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Economic Valuation Models for Insurers

David F. Babbel* and Craig Merrill**

Recently much attention has been given to the approaches insurers undertake in valuing their liabilities and assets. For example, in 1994 the American Academy of Actuaries created a Fair Valuation of Liabilities Task Force to address the issue.[1] In 1997, the Academy established a Valuation Law Task Force and a Valuation Tools Working Group to investigate the various valuation approaches extant and to make recommendations on which models are best suited to the task.

Much of the published work has focused on attributes of the various models, their strengths and shortcomings. Some of the work has addressed the larger questions, but in our view, it is useful and necessary to provide a taxonomy of approaches and evaluate them in a systematic way in accordance with how well they achieve their aims.

In this paper we focus primarily on the economic valuation of insurance liabilities, although we do address some valuation issues for assets. We begin in Section I by defining insurance liabilities. Next, in Section II, we discuss the criteria for a good economic valuation model. This is followed by a taxonomy of valuation models in Section III. In Section IV, we examine insurance liabilities in the context of this taxonomy and identify the minimum requirements of an economic valuation approach that purports to value them adequately. An illustration of the application of a modern valuation model is given in Section V. We conclude in Section VI by discussing some limitations of our analysis, and offer some recommendations for implementation.

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[1] See Reitano et al [1997], Reitano [1997], Babbel [1997 a, b], Babbel and Merrill [1996], and Merrill [1997].

I. Insurance Liabilities

It has been argued elsewhere[2] that a useful starting point for considering the economic valuation of insurance liabilities is to consider their present value. It turns out that this is not only a good starting point, but perhaps also a good ending point insofar as liability valuation is concerned. This can be seen by considering the sources of uncertainty underlying insurance liabilities. We can divide uncertainty into three categories: actuarial, market, and nonmarket systematic risks.

Actuarial Risks

Actuarial risks include such things as casualty, liability, morbidity, and mortality risks. These risks are the insurance company's bread and butter. Exposure to these risks is managed through diversification and by writing large numbers of similar policies. Because an insurance company can reduce actuarial risks to an arbitrarily small level through risk pooling and diversification, we need not account for them, beyond their expected costs, in valuing liabilities.[3] Any residual exposure can be addressed in the process of determining surplus requirements.

Our approach differs from most traditional actuarial *reserving* methods in that they typically adjust for actuarial risk (and sometimes even for indirect expenses) by reducing the discount rates to reflect a risk charge.[4] By entangling the actuarial risk charge with the reserves, the procedure produces conservative estimates of reserves. In essence, reserves so stated are a measure of liabilities together with a portion of the economic surplus deemed necessary to maintain solvency. In contrast to the traditional approach, we suggest separating the value of liabilities from any element of economic surplus. This will allow a closer, more explicit examination of the liabilities' economic value and behavior over time and across various scenarios, and enable a more precise analysis of the amount of economic surplus needed to maintain solvency.

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[2] See Babbel [1997a,b] and Merrill [1997].

[3] We are speaking here, of course, of relative risks and not absolute risks. If the insurer elects not to reduce these risks, investors in insurance stocks can easily do so by diversifying their own portfolios of insurance and other stocks. Hence, they will refuse to pay a premium for the stock of a company that does not reduce its actuarial risks.

[4] Indeed, our approach is not intended to be a *reserving* method at all. It is more closely aligned, in spirit, with producing an actuarial value, and is embraced by that definition. The major difference in what we are proposing from the typical *implementation* of estimating an actuarial value is that we take into account explicitly the impact of interest rate sensitivities of expected cash flows in deriving their economic value.

Market Risks

Market risks include fluctuating interest rates, inflation rates, and exchange rates. Insurance companies are exposed to these market risk factors. Their exposure can be measured and priced because of the existence of an active market for securities that are exposed to the same sources of uncertainty. Then their risk exposure can be managed using Treasury instruments and their derivatives, and forward exchange contracts where necessary.

Nonmarket Systematic Risks

Nonmarket systematic risks include changes in the legal environment, tax laws, or regulatory requirements. It is not generally possible to hedge these risks. Such nonmarket systematic risks are not easily handled in valuation models. However, relative to market risks they generally have a much less significant impact on the present value of insurance liabilities.

The lack of explicit models of these nonmarket systematic risks has not hindered the development of valuation tools such as mortgage-backed security pricing models. The chance of a tax code change removing mortgage interest rate deductibility may be no less remote than a change in the legal environment surrounding insurance claim settlement, yet these potential changes are not explicitly incorporated into any of the valuation models in use today. They may, however, be reflected in the spread over Treasuries that these securities elicit or, more appropriately, a deduction from the expected inflows that the securities will generate to their investors.

