model default occurs at the maturity date of the debt, $t+H$, if the firm's assets, $V_{ji,t+H}$, are less than the face value of the debt at that time, $D_{ji,t+H}$. This is in contrast with the first-passage models where default would occur the first time that $V_{ji,t}$ falls below a default boundary (or threshold) over the period $t$ to $t+H$.[9] The default probabilities are computed with respect to the probability distribution of asset values at the terminal date, $t+H$ in the case of the Merton model, and over the period from $t$ to $t+H$ in the case of the first-passage model. The Merton approach may be thought of as a European option and the first-passage approach as an American option. Our approach can be adapted to both of these models, but in what follows we focus on Merton's specification.

The value of the firm at time $t$ is the sum of debt and equity, namely

$$V_{ji,t} = D_{ji,t} + E_{ji,t}, \text{ with } D_{ji,t} > 0, \tag{1}$$

[7] For a discussion of the power of Merton default prediction models see Falkenstein and Boral (2001) and Gemmill (2002) who find that the Merton model generally does well in predicting default (Falkenstein and Boral) and credit spreads (Gemmill). Duffee (1999) points out that due to the continuous time diffusion processes underlying the Black Scholes formula, short-term default probabilities may be underestimated.

[8] A similar default condition is used by regulators, e.g. in the New Basel Accord. See Section III.F, §146 in BIS (2001).

[9] The first-passage approach is discussed in Black and Cox (1976). For a review see, for example, Duffie and Singleton (2003, Section 3.2). More recent modeling approaches also allow for strategic default considerations, as in Mella-Barral and Perraudin (1997).

or alternatively,

$$\frac{V_{ji,t}}{D_{ji,t}} = 1 + \frac{E_{ji,t}}{D_{ji,t}}. \tag{2}$$

Conditional on time $t$ information, default will take place at time $t + H$ if

$$V_{ji,t+H} \leq D_{ji,t+H},$$

or using (2) if

$$\frac{E_{ji,t+H}}{D_{ji,t+H}} \leq 0. \tag{3}$$

Equation (3) is restrictive in that it requires equity values to be negative before default occurs. Aside from non-trivial practical considerations having to do with arriving at an independent estimate of $V_{ji,t}$, there are several reasons behind relaxing this condition. Because default is costly and violations to the absolute priority rule in bankruptcy proceedings are so common, in practice shareholders have an incentive to put the firm into receivership even before the equity value of the firm hits zero.[10] In fact, several authors have found that in 65% to 80% of bankruptcies, even shareholders receive something without debt-holders necessarily having been fully paid off (see, for instance, Eberhart and Weiss 1998, and references therein). Moreover, we see in practice that equity values remain positive for insolvent firms. Similarly, the bank might also have an incentive of forcing the firm to default once the firm's equity falls below a non-zero threshold, as well as an incentive to bypass the costly proceedings by agreeing to terms that yield positive value to the shareholders themselves.[11] The value of equity incorporates not only the asset values, but an option value that a firm in default may in fact recover before creditors take control of these assets. Finally, default in a credit relationship is typically a weaker condition than outright bankruptcy. An obligor may meet the technical default condition, e.g. a missed coupon payment, without subsequently going into bankruptcy. This distinction is particularly relevant in the banking-borrower relationship we seek to characterize.[12]

In what follows we assume that default takes place if

$$0 < E_{ji,t+H} < C_{ji,t+H}, \tag{4}$$

where $C_{ji,t+H}$ is a positive default threshold which could vary over time and with the firm's particular characteristics (such as region or industry sector). Natural candidates include quantitative factors such as leverage, profitability, firm age and perhaps size (most of which appear in models of firm default), as well as more qualitative factors such as management quality.[13] Obviously some of

[10] See, for instance, Leland and Toft (1996) who develop a model where default is determined endogenously without imposing a positive net worth condition.

[11] For a treatment of this scenario, see Garbade (2001).

[12] An excellent example of the joint borrower-lender decision process is given by Lawrence and Arshadi (1995).

[13] For models of bankruptcy and default at the firm level, see, for instance, Altman (1968), Lennox (1999), Shumway (2001), Chava and Jarrow (2004), Hillegeist, Keating, Cram and Lundstedt (2004), as well as a survey by Altman and Saunders (1997).

these factors will be easier to observe and measure than others. The observable accounting-based factors are at best noisy and at worst could be biased, highlighting the information asymmetry between managers (agents) and share/debt-holders (principals).[14]

Although our objective is not to build a default model per se, we face the same measurement difficulties and information asymmetries. To overcome them, we make use of the credit rating of a firm which we denote by $\mathcal{R}$.[15] This will help us specifically in estimating the default thresholds needed in the determination of the default probabilities. Naturally, rating agencies have access to, and presumably make use of, private information about the firm to arrive at their firm-specific credit rating, in addition to incorporating public information such as balance sheet information and, of course, equity returns. Thus we make the assumption that rating agencies benchmark their ratings on past returns and volatilities of *all* firms that have been rated $\mathcal{R}$ in the past.

Consider then a particular $\mathcal{R}-$rated firm at time $t$, and assume that in arriving at their rating the credit rating agency uses the following standard geometric random walk model of equity values:

$$\ln(E_{\mathcal{R},t+1}) = \ln(E_{\mathcal{R}t}) + \mu_{\mathcal{R}} + \sigma_{\mathcal{R}}\eta_{\mathcal{R},t+1}, \qquad \eta_{\mathcal{R},t+1} \sim IIDN(0,1), \tag{5}$$

with a non-zero drift, $\mu_{\mathcal{R}}$, and idiosyncratic Gaussian innovations with a zero mean and fixed volatility, $\sigma_{\mathcal{R}}$.[16] We assume that conditional on data at time $t$, a firm's rating does not change over the horizon $(t, t+H)$, namely

$$\ln(E_{\mathcal{R},t+H}) = \ln(E_{\mathcal{R}t}) + H\mu_{\mathcal{R}} + \sigma_{\mathcal{R}}\sum_{s=1}^{H}\eta_{\mathcal{R},t+s},$$

and by (4) default occurs if

$$\ln(E_{\mathcal{R},t+H}) = \ln(E_{\mathcal{R}t}) + H\mu_{\mathcal{R}} + \sigma_{\mathcal{R}}\sum_{s=1}^{H}\eta_{\mathcal{R},t+s} < \ln\left(C_{\mathcal{R},t+H}\right), \tag{6}$$

or if the $H$-period change in equity value or return falls below the log-threshold-equity ratio:

$$\ln\left(\frac{E_{\mathcal{R},t+H}}{E_{\mathcal{R}t}}\right) < \ln\left(\frac{C_{\mathcal{R},t+H}}{E_{\mathcal{R}t}}\right). \tag{7}$$

Equation (7) tells us that the relative (rather than absolute) decline in firm value must be large enough over the horizon $H$ to result in default, meaning it is independent of the size of the firm. Firm size is an input to the credit rating determination; a small firm would need a larger equity cushion to withstand a given shocks than a large firm with the same rating.

[14] With this in mind, Duffie and Lando (2001) allow for the possibility of imperfect information about the firm's assets and default threshold in the context of a first-passage model.

[15] $\mathcal{R}$ may take on values such as 'Aaa', 'Aa', 'Baa',..., 'Caa' in Moody's terminology, or 'AAA', 'AA', 'BBB',..., 'CCC' in S&P's terminology.

[16] Clearly non-Gaussian innovations can also be considered. But for quarterly data that we shall be working with Gausssian innovations seems a good first approximation.

Under the assumption that the evolution of firm equity value follows (5), $\ln(E_{\mathcal{R},t+H}/E_{\mathcal{R}t})$ may be approximated by the cumulative returns so that (7) can be re-written as

$$H\mu_{\mathcal{R}} + \sigma_{\mathcal{R}} \sum_{s=1}^{H} \eta_{\mathcal{R},t+s} < \ln\left(\frac{C_{\mathcal{R},t+H}}{E_{\mathcal{R}t}}\right).$$

Therefore, the default probability for the $\mathcal{R}-$rated firms at the terminal date $t+H$ is given by

$$\pi_{\mathcal{R}}(t,H) = \Phi\left(\frac{\ln(C_{\mathcal{R},t+H}/E_{\mathcal{R}t}) - H\ \mu_{\mathcal{R}}}{\sigma_{\mathcal{R}}\sqrt{H}}\right), \tag{8}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. Denote the $H$-period forward log threshold-equity ratio to be $\lambda_{\mathcal{R}}(t,H) = \ln(C_{\mathcal{R},t+H}/E_{\mathcal{R}t})$ so that

$$\lambda_{\mathcal{R}}(t,H) = H\mu_{\mathcal{R}} + Q_{\mathcal{R}}(t,H)\ \sigma_{\mathcal{R}}\sqrt{H},$$

where

$$Q_{\mathcal{R}}(t,H) = \Phi^{-1}\left[\pi_{\mathcal{R}}(t,H)\right]$$

is the quantile associated with the default probability $\pi_{\mathcal{R}}(t,H)$.

An estimate of $\lambda_{\mathcal{R}}(t,H)$ can now be obtained using past observations on returns, $r_{\mathcal{R}t} = \ln(E_{\mathcal{R},t}/E_{\mathcal{R},t-1})$, and the empirical default frequencies, $\hat{\pi}_{\mathcal{R}}(t,H)$, of $\mathcal{R}-$rated firms over a given period of say $t = 1,2,...,T$.[17] Denoting the estimates of $\mu_{\mathcal{R}}$ and $\sigma_{\mathcal{R}}$ by $\hat{\mu}_{\mathcal{R}}$, and $\hat{\sigma}_{\mathcal{R}}$, respectively, we have

$$\hat{\lambda}_{\mathcal{R}}(t,H) = H\hat{\mu}_{\mathcal{R}} + \hat{Q}_{\mathcal{R}}(t,H)\ \hat{\sigma}_{\mathcal{R}}\sqrt{H}, \tag{9}$$

where $\hat{\mu}_{\mathcal{R}}$ and $\hat{\sigma}^2_{\mathcal{R}}$ are the mean and standard deviation of returns of firms with rating $\mathcal{R}$ over the sample period, and[18]

$$\hat{Q}_{\mathcal{R}}(t,H) = \Phi^{-1}\left[\hat{\pi}_{\mathcal{R}}(t,H)\right]. \tag{10}$$

The estimates of $\hat{\mu}_{\mathcal{R}}$ and $\hat{\sigma}_{\mathcal{R}}$ can also be updated using a rolling window of size 7-8 years (the average length of the business cycle).

In practice, $\hat{\pi}_{\mathcal{R}}(t,H)$ might not provide a reliable estimate of $\pi_{\mathcal{R}}(t,H)$ as it is likely to be based on very few defaults over any particular period $(t, t+H)$. One possibility would be to use an average estimate of $\lambda_{\mathcal{R}}(t,H)$ obtained over a reasonably long period of 10 to 20 years (on a rolling basis). For example, based on the sample observations $t = 1,2,...,T$, we would have

$$\hat{\lambda}_{\mathcal{R}}(H) = H\ \hat{\mu}_{\mathcal{R}} + \hat{Q}_{\mathcal{R}}(H)\ \hat{\sigma}_{\mathcal{R}}\ \sqrt{H}, \tag{11}$$

[17] An important source of heterogeneity is likely the large variation in bankruptcy laws and regulation across countries. However, by using rating agency default data, we use their homogeneous definition of default and are thus not subject to these heterogeneities.

[18] In practice where there are many $\mathcal{R}$-rated firms in a given period, average returns across all $\mathcal{R}$-rated firms can be used to estimate $\hat{\mu}_{\mathcal{R}}$. The computation of $\hat{\sigma}^2_{\mathcal{R}}$ is more involved and is described in a note available from the authors on request.

where the (average) quantile estimate $\hat{Q}_{\mathcal{R}}(H)$ is given by

$$\hat{Q}_{\mathcal{R}}(H) = T^{-1} \sum_{t=1}^{T} \left\{ \Phi^{-1} \left[ \hat{\pi}_{\mathcal{R}}(t, H) \right] \right\}, \tag{12}$$

assuming that rating agencies use about a one-year horizon ($H = 4$ quarters) when assessing a firm.

The above framework allows us to obtain estimates of the default threshold-equity ratios by credit ratings. Also, given sufficient data for a particular region or country $i$, one could in principle use default frequencies that vary across regions/countries and estimate default threshold-equity ratios that vary across countries and ratings. However, since a particular firm $j$'s default is typically a unique terminal event, multiple (serial) defaults notwithstanding, firm-specific default threshold-equity ratios can not be obtained independently of that firm's default probability which we aim to compute. This presents us with a fundamental identification problem which we propose to resolve by making the following (identification) condition:

$$C_{ji\mathcal{R},t+H}/E_{ji\mathcal{R}t} = C_{\mathcal{R},t+H}/E_{\mathcal{R}t}, \text{ for all } j, \tag{13}$$

where $E_{ji\mathcal{R}t}$ and $C_{ji\mathcal{R},t+H}$ are respectively the equity and the default threshold values of firm $j$ in region $i$, with the credit rating, $\mathcal{R}$, at time $t$. Condition (13) says that at a given point in time, any two firms with the same credit ratings are assumed to have the same default threshold-equity ratios. Note that we do not assume that they need to have the same threshold levels but just the same ratio; firms with the same credit rating but of different size will have potentially very different threshold levels.

This problem of identification, although quite fundamental, need not be solved by imposing the same default threshold. For example, instead of imposing that all firms with the same credit rating have the same default threshold, one could equally require that all firms with the same credit rating to have the same distance to default ratio, namely $\left[\lambda_{\mathcal{R}}(t, H) - H\mu_{\mathcal{R}}\right] / \sigma_{\mathcal{R}}\sqrt{H}$, to be the same across all firms in a given rating category. Also, instead of using credit rating as the sole type-identifier, one could consider firms grouped by industry or geographical regions as well as by their credit ratings. Clearly, other groupings of firms can also be entertained, so long as one could plausibly make the argument of within-group homogeneity of either the default threshold or the distance to default ratio. Further discussions of these alternative identification schemes and their implications for the credit loss distribution although clearly worthwhile, is beyond the scope of the present paper.

## 2.2 Firm-Specific Defaults

We are now in a position to develop our firm-specific default probability model. Denote the return of firm $j$ in region $i$ over the period $t$ to $t+1$ by $r_{ji,t+1} = \ln\left(E_{ji,t+1}/E_{ji,t}\right)$, and assume that conditional on the information available at time $t$, $\Omega_t$, it can be decomposed as

$$r_{ji,t+1} = \mu_{ji,t} + \xi_{ji,t+1}, \tag{14}$$

where $\mu_{ji,t}$ is the (forecastable) conditional mean, and $\xi_{ji,t+1}$ is the (non-forecastable) innovation component of the return process. The conditional mean will be a function of the regional and global macroeconomic factors, allowing an avenue through which shocks to these factors affect firm returns. The precise form of $\mu_{ji,t}$, and how it relates to national and global risk factors will be specified using the GVAR model to be briefly summarized in Section 3.2. Return correlations across firms are captured through $\mu_{ji,t}$ and $\xi_{ji,t+1}$ as will be made clear in Section 3.2. Following the standard Merton model we shall assume that

$$\xi_{ji,t+1} \mid \Omega_t \sim N(0, \omega^2_{\xi,ji}). \tag{15}$$

The assumption that the conditional variance of returns is time-invariant seems reasonable for quarterly returns, although it would need to be relaxed for returns measured over shorter periods, such as weeks or days.[19]

We can now characterize the separation between a default and a non-default state with an indicator variable

$$I\left(\ln\left(E_{ji,t+1}/E_{ji,t}\right) < \ln\left(C_{ji,t+1}/E_{ji,t}\right)\right), \quad \text{or } I\left(r_{ji,t+1} < \lambda_{ji}(t,1)\right),$$

such that, using (7),

$$\begin{aligned} I\left(r_{ji,t+1} < \lambda_{ji}(t,1)\right) &= 1 \text{ if } r_{ji,t+1} < \lambda_{ji}(t,1) \Longrightarrow \text{ Default}, \\ I\left(r_{ji,t+1} < \lambda_{ji}(t,1)\right) &= 0 \text{ if } r_{ji,t+1} \geq \lambda_{ji}(t,1) \Longrightarrow \text{ No Default}. \end{aligned} \tag{16}$$

Using the same approach as above, the one quarter ahead (with $H = 1$) default probability for firm $j$ is given by

$$\pi_{ji}(t,1) = \Phi\left(\frac{\lambda_{ji}(t,1) - \mu_{ji,t}}{\omega_{\xi ji}}\right). \tag{17}$$

$\mu_{ji,t}$ and $\omega_{\xi ji}$ can be estimated using the firm-specific return regressions. $\lambda_{ji}(t,1)$ will be estimated using the rating information of this firm at time $t$, under the identification condition (13). If the firm is rated $\mathcal{R}$, then $\lambda_{ji}(t,1)$ will be estimated by $\hat{\lambda}_{\mathcal{R}}(t,1)$ as in (9), on the assumption that all $\mathcal{R}-$rated firms have the same threshold-equity ratio. The default condition for firm $j$ with credit rating $\mathcal{R}$ can therefore be written as

$$I\left(r_{ji,t+1} < \hat{\lambda}_{\mathcal{R}}(t,1)\right) = 1 \text{ if } r_{ji,t+1} < \hat{\lambda}_{\mathcal{R}}(t,1) \Longrightarrow \text{ Default}. \tag{18}$$

[19] Volatility in quarterly models is of third order importance. Our framework could easily be adapted to deal with more complex volatility effects by normalizing returns with dynamic volatilities using, for example, the RiskMetrics method or other GARCH specifications.

Note that while the default condition is the same for all $\mathcal{R}-$rated firms, the default *probability* varies by firm. Once again, due to the small number of defaults over a single period $(t, t+1)$, in practice it might be more appropriate to use a (rolling) average estimate such as $\hat{\lambda}_{\mathcal{R}}(H)$ defined by (11).

Under (17) the default probability for firm $j$, and therefore its distance from default, is driven by:

1. The firm's credit rating: the lower the credit rating, the "closer" the default threshold.
2. The volatility of the equity return, $\omega_{\xi,ji}$: the more volatile, the more likely the firm is to cross the threshold.
3. The (unconditional) equity return, $\mu_{ji,t}$: the higher that expected return, the "further" the firm is from default.

Mappings from credit ratings to default probabilities are typically obtained using corporate bond rating histories over many years, often 20 years or more, and thus represent some average across business cycles. The reason for such long samples is simple: default events for investment grade firms are quite rare; for example, the annual default probability of an '$\mathcal{A}$' rated firm is approximately one basis point for both Moody's and S&P rated firms.[20]

In the literature, the use and interpretation of credit ratings are somewhat ambiguous. One interpretation is that they are "cycle-neutral" (Saunders and Allen 2002, Catarineu-Rabell, Jackson and Tsomocos 2002, Amato and Furfine 2004; Carpenter, Whitesell and Zakrajšek 2001 point to some of the ambiguities), meaning that ratings are assigned only on the basis of firm-specific information and not systematic or macroeconomic information.[21] The rating agency's own description of their rating methodology broadly supports this view.

> (Moody's 1999, p.6,7): ".. [O]ne of Moody's goals is to achieve stable *expected* [italics in original] default rates across rating categories and time." ... "Moody's believes that giving only a modest weight to cyclical conditions best serves the interests of the bulk of investors."
>
> (S&P 2001, p.41): "Standard & Poor's credit ratings are meant to be forward looking; ... Accordingly, the anticipated ups and downs of business cycles – whether industry-specific or related to the general economy – should be factored into the credit rating all along." ... "The ideal is to rate 'through the cycle'".

