Pricing the Risk of Recovery in Default with APR Violation

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Abstract

This paper proposes a simple approach to infer the risk neutral density of recovery rates implied by the prices of the debt securities of a firm. The proposed approach is independent of modeling default arrival rates and allows for the violation of absolute priority rule (APR). The paper demonstrates that a new statistic, the adjusted relative spread, captures risk neutral recovery information in debt prices. Interest rates and firm tangible assets are shown to be significant determinants of the price of recovery. An application illustrates the pricing of credit derivatives written on the realized recovery rate.

JEL specification: G130,G330

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1 Introduction

The risk-neutral density of recovery rates in default is a necessary input to pricing credit derivatives (Jarrow and Turnbull, 2000). In this paper we propose a method to extract the parameterized risk-neutral density of default conditional recovery rates from the optionality embedded in the prices of senior and junior debt of a firm. Our approach exploits the fact that relative prices of securities facing identical arrival risks but differing in their default conditional recovery rates are an important source of information on the price of recovery risk. Jarrow (2000) uses debt and equity prices to estimate simultaneously the risk-neutral default probability with constant recovery level. In contrast, our approach follows Madan and Unal (1998) and utilizes senior and junior debt prices to estimate risk-neutral recovery density as well as the risk-neutral default probability.

An important statistic developed in this paper that synthesizes recovery information from market prices is the *adjusted relative spread*. This is defined as the proportion of senior debt times the ratio of the difference between the prices of senior and junior debt to the difference between default-free and junior debt prices. We show that the adjusted relative spread is free of default timing considerations, positively related to recovery levels and negatively related to the variance of the recovery distribution. Recognizing that senior recovery is aggregate recovery less that of the junior claimants, models for the adjusted relative spread are completed on valuing recovery by the junior claimant conditional on default. Because the adjusted relative spread equation is free of default timing risk, we term this equation the *pure recovery framework*. This equation is fundamental to market-based recovery investigations.

Implementation of the framework leads to the construction of a specific parameterized pure recovery model deriving a valuation formula for the default conditional recovery risk embedded in debt prices. Toward this end we follow Black and Cox (1976) and Stulz and Johnson (1985), and express the recovery rate to junior debt holders in terms of the payoff to a call option written on the aggregate default conditional recovery rate. We extend their approach by valuing this call option assuming APR violation.[1] Hence, we are able to develop models for the market observed adjusted relative spread in terms of the mean and variance of the aggregate recovery rate and parameters that capture APR violations. We show that an empirical model can be further developed by expressing the risk-neutral mean of aggregate recovery rate in default in terms of macro and firm specific variables.

Our empirical investigation follows two steps. First, we evaluate whether or not the cross-sectional variation in the adjusted relative spreads reflects the variation of risk neutral recovery rates. Noting that risk neutral recovery rates are related to actual recovery rates adjusted downward for the effects of risk aversion, we anticipate that risk neutral recovery rankings should be comparable to the rankings of physical recovery rates across firms, especially when risk aversion adjustments are not firm specific. Hence, we compare adjusted relative spread rankings with those obtained on the basis of actual recovery rates.

From the Lehman Brothers Fixed Income Data base we calculate adjusted relative spreads for 28 firms identified from 10 different industries. With respect to actual recovery rates observed in practice we utilize Altman and Kishore (1996) estimates.[2] They report the estimates of the

[1] There exists significant evidence documenting APR violation in banruptcy proceedings (See for example, Franks and Torous (1989,1994), Eberhart, Moore, and Roenfeldt (1989), Weiss (1990), Eberhart and Sweeney(1992), Altman and Eberhart(1994), Betker(1995)). As indicated by Weiss (1990) such violation is plausible because bankruptcy law gives junior creditors the ability to delay final resolution. Hence, senior debt-holders will be willing to violate priority not to incur any additional costs by the delay of the bankruptcy resolution. Frank and Torous (1994) provide empirical evidence documenting deviation from absolute priority by creditor class.

[2] There is extensive literature estimating physical recovery rates by seniority and rating. See for example, Franks and Torous (1994), Van de Castle (1999), Keenan, Hamilton and Berthault (2000), and Hamilton, Gupton, and Berthault (2001).

recovery rates on defaulted bonds stratified by Standard Industrial Classification sector. We show that both cross-sectionally and in the time series (for 48 consecutive months) rank orders based on adjusted relative spreads agree with those based on actual recoveries. These findings confirm our basic contention that adjusted relative spreads are capturing variations in risk neutral recovery levels.

We next investigate the determinants of expected risk neutral recovery rates. In this exercise we specify the mean aggregate risk neutral recovery rate as a function of risk-free interest rates and the level of the tangible assets of the firm. We estimate the pure recovery model for 11 firms using time series data. Adjusted relative spreads are significantly related to interest rates and firm tangible assets. Parameter estimates reflecting the APR violation vary significantly across firms indicating that APR violation is not expected uniformly for all firms. As expected, risk neutral mean recovery rates for the sampled firms lie below the physical recovery rate for the respective industry. This finding suggests that recovery risk is actually being priced by the market participants. Methods employing physical recovery estimates in pricing credit risk are thereby called into question as underpricing the credit risk. Given the widespread prevalence of these procedures we suspect that credit risk is being seriously mispriced by a lack of attention on the issues of risk neutral recovery modeling. Hence, it is essential for correct pricing of credit risk that efforts be made to learn about risk neutral recovery using market prices with embedded optionalities.

An important application of risk neutral density estimation is the pricing of options on the underlying risk. The pricing of puts on recovery is an important credit derivative of which the binary credit default swap is an example (Hull and White (2000)). We couple our estimates of risk neutral recovery densities with estimates of default probabilities inferred directly from market prices to derive prices for put options written on realized recovery levels. The pricing of excess losses is an important activity in the evaluation of credit risk from a sound economic perspective. In this regard we offer an easily implementable market based methodology.

The paper is organized as follows: Section 2 develops the pure recovery model. Section 3 provides evidence that the adjusted relative spreads reflect the variation of physical recovery rates in defaulted bonds. Section 4 proposes an empirical specification for the pure recovery model and provides the model estimates. Section 5 presents the details and results on pricing recovery contingent options. Section 6 concludes the paper.

2 The Pure Recovery Model

We present a general statistic termed the adjusted relative spread that one may derive from market prices and employ as a fundamental variable in recovery modeling. This statistic is defined in subsection 2.1. An explicit model for pricing recovery contingent options is developed in subsection 2.2. This model forms the basis of empirical investigations into the risk neutral recovery density. In subsection 2.3 we briefly summarize the comparative static results with respect to the adjusted relative spread.

2.1 The Adjusted Relative Spread

Consider a frictionless economy where two classes of zero-coupon bonds are traded: default-free and defaultable. Default-free bond price with unit face value and maturity $\tau = T - t$, is given by $P(\tau)$. In the case of defaultable bond, bondholders receive the promised unit face at maturity if the firm survives till maturity. The survival probability of the firm is denoted by $G(\tau)$. Default occurs at a random time and debt holders are paid a reduced value of the face. Expected value of this recovery is denoted by $E[y]$. Assuming the default arrival and the recovery processes to be independent, the standard framework to express the price of defaultable bond is[3]:

$$v(\tau) = P(\tau)G(\tau) + P(\tau)(1 - G(\tau))E[y]. \tag{1}$$

To extend this framework to value defaultable senior and junior debt issues of the firm requires an explicit description of the payoff structure of the debt securities facing identical default arrival risk but different conditional recovery. Toward this end, let $\overline{S}(\tau)$ and $\overline{J}(\tau)$ denote the promised face to senior and junior debt with maturity τ, respectively. Further let $\overline{S}$ and $\overline{J}$ denote the sum of the promised face across all maturities to senior and junior debt. Hence, total promised face of all debt outstanding is $\overline{P} = \overline{S} + \overline{J}$,with the largest maturity denoted by $\overline{T}$. At time of default, the firm defaults on all its outstanding debt obligations. In this case, payment to the outstanding senior and junior debt can be expressed as:

$$S = \int_0^{\overline{T}} S(\tau)d\tau, \tag{2}$$

$$J = \int_0^{\overline{T}} J(\tau)d\tau. \tag{3}$$

Thus, total payment to all debt claimants at time of default is $P = S + J$. This payoff structure can also be expressed in terms of recovery rates. Denoting the aggregate recovery rate to all outstanding debt by y we obtain:

$$y = \frac{P}{\overline{P}} = \frac{\overline{S}}{\overline{S} + \overline{J}} y^S + \frac{\overline{J}}{\overline{S} + \overline{J}} y^J. \tag{4}$$

