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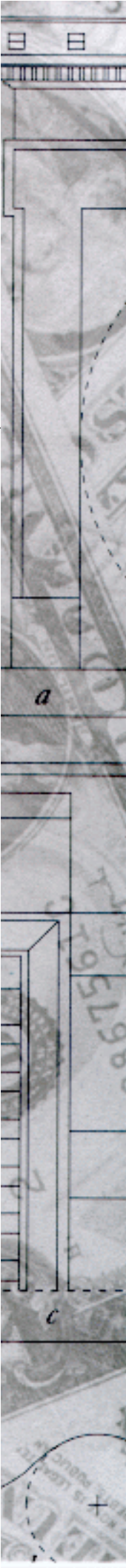
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*The Value of Integrative Risk
Management for Insurance
Products with Guarantees*

by
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01-06

The Wharton School
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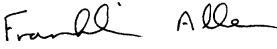



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The Value of Integrative Risk Management for Insurance Products with Guarantees

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Abstract

Insurers increasingly offer policies that converge with the products of the capital markets, and they face a need for integrative asset and liability management strategies. In this paper we show that an integrative approach—based on scenario optimization modelling—adds value to the risk management process, when compared to traditional methods. Empirical analysis with products offered by the Italian insurance industry are presented. The results have implications for the design of competitive insurance policies, and some examples are analyzed.

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	1997	1998	1999	2000	2001	2002
Household total	944.853	1427.999	1781.996	2124.102	2488.154	2877.773
% of household's assets	23.6	31.4	34.6	38.3	41.9	44.8
Mutual funds	368.432	720.823	920.304	1077.360	1237.964	1386.519
Asset Management	375.465	542.205	673.500	781.300	880.450	956.970
Life and general insurance	165.000	202.300	257.400	329.600	433.400	574.000

Table 1: Traded financial assets by Italian households during 1997–2002 in billions of ITL. (Source: ISVAP, the board of regulators for Italian insurers.)

1 Introduction

With the historical low interest and inflation rates of the last decade insurance companies face declining profit margins. The most significant factor contributing to the decline have been the policies offered by insurance companies in the inflationary seventies. In order to compete with the high yields of Treasury bonds of that time, insurance policies were enhanced with both a minimum guaranteed rate of return and a bonus provision when asset fund returns exceed the minimum guarantee. Such policies, known as unit-linked or index-linked, are prevalent among continental European insurance companies, but they are also encountered in the UK, United States and Canada.

In the low-inflation 1990's insurance companies still could not abandon these products. The popularity of mutual funds and asset management creates competitive pressures on the industry to deliver policies that combine traditional insurance against actuarial risks, with attractive returns. Insurers who fail to do so see their market share erode. The statistics for the Italian industry are telling (see Table 1). In the period 1997 to 2000 Italian households more than doubled (125% increase) their traded financial assets. However, assets invested in life and general insurance increased by 99% while assets in mutual funds increased by 190%, and those under asset management by 110%. Insurance companies trail the competition in claiming a share of the household's wallet. The industry expects to reverse this trend by 2002. By that time Italian households are expected to increase their traded assets by 200%, with the insurance policies increasing their share by 250%, mutual funds by 280%, and asset managers by 150%. The main competitive weapon in the arsenal of the insurance firms are policies with guarantees and bonus provisions.

1.1 Main features of policies with guarantees

The key feature of these policies is that they promise a guaranteed return upon maturity. If the asset portfolio performance is below this guarantee the