[20] For an overview of the rating industry, see Cantor and Packer (1995); Jafry and Schuermann (2004) provide detailed default probability estimates by rating.

[21] Amato and Furfine (2003) find little evidence of procyclicality in ratings.

However, there is ample evidence to suggest that credit ratings and associated default probabilities vary systematically with the business cycle (e.g. Nickell, Perraudin and Varotto 2000, Bangia et al. 2002). Moody's itself has changed its rating process in this regard (Moody's 1999, p.6): "Moody's has been striving for some time to increase the responsiveness of its ratings to economic developments." Our mapping from default experience to thresholds allows for this time variation.

The GVAR model provides the link between changes in macroeconomic variables (in region $i$ and globally) through $\mu_{ji,t}$, and it does so uniquely for each firm to allow for firm-specific heterogeneity. The main advantage of using the GVAR as a driver for a credit portfolio model is that it provides the (conditional) correlation structure among macroeconomic variables of the global economy.

# 3 Conditional Credit Risk Modeling

## 3.1 The Macroeconomic Engine: GVAR

The macroeconomic engine driving the credit risk model is described in detail in PSW. We only provide a very brief, non-technical overview here. The GVAR is a global quarterly model estimated over the period 1979Q1-1999Q1 comprising a total of 25 countries which are grouped into 11 regions (shown in bold in Table 1). These countries comprise around 80% of world output (in 1999). The advantage of the GVAR is that it allows for a true multi-country setting; however it can become computationally demanding very quickly. For that reason the seven key economies of the U.S., Japan, China, Germany, U.K., France and Italy are modeled as regions of their own while the other 18 countries are grouped into four regions.[22]

**Table 1**

**Countries/Regions in the GVAR Model**

| **U.S.A.** | **Germany** | **Japan** | **China** |
|---|---|---|---|
| **U.K.** | **Italy** | **France** | |
| **Western Europe** | **South East Asia** | **Latin America** | **Middle East** |
| ·Spain | ·Korea | ·Argentina | ·Kuwait |
| ·Belgium | ·Thailand | ·Brazil | ·Saudi Arabia |
| ·Netherlands | ·Indonesia | ·Chile | ·Turkey |
| ·Switzerland | ·Malaysia | ·Peru | |
| | ·Philippines | ·Mexico | |
| | ·Singapore | | |

In contrast to existing modeling approaches, in the GVAR the use of cointegration is not confined to a single country or region. By estimating a cointegrating model for each country/region separately, the model also allows for endowment and institutional heterogeneities that exist across the

[22] See PSW, Section 8, for details on cross-country aggregation into regions.

different countries. Accordingly, specific vector error-correcting models (VECM) are estimated for individual countries (or regions) by relating domestic macroeconomic variables such as GDP, inflation, equity prices, money supply, exchange rates and interest rates to corresponding, and therefore country-specific, foreign variables constructed exclusively to match the international trade pattern of the country/region under consideration.[23] By making use of specific exogeneity assumptions regarding the 'rest of the world' with respect to a given domestic or regional economy, the GVAR makes efficient use of limited amounts of data and presents a consistently estimated global model.

The GVAR allows for interactions to take place between factors and economies through three distinct but interrelated channels:

- Contemporaneous dependence of domestic on foreign variables and their lagged values;
- Dependence of country specific variables on observed common global effects such as oil prices;
- Weak cross-sectional dependence of the idiosyncratic shocks.

The individual models are estimated allowing for unit roots and cointegration assuming that region-specific foreign variables are weakly exogenous, with the exception of the model for the U.S. economy which is treated as a closed economy model. The U.S. model is linked to the outside world through exchange rates, which in turn are themselves determined by the rest of the region-specific models. PSW show that the careful construction of the global variables as weighted averages of the other regional variables leads to a simultaneous system of regional equations that may be solved to form a global system. They also provide theoretical arguments as well as empirical evidence in support of the weak exogeneity assumption that allows the region-specific models to be estimated consistently.

For policy analysis, one would like to be able to examine how shocking a given macroeconomic variable affects all other macroeconomic variables in the global economy. For example, it might be of interest to determine the effects of a contemporaneous 10% drop in the Japanese equity prices on other macroeconomic variables, and the effects that these have on the credit risk of a given portfolio. Impulse response functions provide us with the tools to carry out this type of analysis. Technical details on the derivation of generalized impulse response functions within the GVAR model are provided in Section C of a Supplement that is available from the authors on request.

The conditional loss distribution of a given credit portfolio can now be derived by linking up the return processes of individual firms, initially presented in equation (14), explicitly to the macro and global variables in the GVAR model. In this way we are able to generate multi-period loss distributions conditional on a baseline macroeconomic forecast as well as loss distributions conditional on macroeconomic shocks.

[23] Theoretical underpinnings of such country-specific models are provided in Garratt et al. (2003a).

## 3.2 Return Regressions: A Heterogeneous Formulation

Firm returns in a multi-factor context are often modeled as a function of macro variables that are specific to the firm's domicile country plus global variables such as changes in oil prices. But such a specification leaves out one of the key features of the GVAR model, namely the foreign-specific variables which could be particularly important in the case of large international corporations. Here we extend the firm return model by incorporating all GVAR factors to take full advantage of the GVAR dynamics. Accordingly, a firm's change in value (or return) is assumed to be a function of contemporaneous changes in the underlying macroeconomic factors (the systematic component), say $k_i$ region-specific domestic and $k_i^*$ foreign macroeconomic variables, the exogenous global variables $\mathbf{d}_t$ (in our application oil prices) and the firm-specific idiosyncratic shocks $\eta_{ji,t+1}$:

$$r_{ji,t+1} = \alpha_{ji} + \boldsymbol{\beta}'_{ji}\Delta\mathbf{x}_{i,t+1} + \boldsymbol{\beta}^{*\prime}_{ji}\Delta\mathbf{x}^*_{i,t+1} + \boldsymbol{\gamma}'_{ji}\Delta\mathbf{d}_{t+1} + \eta_{ji,t+1}, \tag{19}$$

for $j = 1, 2, ..., nc_i$, $i = 0, 1, .., N$, where $\mathbf{x}_{i,t+1}$, $\mathbf{x}^*_{i,t+1}$, and $\mathbf{d}_{t+1}$ are the $k_i \times 1$, $k_i^* \times 1$, and $s \times 1$ vectors of macroeconomic and global factors, $nc_i$ is the number of firms in region $i$, and $N+1$ is the total number of regions, with the U.S. economy being designated as region 0. The endogenous variables for each region, $\mathbf{x}_{i,t+1}$, typically include real output, inflation, interest rate, real equity prices, exchange rate, and real money balances. The foreign variables $\mathbf{x}^*_{it}$ are tailored to be region-specific. The GVAR assumes that each macroeconomic variable in the vector $\mathbf{x}^*_{it}$ is a weighted average of the corresponding macroeconomic variables of all other regions outside region $i$. Taking output as an example:

$$y^*_{it} = \sum_{\ell=0}^{N} w_{i\ell}y_{\ell t}, \text{ with } \sum_{\ell=0}^{N} w_{i\ell} = 1 \text{ and } w_{\ell\ell} = 0,$$

where $y^*_{it}$ is the log of the output of the rest of the world from the perspective of country/region $i$, $y_{\ell t}$ is the log of the output of region $\ell$, and $w_{i\ell}$ is the weight attached to region $\ell$'s output in construction of the rest of the world output as seen by region $i$. Weights for the construction of the region-specific global variables are based on the trade share of region $l$ in the total trade volume of region $i$, although for variables such as equity and interest rates they could be based on capital flows instead.

The GVAR specifies the following augmented vector autoregressive form:

$$\begin{aligned} \mathbf{x}_{it} &= \mathbf{a}_{i0} + \mathbf{a}_{i1}t + \boldsymbol{\Phi}_i\mathbf{x}_{i,t-1} + \boldsymbol{\Lambda}_{i0}\mathbf{x}^*_{it} + \boldsymbol{\Lambda}_{i1}\mathbf{x}^*_{i,t-1} + \boldsymbol{\Psi}_{i0}\mathbf{d}_t + \boldsymbol{\Psi}_{i1}\mathbf{d}_{t-1} + \boldsymbol{\varepsilon}_{it}, \\ t &= 1, 2, ..., T;\ i = 0, 1, 2, ..., N, \end{aligned} \tag{20}$$

where $\mathbf{x}_{it}$ is the $k_i \times 1$ country-specific factors/variables, $\mathbf{a}_{i1}$ is a $k_i \times 1$ vector of linear trend coefficients, $\boldsymbol{\Phi}_i$ is a $k_i \times k_i$ matrix of associated lagged coefficients, $\mathbf{x}^*_{it}$ is the $k_i^* \times 1$ vector of foreign variables specific to country $i$ with $\boldsymbol{\Lambda}_{i0}$ and $\boldsymbol{\Lambda}_{i1}$ being $k_i \times k_i^*$ matrices of fixed coefficients, $\mathbf{d}_t$ is an $s \times 1$ vector of common global variables assumed to be exogenous to the global economy with $\mathbf{\Psi}_{i0}$ and $\mathbf{\Psi}_{i1}$ being $k_i \times s$ matrices of fixed coefficients, and $\boldsymbol{\varepsilon}_{it}$ is a $k_i \times 1$ vector of country-specific shocks assumed to be serially uncorrelated with a zero mean and a non-singular covariance matrix, $\mathbf{\Sigma}_{ii} = (\sigma_{ii,\ell s})$, where $\sigma_{ii,\ell s} = cov(\varepsilon_{i\ell t}, \varepsilon_{ist})$, or written more compactly

$$\boldsymbol{\varepsilon}_{it} \backsim i.i.d.(\mathbf{0}, \mathbf{\Sigma}_{ii}).$$

Although the model is estimated on a regional basis, we allow for the shocks to be correlated across regions. In particular, we assume that

$$\begin{aligned} E\left(\boldsymbol{\varepsilon}_{it}\boldsymbol{\varepsilon}'_{jt'}\right) &= \mathbf{\Sigma}_{ij} \text{ for } t = t', \\ &= \mathbf{0} \text{ for } t \neq t. \end{aligned}$$

The model is specified with one lag, although multi-lag extensions are possible.[24] While we present the model in more detail in the Supplement, note that the set of regional equations can be combined to form a global VaR in $\mathbf{x}_t = (\mathbf{x}'_{0t}, \mathbf{x}'_{1t}, ..., \mathbf{x}'_{Nt})'$ which is the global $k \times 1$ vector, where $k = \sum_{i=0}^{N} k_i$ is the total number of the endogenous variables in the global model, in our case 63:

$$\mathbf{x}_t = \mathbf{b}_0 + \mathbf{b}_1 t + \mathit{F}\mathbf{x}_{t-1} + \mathbf{\Upsilon}_0\mathbf{d}_t + \mathbf{\Upsilon}_1\mathbf{d}_{t-1} + \mathbf{u}_t, \tag{21}$$

where $\mathbf{b}_0$ and $\mathbf{b}_1$ are $k \times 1$ vectors of coefficients, $\mathit{F}$ is a $k \times k$ matrix of coefficients, $\mathbf{d}_t$ is an $s \times 1$ vector of common global variables assumed to be exogenous to the global economy (here to be the oil price) with corresponding $k \times s$ matrices of coefficients, $\mathbf{\Upsilon}_0$ and $\mathbf{\Upsilon}_1$. Finally, $\mathbf{u}_t$ is a $k \times 1$ vectors of (reduced form) shocks that are linear functions of the region-specific shocks ($\boldsymbol{\varepsilon}_{it}$).

As shown in PSW

$$\mathbf{z}_{i,t+1} = \begin{pmatrix} \mathbf{x}_{i,t+1} \\ \mathbf{x}^*_{i,t+1} \end{pmatrix} = \mathbf{W}_i\mathbf{x}_{t+1},$$

where the weight matrix $\mathbf{W}_i$ serves as the 'link' between the endogenous vector of variables in the world economy, $\mathbf{x}'_{t+1}$ and the domestic ($\mathbf{x}_{i,t+1}$) and foreign ($\mathbf{x}^*_{i,t+1}$) variables for region $i$. The non-zero elements of $\mathbf{W}_i$ are given by trade weights of country $i$ relative to all other countries in the GVAR model. Hence we have

$$r_{ji,t+1} = \alpha_{ji} + \mathbf{B}'_{ji}\mathbf{W}_i\Delta\mathbf{x}_{t+1} + \boldsymbol{\gamma}'_{ji}\Delta\mathbf{d}_{t+1} + \eta_{ji,t+1}, \tag{22}$$

where $\mathbf{B}_{ji} = \left(\boldsymbol{\beta}'_{ji}, \boldsymbol{\beta}^{*\prime}_{ji}\right)'$. The GVAR model provides forecasts of all the global variables, $\mathbf{x}_{t+1}$, that directly or indirectly affect the returns, $r_{ji,t+1}$. If the model captures all systematic risk, the idiosyncratic risk components of any two companies in the model would be uncorrelated, namely

[24] Recently, Dees, di Mauro, Pesaran and Smith (2004) have extended the GVAR to more countries (33) and a longer sample length (1979Q1-2003Q4) and proceed to estimate country-specific models with different lag lengths which are then aggregated to a global model along the lines outlined in PSW.

the idiosyncratic risks, $\eta_{ji,t+1}$, ought to be cross-sectionally uncorrelated. The values of the global exogenous variables, $\mathbf{d}_{t+1}$, could either be fixed to represent particular scenarios of interest, such as high or low oil prices, or could be forecast using a sub-model for oil prices (possibly with macroeconomic feedbacks).

Under this specification, due to the contemporaneous dependence of $\Delta\mathbf{x}_{t+1}$ on $\Delta\mathbf{d}_{t+1}$, we re-write (22) as

$$r_{ji,t+1} = \alpha_{ji} + \Gamma'_{ji}\Delta\mathbf{y}_{t+1} + \eta_{ji,t+1}, \tag{23}$$

where $\Gamma'_{ji} = \left(\mathbf{B}'_{ji}\mathbf{W}_i, \boldsymbol{\gamma}'_{ji}\right)$ are the factor loadings, and $\Delta\mathbf{y}_{t+1} = \left(\Delta\mathbf{x}'_{t+1},\ \Delta\mathbf{d}'_{t+1}\right)'$ collects all the observable macroeconomic variables plus oil prices in the global model (totaling 64 in PSW). To be sure, these return regressions are not prediction equations per se as they depend on contemporaneous variables. However, using results provided in a Supplement (available on request), we can write

$$r_{ji,t+1} = \alpha_{ji} + \Gamma'_{ji}\left(\boldsymbol{\mu}+\boldsymbol{\delta}\right) - \Gamma'_{ji}\left(\mathbf{I}-\boldsymbol{\Phi}\right)\left(\mathbf{y}_t - \boldsymbol{\gamma}\ t\right) + \Gamma'_{ji}\mathfrak{D}\boldsymbol{\upsilon}_{t+1} + \eta_{ji,t+1}, \tag{24}$$

which decomposes the individual asset returns into a predictable component, $\Gamma'_{ji}\left(\mathbf{I}-\boldsymbol{\Phi}\right)\left(\mathbf{y}_t - \boldsymbol{\gamma}\ t\right)$, and an unpredictable component, $\Gamma'_{ji}\mathfrak{D}\boldsymbol{\upsilon}_{t+1} + \eta_{ji,t+1}$. This term comprises effects due to common macroeconomic shocks, $\Gamma'_{ji}\mathfrak{D}\boldsymbol{\upsilon}_{t+1}$, where $\mathfrak{D}$ is a $(k+s)\times(k+s)$ matrix of fixed coefficients from the GVAR model, $\boldsymbol{\upsilon}_{t+1} = (\boldsymbol{\varepsilon}'_{t+1}, \varepsilon_{d,t+1})'$ collects the set of all macroeconomic innovations, $\boldsymbol{\varepsilon}_{t+1}$, and the global exogenous factor innovation, $\varepsilon_{d,t+1}$ (in our model the oil price innovation). The firm-specific idiosyncractic innovations, $\eta_{ji,t+1}$, are assumed to be distributed independently of $\boldsymbol{\upsilon}_{t+1}$. The remaining terms are firm-specific fixed effects, $\alpha_{ji}$, and the drift components of the macro factors and the global exogenous variables, $\Gamma'_{ji}\left(\boldsymbol{\mu}+\boldsymbol{\delta}\right)$.

The predictable component is likely to be weak and will depend on the size of the factor loadings, $\Gamma_{ji}$, and the extent to which the underlying global variables are cointegrating. In the absence of any cointegrating relations in the global model, $\boldsymbol{\Phi} = \mathbf{I}$ and none of the asset returns are predictable. As it happens the econometric evidence presented in PSW strongly supports the existence of 36 cointegrating relations in the global model and is, therefore, compatible with some degree of predictability in asset returns. The extent to which asset returns are predicted could reflect time-varying risk premia and does not necessarily imply market inefficiencies. Our modelling approach provides an operational procedure for relating excess returns of individual firms to all the observable macro factors in the global economy.

### 3.3 Expected Loss Due to Default

Given the value change process for firm $j$, defined by (19), and the log threshold-equity ratio, $\hat{\lambda}_{\mathcal{R}}(t,H)$, obtainable from an initial credit rating (see Section 2.1), we now consider the conditions under which the firm defaults. Specifically, we need to define the expected loss to firm $j$ at time $T$ given information available to the lender (e.g. a bank) at time $T$, which we assume is given by $\Omega_T$.

Following (18), default occurs when the firm's value (return) falls below the log-threshold-equity ratio $\hat{\lambda}_{\mathcal{R}}(T,1)$. Expected loss at time $T$ (but occurring at $T+1$), $E_T\left(L_{ji,T+1}\right) = E\left(L_{ji,T+1} \mid \Omega_T\right)$, is given by

$$\begin{aligned} E_T\left(L_{ji,T+1}\right) &= \Pr\left(r_{ji,T+1} < \hat{\lambda}_{\mathcal{R}}(T,1) \mid \Omega_T\right) \times E_T(\mathcal{X}_{ji,T+1}) \times E_T(\mathcal{S}_{ji,T+1}) \\ &\quad + \left[1 - \Pr\left(r_{ji,T+1} < \hat{\lambda}_{\mathcal{R}}(T,1) \mid \Omega_T\right)\right] \times \tilde{L}, \end{aligned} \tag{25}$$

where $\mathcal{X}_{ji,T+1}$ is the maximum loss exposure assuming no recoveries (typically the face value of the loan) and is known at time $T$, $\mathcal{S}_{ji,T+1}$ is the percentage of exposure which cannot be recovered in the event of default (sometimes called loss given default or severity),[25] and $\tilde{L}$ is some future value of loss in the event of non-default at $T+1$ (which we set to zero for simplicity).[26] Typically $\mathcal{S}_{ji,T+1}$ is not known at time of default and will be treated as a random variable over the range $[0,1]$. In the empirical application we make the typical assumption that $\mathcal{S}_{ji,T+1}$ are draws from a beta distribution with given mean and variance calibrated to (pooled) historical data on default severity.[27]

Substituting (22) into (25) and setting $\tilde{L}$ to zero we now obtain:

$$E_T\left(L_{ji,T+1}\right) = \pi_{ji,T+1|T} \times E_T(\mathcal{X}_{ji,T+1}) \times E_T(\mathcal{S}_{ji,T+1}), \tag{26}$$

where

$$\pi_{ji,T+1|T} = \Pr\left(\alpha_{ji} + \Gamma'_{ji}\Delta\mathbf{y}_{T+1} + \eta_{ji,T+1} < \hat{\lambda}_{\mathcal{R}}(T,1) \mid \Omega_T\right),$$

is the conditional default probability over the period $T$ to $T+1$, formed at time $T$. Our modeling framework allows us to derive an explicit expression for $\pi_{ji,T+1|T}$.

