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Scenario modeling of selective hedging strategies*

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Abstract

We study currency risk management in the context of scenario analysis. We develop scenario-based optimization models that jointly determine the portfolio composition and the hedging strategy within each currency. Thus the model prescribes optimal selective hedging policies. We then study empirically the performance of the models. The new elements of our empirical analysis are: various horizons (one month and one semester), various currency bases, explicit incorporation of realistic transaction costs. The results show that transaction costs are very important in determining the profitability of various currency risk management strategies for both stocks and bonds at the one month horizon.

1 Introduction

The possibilities of achieving international diversification of financial portfolios has increased at a rapid rate in the last years, even though many investors do not seem to take full advantage of this, see French and Poterba [11]. If the trend continues in the future we should observe an even larger share of financial investments going towards assets denominated in foreign currencies. Internazionalization of consumption baskets is proceeding at a much slower rate. Most people use a very large share of their income for purchasing goods denominated in the domestic currency. The increasing divergence between the currency denomination of consumption and that of financial wealth underscores the importance of exchange rate risk.

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Single investors have no control of the various exchange rate regimes prevailing at the macroeconomic level. There is no clear tendency to fix exchange rates at the world level, even though recurrent periods of turbulence have motivated various discussions about international target zones among the major currencies. There are efforts to stabilize exchange rates among economic blocks, the most noticeable being the decision of the European Monetary Union to permanently lock the exchange rates by the creation of a European common currency. However there are no similar efforts to fix exchange rates among economic blocks and the efforts on the part of central banks to affect exchange rates are of dubious effects both in the short run and in the long run. For the foreseeable future investors will therefore be left with the difficult task of managing exchange rate risk, see Gruber [14].

The task is complicated by the fact that exchange rates have many roles in an internationally diversified portfolio. Their expected rate of change is a component of the expected return of the various securities denominated in domestic currency. Also, their volatility and correlations are key to determining the overall volatility of the returns of the portfolio. While volatility of exchange rates certainly increases volatility of the portfolio, comovements between exchange rates and asset returns denominated in foreign currency may decrease the volatility of the overall portfolio.

The problem of managing exchange rate risk is made even more complicated by the difficulties connected with understanding the fluctuations of exchange rates, see Frankel and Rose [10] for a general overview. There is a large agreement that a random walk is the best descriptor of exchange rate dynamics and that no structural macroeconometric models can predict better than a random walk, see Meese and Rogoff [18]. A long run perspective does not help much to the extent that even considering a few decades of data does not allow researchers to find clear and stable relations with macroeconomic variables. Basic relations between exchange rates and variables like price levels and interest rates certainly hold better as historical averages than at a monthly horizon, so that the bulk of exchange rates fluctuations is still unexplained. Much research has been produced recently about the short run volatility dynamics of various exchange rates. Various statistical models have been fairly successful at explaining the short run dynamics of volatility, which seems to be well approximated by varieties of the GARCH class. The relation between current and past volatility is strong at the daily horizon and decreases, roughly in line with the theoretical decay properties of GARCH models, at weekly and monthly horizons, see e.g. Baillie and Bollerslev [2].

In the light of the previous considerations, it is no surprise that an extensive body of literature has considered the merit of hedging exchange rate risk. In what follows we reconsider the problem with certain innovations regarding the methodologies, the perspective used in tackling the problem, and the data. As to methodology, we employ for the first time in the exchange rate hedging literature a scenario-based optimization methodology rather than a classic mean variance approach. This is helpful because it does not restrict the probability function to be normal. We determine possible scenarios for returns and exchange rates and feed them into an optimizer to find the optimal portfolios. Moreover, in our dynamic analysis we explicitly account for transaction costs related to portfolio modifications, a factor which turns out to be significant for the empirical results. As to the perspective, we distinguish different types of investors based on various characteristics like their risk aversion, their domestic currency and their time horizon. Both elements are important for determining the frontier of efficient portfolios and the effectiveness of the forecasting models.

After reviewing the principal results obtained with static analysis, we concentrate on a dynamic analysis of portfolio formation and evaluation over time. The empirical analysis considers a data set for stocks and bonds starting in January 1985 and ending in May 1998. The assets that we consider are denominated in Swiss francs, Deutsche mark, US dollars, British pounds and Japanese yen. Our analysis promises therefore to be relevant even in the face of the introduction of the euro if one assumes that the new European currency will inherit the probability distribution properties of the Deutsche mark.

The plan of the paper is as follows. In section 2 we briefly survey the literature on optimal hedging and discuss a few results about the statistical behavior of returns and exchange rates which are crucial to understanding some of the methods and results used in our empirical analysis. In section 3 we present our scenario-based model for selective hedging. Section 4 presents the results and section 5 concludes. To give a quick preview of our achievements, we find that while our results for the static analyses are very similar to those reported in the literature, those for the dynamic strategies are quite different. First, optimal policy is strongly dependent on the base currency, for example complete hedging is always optimal for a Swiss franc investor but not for a US dollar investor. Second, selective hedging of passive portfolios is generally better even for a US investor. Third, choice of hedging strategy is very sensitive to market data, and therefore the inclusion of transaction costs lead to substantially different results from the case where transaction costs are ignored.

2 Selective hedging

The literature presents different views as to the optimal course of action for international portfolio managers. The key elements of the various analyses are: the distinction between static and dynamic strategies, the base currency, the portfolio structure and the hedging policy. Among these, the distinction between static and dynamic strategies is particularly important in light of the empirical analysis carried out in this paper. As a first approximation one can regard the former as being relevant for strategic (long run) portfolio selection and the latter as being relevant for tactical (short run) asset allocation. As to the hedging policy, three major strategies have been discussed in the literature: unitary, partial and selective hedging. Unitary hedging means that there is either complete or no hedging of the exchange rate risk connected with all the assets denominated in foreign currency. In partial hedging, the hedge ratio can be different from zero or one but uniformly across assets. Selective hedging is the more general approach because the hedge ratio may change across currencies and can take any value between zero and one.

Perold and Schulman [20] point out that complete hedging is optimal due to the low impact of currency hedging on expected returns and the substantial reduction in volatility. Their work created the idea that hedging could be a "free lunch". It can be shown that in a mean-variance world, complete hedging is optimal when uncovered interest rate parity holds and there is no covariance between the rate of change of the exchange rate and the rates of return of the various assets denominated in foreign currency. The first condition implies equality between the expected return from hedged and unhedged assets; the second implies that exchange rate fluctuations strictly increase the volatility of the portfolio. As a consequence hedging decreases volatility at no expected penalty.

