Wharton

Financial
Institutions
Center

The Market Value and Dynamic Interest Rate Risk of Swaps

by
Andrew H. Chen
Mohammed M. Chaudhury

96-44

The Wharton School
University of Pennsylvania

THE WHARTON FINANCIAL INSTITUTIONS CENTER

The Wharton Financial Institutions Center provides a multi-disciplinary research approach to the problems and opportunities facing the financial services industry in its search for competitive excellence. The Center's research focuses on the issues related to managing risk at the firm level as well as ways to improve productivity and performance.

The Center fosters the development of a community of faculty, visiting scholars and Ph.D. candidates whose research interests complement and support the mission of the Center. The Center works closely with industry executives and practitioners to ensure that its research is informed by the operating realities and competitive demands facing industry participants as they pursue competitive excellence.

Copies of the working papers summarized here are available from the Center. If you would like to learn more about the Center or become a member of our research community, please let us know of your interest.

Anthony M. Santomero
Director

The Working Paper Series is made possible by a generous grant from the Alfred P. Sloan Foundation

The Market Value and Dynamic Interest Rate Risk of Swaps [1]

This Version: February 1996

Abstract: At the time of initiation, interest rate swaps are of zero market value to the counterparties involved. However, as time passes, the market value of the swap position of each counterpart may become positive or negative. These value changes are stochastic in nature and are primarily driven by stochastic variations of the term structure of interest rates. In this paper, we develop models for determining the market values and dynamic interest rate risks of existing swap positions using the one-factor general equilibrium term structure model of Cox, Ingersoll, and Ross (1985). The valuation and risk measurement framework of this paper should be useful in developing a value turn risk accounting method advocated by Merton and Bodie (1995) for better internal management and reporting purposes and for more effective regulation.

Andrew Chen is from the Cox School of Business, Southern Methodist University, Dallas, TX 75275, (tel: (214) 768-3179, email: achen@mail.cox.smu.edu), and Mohammed Chaudhury is from the College of Commerce, University of Saskatchewan, 25, Campus Dr., Saskatoon, SK, Canada S7N5A7, and the Cox School of Business, Southern Methodist University, Dallas, TX 75275, (tel: (214) 768-2546, email: mchaudhu@mail.cox.smu.edu, chaudhury@sask.usask.ca).

This paper was presented at the Wharton Financial Institutions Center's conference on Risk Management in Banking, October 13-15, 1996.

IN THE PAST TWO decades financial derivatives, including futures, options, and interest rate and currency swaps, have become important and useful instruments in risk management for financial institutions and business firms. After the recent reports of large losses by Proctor and Gamble, Gibson Greetings, Metallgesellschaft, Orange County, and others, a great deal of attention has been given to the discussions of the benefits and costs of financial derivatives. Since their introduction in the early 1980's, interest rate swaps have become one of the most powerful and popular financial tools for transferring and hedging risk for banks and business corporations. The market for interest rate swaps has grown very rapidly in the past fifteen years. As of the end of 1994 the notional amount of outstanding interest rate swaps was more than $8.8 trillion[1]. It is notable that banks are now the major players in the market for interest rate swaps. For instance, as of the end of 1992 the notional amount of outstanding interest rate swaps was $6.0 trillion, and U.S. commercial banks alone held $2.1 trillion of interest rate swaps[2].

Interest rate swaps are simple financial contractual agreements between two parties. In a *plain vanilla* fixed/floating interest rate swap, two counterparties exchange their interest payments on the *notional principal* for a specified length of time. Using an interest swap, a firm can easily create a synthetic liability that has a different maturity, different interest risk and possibly a lower cost than existing liability alternatives to the firm[3]. The growing popularity of interest rate swaps is due in part to the fact that interest rate swaps are simple and easy to execute and they are the relatively inexpensive instruments for hedging or for altering the interest rate risk of a firm's portfolio.

The U.S. commercial banks' dominance in the swap markets has recently raised many concerns about their swap transactions. These include the possibility of the failure of some large banks in the swap market leading to the collapse of the payments and credit systems, known as the *systemic risk.* Besides such dire consequences at the banking system level[4], the swap

---

1 Source: ISDA, see *Risk*, Vol. 8, No. 7, July 1995.

2 See Gorton and Rosen (1995).

3 See Loeys (1985), Bicksler and Chen (1986), Smith, Smithson, and Wakeman (1986), Turnbull (1987), Arak, Estrella, Goodman, and Silver (1988), Wall (1989), Litzenberger (1992), and Titman (1992) for discussions of motivation for interest rate swaps and their applications in hedging interest rate risk and in asset/liability management.

transactions of an individual bank or a business corporation also have important implications for its shareholders, creditors and other stakeholders. Efforts are being made by various regulatory and accounting overseeing agencies to better disclose and monitor the swap and other derivatives related positions of their users. Many users themselves are also instituting internal policies and mechanisms to closely track and manage their positions in the swap and other derivative markets.

An important element for the success of any external or internal effort to better regulate, disclose, or manage the derivatives positions is the understanding of how to determine the marked to market value (simply the market value hereafter) of these positions and how the marked to market value may change as the market environment changes. In other words, measurements of value as well as risk of derivatives positions are necessary. To underscore the importance of and the need for risk measurement, Merton and Bodie (1995, p. 8-10) write,

> "To facilitate measurement, financial accounting must undergo fundamental revisions in the long run... central to those revisions is the creation of a specialized new branch dealing with *risk accounting.* Until a system of *risk accounting* is in place, truly effective regulation will be difficult to implement." (italics added)

Of course, it will be desirable to maintain consistency between measurements of value and risk which calls for a unified treatment of market value and risk of derivatives. For derivatives such as futures and options, there are well-developed valuation models in the finance literature for this purpose. For swaps, much has been done about their valuation when they originate. This includes the literature that deals with the pricing of the credit risks of the counterparties of a swap arrangement. In comparison, to our knowledge, there is no unified theoretical exposition on the determination of the market value and the interest rate or market risk of *previously established* swap positions[5]. One probable reason is the similarity of the fixed-for-floating swaps

4 If the market value of the banks' swap positions were, say, $200 billion (10% of $2 trillion notional), it will mean a liability of $200 billion for the banks if the market situation has moved in an adverse fashion for the banks. A meagre 5% fall in the value of these swaps will drain the banks' market value equity by $10 billion.

to coupon bonds or alternatively to a set of interest rate forward contracts. However, as noted by Litzenberger (1992, p. 831), ".. there is more to these plain vanilla swaps than first meets the eye."

In this paper, we propose a theoretical model for the market value and the interest rate risk of existing or previously established swap positions. As in most theoretical treatments of interest rate contingent claims, our building block is the term structure of interest rates on default-free zero coupon bonds. We incorporate important pricing features of the secondary swap market in deriving the relevant cashflows which are then discounted using the term structure of interest rates to arrive at the market value of an existing swap position. This approach can be implemented easily and provides important insights into the valuation of existing swap positions. For example, the market value of an existing swap position is shown to be related to the value of a reference coupon bond with a fixed coupon rate and unit face value. The coupon rate of this reference coupon bond, however, varies depending on whether the swap position is that of a fixed rate payer or receiver.

Since interest rate swaps are interest rate contingent contracts, their market values are detemined, in equilibrium, ultimately by the same fundamental economic variables or parameters that determine the term structure of interest rates. We explore these important links using the one-factor general equilibrium term structure model of Cox, Ingersoll, and Ross (1985). Despite some limitations and the development of other arbitrage-free models, this model, commonly known as the CIR model, is the most widely used equilibrium valuation model of interest rate contingent securities. Besides examining the equilibrium valuation of existing swap positions, use of the CIR model also allows us to derive a dynamic measure of interest rate risk for the existing swap positions that is similar to the stochastic duration measure of Cox, Ingersoll, and Ross (1979) for coupon bonds and the quasi stochastic duration measure of Chen, Park, and

---

5 There is a growing practice in the industry of marking to market the existing swap positions. Other than Litzenberger (1992), we are not aware of any rigorous discussion about the theoretical basis of the industry practice. Litzenberger, however, emphasizes the role of the unique treatment of swaps under default events in explaining why the industry practice is not sensitive to credit risk and why swap spreads show relatively low cyclical variability.

Wei (1986) for bond futures. This results in a unified theoretical treatment of the market value and risk of previously established swap positions.

The theoretical valuation and dynamic risk measurement framework developed in this paper can be useful from a practical point of view in several regards. *First,* our framework can be applied to evaluate the adequacy of current disclosure requirements with respect to swap positions of firms in general and the financial institutions in particular. For example, banks are currently required to report the replacement value of their aggregate swap positions which may be different from the market value of those positions. This may lead to a distorted picture of a bank's capital adequacy. *Second,* early warning signals to detect severe erosion in equity and excessive risk exposure can be put in place using our framework. The dynamic risk measure derived in this paper should be particularly useful in tracking the risk exposure in a changing market. *Third,* the constructs of our paper can be applied to establish proper hedge by firms wanting to hedge their swap positions or other term structure-sensitive assets or liabilities. In the same vein, the establishment and management of an internal policy of specific risk exposure targeting, such as a target stochastic duration, can be facilitated using our results. *Fourth,* since we use a general (equilibrium) framework for interest rate contingent claims, the market value and risk of economic transactions that are either equivalent or close in nature to swap positions, e.g., parallel loans, can be measured and analyzed using a common framework. Such attempts will be in the spirit of *functional regulation* or similar regulatory treatment of economically equivalent transactions advocated by Merton and Bodie (1995).

The rest of this paper is organized as follows. In Section I, we develop models for determining the market values of previously established swap positions to the counterparties. Equilibrium valuation of existing swap positions using the CIR model is then discussed in Section II. In Section III, we address the interest rate risk of existing swap positions and derive a dynamic measure of this risk. Some methods in swap management are briefly discussed in Section IV. We conclude the paper in Section V.

I. The Market Values of Swap Positions

By market convention, the fixed-rate payer that has a long swap position in a fixed/floating interest rate swap is called the *taker* or *buyer* of the swap, while the floating-rate payer that has a short swap position in the fixed/floating interest rate swap is called the *provider* or *seller* of the swap. The fixed-rate payer and the floating-rate payer of an interest rate swap are called the counterparties of the swap.

At the date of contract initiation of a fixed/floating interest rate swap, the swap contract is usually executed *at-the-money* and the counterparties are said to have positions in a *par value* (or *at-the-money* or *at-market)* swap because there is no initial cash exchange between the two counterparties. Thus, at the date of contract initiation, an interest rate swap contract is neither an asset nor a liability to either counterparty. However, subsequent to its initial date of agreement, any market interest rate movements can cause the market value of a swap contract to become positive to one counterparty and negative to other counterparty. For instance, a fall in the market prices of the fixed/floating interest rate swaps (expressed in terms of the fixed rate of interest on a swap) will make the existing swap contract a liability to the counterparty with a *long* swap position (i.e., the fixed-rate *payer in* the swap) and an asset to the counterparty with a *short* swap position (i.e., the floating-rate *payer* in the swap). Conversely, a rise in the market prices of the fixed/floating interest rate swaps will bring a gain to the counterparty with a *long* swap position (the *buyer)* and a loss to the counterparty with a *short* swap position (the *seller).*

Financial managers should be able to determine at any time the market values of the individual swap contracts held by their firms, if they want to manage the swap positions of their firms in a prudent fashion. In the following, we shall develop and discuss models for determining the market values of existing *long* and *short* swap positions.