Using firm returns as characterized by (24), and after some simplifications, we have

$$\pi_{ji,T+1|T} = \Pr\left(\xi_{ji,T+1} < \hat{\lambda}_{\mathcal{R}}(T,1) - \mu_{ji,T+1|T} \mid \Omega_T\right), \tag{27}$$

where

$$\xi_{ji,T+1} = \eta_{ji,T+1} + \Gamma'_{ji}\mathfrak{D}\boldsymbol{\upsilon}_{T+1}, \tag{28}$$

and

$$\mu_{ji,T+1|T} = \alpha_{ji} + \Gamma'_{ji}\left[\boldsymbol{\mu}+(T+1)\boldsymbol{\delta}\right] - \Gamma'_{ji}\left(\mathbf{I} - \boldsymbol{\Phi}\right)\mathbf{y}_T. \tag{29}$$

These results decompose the return for firm $j$ into its explained (29) and unexplained (28) components, itself containing the idiosyncracratic innovation $\eta_{ji,T+1}$ and the systematic innovations

[25] One would expect loss severity to be higher in recessions than expansions (see Frye (2000) and Altman et al. (2002)). Defaults are pro-cyclical, flooding the market with distressed assets which drive down their price (or increasing severity). However, for simplicity we follow the standard assumption that exposure and severity are independently distributed.

[26] It is common practice in the industry to set $\widetilde{L}$ to zero.

[27] The beta distribution is usually chosen since it is bounded, such as on the unit interval, with two shape parameters which can be expressed in terms of mean and standard deviation of losses.

collected in $\boldsymbol{\upsilon}_{T+1} = (\boldsymbol{\varepsilon}'_{T+1}, \varepsilon_{d,T+1})'$. Note that although the firm in question operates in country/region $i$, its probability of default could be affected by macroeconomic shocks worldwide.

Under the assumption that all these shocks or innovations are jointly normally distributed and the parameter estimates are given, we have the following expression for the probability of default over $T$ to $T+1$ formed at $T$[28]

$$\pi_{ji,T+1|T} = \Phi\left(\frac{\hat{\lambda}_{\mathcal{R}}(T,1) - \mu_{ji,T+1|T}}{\sqrt{Var\left(\xi_{ji,T+1} \mid \Omega_T\right)}}\right), \tag{30}$$

where

$$Var\left(\xi_{ji,T+1} \mid \Omega_T\right) \equiv \omega^2_{\xi,ji} = \omega^2_{\eta,ji} + \Gamma'_{ji}\mathfrak{B}\Gamma_{ji}, \tag{31}$$

$\mathfrak{B} = \mathfrak{D}\boldsymbol{\Sigma}_{\boldsymbol{\upsilon}}\mathfrak{D}'$ is given by (A.18) in the Supplement, $\boldsymbol{\Sigma}_{\upsilon}$ is the variance-covariance matrix of the composite systematic innovations $\boldsymbol{\upsilon}_t$, and $\omega^2_{\eta,ji}$ is the variance of firm's idiosyncractic shock, $\eta_{ji,T+1}$.

Both of the restrictions (given parameter values and joint normality) can be relaxed. Parameter uncertainty can be taken into account by integrating out the true parameters using posterior or predictive likelihoods of the unknown parameters, as in Garratt et al. (2003b). In the presence of non-normal shocks one could simulate the loss distributions assuming fat-tailed distributions such as Student $t$ with a sufficiently low degree of freedom. Alternatively, one can employ non-parametric stochastic simulation techniques by re-sampling from estimated residuals of the GVAR model to estimate $\pi_{ji,T+1|T}$.

The expected loss due to default of a loan (credit) portfolio can now be computed by aggregating the expected losses across the different loans. Denoting the loss of a loan portfolio over the period $T$ to $T+1$ by $L_{T+1}$ we have

$$E_T\left(L_{T+1}\right) = \sum_{i=0}^{N}\sum_{j=1}^{nc_i} \pi_{ji,T+1|T}\; E_T(\mathcal{X}_{ji,T+1})\; E_T(\mathcal{S}_{ji,T+1}), \tag{32}$$

where $nc_i$ is the number of obligors (which could be zero) in the bank's loan portfolio resident in country/region $i$.

### 3.4 Simulation of the Loss Distribution

The expected loss as well as the entire loss distribution can be simulated for the period $T$ to $T+1$ once the GVAR model parameters, the return process parameters in (22) and the thresholds in (11) have been estimated for a sample of observations $t = 1, 2, ..., T$. The key component of the simulations are firm-specific returns defined by (24), which we write as

$$r_{ji,T+1} = \mu_{ji,T+1|T} + \Gamma'_{ji}\mathfrak{D}\boldsymbol{\upsilon}_{T+1} + \eta_{ji,T+1}, \tag{33}$$

[28] Joint normality is sufficient but not necessary for $\xi_{ji,t+1}$ to be approximately normally distributed. This is because $\xi_{ji,t+1}$ is a linear function of a large number of weakly correlated shocks (63 in our particular application).

where the predictable component, $\mu_{ji,T+1|T}$, is given by (29), and the innovations, $\boldsymbol{\upsilon}_{T+1}$ and $\eta_{ji,T+1}$, are distributed independently with zero means and the variances $\boldsymbol{\Sigma}_{\upsilon}$ and $\omega^2_{\eta,ji}$, respectively. In general, firm-specific returns can be simulated under alternative assumptions regarding the probability distribution of the innovations. However, simulations become particularly simple to implement under Gaussian innovations.[29] In this case, $\Gamma'_{ji}\mathfrak{D}\boldsymbol{\upsilon}_{T+1}$ is also distributed as $N\left(0, \Gamma'_{ji}\mathfrak{D}\boldsymbol{\Sigma}_{\upsilon}\mathfrak{D}'\Gamma_{ji}\right)$ and firm-specific returns can be obtained as

$$r^{(r)}_{ji,T+1} = \mu_{ji,T+1|T} + \xi^{(r)}_{ji,T+1}, \tag{34}$$

where $r^{(r)}_{ji,T+1}$denotes the $r^{th}$ replication of the firm-specific returns, and $\xi^{(r)}_{ji,T+1}$ is the $r^{th}$ replication of the composite shock given by

$$\xi^{(r)}_{ji,T+1} = \left(\Gamma'_{ji}\mathfrak{D}\boldsymbol{\Sigma}_{\upsilon}\mathfrak{D}'\Gamma_{ji}\right)^{1/2} Z^{(r)}_0 + \omega_{\eta,ji} Z^{(r)}_{ji} \tag{35}$$

where $Z^{(r)}_0$ and $Z^{(r)}_{ji}$ are independent draws from $N(0,1)$.

The loss can then be simulated in period $T+1$ using (known) loan face values, say $FV_{ji,T}$, as exposures, and draws from a beta distribution for severities (as described above):

$$L^{(r)}_{T+1} = \sum_{i=0}^{N}\sum_{j=1}^{nc_i} I\left(r^{(r)}_{ji,T+1} < \hat{\lambda}_{\mathcal{R}}(T,1)\right) FV_{ji,T}\ \mathcal{S}^{(r)}_{ji,T+1}. \tag{36}$$

The simulated expected loss due to default is given by (using $R$ replications)

$$\bar{L}_{R,T+1} = \frac{1}{R}\sum_{r=1}^{R} L^{(r)}_{T+1}, \tag{37}$$

and as $R \to \infty$ then $\bar{L}_{R,T+1} \xrightarrow{p} E_T\left(L_{T+1}\right)$, The simulated loss distribution is given by ordered values of $L^{(r)}_{T+1}$, for $r = 1, 2, ..., R$. For a desired percentile, for example the 99%, and a given number of replications, say $R = 10,000$, credit value at risk is given as the $100^{th}$ highest loss.

## 3.5 Default and Expected Loss Given Economic Shocks

In credit risk analysis we may also be interested in evaluating quantitatively the relative importance of changes in different macroeconomic variables or factors on the loss distribution. In the argot of risk management this is sometimes called scenario analysis. To this end the loss distribution conditional on a given shock can be compared to a baseline distribution without such a shock. It is difficult to imagine conducting such counterfactuals using a credit risk model that relies on accounting data.

As with all counterfactual experiments it is important that the effects of the shock on other macroeconomic variables are clearly specified. One possibility would be to assume that the other

[29] The simplifying procedure is also applicable when $\boldsymbol{\upsilon}_{T+1}$ has a standard multivariate Student-t distribution.

variables are displaced according to their historical covariances with the variable being shocked. This is in line with the generalized impulse response function (GIRF) analysis discussed in Section C of the Supplement. In this set-up, if variable $\ell$ in country $i$ is shocked by one standard error (i.e. $\sqrt{\sigma_{ii,\ell\ell}}$) in the period from $T$ to $T+1$, on impact the vector of the macroeconomic variables would be displaced by

$$\boldsymbol{\psi}_{i\ell}(\Delta\mathbf{y},1) = \frac{1}{\sqrt{\sigma_{ii,\ell\ell}}}\mathfrak{D}\boldsymbol{\Sigma}_{\boldsymbol{\upsilon}}\mathfrak{s}_{i\ell}, \tag{38}$$

where $\mathfrak{s}_{i\ell}$ is a $(k+s)\times 1$ selection vector with its element corresponding to the $\ell^{th}$ variable in country $i$ being unity and zeros elsewhere.

The above counterfactual, while of some interest, will underestimate the expected loss under both shock scenarios since it abstracts from volatility of the macroeconomic factors. To allow for volatility of macroeconomic factors in the analysis consider the case where the various shocks are jointly normally distributed, and note that

$$r_{ji,T+1} = \mu_{ji,T+1|T} + \Gamma'_{ji}\mathfrak{D}\boldsymbol{\upsilon}_{T+1} + \eta_{ji,T+1},$$

where $\mu_{ji,T+1|T}$ is defined by (29). Following a similar line of argument as in PSW (see also the Supplement, Section C), if the shock is assumed to be anticipated we have

$$r_{ji,T+1}\left|\Omega_T,\ \varepsilon_{iT+1,\ell} = \sqrt{\sigma_{ii,\ell\ell}}\right. \backsim N\left(\mu_{ji,T+1|T} + \Gamma'_{ji}\boldsymbol{\psi}_{i\ell}(\Delta\mathbf{y},1),\ \omega^2_{\xi,ji,i\ell}\right),$$

where $\varepsilon_{i,T+1,\ell} = \mathfrak{s}'_{i\ell}\boldsymbol{\upsilon}_{T+1}$, $\boldsymbol{\psi}_{i\ell}(\Delta\mathbf{y},1)$ is defined by (38) and[30]

$$\omega^2_{\xi,ji,i\ell} = \omega^2_{\eta,ji} + \Gamma'_{ji}\mathfrak{B}_{i\ell}\Gamma_{ji}, \tag{39}$$

where $\Gamma_{ji}$ are the factor loadings and $\mathfrak{B}_{i\ell}$ is given by equation (A.29) in the Supplement. But if the shock is unanticipated (which we consider to be more relevant for credit risk analysis) we have

$$r_{ji,T+1}\left|\Omega_T,\ \varepsilon_{iT+1,\ell} = \sqrt{\sigma_{ii,\ell\ell}}\right. \backsim N\left(\mu_{ji,T+1|T} + \Gamma'_{ji}\boldsymbol{\psi}_{i\ell}(\Delta\mathbf{y},1),\ \omega^2_{\xi,ji}\right),$$

where $\omega^2_{\xi,ji}$ is given by (31).

Therefore, to allow for volatility of the innovations (macroeconomic as well as idiosyncratic), the simulation of the loss distribution needs to be carried out using the draws

$$r^{il,(r)}_{ji,T+1} = \mu_{ji,T+1|T} + \Gamma'_{ji}\boldsymbol{\psi}_{i\ell}(\Delta\mathbf{y},1) + \xi^{(r)}_{ji,T+1} \tag{40}$$

where $\xi^{(r)}_{ji,T+1}$ is defined by (35).

In the case of our empirical application where the log of oil prices is the only global variable in the model, the effect of a unit unanticipated shock to oil prices, $P^o_t$, can be simulated by generating the returns as

$$r^{o,(r)}_{ji,T+1} = \mu_{ji,T+1|T} + \Gamma'_{ji}\boldsymbol{\psi}_{o}(\Delta\mathbf{y},1) + \xi^{(r)}_{ji,T+1},$$

[30] Note that $\mathfrak{s}'_{i\ell}\boldsymbol{\Sigma}\mathfrak{s}_{i\ell} = \sigma_{ii,\ell\ell}$.

where

$$\boldsymbol{\psi}_o(\Delta\mathbf{y},1) = \frac{1}{\sigma_o}\mathfrak{D}\boldsymbol{\Sigma}_{\boldsymbol{\upsilon}}\mathfrak{s}_o = \sigma_o\begin{pmatrix}\boldsymbol{\Upsilon}_0\\ 1\end{pmatrix},$$

$\sigma_o^2$ is the variance of oil price shock, $\varepsilon_{ot}$, $\mathfrak{s}_o$ is a $(k+1)\times 1$ selection vector of zeros except for its last element which is set equal to unity, such that $\mathfrak{s}_o'\boldsymbol{\upsilon}_t = \varepsilon_{ot}$, and $\boldsymbol{\Upsilon}_0$ collects the coefficients for the contemporaneous effect of oil prices on the macroeconomic variables $\mathbf{x}_t$.[31] It is also worth noting that

$$\Gamma_{ji}'\boldsymbol{\psi}_o(\Delta\mathbf{y},1) = \sigma_o\left(\mathbf{B}_{ji}'\mathbf{W}_i\boldsymbol{\Upsilon}_0 + \boldsymbol{\gamma}_{ji}'\right) = \sigma_o\theta_{ji,o}$$

simplifying the oil shock-conditional first period return to

$$r_{ji,T+1}^{o,(r)} = \mu_{ji,T+1|T} + \sigma_o\theta_{ji,o} + \xi_{ji,T+1}^{(r)}. \tag{41}$$

This expression clearly shows that, relative to the baseline, the mean is increased by $\sigma_o\theta_{ji,o}$.

Default occurs if the $r^{th}$ simulated return falls below the threshold-equity ratio $\hat{\lambda}_{\mathcal{R}}(T,1)$ defined by (9), so that for all three cases,

$$\begin{array}{rrcll} \text{Baseline (34)} & r_{ji,T+1}^{(r)} & < & \hat{\lambda}_{\mathcal{R}}(T,1) \Longrightarrow & \text{Default,} \\ \text{Macro-shock-Conditional (40)} & r_{ji,T+1}^{il,(r)} & < & \hat{\lambda}_{\mathcal{R}}(T,1) \Longrightarrow & \text{Default,} \\ \text{Oil–shock-Conditional (41)} & r_{ji,T+1}^{o,(r)} & < & \hat{\lambda}_{\mathcal{R}}(T,1) \Longrightarrow & \text{Default.} \end{array} \tag{42}$$

Using these results in (36), the loss distribution can be simulated for any desired level of accuracy by selecting $R$, the number of replications, to be sufficiently large.

Finally, it might also be of interest to compare the base line default probability, $\pi_{ji,T+1|T}$, given by (30) with the default probability that results under the (unanticipated) shock to $x_{i,T+1,\ell}$, which we denote by $\pi_{ji,T+1|T}^{i\ell}$. We have

$$\pi_{ji,T+1|T} = \Phi\left(\frac{\hat{\lambda}_{\mathcal{R}}(T,1) - \mu_{ji,T+1|T}}{\omega_{\xi,ji}}\right),$$

and

$$\pi_{ji,T+1|T}^{i\ell} = \Phi\left(\frac{\hat{\lambda}_{\mathcal{R}}(T,1) - \mu_{ji,T+1|T} - \Gamma_{ji}'\boldsymbol{\psi}_{i\ell}(\Delta\mathbf{y},1)}{\omega_{\xi,ji}}\right). \tag{43}$$

### 3.6 Simulation of Multi-Step Ahead Loss Distributions

The forecast horizon for computing losses is constrained by the horizon used by the rating agencies (typically one year) when assessing a firm.[32] We take this to be about one year, but certainly not less, commensurate with a one year ahead default probability. We begin by considering the

[31] More details can be found in Section A of the Supplement.

[32] Going beyond this horizon would involve updating forward credit ratings, a topic of current research by the authors.

default conditions two periods (quarters) forward. The Merton model considers default only at the terminal date. Viewed at period $T$, firm $j$ will default if

$$r_{ji,T+1} + r_{ji,T+2} < 2\hat{\mu}_{\mathcal{R}} + \hat{\sigma}_{\mathcal{R}}\sqrt{2}\hat{Q}_{\mathcal{R}}(2),$$

where the quantile estimate $\hat{Q}_{\mathcal{R}}(2)$ is given in (12). Extending this to $H$ periods is straight forward, namely

$$R_{ji,T+H} = \sum_{\tau=1}^{H} r_{ji,T+\tau} < H\hat{\mu}_{\mathcal{R}} + \hat{\sigma}_{\mathcal{R}}\sqrt{H}\hat{Q}_{\mathcal{R}}(H), \tag{44}$$

where $R_{ji,T+H}$ denotes the cumulative $H$-period return. The firm's default probability can now be computed by simulating from the joint probability distribution function of future returns $r_{ji,T+1}, ...,$ $r_{ji,T+H}$, conditional on $\Omega_T$. For details see Section B in the Supplement.

### 3.6.1 Baseline Multi-period Loss Distribution

Of course in our set-up firm returns are serially correlated through their systematic risk factor dependence (the GVAR), and so the loss distribution due to default by firm $j$ in region $i$ over the period $T$ to $T + H$ can now be written as[33]

$$\begin{aligned} L_{ji}(T+1, T+H) \;\; &= \;\; L_{ji,T+1} + \varphi I\left(R_{ji,T+1} \geq \hat{\lambda}_{\mathcal{R}}(T,1)\right)\; L_{ji,T+2} + \\ &\quad ... + \varphi^{H-1}\left[\prod_{\kappa=1}^{H-1} I\left(R_{ji,T+\kappa} \geq \hat{\lambda}_{\mathcal{R}}(T,\kappa)\right)\right]\; L_{ji,T+H}, \end{aligned} \tag{45}$$

where $\varphi$ is a discount factor ($0 \leq \varphi < 1$, which could be set as $\varphi = 1/(1+\rho)$ with $\rho$ being an average real rate of interest), $R_{ji,T+\kappa}$ is defined by (44), and

$$L_{ji,T+\kappa} = I\left(R_{ji,T+\kappa} < \hat{\lambda}_{\mathcal{R}}(T,\kappa)\right)\; \mathcal{X}_{ji,T+\kappa}\; \mathcal{S}_{ji,T+\kappa}, \text{ for } \kappa = 1, 2, ..., H.$$

The multi-period loss expression (45) can be thought of as a survival function which progressively computes loss in period $T+\tau+1$ only if the firm has survived the previous period $T+\tau$. Using this architecture the multi-period baseline loss distribution can be simulated using the draws $r^{(r)}_{ji,T+\tau}$, for $\tau = 1, 2, .., H$ and $r = 1, 2, ..., R$ (see below and (A.22) in the Supplement), and the empirical distribution of $L_{ji}(T+1, T+H)$ can be constructed from $L^{(r)}_{ji}(T+1, T+H)$ where

$$L^{(r)}_{ji}(T+1, T+H) = L^{(r)}_{ji,T+1} + \sum_{t=2}^{H} \varphi^{t-1}\left[\prod_{\kappa=1}^{t-1} I\left(R_{ji,T+\kappa} \geq \hat{\lambda}_{\mathcal{R}}(T,\kappa)\right)\right]\; L^{(r)}_{ji,T+t},$$

and

$$L^{(r)}_{ji,T+\kappa} = I\left(\sum_{\tau=1}^{\kappa} R_{ji,T+\tau} < \hat{\lambda}_{\mathcal{R}}(T,\kappa)\right)\; \mathcal{X}^{(r)}_{ji,T+\kappa}\; \mathcal{S}^{(r)}_{ji,T+\kappa},\; \text{for } \kappa = 1, 2, ..., H.$$

[33] Once again, we assume for simplicity that losses $L$ past the horizon, $H$, are zero.