[3]Madan and Unal (2000) provide a detailed analysis of the assumptions behind this framework.

or

$$y = p_s y^S + (1 - p_s) y^J. \tag{5}$$

In equations (4) and (5) $y^S = \frac{S}{\bar{S}}$ and $y^J = \frac{J}{\bar{J}}$ are the average recovery rates by senior and junior debt holders, respectively. We assume that $y^S = \frac{S}{\bar{S}} = \frac{S(\tau)}{\bar{S}(\tau)}$ and $y^J = \frac{J}{\bar{J}} = \frac{J(\tau)}{\bar{J}(\tau)}$. This assumption implies that at time of default the recovery rate y^S and y^J are applicable to senior and junior debt claimants regardless of maturity. Hence, utilizing the framework of equation (1) we can express the prices of zero-coupon senior $v_S(\tau)$ and junior $v_J(\tau)$ unit face debt instruments of a firm with maturity τ as follows:

$$v_S(\tau) = \left(G(\tau) + (1 - G(\tau))E[y^S]\right) P(\tau), \tag{6}$$

and

$$v_J(\tau) = \left(G(\tau) + (1 - G(\tau))E[y^J]\right) P(\tau). \tag{7}$$

Note that, using equations (6) and (7), the relative spread of the prices of senior to junior debt over the spread of default-free bond to junior debt is:

$$RS = \frac{v_S(\tau) - v_J(\tau)}{P(\tau) - v_J(\tau)} = \frac{E(y^S) - E(y^J)}{1 - E(y^J)}. \tag{8}$$

The relative spread expression, RS, is by design independent of the timing risk ($G(\tau)$). The attractiveness of the RS is that it gives information regarding the market's expectation of the conditions at which default will occur. To see this we simplify the right-hand side of equation (8) such that the relative spread is expressed only in terms of the distribution of the aggregate recovery rate. Note that by definition

$$y^S = \frac{y}{p_s} - \frac{(1-p_s)}{p_s} y^J,$$

and

$$y^S - y^J = \frac{y}{p_s} - \left(\frac{(1-p_s)}{p_s} + 1 \right) y^J. \tag{9}$$

Taking expectations

$$E(y^S) - E(y^J) = \frac{1}{p_s}(E[y] - E(y^J)), \tag{10}$$

and substituting equation (10) in equation (8) we obtain the expression for adjusted relative spread (ARS):

$$ARS = p_s RS = \left(\frac{E(y) - E(y^J)}{1 - E(y^J)} \right). \tag{11}$$

We view equation (11) as *the framework for pure recovery modeling.* In particular one may employ equation (11) to formulate a number of specific recovery models that can be empirically investigated.

2.2 Valuing Post-Default Recovery Contingent Options

The adjusted relative spread may be computed from prices of senior and junior debt claimants and information on the proportion of senior debt. From equation (8) we see that models for this statistic essentially require a parametrization of the risk neutral recovery density and a specification of the payoff structure of the junior claimant.

We begin with the specification of the junior recovery in default as a contingent claim on the aggregate recovery rate y. Toward this end, we first relate y^J to y by the function $y^J = J(y)$.

Next, a specific density, $f(y)$, is proposed for the default conditional aggregate recovery. This results in:

$$E(y^J) = \int_0^1 J(y) f(y) dy. \tag{12}$$

Hence, integrating equation (12) yields the expected value of post-default recovery by junior debt-holders, $E(y^J)$, that is expressed in terms of the parameters of the density $f(y)$.

To specify the payoff function $J(y)$, note that in terms of equation (5), under strict APR, junior debt-holders receive payments only after senior debt-holders are fully paid ($y = p_s$). In this case, the function $J(y)$ can be obtained utilizing Black and Cox (1976). They show that under strict APR, $J(y)$ represents the payoff to a long position on a call option written on the default-conditional recovery rate with a strike equal to the proportion of outstanding senior debt (p_s).[4] In Figure 1, the payoff to senior and junior debt-holders are shown by the bold lines and the p_s is 50 percent. Junior debt-holders receive payments only after the aggregate recovery rate to all debt claimants is above 50 percent.

However, if we allow for APR violation, junior debt-holders receive payments before senior debt-holders are fully paid. Hence, in general we would have a third region where sharing occurs. We capture such sharing by introducing the parameter λ which reflects the argument that junior debt-holders receive nothing ($J(y) = 0$) as long as $y \leq \lambda p_s$ (region 1) and start sharing by receiving payments ($J(y) > 0$) in the region ($y > \lambda p_s$) (region 2). Figure 1 demonstrates such a sharing. We assume $\lambda = .50$. As shown, violation of APR effectively makes the junior debt-holders better off by reducing the strike-price of the call option they are holding and makes the senior-debt-holders worse off. In the region, $y \leq \lambda p_s$, $S(y)$ can be determined by the product of $(\frac{\lambda}{1-(1-\lambda p_s)})$ and y

[4]In the same manner, $S(y)$ represents the payoff to a default-free bond and a short position on a put option written on the firm's default-conditional payout.

which effectively equals $\frac{y}{p_s}$. For example, when $y = \lambda p_s$, senior-debt holders will be paid only 25 percent of their promised amount and junior debt-holders will receive no payment. However, any improvement in y above λ will not totally accrue to the senior debt-holders but will be shared with the junior debt-holders. In region 2, $(y > \lambda p_s)$, $J(y)$ is determined by the product of, $(\frac{1}{1-\lambda p_s})$ and the increment of y over λp_s. However, in region 2, we suppose that the recovery rate to the senior claimant $1/p_s$ is reduced by a constant θ for a value of $\theta < 1$. The specific recovery by the senior claimant in this region starts at λ and increases at the rate θ/p_s and is

$$S(y) = \lambda + \frac{\theta}{p_s}(y - \lambda p_s). \tag{13}$$

Note on this pattern the senior claimant is fully paid off at the aggregate recovery level y^*,

$$y^* = \lambda p_s + \frac{(1-\lambda)p_s}{\theta}. \tag{14}$$

To ensure that $y^* \leq 1$ we must have

$$\theta \geq \frac{p_s - \lambda p_s}{1 - \lambda p_s}. \tag{15}$$

Hence, the recovery rate by the junior must be adjusted as $\frac{(1-\theta)(y-\lambda p_s)}{1-p_s}$. In the region $y > y^*$ (region 3) we clearly have that $S(y) = 1$ and $J(y) = \frac{y-p_s}{1-p_s}$. In summary, the payments to the junior claimant in the three regions are given by

$$J(y) = \begin{cases} 0 & y \leq \lambda p_s \\ \frac{(1-\theta)(y-\lambda p_s)}{1-p_s} & \lambda p_s < y \leq y^* \\ \frac{y-p_s}{1-p_s} & y^* < y \leq 1 \end{cases} \tag{16}$$

Alternatively,

$$J(y) = \frac{1-\theta}{1-p_s}\ Max(y - \lambda p_s, 0) + \frac{\theta}{1-p_s}\ Max(y - y^*, 0)\ . \tag{17}$$

As can be observed, for $\lambda = 1$ (APR enforced), we obtain the Black and Cox (1976) characterization of junior debt-holders holding a call option and acting like equity-holders. With APR violation, ($\lambda < 1$), the value of the call options increase making senior debt-holders worse off and the junior debt-holders better off. Hence, equation (17) show that the junior debt-holders' payoff function can be expressed in terms of two call options written on the firm's expected default-conditional aggregate recovery rate, with strikes λp_s and y^*. They are holding $\frac{1-\theta}{1-p_s}$ units of the first and $\frac{\theta}{1-p_s}$ of the second call option.

The second component to be evaluated in equation (12) is $f(y)$. From this density one can determine the probability of the call options given in equation (17) to be in the money once default occurs. Hence, the integral in equation (12), for example, represents the value of the call option held by the junior debt-holder.