Valuing Liabilities

In our opinion, the focus of insurance valuation models should center on the present value of liabilities, including their market risk sensitivities. The present value of liabilities tells us how much money an insurer would need today to satisfy, on a probabilistic basis across various economic states of the world, the obligations imposed on it through the insurance policies it has written. Here, "economic states" are defined broadly to incorporate market factors such as interest rates and inflation that can be hedged with financial securities. Treasury securities and their derivatives are the natural hedge instruments for such obligations, because they do not introduce credit risk or other risks extraneous to our needs.

An insurer may be able to satisfy its liabilities with fewer assets, if it gets lucky, by making interest rate, equity, or low credit quality bets, but *hope* should not be confused with *expectation.* The present value, properly computed via Treasury-rate-based lattices, simulations, or closed-form solutions to stochastic differential equations, takes into account any interest rate sensitivities in the cash flows. To the extent that lapse, surrender, or other factors cause mortality, morbidity, accident frequency and severity to be related to interest and inflation rates, they are reflected directly in the modeling of cash flows.[5] Economic surplus needed to cushion variations from these interest-sensitive projections is not included in the valuation of liabilities; however, a necessary ingredient for determining the amount of economic surplus needed is a good estimate of the present value of liabilities.[6]

II. Criteria for a Viable Economic Valuation Model

Several criteria can be identified as necessary ingredients for a viable economic valuation model. We propose the following criteria:

1) A model that purports to be an economic valuation model should produce values that are *consistent with the definition of economic value* that is being used so that it will be *relevant* and *meaningful*. This criterion is undoubtedly the most important one, and a precursor for the others. A model should not be substituted for an economic valuation model simply because the former is easier to apply. If its focus is on the wrong thing, or if it simply is not designed to address adequately the dimension of economic valuation, then it should not be used for economic valuation purposes. When practitioners enlist an accounting model or some other such dubious approach to estimate economic value, it reminds us of the legendary intoxicated man who was searching under a street light for his lost keys. An approaching police officer inquired of the hapless man what he was doing, and the man explained that he had lost his keys near a parking meter some distance away, but that it was easier to search for them under the bright light than around the locality at which they were dropped.

2) A viable model should be *implementable*. It almost goes without saying that the reach of a valuation model must not exceed its (and our) grasp. The generality of the model must be tempered by our ability to implement it. While it is common among academicians in the field of financial economics to develop theoretical models that are far removed from any current application, the actuarial profession cannot afford that luxury. Actuaries require models that can be developed readily, and implemented with available computer technology. A model that fails of this front may offer future promise, but cannot hold in abeyance the valuation work that needs to be done today.

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[5] An early example of this approach was first published in 1989 and republished in Asay et al [1993].

[6] It is important to keep in mind that our estimate of the economic value of liabilities must not be misconstrued as an estimate of policy reserves. Were reserves to be set at the level of the liabilities' economic value, they would prove to be inadequate over the short run half of the time, and inadequate over the long run with certainty (see Buhlman, [1970]).

3) A viable model should produce *consistent* prices across all assets and liabilities. Many valuation models in use today are specialized for only one class of assets or liabilities. For example, an option pricing model may be used to price options, an equity valuation model may be used to price common stock, a bond pricing model may be used for bonds, and an actuarial model may be used for insurance liabilities.[7] The only consistency obtained in using such disparate models is that they render a value in dollar terms. It would be far better to have a general valuation model capable of valuing most, or all classes of assets and liabilities. This would be helpful not only for encouraging consistency of assumptions, but is also necessary for generating future economic scenarios for solvency testing and business planning. Parsimony should not be mistaken for efficiency. While it is correct that a particular financial instrument, such as a short-term default-free noncallable bond, may require only a single random factor to generate an appropriate value, that does not imply that such parsimony in modeling should be heralded as the guiding principle in model selection. A more inclusive model may incorporate additional factors that are not particularly germane to the valuation of a particular instrument, but which may allow the valuation of the financial instrument in question as well as a host of other instruments, such as callable corporate bonds, in a consistent fashion.