Aggregating across firms, we finally obtain the time $T$ conditional $H$-step ahead simulated loss distribution of the credit portfolio:

$$L^{(r)}(T+1,T+H)=\sum_{i=0}^{N}\sum_{j=1}^{nc_i}L_{ji}^{(r)}(T+1,T+H),\; r=1,2,...,R.$$

### 3.6.2 Multi-period Loss Distribution Given Economic Shocks

Consider now the effect of a one standard error shock to factor $\ell$ in country $i$ on the multi-period loss distribution. Using the results in the Supplement, Section C on impulse responses we have

$$r_{ji,T+\kappa}^{i\ell,(r)}=\mu_{ji,T+\kappa|T}+\Gamma'_{ji}\boldsymbol{\psi}_{i\ell}(\Delta\mathbf{y},\kappa)+\xi_{ji,T+\kappa}^{(r)},\text{ for }\kappa=1,2,...,H, \tag{46}$$

where $\boldsymbol{\psi}_{i\ell}(\Delta\mathbf{y},\kappa)$ is given by

$$\begin{aligned}\boldsymbol{\psi}_{i\ell}(\Delta\mathbf{y},\kappa) &= \frac{1}{\sqrt{\sigma_{ii,\ell\ell}}}\mathfrak{D}\boldsymbol{\Sigma}_{\boldsymbol{\upsilon}}\mathfrak{s}_{i\ell}\text{ for }\kappa=1,\\ &= \frac{1}{\sqrt{\sigma_{ii,\ell\ell}}}\left(\boldsymbol{\Phi}^{n-1}-\boldsymbol{\Phi}^{n-2}\right)\mathfrak{D}\boldsymbol{\Sigma}_{\boldsymbol{\upsilon}}\mathfrak{s}_{i\ell},\text{ for }\kappa=2,3,...,H,\end{aligned}$$

and

$$\xi_{ji,T+\kappa}^{(r)}=\left(\Gamma'_{ji}\mathfrak{B}\Gamma_{ji}\right)^{1/2}Z_0^{(r)}+\sum_{\tau=1}^{\kappa-1}\left(\Gamma'_{ji}\mathcal{H}_\tau\mathfrak{B}\mathcal{H}'_\tau\Gamma_{ji}\right)^{1/2}Z_\tau^{(r)}+\omega_{\eta,ji}\;Z_{ji\kappa}^{(r)}\;, \tag{47}$$

where $\mathfrak{B}=\mathfrak{D}\boldsymbol{\Sigma}_{\upsilon}\mathfrak{D}'$, and $Z_\tau^{(r)}$ and $Z_{ji\kappa}^{(r)}$ are independent draws from $N(0,1)$ for all $\tau$, $j,i$ and $\kappa$. Clearly, for $\kappa=1$ the above expression reduces to (35). The return simulations for the base-line case are given

$$r_{ji,T+\kappa}^{(r)}=\mu_{ji,T+\kappa|T}+\xi_{ji,T+\kappa}^{(r)},$$

for $\kappa=1,2,...,H$.

# 4 Credit Loss Results

## 4.1 Risk, Return and Default by Credit Rating

In order to obtain estimates of the rating-specific default threshold ratios, we make use of the rating histories from Standard and Poors spanning 1981-1999, roughly the same sample period as is covered by the GVAR model. We use S&P ratings since they are designed to capture default probability, whereas Moody's also incorporates an expectation of recovery into their ratings (Cantor and Packer 1995, BIS 2000, particularly its Annex I.B). The estimates of the one- through four-quarter ahead threshold-equity ratio, $C_{\mathcal{R},t+H}/E_{\mathcal{R}t}$, are computed using $exp(\hat{\lambda}_{\mathcal{R}}(H))$ with $H=1,..,4$, where $\hat{\lambda}_{\mathcal{R}}(H)$ is defined by (11). Empirical default probabilities, $\hat{\pi}_{\mathcal{R}}(t,H)$, are obtained using default intensity-based estimates detailed in Lando and Skødeberg (2002). The transition intensity approach uses techniques from survival analysis which make efficient use of ratings histories to obtain transition probabilities. This becomes especially important for the estimation of the transition from rating $\mathcal{R}$ to default. No default event may have occurred within a particular quarter; that does not, however, necessarily mean that $\hat{\pi}_{\mathcal{R}}(t, H) = 0$. It suffices that an obligor migrated from, say, $\mathcal{AAA}$ to $\mathcal{AA}$ to $\mathcal{A}$, and that a default occurred from $\mathcal{A}$ to contribute probability mass to $\pi_{\mathcal{AAA}}$; see also Jafry and Schuermann (2004). Still, there may be instances when there is no movement at all during a particular quarter. In that case the *estimated* default intensity (and hence probability) would indeed be identically equal to zero.

For each quarter and each rating-specifc default probability, $\hat{\pi}_{\mathcal{R}}(t, H)$, we compute the inverse CDF to obtain a time series of rating specific thresholds.[34] Since S&P rates only a subset of firms (in 1981 S&P rated 1,378 firms of which about 98% were U.S. domiciled; by early 1999 this had risen to 4,910, about 68% U.S.), it is reasonable to assign a non-zero (albeit very small) probability of default, even if the empirical estimate is zero.[35] This is particularly relevant if we wish to infer default behavior for a much broader set of firms than is covered by the rating agencies. With this in mind, we impose a lower bound on the quarterly $\hat{\pi}_{\mathcal{R}}$ set at 0.025 basis points per quarter.

Rating specific average returns, $\hat{\mu}_{\mathcal{R}}$, and their volatility, $\hat{\sigma}_{\mathcal{R}}$, are computed using the cum dividend total return measure from CRSP for all U.S. firms with a credit rating in a given quarter over the sample range 1981Q1 to 1999Q1. The results for $H = 1$ are presented in Table 2 below for the range of ratings that are represented in our portfolio of firms, namely $\mathcal{AAA}$ to $\mathcal{B}$; similar results are obtained for $H = 2, 3$ and 4.

**Table 2**

**Rating-Specific Return and Equity-Threshold Estimation**

| Credit Rating | $\hat{\mu}_{\mathcal{R}}$ | $\hat{\sigma}_{\mathcal{R}}$ | $\hat{\mu}_{\mathcal{R}}/\hat{\sigma}_{\mathcal{R}}$ | $\hat{\pi}_{\mathcal{R}}(t, 1)$ (in bp) | $\widehat{C_{\mathcal{R},1}/E_{\mathcal{R}}}$ | # of obs.[36] |
|---|---|---|---|---|---|---|
| $\mathcal{AAA}$ | 4.54% | 13.87% | 0.33 | 0.026 | 0.56 | 1,177 |
| $\mathcal{AA}$ | 4.06% | 15.16% | 0.27 | 0.369 | 0.52 | 6,272 |
| $\mathcal{A}$ | 4.13% | 15.31% | 0.27 | 0.714 | 0.52 | 12,841 |
| $\mathcal{BBB}$ | 3.80% | 17.38% | 0.22 | 10.63 | 0.50 | 9,499 |
| $\mathcal{BB}$ | 3.21% | 24.72% | 0.13 | 49.21 | 0.42 | 7,002 |
| $\mathcal{B}$ | 2.04% | 34.82% | 0.06 | 351.66 | 0.40 | 6,493 |

$\widehat{C_{\mathcal{R},1}/E_{\mathcal{R}}}$ denotes the sample estimate of the one-quarter ahead default equity ratio,

$\hat{\mu}_{\mathcal{R}}$ and $\hat{\sigma}_{\mathcal{R}}$ are the sample estimates of the quarterly mean and standard deviations of $R$-rated firms

$\hat{\pi}_{\mathcal{R}}(t, 1)$ is the quarterly default probability (in basis points) for $R$-rated firms

[34] While (9) and (10) are written in terms of a standard normal distribution, other distributions such as the Student $t$ can also be used.

[35] Ratings and rating histories are from Standard and Poor's CreditPro Database V. 6.2. We use the sample period 1981Q1-1999Q1.

We note that average quarterly volatility, $\hat{\sigma}_{\mathcal{R}}$, increases monotonically as we descend the rating spectrum to the point where the volatility of a $\mathcal{B}$-rated firm is more than twice than that of an $\mathcal{AA}$-rated firm. Average returns do not keep pace with the increasing volatility, resulting in similarly declining Sharpe ratios ($\hat{\mu}_{\mathcal{R}}/\hat{\sigma}_{\mathcal{R}}$). Quarterly default probabilities display the familiar pattern of increasing dramatically as we descend the credit spectrum, especially once the investment grade boundary is crossed (i.e. $\mathcal{BB}$ and below).

The counter-intuitive pattern of declining expected returns is in line with Dichev (1998) who finds that bankruptcy risk measured by credit ratings is not rewarded by higher returns. On the contrary, his analysis suggests that since 1980 firms with higher bankruptcy risk earn lower than average returns. Vassalou and Xing (2004) argue that rating data might reflect deteriorating financial conditions of a company with too much delay. Using the Merton default model to compute a default likelihood indicator, they find that high default stocks earn significantly higher returns than low default stocks, but only if they are small or have a high book-to-market value. To be sure, $\hat{\mu}_{\mathcal{R}}$ plays at best a secondary role in determining the default threshold $\hat{\lambda}_{\mathcal{R}}$ which is driven primarily by $\hat{\sigma}_{\mathcal{R}}$ and $\hat{\pi}_{\mathcal{R}}$.

Of particular interest is the behavior of the one-quarter forward threshold-equity ratio $\widehat{C_{\mathcal{R},1}/E_{\mathcal{R}}}$ which exhibits relatively little variation across ratings. It ranges from 0.56 for $\mathcal{AAA}$ to 0.40 for $\mathcal{B}$. When we extended the sample to the maximum sample length available, 1981Q1 - 2002Q4, the means, standard deviations and forward threshold-equity ratios remained quite stable. Only the default probabilities exhibited noticeable variation when extending the sample. These probability values are very small (they are reported in basis points!) simply because there are so few defaults for the very high credit grades. Moreover, the years 2001 and 2002 saw record default levels in the corporate bond markets (S&P 2003).

To understand the role of the threshold-equity ratio, take for example a firm rated $\mathcal{BBB}$ and its threshold-equity ratio of 0.50. If this firm has an equity level of 100 today, it would be able to sustain a drop to $0.50 \times 100 = 50$ over one quarter before defaulting on its debt obligations. The likelihood of this event is driven largely by $\hat{\sigma}_{\mathcal{BBB}}$ which is 17.38%. By contrast, a firm rated $\mathcal{B}$ would be able to sustain a drop to $0.40 \times 100 = 40$, but the likelihood of this event, driven by $\hat{\sigma}_{\mathcal{B}} = 34.82\%$, is of course much higher than for the $\mathcal{BBB}$-rated firm.

## 4.2 The Sample Portfolio

We analyze the effects of economic shocks on a fictitious large-corporate loan portfolio which is summarized in Table 3. It contains a total of 119 companies, resident over 10 of the 11 regions. In order for a firm to enter our sample, several criteria had to be met. We restricted ourselves to major, publicly traded firms which had a credit rating from either Moody's or S&P. Thus, for example, Chinese companies are not included for lack of a credit rating. The firms should be represented within the major equity index for that country. We favored firms for which equity return data was available for the entire sample period, i.e. going back to 1979. Typically this would exclude large firms such as telephone operators which in many instances have only been privatized recently, even though they might now represent a significant share in their country's dominant equity index. The data source is Datastream, and we took their Total Return Index variable which is a cum dividend return measure.

The third column in Table 3 indicates the sample range of the equity series available for return regression analysis. We wanted to mimic (broadly) the portfolio of a large, internationally active bank. Arbitrarily picking Germany as the bank's domicile country, the portfolio is relatively more exposed to German firms than would be the case if exposure were allocated purely on a GDP share (in our "world" of 25 countries). For the remaining regions, exposure is more in line with GDP share. Within a region, loan exposure is randomly assigned. The expected severity for loans to U.S. companies is the lowest at 20%, based upon studies by Citibank, Fitch Investor Service and Moody's Investor Service.[37] All other severities are based on assumptions, reflecting the idea that severities are higher in less developed countries. Table 3 gives the portfolio composition, regional weights, individual exposures, expected ($\mu_\beta$) and unexpected ($\sigma_\beta$) severities, as well as average pair-wise quarterly return correlations.[38] We see substantial variation in those correlations across the different countries and regions, ranging from 53% in Germany to 19% in the U.K. and Western Europe. The overall average pair-wise correlation across the whole portfolio is a relatively low 15% owing to the high degree of geographic diversification.

[37] As cited in Saunders and Allen (2002).

[38] Mean severity is assumed to be slightly lower in Germany (as compared to France or U.K., for example), since Germany is assumed to be the bank's domicile country and hence the bank may have some local advantages in the recovery of distress assets. Unexpected severity refers to standard deviation of severity distribution assumed here to be Beta distributed.

**Table 3**

**The Composition of the Sample Portfolio for Regions**

| Region | # Firms | Equity Series[1] Quarterly | Credit Rating[2] Range | Exposure Per cent | Severity[3] Mean ($\mu_\beta$) | S.D. ($\sigma_\beta$) | Avg. Return Correlation[4] |
|---|---|---|---|---|---|---|---|
| U.S. | 14 | 79Q1 - 99Q1 | $\mathcal{AAA}$ to $\mathcal{BBB}-$ | 20 | 20% | 10% | 0.26 |
| U.K. | 9 | 79Q1 - 99Q1 | $\mathcal{AA}$ to $\mathcal{BBB}+$ | 6 | 35% | 15% | 0.19 |
| Germany | 18 | 79Q1 - 99Q1 | $\mathcal{AAA}$ to $\mathcal{BBB}-$ | 21 | 30% | 15% | 0.53 |
| France | 8 | 79Q1 - 99Q1 | $\mathcal{AA}$ to $\mathcal{BBB}$ | 8 | 35% | 15% | 0.24 |
| Italy | 6 | 79Q1 - 99Q1 | $\mathcal{A}$ to $\mathcal{BBB}-$ | 8 | 35% | 15% | 0.31 |
| W. Europe | 12 | 79Q1 - 99Q1 | $\mathcal{AAA}$ to $\mathcal{BBB}+$ | 8 | 35% | 15% | 0.19 |
| Middle East | 4 | 90Q3 - 99Q1 | $\mathcal{B}-$ | 2 | 60% | 20% | 0.38 |
| S.E. Asia | 23 | 89Q3 - 99Q1 | $\mathcal{A}$ to $\mathcal{B}$ | 10 | 50% | 20% | 0.27 |
| Japan | 13 | 79Q1 - 99Q1 | $\mathcal{AAA}$ to $\mathcal{B}+$ | 10 | 35% | 15% | 0.32 |
| L. America | 12 | 89Q3 - 99Q1 | $\mathcal{A}$ to $\mathcal{B}-$ | 5 | 65% | 20% | 0.23 |
| Total | 119 | - | - | 100 | - | - | 0.15 |

1. Equity prices of companies in emerging markets are not available over the full sample period used for the estimation horizon of the GVAR. We have a complete series for all firms only for the U.S., U.K., Germany and Japan. For France, Italy and W. Europe, although some of the series go back through 1979Q1, data was available for all firms from 1987Q4 (France), 1987Q4 (Italy), 1989Q3 (W. Europe). We used that sample range for the multi-factor regressions for those regions. For L. America we have a complete sample range for all firms from 1990Q2.
2. The sample contains a mix of Moody's and S&P ratings, although S&P rating nomenclature is used for convenience.
3. Severity is drawn from a beta distribution with mean $\mu_\beta$ and standard deviation $\sigma_\beta$.
4. Arithmetic average of quarterly pair-wise correlations of firm returns.

The average credit quality, as measured by exposure weighted credit rating, is somewhat higher (better) than the average commercial and industrial (C&I) lending portfolio for large, U.S. banks. Treacy and Carey (2000) report that on average about half of those portfolios are of investment grade quality, meaning having a rating of $\mathcal{BBB}-$ or better, whereas the proportion meeting this threshold is about 80% in our portfolio. We would therefore expect losses in our portfolio to be lower, on average, than losses in a typical C&I portfolio for a U.S. bank.

### 4.3 Return Regressions

#### 4.3.1 Variable Selection Process

The general form of the multi-factor return equations used in this study is given by (19), which links individual firm returns to observed domestic and global macroeconomic risk factors, and can be estimated by least squares under the assumption that the firm-specific shocks, $\eta_{ijt}$, and the set of macro shocks, $\boldsymbol{v}_t$, are uncorrelated. However, since there is likely to be a high degree of correlation between some of the domestic and foreign variables (e.g. real equity prices and interest rates), a more parsimonious version might be desirable for empirical analysis. To this end two possible approaches can be followed. A standard procedure would be to apply regressor selection methods to each of the firm-specific regressions separately. Since we have 119 firms in our portfolio with as many as 13 estimated coefficients each,[39] the application of such a procedure, besides being very time-consuming, can be subject to a considerable degree of specification searches with undesirable consequences. Alternatively, we could view the 119 return regressions as forming a panel with heterogeneous slope coefficients and base the regressor selection procedure on the means of the estimated coefficients, referred to as the mean group estimators (MGE).[40] This approach is clearly more manageable and will be adopted in this study.

Initially, we estimated multi-factor regressions including all the domestic and foreign variables relevant to the firm's domicile region. The variables are output $(\Delta y, \Delta y^*)$, inflation $(\Delta^2 p, \Delta^2 p^*)$, equity price $(\Delta q, \Delta q^*)$, real exchange rates $(\Delta(e-p), \Delta(e-p)^*)$, interest rates $(\Delta r, \Delta r^*)$, and real money balances $(\Delta m, \Delta m^*)$. An asterisk denotes foreign variables. The return equations estimated for the U.S. firms are somewhat different in that the only foreign regressor included is the foreign real exchange rate $(\Delta(e-p)^*)$, but the domestic exchange rate variable is excluded as the U.S. dollar is the numeraire currency. For the non-U.S. regressions, we apply the MGE procedure to remove insignificant variables. Because of the limited number of U.S. firms, we rely on t-statistics and the signs of individual coefficients to choose the best subset of regressors. Finally, recognizing the likely collinearity of $\Delta q$ and $\Delta q^*$ (the domestic and foreign equity series), we run two versions of each model, one with domestic equity and one with foreign. We choose the model with the higher adjusted R-squared, $\bar{R}^2$.[41,42]

[39] One constant, six domestic, five foreign macroeconomic variables plus oil prices.

[40] For further details of the MGE procedure see Pesaran and Smith (1995) and Pesaran, Smith and Im (1996).

[41] Since the two non-nested multi-factor regressions have the same number of coefficients, the same result would follow if other model selection criteria are used.

