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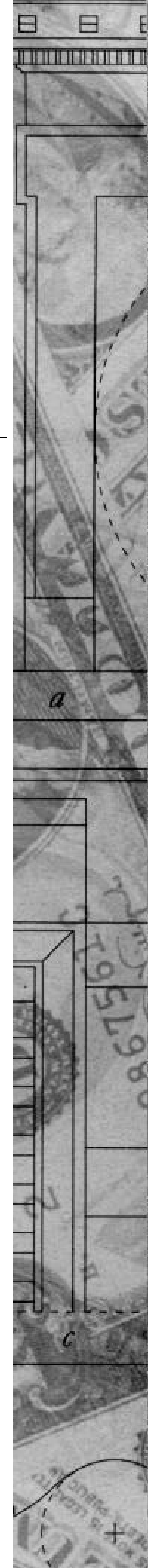

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Overcoming the Inherent Dependency of DEA Efficiency Scores: A Bootstrap Approach

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Abstract

The efficiency scores generated by DEA (Data Envelopment Analysis) models are clearly dependent on each other in the statistical sense. However, this dependency has been ignored in all published uses of these scores when used to make statistical inferences. For example, regression analysis has been widely applied to the analysis of the variation of the DEA efficiency scores. However, the conventional procedure, which has been generally followed in the literature, is invalid. Because of the presence of the inherent dependence among the DEA efficiency scores, one basic model assumption required by regression analysis, independence within the sample, is violated. This paper provides a Bootstrap method to overcome this dependency problem. The core idea is to substitute the incorrect conventional estimators for the standard errors of the regression coefficient estimates with the Bootstrap estimators for the standard errors of these estimates. The method is illustrated using an empirical example from the U.S. health care system.

1. Introduction

Using regression analysis to explain the variations in the distribution of the Data Envelopment Analysis (DEA) efficiency scores has been widely used in the literature. For example, regression analysis has been used to analyze the DEA efficiency scores or some kind of transformations of the DEA efficiency scores in analyzing educational systems (Lovell *et al.* 1994; Ray 1991), health care (Kooreman 1994; Juras and Brooks 1993), the insurance industry (Cummins *et al.* 1999; Carr 1997), and in many other published and unpublished studies.

The use of regression to combine the non-parametric DEA method and parametric statistical methods is recognized in Charnes *et al.* (1994) and Grosskopf (1996). Obviously, regression analysis is among the most useful and most widely used statistical methods. It is a reliable and easy to use tool to determine whether or not certain factors influence the decision-making units' (DMUs) efficiency scores. In practice, it helps to seek the answer to the question we are essentially interested in: inside the black box of an organization, what are those factors that significantly influence the DMUs' efficiency?

A typical way of using regression analysis in the DEA literature is to fit one regression model, in which the variations of the DEA efficiency scores (or some kind of transformation of the efficiency scores) are explained by a group of explanatory variables of interest. The relationship between the efficiency scores and the explanatory variables is then evaluated based on the results of this regression model. We call this kind of method as *direct regression* in this paper. The basic procedure for direct regression can be summarized as followings:

(1) Run one DEA model (or some kind of extension of the conventional DEA model) for each DMU of all the n DMUs in the observation set to calculate the DEA efficiency scores $\theta_1, \theta_2, ..., \theta_n$.

(2) Fit a single regression model, in which the DEA efficiency score θ_i or some kind of transformation of θ_i (e.g., the logarithmic transformation), is the response variable.

(3) Do hypothesis testing: test the individual null hypothesis "$H_0 : \beta_j = 0$" based on the results from fitting the regression model. That is, determine whether or not a certain explanatory variable influences the DMUs' efficiency scores at some pre-specified significant level.

Several problems related to the above procedure are examined in Grosskopf (1996) under the title of "the two-step procedure". For example, if the sample distribution of the DEA efficiency scores is non-normal, a normal distribution may be approximated using the logarithmic transformation of the efficiency score as shown in Lovell *et al.* (1994)[1]. Also, some estimation problems arising from correlation between the input-output vector in the DEA model and the explanatory variable in the regression model are discussed in Deprins and Simar (1989) and Simar, Lovell and Eeckaut (1994).

However, a serious problem with the above procedure has been so ignored in the literature to date. That is, the above procedure violates a basic assumption required by regression analysis: the assumption of independence within the sample. Assume there are n observations in the observation set. If $X_1, X_2, ..., X_n$ were n row vectors of predictors randomly sampled form a population, and responses $Y_1, Y_2, ..., Y_n$ satisfied $Y_i = X_i\beta + \varepsilon_i$, where the errors ε_i's are normally distributed with mean zero and constant variance, independent to each other and independent to the X_i's, then least square regression gives appropriate inferences about the column vector parameter β. However, when $Y_1, Y_2, ..., Y_n$ are the DEA efficiency scores $\theta_1, \theta_2, ..., \theta_n$, the errors ε_i's can not be independent to each other, because the methodology of DEA insures that $\theta_1, \theta_2, ..., \theta_n$ are not independent to each other. The calculation of the DEA efficiency score for one DMU involves all the other DMUs in the observation set. For example, in the following primal input-oriented CCR model, the DEA efficiency score of DMU t in the observed set, where $t \in \{1, 2, ..., n\}$, is calculated by solving the following linear programming problem:

$$\min \ \theta_t \tag{1}$$

$$s.t.$$

$$B_t \le \sum_{i=1}^{n} \lambda_i B_i \tag{2}$$

$$\theta_t A_t \ge \sum_{i=1}^{n} \lambda_i A_i \tag{3}$$

[1] Notice the sample distribution of the DEA efficiency scores is not necessarily non-normal. In the example in Section 4, the Shapiro-Wilk normality test shows that the sample distribution of the DEA efficiency scores as well as the residuals from fitting the regression model in our example are indeed approximately normally distributed.

$$\lambda_i \geq 0,\ i = 1,\ldots,n. \tag{4}$$

In this problem, A_i denotes the input vector of DMU i, B_i denotes the output vector of DMU i, and λ_i is the weight of DMU i. Obviously, the calculation of the DEA efficiency score for DMU t involves all the other DMUs in the observed set. In other words, the reason for the dependency problem is simply the well-known fact that the DEA efficiency score is a relative efficiency index instead of an absolute efficiency index. This is generally true for all kinds of DEA models and the Modified DEA models. Therefore, the above procedure, direct regression, is not valid because of the violation of one of the basic model assumptions required by regression analysis due to the presence of the inherent dependence among the DMUs' DEA efficiency scores.

In this paper, we present a procedure in which the Bootstrap is used with regression analysis to address the above dependency problem. Notice that in the literature of Bootstrap, this method has been used to address different problems in regression analysis; for example, the non-normality of the distribution of the errors in regression analysis. In this paper, Bootstrap is used in to address a different and more specific problem in regression analysis: the dependence among the errors due to the dependence among the responses within the sample, specifically, the DEA efficiency scores. As we will show later in this paper, in hypothesis testing, the method presented in this paper may lead to a conclusion which is contrary to the one drawn from the above direct regression procedure.