A. Notation

To determine the market values of an existing fixed/floating interest rate swap to its counterparties and the market values of a swap portfolio, let us introduce the following notation:

W — the notional principal of the swap;

m — $= T - t$, the remaining number of *semiannual* periods to the maturity of the swap, also known as the *tenor* of the swap when the swap is originated where T is the original maturity date of the swap and t is the swap evaluation date[6];

$P(j,t)$ — the price at time t of a default-free unit discount (zero coupon) bond that matures at time $t+j$, where j is measured in semiannual periods;

$r(m,t)$ — $= 2\,[1-P(m,t)] / [\sum_{j=1,m} P(j,t)]$, **the bond-equivalent annual yield to maturity (BEY) on** the m -period *par value* Treasury bond at time t implied by the zero coupon yield curve[7];

i_0 — the original fixed rate of interest (BEY type) on the swap;

$i_b(m,t)$ — the dealer's bid price (also known as *pay rate)* of the m -period swap at time t; it is the fixed-rate (BEY type) of an at-the money swap when dealer *pays* the fixed rate;

$i_a(m,t)$ — the dealer's ask price (also known as *receive rate)* of the m -period swap at time t; it is the fixed-rate (BEY type) of an at-the money swap when dealer *receives* the fixed rate[8];

$LS(m,t)$ — the market value of an existing swap with m periods to maturity to the *long*- swap-position-holder (the buyer) at time t;

$SS(m,t)$ — the market value of an existing swap with m periods to maturity to the *short*- swap-position-holder (the seller) at time t.

---

6 To simplify our derivations, we assume t to be a reset date for the swap.

7 The $r(m,t)$ calculated in this manner implies absence of coupon stripping or synthetic coupon arbitrage opportunity. However, it is some times argued that the zero coupon bonds are less liquid than the underlying Treasury coupon bond from which the zeros are stripped off. Thus, according to this argument, the yield on the *par value* coupon bond implied by the zeros is an inaccurate (over) estimate of the coupon bond if it were to directly trade at par.

8 In reality, there may be many dealers making market in a given type of swap and their bid and ask quotations may vary albeit by small amounts. For our analysis, *the dealer* is taken to be an average dealer.

The actual process of determination of the swap dealer's bid and ask prices is outlined in Appendix A. Without loss of generality, we can express the bid and ask prices in the following manner:

$$i_b(m,t) = r(m,t) + d_b(m,t), \text{ and}$$

$$i_a(m,t) = r(m,t) + d_a(m,t),$$

where $d_b(m,t)$ and $d_a(m,t)$ are dealer's bid and ask swap spreads. In general, the swap spreads can be functions of the term structure and thus can be an additional source of variation in the bid and ask prices as the interest rate situation changes in the market. This indirect effect of changes in Treasury market yields is however quite small compared to the direct effect through $r(m,t)$ since the spreads themselves are quite small relative to $r(m,t)$. Hence we assume in what follows that the spreads do not depend on the Treasury yields. We, however, allow the spreads to vary with the time to maturity of the swap, m, as is the case in reality.

As was noted earlier, $r(m,t)$ is the BEY of *m-period par value* Treasury bond implied by the term structure of zeros and as such its use is in the spirit of arbitrage-free valuation approaches used in the industry. One added benefit of using the above construct is that the coupon rate of a *par value* Treasury security is equal to its BEY. As we shall see later, this feature simplifies our analysis of market value and risk of swap positions. Further, term structure models are needed to evaluate the market risk of existing swap positions and most well-known theoretical models of the term structure of interest rates offer explicit solutions for $P(j,t)$'s. The chosen $r(m,t)$-*based* construct for swap bid and ask prices allows one to explore with ease the effect of the parameters of term structure models on the market value and risk of existing swap positions.

Our focus in this paper is on the market value and risk of swap positions that were initiated earlier. Hence we treat i_b and i_a (i.e., the spread parameters d_b and d_a) as *given* to us. In particular, we do not directly analyze the effects that the credit risk and the demand and supply of swaps have on the determination of i_b and i_a. and thus on the market value and risk of existing swap positions. This approach for the valuation of existing swap positions is reasonable given the size and liquidity of today's swap market. In today's market a firm can unwind its existing swap position without any noticeable impact on the market price (i_b and i_a). As such the firm can be treated as a price-taker in the context of valuing its existing swap positions.

B. The Market Value of An Existing Swap Position

Most of the existing fixed/floating interest rate swaps, especially those with more active floating indices such as LIBOR and T-bill rates, can be readily traded in the secondary markets. The market value of an existing interest rate swap position is the dealer's evaluation of the lump sum value of the particular swap position at a given time. More specifically, the market value of an existing swap position is the lump sum dollar amount the dealer must receive or pay to be indifferent between stepping into the existing swap position or taking the same side in a new at-the-money swap[9].

If the swap buyer wishes to unwind the long position prematurely at time t, she may ask the dealer to take up her position, which is to pay $i_0/2$ semiannually against 6-month LIBOR flat on a notional principal of W for m semiannual periods. The dealer's current bid price is $i_b(m,t)$, that is if the dealer takes the long side in a new at-the-money swap, she is willing to pay fixed at the rate of $i_b(m,t)/2$ semiannually against 6-month LIBOR flat for m semiannual periods. Thus if the dealer takes over the swap buyer's existing long position instead of taking the long side in a new at-the-money swap, she will experience an incremental cashflow of $W[(i_b(m,t)-i_0)/2]$ for the next m semiannual periods. In general, the incremental cashflow stream will be an *inflow (outflow)* to the dealer if the interest rates have *risen (fallen)* since the time of the existing swap's origination.

Ignoring credit risk, the incremental cashflow stream to the dealer is *certain* since it does not depend on the floating rate. Taking over the swap buyer's position instead of taking the *long* side in a new at-the-money swap is like *buying (short selling,* if incremental cashflow is *negative*) $W[(i_b(m,t)-i_0)/2]$ unit discount bonds of each of the m maturities. Therefore, the value of the swap buyer's position to the dealer is equal to the value of this portfolio of unit discount bonds. At the margin, the swap dealer will be indifferent between taking the *long* side of a new

---

9 This is essentially the same as the ISDA Code's "agreement value."

at-the-money swap *and* taking over the swap buyer's existing *long* swap position and *pay to (receive from,* if incremental cashflow is *negative)* the swap buyer the value of the discount bond portfolio. The signed value of the discount bond portfolio is thus indeed the market value of the swap buyer's existing *long* position:

$$LS(m,t) = W[(i_b(m,t)-i_0)/2]\,[\Sigma_{j=1,m} P(j,t)] \tag{1}$$

Using equation (l), a swap buyer can easily calculate the market value of her position by simply observing the dealer's bid price and the current term structure or the market prices of the zeros[10]. As an approximation for $[\Sigma_{j=1,m} P(j,t)]$, one may try the traditional annuity factor with the *m*- period *par value* Treasury Bond's semiannual yield to maturity as the single interest rate, $[1 - (1+0.5r(m,t))^{-m}]/0.5r(m,t)$. For a flat yield curve, this approach will be exact. For a rising (falling) yield curve, the approximation will lead to an underestimation (overestimation) of the magnitude of $LS(m,t)$. This is because $r(m,t)$ *is* greater (less) than the yield to maturity on just the coupon stream of a par value bond when the yield curve is rising (falling)[11]. The size of the approximation error increases with swap maturity and the absolute value of the yield curve slope. Caution is also warranted, especially in a *rising* yield curve situation, in using $i_b(m,t)$ as the single discount rate to value the incremental cashflow stream of the swap position since by construction it is also a yield to maturity on a coupon bond (and not just the coupons) like $r(m,t)$.[12]

At any given point in time, an existing *long* swap position is of *positive* (an *asset)* or *negative* (a *liability)* market value to the swap buyer depending on whether $i_b(m,t)$ is *greater* than or *less* than i_0. If an existing *long* position is of positive value, $LS(m,t) > 0$, the swap is said to be *in-the-money* to the swap buyer, and the buyer has gained from holding a *long* position in the swap

---

10 Strictly speaking, one would use either the bid price or the ask price of the zeros depending on the sign of the differential cashflow.

11 For example, when the semiannual yields are 0.03 and 0.05 for 6-month and l-year maturity zeros, $r(2,0)/2$ is 0.0495 while the semiannual yield to maturity on just the coupons of the *par value* bond is 0.0430. This leads to an underestimation of the value of coupons by about 0.9%.

12 The industry practice (Marshall and Bansal, 1992, p.432) of marking swap positions to market using zero coupon swaps curve implied by *par value* or *at-market* swaps curve is also questionable. Appendix B contains a brief discussion of the industry practice and its limitations.

transaction. If $LS(m,t) < 0$, the swap is said to be *out-of-the-money* (i.e., the *"underwater"* swap) to the swap buyer, and the buyer has lost from the swap transaction.

Using arguments similar to the valuation of an existing *long* position, the market value of the swap seller's existing *short* position can be derived as:

$$SS(m,t) = W[(i_0 - i_a(m,t))/2] \, [\sum_{j=1,m} P(j,t)] \tag{2}$$

In this case, the swap dealer's incremental cashflow from taking over the swap seller's existing *short* position instead of taking the same side in a new at-the-money swap is $W[(i_0 - i_a(m,t))/2]$ for the next *m* semiannual periods. In general, the incremental cashflow stream will be an *inflow (outflow)* to the dealer if the interest rates have *fallen (risen)* since the time of the existing swap's origination. Taking over the swap seller's position instead of taking the *short* side in a new at-the-money swap is like *buying (short selling,* if incremental cashflow is *negative*) $W[(i_0 - i_a(m,t))/2]$ unit discount bonds of each of the *m* maturities. Therefore, the value of the swap seller's position to the dealer is equal to the value of this portfolio of unit discount bonds. At the margin, the swap dealer will be indifferent between taking the *short* side of a new at-the-money swap *and* taking over the swap seller's existing *short* swap position and *pay to (receive from,* if incremental cashflow is *negative)* the swap seller the value of the discount bond portfolio.

An existing *short* swap position is of *positive* (an *asset)* or *negative* (a *liability)* market value to the swap seller depending on whether $i_a(m,t)$ is *less* than or *greater* than i_0. If an existing *short* position is of positive value, $SS(m,t) > 0$, the swap is said to be *in-the-money* to the swap seller, and the seller has gained from holding a *short* position in the swap transaction. If $SS(m,t) < 0$, the swap is said to be *out-of-the-money* (i.e., the *"underwater"* swap) to the swap seller, and the seller has lost from the swap transaction. At any point in time t, if an existing swap is *in-the-money* to its *buyer,* it will be usually *out-of-the-money* to its *seller,* and vice versa.

Note that the magnitudes of the market values of an existing *long* position and an existing *short* position are different even if their terms (W, m, i_0) are identical. This differential value is what the swap dealer hopes to capture by making the market in swaps:

$$|LS(m,t)-SS(m,t)| = [d_a(m,t)-d_b(m,t)][\Sigma_{j=1,m} P(j,t)] \quad (3)$$

Let us now illustrate how to determine the market values of an existing fixed/floating interest rate swap to its counterparties. Assume that the Bank has on its book a *long (short)* position in a fixed/floating interest rate swap with Counterparty A (B). The terms of the existing swap and the corresponding swap payment schedule for the bank are shown below

	Bank's *long* position With A		Bank's *short* position With B	
(1) Original Swap:				
1. Notional principal (*W*):	$10 million		$10 million	
2. Fixed rate pay/receive (i_0):	7.50%		7.60%	
3. Floating rate receive/pay:	6-month LIBOR		6-month LIBOR	
4. Remaining time to maturity (*m*):	6 periods (3 years)		6 periods (3 years)	
(2) Bank's Swap Payment Schedule:				
Semiannual				
Period	Payment	Receipt	Payment	Receipt
j=1,2,..,6	-$375,000	+ $10 million x (LIBOR/2)	-$10 million x (LIBOR/2)	+$380,000

(3) Current (time *t*) Market Conditions:

a. BEYon 3-year *par value* Treasury Bond (*r(6,t)*): 6.00%

b. 3-year *par value (at-market)* Fixed/Floating swap prices:

a) Dealer's *bid* price : 3-year *par value* T-Bond + 30 bp or 6.30%

b) Dealer's *ask* price : 3-year *par value* T-Bond + 40 bp or 6.40%

c. BEY on Zeros: 4%(j=1), 4.8%(j=2), 5.6%(j=3), 6.4%(j=4), 7.2%(j=5), 5.98%(j=6)

The current market conditions show that the dealer's bid swap price, $i_b(6,t)$, *is* 6.30%, which is the sum of the yield, *r(6,t),* on the 3-year par value Treasury Bond of 6% and the dealer's bid spread $d_b(6,t)$, for the 3-year swap of 30 basis points. Also, the dealer's ask swap price, $i_a(6,t)$, is 6.40%, which is 40 basis points for the ask spread, $i_a(6,t)$, for a 3-year swap plus the 3-year par value Treasury Bond yield of 6%. The incremental cashflow to the dealer of the bank's long position in the existing swap is -$60,000 {= $10 million x [(6.30% - 7.50%)/2]} per semiannual period for the next three years. At the currently observed periodic-specific BEY's on the zeros, **the annuity factor, $\sum_{j=1,6} P(j,t)$, is 5.411744. Using equation (1), the current market value of the** bank's existing *long* swap position is -$324,705.