A straightforward assumption would be to assume that y is normally distributed. However, such an assumption violates two important characteristics of the aggregate recovery rate. First, y lies between 0 and 1 because it is the ratio of recovery to the promised payments to debt claimants at any default time. Second, the mean and variance of y are related because as the mean approaches unity (100 percent recovery rate) or zero the variance of y becomes zero. Hence, we propose that the aggregate recovery rate is related to a normal random variable x by the logit transformation $y = \frac{e^x}{1+e^x}$. Further, we assume that the variable x, which is the logarithm of the default conditional recovery to loss ratio $\left(x = \ln\left(\frac{y}{1-y}\right)\right)$, is normally distributed with mean μ

and variance σ^2. It follows that the conditional density for the aggregate recovery rate, $f(y)$, is:

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}y(1-y)} \exp\left(-\frac{1}{2\sigma^2}\left(\ln\left(\frac{y}{1-y}\right)-\mu\right)^2\right), \qquad 0 < y < 1. \tag{18}$$

The characteristics of the recovery rate are captured in the density given in equation (18). Figure (2) shows the density for the recovery level for $\mu = \pm\, 0.5$ and $\sigma = 0.25$, and 0.5. We observe that the density may be positioned at various points on the unit interval and it may be widely or narrowly spread out.

The mean and variance of the recovery density can also be expressed in terms of μ and σ.

Proposition 1 *The mean and variance of the firm's aggregate recovery rate y, given μ and σ^2 are* :

$$E(y) = 1 - \int_0^1 N\left(\frac{\ln\left(\frac{y}{1-y}\right)-\mu}{\sigma}\right) dy, \tag{19}$$

$$Var(y) = \int_0^1 2(1-y)N\left(\frac{\ln\left(\frac{y}{1-y}\right)-\mu}{\sigma}\right) dy - \left(\int_0^1 N\left(\frac{\ln\left(\frac{y}{1-y}\right)-\mu}{\sigma}\right) dy\right)^2. \tag{20}$$

Proof in the Appendix

Given this density, the value of the call option can be expressed as follows:

Proposition 2 *The call option written on the firm's aggregate recovery with strike k, pays the following conditional on default:*

$$C(k;\mu,\sigma^2) = 1 - k - \int_k^1 N\left(\frac{\ln\frac{y}{1-y}-\mu}{\sigma}\right) dy. \tag{21}$$

Proof in the Appendix.

The integral of $N(\cdot)$ in equation (21) is easily evaluated numerically. Hence, the post-default expected recovery rate for the junior debt allowing for APR violation is given by:

$$\begin{aligned} E(y^J) &= \frac{1-\theta}{1-p_s} C(\lambda p_s; \mu, \sigma^2) + \frac{\theta}{1-p_s} C(y^*; \mu, \sigma^2), \\ y^* &= \lambda p_s + \frac{(1-\lambda)p_s}{\theta}. \end{aligned} \tag{22}$$

Substituting equation (22) in equation (11),we obtain:

$$ARS = \left(\frac{C(0; \mu, \sigma^2) - \frac{1-\theta}{1-p_s} C(\lambda p_s; \mu, \sigma^2) - \frac{\theta}{1-p_s} C(y^*; \mu, \sigma^2)}{1 - \frac{1-\theta}{1-p_s} C(\lambda p_s; \mu, \sigma^2) - \frac{\theta}{1-p_s} C(y^*; \mu, \sigma^2)} \right). \tag{23}$$

Hence, the pure recovery model is fully expressed in terms of option type payoffs with mean μ and variance σ^2, which are related to the mean and variance of the expected aggregate recovery rate using Proposition 1. In other words, once μ and variance σ are estimated using proposition 1 one can easily obtain the risk neutral mean and volatility of aggregate recovery rate in default.

2.3 Parameter Sensitivity of Adjusted Relative Spread

This section evaluates the sensitivity of the pure recovery model to parameters λ, θ, σ,and μ and obtains empirically testable implications. Figure 3 assumes $\lambda = 0.50$, $\theta = 0.50$ and $p_s = 0.50$ and examines the behavior of the ARS as μ and σ vary. First, we observe that ARS is an increasing function of μ. Higher levels of ARS implies higher aggregate recovery.

The second important observation is that ARS reflects the recovery by senior debt. This result is expected because note that the numerator of equation (23) represents the difference between the aggregate recovery and the recovery by the junior debt holders. This difference is nothing but the recovery by senior debt holders. Hence, ARS can be seen as a statistic capturing the recovery by senior debt holders deflated by the premium of the junior debt over the risk-free debt. As σ approaches zero the ARS curve begins reflecting the payoff structure described in Figure 1. This is plausible because a low volatility implies the mean recovery will be realized with certainty. Hence, the curve represents recovery for the senior debt at various levels of mean recovery as depicted by the three different payoff regions. Junior debtholders receive nothing in region 1. Sharing occurs in region 2, that starts after $y = \lambda p_s = 0.25$. For $y \geq y^* = 0.75$ (region 3) senior debt becomes risk-free and ARS becomes 0.5. However, ARS decreases with increased uncertainty of the aggregate recovery rate which is negative news for the senior debt holders. Hence ARS curve shifts down.

Figure 4 displays the impact of APR violation parameter λ on ARS. As λ increases, sharing between senior and junior debtholders starts after a higher portion of senior debt is paid. Hence, senior debt will be more valuable, and ARS will increase. This is what we observe in Figure 4 and the ARS curve shifts up as λ increases. Similarly, senior debt holders benefit as the rate of increase in λ, the θ parameter, increases. This is because a higher θ indicates less of the recovered face value is shared with junior debtholders. Therefore, senior debt is paid out more quickly and is more valuable, which benefits the senior debt holders.

3 Adjusted Relative Spreads and Physical Recovery Rates

The pure recovery model relates the adjusted relative spread, a particular construct of market prices, to the distribution of risk neutral recovery. This suggests that adjusted relative spreads should be related to physical recovery levels as we expect that risk neutral and physical recovery rates are related. Our empirical analysis first investigates whether adjusted relative spreads, are at all related to physical recoveries in default. Next we take up the full fledged estimation of the proposed pure recovery model.

3.1 Data

Corporate bond data are obtained from the Lehman Brothers Fixed Income Data Base. The database provides end-of-month bid price, coupon rate, yield-to-maturity, industry classification and other important information for the bonds constituting the Lehman Bond Index. Putable bonds, nonregular bonds and bonds with sinking fund features are excluded from the sample. We further remove bond observations with more than 10 years and less than 6 months of maturity. Firms with only senior or only junior bonds are also deleted from the sample. We include those callable bonds where we could identify junior and senior bond issue of a firm that are both callable.

Majority of the corporate bond issues are coupon paying bonds and restricting the sample to zero-coupon bonds would have caused very few observations. However, identifying senior and junior debt issue of a firm with identical coupon structure is also very difficult. To include coupon bonds in the study we follow the following matching strategy. For each date, we match a junior bond to another senior bond issued by the same firm with closest possible duration and coupon rate. Our decision criteria for this match is defined by two numbers, $\delta_1 = \dfrac{|d_S - d_J|}{(d_S + d_J)/2}$ and $\delta_2 = |C_S - C_J|$, where C_S and C_J are the coupon rates, and d_S and d_J are the Macaulay durations of senior and junior bonds, respectively. If $\delta_1 \leq 0.3$ and $\delta_2 \leq 0.03$ we accept the match, otherwise the adjusted relative spread is considered to be missing for that junior bond at this date. We calculate zero coupon senior, junior and Treasury bond prices $v_S(\tau)$,$v_J(\tau)$ and

$P(\tau)$ by discounting a \$100 face value with the available yield-to-maturity at $\tau = d_J$.

The resulting sample consists of 33 ARS statistics for 28 companies. The companies are reported in Table 1 together with the industry they represent. The table also reports starting and ending dates of the observations. As can be observed in three cases we are able to determine the ARS statistic using more than one pairings of the bond.