4) A viable model should *calibrate closely* to observable prices. Economic value is the same as market value when the financial instrument in question is tradable in an active market. The standard for a good fit, which has been widely accepted in recent years, is that the model should closely match observed Treasury prices, particularly "on-the-run" Treasury bills, notes and bonds, and the prices of Treasury derivatives, including options on Treasury futures. These prices are important because the instruments are widely traded and very liquid. We have confidence that reported prices are not stale, and that there is sufficient trading volume subject to the discipline of arbitrage forces to ensure that prices are at appropriate levels. The model should also calibrate reasonably well to the market prices of other financial instruments, but insofar as valuing insurance liabilities is concerned, this feature is unnecessary.[8]

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[7]Practitioners on Wall Street sometimes clumsily attempt to apply the same model for pricing Treasury and corporate bonds. Because a model which prices Treasuries well does not fit well the observed market prices of corporate bonds, practitioners often assume a different (lower) volatility of short-term Treasury rates than what is used for pricing the Treasuries themselves. While this *ad hoc* procedure may produce model price estimates that are closer to corporate bond market prices, it clearly violates the consistency standards of accepted valuation principles.

[8] The reason it is unnecessary to have the model calibrate closely to market prices of other financial instruments is that only the default-free rates of interest are used in deriving the economic value of insurance liabilities. See Babbel [1994, 1997a,b].

5) A viable model should be *noncontroversial*. It should be based on well-established principles and accepted valuation practices. It would be unwise for any body of professionals to adopt a valuation model for reporting or managerial purposes that was based on fanciful theory and unproven technology.

6) A viable model should be *specifiable* and *auditable*. By "specifiable" we mean that regulators and managers ought to be able to specify a set of economic parameters to be used in the valuation for the purpose of studying the company. Of course, the best values of the parameters for valuation purposes are those that are inferred from observed market prices and the behavior over time of interest rates and other economic factors that influence value. But a viable model should also be open to a set of parameter specifications from parties who have an interest in promoting solvency and managing a company. Once the parameters are estimated or otherwise specified, the output of the model should be amenable to auditing, to ensure that it is producing scenarios and values consistent with those parameters.

7) A viable model should feature *value additivity*. By this, we mean that the values estimated for various parts of a portfolio of assets or liabilities should be equal to the value of the entire portfolio when summed together.[9] In a world that does not foster costless arbitrage, the sum of the parts may not be exactly equal to the value of the whole, but should fall within some bounds related to transactions costs.

Having discussed seven criteria needed in a viable valuation model, we next proceed to a discussion of the menu of available models, their relation one to another, and their inclusiveness of the factors that determine economic value.

III. A Taxonomy of Valuation Models

There are many seemingly different valuation models in the literature. Some examples include general equilibrium models, partial equilibrium models, contingent claims models, Black and Scholes option pricing model, Arbitrage Pricing Theory (APT), the Capital Asset Pricing Model (CAPM) and its variations, as well as a wide variety of models for pricing interest rate contingent claims. In this section we will introduce a simple taxonomy for understanding the relationships between valuation models.

In any valuation model there will be cash flows, with their associated probabilities of occurrence, and discount rates. A key feature of financial valuation models is how they

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[9] As explained in Section V, while the values must be consistent with the value additivity principle, the risks need not be additive.

account for risk in the cash flows. There are three common methods for dealing with risk: by modifying the cash flows, the discount rate, or the probabilities.

The first is to use a certainty equivalent cash flow.[10] The certainty equivalent cash flow is the cash flow that would make the investor indifferent between a certain outcome and facing a random draw on a set of risky cash flow outcomes. This approach depends on the specification of a utility function. Given an assumed utility function, individuals' risk aversion may be ranked by comparing their certainty equivalent cash flows. This approach is not often used in security valuation models. However, it has been profitably applied to the firm's capital budgeting problem.

The second approach is to embed a risk premium to the discount rate in order to allow for the excess return that investors seek as compensation for bearing risk.[11] In practice, valuation models for fundamental securities like stocks, bonds, or projects within a firm tend to use the risk premium approach. The CAPM and APT are well known models that have been developed specifically to estimate risk premiums.

Third, there is the risk-neutral pricing methodology employed in option pricing. In risk-neutral pricing the probability distribution is typically adjusted to compensate for risk so that cash flows can be discounted at the risk-free rate as if investors were risk neutral. Risk neutral valuation is most commonly applied to derivative securities. It is important to keep in mind that with risk-neutral valuation, the price is determined by the absence of arbitrage. The existence of the risk-neutral measure is synonymous with the absence of arbitrage.[12] Thus, investors with widely varying levels of risk aversion can agree on the price yielded by a risk-neutral model because it represents the price where there are no arbitrage opportunities. However, there need be no inconsistencies between the models. Properly implemented, all approaches will lead to the same valuation.[13]

Our framework for considering valuation models can be represented with a two-by-two matrix depicting two dimensions along which a model may be extended or simplified. This matrix is given in Exhibit 1. The first column of the matrix represents models

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[10] See Robichek and Myers [1965].