The merit of the different prescriptions is an empirical issue. Solnik [23] analyzed the period 1971-1996 from the point of view of a Swiss investor. He found that, first, average unhedged returns are very close to average fully hedged returns, for indices of both stocks and bonds, and, second, that the standard deviation of hedged returns is marginally lower (about 3-4 percentage points in most cases) than the standard deviation of unhedged returns in the case of stock indices. Third he found that the standard deviation of hedged returns is significantly lower (about 8-10 percentage points in most cases) than the standard deviation of unhedged returns in the case of bond indices. The interpretation of these results is that currency risk is significant in the short run, mainly for bonds, while the cost of hedging is insignificant in the long run. These findings might seem to support the position according to which currency hedging is optimal for bond portfolios, even though a proper answer would require a specification of the objective function of the investor, including his time horizon and the amount of risk he can accept in the short run, and an analysis of the various covariances.

Filatov and Rappoport [10] show that complete hedging is dominated by selective hedging. Moreover they show (assuming the validity of uncovered interest rate parity) that the signs of covariances between exchange rates and domestic returns have historically been different across base currencies. For example complete hedging was optimal for a US investor for the period 1980-1989 while selective hedging was optimal for a non-US investor over the same period. For example a US investor looking for the minimum variance bond portfolio should have been 100% invested in foreign bonds with complete hedging, while a German or Japanese investor should have been largely invested in domestic bonds (about 80% of the total) with no hedging. A British investor should have allocated 30% to domestic bonds and should have hedged 76% of the foreign investment. Therefore the optimal hedging strategy is in this context dependent on the currency of denomination of the investor and on the composition of the optimal portfolio, challenging the theory proposed by Black [4] which suggests the existence of a universal hedge ratio that is optimal for all the investors. The general relevance of the results of Filatov and Rappoport [10] may be dubious given the lack of diversification of the portfolios considered by the authors.

The importance of the static analysis for deriving implications for strategic asset allocation is implicitly called into question by results obtained by other researchers on the temporal stability of the efficient frontiers. Jorion [15] finds that international diversification of stocks and bonds is highly beneficial to portfolio returns, but the optimal hedging strategy is highly time-dependent. Abken and Shrikhande [1] confirm that the efficient frontier for a US investor choosing stocks and bonds of seven industrialized countries is unstable across portfolios and across periods. Hedging was the best policy for equities over the period 1980-1985 while no hedging was optimal over the periods 1986-1990, 1991-1996 and for the overall 1986-1996. In contrast, the two policies are complementary in the case of bond portfolios, as the hedged frontier usually covers a risk-return space that is not covered by the unhedged frontier. The optimal policy is therefore a function of the degree of risk aversion of the investor. A conservative investor should hedge while an aggressive investor should not. Abken and Shrikhande [1] however do not report results on joint maximization across stocks and bonds and consider only unitary hedging instead of the more flexible selective hedging. In the empirical section we will provide more evidence on these issues.

Glen and Jorion [12] present a more extensive empirical analysis, whose main findings for the period 1974-1990 are as follows. First, selective hedging improves upon the results obtained by unhedged, hedged and universally hedged optimized portfolios of only-bond and bond-stock while it does not improve upon the results obtained by unhedged, hedged and universally hedged optimized portfolios of stocks. This is consistent with Solnik [23] and Filatov and Rappoport [10] and confirms that the effects of the hedging policy are more relevant for bonds than for stocks. Second, selective hedging does not improve upon the results obtained by unhedged, hedged and partially hedged passive indices of stocks, bonds and stocks and bonds, pointing out the importance of the interaction between optimal hedging and overall portfolio composition.

Glen and Jorion [12] also consider dynamic hedging strategies, which are of particular interest in view of the evidence reported by Abken and Shrikhande about temporal instability of the optimal policy. Such an instability means on the one hand that historical evidence may not give a consistent guideline about the strategic hedging choice and on the other hand that large benefits could be obtained by following a state-contingent tactical policy which varies the level and the structure of hedging depending on the investment opportunity set. Glen and Jorion [12] find that selective hedging is always better than either complete hedging or no hedging or partial hedging for passive indices, with gains of approximately 3%, which are however statistically significant only for the actively hedged bond portfolio against the unhedged (not against the totally hedged) bond portfolio. Notice the contrast with the results from the static analysis, showing that hedging was not helpful in the case of passive indices.

The main results of this large literature may perhaps be summed up as follows. Selective hedging in the context of overall portfolio optimization is relevant mainly for bonds. Static selective hedging of otherwise passive indices is not very relevant, confirming the key role of the interactions between exchange rates and bond returns. Efficient frontiers are unstable and may provide little guidance to future applications of various hedging strategies. There is scant evidence about dynamic hedging policies, which however seem to be effective even in the context of passive portfolios.

3 A scenario-based model for selective hedging

A central issue in any portfolio selection model is the modelling of the uncertain returns of the investment alternatives. Usually this issue is addressed defining either the expected return and the variance of the various assets or a set of possible realizations – scenarios – of the random variables, e.g. Chapter 13 in Censor and Zenios [7]. In the latter case the scenarios can be generated by either a model – see e.g. Mulvey [19] – or past returns, e.g. Grauer and Hakansson [13], or experts' opinions, e.g. Koskosides and Duarte [17]. In what follows we use a scenario-based portfolio selection model, where past returns are employed as equiprobable scenarios. As a risk measure we employ the mean absolute deviation with respect to the expected portfolio return, see Konno and Yamazaki [16].