The inherent dependency problem of the DEA efficiency scores has been ignored in the literature. However, one should be concerned with not only in the regression analysis of the variations of the DEA efficiency scores, but also other statistical inferences based on DEA efficiency scores. For example, in the also widely used non-parametric analysis of DEA efficiency scores, the independence within sample is also required. But this assumption can also not be satisfied due to the inherent dependency of the DEA efficiency scores. However, to the best of our knowledge, this problem has never been acknowledged, nor addressed in the literature. In our opinion, the Bootstrap is a possible tool to fix this problem in the non-parametric analysis of the DEA efficiency scores as well.

The rest of the paper is organized as followings: In Section 2, the Bootstrap method is introduced. In Section 3, a procedure of regression analysis with the Bootstrap in post-DEA analysis is presented. An empirical example is described in Section 4 to illustrate the methodology presented herein. The main results of this paper are summarized in Section 5.

2. The Bootstrap Method

Bradley Efron invented the Bootstrap method in 1979 (See Efron 1979). Since then, it has quickly become a popular and powerful statistical tool used to address some “hard” problems in statistical analysis. This method is a computationally intense method. However the modern computer is more than sufficient for the computation required by this method.

The problem solved by Bootstrap is mainly an estimation problem. Considering a random sample $X = (X_1, X_2, ..., X_n)$ from a population with an unknown distribution F, the goal is to estimate the sampling distribution of some pre-specified random variable R (X, F), based on the real data set x. Here $x = (x_1, x_2, ..., x_n)$ denotes the observed realization of $X = (X_1, X_2, ..., X_n)$.

As described in Efron (1979), the principle of the Bootstrap method is very simple and straightforward:

(1) Construct the sample probability distribution $\hat{F}$, assigning probability 1/n at each point in the observed sample: $x_1, x_2, ..., x_n$.

(2) Draw a random sample of size n with replacement from $\hat{F}$ while $\hat{F}$ is fixed at its observed value. That is,

$$X_i^* = x_i^*, X_i^* \sim_{ind} \hat{F}, \text{ i = 1, 2, …, n.} \tag{5}$$

The sample $X^* = (X_1^*, X_2^*, ..., X_n^*)$ is defined as the *Bootstrap sample*.

(3) The distribution of the random variable R (X, F) is approximated by the bootstrap distribution of

$$R^* = R(X^*, \hat{F}). \tag{6}$$

Behind this principle, the core idea is that given $X = x$, $\hat{F}$ is the central point of F among all the likely F’s, and then R^* should be close enough to R. In theory, when $\hat{F} = F$, it must be the case that $R^* = R$. Theoretically, R^* can be calculated after x is observed. For more details about the methodology of the Bootstrap method, please refer to Efron (1979) and Efron and Tibshirani (1993). Based on well-established facts, the Bootstrap has been shown to work satisfactorily in many estimation problems, such as the estimation of the variance of the sample median and confidence intervals. As we mentioned before, it was also used to estimate the distribution of the regression coefficients when the error terms' distributions *independently* follow some unknown distribution.

Bootstrap was first introduced into the non-parametric frontier analysis field through a pioneering work in Simar (1992). Since then, it has been used to construct the confidence interval for the means of the DEA efficiency scores (Atkinson and Wilson 1995), to derive the confidence intervals and a measure of bias for the DEA efficiency scores (Ferrier and Hirschberg 1997), and to analyze the sensitivity of the DEA efficiency scores related to the variations of the estimated frontier (Simar and Wilson 1998). It has been recognized that the Bootstrap method is a powerful tool to address the statistical aspects of DEA (Grosskopf 1996). All this literature has been focused on the estimation of the distribution of the DEA efficiency scores. To date, no one has addressed the regression question raised in the Introduction.

3. Applying Bootstrap to the Regression Analysis of DEA Efficiency Scores

In this section, we present a procedure for the regression analysis of the DEA efficiency scores by using Bootstrap to solve the dependency problem. Before going into the details of the method, let us first take a closer look into the "dependency problem". What is the real problem if the responses, $Y_1, Y_2, ..., Y_n$, are dependent to each other, or, in other words, correlated to each other?

Basically, when $Y_1, Y_2, ..., Y_n$ are correlated, if we fit the regression model as if they were not correlated, the estimate of the standard error of the regression coefficient estimate, $\hat{se}(\hat{\beta}_j)$, which is obtained by fitting the regression model, is no longer correct. As stated in Goldberger (1991):

> "… the familiar estimator of the variance matrix of the vector of **b** [that is, the vector of regression coefficient estimates] is biased, and so the conventional estimator of the standard errors are not correct measures of the imprecision, and consequently the confidence region and the hypothesis test procedures … will not be valid."(Goldberger 1991)

This means that we are no longer sure about how accurate $\hat{\beta}_j$ (the estimate of regression coefficient β_j by fitting the regression model) is, and the usual formula for $\hat{se}(\hat{\beta}_j)$, the estimator for the standard error of $\hat{\beta}_j$, will typically give the wrong answer. This immediately causes a more serious problem that we need to worry about: the t-ratios and P-values for the hypothesis test of "$H_0 : \beta_j = 0$" are no longer correct since we calculated the test statistic t according to

$$t = \hat{\beta}_j \Big/ \hat{se}(\hat{\beta}_j) \qquad (7)$$

Therefore, the conclusions we reach through this kind of direct regression analysis may be misleading.

To address the dependency issue, consider the following problem:

Assume a random sample of n DMUs is taken from a completely unknown distribution F. This random sample is denoted by $X = (X_1, X_2, \ldots, X_n)$, where $X_i = (U_i, V_i), i = 1, \ldots, n,$ is a (t + m) dimension vector consisting of one t-dimensional vector U_i and one m-dimensional vector V_i. The components of vector U_i are the inputs and outputs of DMU i used in the DEA model. The components of vector V_i are the corresponding values of the explanatory variables associated with DMU i used in the regression model. Consider the DEA model for DMU i as a projection procedure ϕ_i. Then, the efficiency score θ_i is the projection of $U = (U_1, U_2, \ldots, U_n)$ through ϕ_i. That is,

$$\theta_i = \phi_i(U). \tag{8}$$

Suppose that $\beta_j, j = 0,1,\ldots,m$ are the regression coefficients in the following regression model:

$$\theta_i = Y_i = G(\beta, V_i) + \varepsilon_i, \; i = 1,\ldots,n, \; \beta = (\beta_0, \beta_1, \ldots, \beta_j, \ldots, \beta_m). \tag{9}$$

Here ε_i is the error term, i = 1, 2, …, n. Obviously, by estimation of the regression model, we can calculate the estimate for β_j. That is,

$$\hat{\beta}_j = \varphi_j(\theta, V, F), j = 0,1,\ldots,m. \tag{10}$$

Here $\theta = (\theta_1, \theta_2, \ldots, \theta_n)$ and $V = (V_1, V_2, \ldots, V_n)$. Since $\theta_i = \phi_i(U)$ and X = (U, V), the estimate $\hat{\beta}_j$ is essentially just a random variable dependent on (X, F) through a projection procedure R_j. That is:

$$\hat{\beta}_j = \varphi_j((\phi_1(U), \phi_2(U), \ldots, \phi_n(U)), V, F) = R_j(X, F), j = 0,1,\ldots,m. \tag{11}$$

Hence, we can calculate the coefficient estimate $\hat{\beta}_j$ from the observed random sample x. However, since θ_i's are correlated to each other, we are no longer sure about the accuracy of the $\hat{\beta}_j$'s. As we discussed before, $\hat{se}(\hat{\beta}_j)$, the estimator for $se(\hat{\beta}_j)$ (the standard error of $\hat{\beta}_j$) obtained by the direct regression method is no longer correct. In order to get the correct t-ratios and P-values for the hypothesis testing, we then use bootstrap to estimate $se(\hat{\beta}_j)$.