3.2 Cross-sectional and time-series variation in adjusted relative spreads

Altman and Kishore (1996) document recovery rates in bond defaults classified by Standard Industrial Classification (SIC) sectors. We utilize their study to contrast the industry estimates with the ARSs reported in Table 1. Table 2 reports the comparison. We observe that the ranking relationship between ARSs and the recovery rates are remarkably close. Public utilities and chemical and petroleum companies have the highest ARSs, which is consistent with the recovery rates estimated for these industries by Altman and Kishore. Furthermore, the correlation between ARSs at the firm level and the recovery rates of the industry the firm belongs is 0.73 and is significant at the 1 percent level.

To gain further insight, we group firms into high recovery, medium recovery, and low recovery industries using the Altman and Kishore industry recovery estimates. Industries where Altman and Kishore recovery rate estimates exceed 45% are defined as high recovery group, industries with recovery rates below 35% constitute the low recovery group. Hence, industries 1-3, 4-7 and 8-10 in Table 2, constitute the High, Medium and Low recovery groups, respectively. Next we assign firms reported in Table 1 to one of these three portfolios and obtain monthly average ARS for each portfolio. Figure 5 plots the time series pattern of ARSs for the three portfolios. Consistent with our expectations there is a pecking order going from the ARS curve of the high recovery group toward the low recovery group. This difference also persists over time.

Hence, the evidence presented strongly supports the argument that ARSs are indeed related to physical recovery rates. A high level of ARS is associated with higher recovery level and this prediction holds for cross-sectional as well time series behavior of adjusted relative spreads.

4 Estimating the Pure Recovery Model

4.1 Empirical Specification

The relative spread model of equation (23) may be adapted to analyze the conjectured dependence of recovery rates on the business cycle and on appropriate firm specific information. For such an exercise we denote by x_t a time series on a vector of macro and firm specific variables that are presumed to affect recovery levels. We then consider the model

$$\mu_t = \beta_0 + \beta' x_t, \tag{24}$$

and summarize the model of equation (23) by the relation

$$ARS_t = \Phi(\lambda, \theta, \mu_t, \sigma, p_s) + \varepsilon_t, \tag{25}$$

where it is supposed that the error term represents uncorrelated statistical noise.

Equation (25) in conjunction with equation (24) constitutes a potentially estimable econometric model permitting estimation of the recovery model of equation (24) together with the APR violation parameters, λ, θ and the volatility of the log recovery to loss ratio, σ.

To choose plausible firm specific variables we follow the study by Altman and Kishore (1996).

They argue that recovery rates are related to the asset structure of firms and provide evidence that firms with more tangible and liquid assets have a higher liquidation value, and therefore higher recovery rates upon default. In addition, there exists evidence showing that recovery rates vary with macro-economic variables (Franks and Torous(1994)). As a result, we employ the following two factor model to capture impact of firm specific and macroeconomic variables on mean recovery rates.

$$\mu_t = \beta_0 + \beta_1 RF_t + \beta_2 TANG_t. \tag{26}$$

The model is estimated is estimated using time series data. $TANG$ represents the tangible assets of the firm. We define tangible assets as the sum of current assets (COMPUSTAT quarterly item 40) and net plant property and equipment (COMPUSTAT quarterly item 42) divided by total assets (COMPUSTAT quarterly item 44). We predict a positive relationship between $TANG$ and implied recovery rates. RF is the risk free rate and controls for the interest rate risk environment. The risk free rates at the desired date and maturity are calculated from daily treasury bond yields that come from the H15 release of the Federal Reserve System. The yield curve is spanned with cubic spline method to find the risk free rate at any maturity.

The requirement that data availability in COMPUSTAT files for firms whose adjusted relative spreads are reported in Table 1 causes further shrinkage in our sample. We identify 11 out of 28 firms to have data in both sources and have sufficient time series data available for the ARSs.

Table 3 provides descriptive statistics about the time series data for the firms used in the estimation. We observe that nine firms have speculatively rated bonds (B and BB). The average duration of the bonds is approximately 5 years. In nine cases firms have a tangible asset ratio of more than 60 percent and senior debt ratio varies between 38 percent and 90 percent. Finally, we note that there is a economically significant spread between between senior bond and treasuries and senior bond and junior bonds.

4.2 Results

The nonlinear least squares estimate of the pure recovery model is reported in Table 4 for the sample firms. The first three columns report estimates of the hypothesized determinants of the aggregate recovery rate. The risk-free rate is positive and significant in six cases. This is plausible given that rising interest rates benefit the assets by increasing cash and earnings implying a higher recovery in case of default. The estimates relating to the tangible assets are also as expected. They are all positive and in 9 cases the t-values are significant. This confirms Altman and Kishore (1996) arguments that recovery rates are higher for firms having higher tangible assets.

Column 4 and 5 report estimates of the APR violation parameters. First we observe that the estimated values vary significantly across firms. This can be construed as evidence that ex ante there is no uniform expectation by the market participants about how APR will be violated conditional on default, across firms. Column 6 shows the estimate of the volatility term, σ. The uncertainty related to the recovery rate can vary significantly across firms.

The parameters λ, θ, and σ are structural and reflect variations in the functional form of the dependence of adjusted relative spreads to the data on the explanatory variables (interest rates and the level of tangible assets). The exact functional form is not identified with precision and this is reflected in high standard errors for the estimates of λ, θ, and σ. Hence, the t-statistics reported for the explanatory variables are conditional on the estimated values for λ, θ, and σ.

4.3 Applications of the model

4.3.1 Risk Neutral Mean and Volatility of Recovery in Default

For the 11 firms we employ the parameter estimates reported in Table 4 along with equations (19) and (20) to construct risk neutral mean recovery and its volatility. These are reported in Table 5 alongside with the corresponding mean physical recovery level for the industry. We observe that in 9 of the 11 cases the risk neutral means are significantly below their physical counterparts.

We note that these distributional moments are risk neutral entities that are differentiated from their physical counterparts. Specifically we anticipate that risk aversion reduces the risk neutral recovery rate below the expected physical recovery and also raises the risk neutral variance above the true variance. These differences between the risk neutral and physical outcomes are the commonly observed impacts of risk aversion in options markets as documented by lower risk neutral rates of return and higher implied volatilities. [5]

It is recognized that in the absence of systematic risk in default recoveries, risk neutral and physical recovery rates should be identical (Hull and White (2000)). However, given the particularity in time of default occurrences, and their substantial size, it is difficult to see how the risk of recovery in default can be diversified away. In the absence of such diversification, risk aversion considerations predict that risk neutral expected recoveries would be lower and simultaneously default probabilities would be higher. The comparison of our risk neutral estimates with physical historical industry averages on both dimensions supports our contention that systematic risk is an element of risk neutral recovery.

[5] For an excellent discussion on the differences between risk neutral and physical probabilities see Saunders(1999), Chapter 9.

4.3.2 Put Options on Realized Recovery in Default

An important application made possible by our identification of the risk neutral recovery density is the ability to price credit derivatives written on the realized recovery rate. We illustrate the calculation by directly applying our risk neutral density identifications to pricing put options written on the level of post-default recovery levels.

Suppose that a counterparty has a claim to an agreed upon principal from an economic entity. This may take the form of present values of promised coupons in a swap contract. Default by the economic entity in question is then a serious issue for the counterparty and they could be interested in insurance against excessive losses that exceed capital reserves set aside to carry such a loss. The counterparty may then cover this potentially negative situation by purchasing a put option that pays out in proportion to the shortfall of recovery levels below a prespecified strike (the capital reserve).

This insurance contract can be priced as a put option written on the recovery rates of the economic entity. Suppose the level of recovery measured in final dollars is y on debt with maturity date T. For a notional principal of K the insurance pays the equivalent of

$$I = K\,(k - y)^{+}\,, \tag{27}$$

with strike k in time T dollars if there is default at time $z \leq T$ with a recovery of y. The actual dollar payment is the value of Treasury bonds at time z with a time-to-maturity of τ and a face value of F.