We use the following notation

I	Set of unhedged markets, $I = \{1, \dots, i, \dots, N\}$,
J	Set of hedged markets, $J = \{1, \dots, j, \dots, M\}$,
K	Set of all markets, $K = \{1, \dots, N, N+1, \dots, N+M\}$,
Ω	Set of scenarios, $\Omega = \{1, \dots, s, \dots, S\}$,
u_i	Position in unhedged market i (in percentage of total wealth),
h_j	Position in hedged market j, (in percentage of total wealth),
r_i^s	Domestic currency return of asset i under scenario s in local currency,
e_i^s	Currency return of asset i under scenario s,
f_j^s	Forward return of asset j under scenario s.
x_k	Position in market k
ξ_k^s	Scenario dependent returns of the various assets

The mean-absolute deviation (MAD) portfolio selection model can be written as:

$$\text{Minimize} \qquad \frac{1}{S}\sum_{s=1}^{S}\left|\sum_{k=1}^{N+M}\xi_k^s x_k - \left(\frac{1}{S}\sum_{s=1}^{S}\sum_{k=1}^{N+M}\xi_k^s x_k\right)\right| \tag{1}$$

$$\frac{1}{S}\sum_{s=1}^{S}\sum_{k=1}^{N+M}\xi_k^s x_k \geq \rho \tag{2}$$

$$\sum_{k=1}^{N+M} x_k = 1 \tag{3}$$

$$x_k \geq 0 \qquad \text{for all } k \in K \tag{4}$$

The decision variables of the problem are the portfolio weights u_i invested in the unhedged markets and the portfolio weights h_j invested in the hedged markets, which are summarized in the decision vector $\mathbf{x}$ which is the concatenation of the unhedged and hedged vectors, i.e. $\mathbf{x}' = (\; u' \quad h' \;)'$. The problem, as expressed by equation (1), is to find a portfolio that minimizes the mean absolute deviation, over all scenarios, with respect to the portfolio expected return. Equation (2) requires the expected portfolio return to be greater than a user-specified target level ρ. Varying this level will generate the efficient frontiers. Equations (3) imposes the portfolio weights to sum up to one, while equation (4) forces the portfolio holdings to be nonnegative, disallowing short sales. The scenario-dependent returns of the various assets, ξ_k^s, are defined as follows: for the unhedged markets I the return under scenario s is given by $\xi_i^s = r_i^s + e_i^s$, for all $i \in I$, whereas for the hedged markets J, the return under scenario s is given by $\xi_j^s = r_j^s + f_j^s$, for all $j \in J$. The forward return under scenario s is $f_j^s = \ln\left[(1 + r_d^s) / \left(1 + r_f^s\right)\right]$, with r_d^s and r_f^s representing the domestic, respectively the foreign, one month interest rate under scenario s.

The optimization structure will in general suggest a different hedge ratio for assets denominated in different currencies. It may find as a particular case an hedge ratio which is equal across currencies and which is equal to one or zero. The model is therefore general enough to encompass most of the empirical exercises that have been proposed in the literature and have been reviewed in the previous section.

Another dimension of generality is that the model my be used on the basis of all the available observations or on the basis of various subsets of observations. In the context of a static framework we consider the whole data sample and build as many scenarios as there are observations. The efficient frontier resulting from this exercise may be interpreted in terms of a long run strategic view based on the unconditional moments estimated on the basis of long time series. In reality investors take sequential decisions at different points of time, conditioning on the investment opportunity set. The resulting tactical portfolio may deviate from the strategic portfolio to exploit perceived over- and under-valuation of the assets. A mean-variance utility investor may implement a tactical dynamic asset allocation by continuously reestimating the efficient frontier and changing the asset allocation accordingly. The relevant issue in such a dynamic context is understanding which elements of the investment opportunity set are relevant for the investment decisions, and how they should be interpreted.

As we pointed out in the introduction, it is hard to relate in a systematic way future returns to observable instrumental variables, especially at short run horizons. At the monthly level it is not unusual to find that few percentage points of the overall volatility of stocks, currencies and bonds are explained by quantitative forecasting models. Even correlations are fairly unstable, see Solnik, Boucrelle and Le Fur [21]. On the basis of these findings one might be skeptical about the effectiveness of conditional hedging strategies which depend in an essential way on the possibility to forecast the conditional distribution of returns and exchange rate.

Glen and Jorion [12] use a regression of the change in the exchange rate on interest rate differentials to retrieve a forecast of the returns on forward contracts. It is hard to evaluate the effectiveness of the forecasting model because the authors do not report the estimated coefficients of their regressions. However, estimates reported by Fishwick [9] for almost the same sample period show that the estimated coefficients in closely connected uncovered interest rate parity regressions signal that high interest rate currency tend to appreciate rather than depreciate for the holding period considered by the interest rates. The same evidence is recently reported by Bansal [6], showing that the exchange rate variability which is explained by interest rate regressions is rather low at the monthly level, even accounting for a (significant) effect of the sign of the interest rate differential. In our sample period the coefficients of determination for a regression similar to that used by Bansal are 6% for the dollar-yen, 1% for the dollar-mark, 3% for the dollar- pound, 2% for the dollar-franc, 2% for the franc-yen, 3% for the franc-mark and 2% for the franc-pound. This confirms that at the monthly level the interest rate differential has only a weak capacity to explain future exchange rates.

Similar findings have led various researchers, among whom Grauer and Hakansson[13], to use a non-parametric method aimed at estimating at each point in time the empirical joint density function of returns by means of previous observations. The methodology uses the observations of the last N months on a set of returns as the empirical distribution function. In the simplest version of scenario analysis, the one we use in this paper, each of this is assigned an equal probability of 1/N. The difference between static and dynamic analyses is therefore that in the latter we use only past information to estimate relevant parameters and update the estimates as we move forward in time. To be more precise, our dynamic experiments proceed as follows:

- Step 1: For a given month compute the efficient frontier for the asset class under study using as scenarios the past five years of data. Consider only the local currency market return and the currency return as random variables since the interest rate differential is known at the beginning of the month. Obtain the unhedged, the hedged and the selectively hedged efficient frontiers.
- Step 2: On each of the three frontiers (hedged, unhedged and selective) choose the portfolio which maximizes the Sharpe ratio.
- Step 3: Compute the monthly return for each of these three portfolios.
- Step 4: Go back to Step 1 and repeat the experiment for the following month.

The goal of this analysis is to check whether or not the efficiency of the partial hedging strategy is warranted also in a dynamic framework, and to understand whether one may find implications which are general across currencies and across portfolios. The analyses are performed for two currencies (US dollar and Swiss franc), two time horizons (one month and six month), with and without transaction costs, for active and fixed portfolios of bonds-only and stocks-only. We have estimated transaction costs in a realistic way, applying to each transaction size a specific cost which differs for stocks and bonds. These are the assumptions we have made, having in mind the Swiss franc as a unit of measurement:

Size of transaction	Bonds	Stocks
25,000	1.65%	2.05%
50,000	1.60%	2.00%
100,000	1.48%	1.88%
150,000	1.40%	1.77%
200,000	1.26%	1.64%
250,000	1.16%	1.54%
500,000	0.76%	1.20%
1000,000	0.50%	0.97%
2000,000	0.38%	0.85%

Evaluating the optimization results by means of a methodology which takes into account transaction costs is in our view a significant generalization over the exercises which have been performed in the literature. In a dynamic portfolio optimization, where the structure of the portfolio may be modified every month, it may become very relevant to take into account of the costs of the turnover associated with the portfolio formation rule.