Based on Algorithm 6.1: "The Bootstrap Algorithm for Estimating Standard Errors" in Efron and Tibshirani (1993), we present the following procedure to estimate $se(\hat{\beta}_j)$ using Bootstrap:

Step 1: Construct the sample probability distribution $\hat{F}$ by assigning probability of 1/n at each DMU in the observed sample: $(x_1, x_2, ..., x_n)$.

Step 2: Draw c (c is a constant) random samples of size n with replacement from the original sample $(x_1, x_2, ..., x_n)$: [2]

$$S_k = (x_{k1}, x_{k2}, ..., x_{kn}),\ k = 1, ..., c, \qquad (12)$$

where $x_{ki} = (u_{ki}, v_{ki})$, $i = 1,...,n$. S_k is the so-called *Bootstrap sample*.

Step 3: For each Bootstrap sample S_k, $k = 1,...,c$, run the DEA model and recalculate the efficiency scores for all n DMUs:

$$\theta_{ki} = \phi_i(u_k),\ i = 1, ..., n, \qquad (13)$$

where ϕ_i represents the DEA model for DMU i.

Step 4: For each Bootstrap sample S_k, $k = 1,...,c$, evaluate the *Bootstrap replication* $\hat{\beta}_{kj}, k = 1,...,c,\ j = 0,1,...,m$ by fitting the regression model:

$$\theta_{ki} = G(\beta_k, v_{ki}) + \varepsilon_{ki},\ i = 1,...,n\ ,\ \beta_k = (\beta_{k0}, \beta_{k1}, ..., \beta_{kj}, ..., \beta_{km}) \qquad (14)$$

Step 5: Estimate the standard error $se(\hat{\beta}_j)$ by the sample standard deviation of the c Bootstrap replications of $\hat{\beta}_j$:

$$\hat{s}e_c(\hat{\beta}_j) = \left\{ \sum_{k=1}^{c} (\hat{\beta}_{kj} - \overline{\beta}_j)^2 \Big/ (c-1) \right\}^{\frac{1}{2}},\ j = 0,1,...,m, \qquad (15)$$

where

$$\overline{\beta}_j = \frac{\sum_{k=1}^{c} \hat{\beta}_{kj}}{c}, j = 0,1,...,m. \qquad (16)$$

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[2] For more details about our way of bootstrapping the whole original sample please refer to Section 9.5 in Efron and Tibshirani (1993).

We call $\hat{s}e_c(\hat{\beta}_j)$ the *Bootstrap estimator* for the standard error of $\hat{\beta}_j$.

Now, we are ready to use a t-test to test the following hypothesis:

$$H_0 : \beta_j = 0,\ vs.\ H_a : \beta_j \neq 0.$$

Calculate the test statistic according to:

$$t = \frac{\hat{\beta}_j}{\hat{s}e_c(\hat{\beta}_j)}, \tag{17}$$

and compare t to the critical value $t_{\alpha/2}$ from the student t distribution with degrees of freedom equal to (n-m-1) . If $|t| > t_{0.025}$, reject the null hypothesis $H_0 : \beta_j = 0$ in favor of $H_a : \beta_j \neq 0$, at $\alpha = 0.05$ significant level. Otherwise, the null hypothesis $H_0 : \beta_j = 0$ is tenable at $\alpha = 0.05$ significant level.

The above procedure, unlike the direct regression method, correctly implements Efron's Bootstrap method to give appropriate standard errors when the n original DMU's, X_i, i = 1,2,.., n, are independently sampled from F, even though the efficiency scores computed from the X's are dependent.

4. Empirical Illustration

In this section, we will use a data set from the health care system in the United States to illustrate the procedure presented in Section 3.

4.1 The Data Set

We take our data sample from the "Hospital Cost Report Data" for the fiscal year 1994-1995 released by the Health Care Financing Administration's (HCFA) (See the Appendix: Table A1). There are 100 hospitals in our sample. The inputs and outputs for the DEA model are:

Inputs:

1. the number of full time employees,
2. costs($ million);

Outputs:

1. the number of patient days produced by the hospital,
2. the number of discharges from the hospital.

The following data will be used as the explanatory variables in the regression model:

1. BEDS: the number of beds in the hospital, which is a measurement of the size of the hospital;
2. FORPROF: a dummy variable where FORPROF = 1 if the hospital is a for-profit hospital, FORPROF = 0 otherwise;
3. TEACH: a dummy variable where TEACH = 1 if the hospital is a teaching hospital , TEACH = 0 otherwise.
4. RES: the number of residents in the hospital.

One of the interesting questions arising from this dataset is: "Are the for-profit hospitals more efficient than the not-for-profit hospitals?" Economic theory might suggest this, and the individual test for the coefficient of FORPROF in the regression model will provide the evidence about the hypothesis.

4.2 Model Description

First, following the literature (Pina and Torres 1992 and Thanassoulis 1993), we choose to use a constant returns-to-scale, input-oriented DEA model (CCR-I) for this illustrative example. However, the method presented in Section 3 is independent of the chosen model orientation and returns-to-scale assumption and thus, is applicable to any other DEA model employed by an analyst.

Second, we choose linear ordinary least square (OLS) regression model in this empirical illustration to simplify the exposition. However, the method presented here is applicable to many other types of regression models. As we will show later, the main reason for choosing the linear OLS regression model is that the Shapiro-Wilk normality tests show that both the DEA efficiency scores and the residuals from fitting the linear OLS regression model in our example are indeed approximately normally distributed.

4.3 Hypotheses

In this example, we are interested in the following four hypotheses:

(1) Is the size of the hospital a factor influencing the efficiency of the hospital?

(2) Does the fact that one is a for-profit hospital or not influence the efficiency of the hospital?

(3) Does the fact that one is a teaching hospital or not influence the efficiency of the hospital?

(4) Is the number of the residents a factor influencing the efficiency of the hospital?

In order to answer these questions, we fit the following OLS regression model:

$$\theta_i = \beta_0 + \beta_1 BEDS_i + \beta_2 FORPROF_i + \beta_3 TEACH_i + \beta_4 RES_i + \varepsilon_i, \qquad (18)$$

and test the following hypothesis concerning the regression coefficients:

(1) $H_0^1 : \beta_1 = 0$, vs. $H_a^1 : \beta_1 \neq 0$.

(2) $H_0^2 : \beta_2 = 0$, vs. $H_a^2 : \beta_2 \neq 0$.

(3) $H_0^3 : \beta_3 = 0$, vs. $H_a^3 : \beta_3 \neq 0$.