[42] Of course, there are other approaches to choosing an multi-factor specification for each firm. We considered (and, in fact, carried out) alternative approaches, including one which began with only domestic variables (plus oil) in the multi-factor regressions, slimming down via MGE, and then potential substitution of foreign for domestic variables if the significance or sign of the domestic variable was called into question. In the end, we felt that taking an approach that was more consistent with the framework of the GVAR model (i.e. beginning with all of the GVAR models and then paring the model down) was more appropriate.

### 4.3.2 Return Regression Results

A summary result of the initial multi-factor regressions are provided in Table 4, where the proportion of firms with significant multi-factor regressions (using an F-test at the 5% level) and significant t-ratios for individual factors are given across different countries/regions. Around 90% of the return regressions were significant (using the F-test) at the 5% level. The F-test values in the first row of Table 4 suggest that changes in the macroeconomic factors have a significant influence on equity returns. The t-statistics for the coefficients of individual macroeconomic factors clearly single out two important ones: the domestic and foreign real equity returns.[43] For regions where no full equity series could be incorporated in the GVAR, i.e. the Middle East, we cannot identify one dominant macroeconomic factor. In South East Asia, both domestic and foreign output matter, as does the exchange rate. Oil price changes are significant in about a quarter of the regressions.

**Table 4**

**Results from Firm Multi-Factor Regressions: % of firms significant at 5% level[44]**

| | U.S.A. | U.K. | Germany | France | Italy | W. Europe | Mid East | S. E. Asia | Japan | Latin America |
|---|---|---|---|---|---|---|---|---|---|---|
| F-test | 93% | 100% | 94% | 88% | 67% | 100% | 75% | 65% | 92% | 25% |
| const.[45] | 21% | 44% | 6% | 25% | 33% | 8% | 25% | 30% | 15% | 17% |
| $\Delta y$ | 14% | 11% | 0% | 13% | 0% | 0% | 0% | 35% | 8% | 8% |
| $\Delta^2 p$ | 21% | 11% | 0% | 13% | 0% | 0% | 25% | 13% | 0% | 8% |
| $\Delta q$ | 93% | 44% | 11% | 38% | 83% | 92% | – | 74% | 85% | 25% |
| $\Delta(e-p)$ | – | 11% | 0% | 0% | 0% | 17% | 25% | 35% | 38% | 25% |
| $\Delta r$ | 0% | 0% | 0% | 0% | 17% | 8% | 50% | 4% | 0% | 0% |
| $\Delta m$ | 14% | 0% | 0% | 0% | 0% | 17% | 25% | 13% | 0% | 17% |
| $\Delta y^*$ | – | 0% | 0% | 0% | 17% | 0% | 0% | 35% | 8% | 8% |
| $\Delta^2 p^*$ | – | 11% | 0% | 13% | 17% | 0% | 0% | 0% | 0% | 17% |
| $\Delta q^*$ | – | 56% | 100% | 63% | 33% | 50% | 25% | 17% | 15% | 8% |
| $\Delta(e-p)^*$ | 7% | – | – | – | – | – | – | – | – | – |
| $\Delta r^*$ | – | 22% | 0% | 13% | 33% | 0% | 25% | 0% | 0% | 17% |
| $\Delta m^*$ | – | 0% | 0% | 0% | 0% | 17% | 0% | 17% | 0% | 0% |
| $\Delta p^o$ | 21% | 22% | 33% | 38% | 17% | 8% | 50% | 17% | 0% | 25% |
| avg. $R^2$ | 0.30 | 0.34 | 0.38 | 0.43 | 0.49 | 0.51 | 0.48 | 0.47 | 0.40 | 0.39 |
| avg. $\bar{R}^2$ | 0.23 | 0.22 | 0.27 | 0.31 | 0.37 | 0.41 | 0.26 | 0.30 | 0.29 | 0.11 |

[43] Thus, it seems plausible to reduce the multi–factor approach to a single factor CAPM-type approach for regions where an equity series is available.

[44] We use the maximum sample length available to all firms in one region.

[45] The remaining are t-tests.

Across the ten regions, variation in the macroeconomic factors explains between 11% and 41% of the total variations in firm returns, as measured by $\bar{R}^2$. If we have captured overall systematic risk reasonably well, the diversification benefits in an all-U.K. portfolio (average $\bar{R}^2 = 0.22$) should thus be greater than for an all- South East Asian portfolio (average $\bar{R}^2 = 0.30$), which seems to be more driven by systematic risk. Consequently, similarly sized macroeconomic shocks should affect loans to South East Asian obligors to a higher extent than loans to U.K. obligors.

We now employ the MGE procedure in order to determine the overall significance of the factors. The results are summarized in Table 5.

**Table 5**
**Mean Group Estimates of Factor Loadings in Return Regressions**

| Factors | MGE $\hat{\beta}_\ell$ $(\hat{\beta}^*_\ell)$ | S.E. of MGE $\sqrt{\widehat{Var}(\hat{\beta}_\ell)}$ | t-ratios $t_\ell$ $(t^*_\ell)$ | Number of Coefficients $\sum_{i=0}^{N} nc_i$ |
|---|---|---|---|---|
| constant | 0.05 | 0.01 | 4.97 | 119 |
| $\Delta y$ | 0.25 | 0.47 | 0.54 | 119 |
| $\Delta^2 p$ | -0.67 | 0.33 | -2.02 | 119 |
| $\Delta q$ | 0.59 | 0.06 | 9.73 | 115 |
| $\Delta(e-p)$ | -0.07 | 0.09 | -0.73 | 105 |
| $\Delta r$ | -1.96 | 0.71 | -2.76 | 119 |
| $\Delta m$ | -0.14 | 0.24 | -0.59 | 119 |
| $\Delta y^*$ | -2.38 | 0.94 | -2.54 | 105 |
| $\Delta^2 p^*$ | -1.50 | 1.07 | -1.41 | 105 |
| $\Delta q^*$ | 0.49 | 0.10 | 4.95 | 105 |
| $\Delta(e-p)^*$ | 0.10 | 0.15 | 0.62 | 14 |
| $\Delta r^*$ | 0.63 | 3.23 | 0.20 | 105 |
| $\Delta m^*$ | -0.88 | 0.45 | -1.97 | 105 |
| $\Delta p^o$ | 0.29 | 0.06 | 4.67 | 119 |

Based on the MG test results the statistically most significant factors are, perhaps not surprisingly, changes in domestic and foreign real equity prices ($\Delta q$ and $\Delta q^*$). The MGE of equity prices have the expected signs and their magnitudes seem plausible. For example, the estimated coefficients of changes in domestic and foreign equity prices add up to 1.08, suggesting that the composition of the loan portfolio closely matches that of a global market portfolio. Domestic inflation (and to a lesser extent foreign inflation) and oil prices were also statistically significant. Both domestic and foreign inflation have negative effects on returns, as to be expected. The overall effect of the oil price changes is, however, positive. This seems a reasonable outcome for energy and petrochemical companies and for some of the banks, although one would not expect this result to be universal. In fact we do observe considerable variations in the individual estimates of the coefficients of oil prices changes across different firms in our portfolio. In the final regressions, of the 119 firm regressions, the coefficient on oil price changes was positive for 89 firms (about 75% of the total), and negative for the remaining firms. The MGE for each subset was also significant.

Among the remaining factors, interest rates and foreign output are also significant. The latter is difficult to explain, particularly considering that domestic output is not statistically significant and foreign output has a wrong sign. In view of this we decided to exclude both of the output variables from our subsequent analysis. Of the two interest rate variables we included the domestic rate which had the correct sign.

Our concerns regarding multicollinearity were confirmed by the regression results. Initially, we included both foreign and domestic equity variables but found implausible (negative) estimates for some of the multi-factor regressions, which we believe partly reflects the high correlation of $\Delta q$ and $\Delta q^*$ in some regions. Working with multi-factor regressions with perversely signed estimated coefficients is particularly problematic for the analysis of shock scenarios where the coefficient of equity prices plays a critical role in the transmission of shocks to the loss distribution. We ran two sets of multi-factor regressions (including inflation, interest rate and the oil price variables); one with $\Delta q$ and another with $\Delta q^*$, and selected the regression with higher $\bar{R}^2$. The summary of the final set of multi-factor regressions and the associated MG estimates are given in Table 6. In this specification inflation, equity price changes and oil price changes remain the key driving factors in the multi-factor regressions.

**Table 6**
**Mean Group Estimates of Factor Loadings**
**The Preferred Model**

| Factors | MGE $\hat{\beta}_\ell$ $(\hat{\beta}^*_\ell)$ | S.E. of MGE $\sqrt{\widehat{Var}(\hat{\beta}_\ell)}$ | t-ratios $t_\ell$ $(t^*_\ell)$ | Number of Coefficients $\sum_{i=0}^{N} nc_i$ |
|---|---|---|---|---|
| constant | 0.02 | 0.004 | 6.33 | 119 |
| $\Delta^2 p$ | -0.71 | 0.28 | -2.52 | 119 |
| $\Delta q/\Delta q^*$ | 1.01 | 0.04 | 25.07 | 119 |
| $\Delta(e-p)^*$ | 0.07 | 0.15 | 0.46 | 14 |
| $\Delta r$ | -1.82 | 0.65 | -2.79 | 119 |
| $\Delta p^o$ | 0.39 | 0.07 | 5.51 | 119 |

It is important to note that the simulation of the loss distributions are not based on the MG estimates, but are computed using the 119 individual firm return regressions that allow for fully heterogeneous factor loadings across firms. The individual estimates display a considerable degree of heterogeneity across firms, both in sign and in magnitude. The MG estimates and their standard errors provide a useful summary of the extent to which parameter heterogeneity matters. They also allow us to streamline the search process across alternative specifications of the return regressions. Otherwise, we would have faced the daunting task of searching across all the 119 return regressions separately. In addition to being highly time-consuming, such a procedure is also likely to be subject to a much higher degree of pre-testing as compared to the selection procedure adopted above.

### 4.4 Simulated Conditional Loss Distributions

With the estimated GVAR model serving as the economic scenario generator and the fitted multi-factor regressions as the linkage between firms and the economy, we simulated loss distributions one through four quarters ahead.[46] A one year horizon is typical for credit risk management and thus of particular interest. In addition to the loss distribution implied by the baseline forecast, for each horizon we examined the impact of several shock scenarios:[47]

- a $-2.33\sigma$ shock to real U.S. equity, corresponding to a quarterly drop of 14.28%,
- a $+2.33\sigma$ shock to real German output, corresponding to a quarterly rise of 2.17%,
- a $-2.33\sigma$ shock to real S.E. Asian equity, corresponding to a quarterly drop of 24.77%,
- a $+2.33\sigma$ shock to Japanese real money supply, corresponding to a quarterly rise of 2.87%.
- a $+2.33\sigma$ shock to the price of crude oil, corresponding to a quarterly rise of 16.01%.[48]

We also experimented with symmetric positive shocks to U.S. and S.E. Asian equity prices, and a symmetric negative shock to the price of crude oil. These are of particular interest here since their impacts on losses will not be (negatively) symmetric due to the nonlinearity of the credit risk model. In addition we consider a stress scenario for the U.S. equity market as reported in PSW, namely an adverse shock of $8.02\sigma$. Such a large shock corresponds to a quarterly drop of 49% which is the largest quarterly drop in the S&P 500 index since 1928 (which occurred in the three months to May, 1932). It also corresponds to the recent decline from their peak in 2000 to a recent low (in early October, 2002). Finally we include an intermediate negative equity shock of $-5\sigma$.

We carried out 200,000 replications for each shock scenario using Gaussian (compound) innovations. For the forecasts and shock scenarios, we computed expected loss results using the analytic

[46] The important issue of credit risk model evaluation, especially for a regulator under BIS 2, is beyond the scope of this paper; we plan to address it in subsequent work. See also Lopez and Saidenberg (2000). Lucas (2001) illustrates the difficulty of this validation process in the easier context of market risk.

[47] $2.33\sigma$ corresponds, in the Gaussian case, to the 99% Value-at-Risk (VaR), a typical benchmark in risk management.

[48] The price at the end of 1999Q1 was \$12.31 a barrel (Brent Crude). A $+2.33\sigma$ shock would raise the price to \$14.45.

formula (using (32)) as well as by stochastic simulations, (37). The two sets of estimates turn out to be within 0.5% (analytic baseline EL is 14.55bp for the first quarter) so we only report the simulated ones. The simulated expected and unexpected loss results are summarized in Tables 7a and 7b, respectively, where each column represents a particular scenario. The scenarios are ordered roughly in descending order (left to right) of loss impact.

[Insert Tables 7a and 7b about here]

We begin our discussion by looking at baseline losses. The expected baseline loss, seen in the middle of Table 7a, over the course of four quarters is about 57.8bp (basis points) of exposure. Losses occur more or less evenly throughout the four quarters, though the last generates the highest losses (16.0bp) and the second quarter the lowest (12.8bp). As a basis of comparison, the average net charge-offs (loans charged off less amount recovered over total loans) for the U.S. banking industry from 1987 to 2003 was about 89bp.[49] Another point of comparison is with industry charge-offs in 2000Q1 since the expected losses of our portfolio are essentially a one-year forecast to 2000Q1 (our sample ends in 1999Q1). Those were 56bp (at an annual rate). These results are quite realistic and are consistent with our expectations raised in Section 4.2 that our average losses would be a bit lower than the U.S. average since the average credit quality of our portfolio is higher: about 80% investment grade instead of the typical 50%. Of course, our portfolio has broad international exposure, so we would not really expect to match the U.S. loss experience exactly.

From a risk perspective, it is not so much expected as *un*expected loss which matters. This is captured by the volatility or the standard deviation of losses summarized in Table 7b. For the baseline, unexpected losses are almost double expected losses, about 106.0bp. The quarterly loss volatilities are increasing over the four quarters; note that they are not additive over time as is the case with average (expected) losses.

We now turn to the effect of shock scenarios. A $2.33\sigma$ drop of real U.S. equity prices results in an expected loss of 80bp over four quarters, about 38% above the baseline (Table 7a). The increase in loss volatility is similar as can be seen in Table 7b: a 35% increase to 143bp. An inflationary shock to Japanese real money supply of the same probabilistic size, namely $2.33\sigma$, results in only a less than 3% increase in both EL (to 59.3bp) and UL (to 107.5bp). The adverse shock to S.E. Asian real equity prices is closer, in EL and UL terms, to the impact of the equiprobable adverse U.S. equity shock. A benign shock to German real output reduced four quarter expected losses by about 5% to 54.7bp, and unexpected losses by about 6% to 99.6bp.

[Insert Figure 1 about here]

[49] These figures were calculate by the authors using U.S. bank regulatory reports Y-9C, the so-called "Call Reports." Details are available upon request.

Of principal interest to risk managers is the tail of the loss distribution. Several of the shock scenarios for the tails are summarized in Figure 1. The basic pattern established in the discussion of expected and unexpected losses is confirmed in the 99% and beyond tail of the loss distribution. An inflationary shock to Japanese money supply increases credit losses somewhat relative to the baseline loss distribution. A positive shock to German output reduces tail losses only modestly, while adverse shocks to S.E. Asian and U.S. real equity prices increase tail losses, proportionately similar to their impact on unexpected losses. For example, 99.5% baseline value-at-risk (VaR) is about 6.86%, while it is 30% higher for the S.E. Asian equity shock scenario (at 8.91%) and 44% higher for the U.S. equity shocks scenario (at 9.87%).

[Insert Table 8 about here.]

The tail values are around eight (99%) to twelve (99.5%) times expected losses for each scenario, and around four to six times unexpected losses. The EL multiples are much higher than the re-sampling based results in Carey (2002) who reported multiples closer to four for the case of the U.S.; see especially his Table 3. Our portfolio is rather small, only 119 firms (Carey had about 500), and therefore has considerable potential for diversifying firm-specific risks. To evaluate the consequences of reducing the idiosyncratic risk for the EL multiple, we focused on one quarter ahead and carried out the simulations with each of the firms in the portfolio 'copied' 10 and 100 times to arrive at an effective portfolio of 1190 and 11,900 firms, respectively. The results are summarized in Table 8 and show sizeable reductions in unexpected losses, from 30.83bp to 12.59bp for 10 copies and just 8.85bp for 100 copies. Similar reductions can be seen in the tails. For instance, first quarter 99.5% VaR declines from 153.06bp to 63.14bp (53.67bp) when $C = 10$ (100). EL, of course, remains virtually unchanged and very close to the analytic value of 14.55bp, as it should. Thus much of the fat-tailedness of our loss distribution is due to the presence of significant amount of (diversifiable) idiosyncratic risk, and once this is controlled for, the EL to VaR multiples are in line with Carey's results.

[Insert Figure 2 about here]

Symmetric shocks do not result in symmetric outcomes, namely positive and negative shocks of the same absolute size do not have the same absolute effects on loss distributions. Consider first expected and unexpected losses. While a negative shock to, say, S.E. Asian real equity prices increased expected losses (Table 7a) by 28%, a positive shock reduces them only by 18%, namely from 57.8bp to 47.4bp. Similarly, unexpected losses (Table 7b) increase by 23% under an adverse scenario (from 106bp to 130bp) while they decrease just 16% (to 89.3bp) under a benign scenario. This is confirmed in the tails of the loss distribution for the pair of S.E. Asian real equity shock scenarios displayed in Figure 2.

The asymmetric reaction of losses to symmetric risk factor shocks is vividly illustrated with the oil price shocks. Recall that the mean group estimate of oil price changes in the return regression equation was found to be positive at 0.39 (see Table 6). Thus in Table 7a we see that a $2.33\sigma$ drop in oil prices increases expected losses by 27% (from 57.8bp to 73.3bp) while a commensurate increase reduces expected losses by only 12% (to 50.9bp). The relative impact is less dramatic on loss volatility as seen in Table 7b. The $-2.33\sigma$ oil price shock increases loss volatility by only 10% (106.0bp to 116.7bp), the positive shock decreases loss volatility by just 6% (to 99.5bp).

[Insert Figure 3 about here]

Figure 3 shows the loss distribution of the two oil price shocks together with the baseline distribution for the $95^{th}$ percentile and beyond. For the majority of firms in our sample, an upward shock to the oil price has benign effects. Yet, there are also firms which move close to default in the presence of an upward shock to the oil price (as one would expect for many industries). As we have already seen, symmetric shocks do not result in symmetric changes to the loss distribution. The increase in credit loss from an adverse shock is disproportionately larger than loss mitigation from a benign shock of the same magnitude. While oil price shocks may have opposite effects on individual firm default risk, the adverse effect tends to outweigh the benign one. Thus, it is plausible within the portfolio context of our model that positive and negative shocks to the same variable may both result in adverse effects on credit loss.

One can clearly see just how steep, in this display manner, the loss curve becomes in the far tail. Past a certain point, about the $99^{th}$ percentile, losses increase dramatically. It is no accident that credit risk managers focus on this region.

[Insert Figure 4 about here]

What happens as the shocks become more and more extreme? This question is addressed in Figure 4 for different U.S. real equity shock scenarios: $-5.00\sigma$ and $-8.02\sigma$, the latter matching the largest quarterly drop in the S&P 500 index since 1928. We also display the baseline loss distribution for comparison. To be sure, a shock as extreme as $-8.02\sigma$ is, of course, outside the bounds of the estimated model. It would be unreasonable to believe that such a large shock would not result in changes of the underlying parameters. In particular, we note that the volatilities remain constant, and while stressing volatilities would certainly be of interest (especially in light of the nonlinearities already in the model), they require care so as to maintain coherence across the error covariance matrix; such stress testing is beyond the scope of the present work. Nonetheless, it is still instructive to examine the impact of an extreme shock given the constant volatility as one way of stressing a credit risk model.