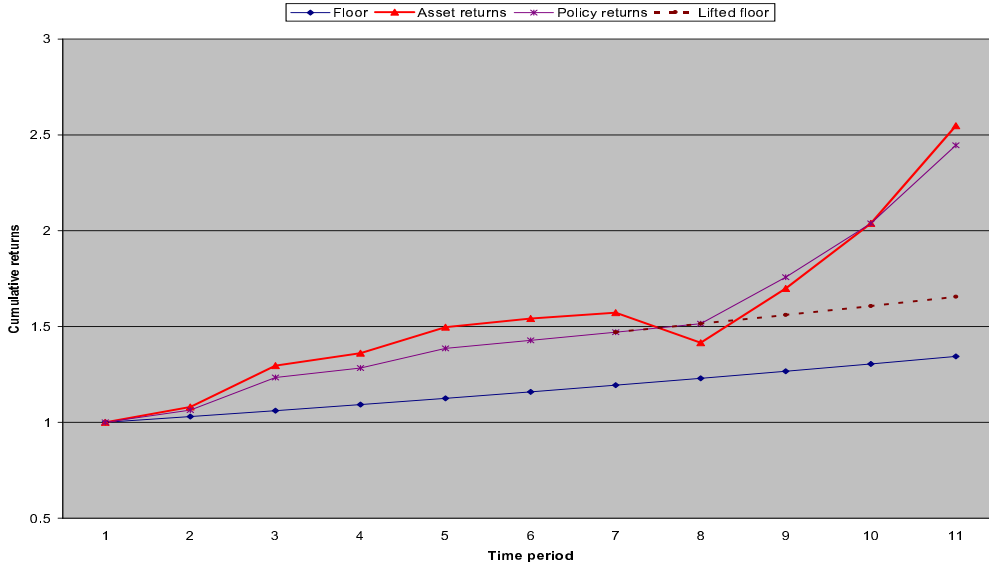


Figure 1: Typical returns of the asset portfolio and the participating policy with a minimum guarantee 3% and participation rate 80%. The minimum guarantee applies to a liability that is lifted every time a bonus is paid as illustrated at period seven. The asset portfolio experienced substantial losses at period seven while the liability grew at the 3% guaranteed rate. Subsequent superior returns of the assets allowed the firm to recover its losses by the tenth period and achieve a positive net return at maturity.

company must compensate for the shortfall with its own capital. When the asset portfolio does better than the guarantee then a fraction—say 80%—of the asset return is given as bonus to the policyholders, with the remaining part contributing to the revenues. In the Italian policies the minimum guaranteed return applies to the bonus as well. What is given can not be taken away, and the liability is lifted every time a bonus is paid. Figure 1 illustrates the growth of a typical liability.

The dynamics of the value of the liability are denoted by the random variable \tilde{L}_t , where t ranges from 0 (today) to T (maturity). Similarly the dynamics of the assets are denoted by \tilde{A}_t and the dynamics of the firm's capital by \tilde{E}_t . The firm collects a premium L_0 by issuing a policy, invests its own capital according to a regulatory ratio, $E_0 = \rho L_0$, and purchases assets $A_0 = L_0(1 + \rho)$. Shareholder value upon maturity of the policy is measured by the excess return on equity

$$\text{exROE} = \frac{\tilde{A}_T - \tilde{L}_T}{\tilde{E}_T}.$$

This quantity is also a random variable and in order to compare alternative policies we will compute their certainty equivalent excess return on equity.

$$U(\text{CEexROE}) = \mathcal{E} \left[U \left(\frac{\tilde{A}_T - \tilde{L}_T}{\tilde{E}_T} \right) \right], \quad (1)$$

$\mathcal{E}[\cdot]$ denotes expectations of the random variable and $U\{\cdot\}$ denotes the utility function. CEexROE is a measure of the reward of the firm for assuming the risk of the guarantee. The cost of the guarantee is the downside risk for the firm. This is the cost of the firms' own capital when a shortfall is realized, which is what transpired between periods seven and ten in our example. More on this cost is said in the next section.

1.1.1 Challenges for the manager of policies with guarantees

The first challenge in managing endowments with guarantees is to price them correctly. Recent development in financial pricing of insurance products (Babbel and Merrill, 1999, Embrechts, 2000) serve the industry well in pricing the options embedded in these products. The minimum guarantee together with the actuarial risk can be priced as a straight bond with standard actuarial pricing tools. However, the bonus returns that are paid at maturity create an embedded European put option: at maturity the policyholder can sell the policy to the firm for the guaranteed amount. Policyholders are also granted the right to surrender their policies before maturity, perhaps at some fee. This right creates an embedded American option: the policy can be sold to the firm at any point before maturity at the guaranteed amount minus a surrender fee. Significant advances have been made since the seventies in pricing these embedded options either as a portfolio (Brennan and Schwartz, 1976, Boyle and Schwartz, 1977) or as distinct derivatives (Grosen and Jorgensen, 1999). These advances have been gaining increasing attention from industry (Giraldi et al., 2000).

The second challenge is to create an integrative asset and liability management strategy to support the policy. Reliance on fixed income assets is unlikely to yield the minimum guarantee. For instance, Italian guaranteed rates after 1998 are at 3%, differing only by 1% from the ten-year yield. With costs of the order of 1 to 1.5% this margin is not adequate. In Germany the guaranteed rates after 1998 are at 3.5%, a mere .5% difference from the ten-year yields.

Equities, on the other hand, are likely to return well in excess of the minimum guarantee, within the time horizon of the policy's maturity. For instance, the Italian stock index has averaged 13% return (per annum) during the last decade. However, equities are not the answer. Excessive reliance on

equities creates a volatile asset portfolio, with the upside being shared by the policyholder as a bonus, and the downside limited by the guarantee. The embedded options become, in this case, expensive, and the company must compensate any shortfalls with its own capital. Siglienti (2000) argues that portfolios with more than 10 to 15% in equities are likely to destroy shareholder value.

Perhaps the most prominent example of the challenges facing the industry in managing these products is that of Nissan Mutual Life. The company failed on a \$2.56 billion liability arising from a 4.7% guaranteed investment. The risk management practices of Nissan failed to properly account for the value of the options sold to policyholders as part of the minimum guarantee policy.