(4) $H_0^4 : \beta_4 = 0$, vs. $H_a^4 : \beta_4 \neq 0$.

4.4 Application of the Bootstrap Method[3]

The application of the bootstrap method to our analysis consists of the following phases:

Phase 1: Estimate the point estimate: $\hat{\beta}_j, j = 0,1,...,m$.

Run the CCR-I model with the observed sample ($x_1, x_2,..., x_n$) and obtain the efficiency scores $\theta_1, \theta_2,..., \theta_n$. Get the least-squares estimate $\hat{\beta}_j$ for $\beta_j, j = 0,1,...,m$, by fitting the linear OLS regression model (18) within the observed sample.

Phase 2: Estimate the standard error of the LS estimate $\hat{\beta}_j$: $se(\hat{\beta}_j),\ j = 0,1,2,...,m$.

Step 1: Construct the sample probability distribution $\hat{F}$, assigning probability of 1/100 at each DMU in the observed sample: $x_1, x_2,..., x_{100}$.

Step 2: Take 1000 random samples of size 100 with replacement from the observed sample of 100 hospitals. These samples are the Bootstrap samples.

Step 3: Run the CCR input-oriented DEA model for each Bootstrap sample.

Step 4: Within each Bootstrap sample, fit the following linear OLS regression model:

$$\hat{\theta}_{ki} = \hat{\beta}_{k0} + \hat{\beta}_{k1} BEDS_{ki} + \hat{\beta}_{k2} FORPROF_{ki} + \hat{\beta}_{k3} TEACH_{ki} + \hat{\beta}_{k4} RES_{ki}, \quad (19)$$

for $i = 1,2,...,100;\ k = 1,\ 2,...,1000$.

Here θ_{ki} is the DEA efficiency score for Hospital i in Bootstrap Sample k, and $\hat{\beta}_{kj}$, j = 0, 1, 2, 3, 4, are the Bootstrap replications for $\hat{\beta}_j$ in Bootstrap Sample k.

Step 5: Estimate the standard error $se(\hat{\beta}_j)$ by the sample standard deviation of the c Bootstrap replications:

[3] The DEA models are solved by GAMS and all the regression analysis is completed by S-plus, a statistical language.

$$\hat{s}e_c(\hat{\beta}_j) = \left\{ \sum_{k=1}^{c} (\hat{\beta}_{kj} - \overline{\beta}_j)^2 \Big/ (c-1) \right\}^{\frac{1}{2}}, \quad j = 0,1,...,4, \qquad (20)$$

where

$$\overline{\beta}_j = \frac{\sum_{k=1}^{c} \hat{\beta}_{kj}}{c}, j = 1,...,4 \text{, c = 100, 200, …, 1000.} \qquad (21)$$

Phase 3: Use t-test to test the individual hypothesis "$H_0 : \beta_j = 0$" at $\alpha = 0.05$ significant level with a two-sided alternative "$H_a : \beta_j \neq 0$".

Calculate the test statistic t according to (13), and then compare the calculated t to the critical value $t_{0.025}$ from the student t distribution with degree of freedom equal to (100-4-1) = 95. If $|t| > t_{0.025}$, reject the null hypothesis $H_0 : \beta_j = 0$ in favor of $H_a : \beta_j \neq 0$, and conclude that the jth factor influences the hospitals' efficiency at $\alpha = 0.05$ significant level. Otherwise, the null hypothesis $H_0 : \beta_j = 0$ is tenable and we cannot reject the null hypothesis that the jth factor does not influence the hospitals' efficiency at $\alpha = 0.05$ significant level.

4.5 Results

First, we fit the linear OLS regression model with the DEA efficiency scores and the explanatory variables corresponding to the observed sample (Appendix: Table A1-A2). In order to compare the results from the direct regression and the Bootstrap method presented in Section 3, we then tested the hypotheses based on the results of this direct regression.

In order to choose the appropriate regression model, we did normal quantile and normal curve plot (Figure 1) and the Shapiro-Wilk normality test for the DEA efficiency scores for the observed sample.

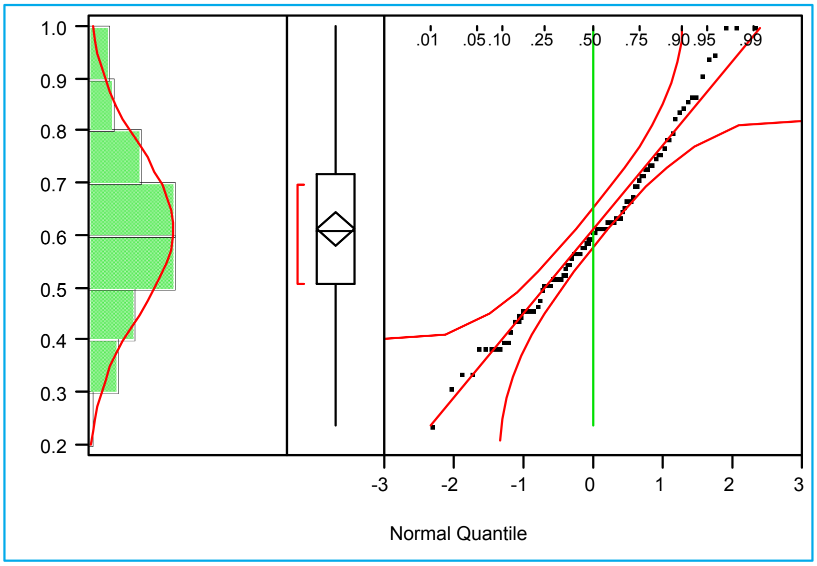

Figure 1. Normal Curve and Normal Quantile Plot for the DEA Efficiency Scores

In the normal quantile plot in Figure 1, the points tend to follow a straight line, which suggests that the efficiency scores are normally distributed. As a more rigorous proof, the result from the Shapiro-Wilk normality test confirmed the normality assumption about the efficiency scores' distribution. The value of the W statistic is equal to 0.970961 and the P-value of the test is 0.1622, which is much higher than 0.05. Therefore, the null hypothesis about the normality distribution of the DEA efficiency scores is clearly not rejected at $\alpha = 0.05$ significant level. Hence, we choose to fit the linear ordinary least square regression model.

In order to double check whether this choice is appropriate or not, we then did normal quantile and normal curve plot (Figure 2) and the Shapiro-Wilk normality test for the residuals of DEA efficiency scores after we fit the linear OLS regression model.