The incremental cashflow to the dealer of Counterparty A's existing *short* swap position is $55,000 {= $10 million x [(7.50% - 6.40%)/2]} per semiannual period for the next three years. Using equation (2), the current market value of Counterparty A's existing *short* swap position is $297,646. While the swap was *at-the-money* at origination (no cash changed hands), the bank's *long* position has since gone *under water* and Counterparty A's *short* position has become *in-the-money*. The difference, $27,059, in the market values of the Bank's *long* position and Counterparty A's *short* position is the value (at the current yield curve for zeros) of the swap dealer's spread of 5 basis points ((40 bp -30 bp)/2) on $10 million notional for 6 semiannual periods. The swap dealer can capture this value if both the Bank and Counterparty A unwind their respective positions through the dealer.

The current market value of the Bank's existing *short* position (vis-à-vis Counterparty B) is $324,705, which is the value (at the current yield curve) of the incremental cashflow of $60,000 {=$10 million x [(7.60% - 6.40%)2]} for 6 semiannual periods to the dealer. Counterpart B's existing *long* position has an incremental cashflow of -$65,000 {= $10 million x [(6.30% - 7.60%)/2]} and is valued at $351,763. The difference of $27,058 in value once again belongs to the swap dealer if the Bank and Counterparty B unwind their respective positions through the dealer.

The changes in the market value of the Bank's two positions (*long* with Counterparty A and *short* with Counterparty B) offset each other. If the Bank itself is the swap dealer in question and the two counterparties decide to unwind their respective positions, the Bank will pay Counterparty A $297,646 and receive $351,763 from Counterparty B as lump sums. These transactions will leave the Bank (as a swap dealer) with $54,117 and all of its swap positions closed. To dispose of its *long* position with Counterparty A, the Bank had to pay $324,705 to another dealer, but being the swap dealer itself, the Bank is getting away with paying Counterparty A $297,646. Similarly, the Bank would have received only $324,705 if its *short* position with Counterparty B was sold to another dealer, but it is now receiving $351,763 from Counterparty B. As a swap dealer, the Bank has thus picked up the value of 5 basis points (semiannual basis) swap spread from each of A and B. However, the net gain is actually only 5 basis points (semiannual basis) since in the process the Bank's original fixed rate spread of 5 basis points (semiannual basis) has been lost. This original spread would also have been lost if the Bank decided to unwind its positions through another dealer with no net payment to the dealer.

C. The Determinants of the Market Value of An Existing Swap Position

To gain more insights into the determinants of the market value of swap positions, let us substitute for $i_b(m,t)$ in equation (1) and express the value of an existing *long* position in terms of the value of a coupon bond:

$$LS(m,t) = W\,[1 - B_{IL}(m,t)\,] \qquad \text{(1a),}$$

$$\text{where } B_L(m,t) = P(m,t) + [(\,i_0 - d_b(m,t))/2][\textstyle\sum_{j=1,m} P(j,t)] \qquad \text{(1b).}$$

$B_{IL}(m,t)$ is the price of an m-period coupon bond with unit face value and semiannual coupon of $C_L(m,t) = (\,i_0 - d_b(m,t))/2$. This m-period coupon bond would sell at par ($1.00) if it had a BEY of $R_{BI}(m,t) = i_0 - d_b(m,t)$.

Substituting for $i_a(m,t)$ in equation (2), we can express the value of an existing *short* position in terms of the value of a different coupon bond:

$$SS(m,t) = W\,[Bs(m,t) - 1] \qquad \text{(2a),}$$

$$\text{where } B_S(m,t) = P(m,t) + [(\,i_0 - d_a(m,t))/2][\textstyle\sum_{j=1,m} P(j,t)] \qquad \text{(2b).}$$

$B_S(m,t)$ is the price of an m -period coupon bond with unit face value and semiannual coupon of $C_S(m,t) = (\,i_0 - d_a(m,t))/2$. This m-period coupon bond would sell at par ($1.00) if it had a

BEY of $R_{B/S}(m,t) = i_0 - d_a(m,t)$. Hereafter, we shall refer to the bonds in (1b) and (2b) as the reference coupon bonds.

Equations (1a), (lb), (2a), and (2b) confirm the common knowledge that a *long (short)* swap position behaves like a *short (long)* position in a coupon bond. These equations, however, clearly specify that the related coupon bond is not the same for the *long* and the *short* positions in the presence of swap dealer's spreads. The specific terms of these related bonds and more importantly the pricing structure of these bonds are delineated in these equations. As shown in these questions, the broad determinants of the prices of these related bonds and hence the market values of the existing swap positions per dollar of notional principal are the following: (I) the original fixed-rate of interest, i_0; (ii) the dealer's current bid or ask swap spread, $d_b(m,t)$ or $d_a(m,t)$; (iii) the remaining time to maturity of the swap, m; and (iv) the term structure of interest rates or discount bond prices, $P(j,t)$'s, $j=1,2, ..,m$.

The effect of the original fixed rate, i_0, is obvious. Other things equal, a *higher* i_0 increases the value of both reference coupon bonds, $B_{/L}(m,t)$ and $B_S(m,t)$, and hence leads to a *lower (higher)* market value of an existing *long (short)* swap position. The effect of the dealer's spreads are also apparent. *Larger spreads* decrease the value of both reference coupon bonds, $B_{/L}(m,t)$ and $B_S(m,t)$, and hence leads to a *higher (lower)* market value of an existing *long (short)* swap position. The original fixed rate, i_0, and dealer's spreads, $d_b(m,t)$ and $d_a(m,t)$, influence the market values of existing swap positions through their effect on the coupon rates of the related hypothetical bonds, $C_L(m,t)$ and $C_S(m,t)$.

Unlike the original fixed rate, i_0, dealer's spreads, $d_b(m,t)$ and $d_a(m,t)$, vary over the life of a swap. Part of the time variation in the spreads is predictable and the other part is stochastic. The predictable part arises from the real life observation that both the bid spread, $d_b(m,t)$, and the ask spread $d_a(m,t)$, usually increase with swap maturity. As the swap gets closer to its maturity, the spreads will decline leading to an *increase in* the value of both reference coupon bonds, $B_{/L}(m,t)$ and $B_S(m,t)$, by increasing their coupon rates, $C_L(m,t)$ and $C_S(m,t)$. The predictable time variation of the spreads thus works *against (in favor of)* the holder of the *long (short)* position.

The spreads may experience stochastic variations over the life of a swap to the extent the swap dealers adjust their spreads in response to changing demand and supply conditions in the swap market[13]. This will introduce some random variations in the coupon rates of the related hypothetical bonds as time passes. For *two* reasons, however, we do not address the random variation of the spreads. *First*, the predominant source of the random variation in the spreads is the changing demand and supply conditions in the swap market, which are in turn largely induced by unanticipated movements in the term structure of interest rates. We are of course going to discuss the effect of the stochastic variations in the term structure on the market value and risk of swap positions. *Second,* the spreads are much smaller in size compared to the original fixed interest rate, i_0. Hence any realistic degree of variation in the spread will have a negligible impact on the coupons of the two hypothetical bonds and thus on the market value and risk of the swap positions[14].

Other than the minor effect of time to maturity, *m*, via the spreads, its primary impact on the market value of the swap positions is intertwined with the effect of the term structure of interest rates. Specific comments about these effects can only be made in the context of a given term structure model of which the finance literature has many.

A close link of the market value of swap positions to the term structure is expected given that the interest rate swaps are after all interest rate derivatives with multiperiod cashflows. Looking at the valuation equations (1) and (2), it may first appear that the only role of the term structure is to provide a composite valuation operator (the annuity factor, $\sum_{j=1,m} P(j,t)$) to be applied to the

---

13 The creditworthiness of the counterparties may also change in an unanticipated fashion over the life of the swap and hence may contribute to the (ex ante) random variation of the spreads. In this paper, however, we do not analyze the effect of the credit risk on the market value of swap positions. Hull (1989), Cooper and Mello (1991), Duffie and Huang (1996), and Sun and Wang (1996), among others, analyze the effect of credit risk on swap pricing.

14 In case the credit situation of a counterpart deteriorates substantially and no informed dealer will take over the swap position of the other counterparty, the spread becomes infinite theoretically speaking. Common knowledge has it that both the primary and the secondary swap markets operate more on an availability basis than on a price discrimination basis when it comes to credit risk.

constant multiperiod cashflow of an existing swap position. It is also tempting to ignore this role of the term structure and use the dealer's current bid or ask price as the single discount rate in the traditional annuity factor formula. However, we have mentioned earlier the obvious systematic error in valuation that results from using the bid or ask price or even the *par value* coupon bond's yield to maturity as the single discount rate. Thus the discounting role of the term structure should be preserved. No less important is the role of the term structure in determining the bid and the ask prices. These prices are determined on the basis of the *par value* coupon bond's yield to maturity which itself is contrived from the term structure. When the term structure changes, two things happen: the incremental cashflow stream of a swap position changes (the bid and the ask prices change as the *par value* coupon bond's yield to maturity changes), and the value of the incremental cashflow stream changes (the annuity factor changes). These two changes are not necessarily in the same direction.

While the currently observed term structure is enough to price the existing swap positions, a term structure model serves several useful purposes. *First,* the economywide factors and parameters that determine the term structure and its movement are the ultimate determinants of the market value and risk of swap positions. A term structure model allows us to explore the influence of these ultimate determinants on the market value of swap positions. *Second,* the market risk of swap positions arise from unanticipated movements in the term structure. A term structure model permits us to describe the stochastic evolution of swap positions as a function of the fundamental uncertainties in an economy and to derive a stochastic market risk measure. *Third,* what effect the time to maturity has on a swap position's value depends on the term structure dynamics. A term structure model can help predict the effect of swap's time to maturity. *Last,* a swap portfolio may comprise of swap positions of varying maturities and there is no natural choice for a single yield measure as a determinant of the market value and risk of the swap portfolio. A (one factor) term structure model can provide such an yield measure.

The benefits of a term structure model are to be weighed against the costs which are not clear. The main limitation is not knowing for sure which term structure model best captures the features of term structure movements in reality. As a result, the predictions of a term structure model are always conditional on its ability to describe term structure movements in reality. Unfortunately, there is no apparent solution to this dilemma and it is unlikely there will ever be one[15]. Therefore, the usual caveat applies to our discussions based on a specific term structure model.

II. Equilibrium Term Structure Theory and The Effects on the Market Value of Swap Positions

The finance literature is rich with term structure models[16]. In this paper, we use the one-factor general equilibrium term structure model of Cox, Ingersoll, and Ross (1985), widely known as the CIR model. This section starts with a brief presentation of the CIR model. This is followed by a discussion of the effects of the spot interest rate, the time to maturity of the swap position, and the equilibrium valuation parameters on the value of *long* and *short* swap positions.