[11] See Robichek and Myers [1965], and Rubinstein [1976].

[12] Jarrow and Turnbull [1996] state this proposition on page 163 in the context of a binomial pricing model. A more general proof is given in Harrison and Pliska [1979].

[13] See Singleton [1989], and Babbel and Merrill [1996, pp. 40-44], for a more thorough discussion of the relationship between risk-neutral pricing and pricing based on a risk premium.

with deterministic, or known, cash flows. The second column represents models with stochastic cash flows. Similarly, the rows represent the deterministic or stochastic nature of interest rates as used in a model. Therefore, moving to the right or down in the matrix represents an increase in the complexity of the valuation model. Cell D would be the most general model, featuring both stochastic cash flows and interest rates. We will present a valuation equation for each of the four cells. Each valuation equation will be in terms of a single cash flow. Securities with multiple cash flows are treated as portfolios of single cash flows.

Exhibit 1
Matrix of Valuation Model Complexity

	Deterministic Cash Flows	*Stochastic Cash Flows*
Deterministic Interest Rates	**A**	**B**
Stochastic Interest Rates	**C**	**D**

In our matrix, cell A represents the simplest class of models. Both the sequence of future interest rates and the future cash flows are known with certainty today, at time zero. Let x_t be a cash flow at time t. Let $r_t(n)$ represent an n-period spot rate of interest at time t. Define the short rate to be the spot rate that applies to the shortest time increment. For example, in discrete time the one-period spot rate, $r_t = r_t(1)$, or more simply, r_t, would be the short rate of interest. In continuous time, the instantaneous spot rate, $r_t = r_t(\Delta t)$, would be the short rate. In the equations below, we will render the continuous-time formulations.

Given these definitions, our valuation equation for cell A is

$$V_0 = x_t \exp\left(- \int_0^t r_s ds \right). \tag{1}$$

This equation includes the simple present value formula from introductory finance, where interest rates are assumed constant as a special case. That is, $V_0 = x_t\exp[-r_0(t)t]$.

In cell B we assume that the cash flow is stochastic and possibly affected or determined by other stochastic factors. The cash flow is denoted by $x_t(\mathbf{y}_t)$, where $\mathbf{y}_t$ is a vector of sources of randomness. For example, payments to an auto insurance policyholder are determined by the random losses that occur during a period of coverage. The loss distribution is usually described by frequency and severity distributions. In this case the vector, $\mathbf{y}_t$, might be composed of two components: a random variable for frequency and a random variable for severity. The actual payment, $x_t(\mathbf{y}_t)$, would depend on realized losses as well as on policy provisions such as deductibles and limits of liability.

Another example can be found in option pricing. Consider an equity option-pricing model. In this case $\mathbf{y}_t$ would be a scalar representing the stock price at time t. The option's cash flows are deterministically linked to the price of the stock. Thus, $x_t(\mathbf{y}_t)$ would represent the cash flow to the option.

The valuation equation for cell B is

$$V_0 = \mathrm{E}^*\left[x_t\left(\mathbf{y}_t\right)\right]\exp\left(-\int_0^t r_s ds\right), \tag{2}$$

where the asterisk denotes that the expectation is being taken over the risk-neutral probability measure of the stochastic factors that determine the cash flow. If we can exactly replicate the cash flows of a security using a portfolio of other securities, then the security of interest and its replicating portfolio should have the same price. Otherwise, there would be an arbitrage opportunity. As we stated above, Harrison and Pliska [1979] show that if there are no arbitrage opportunities then we can construct a probability measure under which we can price risky cash flows as if investors were risk neutral. This is the risk-neutral, or equivalent martingale, measure. The risk-neutral measure is the probability measure that sets equation (2) equal to the price of the replicating portfolio. A risk-neutral measure can be used in place of the realistic, or "true" probability distribution because with perfect replication of cash flows, the replicating portfolio and the security being valued must have the same price or else there would be an arbitrage opportunity. Thus, all investors, regardless of their level of risk aversion, will agree on the price.

Equation (2) says that the value of a stochastic future cash flow is the present value, discounting at the risk-free rate, of the risk-neutral expected cash flow. Alternatively, we could specify this valuation equation as

$$V_0 = \mathrm{E}\left[x_t\left(\mathbf{y}_t\right)\right]\exp\left(-\int_0^t \left[r_s - \lambda(s)\right]ds\right), \tag{2'}$$

where the expectation is over the true probability distribution and $\lambda(t)$ is the (possibly time varying) risk premium. In complete markets where the components of $\mathbf{y}_t$ are actively traded, equation (2) and (2′) will yield identical valuations. For example, options could be valued using either equation. However, when the underlying source of uncertainly is not actively traded, equation (2′) is more likely to be used. Thus, when valuing auto insurance where $\mathbf{y}_t$ would be composed of the random variables representing frequency and severity, equation (2′) is used.