Finally, the significance of the results of the profitability of the various hedging policies is evaluated by means of a statistical testing methodology applied to a variant of the active asset allocation proposed by Solnik [22]. According to our interpretation of this methodology one can compare the results obtained by two active portfolios in the following way. Let $w_{A,t-1}$ and $w_{B,t-1}$ be the weights of portfolios A and B chosen at time $t-1$. The unconditionally unexpected difference between the returns of the two portfolios at time t is therefore:

$$u_t = (w_{A,t-1} - w_{B,t-1})'(r_t - \frac{\sum_{v=1}^{T} r_v}{T})$$

while the unconditional standard deviation of the differential return is:

$$\sigma_t = (w_{A,t-1} - w_{B,t-1})'\Sigma(w_{A,t-1} - w_{B,t-1})$$

The t-statistic proposed by Solnik [22] can then be written as:

$$t = \sum_{\tau=1}^{T} \frac{u_\tau}{\sigma_\tau} / \sqrt{T}$$

In our application we have considered the estimate of the means and the variance-covariance matrix of returns obtained for the whole sample while the weights of the dynamic strategies depend on the information obtained at the various information sets and by the rules used to select specific portfolios from the conditionally efficient frontier.

4 Empirical results

In the static analysis we use all the data set to build an unconditional efficient frontier. We therefore use 168 scenarios built from all the months in the sample and solve the problem described in the previous section. In the dynamic analysis case we build a time series of conditional frontiers. At each time t the parameters of the frontiers are estimated on the basis of the thirty previous months. In doing so we pay attention to the timing of the variables. The interest rates are supposed to be known at the beginning of the month, so that the scenarios are only used for estimating moments of exchange rates and asset returns.

In our analysis we consider an investment universe consisting of two asset classes – bonds and stocks – in five different countries – United States, Japan, United Kingdom, Europe[1] and Switzerland. The period considered goes from January 1985 to May 1998 and data are on a monthly basis. As a proxy for the bonds we take the Salomon Brothers World Government Bond Index, all maturities, while for the stocks we use the local currency Morgan Stanley MSCI Index, with dividends reinvested. To compute the returns of the hedged markets we use the one month euro-interest rate, provided, as all market data, from DataStream®.

4.1 Static analysis

We start with the computation of several efficient frontiers, using historical monthly returns from 1985 to 1998. We compute the frontiers from the point of view of both a US dollar based investor and a Swiss franc based investor. For the US dollar we also consider the two subperiods 1985-1991 and 1992-1998. The analysis is done for bonds-only and stocks-only frontiers.

Figures 1 and 2 show the efficient frontiers for bonds-only portfolios for investors based in US dollar and Swiss franc respectively for the sample period 1985-1998. We observe that low levels of risk can be achieved only through hedging for the US dollar investor, while a Swiss franc based investor can achieve low levels of risk also without hedging. This is a first indication of the relevance of the currency of denomination: certain trade-offs between risk and return can be achieved in some cases only with hedging.

Figure 1 here

[1] As a proxy for the European markets we use the German market for the bonds and a reconstruction of the DJ Euro Stoxx 50 for the stocks. We use the the Deutsche mark as European currency.

Figure 2 here

Secondly, we observe that selective hedging envelopes the hedged and unhedged efficient frontiers. Selective hedging coincides with full hedging for low volatility portfolios and with no hedging for higher levels of volatility.

The picture looks different for the Swiss investor, where the gains from selective hedging are more evident for a large set of portfolios. This suggests that the value added by a selective hedging model with respect to a no hedging or a complete hedging strategy depends on the reference currency. As before the coincidence between selective hedging and full hedging for low volatility portfolios suggests that the maximal Sharpe ratio would be unlikely to change with the introduction of selective hedging.

Figure 3 here

Figure 3 shows the complete-hedging and the no-hedging efficient frontiers for a US dollar investor, based on the data from 1985 to 1991 and from 1992 to 1998. As for the whole period, low risk portfolios can only be obtained by complete hedging. It is interesting to notice the downward shift of the unhedged frontier from the first subperiod to the second and the upward shift of the completely hedged frontier.

For a given level of risk aversion, the optimized hedging policies depend on the period analyzed. For instance, a target MAD of 12.5 can be achieved with complete hedging over the period 1985-1998 and the subperiod 1985-1991. For the most recent subperiod 1991-1998 the target MAD can be achieved only with an unhedged portfolio.

Figures 4 and 5 show the efficient frontiers for an US dollar and a Swiss franc based stock-only investor. We can observe that for the US investor the unhedged portfolios dominate the hedged ones and coincide with selective hedging. From the Swiss investor perspective things are different, in that complete hedging is optimal and coincides with selective hedging. This confirms, once again, the importance of the base currency in the hedging decision. Also, it shows that selective hedging here is useful mainly for determining which one of the possible extreme policies are optimal.

Figure 4 here

Figure 5 here

The main results of the static analysis can be summarized as follows. First, hedged bond portfolios cover only a small region of the global risk-return tradeoff, limiting in an important way the choices of a bond investor. Second, selective hedging improves only marginally upon the trade-offs available from complete or no hedging: an investor choosing the hedging policy in a way intuitively connected with his risk aversion (that is highly risk averse investors picking hedged low volatility portfolio, otherwise picking an unhedged high volatility portfolio) would probably do as well as an investor using a sophisticated optimizer. Third, the efficient frontier is unstable across time periods. Fourth, efficient frontiers for stocks are rather flat. The results offered by selective hedging are very close to those obtained with either hedging or not hedging, even though the optimal policy varies with the currency of denomination, i.e. it is a no hedging policy for a dollar investor and a complete hedging policy for a franc investor.

4.2 Dynamic analysis

We have started from a portfolio of nominal value equal to one million Swiss francs and have followed the indications provided by the dynamic strategies. The results are reported in tables 1 and 2.

Table 1 here

Table 2 here

Complete hedging is the best policy for a Swiss franc investor, regardless of portfolio composition, transaction costs and time horizon. Only in the case of a fixed bonds index selective hedging is better than complete hedging at the six month horizon. This result may have to do with the interest rate differential over the sample period. The cost of hedging is equal to the domestic interest rate minus the foreign interest rate. For a Swiss investor the domestic interest rate has been low in the sample and the cost of hedging was mainly negative. This seems to provide a useful rule of thumb for a portfolio manager: if based in a low interest rate currency, he should hedge regardless of the specific portfolio he is looking at.