A. The CIR Model

The CIR model has been extensively used in the literature to value interest rate contingent claims. Some key advantages of the CIR model are: (a) it implies non-negative interest rates; (b) interest rate volatility is heteroskedastic conditional on the interest rate level; (c) the full effect of a shift in the term structure on a portfolio of zeros can be captured since yields on all maturities are allowed to be stochastic, an important feature for swaps; (d) since the market price of risk is obtained as part of the equilibrium, the CIR model avoids internal inconsistencies and arbitrage opportunities; and (e) the basic one-factor CIR model can be easily extended to the case of two

---

15 Practitioners and regulators have been toiling with a similar dilemma in using *value at risk (VAR)* as a measure of market risk of involvements in derivatives (Reed, 1995). Apparently, the *VAR* of an institution depends on the specific models that are used for valuing the the derivatives. There is no uniform industry standards for such models and the regulators are equally reluctant to impose such standards.

16 See Rogers (1995) for an interesting recent review of the well known term structure models.

(Cox, Ingersoll, and Ross (1985), Longstaff and Schwartz (1992)) or more factors (Chen and Scott (1995)) and thus can be adapted to fit multiple points on the initial term structure using the approach of Hull and White (1990)[17].

In the one-factor CIR model, the instantaneous default-free rate, *r(t),* alternatively referred to as the spot rate or the short rate, is the instrumental variable for the underlying single state variable that captures the fundamental stochastic characteristics of an economy. The dynamics of the spot rate is given by:

$$dr(t) = \kappa\,(\theta - r(t))\,dt + \sigma \sqrt{r(t)}\;dz \qquad (4)$$

where dz is one-dimensional Wiener process, $\kappa > 0$, and $\theta > 0$. Known as the mean-reverting square root model of interest rate, the process in equation (4) is a continuous time first-order autoregressive process where the randomly moving spot rate is pulled toward its stationary mean, θ; at an expected rate (speed of adjustment) of κ. The instantaneous drift and variance of the spot rate are respectively $\kappa\,(\theta - r(t))$ and $\sigma^2 r(t)$.

The time *t* equilibrium price of a *j* -period (matures at *t+j)* default-free unit discount bond in the CIR model is given by:

$$P(j,t) = G(j,t)\, exp[-r(t)H(j,t)] \qquad (5)$$

where

$G(j,t) = [\,2\,\gamma\, exp\{(\gamma + \kappa + \lambda)j/2\} \,/\, \{(\gamma + \kappa + \lambda)(exp(\gamma j)-1)+2\gamma\}\,]^{2\kappa\theta/\sigma^2}$,

$H(j,t) = 2(exp(\gamma j)-1) \;/\; \{(\gamma + \kappa + \lambda)(exp(\gamma j)-1)+2\gamma\}$,

$\gamma = \sqrt{\{(\kappa + \lambda)^2 + 2\sigma^2\}}$, and

$\lambda r(t)$ is the covariance of spot rate changes with percentage changes in optimally invested wealth (by a representative agent with constant absolute risk aversion). The parameter λ is considered as a preference or risk premium parameter since the instantaneous expected return on a bond is $r(t) + \lambda\,[r(t)\;\; P_r(j,t) / P(j,t)]$, where the subscript r denotes partial derivative of the unit discount

---

[17] See Longstaff (1993, footnote 3, p. 29) for the limitations of exactly fitting the whole initial term structure.

bond price with respect to the spot rate, *r(t)*, and the expression in parenthesis is the spot rate elasticity of the discount bond price.

The equilibrium term structure in terms of the yields to maturity on the unit discount bonds of various maturities, or the equilibrium yield curve, is given by:

$$R(j,t) = [r(t)\,H(j,t) - \ln(G(j,t))]\,/\,j \tag{6}$$

The yield curve is rising if $r(t)$ is below the long-term (infinite maturity) yield, $2\kappa\theta\,/(\gamma+\kappa+\lambda)$. For $r(t)$ in excess of $\kappa\theta\,/(\kappa+\lambda)$, the yield curve is falling. For $2\kappa\theta\,/(\gamma+\kappa+\lambda) < r(t) < \kappa\theta\,/(\kappa+\lambda)$, the yield curve is humped. The yield curve is monotonically increasing in the spot rate, *r(t)*. For small increases (decreases) in the spot rate, the yield curve shifts up (down) in a non-parallel fashion as the change is greater for shorter maturities.

As shown by equations (5) and (6), the determinants of the equilibrium bond prices and the term structure in the CIR model are: the spot rate, $r(t)$; the stationary mean of the spot rate, θ; the instantaneous variance of the spot rate, σ^2; the preference or risk premium parameter, λ; and the speed of adjustment parameter, κ. The (general equilibrium) market value of a swap position is a function of the equilibrium term structure or the set of equilibrium discount bond prices, *P(j,t)'s* [18]. Hence the spot rate, its stationary mean and its instantaneous variance, the risk premium parameter, and the speed of adjustment parameter are the economy-wide or fundamental determinants of the equilibrium market value of swap positions. In what follows, we assume that the observed yield curve or the set of discount bond prices is always at the equilibrium level, which however changes over time. We, therefore, omit the adjective *equilibrium* in referring to the term structure or the market value of swap positions.

---

18 The equilibrium term structure is completely specified by the equilibrium discount bond prices. Hence, we use the terms *yield curve* or *discount bond prices* interchangeably in referring to the term structure.

B. Effect of the Spot Rate?

Of the fundamental determinants, the key one is the spot rate, *r(t),* which contains all the value-relevant information about the current state of the economy in the one-factor CIR model. Since the whole yield curve is a (monotonically increasing) function of the spot rate, the market values of all swap positions, irrespective of their maturity, depend on this single interest rate alone. Albeit the *par value* coupon bond's yield to maturity will vary across swap positions, but all these *par value* yields are determined by the current spot rate alone.

Let us now see how the market value of an existing *long* swap position $LS(m,t)$, is related to the current spot rate, $r(t)$. In our one-factor CIR model, discount bond price of any maturity in equation (5) is a decreasing convex function ($P_r(j,t)<0$, $P_{rr}(j,t)>0$) of the spot rate. Thus, the price of the reference coupon bond, $B_{/L}(m,t)$, in equation (lb), is also a decreasing convex function of the spot rate. Therefore, according to equation (la), the market value of an existing *long* swap position, $LS(m,t)$, is an *increasing concave* ($LS_r(m,t)>0$, $LS_{rr}(m,t)<0$) function of the spot rate, $r(t)$. It can be shown that the par value coupon bond's yield to maturity is increasing in the spot rate ($r_r(m,t)>0$), and therefore, $LS(m,t)$ is increasing in $r(m,t)$. These results will hold for any one-factor term structure model where the discount bond price is a decreasing convex function of the spot rate for all maturities[19].

The direct relationship between the spot rate and the market value of an existing *long* swap position is in fact the net result of two opposing influences. Looking at equation (l), when the spot rate changes, the constant incremental cashflow ($W[(i_b(m,t)-i_0)/2]$) and the composite valuation operator or annuity factor ($\sum_{j=1,m} P(j,t)$) change in opposite directions. For an increase in the spot rate, the *par value* coupon bond's yield to maturity, $r(m,t)$, goes up leading to an increase in the bid price, $i_b(m,t)$. This enhances the constant incremental cashflow to the dealer

19 Two well-known examples of such one-factor term structure models are Vasicek (1977) and Dothan (1978). This result, however, may not hold universally in a two-factor model. As shown by Longstaff and Schwartz (1992), in a two-factor CIR model where the instantaneous variance of the spot rate is the second factor, the price of longer maturity discount bonds may be increasing in the spot rate.

and hence an increase in the value of an existing *long* swap position using the old (pre-change) annuity factor. Let us call this the *coupon effect.* For an increase in the spot rate, the composite **annuity factor, $\sum_{j=1,m} P(j,t)$, goes down, making** the old incremental cashflow stream less valuable. Let us call this the *discounting effect.*

To recapitulate, when the spot rate goes up, the incremental cashflow to the dealer goes up but the stream is less valuable. The *positive coupon effect,* however, dominates the *negative discounting effect* of an increase in the spot rate leading to the *net effect* of an *increase* in the market value of an existing *long* swap position. The intuitive reason behind this is that when the spot rate changes, the yield to maturity on the *par value* coupon bond changes *more* than the yield to maturity on the annuity of just the coupon stream. At higher levels of the spot rate and thus yield to maturity, the differential between the yields to maturity of the coupon bond and the coupon annuity narrows in a relative sense. This is at the source of concavity of the relationship between the spot rate and the market value of an existing *long* swap position[20].

If we plot the market value of an existing *long* swap position, *LS(m,t),* as a function of the spot rate, *r(t),* the steepness or slope of the function will indicate the magnitude of the effect of a spot rate change. A key determinant of the magnitude of the spot rate effect is the swap position's time to maturity, *m.* To illustrate this matter, we plot in Figure 1, the value of an existing *long* swap position as a function of the spot rate for three different swap maturities, *m* = 1 (0.5 years), 8 (4.0 years), and 14 (7.0 years). The assumed values of the other parameters are: $\kappa=0.1$, $\theta=0.04$, $\lambda=0.0$, $\sigma^2=0.0025$, $i_0=0.05$, and $d_b(m,t)=0.0012$ for $m=1,8$, and 14.

Figure 1 shows that the function gets steeper as the swap maturity gets longer, that is the spot rate effect is *stronger* for *longer* maturity swaps[21]. This is because the *positive coupon effect* of a spot rate increase is much more dominant relative to the *negative discounting effect* for *longer*

---

20 In equation (la), the netting out of the two effects lead to a short position in the hypothetical coupon bond with a coupon rate that does not depend on the spot rate. Hence the well known decreasing convexity of a coupon bond's price lead to increasing concavity of the market value of an existing *long position.*

21 As expected, the functions are concave, although the degree of concavity is negligible given the assumed parameter combination.

maturity *long* swap positions (with the same original fixed rate). The economic intuition behind this pattern is that the yield differential between the *par value* coupon bond and the coupon annuity is relatively more for longer maturity bonds in sloped yield curve situations. When the spot rate increases, yields to maturity on both *par value* coupon bond and coupon annuity increase for all maturities. However, the increase in the yield to maturity on *par value* coupon bond relative to that on coupon annuity is *greater* when the maturity is *longer,* thus causing the greater relative dominance of the coupon effect. Once again the net effect can be seen more clearly in equations (la) and (lb). The price of the reference coupon bond with a constant (unrelated to the spot rate) coupon is more responsive to the spot rate (or *par value* coupon bond's yield) for a longer maturity. This results in a greater responsiveness for longer maturity existing *long* positions.

Using similar arguments, we find that the market value of an existing *short* position is a decreasing convex function of the spot rate (or *par value* coupon bond's yield to maturity). The steepness effect of the swap maturity is similar to the case of a *long* position. In other words, longer maturity *short* swap positions are more responsive to spot rate changes than shorter maturity *short* swap positions.

The above discussion of the spot rate effect on the market value of existing swap positions may come across as if it was the effect of a change in just one point on the yield curve or the term structure. The careful reader will note that the spot rate effect is in fact the effect of a (non-parallel) shift in the whole yield curve. This is because in our one-factor CIR model, when the spot rate changes, it induces a change (in the same direction) in the yields on all maturities, with the shorter maturity yields changing more than the longer maturity yields. We should also bear in mind that the spot rate effect discussed above relates only to small changes in the spot rate; by assumption, the spot rate follows a diffusion process without any jump. While a more general model may be desirable, it is beyond the scope of this paper.

C. Effect of Swap Maturity

While we have mentioned the influence of the swap's time to maturity on the spot rate effect, we should now look into the maturity effect by itself. To do so, we shall first describe the stochastic processes for the reference coupon bonds.