The market price of risk must be estimated. Specifying a utility function for a representative investor usually does this. The risk aversion associated with the assumed utility function determines $\lambda(t)$. The market price of risk can then be determined empirically by calibrating the model to market data – that is, finding the $\lambda(t)$ that best equates model prices to market prices.

Examples of models that would fall within cell B are the CAPM, APT, Black-Scholes option pricing model, and many traditional insurance pricing models. In each case, some sort of distribution of future cash flows is assumed and the expectations of those cash flows are discounted at deterministic, and often constant, interest rates. While the CAPM and APT are referred to as pricing models, they actually determine an appropriate discount rate to compensate for risk when discounting risky cash flows. Thus, they fall within the category of risk-premium models. The Black-Scholes model, on the other hand, is the classic example of equation (2). Their pricing model is derived using the absence of arbitrage between the option and a dynamically rebalanced portfolio that exactly replicates the option's potential cash flows.

In cell C we assume that cash flows are known in advance but that interest rates are stochastic. This is the approach taken in fixed income valuation models, such as the Cox, Ingersoll and Ross [1985], Brennan and Schwartz [1977], or Hull and White [1990] models. The valuation equation in this case is

$$V_0 = x_t \mathrm{E}^*\left[\exp\left(-\int_0^t r_s\, ds\right)\right], \tag{3}$$

where the asterisk once again represents the risk-neutral expectation operator. If the expectation is taken over the true probability distribution and we incorporate a risk premium, then the valuation equation would be

$$V_0 = x_t \mathrm{E}\left[\exp\left(-\int_0^t [r_s - \lambda(s)]ds\right)\right]. \tag{3′}$$

The various theories of the term structure of interest rates put forth possible formulations for $\lambda(t)$.[14] It should be noted that the local expectations hypothesis (LEH) suggests that $\lambda(t) = 0$. This means that the expected return over the next period on all interest rate contingent securities is the risk-free rate. If the LEH holds, then equations (3) and ($3'$) are identical.

Finally, cell D is the most general case. Here, both cash flows and interest rates are treated as stochastic. In addition, cash flows may depend on or be influenced by interest rates. The valuation equation for cell D is

$$V_0 = \mathrm{E}_r^*\left[\exp\left(-\int_0^t r_s\,ds\right)\mathrm{E}_y^*\left[x_t\left(\mathbf{r}_t, \mathbf{y}_t\right)\right]\right] \tag{4}$$

where $\mathbf{r}_t$ denotes a vector of short rates that have occurred from time zero to time t and the subscript on the risk-neutral valuation operator indicates over which variable the expectation is being taken.

This model is the most general in the matrix. Examples of potential applications of this formula include options on bonds, interest rate caps and floors, mortgage-backed securities, catastrophe bonds that are used to fund reinsurance, and insurance liabilities. The analog to equation (4) that incorporates risk premiums in the discount rate is

$$V_0 = \mathrm{E}_r\left[\exp\left(-\int_0^t\left[r_s - \lambda_r(s) - \lambda_y(s)\right]ds\right)\mathrm{E}_y\left[x_t\left(\mathbf{r}_t, \mathbf{y}_t\right)\right]\right] \tag{4'}$$

where the subscript on λ denotes the risk premium relative to interest rates or the factors contained in $\mathbf{y}_t$.

In reality, cells A, B and C are just special cases of cell D which is codified in equation (4). Therefore, we can treat equation (4) as the valuation model in almost all cases. However, in the case of pricing a simple bond, cash flows are deterministic and there is no necessity to deal with the added complexity of equation (4). We can think of equation (4) as a model with optional features that can be turned on or off as needed. When we have deterministic cash flows we can "toggle off" the stochastic cash flow portion of the model and use only what is left in cell C or equation (3). On the other hand,

[14] See Santomero and Babbel [1997], pp. 88-101 for a discussion of the various term structure hypotheses.

when we value short-lived equity options the important feature is the stochastic cash flow. Modeling stochastic interest rates just does not impact estimated value enough to justify injecting the added complexity. So we can "toggle off" the stochastic interest rate portion of (4) and use cell B or equation (2). Finally, when we teach present value, the stochastic portion of (4) can become quite confusing. So we use cell A or equations similar to (1) to teach discounting and capital budgeting. At the core, though, a very wide variety of valuation models are encompassed in cell D or equation (4).