It is precisely in the area of risk management that we focus the discussion of our paper. We start first with the application of a traditional portfolio diversification approach, and show that it fails to capture some important characteristics of the problem. There is nothing efficient about efficient portfolios when the nonlinearity of the embedded options is properly accounted for. We then discuss a novel model based on scenario optimization that integrates the asset management problem with the characteristics of liabilities with minimum guarantee. The value of integrated financial product management is extensively argued in practice, but case studies showing that an integrative perspective adds value are scant, see Holmer and Zenios (1995) for some examples. In this paper we use a model developed by Consiglio, Cocco and Zenios (2000) to show how an integrative approach adds value to the risk management process for these complex insurance products.

2 The traditional approach: Efficient portfolios

Diversified portfolios of stocks and bonds for an Italian insurance firm are built using Markowitz mean-variance models. Using indices of short, medium, and long-term debt of the Italian government, and stock indices of the major industrial sectors traded in the Milano stock exchange we obtain the efficient frontier illustrated in Figure 2.

Should an insurance firm offering a minimum guarantee product choose portfolios—based on its appetite for risk—from the set of efficient portfolios? Let us plot each one of the efficient portfolios in the space of CEexROE (shareholder's reward) versus cost of the guarantee (the firm's risk). The results are shown in Figure 3. There is nothing efficient about efficient portfo-

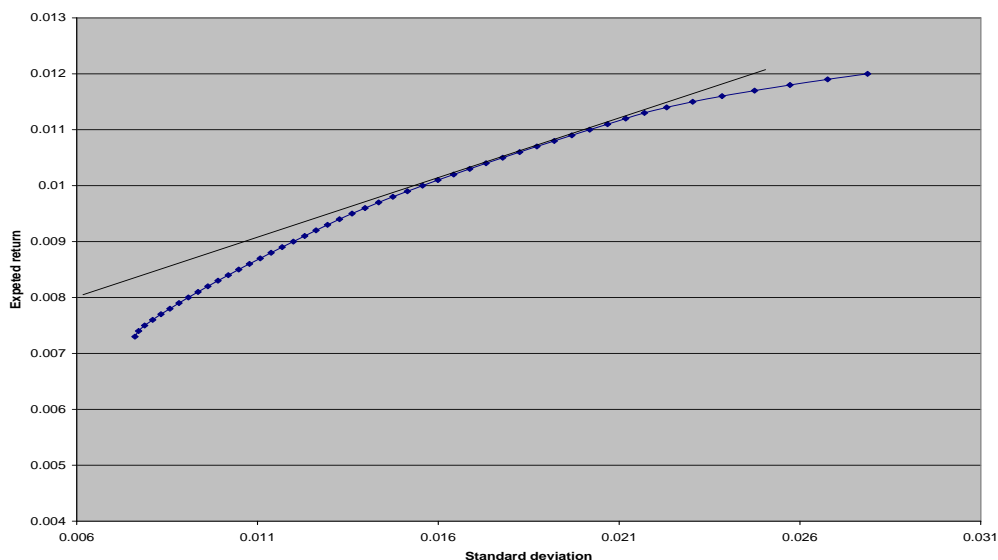


Figure 2: Mean-variance efficient portfolios of Italian stocks and bonds and the capital market line.

lios when the liability created by the minimum guarantee policy is accounted for. Portfolios from A to G are on the mean-variance frontier that lies below the capital market line. It is not surprising that they are not efficient in the CExROE vs Cost-of-guarantee space. However, the tangent portfolio G is also inefficient. A more aggressive portfolio strategy is needed in order to achieve the minimum guaranteed return and deliver excess return to shareholder. And still this increasing appetite for higher but risky returns is not monotonic. As we move away from portfolio G towards the most risky portfolio B we see at first the cost of the guarantee declining and CExROE improving. But as we approach B shareholder value erodes, just as Siglienti found out from his simulations. For these very volatile portfolios the embedded option is deep in-the-money, and shareholders money are used to compensate for the shortfalls without realizing any excess returns.

As a first step of our analysis we have shown that it is important to take an integrative view of the asset allocation problem of firms issuing products with guarantees. Properly accounting for the cost of the guarantee is important, if the firm is to avoid unnecessary risk exposures and destroy shareholder value. The literature for pricing the embedded options is useful in quantifying the tradeoffs between the upside potential of a policy (as measured by CExROE) and the downside risk (as measured by the cost of the guarantee).