Following Cox, Ingersoll, and Ross (1979, 1985), it can be shown that the price, $B(r,\tau)$, of a default-free bond with a coupon stream $C(\tau)$, continuous or discrete, with time to maturity τ, follows the dynamics:

$$dB(r,\tau) = [\alpha(r,\tau)\, B(r,\tau) - C(\tau)]dt + \delta\,(r,\tau)\, B(r,\tau)\, dz \tag{7}$$

where $\alpha(r,\tau)$ is the instantaneous expected rate of return on the bond, $\alpha(r,\tau)B(r,\tau) - C(\tau)$ is the drift or expected instantaneous change in the bond price, $\delta\,(r,\tau)$ is the instantaneous standard deviation of return, and according to Ito's Lemma:

$$\alpha(r,\tau)B(r,\tau) - C(\tau) = 0.5\,\sigma^2\, r\, B_{rr} + \kappa\,(\theta - r)\, B_r - B_\tau \tag{8a}$$

$$\delta\,(r,\tau) = |B_r\, \sigma\sqrt{r}\, /\, B(r,\tau)| \tag{8b}$$

Substituting the general equilibrium expected rate of return on the bond[22], $\alpha(r,\tau) = r + (\lambda r B_r / B(r,\tau))$, in equation (8a) results in the fundamental PDE that the equilibrium bond price must follow:

$$B_\tau = 0.5\,\sigma^2\, r\, B_{rr} + \kappa\,(\theta - r)\, B_r - (rB(r,\tau) + \lambda\, r\, B_r - C(\tau)) \tag{9}$$

The price dynamics and the maturity derivative (negative of time derivative) above for a discount bond can be obtained as a special case where $C(\tau)=0.0$ for all τ. In our one-factor CIR model, the maturity derivative, $P_\tau(j,t)$, is *negative*. That is, with $C(\tau)=0.0$, the right hand side of equation (9) is negative. This is expected since the discount bond price approaches the *par value* from below as the bond gets closer to maturity. With $C(\tau) > 0.0$, the equilibrium expected growth in price is reduced resulting in a less negative maturity derivative. This is also expected as the terminal boundary condition of price being equal to *par value* applies to coupon bond too. The maturity derivative can become positive with a coupon large enough relative to

---

22 Alternatively, one can risk-adjust the drift of r and replace the expected rate of return on the bond by r.

the equilibrium expected return on the bond. Thus, the sign of the maturity derivative depends on **the parameter combination, specially on r, λ, and $C(\tau)$.**

For a given coupon rate, in general we would expect the (signed) maturity derivative of a coupon bond to decrease with the spot rate and to be positive (negative) for small (large) values of the spot rate. Thus, as the swap maturity, *m*, gets longer, we would expect the reference coupon bond values, $B_L(m,t)$ and $B_S(m,t)$, to become larger (smaller) at low (high) levels of the spot rate, *r(t)*[23]. Hence, according to equation (la), we would expect the market value of existing *long* swap position, *LS(m,t),* to be higher (lower) for longer maturity swaps at high (low) levels of the spot rate. The market value of existing *short* swap position, *SS(m,t),* would tend to be lower (higher) for longer maturity swaps at high (low) levels of the spot rate.

The above pattern for *long* swap positions can be observed in Figure 1. When λ=0.0, the benchmark for high or low levels of *r(t)* is roughly the annual coupon rate on the reference **coupon bond, $i_0 - d_b(m,t)$. Thus, roughly at spot rates below (above) 5%, we see a longer** maturity *long* swap position to have a relatively lower (higher) value. This maturity effect can be seen more clearly in Figure 2 where we plot the market values of *long* swap positions against swap maturity (ranging from 6 months to 10 years) for three alternative levels (1%, 4%, 10.5%) of the spot rate.

The above swap maturity effect implies, in *general,* that *in-the-money (out-of-the-money) long (short)* swap positions will tend to *decline (increase)* in market value as they approach maturity. The opposite is true for *out-of-the-money (in-the-money) long (short)* swap positions. This general type of swap maturity effect is similar to that *usually* found for the coupon bonds that sell at discount or premium from their *par value.* An *in-the-money (out-of-the-money)* swap position is like a *premium (discount)* coupon bond, as is evident from equations (la) and (2a). The intuitive reason behind this usual maturity effect is simple, it is namely the gravitational pull toward the terminal boundary condition: bonds or swaps selling below or above *par* before maturity will have to sell at *par* at maturity.

---

23 What is a low or a high level for the spot rate in this context depends on the parameter combination.

In our one-factor CIR model, given the original fixed rate (or coupon for reference coupon bond), the current level of the spot rate determines whether the swap position is in-the-money or out-of-the-money (reference coupon bond being premium or discount). As such the level of the spot rate is the key determinant of the direction of the swap maturity effect. It should be noted that given the general equilibrium pricing parameters, κ, θ, λ, the level of the spot rate determines whether the yield curve is rising, humped or falling. The general equilibrium pricing parameters, κ, θ, λ, determine the slope or steepness of the yield curve.

At a low level of the spot rate, the term structure is most likely rising. If we compare two maturities, *m* and *m* -1, usually the value of the shorter maturity zero coupon bond is greater, that is, $P(m\text{-}1,t)>P(m,t)$. The value of the reference coupon bond's face value hence increases by $P(m\text{-}1,t)\text{-}P(m,t)$ as the swap maturity gets shorter. However, the value of the reference coupon bond is reduced by the value of the coupon lost, $(i_0 - d_b(m,t))P(m,t)/2$ [24]. Unless the term structure is quite steep and/or the original fixed rate is very low, the lost coupon effect will dominate and we shall see a positive (negative) swap maturity (time) effect. In other words, the *longer maturity long* swap positions (reference coupon bonds) will be *deeper-out-of-the-money* (at a greater premium). If the spot rate does not change, this means that the market value of the *long* swap position will monotonically approach the par value from below over time.

At a high level of the spot rate, the term structure is most likely falling. The direction of the two effects will still be the same unless the yield curve is too steep. However, the face value effect will dominate since the change in the value of the face amount (due to maturity change) is substantial at high yields, and specially so for shorter maturity swaps. The net result is that the *longer maturity long* swap positions (reference coupon bonds) will be *deeper-in-the-money* (at a greater discount). If the spot rate does not change, this means that the market value of the *long* swap position will monotonically approach the par value from above over time.

It should be mentioned, however, that while the above pattern of maturity effect is usually the case, it is by no means universal. It is possible to have a non-monotonic relationship between

---

24 We are ignoring here the effect of the bid spread differences for different maturities.

swap maturity and the market value of an existing swap position. See, for example, the 1% spot rate curve in Figure 3 and the 10.5% spot rate curve in Figure 4. The only assumption that is different between these two figures and Figure 2 is that relating to the value of the risk premium parameter, λ. Similar examples can be constructed by varying the other general equilibrium pricing parameters, θ and κ.

D. Effects of Other CIR Model Parameters

Lastly, in this section, we look at the effects of the parameters, λ, κ, and θ, on the current market value of an existing swap position. In the one-factor CIR model, the zero coupon bond price, $P(j,t)$, is an increasing ($P_{\lambda}(j,t)>0$) concave ($P_{\lambda\lambda}(j,t)<0$) function of the preference or risk premium parameter, λ[25]. Consequently, the reference coupon bond prices, $B_{L}(m,t)$ and $B_{S}(m,t)$, are greater when λ is higher. This means that the market value of an existing *long* swap position, $LS(m,t)$, is a decreasing ($LS_{\lambda}(m,t)<0$) convex ($LS_{\lambda\lambda}(m,t)>0$) function of the preference or risk premium parameter, λ. By the same arguments, the market value of an existing *short* swap position, $SS(m,t)$, is an increasing ($SS_{\lambda}(m,t)>0$) concave ($SS_{\lambda\lambda}(m,t)<0$) function of the preference or risk premium parameter, λ.

The effect of the speed of mean reversion in the spot rate depends on whether the current spot rate, $r(t)$, is below or above its long-term mean, θ. If the current spot rate is low, that is below θ, the zero coupon bond price, $P(j,t)$, is a decreasing ($P_{\kappa}(j,t)<0$) convex ($P_{\kappa\kappa}(j,t)>0$) function of κ. Accordingly, when the spot rate is low by historical standards, the market value of an existing *long* swap position, $LS(m,t)$, is an increasing ($LS_{\kappa}(m,t)>0$) concave ($LS_{\kappa\kappa}(m,t)<0$) function of κ. Under the same circumstances, the market value of an existing *short* swap position, $SS(m,t)$, is a decreasing ($SS_{\kappa}(m,t)<0$) convex ($SS_{\kappa\kappa}(m,t)>0$) function of κ. In other

25 With a greater signed covariance of the equilibrium spot rate with the percentage change in optimally invested wealth (the *market portfolio),* it is more likely that the zero coupon bonds will be more valuable (lower yields) when wealth is low and hence, the marginal utility of wealth is high.

words, when the spot rate is relatively low, a slow reversion of the spot rate to its long-term mean is detrimental (beneficial) to the holder of an existing *long (short)* swap position. The opposite is the implication of slowly mean-reverting interest rates for the swap counterparties when the current rates are high by historical standards.

This brings us to the effect of the long-term mean, θ, for the spot rate, that is unequivocal in our one-factor CIR model. The zero coupon bond price, $P(j,t)$, is a decreasing ($P_\theta(j,t)<0$) convex ($P_{\theta\theta}(j,t)>0$) function of θ. Therefore, the market value of an existing *long* swap position, $LS(m,t)$, is an increasing ($LS_\theta(m,t)>0$) concave ($LS_{\theta\theta}(m,t)<0$) function of θ. On the other hand, the market value of an existing *short* swap position, $SS(m,t)$, is a decreasing ($SS_\theta(m,t)<0$) convex ($LS_{\theta\theta}(m,t)>0$) function of θ.

III. Dynamic Interest Rate Risk of Swap Positions

Credit risk and interest rate or market risk are the two major types of risk inherent in an interest rate swap position. In this section, some brief comments on the credit risk are followed by a more detailed examination of the interest rate risk.

Since interest rate swaps are private contractual agreements between two counterparties, they are of course subject to a credit or default risk: the counterpartymight not meet its interest payment obligation. However, it should be pointed out that the credit risks in interest rate swaps are relatively unimportant for two reasons. *First,* because entering into an interest rate swap agreement is a voluntary market transaction performed by two counterparties, a counterparty's credit standing must be acceptable to the other counterparty If one counterparty's credit standing has not reached the par, then a letter of credit from a guarantor is usually required before the signing of the swap contract. *Secondly,* an interest rate swap contract calls for a periodic payment of the *net* amount of the difference between the fixed and the floating interests on the notional principal. Thus, the amount that might be defaulted is relvely small in relation to the notional amount of the interest-rate swap.

As we have noted earlier, an interest rate swap has a zero market value to its counterparties when the swap originates. However, a subsequent change in market interest rates can cause a change in the market value of the swap contract, making the swap position an asset with positive market value to one counterpartyand a liability with negative market value to the other counterparty To the extent market interest rates change in a predictable fashion, it does not pose a risk to the holder of a swap position since the induced changes in the market value of the swap position are foreseen. It is the unanticipated or stochastic variations in the market interest rates leading to variations in the value of a swap position that are at the source of the interest rate or market risk of a swap position.

In the following, we derive a dynamic measure of interest rate risk for existing swap positions using the framework of Cox, Ingersoll, and Ross (1979). The behavior of this risk measure as it relates to the swap-specific factors and the equilibrium valuation parameters of the one-factor CIR model is then examined.