The valuation formulae presented above are applicable to multi-factor models as well. In most multi-factor interest rate models the additional factors relate to components of the stochastic evolution of the short rate of interest. For example, Fong and Vasicek [1991] assume that the volatility of the short rate is itself stochastic. Thus, the two factors, or sources of uncertainty, are random shocks to the short rate itself and random changes in the volatility of the short rate. Similarly, most models of the evolution of the short rate of interest incorporate mean reversion. Hull and White [1994] assume that the level to which rates revert is stochastic. In this case, the two source of uncertainty are random shocks to the short rate and random shocks on the level to which the short rate reverts. Exhibit 2 provides a classification of many of the models in popular use today.

One last comment on discrete-time vs. continuous-time models is in order. The impact on the valuation equations given above is in the probability distributions and the integral over short rate paths. For a discrete-time model, the expectation would be over a discrete distribution and the interest rates would be for discrete periods of time. Therefore, the integral in the exponential function would have to be changed to a summation.

IV. The Valuation Model Needed for Insurance Liabilities

In reviewing the taxonomy of economic valuation models, it appears that we must opt in favor of a model from Class D. Only models of this class are capable of producing viable estimates of economic values for financial instruments that feature stochastic cash flows which are influenced by stochastic interest rates.

We have argued elsewhere[15] that virtually all insurance company liabilities exhibit these characteristics. Accordingly, if an economic valuation of insurance liabilities is our main objective, and we wish to satisfy criterion (1), we are relegated to use models from Class D to obtain viable values.

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[15] See Babbel [1990].

While it is possible to enlist a valuation model from another class for the task of valuing insurance liabilities, the gerryrigging and contorting of model parameters needed in order to approximate a proper economic value are themselves inconsistent with financial valuation principles and subject to imprecision, mischief, controversy, and a dubious end result. It would be only by accident that they would produce a number which could address the question: "How much money would an insurer need today to satisfy, on a probabilistic basis across various economic states of the world, the obligations imposed on it through the insurance policies it has written?" Moreover, one could not determine whether the "values" produced by such models were close approximations to true economic values without first computing them with a Class D model.

We are fortunate indeed if we opt for an economic valuation model from that class, because it is also the best model for valuing the myriad of disparate financial assets held by most insurers. While it is true that models from Classes A, B and C could produce viable estimates of value for some asset categories, only Class D models embrace the valuation technology that can be used with virtually all asset categories. Moreover, Class D models produce output that is *relevant* and *meaningful*. The number produced serves as a threshold above which an insurer must operate if it is to stay in business long. This is a useful number to regulators and insurers alike, who need to know how much it should take to defease fully the liabilities the insurer has underwritten. It is a number that is easily compared among insurers, and meaningfully related to the market value of assets supporting the liabilities. It also is a number that serves as a sound starting point for the analysis of the amount of economic surplus needed to maintain solvency.

Now let us revisit the remaining six criteria for a viable economic valuation model to see how well Class D models may satisfy them. Criterion (2) is met because these models are clearly *implementable*. Wall Street has been using Class D models for nearly a decade in the valuation of massive amounts of mortgage-backed securities. Like insurance liabilities, mortgage-backed securities are subject to a considerable amount of cash flow uncertainty, most of which devolves from fluctuating prepayment rates occasioned by changing interest rates. In the case of insurance liabilities, some of the uncertainty, such as the incidence of lapse and surrender, devolves from the vacillation of future interest rate levels and paths. This uncertainty can be modeled in a fashion similar to mortgage-backed security prepayments. To the extent that uncertainty stemming from mortality, morbidity, accident experience and some base levels of lapses and surrender is not related directly to interest rates, it can be reflected directly in the expected cash flows input into the valuation model.

With respect to criterion (3), Class D models are capable of producing *consistent prices* across all assets and liabilities. Of course, to achieve full consistency, one must use the same general valuation model for both assets and liabilities, and not merely one or another member of the class.[16] Perhaps the greatest benefit to practitioners of using a single Class D model is in the generation of future scenarios and distributions of values. By having all liabilities and assets valued using a single model, we can be assured of consistency when it comes to modeling the effect of a change in one or more parameters on the economic well being of the company.