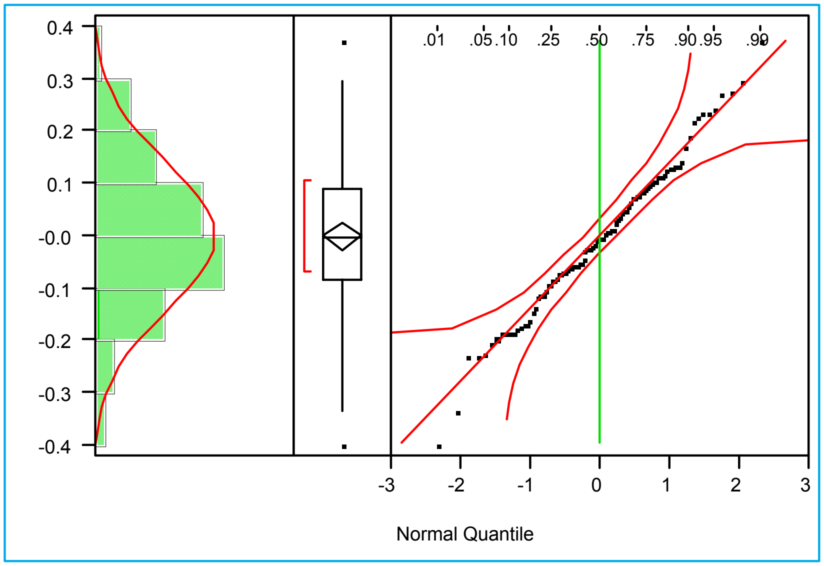

Figure 2. Normal Curve and Normal Quantile Plot for the Residuals of the DEA Efficiency Scores

Once again, we see that in the normal quantile plot, the points tend to follow a straight line and the mean of the residuals is zero, which suggests that the residuals of the efficiency scores follow a normal distribution with mean zero. The value of the W test statistic for the Shapiro-Wilk normality test is equal to 0.989371 and the P-value of the test is 0.9410, which is far more than 0.05. Hence, the model assumption of the linear OLS regression model concerning the normal distribution of the residuals is satisfied. This verifies the appropriateness of the choice of the linear OLS model in our example.

The results of the direct regression are summarized in Table 1 (we test the hypotheses at $\alpha = 0.05$ significance level with two-sided alternatives):

Table 1. Results of Direct Regression

	Estimate	Std. Error	t value	Pr ($>\|t\|$)	Hypothesis Testing
INTERCEPT	0.6084	0.0352	17.278	0	
BEDS	0.0001	0.0001	0.8126	0.4185	H_0^1 tenable
FORPROF	0.0995	0.0418	2.3834	0.0191	**reject** H_0^2
TEACH	-0.056	0.0393	-1.4227	0.1581	H_0^3 tenable
RES	-0.001	0.0003	-3.1483	0.0022	reject H_0^4

Then we applied the Bootstrap method presented in Section 3 with c = 100, 200, …, 1000; the results are shown in Tables 2 through 11.

Table 2. Results of Bootstrap Regression with C=100 Samples

	$\hat{\beta}_j$	$\overline{\beta}_j$	$\hat{se}_{100}(\hat{\beta}_j)$	t value	Pr(>|t|)	Hypothesis Testing
Intercept	0.6084	0.6365	0.0365	16.6462	0	
BEDS	0.0001	0.0001	0.0001	0.9625	0.3382	H_0^1 tenable
FORPROF	***0.0995***	***0.1008***	***0.0571***	***1.7441***	***0.0844***	H_0^2 ***tenable***
TEACH	-0.056	-0.0564	0.036	-1.5533	0.1237	H_0^3 tenable
RES	-0.001	-0.0011	0.0002	-4.2911	0	reject H_0^4

Table 3. Results of Bootstrap Regression with C=200 Samples

	$\hat{\beta}_j$	$\overline{\beta}_j$	$\hat{se}_{200}(\hat{\beta}_j)$	t value	Pr(>|t|)	Hypothesis Testing
Intercept	0.6084	0.6369	0.0377	16.1294	0	
BEDS	0.0001	0.0001	0.0001	1.0171	0.3117	H_0^1 tenable
FORPROF	***0.0995***	***0.0991***	***0.0619***	***1.6088***	***0.111***	H_0^2 ***tenable***
TEACH	-0.056	-0.0561	0.0353	-1.584	0.1165	H_0^3 tenable
RES	-0.001	-0.0011	0.0002	-4.2548	0	reject H_0^4

Table 4. Results of Bootstrap Regression with C=300 Samples

	$\hat{\beta}_j$	$\overline{\beta}_j$	$\hat{s}e_{300}(\hat{\beta}_j)$	t value	Pr(>|t|)	Hypothesis Testing
Intercept	0.6084	0.6385	0.0409	14.8658	0	
BEDS	0.0001	0.0001	0.0001	0.9622	0.3384	H_0^1 tenable
FORPROF	***0.0995***	***0.0967***	***0.0603***	***1.6494***	***0.1024***	H_0^2 ***tenable***
TEACH	-0.056	-0.0539	0.0356	-1.5731	0.119	H_0^3 tenable
RES	-0.001	-0.0011	0.0002	-4.3708	0	reject H_0^4

Table 5. Results of Bootstrap Regression with C=400 Samples

	$\hat{\beta}_j$	$\overline{\beta}_j$	$\hat{s}e_{400}(\hat{\beta}_j)$	t value	Pr(>|t|)	Hypothesis Testing
Intercept	0.6084	0.6386	0.0408	14.9002	0	
BEDS	0.0001	0.0001	0.0001	0.9342	0.3526	H_0^1 tenable
FORPROF	***0.0995***	***0.0978***	***0.0611***	***1.6281***	***0.1068***	H_0^2 ***tenable***
TEACH	-0.056	-0.0527	0.036	-1.5548	0.1233	H_0^3 tenable
RES	-0.001	-0.0011	0.0002	-4.4173	0	reject H_0^4

Table 6. Results of Bootstrap Regression with C=500 Samples

	$\hat{\beta}_j$	$\overline{\beta}_j$	$\hat{s}e_{500}(\hat{\beta}_j)$	t value	Pr(>|t|)	Hypothesis Testing
Intercept	0.6084	0.6384	0.0407	14.9512	0	
BEDS	0.0001	0.0001	0.0001	0.9364	0.3514	H_0^1 tenable
FORPROF	***0.0995***	***0.0967***	***0.0629***	***1.5833***	***0.1167***	H_0^2 ***tenable***
TEACH	-0.056	-0.0532	0.0372	-1.5026	0.1363	H_0^3 tenable
RES	-0.001	-0.0011	0.0002	-4.2643	0	reject H_0^4

Table 7. Results of Bootstrap Regression with C=600 Samples

	$\hat{\beta}_j$	$\overline{\beta}_j$	$\hat{s}e_{600}(\hat{\beta}_j)$	t value	Pr(>|t|)	Hypothesis Testing
Intercept	0.6084	0.6384	0.0418	14.5502	0	
BEDS	0.0001	0.0001	0.0001	0.9355	0.3519	H_0^1 tenable
FORPROF	***0.0995***	***0.098***	***0.0629***	***1.5821***	***0.1169***	H_0^2 ***tenable***
TEACH	-0.056	-0.0539	0.0371	-1.5077	0.1349	H_0^3 tenable
RES	-0.001	-0.0011	0.0002	-4.2892	0	reject H_0^4