The picture is much more heterogeneous for a US dollar investor. Selective hedging is the best for fixed portfolios of stocks and mix at one month horizon, for all the portfolio structures, when transaction costs are ignored. Selective hedging is optimal for a fixed portfolio of bonds. Selective hedging is optimal at the six month horizon for all types of fixed portfolios. However selective hedging is not best when other factors are brought into the picture. Complete hedging is best (for bonds and mix) at the one month horizon when the structure of the whole portfolio is chosen optimally, even taking into account transaction costs. Also, there is no best policy at the six month horizon for active portfolios: no hedging is best for mix, selective hedging is best for stocks and complete hedging is best for bonds.

So the second general results that we obtain is that the model is indeed able to adapt in a flexible way to the various configurations of base currencies, time horizons and transaction costs. The optimization methodology sometimes suggests extreme polices of no hedging or complete hedging and sometimes suggests the adoption of an hedge ration which varies across currencies. On the basis of this flexibility we are able to show that effectiveness of selective hedging is not universal. It depends on the currency, on the specific assets which are chosen by the optimizer, on the overall optimization problem. We find no clear evidence for the superiority of selective hedging for fixed portfolios of bonds at the monthly horizon. Moreover we do not find much evidence for the view that selective hedging is optimal for optimized portfolios. Indeed at the monthly horizon complete hedging is optimal for bonds even without transaction costs, and with transaction costs complete hedging is optimal also for optimized stock portfolios. At the six month horizon complete hedging is again optimal for bonds, regardless of transaction costs.

A third general result is that transaction costs are very relevant at the one month horizon in the US case, when selective hedging is never optimal with transaction costs. Transaction costs are not relevant anymore (in terms of affecting the ordering of the three policies that we consider) at the six month horizon. At this frequency selective hedging remains optimal for fixed portfolios even after transaction costs and the ordering is not affected also in the case of active portfolios.

We now turn to the results of statistical significance. We have applied the tests only to the one-month strategies because of scarcity of observations for the six-month strategies. The results for the t-tests for stocks are the following:

Table 3 here

Table 4 here

A few patterns emerge. First is the relevance of transaction costs. In the US case the t-tests are not significant at conventional levels when transaction costs are ignored, while they are significant with transaction costs. In particular, and quite intuitively, transaction costs penalize the dynamic selective strategy which does significantly worse than both the covered and unhedged strategies, for both stocks and bonds. The policy of systematically covering exchange rate risk was the best for dollar-based portfolios. The same pattern emerges for the Swiss based investor, even though with less force. Again, the policy of selective hedging does significantly worse than the others for both stocks and bonds, which are however not significantly different from each other. This calls into the question the empirical results of Glen and Jorion [12], since their analysis ignored transaction costs. With the introduction of transactions costs selective hedging is not always optimal.

Second, the relevance of transaction costs emerges also in the analysis of bond portfolios, even though we have assumed that transaction costs lower for bonds than for stocks.

Third, is the coherence of the results across the types of portfolios. In the dollar case covering exchange rate risk is always optimal while in the Swiss franc case hedging is inferior to not hedging, but not significantly so. Stability of the currency makes the choice of hedging irrelevant for all the portfolios considered.

5 Conclusion

The decision about the currency of denomination of the various assets is important for portfolio managers. Theory suggests the validity of various systematic approaches, including no hedging, selective hedging and complete hedging. The superiority of one strategy over the others depend on various elements among which the expected returns and covariances of returns and the relation between exchange rate movements and interest rates. Ultimately, the choice among the three strategies is an empirical problem.

We have carried out a thorough empirical investigation of this issue which extends the results currently available in the literature mainly for using an optimization model which may generate various hedging policies as particular cases of the overall framework and for taking into account transaction costs in the context of the evaluation of the profitability of dynamic portfolio reallocation rules. Our results therefore complement and extend those currently available in the literature and can be summarized in the following way.

At the strategic (unconditional) level there seems to be little ground for giving a general prescription about hedging policy. This result underlines the importance of using a flexible optimization model allowing the investor to choose the hedging policy in connection with scenarios related to exchange rates, returns and interest rates. Of course the investor choosing the strategic portfolio on the basis of long time series of data and of a long holding period cannot hope that the hedging policy holds well also when the value of the portfolio is evaluated at some intermediate stage. Indeed, the data show the existence of much instability in the efficient frontiers, meaning that the strategic portfolios may conditionally be very far from the tactical portfolio. This implies that the optimization model must be evaluated frequently to take into account of more recent market conditions.

The four somewhat general results that we have found from the dynamic analysis are the following: (a) an investor based in a low interest rate currency should always hedge his portfolio, regardless of time horizon, transaction costs and the asset mix; (b) selective hedging is not always the best policy, since a simple fixed rule based on complete hedging may be better from a dynamic point of view; (c) transaction costs are always important in determining the overall profitability, but the ordering of the policies is affected by them only at the monthly horizon; (d) transaction costs are so important at the monthly horizon as to generally make selective hedging an inferior policy.

The results are relevant from several points of view. First, transaction costs are important in dynamic strategies in determining which strategy is the most appropriate under varying market conditions. For this reason, transaction costs provide a motivation for intertemporal optimization methods which take into account the possibility and the cost of portfolio modification at future horizons. An investor taking into account the future costs of modification of the portfolio will make in general different choices from an investor ignoring such costs. This consideration clarifies the relevance of a fully forward-looking dynamic analysis of portfolio optimization.

Several elements remain to be investigated. First, it is interesting to evaluate the problem with more efficient forecasters of returns for stocks, bonds and currencies. Here we have considered a very simple forecast but it would not be difficult to retrieve the forecasts from a more sophisticated econometric model. This is a major topic of research that we have not carried out here because we wanted to evaluate the various strategies in light of very simple and widely available forecasting rules. Second is the comparison between our methodological approach based on empirical distribution functions with a standard one based on means and covariances of returns. The former should be much more robust than the latter to deviations of the probability distribution from normality.