Equation 28 gives the price, $w(t;\tau,k,K)$, of such an insurance contract as

$$w(t;\tau,k,K)=P(\tau)K(1-G(\tau))E((k-y)^{+})\,, \tag{28}$$

where the expectation is taken with respect to the risk neutral distribution on recovery. Note that the expectation in equation (28) denotes a put option written on recovery rate y with a strike of k . Using put-call parity, the price of the insurance becomes

$$w(\tau,k,K)=P(\tau)K(1-G(\tau))\int_{0}^{k}N\left(\frac{\ln\left(\frac{y}{1-y}\right)-\mu_t}{\sigma}\right)dy. \tag{29}$$

To complete the exercise on pricing the insurance we need to determine the risk neutral default probability, $G(\tau)$. We determine $G(\tau)$ directly from market prices without formulating a model for the arrival rate of default. Note that equations (5), (6), and (7) yield:

$$p_s v_S(\tau)+(1-p_s)v_J(\tau)=(G(\tau)+(1-G(\tau))E[y])P(\tau), \tag{30}$$

or equivalently,

$$G(\tau)=\frac{p_s v_S(\tau)+(1-p_s)v_J(\tau)-E[y]P(\tau)}{P(\tau)(1-E[y])}. \tag{31}$$

Hence, combining (31) with equation (29) we can price the option on post default recovery. Table 6 provides estimates for our sample firms assuming the amount of capital reserves are set at 8%, or $k=0.92$. The price of this insurance varies between \$3 and \$21 per \$100 of notional amount.

These put options may be used by market participants to transform risky loans back to risk free ones. The cost of the insurance raises the interest rate on the loan and builds in an implied risky yield spread. By way of example we note that the yield spread associated with the put option prices for AMC, Flagstar, and Valassis Inserts are 226, 592 and 60 basis points respectively. The corresponding ratings are $B, B-$, and $BBB-$. These spreads assume that the lender buys the insurance and receives on average 96 dollars on a risk free basis. The risk free rate is assumed to be 6% with maturities as given in Table 3. These spreads are broadly consistent with the associated ratings.

5 Conclusion

This paper proposes a parsimonious way to extract the parameterized risk neutral density of default conditional recovery rates from data on a firm's senior and junior debt prices, the level of the senior debt, tangible assets, and risk free interest rates. This is an important advance in understanding the determinants of default spreads as there is little possibility of direct observation of the quantities of interest, given the absence of the occurrence of the event, ex ante.

The empirical experiments reported ascertain market sentiments on the recovery dimension of default. An important contribution of the paper is to demonstrate that a new statistic, the *adjusted relative spread*, captures recovery information embedded in debt prices. Risk-neutral mean recovery-rate estimates for a sample of industrial firms show that the recovery rankings for these firms are comparable to the industry-level recovery rankings reported in Altman and Kishore (1996). However, the estimated risk neutral means are significantly below their physical counterparts. This raises the concern that the use of physical recovery levels in pricing credit risk may seriously underprice the risks involved. As an illustration we demonstrate the use of the risk-neutral mean and volatility estimates to price put options written on the recovery risk.

Hence, we conclude that it is essential for correct pricing of credit risk that efforts be made to learn about risk neutral recovery using market prices with embedded optionalities.

6 Appendix

Proof of Proposition 1*:* The mean of recovery rate y is defined as:

$$E(y) = \int\limits_0^1 y f(y) dy. \tag{32}$$

Applying integration by parts to (32):

$$E(y) = yF(y)\Big|_0^1 - \int_0^1 F(y)dy = 1 - \int_0^1 F(y)dy. \tag{33}$$

We determine $F(y)$ in terms of the standard normal distribution function $N(\cdot)$. For any real number u, $0 \leq u \leq 1$,

$$\begin{aligned} F(u) &= \text{Prob}(y < u) \\ &= \text{Prob}\left(\frac{e^x}{1+e^x} < u\right) \qquad (34) \\ &= \text{Prob}\left(e^x < \frac{u}{1-u}\right) \\ &= \text{Prob}\left(x < \ln\left(\frac{u}{1-u}\right)\right). \qquad (35) \end{aligned}$$

Assuming $x \simeq N(\mu, \sigma^2)$

$$= N\left(\frac{\ln\left(\frac{u}{1-u}\right) - \mu}{\sigma}\right). \tag{36}$$

$$E(y) = 1 - \int\limits_0^1 N\left(\frac{\ln\left(\frac{y}{1-y}\right) - \mu}{\sigma}\right) dy. \tag{37}$$

Variance of recovery rate y, can be expressed as a difference of two terms:

$$Var(y) = E(y^2) - E(y)^2. \tag{38}$$

Evaluating the first term in (38):

$$E(y^2) = \int_0^1 y^2 f(y) dy. \tag{39}$$

Applying integration by parts to equation (39) we obtain:

$$E(y^2) = y^2 F(y) \Big|_0^1 - \int_0^1 yF(y)dy = 1 - \int_0^1 yF(y)dy. \tag{40}$$

Substituting the expression for $F(y)$ into equation (40):

$$E(y^2) = 1 - \int_0^1 yN\left(\frac{\ln\left(\frac{y}{1-y}\right) - \mu}{\sigma}\right) dy. \tag{41}$$

Substituting this result and the expression for $E(y)$ from equation (37) into equation (38) and rearranging we obtain:

$$Var(y) = \int_0^1 2(1-y)N\left(\frac{\ln\left(\frac{y}{1-y}\right) - \mu}{\sigma}\right) dy - \left(\int_0^1 N\left(\frac{\ln\left(\frac{y}{1-y}\right) - \mu}{\sigma}\right) dy\right)^2. \tag{42}$$

Q.E.D.

Proof of Proposition 2: To obtain the pricing expression for the call option, note that for strike k we can express the price of a call option written on the underlying asset y as follows:

$$C(k) = \int_k^1 (y-k) f(y) dy, \tag{43}$$

where $f(y)$ is the probability density of y. Equation 43 can be expressed as:

$$C(k) = \int_k^1 y f(y) dy - k \int_k^1 f(y) dy. \tag{44}$$

We evaluate the second integral first. Note that this term is equal to

$$\int_k^1 f(y) dy = \text{Prob}(y > k) = 1 - F(k), \tag{45}$$

where $F(y)$ is the distribution function of y. Using the expression for $F(y)$ from Proposition 1, equation (36) the second term in equation (44) becomes

$$k \int_k^1 f(y) dy = k - kN\left(\frac{\ln\left(\frac{k}{1-k}\right) - \mu}{\sigma}\right). \tag{46}$$

The first term in equation (44) is evaluated as:

$$\begin{aligned} \int_k^1 y f(y) dy &= yF(y)\Big|_k^1 - \int_k^1 F(y) dy \\ &= 1 - kF(k) - \int_k^1 F(y) dy. \end{aligned} \tag{47}$$

It follows from equation (36) that

$$\int_k^1 y f(y) dy = 1 - kN\left(\frac{\ln\left(\frac{k}{1-k}\right) - \mu}{\sigma}\right) - \int_k^1 N\left(\frac{\ln\left(\frac{y}{1-y}\right) - \mu}{\sigma}\right) dy. \tag{48}$$

Substituting equation (46) and equation (48) into equation (44) we obtain the call option valuation expression given in Proposition 2:

$$C(k;\mu,\sigma^2) = 1 - k - \int_k^1 N\left(\frac{\ln\left(\frac{y}{1-y}\right) - \mu}{\sigma}\right) dy. \tag{49}$$

Q.E.D.

7 References

Altman, E.I., Eberhart, A.C.,1994. Do seniority provisions protect bondholders' investments?. Journal of Portfolio Management 20, 67-75.

Altman, E.I., Kishore, V. M.,1996. Almost everything you wanted to know about recoveries on defaulted bonds, Financial Analyst Journal 52, 57-64.

Betker, B. L., 1995. Management's incentives, equity's bargaining power, and deviations from absolute priority in Chapter 11 bankruptcies. Journal of Business 68, 161-193.

Black, F., Cox, J. C. 1976. Valuing corporate securities: Some effects of bond indenture provisions. Journal of Finance 31, 351-367.

Eberhart, A.C., Moore,W. T., Roenfeldt, R. I., 1990. Security pricing and deviations from the absolute priority rule in bankruptcy proceedings. Journal of Finance 45, 1457-1469.

Eberhart, A. C., Sweeney, R. J., 1992. Does the bond market predict bankruptcy settlements. Journal of Finance 47,943-980.