Class D models calibrate well to observable market prices, where available. Thus, criterion (4) is satisfied. Not all Class D models are created equal, however. Some calibrate more closely than others. For example, single-factor models typically calibrate poorly to observable prices without relying on deterministic time-varying model parameters which are difficult to justify theoretically.[17] Additionally, they often imply perfect correlation of movements between short-term and long-term interest rates over time, which gives a very misleading picture of interest rate risk exposure. Finally, if the single-factor models do not feature time-varying parameters, they typically require substantial distortion of parameter values in order to achieve a good fit to observed prices. On the other hand, multi-factor models can achieve a far better fit to observed prices and observed economic phenomena, without imposing perfect correlation in movements across the term structure and without distorting the parameters to unrealistic levels. Thus, while they are more complex and painstaking to run, their valuation capabilities and scope make them far more useful for financial institutions such as insurers. Indeed, we would be so bold to say that anything less than a two-factor model is inadequate for modeling most insurance liabilities.

Class D models have been in use since 1979, and are now so ubiquitous that their use is noncontroversial from a practitioner's viewpoint. From a theoretical point of view, there are currently no valuation models which are received more favorably, despite their drawbacks. Truly today, it is far more controversial to enlist a model from another class for valuing an insurer's assets and liabilities, because it would not capture the economic importance and valuation impact of the interplay between stochastic cash flows and stochastic interest rates. Accordingly, we can say that criterion (5) is fulfilled.

Another attractive aspect of Class D models is that they are specifiable and auditable. After a particular model has been found with a suitable set of attributes, its parameters may be estimated or otherwise specified, and the model's output is ready for audit.

[16] This is not to say that one cannot produce good estimates of economic values using different valuation models; rather, that it is preferable to use a unitary model capable of valuing types of financial instruments under consideration, both assets and liabilities, in order to assure consistency.

[17] See, for example, Black, Derman and Toy [1990].

Once a software implementation of the model has been certified for accuracy, most of the auditing attention can be turned toward the model's inputs and assumptions. Because there is so little that is subjective in Class D models, it meets well criterion (6). And those elements that are subjective (e.g., surrender rates as a function of interest rates) can be the focus of additional scrutiny and sensitivity testing.

Finally, we consider criterion (7), the value additivity principle. The way in which Class D models compute value for a single financial instrument is based strictly on value additivity of its component parts. Moreover, if the same general valuation model is used for a portfolio of assets or block of liabilities, value additivity is automatically assured.

V. An Illustration

To be concrete about the application of Class D models to insurance liabilities, we provide two charts. In Exhibit 3, we show a lattice of short-term interest rates (i.e., "short rates") as they evolve in discrete steps over the time period from t to $t+12$. Each jump in interest rates is either 10 percent above or below its prior level. Below the lattice we show the probability distribution of interest rates at time $t+12$, with the scale on the horizontal axis being ordinal only. At the bottom of the exhibit, we show using a cardinal scale how our simple multiplicative stochastic interest rate process, when smoothed, converges to a lognormal distribution.

Exhibit 4 continues with our illustration to demonstrate how to interface the insurance cash flows with the lattice of short rates. At two of the nodes, we have shown how the cash flows may be modeled using an arctangent lapse function fitted to lapse data. Depending on where along the arctangent curve the interest rate spreads lie, we could get a different distribution of lapse rates. Nonetheless, the expected cash flows given by the arctangent function associated with each node would be used as inputs to the valuation process, and their value would be determined by applying the sequences of short rates that could give rise to those cash flows. In some models, a change of probability measure is enlisted and applied to each node to achieve arbitrage-free pricing.

What these exhibits show is how uncertainty arises from several sources. First, there is the uncertainty surrounding future interest rates. The economic valuation of liabilities would reflect this interest rate risk (as captured by the arctangent function). Second, there is additional uncertainty associated with the lapse rates at each node. This residual lapse risk is not priced explicitly, but would need to be reflected in the level of economic surplus needed to maintain solvency.[18] These, and other risks may not be additive

[18] The economic surplus elicited will manifest itself, in an accounting sense, in the insurance reserves and statutory surplus that the insurer holds on its books.

because of their correlations one with another; accordingly, the overall uncertainty should be assessed in order to determine appropriate surplus levels. Furthermore, there is additional uncertainty stemming from the use of the model itself, and the assumed distributions of interest rates and cash flows. Such uncertainty would increase the amount of surplus deemed to be prudent. Finally, by construction, the model assumes that the asset managers are fully aware of the attributes of the liabilities and have taken every effort to hedge their market risks. If, instead, the investment department has strayed from matching its assets to the needs of the liabilities, additional surplus would be needed to accommodate its choices. Yet the liability valuation model provides a useful benchmark that is a starting point for determining the level of surplus adequacy.

VI. Limitations and Recommendations

In the final analysis, we can say that all of the seven listed criteria for a viable insurance valuation model are met with Class D models. Two questions remain. What are the drawbacks of using such a model, and which model is most suitable for insurers?