While a $-2.33\sigma$ shock results in $99^{th}$ percentile loss that is about 50% more than the baseline, 4.78% to 7.47%, a $-5.00\sigma$ shock more than doubles 99% VaR, to 11.15%, and the extreme $-8.02\sigma$

shock more than triples losses at this level, to 15.25% of exposure. Carey (2002), in his "Great Depression" scenario finds that losses at the $99^{th}$ percentile would be about triple his base case.

## 5 Concluding Remarks

Financial institutions are ultimately exposed to macroeconomic fluctuations in the global economy. Their portfolios are typically sufficiently large that idiosyncratic risk is diversified away, leaving exposure to systematic risk. If business cycles are not perfectly correlated across countries and regions, diversification benefits can be obtained by internationalizing one's exposure. In this paper we develop a Merton-type credit risk model which is linked to a global macroeconometric model that explicitly allows for the interdependencies that exist between national and international factors. A key challenge, which we address, is to allow for firm-specific business cycle effects and heterogeneous default thresholds.

The first step in developing such a model is to build an economic engine reflective of the environment faced by an internationally active global bank which is done in Pesaran, Schuermann and Weiner (2004). For the credit portfolio component of our model we use a simple Merton-type framework, modeling credit risk as a function of correlated equity returns of the obligor companies. Equity returns are linked to the correlated macroeconomic variables contemporaneously through return regressions. We then use the estimated global model as the economic engine for generating a multi-period conditional loss distribution of a credit portfolio using stochastic simulation. Sampling takes place along three lines: correlated random draws of macroeconomic factors; draws of firm-specific risk components; and draws of stochastic loan loss severities. Finally we analyze the impact of a shock to a set of specific macroeconomic variables on the loss distribution, allowing us to analyze the effect of a particular macroeconomic shock in one region on credit portfolios concentrated in other regions, as well as shocks to risk factors, e.g. oil prices, affecting all regions.

Our credit risk modeling approach has two other features of particular relevance for risk managers: exploration of scale and symmetry of shocks on credit risk, and the ranking of shock impacts on credit risk. First, we show that shocks not only have an asymmetric but also non-proportional impact on credit risk due to the nonlinearity of the credit risk model. Because the Merton model is an option-theoretic model, these traits echo characteristics of the options markets: large movements in the underlying prices have disproportional effects on the value of the option portfolio.

Second, the model allows us to rank the effects of different shocks on a global portfolio. Not surprisingly, shocks to real equity prices seem to have the most significant effect on implied credit losses, followed here by shocks to the price of crude oil. This ability for the model to produce relative ranking of the impact of a variety of shocks on the credit portfolio may be of particular interest to risk managers, who typically perform scenario analyses on a quarterly (or perhaps even more frequent) basis. Moreover, the results of the risk impact analysis offers natural hedging strategies and allow the manager to consider alternative strategies such as reallocation or derivative solutions to managing the largest risks associated with a portfolio.

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**Table 7a**

**Mean (Expected Loss) of Simulated Losses for 1 through 4 Quarters Ahead**

**(in Basis Points of $ Exposure)[1]**

| | -8.02σ U.S. Equity | -5.00σ U.S. Equity | -2.33σ U.S. Equity | -2.33σ SEA Equity | -2.33σ Oil | +2.33σ Japanese Money | Base-line | +2.33σ German Output | +2.33σ Oil | +2.33σ SEA Equity | +2.33σ U.S. Equity |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4Q | 235.2 | 123.3 | 80.0 | 74.2 | 73.3 | 59.3 | **57.8** | 54.7 | 50.9 | 47.4 | 43.3 |
| Q1 | 105.9 | 37.6 | 21.2 | 19.3 | 23.8 | 14.7 | **14.5** | 14.2 | 11.8 | 11.8 | 10.2 |
| Q2 | 51.3 | 28.0 | 17.8 | 16.4 | 16.3 | 13.3 | **12.8** | 12.2 | 11.1 | 10.5 | 9.7 |
| Q3 | 40.8 | 28.7 | 19.7 | 18.4 | 16.5 | 14.9 | **14.5** | 13.4 | 12.8 | 11.9 | 11.0 |
| Q4 | 37.1 | 28.9 | 21.3 | 20.0 | 16.7 | 16.4 | **16.0** | 14.9 | 15.2 | 13.2 | 12.3 |

[1] All simulations were made with 200,000 replications

**Table 7b**

**Standard Deviation (Unexpected Loss) of Simulated Losses for 1 through 4 Quarters Ahead (in Basis Points of $ Exposure)[2]**

| | -8.02σ U.S. Equity | -5.00σ U.S. Equity | -2.33σ U.S. Equity | -2.33σ SEA Equity | -2.33σ Oil | +2.33σ Japanese Money | Base-line | +2.33σ German Output | +2.33σ Oil | +2.33σ SEA Equity | +2.33σ U.S. Equity |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4Q | 304.6 | 205.0 | 143.0 | 130.0 | 116.7 | 107.5 | **106.0** | 99.6 | 99.5 | 89.3 | 80.9 |
| Q1 | 103.2 | 51.9 | 37.4 | 35.0 | 37.7 | 30.9 | **30.8** | 30.6 | 28.8 | 28.0 | 25.9 |
| Q2 | 98.2 | 62.7 | 41.1 | 37.2 | 35.5 | 31.8 | **31.2** | 30.1 | 29.0 | 27.8 | 25.9 |
| Q3 | 84.6 | 71.5 | 53.7 | 49.4 | 42.1 | 41.3 | **40.9** | 38.2 | 38.7 | 34.7 | 31.9 |
| Q4 | 78.4 | 72.2 | 60.2 | 56.8 | 46.2 | 49.1 | **48.6** | 45.5 | 49.7 | 41.7 | 38.8 |

[2] All simulations were made with 200,000 replications.

## Table 8

**Granularity and Firm-specific Risk**

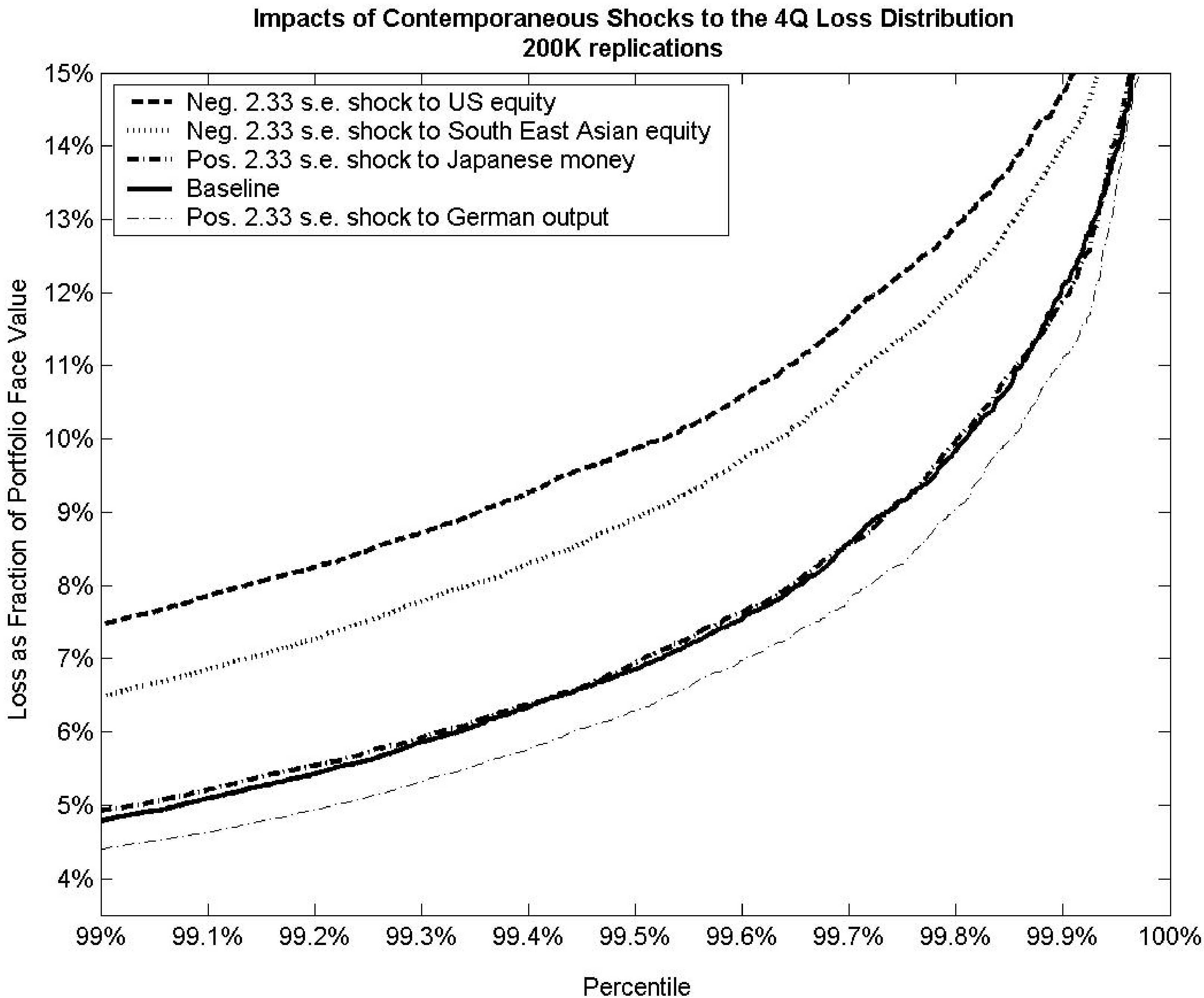
**One Quarter Ahead Losses, Each Firm Copied C Times for a Total of 119×C Exposures**

**(in Basis Points of $ Exposure)**[3]

| | C = 1 | C = 10 | C = 100 |
|---|---|---|---|
| EL | 14.56 | 14.54 | 14.56 |
| UL | 30.83 | 12.59 | 8.85 |
| 99.0% VaR | 134.79 | 55.56 | 46.45 |
| 99.5% VaR | 153.06 | 63.14 | 53.67 |
| 99.9% VaR | 197.10 | 82.81 | 71.31 |

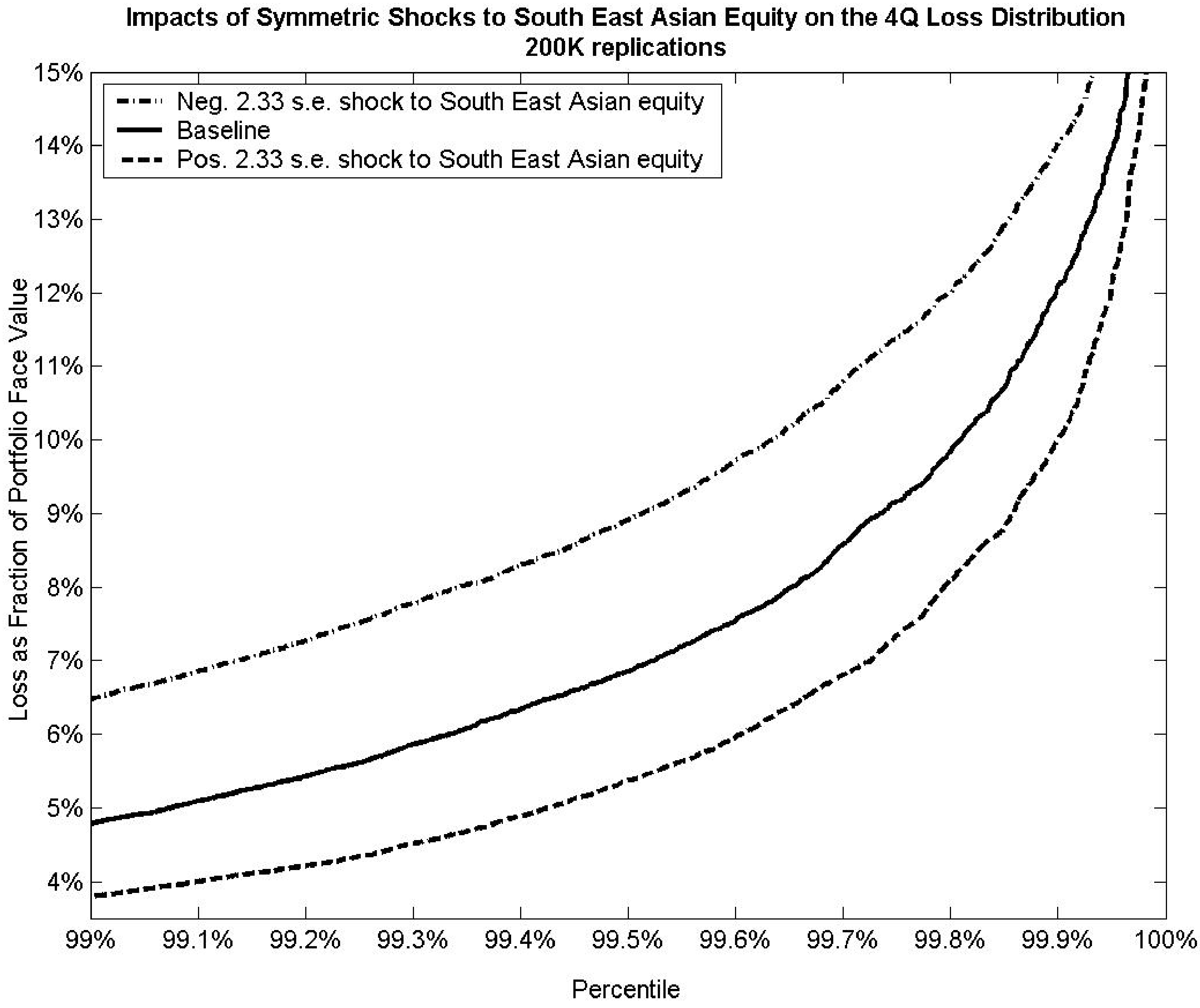
[3] All simulations were made with 200,000 replications. Analytic EL = 14.55bp.

Figure 1



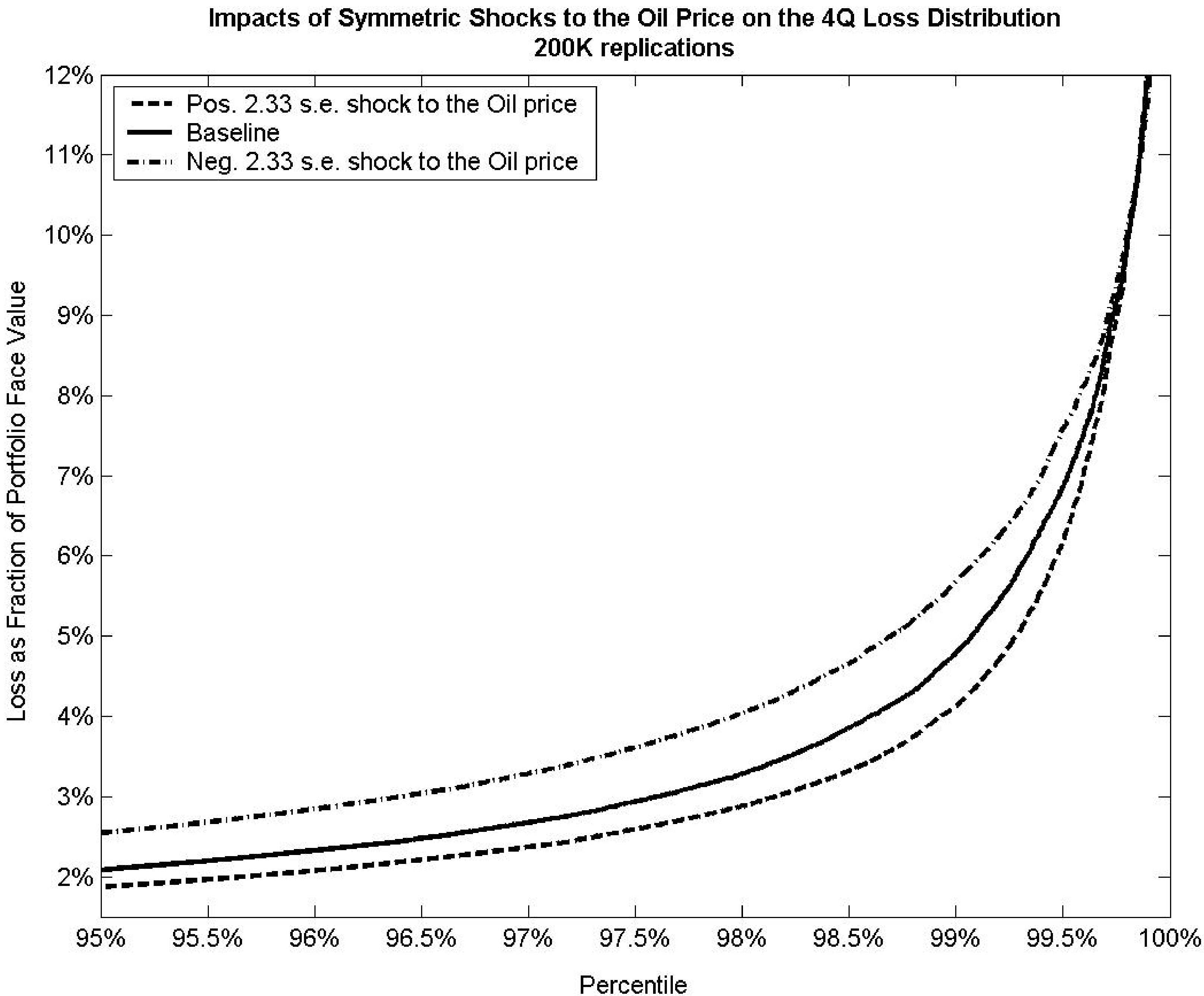
Impacts of Contemporaneous Shocks to the 4Q Loss Distribution
200K replications
Neg. 2.33 s.e. shock to US equity
Neg. 2.33 s.e. shock to South East Asian equity
Pos. 2.33 s.e. shock to Japanese money
Baseline
Pos. 2.33 s.e. shock to German output
Loss as Fraction of Portfolio Face Value
15%
14%
13%
12%
11%
10%
9%
8%
7%
6%
5%
4%
99%
99.1%
99.2%
99.3%
99.4%
99.5%
99.6%
99.7%
99.8%
99.9%
100%
Percentile

Figure 2



Impacts of Symmetric Shocks to South East Asian Equity on the 4Q Loss Distribution
200K replications
Neg. 2.33 s.e. shock to South East Asian equity
Baseline
Pos. 2.33 s.e. shock to South East Asian equity
Loss as Fraction of Portfolio Face Value
15%
14%
13%
12%
11%
10%
9%
8%
7%
6%
5%
4%
99%
99.1%
99.2%
99.3%
99.4%
99.5%
99.6%
99.7%
99.8%
99.9%
100%
Percentile