Franks, J. R., Torous, W. N., 1989. An empirical investigation of US. firms in reorganization. Journal of Finance 44, 747-769.

Franks, J. R., Torous, W. N., 1994. A comparison of financial recontracting in distressed exchanges and Chapter 11 reorganizations. Journal of Financial Economics 35, 349-370.

Hamilton, D.T., Gupton, G., Berthault, A., 2001. Default and recovery rates of corporate bond issuers: 2000. Moody's Investor Service, February 2001.

Hull, J. White, A., 2000. Valuing Credit Default Swaps I: No Counterparty Default Risk. Working paper, University of Toronto.

Jarrow, R., 2000. Estimating recovery rates and (Pseudo) default probabilities implicit in debt and equity prices: Theory and empirical. Working Paper, Cornell University.

J.P. Morgan, 1997. CreditMetrics-Technical Document, New York.

Keenan, S.C., Hamilton, D.T., Berthault, A., 2000. Historical default rates of corporate bond issuers, 1920-1999. Moody's Investor Service, January 2000.

Madan, D., Unal, H., 1998. Pricing the risks of default. Review of Derivatives Research 2, 121-160.

Madan, D., Unal, H., 2000. A Two-Factor Hazard Rate Model for Pricing Risky Debt and the Term Structure of Credit Spreads. Journal of Financial and Quantitative Analysis, 43-65.

Saunders, A., 1999. Credit Risk Measurement : New approaches to Value at Risk and other Paradigms. John Wiley and Sons, New York.

Stulz, R. M., Johnson, H., 1985. An analysis of secured debt. Journal of Financial Economics 14, 501-521.

Van de Castle, K., Keisman, D., 1999. Recovering Your Money: Insights Into Losses from Defaults. Standard and Poor's CreditWeek, June 16 1999

Weiss, L. A., 1990. Bankruptcy resolution: Direct costs and violation of priority of claims. Journal of Financial Economics, 27, 285-314.

Table 1: The sample

This table reports for each firm the issuer name, sample period, number of observations (T), and sample averages of adjusted relative spread (ARS).ARS is the product of the senior debt ratio, p_s, and the relative spread (price difference between senior and junior bond divided by the price difference between default-free bond and junior bond).

2 Digit SIC Code	Company Name	Sample Period	T	Average of ARS
78	AMC Entertainment Inc.	9208-9601	42	0.250
80	American Medical Intn'l	9111-9503	40	0.123
49	Coastal Corporation	9002-9305	38	0.614
15	Del Webb Corp	9305-9703	30	0.262
75	Envirotest Systems	9403-9712	46	0.343
58	Family Restaurants	9402-9712	11	0.237
58	Flagstar	9309-9712	42	0.126
30	Foamex L.P.	9410-9706	24	0.284
58	Foodmaker, Inc	9206-9712	35	0.352
54	Grand Union	9207-9506	30	0.181
75	Hertz Corp	9105-9705	48	0.390
33	Kaiser Alum. and Chemical	9302-9712	47	0.140
54	Kroger I	9402-9701	36	0.148
54	Kroger II	9402-9510	21	0.105
54	Kroger III	9208-9712	28	0.162
48	Lenfest Communictions Inc.	9610-9712	15	0.092
37	Newport News Shipbuilding	9706-9712	7	0.057
76	Prime Hospitality Corp	9706-9712	7	0.141
26	Printpack Inc	9704-9712	9	0.178
54	Ralphs Grocery Co I	9506-9712	31	0.154
54	Ralphs Grocery Co II	9506-9712	31	0.185
28	Revlon Consumer Products	9308-9712	52	0.382
26	Riverwood International	9206-9606	45	0.190
54	Safeway Stores Inc.	9703-9712	8	0.033
44	Sea Containers	9412-9712	37	0.175
37	Sequa Corp	9312-9712	49	0.390
26	Stone Container Corp I	9204-9712	59	0.095
26	Stone Container Corp II	9705-9712	8	0.125
30	Sweetheart Cup	9309-9712	35	0.485
59	Thrifty Payless Holding	9404-9505	12	0.058
59	Thrifty Payless	9404-9604	25	0.087
37	UNC Inc	9611-9712	11	0.383
73	Valassis Inserts	9203-9712	60	0.143

Table 2: Variation of adjusted relative spread across industries

This table classifies the firms in the sample into ten different industry groups and presents averages of actual recoveries and adjusted relative spreads(ARS).ARS is the product of the senior debt ratio, p_s, and the relative spread (price difference between senior and junior bond divided by the price difference between default-free bond and junior bond). Industry classifications, and industry average recovery rates in Column 5 are obtained from Table 3 in Altman and Kishore(1996). Industry averages of ARS are calculated by averaging ARS statistic first across time, then across firms.

Industry Number	Industry Name	2 Digit SIC Codes	Number of firms	Recovery rates by industry	Average of ARS by industry
1	Public utilities	49	1	0.705	0.614
2	Chemicals, petroleum,rubber and plastic products	28-30	3	0.627	0.383
3	Machinery, instruments and related products	35,36,38	3	0.462	0.292
4	Building materials,metals and fabricated products	32-34	1	0.388	0.140
5	Transportation and transportation equipment	37,41,42,45	4	0.384	0.251
6	Communication,broadcasting,movie production, printing and publishing	27,48,78	2	0.371	0.171
7	Construction and real estate	15,65	1	0.353	0.261
8	General merchandise stores	53-59	12	0.332	0.152
9	Wood, paper and leather products	24-26,31	4	0.298	0.147
10	Lodging, hospitals and nursing facilities	70-89	2	0.265	0.132

Table 3: Descriptive statistics

This table reports descriptive statistics for 11 sample firms for which the pure recovery model is estimated. N denotes the number of observations used for each estimation. The average S&P Rating of the firm is calculated by by weighting the senior and junior debt rating by the senior debt ratio. For each statistic the reported number represents averages over the sample period.

Company	Sample Period	N	S&P Rating	Duration (years)	Senior Debt Ratio (p_S)	Risk Free Rate	Tangible Asset Ratio	Treasury Price, $P(\tau)$	Senior Debt Price, $v_S(\tau)$	Junior Debt Price, $v_J(\tau)$
AMC	9208-9601	42	B	5.30	50.8%	6.1%	83.0%	$73.19	$64.68	$57.45
American Medical	9111-9412	37	B+	5.20	53.7	6.1	56.8	73.72	61.60	57.75
Coastal Corp	9002-9305	38	BBB-	4.62	90.1	6.8	87.0	73.61	70.38	64.12
Envirotest Systems	9403-9712	46	B	4.92	63.0	6.4	78.9	73.61	63.56	52.52
Flagstar	9309-9702	42	B-	4.98	42.7	6.3	86.3	73.88	55.77	46.93
Revlon	9308-9712	52	B	5.07	68.6	6.3	72.8	73.61	66.81	59.59
Sequa Corp	9312-9712	49	BB-	5.55	61.9	6.4	69.2	70.91	65.91	58.61
Stone Container	9204-9712	59	B+	4.80	66.0	6.3	72.7	74.59	63.14	60.04
Sweetheart Cup	9412-9607	20	B	5.40	63.0	6.4	97.0	71.74	69.48	57.73
Valassis Inserts	9203-9712	60	BBB-	3.32	70.0	5.8	66.6	83.29	78.26	77.13
Del Webb Corp	9305-9511	30	B	5.80	38.1	6.3	3.0	70.35	66.23	56.27

Table 4: Time-series estimation of the pure recovery model

The pure recovery model is $ARS_t = \left(\frac{C(0;\mu,\sigma^2) - \frac{1-\theta}{1-p_s} C(\lambda p_s;\mu,\sigma^2) - \frac{\theta}{1-p_s} C(y^*;\mu,\sigma^2)}{1 - \frac{1-\theta}{1-p_s} C(\lambda p_s;\mu,\sigma^2) - \frac{\theta}{1-p_s} C(y^*;\mu,\sigma^2)} \right)$ where $y^* = \lambda p_s + \frac{(1-\lambda)p_s}{\theta}$ and $\mu_t = \beta_0 + \beta_1 RF_t + \beta_2 TANG_t$. The dependent variable ARS is the product of the senior debt ratio, p_s, and the relative spread (price difference between senior and junior bond divided by the price difference between default-free bond and junior bond). The parameters λ and θ capture APR violation. . The Treasury rate is RF and $TANG$ is the sum of current assets and net property,plant and equipment divided by total assets. The model is estimated by non-linear least squares. The Root Mean Squared Error is $RMSE \equiv \sqrt{\frac{1}{T}\sum_{t=1}^{T}(ARS_t - \widehat{ARS}_t)^2}$. For each company the first row reports parameter estimates and the second row gives conditional t-statistics for β_1 and β_2.