In a nutshell, we have seen that the management of minimum guarantee

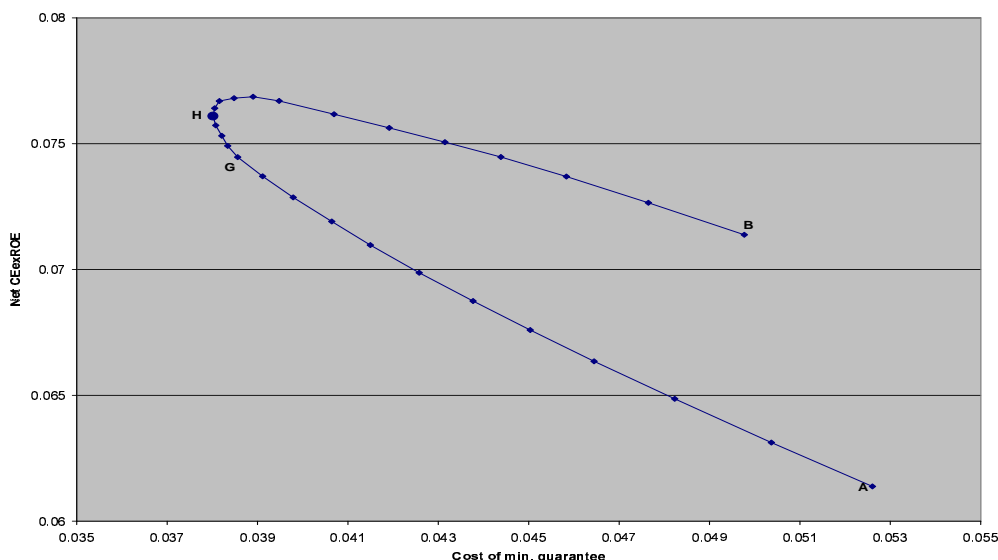


Figure 3: Certainty equivalent excess return of equity (CExROE) to shareholders vs cost of the minimum guarantee for the mean-variance efficient portfolios.

products is a balancing act! Too much reliance on bonds and the guarantee is not met. Excessive reliance on stocks and shareholder value is destroyed.

Is it possible to incorporate the random liability in a Markowitz model, and develop efficient portfolios in the CExROE vs Cost-of-guarantee space? Unfortunately, the return of the liability depends on the return of the asset portfolio and this is not known without determining simultaneously the structure of the asset portfolio. The return of the liability is endogenous to the portfolio selection model. Furthermore, the liability return has a floor—the minimum guarantee. This creates nonlinearities in the model, and highly asymmetric returns that are not conducive to mean-variance type of modelling. An integrative asset and liability management strategy needs an alternative modelling approach.

3 An integrative approach based on scenario optimization

Scenario optimization is a powerful, flexible, and natural paradigm for integrative risk management. It has been gaining popularity as a framework for risk management both in industrial settings (e.g., Dembo et al. 2000)

and in the academic literature (Zenios, 1993, Ziemba and Mulvey, 1999). In a scenario optimization framework it is assumed that the random variables $\tilde{A}_t, \tilde{L}_t, \tilde{E}_t$ take discrete and finite values. We denote these values by a superscript l from a set of scenarios Ω , i.e., A_t^l, L_t^l, E_t^l .

Assuming that the asset returns are given by r_{it}^l in each scenario l then we can write the portfolio return as

$$R_{pt}^l = \sum_i x_i r_{it}^l, \text{ for } t = 1, 2, \dots, T, \text{ and for all } l \in \Omega, \quad (2)$$

where x_i are the proportional holdings in the i th asset.

We can now model the liabilities as a function of the portfolio return. The fact that liability returns are endogenous does not pose any significant difficulties. Liabilities will grow at a rate which is at least equal to the minimum guarantee. Excess returns over \bar{g} are returned to the policyholders according to the participation rate α . The dynamics of the liability are given by

$$L_t^l = (1 - \Lambda_t^l) L_{t-1}^l (1 + \max[\alpha R_{pt}^l, \bar{g}]), \text{ for } t = 1, 2, \dots, T, \text{ and for all } l \in \Omega. \quad (3)$$

Λ_t^l denotes probabilities of actuarial events or policy surrender. The max operator introduces a discontinuity in the model. This is a technical difficulty that can be resolved with a suitable model reformulation. See the Appendix for further details of this model.

The resulting optimization model is more complex than the Markowitz mean-variance quadratic program. It is still solvable, though, with standard optimization packages. We repeated the asset allocation modelling using the Italian data for a liability with different levels of minimum guarantee (\bar{g}) and 80% participation rate (α). The probabilities Λ_t^l were obtained from the Italian mortality tables, but probabilities of policy surrender were ignored. The results are shown in Figure 4. This figure identifies the best asset allocation for each level of minimum guarantee. Best here is defined as the allocations that provides the higher return to shareholders for the lowest downside risk. On the same figure we plot the tradeoff between CExROE and cost of the guarantee from the portfolios of Figure 3. We see that even portfolio H is dominated by the portfolios obtained by an integrative model. The traditional approach of portfolio diversification—Figure 2—followed by a post optimality analysis to incorporate the minimum guarantee liability and its cost—Figure 3—yields suboptimal results. The integrative approach adds value which appears, from our analysis with market data for a real policy, to be quite substantial.