A. Dynamic Measure of Interest Rate Risk

If interest rates are stochastic, as they clearly are, all interest rate derivatives including zero coupon bonds, coupon bonds, and swap positions are exposed to interest rate risk (unanticipated change in their values). The relevant question is, therefore, how to make a meaningful cross-sectional comparison of the interest rate risk of interest-rate derivatives in general and interest rate swaps in particular. A well-known measure of the interest rate risk of a bond, in this regard, is its Macaulay duration. However, the Macaulay duration is a valid measure of interest-rate risk ".. only if the current spot rate and the yield on all bonds of all maturities change by an equal amount, which is possible only with a flat yield curve" (Cox, Ingersoll, and Ross (1979), p.53). As is well-known, Macaulay duration also allows only *one-time* change in the yield curve.

To address the limitations of the original Macaulay duration and subsequent variants closely associated with it, Cox, Ingersoll, and Ross (1979) proposed a dynamic duration measure, called the *stochastic duration,* in the context of continuous time term structure movements represented by a one-factor term structure model. They illustrate the *stochastic duration* measure using the one-factor CIR model we have discussed earlier. Hence, the *stochastic duration* measure has the added benefit of directly linking the stochastic variation of interest-rate contingent contracts to the fundamental uncertainty in an economy. In this sense, the *stochastic duration* measure may also be viewed as the *systematic* or *market risk* of an interest-rate contingent contract.

From equation (8b), we see that the square root of the diffusion coefficient of a bond (with or without coupon) is proportional to the square root of the diffusion coefficient of the spot rate:

$$\delta(r,\tau) = |B_r/B|\,\sigma\sqrt{r} \tag{8b}$$

The proportionality factor, $|B_r/B|$, is called the *relative variation* of the bond. As pointed out by Cox, Ingersoll, and Ross (1979), this is the correct metric of risk in a stochastic interest rate environment. If two bonds with different maturities and coupons have the same *relative variation* measure, their diffusion coefficients will be equal meaning that their values will be equally responsive to the stochastic variation of the spot rate (and thus the term structure in the one-factor model). Since one of these two bonds can be a zero coupon bond, we can always find a zero coupon bond of such a maturity that its *relative variation* is the same as the *relative variation* of a coupon bond. The *stochastic duration* of a coupon bond is defined as the time to maturity of a zero coupon bond that has the same *relative variation* (and thus the same diffusion coefficient).

In the one-factor CIR model, the *relative variation* of a zero coupon bond is given as a function of its time to maturity, *j:*

$$|P_r(j,t)/P(j,t)| = H(j,t) = 2(exp(\gamma j)-1)\ /\ \{(\gamma + \kappa + \lambda)(exp(\gamma j)-1)+2\gamma\} \tag{10}$$

where $\gamma = \sqrt{\{(\kappa + \lambda)^2 + 2\sigma^2\}}$. From equation (1b) or (2b), if we calculate the *relative variation* of a coupon bond to be X, then the *stochastic duration* at time *t*, *SD(t),* of the coupon bond can be found by inverting equation (10):

$$SD(t) = H^{1}(X) = ln\ [1+\{2\gamma X\ /(2-pX)\}]\ /\ \gamma \tag{11}$$

where $p=\lambda+\kappa+\gamma$. In other words, *SD(t)* is that value of *j* for which *H(j,t)* =X. The *stochastic* *duration* of a zero coupon bond is thus its maturity by definition. Since *H(j,t)* is increasing in *j,* if a coupon bond has a higher relative variation, its *SD(t)* will be higher as well meaning that the magnitude of its interest rate risk (measured by its diffusion coefficient) is also higher. Also note that the *stochastic duration* measure of interest rate or market risk is truly dynamic and stochastic. As a coupon bond matures, its value and *relative variation* change because of the non-constant maturity effect and the stochastic evolution of the spot rate (and thus the term structure), and so does its *stochastic duration.*

The reference coupon bonds that we used in valuing the swap positions provide the essential gateway to apply the *stochastic duration* concept in deriving a dynamic and stochastic cross-sectional interest rate or market risk measure of swap positions. For notational convenience, let us normalize the notional principal of a swap position to one dollar. The stochastic differentials of the market values are as follows (dropping the swap maturity and time subscripts):

$$d(LS) = [-\alpha_L(r,\tau)LS + C_L(\tau)]dt + \delta_{LS}(r,\tau)\, LS\, dz \qquad (12)$$

$$d(SS) = [\alpha_S(r,\tau)SS - C_S(\tau)]dt + \delta_{SS}(r,\tau)\, SS\, dz \qquad (13)$$

where $\alpha_L(r,\tau) = r + (\lambda r B_{L,r} / B_{lL}(r,\tau))$, $\alpha_S(r,\tau) = r + (\lambda r B_{S,r} / B_S(r,\tau))$, and the square root of the diffusion coefficient of the swap value is proportional to the square root of the spot rate's diffusion coefficient:

$$\delta_{LS}(r,\tau) = |LS_r / LS(r,\tau)|\sigma\sqrt{r} = |B_{L,r} / (1 - B_L(r,\tau)|\sigma\sqrt{r} \qquad (14)$$

$$\delta_{SS}(r,\tau) = |SS_r / SS(r,\tau)|\sigma\sqrt{r} = |B_{S,r} / (B_S(r,\tau) - 1)|\sigma\sqrt{r} \qquad (15)$$

Equations (14) and (15) are in fact expressions for the instantaneous standard deviations of the relative change or rate of return of swap positions. Since the instantaneous standard deviation, $\sigma\sqrt{r}$, of changes in the spot rate is common to all interest rate contingent claims, the proper metric of relative risk of interest rate contingent claims is the *relative variation.* For existing swap positions, the *relative variations* are:

$$RV_{LS} = |LS_r / LS(r,\tau)| = |B_{L,r} / (1 - B_L(r,\tau)| \qquad (16)$$

$$RV_{SS} = |SS_r / SS(r,\tau)| = |B_{S,r} / (B_S(r,\tau) - 1)| \qquad (17)$$

The stochastic duration of the swap positions at time *t* can be defined as:

$$SD_{LS}(t) = H^{-1}(RV_{LS}) = \ln\, [1 + \{2\gamma RV_{LS} / (2 - p\, RV_{LS})\}] / \gamma \qquad (18)$$

$$SD_{SS}(t) = H^{-1}(RV_{SS}) = \ln\, [1 + \{2\gamma RV_{SS} / (2 - p\, RV_{SS})\}] / \gamma \qquad (19)$$

To calculate the *stochastic duration* of a *long* swap position at time *t*, one will calculate the *relative variation* from equation (16) and then simply use that *relative variation* value in equation (18). For a *short* swap position, equations (17) and (19) will be used instead.

Several points should be noted about the *relative variation* and the *stochastic duration* measures of interest rate risk for swap positions. *First,* the *relative variation* measures are not defined if the swap positions are exactly *at-the-money* (reference coupon bonds are at par). This, however, does not mean that the risk of a swap position that is close to being *at-the-money* is negligible. In fact, to the contrary, swap positions that are close to being *at-the-money* are the riskiest in proportional terms. Intuitively, this situation is like the behavior of the discount on a zero coupon bond as it approaches maturity. The discount will be fairly small relative to the price of the zero coupon bond which will be close to its face value. For a change in the interest rate, the discount and the zero coupon bond price will always move by the same dollar magnitude albeit in opposite directions. However, this same dollar variation will loom extremely large as a proportion of the discount. Equations (la) and (2a) show that the market value of an existing swap position is in fact like discount or premium over face value of the corresponding reference coupon bond. Thus the size of the market value of an existing swap position is small relative to the market value of the corresponding reference coupon bond, specially when the latter is close to par, that is the swap position is close to being *at-the-money.*

In reality, existing (previously established) swap positions will rarely be exactly *at-the-money.* So the aforementioned problem of dynamic risk measurement may not arise at all. In case it does, one way to handle this measurement problem will be to assign a small nonzero value for the swap position in calculating the *relative variation* measure. Our simulations show that the *relative variation* of swaps that are close to being *at-the-money* are distinctively and substantially large. Thus in a cross-sectional comparison and for hedging or other risk management purposes, the exactly *at-the-money* swap positions can be classified in the riskiest category.

***Second*, for a given combination of λ, κ, θ, and σ, there is a maximum value, $X_{max}(\lambda,\kappa,\theta,\sigma)$** of *relative variation* for which the *stochastic duration* measure is meaningful[26]. There is no guarantee that the *relative variation* of swap positions will be bounded by this ceiling. There are two ways this problem can be handled. One way is to calculate the *stochastic duration* measures using the following slightly modified versions of equations (18) and (19):

$$SD_{LS}(t) = H^{-1}(rv_{LS}) = A \ ln\ [1+\{2\gamma\ rv_{LS}(t)\ /(2-\ p\ rv_{LS})\}]\ /\ \gamma \tag{20}$$

$$SD_{SS}(t) = H^{-1}(rv_{SS}) = A \ ln\ [1+\{2\gamma\ rv_{SS}(t)\ /(2-\ p\ rv_{SS})\}]\ /\ \gamma \tag{21}$$

where $rv_{LS} = X_{max}(\lambda,\kappa,\theta,\sigma)\ (RV_{LS}/U)$, $rv_{SS} = X_{max}(\lambda,\kappa,\theta,\sigma)\ (RV_{SS}/U)$, U is an arbitrary large number, and A is a scaling factor for suitable presentation of the *stochastic duration* measures[27].

The use of equations (20) and (21) leaves the cross-sectional ranking of the dynamic risk of swap positions intact. Hence equations (20) and (21) can be used without any qualification for comparisons dealing with swaps alone. The *stochastic duration* measures from equations (20) and (21) cannot, however, be used meaningfully for a comparison of swap positions to other interest rate contingent claims including the zero coupon bonds. This is because the *stochastic duration* values from equations (20) and (21) do not any more mean that the swap positions have the same degree of interest rate risk as the zero coupon bonds of maturity equal to the calculated stochastic duration values.

In a situation where the interest rate risk of swap positions are to be compared against other interest rate contingent claims, an alternative will be to simply compare the unadjusted *relative variation* measures from equations (16) and (17) for swaps and the *relative variation* measures of other claims. After all, *relative variation* is the proper metric of relative risk, its monotonic transformation, the *stochastic duration* measure, is merely intended to represent the riskiness in time units.

---

26 This point was previously noted by Chen, Park, and Wei (1986).

27 In simulations over many parameter combinations, we find reasonable values for *U* and *A* to be 100,000 and 1,000 respectively.

Third, since the *stochastic duration* measure is a positive monotonic transformation of the *relative variation* measure, the direction of effect of the various parameters or variables of interest on the *stochastic duration* or interest rate risk can be observed from the *relative variation* measure.

B. The Behavior of Relative Variation and Stochastic Duration

In this subsection, we first compare the dynamic interest rate risk of an existing swap position to that of the reference coupon bond. The effects of the spot rate and the swap maturity on the dynamic interest rate risk of an existing swap position are then discussed.For the sake of brevity, we limit our discussion to the case of an existing *long* swap position.

B.1. Long swap position vs. reference coupon bond

Since plain vanilla swaps are commonly associated with same maturity coupon bonds and since dealers quote swap prices as spreads over sam maturity on-the-run Treasury bonds, it is of both theoretical and practical interest to compare the *relative variation* of an existing *long* swap position and the corresponding reference coupon bond. The reference coupon bond's maturity is the same as the swap's remaining time to maturity and its coupon rate (paid semiannually) is equal to the swap's original fixed rate less the swap bid spread.

For a given change in the spot rate, the percentage change in the *long* swap's value is equal to its *relative variation* times the change in the spot rate multiplied by 100. Table 1 presents the percentage changes in the market values of the *long* swap and the corresponding reference coupon bond for a change of 10 basis points in the spot rate for swap maturities ranging from 6 months to 10 years and for initial spot rate varying from 1% to 10%. To highlight the effect of interest rate uncertainty, $\lambda = 0.0$ is assumed in Table 1.