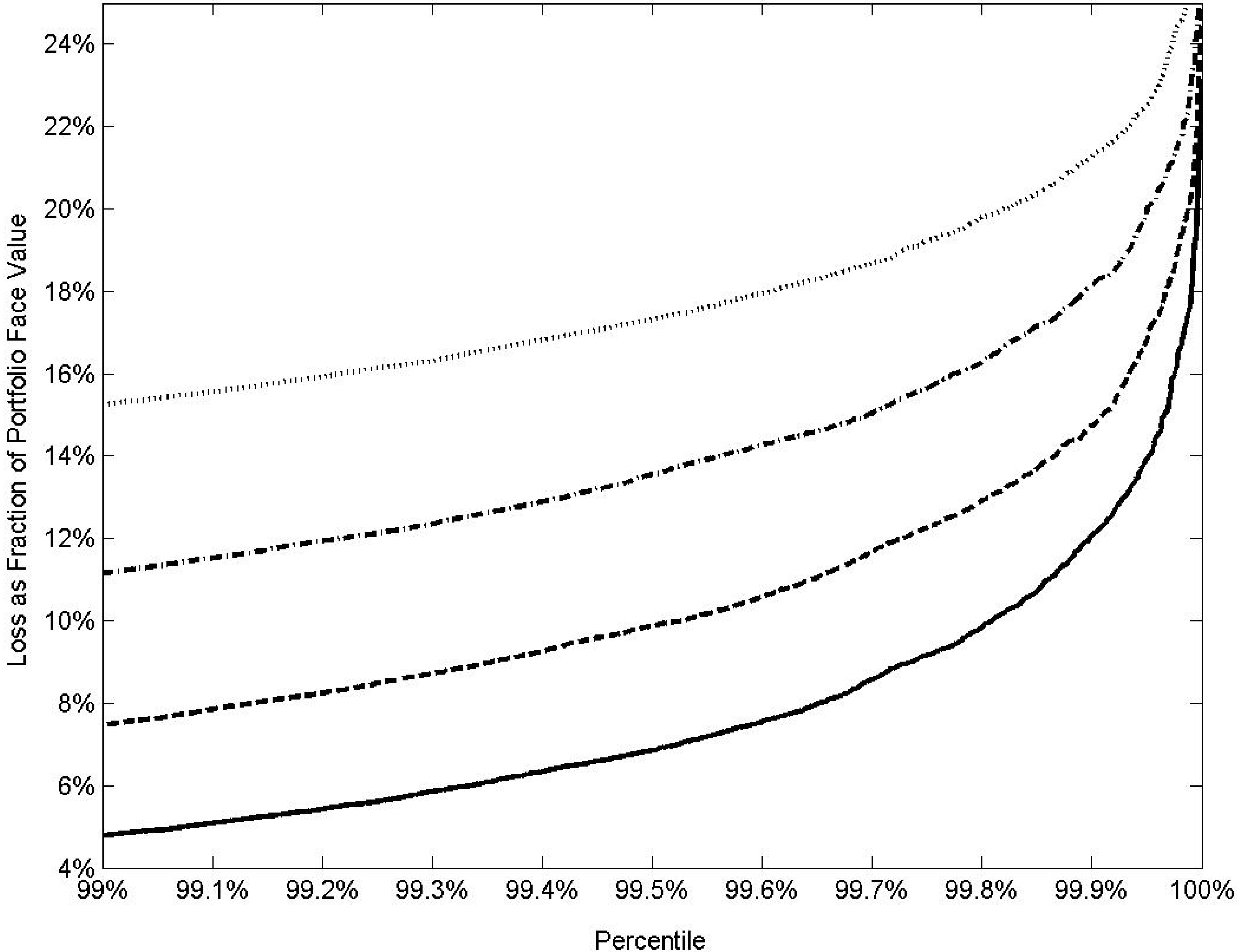
Figure 3



Impacts of Symmetric Shocks to the Oil Price on the 4Q Loss Distribution
200K replications
Pos. 2.33 s.e. shock to the Oil price
Baseline
Neg. 2.33 s.e. shock to the Oil price
Loss as Fraction of Portfolio Face Value
12%
11%
10%
9%
8%
7%
6%
5%
4%
3%
2%
95%
95.5%
96%
96.5%
97%
97.5%
98%
98.5%
99%
99.5%
100%
Percentile

Figure 4

**Impacts of Contemporaneous Shocks to US Equity on the 4Q Loss Distribution**
**200K replications**



# A Supplement: An Overview of the GVAR Framework

This supplement presents a synopsis of the global vector autoregressive model (GVAR) as a generator of global macroeconomic dynamics and scenarios. It gives an overview of the framework underlying the GVAR without going into the details of estimation techniques.[1]

## A.1 Country/Region Specific Models

The GVAR assumes that there are $N+1$ country/regions in the global economy, indexed by $i = 0, 1, \ldots, N$, where 0 is the reference country or region (taken to be the U.S.).[2] Macroeconomic variables of each region are modeled as a function of both their own past and the global economy's current and past state. It is assumed that the regional variables are related to deterministic variables (such as a time trend), foreign variables (which are region-specific weighted averages of the rest of the world) and variables that are taken to be exogenous to this global economy, such as the oil price. We specify the following vector autoregressive form for $k_i$ variables:[3]

$$\begin{aligned} \mathbf{x}_{it} &= \mathbf{a}_{i0} + \mathbf{a}_{i1}t + \mathbf{\Phi}_i\mathbf{x}_{i,t-1} + \mathbf{\Lambda}_{i0}\mathbf{x}^*_{it} + \mathbf{\Lambda}_{i1}\mathbf{x}^*_{i,t-1} + \mathbf{\Psi}_{i0}\mathbf{d}_t + \mathbf{\Psi}_{i1}\mathbf{d}_{t-1} + \boldsymbol{\varepsilon}_{it}, \\ t &= 1, 2, ..., T;\ i = 0, 1, 2, ..., N, \end{aligned} \tag{A.1}$$

where $\mathbf{x}_{it}$ is the $k_i \times 1$ country-specific factors/variables, $\mathbf{a}_{i1}$ is a $k_i \times 1$ vector of linear trend coefficients, $\mathbf{\Phi}_i$ is a $k_i \times k_i$ matrix of associated lagged coefficients, $\mathbf{x}^*_{it}$ is the $k^*_i \times 1$ vector of foreign variables specific to country $i$ (to be defined below) with $\mathbf{\Lambda}_{i0}$ and $\mathbf{\Lambda}_{i1}$ being $k_i \times k^*_i$ matrices of fixed coefficients, $\mathbf{d}_t$ is an $s \times 1$ vector of common global variables assumed to be exogenous to the global economy with $\mathbf{\Psi}_{i0}$ and $\mathbf{\Psi}_{i1}$ being $k_i \times s$ matrices of fixed coefficients, and $\boldsymbol{\varepsilon}_{it}$ is a $k_i \times 1$ vector of country-specific shocks assumed to be serially uncorrelated with a zero mean and a non-singular covariance matrix, $\mathbf{\Sigma}_{ii} = (\sigma_{ii,\ell s})$, where $\sigma_{ii,\ell s} = cov(\varepsilon_{i\ell t}, \varepsilon_{ist})$, or written more compactly

$$\boldsymbol{\varepsilon}_{it} \backsim i.i.d.(\mathbf{0}, \mathbf{\Sigma}_{ii}). \tag{A.2}$$

Although the model is estimated on a regional basis, we allow for the shocks to be correlated across regions. In particular, we assume that

$$\begin{aligned} E\left(\boldsymbol{\varepsilon}_{it}\boldsymbol{\varepsilon}'_{jt'}\right) &= \mathbf{\Sigma}_{ij} \text{ for } t = t', \\ &= \mathbf{0} \text{ for } t \neq t. \end{aligned}$$

Interactions take place through three distinct, but interrelated channels:

1. Direct dependence of $\mathbf{x}_{it}$ on $\mathbf{x}^*_{it}$ and its lagged values.

[1] These can be found in Pesaran, Schuermann and Weiner (2004), hereafter referred to as PSW.

[2] For simplicity we will refer to regions only. For more on country to region aggregation, see PSW.

[3] Although easily extended to incorporate lags greater than one, the GVAR(1) specification given above is seen as sufficient for the illustrative purposes of this paper. Typical values for $k_i$ are 5 or 6.

2. Dependence of the region-specific variables on common global exogenous variables such as oil prices.

3. Non-zero contemporaneous dependence of shocks in region $i$ on the shocks in region $j$, measured via the cross country covariances, $\mathbf{\Sigma}_{ij}$.

The individual models are estimated allowing for unit roots and cointegration assuming that region-specific foreign variables are weakly exogenous, with the exception of the model for the U.S. economy which is treated as a closed economy model.[4] The U.S. model is linked to the outside world through exchange rates themselves being determined in rest of the region-specific models. While models of the form in equation (A.1) are relatively standard, PSW show that the careful construction of the global variables as weighted averages of the other regional variables leads to a simultaneous system of regional equations that may be solved to form a global system. They also provide theoretical arguments as well as empirical evidence in support of the weak exogeniety assumption that allows the region-specific models to be estimated consistently.

## A.2 The Global Model and Multi-step Ahead Forecasts

In view of the contemporaneous dependence of the domestic variables, $\mathbf{x}_{it}$, on the foreign variables, $\mathbf{x}_{it}^*$, the region-specific VAR models (A.1) still need to be solved simultaneously for all the domestic variables, $\mathbf{x}_{it}$, $i = 0, 1, ..., N$. The global solution to the model yields a $k \times 1$ vector $\mathbf{x}_t$, which contains the macroeconomic variables of all regions, such that $\mathbf{x}_t$ is a function of time, the lagged values of all macroeconomic variables $\mathbf{x}_{t-1}$ and the exogenous variables common to all regions (and their lags):

$$\mathbf{x}_t = \mathbf{b}_0 + \mathbf{b}_1 t + \mathit{F}\mathbf{x}_{t-1} + \mathbf{\Upsilon}_0 \mathbf{d}_t + \mathbf{\Upsilon}_1 \mathbf{d}_{t-1} + \mathbf{u}_t, \tag{A.3}$$

$\mathbf{x}_t = (\mathbf{x}_{0t}', \mathbf{x}_{1t}', ..., \mathbf{x}_{Nt}')'$ is the global $k \times 1$ vector, where $k = \sum_{i=0}^{N} k_i$ is the total number of the endogenous variables in the global model, $\mathbf{b}_0$ and $\mathbf{b}_1$ are $k \times 1$ vectors of coefficients, $\mathit{F}$ is a $k \times k$ matrix of coefficients, $\mathbf{d}_t$ is an $s \times 1$ vector of common global variables assumed to be exogenous to the global economy (here to be the oil price) with corresponding $k \times s$ matrices of coefficients, $\mathbf{\Upsilon}_0$ and $\mathbf{\Upsilon}_1$.[5] Finally, $\mathbf{u}_t$ is a $k \times 1$ vectors of (reduced form) shocks that are linear functions of the region-specific shocks ($\boldsymbol{\varepsilon}_{it}$). In particular, we have $\mathbf{u}_t = \mathbf{G}^{-1}\boldsymbol{\varepsilon}_t$, where $\boldsymbol{\varepsilon}_t = (\boldsymbol{\varepsilon}_{0t}', \boldsymbol{\varepsilon}_{1t}', ..., \boldsymbol{\varepsilon}_{Nt}')'$, and the $k \times k$ matrix of coefficients $\mathbf{G}$ is defined in Section 3 of PSW. We also have

$$Var\left(\mathbf{u}_t\right) = \mathbf{G}^{-1}\mathbf{\Sigma}_{\varepsilon}\mathbf{G}'^{-1}, \tag{A.4}$$

where $\mathbf{\Sigma}_{\varepsilon} = Var\left(\boldsymbol{\varepsilon}_t\right)$.

[4] Problems of estimation and testing of cointegrating models with weakly exogenous variables is discussed in Pesaran, Shin and Smith (2000).

[5] The exact relationships between the parameters of the GVAR model in (A.3), and those of the underlying region-specific models (A.1) are given in PSW.

In what follows we assume that the GVAR model is estimated over the period $t = 1, 2, ..., T$, and the objective of the exercise is to generate forecasts, both unconditionally as well as conditional on a particular shock scenario, over the period $t = T+1, ..., T+n$, with $n$ being the forecast horizon. Accordingly, all forecasts and loss distributions at different forecast horizons, $n = 1, 2, ...$, will be conditioned on the state of the economy as characterized by the GVAR model and all the available information at the end of the sample period (i.e. time $T$), namely $\Omega_T = (\mathbf{x}_T, \mathbf{d}_T, \mathbf{x}_{T-1}, \mathbf{d}_{T-1}, ...)$.

For multi-step ahead forecasting and impulse response (or shock scenario) analysis the above solution to the GVAR model needs to be augmented with a model for the common global variables $\mathbf{d}_t$. To this end we adopt the following autoregressive specification

$$\mathbf{d}_t = \boldsymbol{\mu}_d + \boldsymbol{\Phi}_d \mathbf{d}_{t-1} + \boldsymbol{\varepsilon}_{dt}, \text{ for } t = T+1, T+2, ..., T+n, \tag{A.5}$$

where $\boldsymbol{\varepsilon}_{dt} \backsim i.i.d.\ (\mathbf{0}, \boldsymbol{\Sigma}_d)$, which are assumed to be distributed independently of the macroeconomic shocks, $\boldsymbol{\varepsilon}_t$, $t = T+1, T+2, ..., T+n$. We shall assume that all the eigen values of $\boldsymbol{\Phi}_d$ lie on or inside the unit circle and $\Delta\mathbf{d}_t$ is stationary with a constant mean.

For multi-step analysis it is convenient to stack up the macroeconomic (A.3) and global (A.5) equations, and solve out the contemporaneous effect of $\mathbf{d}_t$ on $\mathbf{x}_t$ to yield

$$\mathbf{y}_t = \boldsymbol{\mu} + \boldsymbol{\delta}\ t + \boldsymbol{\Phi}\mathbf{y}_{t-1} + \mathfrak{D}\ \boldsymbol{\upsilon}_t, \tag{A.6}$$

where

$$\mathbf{y}_t = \begin{pmatrix} \mathbf{x}_t \\ \mathbf{d}_t \end{pmatrix},\ \boldsymbol{\mu} = \begin{pmatrix} \mathbf{b}_0 + \boldsymbol{\Upsilon}_0 \boldsymbol{\mu}_d \\ \boldsymbol{\mu}_d \end{pmatrix},\ \boldsymbol{\delta} = \begin{pmatrix} \mathbf{b}_1 \\ \mathbf{0} \end{pmatrix},\ \boldsymbol{\upsilon}_t = \begin{pmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{\varepsilon}_{dt} \end{pmatrix}, \tag{A.7}$$

$$\boldsymbol{\Phi} = \begin{pmatrix} F & \boldsymbol{\Upsilon}_1 + \boldsymbol{\Upsilon}_0 \boldsymbol{\Phi}_d \\ \mathbf{0} & \boldsymbol{\Phi}_d \end{pmatrix}, \text{ and } \mathfrak{D} = \begin{pmatrix} \mathbf{G}^{-1} & \boldsymbol{\Upsilon}_0 \\ \mathbf{0} & \mathbf{I}_s \end{pmatrix}. \tag{A.8}$$

The $(k+s) \times 1$ vector, $\boldsymbol{\upsilon}_t$, augments the region-specific shocks of interest, $\boldsymbol{\varepsilon}_t$, with the common global shocks, $\boldsymbol{\varepsilon}_{dt}$. In the presence of unit root and cointegration it is desirable to ensure that the trend coefficients, $\boldsymbol{\delta}$, are restricted so that the trend characteristics of $\mathbf{y}_t$ are not affected by the number of unit roots in $\boldsymbol{\Phi}$. This is achieved by setting

$$\boldsymbol{\delta} = (\mathbf{I} - \boldsymbol{\Phi})\boldsymbol{\gamma}, \tag{A.9}$$

where $\boldsymbol{\gamma}$ is a vector of unrestricted coefficients.[6] Also, in view of the independence of these shocks we have

$$Var\left(\boldsymbol{\upsilon}_t\right) = \boldsymbol{\Sigma}_{\boldsymbol{\upsilon}} = \begin{pmatrix} \boldsymbol{\Sigma}_\varepsilon & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_d \end{pmatrix}.$$

[6] For further details and discussions see Section 4 in PSW.

Solving the above difference equation forward from $\mathbf{y}_T$, we now obtain

$$\begin{aligned}\mathbf{y}_{T+n} &= \boldsymbol{\Phi}^n \mathbf{y}_T + \sum_{\tau=0}^{n-1} \boldsymbol{\Phi}^\tau \left[\boldsymbol{\mu} + (T+n-\tau)\boldsymbol{\delta}\right] \\ &\quad + \sum_{\tau=0}^{n-1} \boldsymbol{\Phi}^\tau \mathfrak{D} \boldsymbol{\upsilon}_{T+n-\tau}. \end{aligned} \tag{A.10}$$

This solution has three distinct components: The first component, $\boldsymbol{\Phi}^n \mathbf{y}_T$, measures the effect of initial values, $\mathbf{y}_T$, on the future state of the system. The second component captures the deterministic trends embodied in the underlying VAR model. Finally, the last term in (A.10) represents the stochastic (unpredictable) component of $\mathbf{y}_{T+n}$.

As we shall see below, for the purpose of simulating the loss distribution of a given portfolio, the conditional probability distribution of $\Delta \mathbf{y}_{T+n}$ is needed.[7] Using (A.10) and after some algebra we obtain

$$\Delta \mathbf{y}_{T+n} = \left(\boldsymbol{\Phi}^n - \boldsymbol{\Phi}^{n-1}\right) \mathbf{y}_T + \mathfrak{g}(T,n) + \mathcal{U}_{T+n}, \tag{A.11}$$

where

$$\mathfrak{g}(T,n) = \boldsymbol{\Phi}^{n-1} \left[\boldsymbol{\mu} + (T+1)\,\boldsymbol{\delta}\right] + \sum_{\tau=1}^{n-1} \boldsymbol{\Phi}^{\tau-1} \boldsymbol{\delta}, \tag{A.12}$$

and

$$\mathcal{U}_{T+n} = \mathfrak{D} \boldsymbol{\upsilon}_{T+n} + \sum_{\tau=1}^{n-1} \left(\boldsymbol{\Phi}^\tau - \boldsymbol{\Phi}^{\tau-1}\right) \mathfrak{D} \boldsymbol{\upsilon}_{T+n-\tau}. \tag{A.13}$$

Hence

$$E\left(\Delta \mathbf{y}_{T+n} \mid \Omega_T\right) = \left(\boldsymbol{\Phi}^n - \boldsymbol{\Phi}^{n-1}\right) \mathbf{y}_T + \mathfrak{g}(T,n), \tag{A.14}$$

$$Var\left(\Delta \mathbf{y}_{T+n} \mid \Omega_T\right) = \mathfrak{D} \boldsymbol{\Sigma}_\upsilon \mathfrak{D}' + \sum_{\tau=1}^{n-1} \left(\boldsymbol{\Phi}^\tau - \boldsymbol{\Phi}^{\tau-1}\right) \left(\mathfrak{D} \boldsymbol{\Sigma}_\upsilon \mathfrak{D}'\right) \left(\boldsymbol{\Phi}^\tau - \boldsymbol{\Phi}^{\tau-1}\right)'. \tag{A.15}$$

If it is further assumed that the region-specific shocks, $\boldsymbol{\varepsilon}_t$, and the common global shocks, $\boldsymbol{\varepsilon}_{dt}$, are normally distributed, we then have[8]

$$\Delta \mathbf{y}_{T+n} \mid \Omega_T \backsim N\left\{\left(\boldsymbol{\Phi}^n - \boldsymbol{\Phi}^{n-1}\right) \mathbf{y}_T + \mathfrak{g}(T,n),\ \boldsymbol{\Psi}_n\right\}, \tag{A.16}$$

where

$$\boldsymbol{\Psi}_n = \mathfrak{B} + \sum_{\tau=1}^{n-1} \left(\boldsymbol{\Phi}^\tau - \boldsymbol{\Phi}^{\tau-1}\right) \mathfrak{B} \left(\boldsymbol{\Phi}^\tau - \boldsymbol{\Phi}^{\tau-1}\right)', \tag{A.17}$$

and

$$\mathfrak{B} = \mathfrak{D} \boldsymbol{\Sigma}_\upsilon \mathfrak{D}' = \begin{pmatrix} \mathbf{G}^{-1} \boldsymbol{\Sigma} \mathbf{G}'^{-\mathbf{1}} + \boldsymbol{\Upsilon}_0 \boldsymbol{\Sigma}_d \boldsymbol{\Upsilon}_0' & \boldsymbol{\Upsilon}_0 \boldsymbol{\Sigma}_d \\ \boldsymbol{\Sigma}_d \boldsymbol{\Upsilon}_0' & \boldsymbol{\Sigma}_d \end{pmatrix}. \tag{A.18}$$

[7] That is because returns are modeled as being driven by changes in systematic factors in Section 3.