The balancing act of the insurance policy manager is best seen in the structure of the optimal portfolios that back policies with different levels

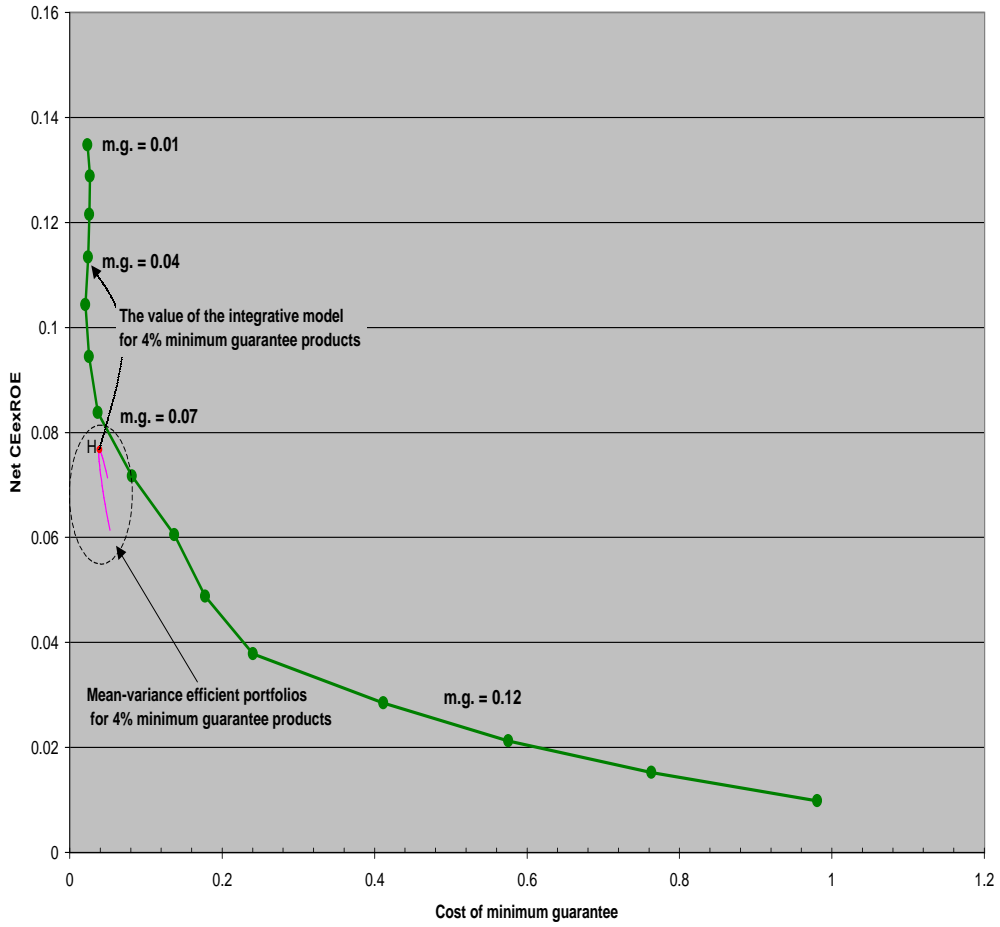


Figure 4: Certainty equivalent excess return of equity (CEexROE) to shareholders vs cost of the minimum guarantee for the integrated portfolios at different levels of minimum guarantee, and for the mean-variance efficient portfolios.

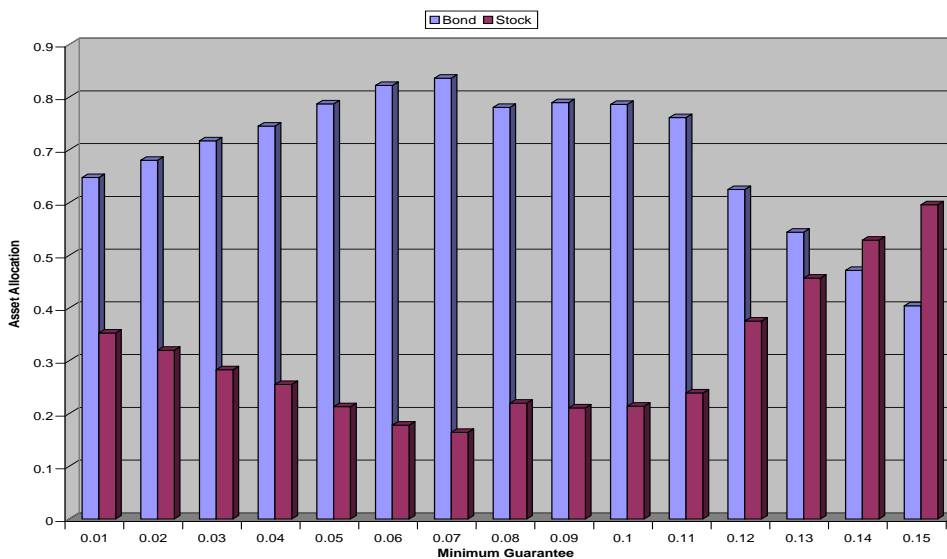


Figure 5: The optimal portfolio composition for different levels of minimum guarantee.