Company	β_0 (CONST)	β_1 (RF)	β_2 (TANG)	λ	θ	σ	$RMSE$
AMC	-12.293	23.082 3.32	11.696 22.75	0.727	0.798	0.862	0.042
American Medical	-3.185	3.948 0.54	1.607 2.02	0.800	0.800	0.500	0.037
Coastal Corp	-11.645	36.012 3.89	11.229 17.15	0.782	0.745	0.010	0.100
Envirotest Systems	-0.552	-37.983 -5.00	2.947 4.88	0.960	0.798	0.118	0.075
Flagstar	-2.174	0.926 0.11	0.008 0.01	0.786	0.806	0.713	0.045
Revlon	-35.596	19.259 2.43	46.636 66.75	1.000	0.999	0.447	0.083
Sequa Corp	-41.105	4.337 1.01	58.437 136.77	0.997	0.393	0.097	0.079
Stone Container	-17.391	-0.395 -0.02	20.466 13.79	0.008	0.979	0.113	0.082
Sweetheart Cup	-67.898	10.808 0.97	69.566 94.38	0.661	0.814	0.124	0.064
Valassis Inserts	-9.991	40.970 2.55	8.911 6.36	0.270	0.720	0.010	0.086
Del Webb Corp	-3.346	43.224 10.51	-0.763 -0.09	0.855	0.858	1.089	0.026

Table 5: Estimating the risk neutral mean and volatility of recovery in default

This table uses the results of the pure recovery model estimation and calculates the mean and volatility of risk neutral recovery for each firm. . Mean recovery is calculated as, $E(y) = 1 - \int_0^1 N\left(\frac{\ln\left(\frac{y}{1-y}\right)-\mu}{\sigma}\right) dy$. Volatility of recovery is estimated by, $Vol(y) = \sqrt{Var(y)} = \sqrt{\int_0^1 2(1-y)N\left(\frac{\ln\left(\frac{y}{1-y}\right)-\mu}{\sigma}\right) dy - \left(\int_0^1 N\left(\frac{\ln\left(\frac{y}{1-y}\right)-\mu}{\sigma}\right) dy\right)^2}$. The integrals in both expressions are evaluated numerically. Each statistic is calculated for each month and then averaged across time. Industry recovery averages in bond defaults are obtained from Altman and Kishore(1996).

Company	Estimated Mean Recovery Rate $E(y)$	Estimated Volatility of Recovery Rate $Vol(y)$	Industry Average of Historical Recovery Rates
AMC	27.3 %	15.0 %	37.1 %
American Medical	12.6	5.7	26.5
Coastal Corp	63.3	0.9	70.5
Envirotest Systems	34.3	2.9	46.2
Flagstar	12.7	8.0	33.2
Revlon	40.4	9.6	62.7
Sequa Corp	40.1	2.4	38.4
Stone Container	9.6	1.7	29.8
Sweetheart Cup	56.7	3.2	62.7
Valassis Inserts	19.1	1.3	46.2
Del Webb Corp	37.6	20.7	35.3

Table 6: Pricing put options on realized recovery in default

This table uses the estimation results of the pure recovery model in Table 4 and prices the put options written on realized recovery rates for each firm. Risk neutral default probability is $1-G(\tau)$, where $G(\tau) = \frac{p_s v_S(\tau)+(1-p_s)v_J(\tau)-E(y)P(\tau)}{P(\tau)(1-E(y))}$. The put options are written on recovery rates. All options have a constant strike, $k = 0.92$ and a notional principal, $K = \$100$. The prices of options are calculated by $w(\tau, k, K) = P(\tau)K(1-G(\tau))\int_0^k N\left(\frac{\ln\left(\frac{y}{1-y}\right)-\mu}{\sigma}\right) dy$. The prices are calculated for each month and then averaged across time.

Company	Probability of Default $1-G(\tau)$	Price of the Put Option $w(\tau, 0.92, 100)$
AMC	22.2 %	\$ 10.79
American Medical	21.7	12.72
Coastal Corp	13.6	3.04
Envirotest Systems	28.8	12.48
Flagstar	35.7	21.16
Revlon	20.5	7.94
Sequa Corp	18.1	6.78
Stone Container	19.6	11.91
Sweetheart Cup	21.1	5.39
Valassis Inserts	7.8	4.82
Del Webb Corp	24.0	9.05

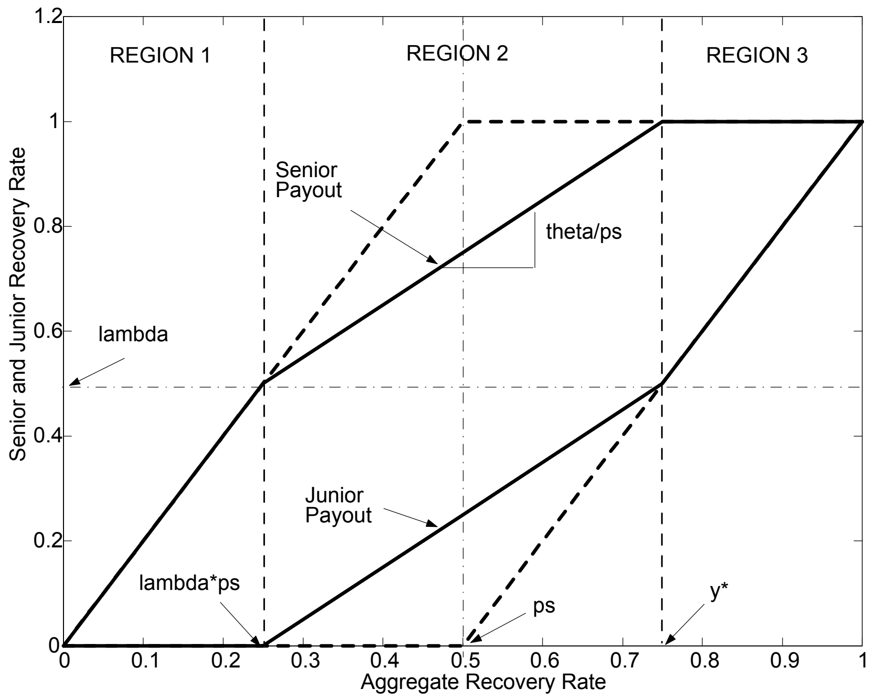

Figure 1: **Senior and junior debt recovery structure** The aggregate recovery rate to debt-holders conditional on default is y and is shown on the horizontal axis. The vertical axis shows the recovery rate to senior and junior debt. Solid (dashed) lines depict the payout structure under(without) APR violation. p_s denotes the strike price of the call option junior debt-holders are holding. $\lambda(lambda)$ represents the recovery level at which absolute priority rule (APR) is violated. Hence, λp_s represents the exercise price of the call option under APR violation assumption. The senior debt holders receive payments at the rate of $\theta(theta)/p_s$ once the APR is violated. y^* represents the strike at which junior debt holders receives payment once the senior debt holders are fully paid. To obtain the curves for senior and junior debt recovery, the parameters are set to $p_s = 0.5$, $\lambda = 0.5$, $\theta = 0.5$ under APR violation ($p_s = 0.5$, $\lambda = 1$, $\theta = 0$ without APR violation)