The major drawbacks in using such models for insurance liability valuation purposes are manifold:

1) Class D models require a higher level of analytical capability than is found at some insurance companies. However, most actuaries who are not already conversant with these models could adopt them with a little bit of directed training, because their curriculum already equips them to undertake tasks of equal complexity.

2) They elicit valuation inputs which may not be readily available, such as lapse functions and crediting rate algorithms. But these inputs are also necessary with far less sophisticated models. The only nuance is to model these inputs as (perhaps fuzzy) functions of the underlying factors (such as interest rates and inflation) that are driving the valuation.

3) The computer requirements for data analysis using stochastic methods are more extensive than for simpler accounting models. As most insurers already have tremendous data processing capacity, we do not see this as a binding constraint.

4) Regulators may not embrace the output of these models until they become more familiar with them, and may therefore require an additional layer of scenario testing using more primitive models. Over time, however, as regulators’ comfort level increases, the advantages and insights that can be garnered through reliance on Class D models will surely impel regulators to use them. When you have to travel from Boston to Hartford, most people would opt for the Mercedes over the roller skates, especially if the skates were never designed to go that far.

5) There are additional short-run costs when going from what has traditionally been an accounting focus to an economic focus – software costs, data assembly and modeling costs, and training costs. Yet we would suggest that in a competitive environment, the insurance companies that delay adopting the economic focus will in the end incur the greater costs due to mispricing of policies and asset/liability imbalances.

With regard to the selection of a suitable Class D model, we are reluctant here to get into a discussion of the competing "brands" of available products and products under development. Therefore, we will restrict our comments to some general observations which may help guide an insurer in its search for a suitable model for valuing its liabilities.

Numerous variants of Class D models exist. They may be distinguished one from another by the choices they make in deriving the valuation models. In Exhibit 5 we list the issues that confront the insurer, and the choices available. The first decision that needs to be made is whether an equilibrium approach will be used, or an arbitrage-free pricing approach. The equilibrium approach begins with some assumptions and observations about the general economy, and from them derives implications for the behavior of the term structure of interest rates. This approach is suitable for scenario testing and "broad brush stroke" valuation. It is especially adept at producing reasonable future scenarios which should be used in the management of insurance financial risk. However, an arbitrage-free approach is more suitable for daily trading. Such an approach is less likely to produce helpful future economic scenarios for solvency testing, because the evolution of the term structure of interest over time under this kind of model often does not reflect certain stable economic relationships that are observed in practice.

The next element which we feel is important is that the model be congruent with valuation of all classes of liabilities. For many insurers, whose predominant asset holdings are publicly traded securities, the importance of a model to value the assets is less important, as such securities have readily observable market values. What is important, however, is that the liability valuation model be consistent with observed prices on Treasury securities and their derivatives. This feature enables assets and liabilities to be simulated harmoniously over time based on random movements in the factors that determine Treasury prices. Models that tend to accommodate these needs are most often based on discrete-time approaches.

We feel that at least two random factors is a minimum configuration for a suitable liability valuation model. Which two factors should be modeled explicitly depends on the nature on one's business. For example, companies with long-tailed property/casualty lines or defined benefit pension business should probably opt for a model whose focus is on inflation and a reference real or nominal rate of interest. This is because elements of the liabilities are strongly influenced by inflation, as well as interest rates. (Such models would also work for other types of insurance companies, but the inflation factor would be a less direct way to model nominal interest rates.) For a life insurer without pension business, it would be sufficient to have both factors focusing on nominal interest rates. Available models have factors related to short-term and long-term nominal interest rates, short-term rates and the spread between short-term and long-term rates, short-term rates and random volatility, short-term rates and a random level for mean reversion, and so forth. Models incorporating more factors than two are also available.

One criterion to guide in the selection of a model is whether it features closed-form solutions for Treasury bonds. For models with such capabilities, the economies in modeling and computer intensity are simply enormous, especially when running scenario analyses. While all Class D models can produce present values, either through simulation, lattices, or closed-form solutions, the advantage of a model with closed-form solutions is that it allows for the modeling of future values with little extra effort. Such models compute the entire term structure of interest rates at each future point of time under each simulation scenario without incurring any additional simulation runs. Without this feature, a model would need to incur tens and tens of millions of additional simulation runs to achieve a similar level of richness, and to provide for a rigorous and consistent depiction of asset/liability management.

A final element that would be useful in a valuation model is for there to be a completely open architecture. New asset and liability instruments are continually being introduced, and no valuation program without an open architecture would be useful for very long in today's dynamic economic environment.

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