of guarantee. Compare the portfolio structures in Figure 5 with the CEexROE of different policies in Figure 4. For low levels of minimum guarantee large equities content of the portfolio is translated to high shareholder value. The volatile returns of the equities pass, mostly, to shareholders. For very large values of minimum guarantee the equities contents is also large. But shareholder value is reduced. Equities is the only way to back very large guarantees, but shareholders value erodes as the firm's equity is used to fund shortfalls. We also note that the optimized portfolios show that the Italian insurers will do best to shift their portfolios towards 20 to 25% in equities in order to back 3% guarantees. The outlook from the integrative model favors more risky portfolios than Siglienti's simulations. The difference is partly due to the fact that we use data from the past ten bullish years and to slight differences in the policies analyzed. But the difference is also due to the integrative model we used, as opposed to simulations, that allow us to be more creative in structuring optimal portfolios. More aggressive strategies can be structured without unnecessary risk exposure.

We extended our results to allow for investments in international bonds. This practice is allowed in the Italian insurance industry, for foreign currency exposures in the range of 5 to 10% of the value of the portfolio. Results in Figure 6 show that, for a given level of minimum guarantee, the CEexROE vs cost-of-guarantee is shifted to the left. For instance, a guarantee of 4%

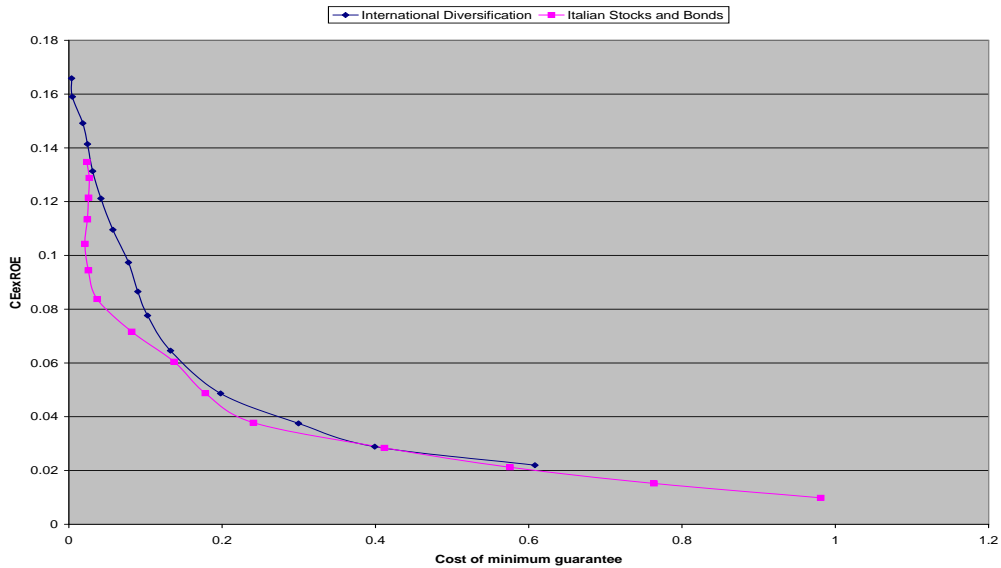


Figure 6: Certainty Equivalent Excess Return of Equity to shareholders vs cost of the minimum guarantee for integrated portfolios in domestic and international markets.

can be achieved by an international portfolio at a cost of 0.02 and CEexROE of 0.14. By contrast a portfolio invested only in the Italian market to back the same 4% guarantee will achieve a CEexROE of 0.11 at the same cost. The difference can be translated into a competitive advantage. But first the firm may wish to sacrifice some of the excess return to hedge currency risk.

4 Designing competitive policies

Equipped with a model to optimally fund a given policy, we are in a position to carry out sensitivity analysis to the policy parameters. For instance, different surrender fees will change the probabilities Λ and the resulting CEexROE vs cost-of-guarantee for a given policy, and consequently the appropriate asset structure. Similarly, the insurer has to decide how much of the cost of the guarantee should be passed on to policyholders by front loading the insurance premium or should be absorbed by the firm as deferred CEexROE. The scenario based model can handle such questions that are central to the design of competitive policies, see the Appendix. Figure 7 illustrates the tradeoffs between the charges to the policyholders and the CEexROE to shareholders for different products. Where exactly to operate on this space is a marketing question. The model quantifies the tradeoffs to aid the specification of a