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Table 1: annualized Sharpe ratios, one month analysis

Panel 1a: fixed portfolios

1 MONTH	without t.c.	with t.c.
mix franc	C=1.90, S=1.78, N=0.98	C=1.84, S=1.46, N=0.94
stocks franc	C=1.49, S=1.43, N=0.99	C=1.46, S=1.16, N=0.96
bonds franc	C=2.28, S=2.08, N=0.85	C=2.24, S=1.53, N=0.82
mix dollar	S=1.63, C=1.61, N=1.50	C=1.55, N=1.43, S=1.43
stocks dollar	S=1.24, N=1.20, C=1.15	N=1.15, C=1.11, S=1.09
bonds dollar	C=2.44, S=1.88, N=1.41	C=2.40, S=1.72, N=1.37

Panel 2b: optimal portfolios

1 MONTH	without t.c.	with t.c.
mix franc	C=1.65>S=1.54>N=0.38	C=0.33>S=-0.14>N=-0.25
stocks franc	C=1.38, S=1.38, N=0.93	C=1.11, S=0.93, N=0.73
bonds franc	C=1.77, S=1.63, N=0.76	C=0.90, S=0.87, N=0.41
mix dollar	C=1.13>N=0.79>S=0.75	N=0.31>C=0.21>S=-0.18
stocks dollar	S=1.04, C=1.04, N=0.97	C=0.80, N=0.52, S=0.48
bonds dollar	C=2.35, S=1.77, N=0.87	C=1.52, S=0.55, N=0.35

The table reports the annualized Sharpe ratios for US dollar and Swiss franc base currencies for different portfolio structures. The securities which are considered are stock and bond indices for the following countries: United States, Japan, United Kingdom, Europe and Switzerland. The period goes from January 1985 to May 1998 and data are on a monthly basis. As a proxy for the bonds we take the Salomon Brothers World Government Bond Index, all maturities, while for the stocks we use the local currency Morgan Stanley MSCI Index, with dividends reinvested. To compute the returns of the hedged markets we use the one month euro-interest rate, provided, as all market data, from DataStream®. We consider three portfolio structure: stocks refers to the results of an optimization over stock indices, bonds refers to the results of an optimization performed over bond indices, mix refers to an optimization based on both bond and stock indices. The first column reports results of a profitability analysis excluding transaction costs while the second column includes transaction costs. Each cell reports three Sharpe ratios: C refers to complete hedging of foreign denominated assets, N refers to no hedging of foreign denominated assets, S refers to selective hedging of foreign denominated assets. Selective hedging means that a different hedge ratio may be potentially chosen by the optimizer. The numerator of the annualized Sharpe ratios is constructed from monthly data by multiplying average monthly continuously compounded returns by twelve. The denominator of the annualized Sharpe ratios is constructed from monthly data by multiplying the standard deviation of monthly continuously compounded returns by square root of twelve. The table refers to an evaluation horizon of one month.

Table 2: annualized Sharpe ratios, six month analysis

Panel 2a: fixed portfolios

6 MONTHS	without t.c.	with t.c.
mix franc	C=2.16, S=2.15, N=1.33	C=2.12, S=2.09, N=1.31
stocks franc	C=2.23, S=2.18, N=1.50	C=2.20, S=2.12, N=1.48
bonds franc	S=1.44, C=1.24, N=0.92	S=1.40, C=1.22, N=0.90
mix dollar	S=2.65, N=2.35, C=2.26	S=2.51, N=2.30, C=2.23
stocks dollar	S=2.88, N=2.81, C=2.25	S=2.83, N=2.78, C=2.23
bonds dollar	S=1.71, C=1.58, N=1.02	S=1.62, C=1.56, N=1.00

Panel 2b: optimal portfolios

6 MONTHS	without t.c.	with t.c.
mix franc	C=1.80, S=1.78, N=1.09	C=1.63, S=1.57, N=0.83
stocks franc	C=2.06, S=2.06, N=1.51	C=2.05, S=2.05, N=1.43
bonds franc	C=1.17, S=0.69, N=0.59	C=1.09, S=0.50, N=0.44
mix dollar	N=1.99, C=1.95, S=1.53	N=1.88, C=1.80, S=1.25
stocks dollar	S=2.47, N=2.45, C=2.04	S=2.26, N=2.26, C=2.04
bonds dollar	C=1.45, S=1.44, N=0.83	C=1.40, S=1.31, N=0.67

The table reports the annualized Sharpe ratios for US dollar and Swiss franc base currencies for different portfolio structures. The securities which are considered are stock and bond indices for the following countries: United States, Japan, United Kingdom, Europe and Switzerland. The period goes from January 1985 to May 1998 and data are on a monthly basis. As a proxy for the bonds we take the Salomon Brothers World Government Bond Index, all maturities, while for the stocks we use the local currency Morgan Stanley MSCI Index, with dividends reinvested. To compute the returns of the hedged markets we use the one month euro-interest rate, provided, as all market data, from DataStream®. We consider three portfolio structure: stocks refers to the results of an optimization over stock indices, bonds refers to the results of an optimization performed over bond indices, mix refers to an optimization based on both bond and stock indices. The first column reports results of a profitability analysis excluding transaction costs while the second column includes transaction costs. Each cell reports three Sharpe ratios: C refers to complete hedging of foreign denominated assets, N refers to no hedging of foreign denominated assets, S refers to selective hedging of foreign denominated assets. Selective hedging means that a different hedge ratio may be potentially chosen by the optimizer. The numerator of the annualized Sharpe ratios is constructed from monthly data by multiplying average monthly continuously compounded returns by twelve. The denominator of the annualized Sharpe ratios is constructed from monthly data by multiplying the standard deviation of monthly continuously compounded returns by square root of twelve. The table refers to an evaluation horizon of six months.

Table 3: test statistics, one month, stocks

Panel 3a: fixed portfolios, one month horizon

stocks	no t.c.	with t.c.
selective-complete, franc	-0.50	-35.74
selective-unhedged, franc	-0.07	-0.83
complete-unhedged, franc	-0.03	-0.01
selective-complete, dollar	0.88	-5.38
selective-unhedged, dollar	1.03	-0.53
complete-unhedged, dollar	-0.62	-0.59

Panel 3b: optimized portfolios

stocks	no t.c.	with t.c.
selective-complete, franc	-1.66	-5.91
selective-unhedged, franc	0.78	-1.46
complete-unhedged, franc	0.78	-0.30
selective-complete, dollar	-0.28	-7.59
selective-unhedged, dollar	1.16	-2.58
complete-unhedged, dollar	0.68	2.59

The table reports the t statistic for statistical significance of the annualized Sharpe ratios for US dollar and Swiss franc base currencies for different portfolio structures. The securities which are considered are stock indices for the following countries: United States, Japan, United Kingdom, Europe and Switzerland. The period goes from January 1985 to May 1998 and data are on a monthly basis. To measure stock returns we use the local currency Morgan Stanley MSCI Index, with dividends reinvested. To compute the returns of the hedged markets we use the one month euro-interest rate, provided, as all market data, from DataStream®. We consider two portfolio structure: optimized stocks refers to the results of an optimization over stock indices and forward contracts, fixed stocks refers to the results of an optimization over forward contracts maintaining the composition of the stock portfolio with shares chosen on the basis of the relative capitalization.