Table 8. Results of Bootstrap Regression with C=700 Samples

	$\hat{\beta}_j$	$\overline{\beta}_j$	$\hat{s}e_{700}(\hat{\beta}_j)$	t value	Pr(>|t|)	Hypothesis Testing
Intercept	0.6084	0.6393	0.0424	14.3431	0	
BEDS	0.0001	0.0001	0.0001	0.9361	0.3516	H_0^1 tenable
FORPROF	***0.0995***	***0.0975***	***0.0623***	***1.5977***	***0.1134***	H_0^2 ***tenable***
TEACH	-0.056	-0.0543	0.0371	-1.5077	0.135	H_0^3 tenable
RES	-0.001	-0.0011	0.0002	-4.2761	0	reject H_0^4

Table 9. Results of Bootstrap Regression with C=800 Samples

	$\hat{\beta}_j$	$\overline{\beta}_j$	$\hat{s}e_{800}(\hat{\beta}_j)$	t value	Pr(>|t|)	Hypothesis Testing
Intercept	0.6084	0.6395	0.0417	14.5727	0	
BEDS	0.0001	0.0001	0.0001	0.9436	0.3478	H_0^1 tenable
FORPROF	***0.0995***	***0.0972***	***0.0617***	***1.6131***	***0.11***	H_0^2 ***tenable***
TEACH	-0.056	-0.0544	0.037	-1.5129	0.1336	H_0^3 tenable
RES	-0.001	-0.0011	0.0002	-4.3147	0	reject H_0^4

Table 10. Results of Bootstrap Regression with C=900 Samples

	$\hat{\beta}_j$	$\bar{\beta}_j$	$\hat{se}_{900}(\hat{\beta}_j)$	t value	Pr(>|t|)	Hypothesis Testing
Intercept	0.6084	0.6391	0.0419	14.5136	0	
BEDS	0.0001	0.0001	0.0001	0.948	0.3455	H_0^1 tenable
FORPROF	***0.0995***	***0.0977***	***0.0607***	***1.6397***	***0.1044***	H_0^2 ***tenable***
TEACH	-0.056	-0.0549	0.0366	-1.5301	0.1293	H_0^3 tenable
RES	-0.001	-0.0011	0.0002	-4.2651	0	reject H_0^4

Table 11. Results of Bootstrap Regression with C=1000 Samples

	$\hat{\beta}_j$	$\bar{\beta}_j$	$\hat{se}_{1000}(\hat{\beta}_j)$	t value	Pr(>|t|)	Hypothesis Testing
Intercept	0.6084	0.6399	0.0423	14.3784	0	
BEDS	0.0001	0.0001	0.0001	0.95	0.3445	H_0^1 tenable
FORPROF	***0.0995***	***0.0986***	***0.0609***	***1.6345***	***0.1055***	H_0^2 ***tenable***
TEACH	-0.056	-0.0552	0.0365	-1.5318	0.1289	H_0^3 tenable
RES	-0.001	-0.0011	0.0002	-4.2893	0	reject H_0^4

Comparing the results in Tables 2-11 and the results in Table 1, one significant change occurs in the hypothesis testing concerning FORPROF:

$H_0^2 : \beta_2 = 0$.

Whether or not one is a for-profit hospital does NOT significantly influence the efficiency of the hospital.

versus

$H_a^2 : \beta_2 \neq 0$.

Whether or not one is a for-profit hospital DOES significantly influence the efficiency of the hospital.

The conclusion drawn from the Bootstrap method is in sharp contrast to the one drawn from the direct regression method. The P-value for the t-test from direct regression shown in Table 1 is 0.0191, which leads to the rejection of the null hypothesis H_0^2 while in favor of H_a^2 at the $\alpha = 0.05$ significant level. Thus, the conclusion is that for-profit hospitals are more efficient. However, as shown in Tables 2-11, the P-values for the t-test are very stable and are around 0.11. In the 1000 samples case, the P-value is 0.1055, which is about 5.524 times the P-value of 0.0191 in Table 1 and much higher than 0.05. Hence, the null hypothesis should be tenable at the $\alpha = 0.05$ significant level. We must therefore conclude that to be a for-profit hospital or not may not influence the efficiency of the hospital, which is in sharp contrast to the incorrect conclusion drawn by the commonly used direct regression method.

In this example, the theoretically correct bootstrap method yields conclusions that are qualitatively different from the commonly used, but nonetheless incorrect, direct regression method.

4.6 Analysis of Results

Comparing Table 1 and Tables 2-11, we find that the Bootstrap estimations for the standard error for the coefficient estimate of FORPROF, $\hat{se}_c(\hat{\beta}_2)$, when c =100, 200, …, 1000, are all much larger than the value of the conventional estimator given by the direct regression, $\hat{se}(\hat{\beta}_2)$. Actually, it is this larger standard error estimate that caused the lower t-ratio and the higher P-value, thus leading to the change in our hypothesis testing.

The Bootstrap method suggests that the point estimate of $\hat{\beta}_2$ is not so stable as indicted by the results of the direct regression method. Is this true and how could it happen?

In the originally observed sample, there are 15 for-profit hospitals and 85 not-for-profit hospitals (see Table A1 in the Appendix). When we evaluate the efficiency for the hospitals within the observed sample with the CCR-I model, there are only three efficient hospitals, which are Hospital 51, Hospital 55 and Hospital 74 (Table A2 in the Appendix). As economic theory might suggest, two out of the three efficient hospitals are for-profit hospitals: Hospital 51 and Hospital 74. Hence, these two hospitals have a strong effect on the efficiency scores of many other hospitals. The dependence between the efficiency scores is created in part, by these hospitals.

To make this insight more tangible, let us take a look at the first five bootstrap samples and what happens to the efficiency scores for just ten of the 100 hospitals. The ten hospitals and their efficiency scores in each sample as well as in the observed sample created in Phase 1 of the analysis are shown in Table 12.

Table 12. Variations in Efficiency Scores

Hospital	Observed Sample	Bootstrap Sample 1	Bootstrap Sample 2	Bootstrap Sample 3	Bootstrap Sample 4	Bootstrap Sample 5
12	0.73	0.73	0.78	0.73	0.73	0.73
16	0.48	0.48	0.51	0.57	0.48	0.51
30	0.77	0.77	0.81	0.77	0.77	0.77
38	0.46	0.46	0.49	0.46	0.46	0.46
48	0.53	0.53	0.56	0.55	0.53	0.54
55	1	1	1	1	1	1
73	0.73	0.73	0.77	0.73	0.73	0.73
85	0.52	0.52	0.55	0.52	0.52	0.52
94	0.64	0.64	0.68	0.64	0.64	0.64
60	0.95	/	1	/	/	1
Is Hospital 55 included?	Yes	Yes	Yes	Yes	Yes	Yes
Is Hospital 51 included?	Yes	Yes	No	Yes	Yes	Yes
Is Hospital 74 included?	Yes	Yes	No	No	Yes	No
Is Hospital 60 included?	Yes	No	Yes	No	No	Yes

Note: “/ ” in the table means that the hospital is not included in that sample.

From Table 12, we find:

(1) Hospital 55 is evaluated as efficient in each of the five Bootstrap samples. This is reasonable since Hospital 55 is on the efficiency surface even in the observed sample.

(2) When all the three efficient hospitals are in the Bootstrap sample (e.g., Bootstrap Sample 1 and Bootstrap Sample 4), all the hospitals included in the Bootstrap sample have the same individual efficiency scores as in the observed sample.