There are two noticeable features of Table 1. *First,* the *long* swap position's variation induced by an unanticipated change in the spot rate is at least a few times greater than the reference coupon bond's variation across all initial spot rate situations and all maturities. The reference coupon bond's value changes by less than 1% for a change of 10 basis points in the spot rate in all the scenarios in Table 1. In comparison, the *long* swap's value changes by more than l% in all cases and by 9% or more for spot rates between 4% to 6%. The latter group represents scenarios where the long swap is close to being at-the-money. While we do not report the *short* swap results here, the percentage changes in a *short* swap's value are marginally greater than the *long* swap figures in Table 1. These results clearly indicate how volatile the plain vanilla swap positions are compared to the coupon bonds. Even a very short maturity, say 1 year, swap position is much more volatile than a 10 year coupon bond.

Second, while the *relative variation* of the reference coupon bond varies marginally with the initial spot rate and a bit more so with maturity, the *relative variation* of a swap position can be significantly different depending on both the spot rate and the swap maturity. The reason behind the lack of sensitivity of the reference coupon bond's *relative variation* to the initial spot rate is that *H(j,t)'s,* the *relative variations* of zero coupon bonds, are all independent of the spot rate, *r(t),* in the one-factor CIR model. When the spot rate changes, the relative weights of the cashflows of various maturities change somewhat and this produces only a minor impact on the *relative variation* of the coupon bond. For the swap position, an additional effect of the spot rate change is that the constant size of the incremental cashflow changes as the par value (not the hypothetical) coupon bond's bond-equivalent yield to maturity changes. As we noted earlier in this paper, this cashflow effect is quite dominant for swap positions. This is a distinct risk characteristic of swap positions compared to their coupon bond counterparts.

B.2. Spot rate and long swap's relative variation

Let us now examine the nature of relationship between the spot rate, *r,* and the *relative variation,* RV_{LS}, of an existing *long* swap position. Differentiating RV_{LS} with respect to *r* and rearranging, we find that the direction of the relationship depends on:

$$DIFF_r = (|LS_r|_r / |LS_r|) - (|LS|_r / |LS|)$$

This is the difference between the proportionate or relative change in the *absolute* first partial of the *long* swap value and the relative change in the *absolute* value of the *long* swap. If the relative change in the *absolute* first partial of the *long* swap value exceeds (falls short of) the relative change in the *absolute* value of the *long* swap, an increase in the spot rate will enhance (reduce) the *relative variation* and *stochastic duration* of a long swap position of a given maturity. The relative changes in question depend on the level of the spot rate and the swap maturity in a complicated fashion.

To illustrate the relationship between the spot rate and the *relative variation* and *stochastic duration,* we present in Figure 5 the $DIFF_r$ for a 50 basis point change in the spot rate and the **adjusted *stochastic duration*, $H^1(rv_{LS})$ [28], of a *long* swap position as a function of the spot rate** for three swap maturities (1 year, 5 year, and 10 year). While the magnitudes vary, the pattern of the relationship between the spot rate and $DIFF_r$ and *stochastic duration* is the same across all three maturities. The *stochastic duration* increases (declines) as the spot rate goes up when $DIFF_r$ is positive (negative), that is, when relative change in the *absolute* first partial of the *long* swap value exceeds (falls short of) relative change in the *absolute* value of the *long* swap. Roughly speaking, $DIFF_r$ *is* positive (negative) when the *long* swap is *out-of-the-money (in-the-money).* In other words, the relative interest rate risk of an existing *long* swap position is an increasing function of the spot rate for *out-of-the-money* swaps and is a decreasing function of the spot rate for *in-the-money* swaps. Swaps that are close to being *at-the-money* carry the greatest interest rate risk.

As we saw in Figure 1, there is no pronounced curvature in the relationship between the spot rate and the *long* swap value. Since the long swap value is a monotonic positive function of the spot rate, the absolute value and the signed value of the first partial with respect to the spot rate are the same. For a given maturity, the first partial does not vary a lot as the spot rate changes. The change in the first partial as a proportion of itself is small and varies only marginally as the spot rate changes.

The nonlinear relationships that we see in Figure 5 are primarily driven by the relative change in the *absolute* value of the *long* swap. While the value of the long swap is monotonic increasing in the spot rate, its *absolute* value is not. When the swap is *out-of-the-money,* its absolute value

---

28 *U* and *A* were set to 100,000 and 1,000 respectively.

in fact decreases with an increase in the spot rate. In absolute terms, the first partial of the *absolute* value of the *long* swap is the same as the first partial of the *long* swap value, and accordingly shows minor variations only. However, the *absolute* value of the swap is quite high when it is deep *out-of-the-money* and continues to fall as the swap approaches the *at-the-money* point. Accordingly, the relatively constant first partial of the *absolute* value of the long swap steadily increases as a proportion of the *absolute* value of the long swap. Once the swap becomes *in-the-money,* the absolute value of the swap starts increasing and this leads to a falling first partial as a proportion of the *absolute* value of the swap. The transition between the rising and falling *stochastic duration* regions is not smooth due to the fact the *relative variation* is not defined for an exactly *at-the-money* swap. Also, the absolute value of the swap has a kink at the exactly *at-the-money* point.

One implication of the pattern in Figure 5 is that swaps that are *out-of-the-money* or *in-the-money* by the same amount have similar interest rate or market risks. This contrasts with the commonly held view among regulators and practitioners that the holder of an *in-the-money* swap has more to lose. This view is usually based on the notion that the interest rate movements that have proved to be ble to one side of the swap mean potential distress for the other side of the swap and hence an increased possibility of default by the latter party. Our analysis shows that too much emphasis on the default risk of existing swaps may seriously distort the relative market or interest rate risk of swaps. The holder of an existing *out-of-the-money* swap position has as much to lose from adverse interest rate movements as the holder of a swap position that is *in-the-money* by the same amount. Also, close to *at-the-money* positions which may appear neutral at first glance are the most susceptible to unanticipated variations in the market interest rate situation or the term structure of interest rates.

B.3. Swap maturity and long swap's relative variation

We shall now examine the nature of relationship between the swap maturity, *m*, and the *relative variation*, RV_{LS}, of an existing *long* swap position. Differentiating RV_{LS} with respect to *m* (or τ) and rearranging, we find that the direction of the relationship depends on:

$$DIFF_m = (|LS_r|_m / |LS_r|) - (|LS|_m / |LS|)$$

This is the difference between the relative change in the *absolute* first partial (with respect to the spot rate) of the *long* swap value and the relative change in the *absolute* value of the *long* swap, both relative changes occurring as the swap maturity increases. If the relative change in the *absolute* first partial of the *long* swap value exceeds (falls short of) the relative change in the *absolute* value of the *long* swap, a longer maturity swap position will have a greater (smaller) *relative variation* and *stochastic duration* at a given spot rate level.

To illustrate the relationship between the swap maturity and the *relative variation* and *stochastic duration,* we present in Figure 6 the $DIFF_m$ for a 0.5 year or 6 month change in the swap maturity and the adjusted *stochastic duration,* $H^1(rv_{LS})$[29], of a *long* swap position as a function of the swap maturity for three spot rate levels (1%, 4%, and 10.5%). The relationship is markedly different across the three spot rate levels.

At the 1% level of the spot rate, $DIFF_m$ *is* positive for swaps up to 3.5 year maturity and the interest rate risk increases with maturity over this range. Four year and longer maturity swaps have a negative $DIFF_m$ and the interest rate risk declines with maturity over this range. The minimum risk swap's maturity is somewhere between 3.5 and 4 years[30]. Both the first partial with respect to the spot rate and the absolute value of the swap increases with swap maturity, as can be seen in Figures l and 2. Both relative changes, on the other hand, are high at short maturity and decline as the maturity gets longer. However, the relative change in the first partial with respect to the spot rate outpaces (falls shy of) that in the absolute value of the swap when the swap maturity is short (long).

---

29 *U* and *A* were set to 100,000 and 1,000 respectively.

30 By setting $DIFF_m = 0.0$, the minimum risk swap maturity can be found numerically.

At 4% level of the spot rate, the relative change in the first partial with respect to the spot rate consistently falls shy of that in the absolute value of the swap over the entire maturity spectrum. Accordingly, the interest rate risk is monotonic decreasing in swap maturity when the spot rate is around 4%.

When the spot rate is quite high at 10.5%, a situation opposite to that of the low spot rate obtains. Now, $DIFF_m$ is negative for swaps up to 6.0 year maturity and the interest rate risk decreases with maturity over this range. Six and a half year and longer maturity swaps have a positive $DIFF_m$ and the interest rate risk increases with maturity over this range. The minimum risk swap's maturity is somewhere between 6.0 and 6.5 years. This pattern is due to the fact that the relative change in the first partial with respect to the spot rate falls shy of (outpaces) that in the absolute value of the swap when the swap maturity is short (long).

One difference between the low and the high spot rate situations is that the first partial with respect to the spot rate is relatively high when the spot rate is low. This helps the high relative change in the first partial with respect to the spot rate over short maturity range to outweigh the relative change in the absolute value of the swap in a low spot rate situation. As the first partial with respect to the spot rate gets smaller with higher spot rates, the high relative change in the absolute value of the swap over short maturity range takes over in higher spot rate environments. This contributes to the reciprocal interest rate risk patterns in low and high interest rate environments.

One key insight from our analysis is that longer maturity swaps are not necessarily riskier than shorter maturity swaps with identical fixed rates. The interest rate risk vs. maturity profile of existing swap positions depends critically on the current level of the spot rate. As the spot rate (and hence the term structure) evolves stochastically, the interest rate or market risk structure of different maturity swaps may completely reverse itself. This underscores the distinction between swaps and bonds as well as the need for dynamic measurement of the market risk of swaps.

IV. Secondary Markets and Swap Management[31]

The secondary swap markets are now very active, and they provide a great deal of liquidity to swap participants. This was due to the formation of the International Swap Dealers Association (ISDA) and the publication of the ISDA Code of Swaps which standardized some technical aspects of swap transactions in 1985. The round lot transaction for interest rate swaps has now decreased to as little as $5 million in notional amount. Now the major swap dealers themselves trade with the interested parties and warehouse a large numbers of interest rate swap contracts in order to avoid having to search for matching counterparties at any point in time.

Entering into a swap agreement in the *primary* swap markets is similar to a portfolio selection decision, while making a swap position adjustment in the *secondary* swap markets is similar to a portfolio revision decision. As the circumstances which originally give rise to an interest rate swap change, the counterparties of the swap will find it beneficial or even necessary to unwind that swap. For example, a counterparty may desire to unwind a swap because of (1) changes in its balance sheet that alter its needs to hedge the asset/liability mismatch; (2) changes in the future interest rate expectation that lead to remove the interest rate protection with a swap position, and (3) a desire to recognize profit or loss from the swap position and to reflect that profit or loss in the current period. A counterparty can unwind a swap position with one of the three major types of interest rate swap transactions in the secondary markets: (1) the swap reversal; (2) the swap termination; and (3) the swap assignment.

(1) Swap Reversal

In an interest rate swap reversal, the holder *(long* or *short* position) of the original swap simply executes a new interest rate swap that is opposite to the original one. For example, a bank with a *long* swap position can enter into a new interest rate swap agreement with a third

---

31 Various methods for swap management are discussed and illustrated in details in Chen and Millon (1989).

party under which it will pay the floating-rate of the same index and receive the fixed-rate of interest payments determined by the current swap market prices. Therefore, the net result of a swap reversal for the counterparty with a *long* swap position will incur a net cash inflow of $(W/2)[i_b(m,t) - i_0]$ per semiannual period for the next m semiannual periods. The disadvantages of the swap reversal are: (i) it does not generally involve an immediate lump sum payment representing profit or loss from the swap transaction, and (ii) it does not eliminate credit exposure to a given counterparty. If the swap reversal is completed with an new counterparty, the credit exposure has increased.