The first column reports results of the t statistic for an annualized Sharpe ratio excluding transaction costs while the second column includes transaction costs. Each raw reports the comparison between the annualized Sharpe ratios of two hedging strategies among complete hedging of foreign denominated assets, no hedging of foreign denominated assets, selective hedging of foreign denominated assets. Selective hedging means that a different hedge ratio may be potentially chosen by the optimizer. The numerator of the annualized Sharpe ratios is constructed from monthly data by multiplying average monthly continuously compounded returns by twelve. The denominator of the annualized

Sharpe ratios is constructed from monthly data by multiplying the standard deviation of monthly continuously compounded returns by square root of twelve. The table refers to an evaluation horizon of one month.

Table 4: test statistics, one month, bonds

Panel 4a: fixed portfolios

bonds	no t.c.	with t.c.
selective-complete, franc	-0.02	-5.68
selective-unhedged, franc	-0.06	-0.77
complete-unhedged, franc	-0.06	-0.01
selective-complete, dollar	1.3	0.05
selective-unhedged, dollar	0.64	-0.04
complete-unhedged, dollar	-0.44	-0.35

Panel 4b: optimized portfolios

bonds	no t.c.	with t.c.
selective-complete, franc	-0.66	-9.17
selective-unhedged, franc	0.64	-0.54
complete-unhedged, franc	0.65	-0.77
selective-complete, dollar	-0.81	-12.04
selective-unhedged, dollar	1.37	-1.80
complete-unhedged, dollar	1.09	2.57

The table reports the t statistic for statistical significance of the annualized Sharpe ratios for US dollar and Swiss franc base currencies for different portfolio structures. The securities which are considered are bond indices for the following countries: United States, Japan, United Kingdom, Europe and Switzerland. The period goes from January 1985 to May 1998 and data are on a monthly basis. As a proxy for the bonds we take the Salomon Brothers World Government Bond Index, all maturities. To compute the returns of the hedged markets we use the one month euro-interest rate, provided, as all market data, from DataStream®. We consider two portfolio structure: optimized bonds refers to the results of an optimization performed over bond indices and forward contracts, fixed bonds refers to the results of an optimization performed over forward contracts maintaining the composition of the bond portfolio with shares chosen on the basis of the relative capitalization.

The first column reports results of the t statistic for an annualized Sharpe ratio excluding transaction costs while the second column includes transaction costs. Each raw reports the comparison between the annualized Sharpe ratios of two hedging strategies among complete hedging of foreign denominated assets, no hedging of foreign denominated assets, selective hedging of foreign denominated assets. Selective hedging means that a different hedge ratio may be potentially chosen by the optimizer. The numerator of the annualized Sharpe ratios is constructed from monthly data by multiplying average monthly continuously compounded returns by twelve. The denominator of the annualized

Sharpe ratios is constructed from monthly data by multiplying the standard deviation of monthly continuously compounded returns by square root of twelve. The table refers to an evaluation horizon of one month.

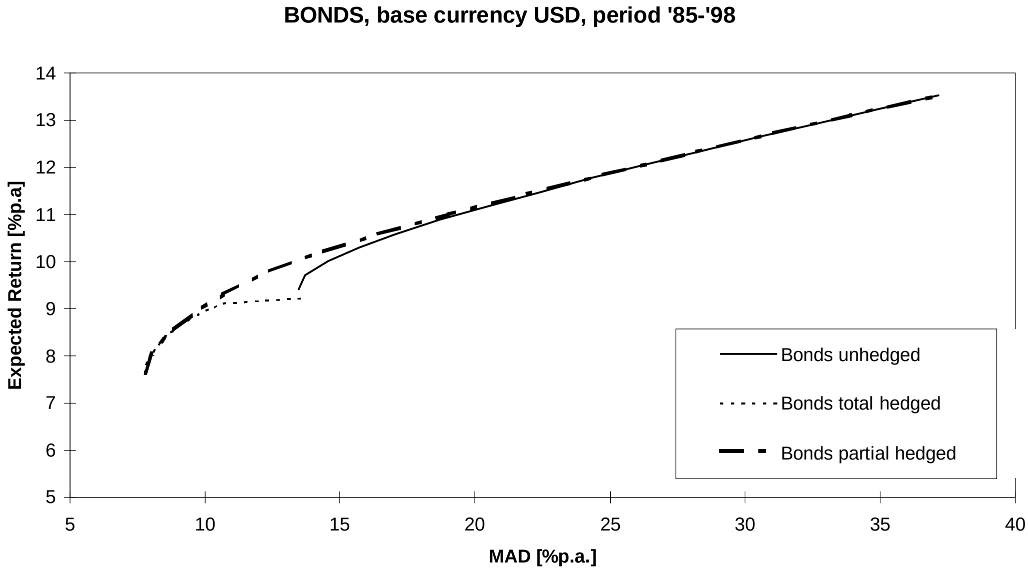

Figure 1: Efficient frontiers for bonds portfolios, base currency US dollar, 1985-1997

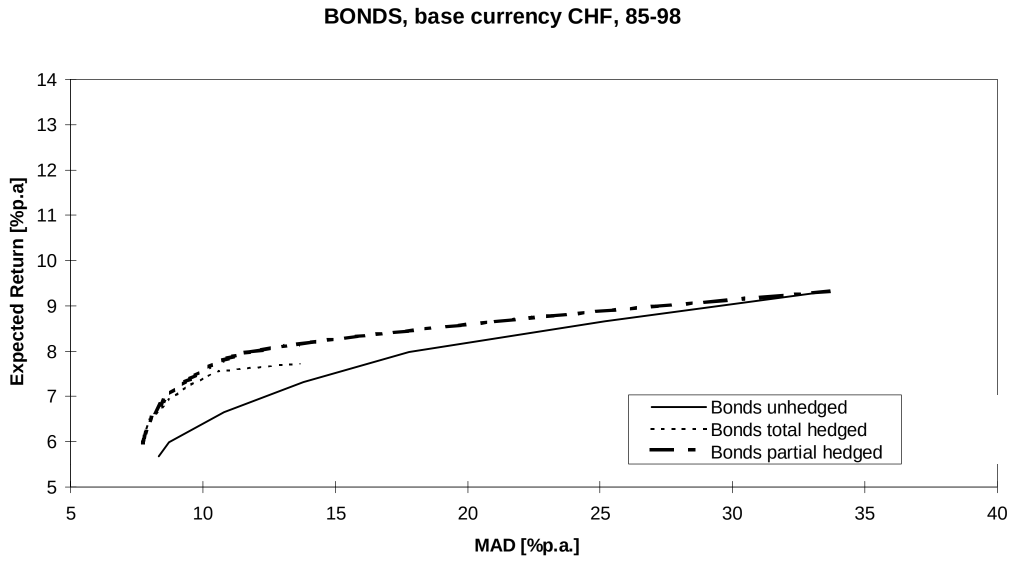

Figure 2: Efficient frontiers for bonds portfolios, base currency Swiss franc, 1985-1997