(3) However, when neither of the two efficient for-profit hospitals is included in the Bootstrap sample (e.g., Bootstrap Sample 2), large changes occur to all the inefficient hospitals' efficiency scores. When these two hospitals are removed, the other hospitals seem to be more efficient. The changes range from 0.03 to 0.05, with an average of 0.0378.

(4) When one of the two efficient for-profit hospitals (in this case, Hospital 74), is not included in the Bootstrap sample (e.g., Bootstrap Sample 3 and Bootstrap Sample 5), noticeable changes happen to the efficiency scores of Hospital 16 and Hospital 48. Compared to their efficiency scores in the observed sample, the change ranges from 0.01 to 0.09, respectively.

(5) The efficiency score for Hospital 60, which is also a for-profit hospital, in the original sample is evaluated as 0.95, which suggests that Hospital 60 is not on the efficiency surface but very close to it. In Bootstrap Sample 2 and 5, Hospital 60 is included. Because Hospital 51 and Hospital 74 are removed from Bootstrap Sample 2 while Hospital 74 is removed from Bootstrap Sample 5 alone, Hospital 60 is now on the new efficiency surface and evaluated as efficient in Bootstrap Sample 2 and Bootstrap Sample 5.

(6) Notice that not only the originally efficient hospitals can influence the originally inefficient hospitals, but it is also possible for the originally inefficient ones to influence the other inefficient DMUs when the efficiency surface moves. Comparing Bootstrap Sample 3 and Sample 5, we notice that Hospital 16 has efficient score of 0.57 in Sample 3 while having a lower efficiency score of 0.51 in Sample 5, even though in these two samples, both Hospital 51 and Hospital 55 are included while Hospital 74 is excluded. Upon closer examination, we find that Hospital 60 is included in Bootstrap Sample 3 but not in Bootstrap Sample 5. In Bootstrap Sample 2 and Bootstrap Sample 5, in which Hospital 60 is evaluated as efficient partly due to the removal of some of the three originally efficient ones, Hospital 16 has the same efficiency score of 0.51. But in Bootstrap Sample 3, while Hospital 60 is removed, Hospital 16's efficiency score goes up to 0.57. This suggests that the efficiency score of Hospital 16 is influenced by the originally inefficient Hospital 60 in addition to being influenced by the three originally efficient ones. This shows that in terms of the DEA

efficiency scores, the dependency exists not only between the efficient hospitals and the inefficient ones, but also among the inefficient ones.

All these changes happening here are directly or indirectly related to whether these two efficient for-profit hospitals are included in the Bootstrap sample or not. When one or both of the two efficient for-profit hospitals are not included in the Bootstrap sample, the efficiency surface moves, which causes dramatic changes of the inefficient hospitals' efficiency scores. As we can see, the inherent dependency among the hospitals' efficiency scores, especially the fact that the inefficient hospitals' efficiency scores heavily depend on the few efficient ones, resulted in big variations of the DEA efficiency scores in the set. Given that we have only examined a small subset of the hospitals and the Bootstrap samples, the variations we see are quite remarkable.

When we fit the regression model, we try to explain the variation of DEA efficiency scores by the differences in the explanatory variables. As discussed above, a large portion of the variation of the DEA efficiency scores is related to the two efficient, for-profit hospitals. Consequently, the coefficient estimate of FORPROF, $\hat{\beta}_2$ will also be strongly influenced by the two efficient for-profit hospitals. This implies that $\hat{\beta}_2$ cannot be as stable as what is indicated by $\hat{s}e(\hat{\beta}_2)$, the estimation of the standard errors of $\hat{\beta}_2$ given by the direct regression. This kind of instability of $\hat{\beta}_2$, which is essentially caused by the inherent dependency of the DEA efficiency scores, is correctly explained by $\hat{s}e_c(\hat{\beta}_2)$, the Bootstrap estimator for $se(\hat{\beta}_2)$. Because the inherent dependency exists among the DMUs in terms of their efficiency scores, when Bootstrap periodically discards the efficient hospitals from the sample, large changes in the efficiency scores occurred and are correctly captured in the standard error estimations provided by the Bootstrap estimator. Therefore, in this sense, the Bootstrap method presented in Section 3 is shown to work satisfactorily in terms of overcoming the inherent dependency problem in the regression analysis of the variations of the DEA efficiency scores.

5. Summary

In this paper, we use the Bootstrap method to obtain a theoretically appropriate solution to the problem posed in the regression analysis of the DEA efficiency scores due to the inherent dependency among the DMUs' efficiency scores. With the help of Bootstrap, we showed that the broken bridge connecting a relatively new and promising non-parametric analysis method, DEA, and one of the oldest and most powerful parametric analysis methods, regression analysis, could be fixed. As shown in Section 2 and Section 3, the procedure and theory of the Bootstrap method require only the randomness of the observed sample. In our case, this requires the independence among the DMUs in terms of their inputs, outputs, and the explanatory variables but not the independence of their DEA efficiency scores. By showing that each regression coefficient estimate is essentially a projection of the random sample from a population with unknown distribution, we are no longer bounded by the dependency problem and thus, are able to use the Bootstrap method to estimate the standard errors of the regression coefficient estimates.

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Appendix

Table A1. Data for Illustrative Health Care Example

Hospital	FTE	Costs($million)	PTDAYS	DISCH	BEDS	FORPROF	TEACH	RES
1	1571.86	174	71986	12665	365	0	0	0
2	816.54	69.9	53081	5861	224	0	0	0
3	533.74	61.7	25030	4951	286	1	0	0
4	805.2	75.4	34163	11877	256	0	0	0
5	3908.1	396	187462	42735	829	0	1	136.8
6	727.72	63.9	31330	8402	194	0	0	0
7	2571.75	220	130077	26877	620	0	1	42.81
8	521	89.1	43390	8598	290	1	0	0
9	718	50	27896	6113	150	0	1	23.21
10	1504.85	121	75941	16427	393	0	0	0
11	1234.49	84.6	57080	14180	317	0	0	0
12	873	68.8	48932	12060	281	0	0	0
13	1067.17	85.8	50436	11317	278	0	0	0
14	668	47.5	67909	6235	244	0	0	0
15	452.35	36.4	25200	6860	155	1	1	13.31
16	1523	97.4	59809	13180	394	0	0	0
17	3152	198	108631	22071	578	0	1	195.67
18	871.96	30.7	17925	4605	160	0	0	0
19	2901.86	290	130004	24133	549	0	1	126.89
20	902.4	78.2	35743	8664	236	0	1	12.08
21	194.69	10.9	15555	1530	132	0	0	0
22	713.51	62.6	32558	8966	138	0	0	0
23	557.36	23.8	12728	2291	276	1	0	0
24	2259.2	120	74061	12942	348	0	1	14.52
25	462.22	32.4	28886	6101	134	0	0	0
26	1212.1	97.3	74194	12681	342	0	0	0
27	2391.94	192	89843	18396	336	0	1	229.19
28	1637	162	80468	21345	415	0	0	0
29	501	37.9	26813	4594	166	1	0	0
30	412.1	40.2	23217	6044	160	1	0	0
31	738.56	27	11514	3052	144	1	0	0
32	414.1	35.7	55611	4354	200	0	0	0
33	1097	105	59443	13101	242	0	1	26.32
34	742	62.8	42542	8739	172	0	0	0
35	1010	97.1	47246	12073	269	0	1	1.1
36	440.6	34.2	30773	4305	201	0	0	0
37	1203.3	85.4	50710	11470	247	0	1	13.82
38	2558.01	195	128450	20441	571	0	1	5.42
39	215.45	8.409936	65743	578	238	0	0	0
40	599.3	30.4	23299	5338	173	0	0	0
41	480.55	29.5	34279	6560	169	1	0	0
42	634.51	29.9	27157	5198	141	0	0	0