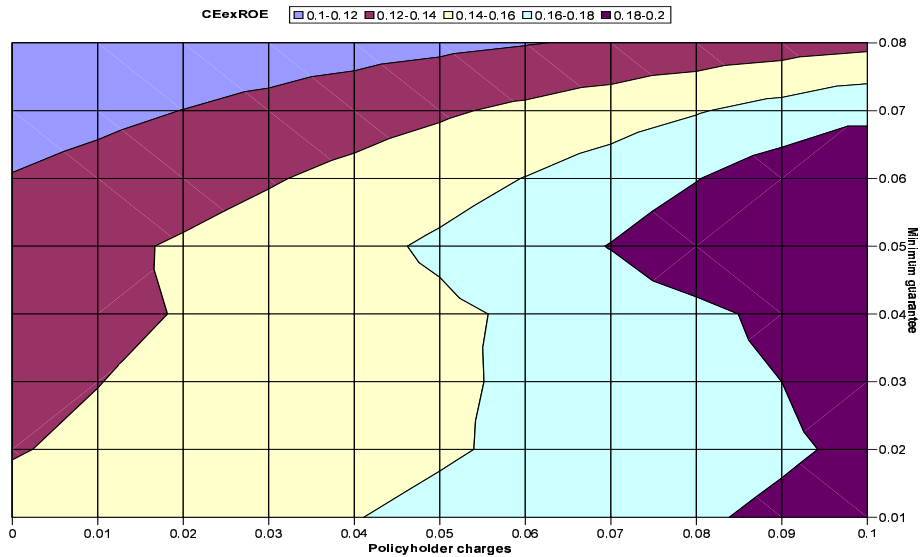


Figure 7: The design of competitive policies can be based on a study of the tradeoffs between the charges to the policyholders, the CEExROE to shareholders, and the guaranteed rate.

competitive policy.

Another key question is to identify the highest minimum guarantee that can be offered to policyholders, and then be backed by portfolios that do not destroy shareholder value. Of course the answer to this question depends on the anticipated returns and volatilities of the available assets. The developed models can be used to identify the highest guarantee that is consistent with creating shareholder value and with the insurer's views about asset returns.

5 Conclusions

We have seen how a scenario based model can provide integrative asset and liability management strategies for products with minimum guarantees. An integrative approach adds value to the risk management process, better policies can be designed and optimal asset portfolios structured to back them. International diversification can be readily incorporated in this modelling framework.

Other aspects of the problem can readily be analyzed too. For instance, alternative reserving methods have been analyzed using a simulation approach by Boyle and Hardy (1997), and the scenario based model by Consiglio, Cocco and Zenios (2000) discussed here. The effects of lapse behavior

on the cost of the guarantee can readily be studied in the context of this formulation as well. Proper surrender fees can thus be estimated.

A The scenario optimization model

We give in this appendix the integrative model. We let Ω denote the set of scenarios $l = 1, 2, \dots, S$, \mathcal{A} the universe of available asset instruments, and $t = 1, 2, \dots, T$ discrete points in time from today ($t = 0$) until maturity T . The data of the problem are as follows.

r_{it}^l , rate of return of asset i during the period $t - 1$ to t in scenario l .

r_{ft}^l , risk free rate during the period $t - 1$ to t in scenario l .

\bar{g} , minimum guaranteed rate of return.

α , participation rate indicating the percentage of portfolio return paid back to the policyholders.

ρ , regulatory equity to debt ratio.

Λ_t^l , probability of abandon of the policy due to lapse or death.

$\phi^l(t, T)$, cumulative return of the short rate during the period from t to T in scenario l .

The variables of the model are defined as follow:

x_i , percentage of initial capital invested in the i th asset.

y_{At}^l , expenses due to lapse or death at time t in scenario l .

z_t^l , shortfall below the minimum guarantee at time t in scenario l .

A_t^l , asset value at time t in scenario l .

E_t^l , total equity at time t in scenario l .

L_t^l , liability value at time t in scenario l .

R_{pt}^l , portfolio rate of return at time t under scenario s .

y_t^{+l} , excess return over \bar{g} at time t in scenario l .

y_t^{-l} , shortfall return under \bar{g} at time t in scenario l .

With this notation we can now define the model. All variables are constrained to be nonnegative except R_{pt}^l , thus short sales are not allowed. We invest the premium collected (L_0) and the equity required by the regulators ($E_0 = \rho L_0$) in the asset portfolio. Our initial endowment $A_0 = L_0(1 + \rho)$ is allocated to assets in proportion x_i such that

$$\sum_{i \in \mathcal{A}} x_i = 1. \quad (4)$$

The dynamics of the portfolio value are given by

$$R_{Pt}^l = \sum_{i \in \mathcal{A}} x_i r_{it}^l, \text{ for } t = 1, 2, \dots, T, \text{ and for all } l \in \Omega. \quad (5)$$

The dynamics for the value of the liability are given by equation (3). To circumvent the discontinuity introduced by the max operator we introduce variables y_t^{+l} and y_t^{-l} to measure the portfolio excess return over the minimum guarantee and the shortfall below the minimum guarantee, respectively. They satisfy:

$$\alpha R_{Pt}^l - \bar{g} = y_t^{+l} - y_t^{-l}, \text{ for } t = 1, 2, \dots, T, \text{ and for all } l \in \Omega, \quad (6)$$

$$y_t^{+l} \geq 0, y_t^{-l} \geq 0, \text{ for } t = 1, 2, \dots, T, \text{ and for all } l \in \Omega. \quad (7)$$

The dynamics for the value of the liability can be written as:

$$L_t^l = (1 - \Lambda_t^l) L_{t-1}^l (1 + \bar{g} + y_t^{+l}), \text{ for } t = 1, 2, \dots, T, \text{ and for all } l \in \Omega. \quad (8)$$

Note that liabilities grow at least at the rate of \bar{g} . Any excess return is added to the liabilities and the minimum guarantee applies to the lifted liabilities in subsequent time periods.