[8] It is also possible to work with non-Gaussian shocks.

Finally, in the present application where the underlying GVAR model admits unit roots and cointegration, the limit distribution of $\Delta \mathbf{y}_{T+n} \mid \Omega_T$ exists and is finite if $\boldsymbol{\delta} = (\mathbf{I} - \boldsymbol{\Phi})\,\boldsymbol{\gamma}$, otherwise $\mathfrak{g}\,(T, n)$ increases without bound as $n \to \infty$. Under $\boldsymbol{\delta} = (\mathbf{I} - \boldsymbol{\Phi})\,\boldsymbol{\gamma}$, using (A.12) we have

$$\mathfrak{g}\,(T, n) = \boldsymbol{\Phi}^{n-1}\boldsymbol{\mu} + (T+1)\left(\boldsymbol{\Phi}^{n-1} - \boldsymbol{\Phi}^{n}\right)\boldsymbol{\gamma} + \left(\mathbf{I} - \boldsymbol{\Phi}^{n-1}\right)\boldsymbol{\gamma}, \tag{A.19}$$

and it is easily seen that

$$\lim_{n\to\infty}\left[\mathfrak{g}\,(T, n)\right] = \boldsymbol{\Phi}^{*}\boldsymbol{\mu} + (\mathbf{I} - \boldsymbol{\Phi}^{*})\,\boldsymbol{\gamma},$$

where $\boldsymbol{\Phi}^{*} = \lim_{n\to\infty}(\boldsymbol{\Phi}^{n})$ is finite under our assumptions. More specifically, if $\boldsymbol{\delta} = (\mathbf{I} - \boldsymbol{\Phi})\,\boldsymbol{\gamma}$ we have

$$\lim_{n\to\infty} \Delta \mathbf{y}_{T+n} \mid \Omega_T \backsim N\left\{\boldsymbol{\Phi}^{*}\boldsymbol{\mu} + (\mathbf{I} - \boldsymbol{\Phi}^{*})\,\boldsymbol{\gamma},\ \boldsymbol{\Psi}^{*}\right\},$$

where[9]

$$\boldsymbol{\Psi}^{*} = \mathfrak{B} + \sum_{\tau=1}^{\infty}\left(\boldsymbol{\Phi}^{\tau} - \boldsymbol{\Phi}^{\tau-1}\right)\mathfrak{B}\left(\boldsymbol{\Phi}^{\tau} - \boldsymbol{\Phi}^{\tau-1}\right)'.$$

Therefore, as argued in Section 4 of PSW, it is important that the GVAR model is estimated subject to the restrictions, $\mathbf{b}_1 = (\mathbf{I} - F)\boldsymbol{\gamma}_1$, which in conjunction with the model for the common global variables, (A.5), ensure that $\boldsymbol{\delta} = (\mathbf{I} - \boldsymbol{\Phi})\,\boldsymbol{\gamma}$.

In summary, the GVAR's sequential regional estimation and global aggregation methodology allows for the practitioner to solve for the conditional distribution of the macroeconomic factors globally, whereas single-stage estimation of the global system in equation (A.3) would be prohibitive due to the very large number of coefficients and generally thin data sets. As a result, the model allows us to examine the effects of a shock in one region on the macroeconomic factors that describe the system globally, as our discussion of impulse response functions below shows.

# B Simulation of Multi-period Returns

Using (23), we have

$$R_{ji,T+\kappa} = \sum_{\tau=1}^{\kappa} r_{ji,T+\tau} = \sum_{\tau=1}^{\kappa} \mu_{ji,T+\tau|T} + \xi_{ji,T+\tau},\ \text{for } \kappa = 1, 2, ..., H, \tag{A.20}$$

where

$$\begin{aligned} \mu_{ji,T+\kappa|T} &= \alpha_{ji} + \Gamma_{ji}'\left[\left(\boldsymbol{\Phi}^{\kappa} - \boldsymbol{\Phi}^{\kappa-1}\right)\mathbf{y}_T + \mathfrak{g}\,(T, \kappa)\right], \\ \xi_{ji,T+\kappa} &= \Gamma_{ji}'\mathcal{U}_{T+\kappa} + \eta_{ji,T+\kappa}, \end{aligned} \tag{A.21}$$

$$\mathcal{U}_{T+\kappa} = \mathfrak{D}\boldsymbol{\upsilon}_{T+\kappa} + \sum_{\tau=1}^{\kappa-1}\left(\boldsymbol{\Phi}^{\tau} - \boldsymbol{\Phi}^{\tau-1}\right)\mathfrak{D}\boldsymbol{\upsilon}_{T+\kappa-\tau}.$$

[9] Notice that all the elements of $\left(\boldsymbol{\Phi}^{\tau} - \boldsymbol{\Phi}^{\tau-1}\right)$ decay exponentially with $\tau$ even under unit roots and hence $\boldsymbol{\Psi}^{*}$ exists and is finite.

and $\mathfrak{g}(T,n)$ is given by (A.19). It is clear that at time $T$, the conditional mean returns, $\mu_{ji,T+\kappa|T}$, $\kappa = 1,2,...,H$, are known insofar as they are forecast by the GVAR. It is also easily seen that the unpredictable components of the returns over the different horizons have the following recursive structure: Given forecasts from the GVAR model,

$$\begin{aligned}
\xi_{ji,T+1} &= \Gamma'_{ji}\mathcal{H}_0\mathfrak{D}\boldsymbol{\upsilon}_{T+1} + \eta_{ji,T+1}, \\
\xi_{ji,T+2} &= \Gamma'_{ji}\mathcal{H}_1\mathfrak{D}\boldsymbol{\upsilon}_{T+1} + \Gamma'_{ji}\mathcal{H}_0\mathfrak{D}\boldsymbol{\upsilon}_{T+2} + \eta_{ji,T+2}, \\
&\vdots \\
\xi_{ji,T+H} &= \Gamma'_{ji}\mathcal{H}_{H-1}\mathfrak{D}\boldsymbol{\upsilon}_{T+1} + \Gamma'_{ji}\mathcal{H}_{H-2}\mathfrak{D}\boldsymbol{\upsilon}_{T+2} + .... + \Gamma'_{ji}\mathcal{H}_0\mathfrak{D}\boldsymbol{\upsilon}_{T+H} + \eta_{ji,T+H},
\end{aligned}$$

where

$$\mathcal{H}_\kappa = \mathbf{\Phi}^\kappa - \mathbf{\Phi}^{\kappa-1},\ \kappa = 1,2,...,H \text{ and } \mathcal{H}_0 = \mathbf{I}_{k+s}.$$

Recall that the matrix $\mathbf{\Phi}$ collects all the GVAR coefficients other than constants and trends and thus characterizes the effect of initial values $\mathbf{y}_T$ on the future state of the macroeconomic system as given by (A.6). Therefore, the conditional distribution of the returns across the different forecast horizons are correlated, and in the simulation of the loss distribution one needs to draw from the *joint* distribution of $\mathbf{r}_{ji}(H) = (r_{ji,T+1}, r_{ji,T+2}, ..., r_{ji,T+H})'$. For this purpose we note that $\xi_{ji,T+\kappa}$, $\kappa = 1,2,..,H$, have zero means and a variance covariance matrix $Var(\mathbf{r}_{ji}(H))$ whose $(\mathfrak{w},\mathfrak{n})$ element is given by

$$\begin{aligned}
\Gamma'_{ji}\left(\sum_{\tau=1}^{\mathfrak{m}}\mathcal{H}_{\mathfrak{w}-\tau}\mathfrak{B}\mathcal{H}'_{\mathfrak{w}-\tau}\right)\Gamma_{ji} + \omega^2_{\eta,ji},\ \text{if } \mathfrak{w} &= \mathfrak{n}, \\
\Gamma'_{ji}\left(\sum_{\tau=1}^{Min(\mathfrak{m},\mathfrak{n})}\mathcal{H}_{\mathfrak{m}-\tau}\mathfrak{B}\mathcal{H}'_{\mathfrak{n}-\tau}\right)\Gamma_{ji}, \qquad \text{if } \mathfrak{w} &\neq \mathfrak{n},
\end{aligned}$$

where $\mathfrak{B} = \mathfrak{D}\mathbf{\Sigma}_\upsilon\mathfrak{D}'$.

Alternatively, when the shocks are Gaussian the returns can be simulated using the relations

$$r^{(r)}_{ji,T+\kappa} = \mu_{ji,T+\kappa|T} + \xi^{(r)}_{ji,T+\kappa},\ \text{for } \kappa = 1,2,...,H, \tag{A.22}$$

where

$$\xi^{(r)}_{ji,T+\kappa} = \sum_{\tau=0}^{\kappa-1}\left(\Gamma'_{ji}\mathcal{H}_\tau\mathfrak{B}\mathcal{H}'_\tau\Gamma_{ji}\right)^{1/2}Z^{(r)}_\tau + \omega_{\eta,ji}\ Z^{(r)}_{ij\kappa}, \tag{A.23}$$

where $Z^{(r)}_0, Z^{(r)}_1, ..., Z^{(r)}_{H-1}; Z^{(r)}_{ij1}, Z^{(r)}_{ij2}, ..., Z^{(r)}_{ijH}$ are independent draws from $IID\ N(0,1)$ for all $i$ and $j$.

# C Shock Scenario Analysis through GIRFs

For policy analysis, one would like to be able to examine how an isolated contemporaneous shock to one macroeconomic variable affects all other macroeconomic variables in the global economy.

For example, it might be of interest to determine the effects of a contemporaneous 10% drop in the Japanese equity prices on other macroeconomic variables, and the effects that these have on the credit risk of a given portfolio. Impulse response functions provide us with the tools to carry out this type of analysis. In so doing, it is of course important that the correlations that exists across the different shocks, both within and across regions, are properly taken into accounted. However, in a model which consists only of regional VAR's (as in equation (A.1)) which are not integrated as in the GVAR, it is impossible to uncover these effects because the interdependencies within regions are lost. On the other hand, single-stage estimation of the global model (A.3) is extremely difficult, and even if it were possible (and consistent), it would be impossible to construct a regional shock (a shock to $\boldsymbol{\varepsilon}_{it}$) within the context of such a global model. Only with the GVAR can both of these challenges be adequately addressed.

In the traditional VAR literature this is accomplished by means of the orthogonalized impulse responses (OIR) à la Sims (1980), where impulse responses are computed with respect to a set of orthogonalized shocks, say $\boldsymbol{\xi}_t$, instead of the original shocks, $\boldsymbol{\varepsilon}_t$. The link between the two sets of shocks are given by

$$\boldsymbol{\xi}_t = \mathbf{P}^{-1}\boldsymbol{\varepsilon}_t,$$

where $\mathbf{P}$ is a $k \times k$ lower triangular Cholesky factor of the variance covariance matrix, $Var(\boldsymbol{\varepsilon}_t) = \boldsymbol{\Sigma}_\varepsilon$, namely

$$\mathbf{P}\mathbf{P}' = \boldsymbol{\Sigma}_\varepsilon.$$

Therefore, by construction $E(\boldsymbol{\xi}_t\boldsymbol{\xi}_t') = \mathbf{I}_k$. However, the drawback of using OIR is that the outcome is dependent on the order of the variables.[10] Koop, Pesaran and Potter (1996) and Pesaran and Shin (1998) have developed an approach which is invariant to the order of the variables, known as the generalized impulse response function (GIRF). The GIRF can be applied to region-specific shocks as well as to the common global shocks. For example, if factor $\ell$ in country $i$ is (purposefully) shocked by one standard error (i.e. $\sqrt{\sigma_{ii,\ell\ell}}$) in the period from $T$ to $T+1$, the GIRF of $\mathbf{y}_{T+n}$ is given by

$$\boldsymbol{\psi}_{i\ell}(\mathbf{y}, n) = E\left(\mathbf{y}_{T+n} \mid \Omega_T,\ \varepsilon_{i,T+1,\ell} = \sqrt{\sigma_{ii,\ell\ell}}\right) - E\left(\mathbf{y}_{T+n} \mid \Omega_T\right).$$

The first term captures the expected effect of the shock, while the second term represents the baseline scenario in the absence of the shock. In the case of the GVAR model, using (A.10) we have

$$\boldsymbol{\psi}_{i\ell}(\mathbf{y}, n) = \boldsymbol{\Phi}^{n-1}\mathfrak{D}E\left(\boldsymbol{\upsilon}_{T+1} \mid \Omega_T,\ \varepsilon_{i,T+1,\ell} = \sqrt{\sigma_{ii,\ell\ell}}\right),$$

which yields

$$\boldsymbol{\psi}_{i\ell}(\mathbf{y}, n) = \frac{1}{\sqrt{\sigma_{ii,\ell\ell}}}\boldsymbol{\Phi}^{n-1}\mathfrak{D}\boldsymbol{\Sigma}_{\boldsymbol{\upsilon}}\mathfrak{s}_{i\ell},\ n = 1, 2, ..., \tag{A.24}$$

[10] This is due to the non-uniqueness of the Cholesky decomposition. While OIR are suitable for low-dimensional models where variables can be arranged in causal order, they are not suitable for large dimensional GVAR models.

where $\mathfrak{s}_{i\ell}$ is a $(k+s) \times 1$ selection vector with its element corresponding to the $\ell^{th}$ variable in country $i$ being unity and zeros elsewhere. A similar expression can also be derived for the effect of shocking one of the common global variables by an appropriate choice of the selection vector, $\mathfrak{s}$, and by replacing $\sqrt{\sigma_{ii,\ell\ell}}$ with the one standard error of the common global variable being shocked.[11]

The GIRF of the changes in the $n$-period ahead forecast, $\Delta \mathbf{y}_{T+n}$, can also be derived directly using (A.11) and is given by

$$\begin{aligned} \boldsymbol{\psi}_{i\ell}(\Delta \mathbf{y}, n) &= \frac{1}{\sqrt{\sigma_{ii,\ell\ell}}} \mathfrak{D} \boldsymbol{\Sigma}_{\boldsymbol{\upsilon}} \mathfrak{s}_{i\ell} \text{ for } n = 1, \\ &= \frac{1}{\sqrt{\sigma_{ii,\ell\ell}}} \left( \boldsymbol{\Phi}^{n-1} - \boldsymbol{\Phi}^{n-2} \right) \mathfrak{D} \boldsymbol{\Sigma}_{\boldsymbol{\upsilon}} \mathfrak{s}_{i\ell}, \text{ for } n = 2, 3, .. \end{aligned} \tag{A.25}$$

Clearly, on impact (for $n = 1$), $\boldsymbol{\psi}_{i\ell}(\mathbf{y}, n) = \boldsymbol{\psi}_{i\ell}(\Delta \mathbf{y}, n)$, but the two impulse response functions deviate at higher order horizons.

Finally, to analyze the impact of shock scenarios on the loss distribution, we also need to consider the effect of region-specific and common global shocks on the whole probability distribution function of $\Delta \mathbf{y}_{T+n}$ conditional on $\Omega_T$. For this purpose we assume that the magnitude and the nature of the shock is not such as to alter the probability distribution function of $\boldsymbol{\upsilon}_{T+1}$, and distinguish between the cases where the change in $\varepsilon_{i,T+1,\ell}$ is pre-announced or anticipated, as compared to the case where the change is unanticipated. The former could be relevant in the case of policy announcements such as specific tax changes or general changes to the monetary policy. But for risk analysis unanticipated forms of shocks seem more relevant. Assuming that the errors, $\boldsymbol{\upsilon}_{T+1}$, are distributed as multivariate normal (even after the system is hit by the shock), the probability distribution in the presence of an unanticipated unit shock to $\ell^{th}$ factor in country $i$ is given by

$$\Delta \mathbf{y}_{T+n} \mid \Omega_T, \varepsilon_{i,T+1,\ell} = \sqrt{\sigma_{ii,\ell\ell}} \backsim N\left( \left( \boldsymbol{\Phi}^n - \boldsymbol{\Phi}^{n-1} \right) \mathbf{y}_T + \mathfrak{g}(T, n) + \boldsymbol{\psi}_{i\ell}(\Delta \mathbf{y}, n),\ \boldsymbol{\Psi}_n \right), \tag{A.26}$$

where $\boldsymbol{\psi}_{i\ell}(\Delta \mathbf{y}, n)$, and $\boldsymbol{\Psi}_n$ are defined by (A.24) and (A.17). Here we are assuming that the shock, if unanticipated, does not change the conditional covariance matrix of $\boldsymbol{\upsilon}_{T+1}$.[12]

When the shock is anticipated its variance as well as its covariances with the other components of $\boldsymbol{\upsilon}_{T+1}$ will be zero on impact and we have

$$\Delta \mathbf{y}_{T+n} \mid \Omega_T, \varepsilon_{i,T+1,\ell} = \sqrt{\sigma_{ii,\ell\ell}} \backsim N\left( \left( \boldsymbol{\Phi}^n - \boldsymbol{\Phi}^{n-1} \right) \mathbf{y}_T + \mathfrak{g}(T, n) + \boldsymbol{\psi}_{i\ell}(\Delta \mathbf{y}, n),\ \boldsymbol{\Psi}_{n,i\ell} \right), \tag{A.27}$$

where

$$\begin{aligned} \boldsymbol{\Psi}_{n,i\ell} &= \mathfrak{B}_{i\ell}, \qquad \text{for } n = 1, \\ \boldsymbol{\Psi}_{n,i\ell} &= \mathfrak{B}_{i\ell} + \sum_{\tau=1}^{n-1} \left( \boldsymbol{\Phi}^{\tau} - \boldsymbol{\Phi}^{\tau-1} \right) \mathfrak{B} \left( \boldsymbol{\Phi}^{\tau} - \boldsymbol{\Phi}^{\tau-1} \right)', \text{ for } n = 2, 3, ... \end{aligned} \tag{A.28}$$

[11] The GIRF are identical to the orthogonalized impulse response function only when $\boldsymbol{\Sigma}_{\boldsymbol{\upsilon}}$ is diagonal and/or when the focus of the analysis is on the impulse response function of shocking the first element of $\boldsymbol{\upsilon}_t$. See Pesaran and Shin (1998).

[12] In principle it is possible to allow for simultaneous mean and variance change, for example, by adopting GARCH-in-mean type models where conditional variance is assumed to be depend on the conditional mean of the errors.

$$\mathfrak{B}_{i\ell} = \mathfrak{D}\left[\boldsymbol{\Sigma}_{\upsilon} - \boldsymbol{\Sigma}_{\upsilon}\mathfrak{s}_{i\ell}\left(\mathfrak{s}'_{i\ell}\boldsymbol{\Sigma}_{\upsilon}\mathfrak{s}_{i\ell}\right)^{-1}\mathfrak{s}'_{i\ell}\boldsymbol{\Sigma}_{\upsilon}\right]\mathfrak{D}'. \tag{A.29}$$

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