Table A1. Data for Illustrative Health Care Example (continued)

Hospital	FTE	Costs($million)	PTDAYS	DISCH	BEDS	FORPROF	TEACH	RES
43	1211.9	91.4	90008	17666	320	0	1	6.25
44	285.5	23.9	16473	2873	135	0	0	0
45	1030.36	67.1	43486	9467	235	0	1	6.44
46	1374.81	95.5	74279	11862	284	0	0	0
47	953.56	49.8	47934	10553	207	0	0	0
48	561.11	41.7	24800	5498	132	0	0	0
49	644	57.1	39663	8604	260	0	0	0
50	376.55	19.6	22003	4759	143	0	0	0
51	404.79	32.8	27566	7871	190	1	0	0
52	397.9	29.4	26072	4248	170	0	0	0
53	374.2	3.944649	4179	819	156	0	0	0
54	1702	100	114603	17235	438	0	1	11.81
55	148.09	5.013379	51660	771	172	0	0	0
56	253.48	16.9	17599	4044	178	0	0	0
57	1445.68	99.3	81041	12912	475	0	1	17.53
58	414.1	26.5	20432	4068	129	0	0	0
59	642.58	48.5	42733	5983	181	1	0	0
60	203.75	13	16923	3467	146	1	0	0
61	421.8	18.3	16179	2840	160	0	0	0
62	320.62	17.3	18882	3370	160	0	0	0
63	679.79	25.6	27561	4447	308	0	1	11.33
64	2382	226	166559	26019	787	0	1	7.08
65	559.29	58.1	40534	8806	342	1	0	0
66	568.15	35	37120	7242	158	0	0	0
67	2408.04	155	70392	9538	266	0	1	111.33
68	632.34	54.6	37228	6359	175	0	0	0
69	917.22	55.2	42135	7294	215	0	0	0
70	554.34	56.9	32352	3320	205	0	1	1
71	780	75.9	39213	7154	172	0	0	0
72	663.82	56.9	34180	5284	200	0	0	0
73	1424	146	107457	18198	432	0	1	2.75
74	313	20.7	20110	5967	165	1	0	0
75	778	78.4	51496	12302	390	0	0	0
76	863.37	62	50957	10557	228	0	0	0
77	3509.12	290	109673	19213	469	0	1	290.53
78	1593.82	152	82400	17707	474	0	1	11.64
79	466	40.1	30647	7265	164	1	0	0
80	666.38	48.2	28048	5182	153	0	0	0
81	998.8	121	45513	6855	238	0	1	88.86
82	1018	98.2	61176	11386	350	0	0	0
83	3238.28	326	122118	19068	514	0	1	146.33
84	1431.1	107	48900	10623	208	0	0	0

Table A1. Data for Illustrative Health Care Example (continued)

Hospital	FTE	Costs($million)	PTDAYS	DISCH	BEDS	FORPROF	TEACH	RES
85	1735.99	273	84118	16458	278	0	1	158.4
86	1769	190	105741	19256	478	0	1	0.93
87	484.56	36.2	24070	6464	125	0	0	0
88	204.7	13.9	28137	1615	135	1	0	0
89	1706.58	287	75153	13465	367	0	1	91.56
90	1029.11	71.9	49993	6690	252	0	1	4
91	1167.2	111	75004	21334	350	0	0	0
92	1657.58	116	77753	17528	413	0	0	0
93	1017.16	88.5	64147	11135	316	0	0	0
94	1532.7	153	99998	17391	395	0	1	4.8
95	1462	113	119107	16053	484	0	1	0.5
96	1133.8	109	55540	15566	355	0	1	8.51
97	609	48.2	71817	5639	376	0	1	1
98	301.31	20.2	43214	2153	141	0	0	0
99	1930.08	201	87197	19315	418	0	0	0
100	1573.3	177	88124	19661	458	0	1	69.71

Notation used in Table A1:

FTE: The number of full time employees in the hospital in FY 1994-95
Costs: The expenses of the hospital ($million) in FY 1994-95
PTDAYS: The number of the patient days produced by the hospital in FY 1994-95
DISCH: The number of patient discharges produced by the hospital in FY 1994-95
BEDS: The number of patient beds in the hospital in FY 1994-95
FORPROF: Dummy variable, which is equal to one if it is for-profit hospital, zero otherwise
TEACH: Dummy variable, which is equal to one if it is teaching hospital, zero otherwise
RES: The number of the residents in the hospital in FY 1994-95

Table A2. CCR-I DEA Efficiency Scores for the Observed Sample

Hospital	Score	Hospital	Score	Hospital	Score	Hospital	Score
1	0.45	***26***	0.59	***51***	1	***76***	0.67
2	0.46	***27***	0.42	***52***	0.61	***77***	0.31
3	0.51	***28***	0.68	***53***	0.74	***78***	0.6
4	0.76	***29***	0.52	***54***	0.62	***79***	0.83
5	0.58	***30***	0.77	***55***	1	***80***	0.44
6	0.6	***31***	0.39	***56***	0.87	***81***	0.4
7	0.57	***32***	0.76	***57***	0.52	***82***	0.62
8	0.91	***33***	0.64	***58***	0.55	***83***	0.34
9	0.46	***34***	0.64	***59***	0.55	***84***	0.4
10	0.59	***35***	0.63	***60***	0.95	***85***	0.52
11	0.62	***36***	0.58	***61***	0.56	***86***	0.61
12	0.73	***37***	0.52	***62***	0.7	***87***	0.7
13	0.57	***38***	0.46	***63***	0.63	***88***	0.65
14	0.63	***39***	0.87	***64***	0.63	***89***	0.44
15	0.79	***40***	0.62	***65***	0.85	***90***	0.39
16	0.48	***41***	0.79	***66***	0.74	***91***	0.94
17	0.39	***42***	0.62	***67***	0.24	***92***	0.57
18	0.52	***43***	0.8	***68***	0.57	***93***	0.62
19	0.46	***44***	0.57	***69***	0.47	***94***	0.64
20	0.51	***45***	0.51	***70***	0.39	***95***	0.66
21	0.53	***46***	0.5	***71***	0.51	***96***	0.71
22	0.65	***47***	0.75	***72***	0.46	***97***	0.67
23	0.34	***48***	0.53	***73***	0.73	***98***	0.63
24	0.39	***49***	0.72	***74***	1	***99***	0.54
25	0.72	***50***	0.86	***75***	0.84	***100***	0.67