At each period the insurance company will face a cash outflow due to policyholders abandoning their policies either because of death or lapse. The amount to be reimbursed is given by the liability value times the probability of abandon:

$$y_{At}^l = \Lambda_t^l L_{t-1}^l (1 + \bar{g} + y_t^{+l}), \text{ for } t = 1, 2, \dots, T, \text{ and for all } l \in \Omega. \quad (9)$$

Whenever the portfolio experiences a shortfall below the minimum guarantee we need to infuse cash into the asset portfolio in order to meet the final liabilities. Shortfalls are modeled by the dynamics:

$$z_t^l = y_t^{-l} L_{t-1}^l, \text{ for } t = 1, 2, \dots, T, \text{ for all } l \in \Omega. \quad (10)$$

We consider first the case where shortfall are funded through equity. With the modelling construct (6)–(10) it is assumed that equity is reinvested at the

risk-free rate and is returned to the shareholders at the end of the planning horizon. (This is not all the shareholders get; they also receive dividends.) The dynamics of the equity are given by:

$$E_t^l = E_{t-1}^l(1 + r_{ft}^l) + z_t^l, \text{ for } t = 1, 2, \dots, T, \text{ and for all } l \in \Omega. \quad (11)$$

We now have all the components needed to model the asset dynamics. The equation describing the asset dynamics has to take into account the equity infusion that funds the shortfall, z_t^l , and the outflow due to actuarial events, y_{At}^l . Thus, we have:

$$A_t^l = A_{t-1}^l(1 + R_{Pt}^l) + z_t^l - y_{At}^l, \text{ for } t = 1, 2, \dots, T, \text{ and for all } l \in \Omega. \quad (12)$$

In order to satisfy the regulatory constraint the ratio between the equity value and liabilities must exceed ρ , that is,

$$\frac{V_{ET}^l}{L_T^l} \geq \rho, \text{ for all } l \in \Omega, \quad (13)$$

where V_{ET}^l is the value of equity at the end of the planning horizon T . If the company sells only a single policy the value of its equity (V_{ET}^l) will be equal to the final asset return—which includes the equity needed to fund shortfall—minus the final liability due to the policyholders, so we have

$$V_{ET}^l = A_T^l - L_T^l. \quad (14)$$

Finally we define an appropriate objective function. We model the goal of a for-profit institution to maximize the return shareholder value. We use return on equity, after liabilities are paid, is a proxy for this. Since return on equity is scenario dependent we maximize the expected value of the utility of excess return. This expected value is converted into a certainty equivalent value for easy reference. The objective function of the model is to compute the maximal Certainty Equivalent Excess Return on Equity (CEexROE) given by:

$$\text{CEexROE} \doteq \mathcal{U}^{-1} \left\{ \underset{x}{\text{Maximize}} \sum_{s \in \Omega} p^s \mathcal{U} \left[\frac{A_T^l - L_T^l}{E_T^l} \right] \right\}, \quad (15)$$

where $\mathcal{U}\{\cdot\}$ denotes the decision maker's utility function and p^l are the statistical probabilities of the scenarios.

How much does a given level of minimum guarantee cost? (This is the question addressed through an options pricing approach in the literature cited above, Brennan-Schwartz, Boyle-Schwartz, Bacinello, Grosen-Jorgensen). The cost of the minimum guarantee is the total amount of reserves required to fund shortfall due to portfolio performances below the

minimum guarantee. The dynamic variable E_t^l models precisely the total funds required up to time t , valued at the risk-free rate. However, E_t^l also embeds the initial amount of equity required by the regulators. This is not a cost and it must be deducted from E_t^l . Thus, the cost of the minimum guarantee is given as the expected present value of the final equity E_T^l adjusted by the regulatory equity, that is,

$$\bar{O}_G = \sum_{l \in \Omega} p^l \left(\frac{E_T^l}{\prod_{t=1}^T (1 + r_{ft}^l)} - \rho L_0 \right). \quad (16)$$

The mathematical programming model defined by equations (4)–(15) is a nonlinearly constrained optimization model which is computationally intractable for large scale applications. However, the nonlinear constraints (8)–(12) are definitional constraints which determine the value of the respective variables at the end of the horizon. These dynamic equations can be solved analytically and the model is reformulated to a linearly-constrained model which is solved with standard optimization packages. Details of the reformulation and an analysis of the model are given in Consiglio, Cocco and Zenios (2000). The same reference discusses alternative reserving methods for funding shortfalls—equity, short-term or long-term debt—and tradeoffs between shareholders value and policy holders cost. The effects of leverage on shareholder value are also analyzed.

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