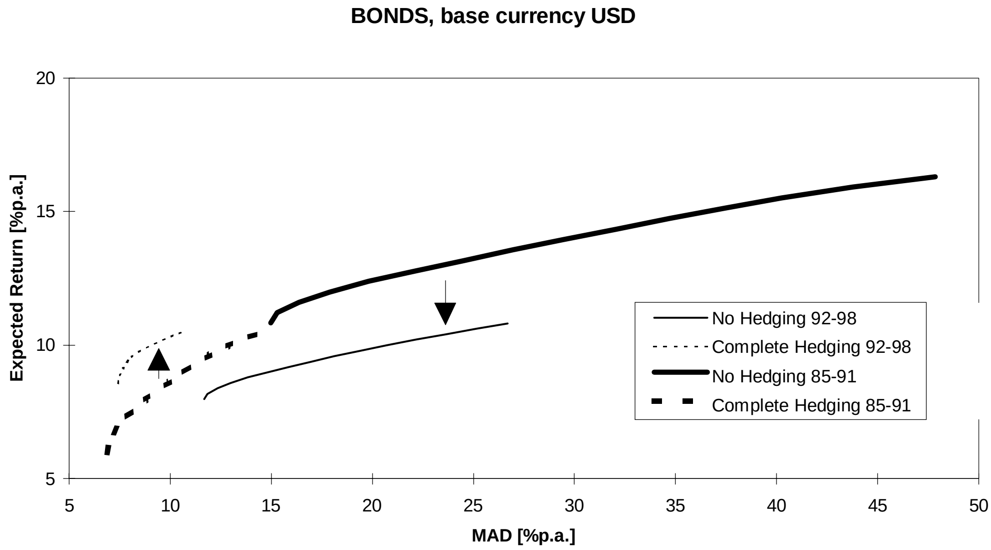

Figure 3: Efficient frontiers for bonds, US Dollar, various subperiods

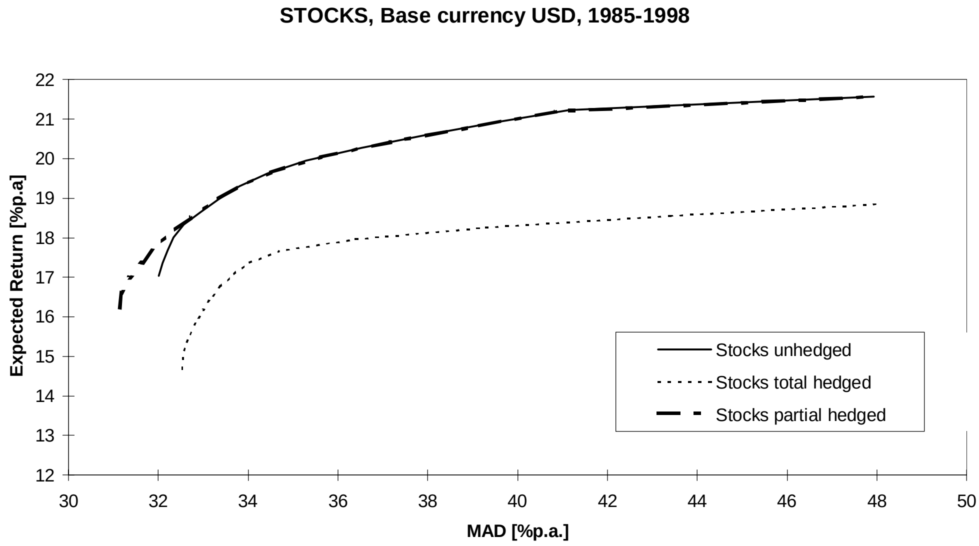

Figure 4: Efficient frontiers for stocks portfolios, US dollar, 1985-1998

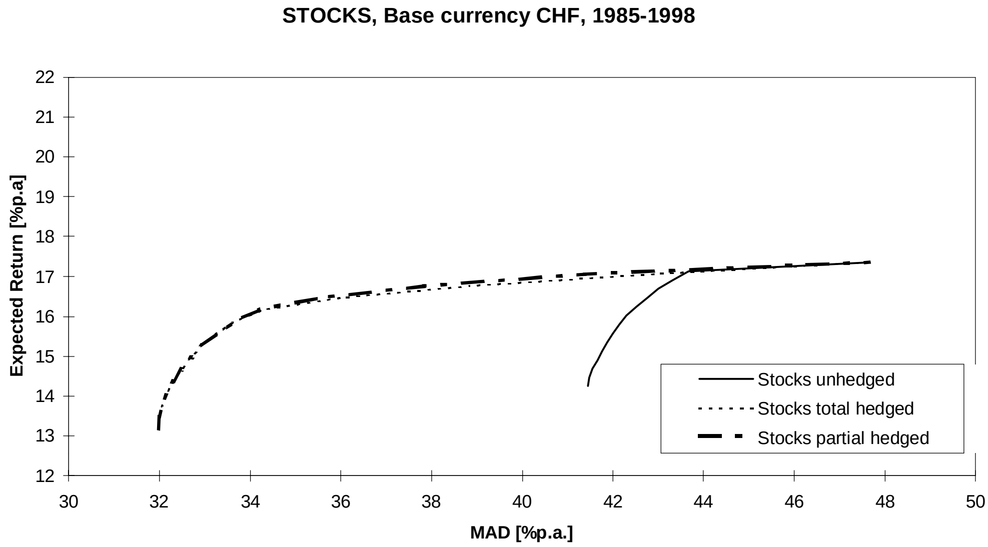

Figure 5: Efficient frontiers for stocks portfolios, Swiss franc, 1985-1998