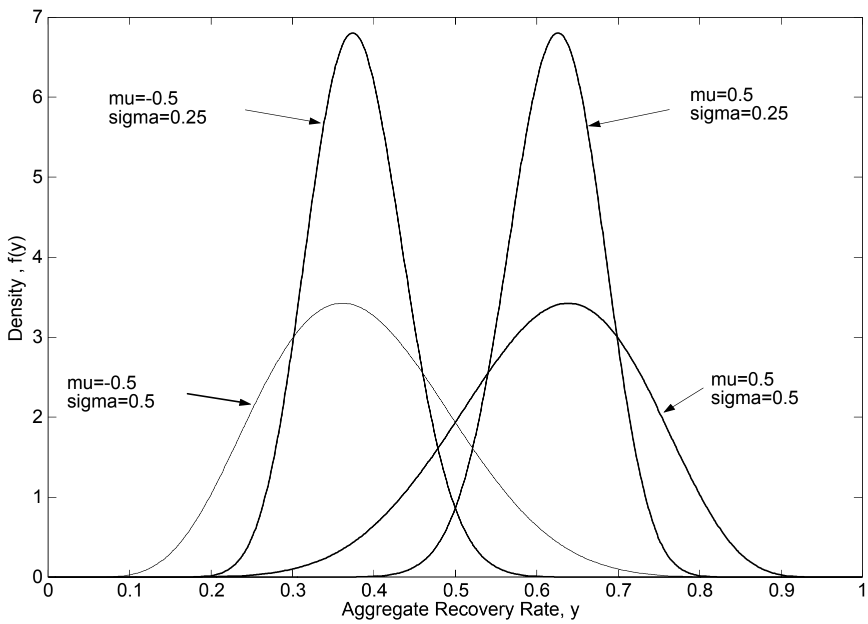

Figure 2: **Recovery density for different parameter values** The density of recovery rate $f(y)$ at any default time lies between 0 and 1. The mean and standard deviation of the density is obtained by utilizing $\mu(mu)$ and $\sigma(sigma)$.

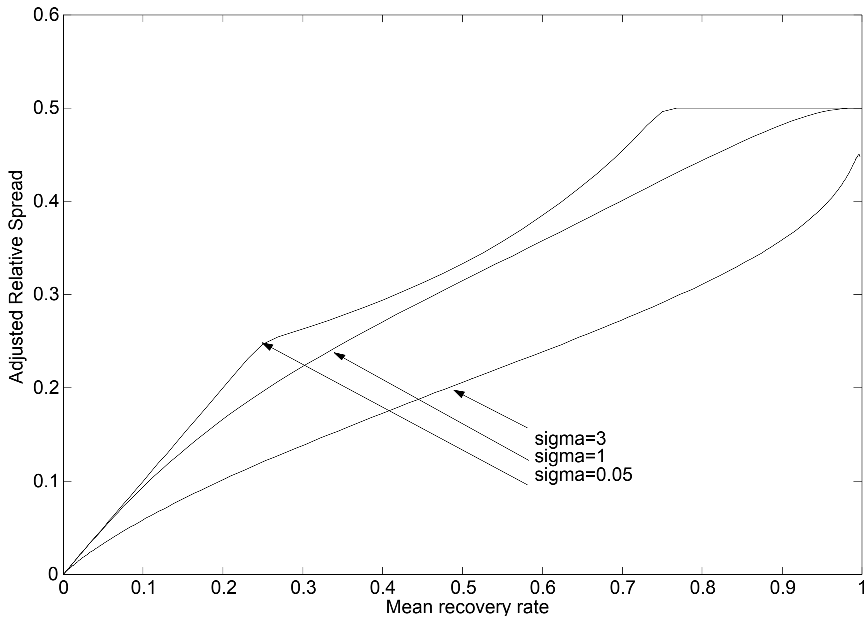

Figure 3: **Adjusted relative spread**(ARS) **sensitivity to payout volatility of recovery in default** ARS is the product of the senior debt ratio, p_s, and the relative spread (price difference between senior and junior bond divided by the price difference between default-free bond and junior bond). For all three curves $\theta = 0.5$, $\lambda = 0.5$ and $p_S = 0.5$.

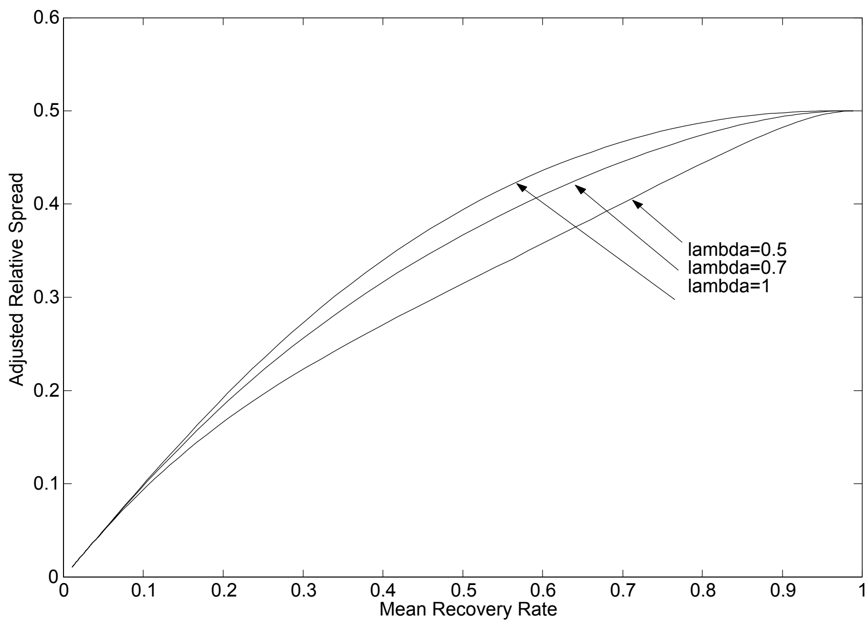

Figure 4: **Adjusted relative spread** (ARS) **sensitivity to APR violation level,** λ ARS is the product of the senior debt ratio, p_s, and the relative spread (price difference between senior and junior bond divided by the price difference between default-free bond and junior bond). For all three curves θ=0.5, p_S=0.5, σ=1.

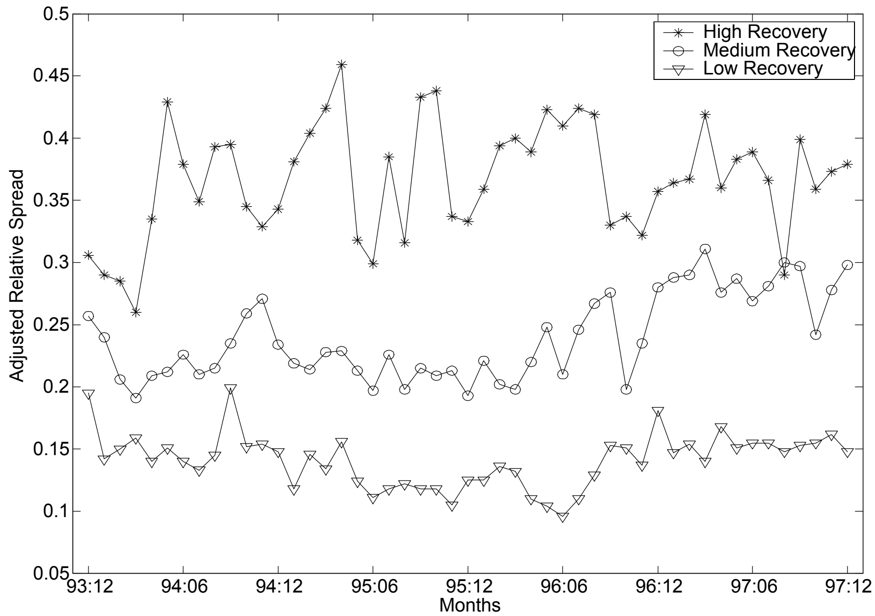

Figure 5: **Time series behavior of adjusted relative spread** (ARS) This figure plots the time series graphs of ARS for High, Medium and Low recovery groups from 93/12 to 97/12. ARS is the product of the senior debt ratio, p_s, and the relative spread (price difference between senior and junior bond divided by the price difference between default-free bond and junior bond). Industries where Altman and Kishore recovery estimates exceed 45% are defined as High recovery group, industries with recovery rates below 35% constitute the Low recovery group. Curves are obtained by averaging ARS statistics across the firms in each recovery group.