(2) Swap Termination Or Closeout

A swap termination or closeout is different from a swap reversal in that all the obligations under the existing swap are extinguished upon the swap termination or closeout. A swap termination is completed upon a cash settlement between the two counterparties of the swap equal to an amount at which the counterparties of the swap are indifferent to staying in the existing swap or entering into a new *par value* swap. As we have seen earlier, the market value of a swap to its buyer is $LS(m,t)$ and the market value of the swap to its seller is $SS(m,t)$, at time t. Thus, the acceptable amount of cash settlement for a swap termination should fall between the absolute values of the above two market values of the swap position to its buyer and its seller.

(3) Swap Assignment

In a swap termination, a counterparty of the swap obtains a termination by paying or receiving the swap buyout price from the other counterparty of the original swap contract. As a result of swap termination, neither party has any further obligations in the swap after the cash settlement is made and the termination is completed. However, in a swap assignment, a cash payment is made to a third party and the original swap agreement remains intact for one counterparty with a new counterparty stepping into the assignor’s position. It should be noted that virtually all swap contracts require a consent of the counterparty on the assignment and the acceptance of the credit of the assignee.

V. Conclusions

In this paper we have developed models for determining the market values of a long swap position as well as a short swap position to its counterparties. The market values of existing (previously established) swap positions are shown to be functions of swap-specific factors that include the relative sizes and the different remaining maturities of the swap contracts, the original fixed interest rates, and the current market prices for the *par value* swaps with the same maturities. Using the general equilibrium term structure model of Cox, Ingersoll, and Ross (1985), we have explored how the values of the swap positions are related to the market factors or the parameters of the equilibrium interest rate process. Following Cox, Ingersoll, and Ross (1979), we have also shown how to measure the interest rate or market risk of an existing swap position. This risk measure is dynamic in the sense that it changes over time as a function of the stochastic evolution of the spot rate and the term structure of interest rates. Additionally, we have discussed some methods for swap management.

Our models of market value (marked to market value) and market (interest rate) risk are mutually consistent and thus offer a unified framework for a *value cum risk accounting method* for swaps. This framework is also general enough to be applicable to other functionally equivalent transactions of a firm. The valuation and risk measurement framework of this paper should thus help the development of a more effective reporting and regulatory system for derivatives transactions as suggested by Merton and Bodie (1995).

The market value and risk of interest rate swaps are intimately linked to those magnitudes for Treasury securities as indicated by the market convention of quoting the swap prices as spreads over the corresponding maturity Treasury yields. However, our analysis indicates important differences between the values and the interest rate risks of bonds and existing swap positions. The value of a swap position is shown to be the discount or premium, as the case may be, from the face value (equal to the notional principal of swap) of a reference coupon bond. Thus the swap positions behave like the discount or premium rather than the price itself of a coupon bond. The importance of this difference is clearly visible in the behavior of the interest rate risk. The simulation results of this paper show that the interest rate risk of a swap position is substantially greater than that of the same maturity coupon bond. Swap positions that are closer to being at-the-money seem to carry the greatest interest rate risk. The interest rate risk diminishes as the swap position becomes either deeper out-of-the-money or deeper in-the-money. Unlike coupon bonds and contrary to our initial perceptions, we find that the shorter maturity swaps can often exhibit greater volatility to unanticipated interest rate variations than the longer maturity ones. Thus, as noted by Litzenberger (1992, p.831), *indeed* ".. there is more to these plain vanilla swaps than first meets the eye."

Much work remains to be done in this area. In a separate paper, we are pursuing the disclosure and regulatory implications of the results in this paper. The sensitivity of the results that rely upon the one-factor CIR model need to be looked into under alternative term structure assumptions. Analyses of more complex swaps, e.g., differential swaps, amortizing swaps, etc., and swap derivatives are expected to reveal more intricacies and perhaps surprises in terms of their relationship to bonds. In this paper, we have ignored the effect of time varying credit risk on the stochastic variations of the swap positions by treating the current term structure of swap spreads as given and constant. The importance of this assumption can be investigated. Given data availability, this paper's predictions regarding the market value and risk of existing swap positions may also be empirically tested.

REFERENCES

Arak, Marcelle, Artro Estrella, Laurie Goodman, and Andrew Silver (1988), Interest rate swaps: An alternative explanation, *Financial Management* 17, 12-18.

Bautista, Carl and Kris Mahabir (1994), Managing risk and capturing opportunities with derivative securities, in Dwight Churchill ed.: *Innovations in Fixed Income* (Association for Investment Management and Research, Charlottesville, VA), 76-82.

Bicksler, James and Andrew H. Chen (1986), An economic analysis of interest rate swaps, *Journal of Finance* 41, 645-655.

Brooks, Robert and D.K Malhotra (1994), Components of the bid-asked spread of default-risky interest rate swaps, *Advances in Futures and Options Research* 7, 237-249.

Chen, Andrew H., Hun Y. Park, and K John Wei (1986), Stochastic duration and dynamic measure of risk in financial futures, *Advances in Futures and Options Research* lB, 93-111.

Chen, Andrew H. and Marcia Millon (1989), The secondary market and dynamic swap management, *Recent Development of International Banking and Finance* 3, 131-148.

Chen, Andrew H. and Arthur K. Selender (1995), Determination of swap spreads: An empirical analysis, Unpublished manuscript, Cox School of Business, Southern Methodist University, February.

Chen, Ren-Raw and Louis Scott (1995), Interest rate options in multi-factor Cox-Ingersoll-Ross models of the term structure, forthcoming in the *Journal of Derivatives.*

Cooper, Ian and Antonio Mello (1991), The default risk of swaps, *Journal of Finance* 46, 597-620.

Cox, John, Jonathan Ingersoll, Jr., and Stephen Ross (1979), Duration and the measurement of basis risk, *Journal of Business* 52, 51-61.

Cox, John, Jonathan Ingersoll, Jr., and Stephen Ross (1985), A theory of the term structure of interest rates, *Econometrica* 53, 385-408.

Dothan, Uri (1978), On the term structure of interest rates, *Journal of Financial Economics* 6, 59-69.

Duffie, Darell and Ming Huang (1996), Swap rates and credit quality, Paper presented to the 1996 American Finance Association Meetings, San Francisco.

Gorton, Gary and Richard Rosen (1995), Banks and derivatives, forthcoming, *1995 NBER Macroeconomics Annual*, March.

Hull, John (1989), Assessing credit risk in a financial institution's off-balance commitments, *Journal of Financial and Quantitative Analysis* 24, 489-501.

Hull, John and Alan White (1990), Pricing interest-rate derivative securities, *Review of Financial Studies* 3, 573-592.

Litzenberger, Robert (1992), Swaps: plain and fanciful, *Journal of Finance* 47, 831-850.

Loeys, Jan (1985), Interest rate swaps: A new tool for managing risk, *Federal Reserve Bank of Philadelphia Business Review*, May/June.

Longstaff, Francis and Eduardo Schwartz (1992), Interest rate volatility and the term structure: A two-factor general equilibrium model, *Journal of Finance* 47, 1259-1282.

Longstaff, Francis (1993), The valuation of options on coupon bonds, *Journal of Banking and Finance* 17, 27-42.

Marsall, John and Kenneth Kapner (1993), *Understanding Swaps* (John Wiley, New York).

Marshall, John and Vipul Bansal (1992), *Financial Engineering: A Complete Guide to Financial Innovation* (New York Institute of Finance, New York).

Merton, Robert and Zvi Bodie (1995), Financial infrastructure and public policy: a functional perspective, Working Paper #95-064, Division of Research, Harvard Business School.

Reed, Nick (1995), Disclosure deficits, *Risk* 8 (10), October, 25-32.

Rogers, L.C.G. (1995), Which model for term structure of interest rates should one use?, in Mark Davis, Darrell Duffie, Wendell Fleming, and Steven Shreve, eds.: *Mathematical Finance* (Springer-Verlag: New York), 93-115.

Smith, Clifford, Charles Smithson, and Lee Wakeman (1986), The evolving market for swaps, *Midland Corporate Finance Journal,* Winter.

Sun, Tong-sheng and Ching Wang (1996), A model for corporate bonds and the pricing of interest rate swaps with credit risk, *International Journal of Business* l(l), 1-25.

Titman, Sheridan (1992), Interest rate swaps and corporate financing choice, *Journal of Finance* 47, 1503-1516.

Turnbull, Stuart (1987), Swaps: A zero sum game?, *Financial Management* 16, 15-21.

Vasicek, Oldrich (1977), An equilibrium characterization of the term structure, *Journal of Financial Economics* 5, 177-188.

Wall, Larry (1989), Interest rate swaps in an agency theoretic model with uncertain interest rates, *Journal of Banking and Finance* 13, 261-270.

Appendix A: Determination of Dealer's Quotes on Swap Prices

By market convention, dealer's bid and ask prices are quoted as spreads over the bond equivalent asked yield to maturity, $r_{on}(m,t)$, of the corresponding maturity *on-the-run* (most recently issued) Treasury security

$$i_b(m,t) = r_{on}(m,t) + d_{b,on}(m,t), \text{ and}$$

$$i_a(m,t) = r_{on}(m,t) + d_{a,on}(m,t),$$

where $d_{a,on}(m,t) - d_{b,on}(m,t)$ is the spread dealer hopes to make as a market maker[32]. For example, a swap dealer's quote for 5-year (m = 10) fixed-for-floating swap may be like T+ 45 - T+41, which means the swap dealer is willing to receive (pay) a fixed rate for 5 years at the current BEY on the 5-year *on-the-run* Treasury note plus 45 (41) basis points against paying (receiving) the floating 6-month LIBOR flat. If the current BEY on 5-year *on-the-run* Treasury note, $r_{on}(10,0)$, is 7%, in our notation, $d_{b,on}(10,0)=0.0041$, $d_{a,on}(10,0)=0.0045$, $i_b\,(10,0) = 0.0741$, and $i_a(10,0) = 0.0745$.

While the dealers quote their bid and ask prices as spreads over the BEY of *on-the-run* Treasury security, these prices are first arrived at by subtracting and adding spreads over what is called the swap *midrate*, $r_{mid}(m,t)$:

$$i_b(m,t) = r_{mid}(m,t) - d_{b,mid}(m,t), \text{ and}$$

$$i_a(m,t) = r_{mid}(m,t) + d_{a,mid}(m,t),$$

where usually $d_{a,mid}(m,t) = d_{b,mid}(m,t)$[33]. To continue our example, the swap midrate, $r_{mid}(m,t)$, was first calculated at 7.43%, and then 2 basis points ($=d_{a,mid}(m,t)=d_{b,mid}(m,t)$) were subtracted and added to arrive at the quoted bid and ask prices of $i_b = 0.0741$ *and* $i_a = 0.0745$ respectively.

---

32 With the massive growth in the swap market and increased competition among dealers, the bid spread and the ask spread over Treasury have declined overtime. The market-making spread (ask-bid) of the dealer has also narrowed over the years and is now typically less than 10 basis points. See the recent study of Brooks and Malhotra (1994).

33 See Marshall and Kapner (1993) for industry practices in the swap market.

To calculate the swap *midrate*, $r_{mid}(m,t)$, dealers typically employ arbitrage-free valuation approach. For short-dated (maturity less than 2 years) fixed-for-floating swaps, dealers generally use the forward LIBOR rates implied by the Eurodollar strip (strip of Eurodollar futures contracts), to calculate the no-arbitrage fixed rate, i.e., the swap *midrate* [34]. The rationale is that the dealer can hedge the floating LIBOR exposure (pay or receive) by taking appropriate position in the Eurodollar strip.

Whether short-dated or long-dated dealers can also hedge their unmatched swap positions by taking appropriate positions in the Treasury securities, cash and/or futures. For example, if the dealer is paying fixed rate (5-year Treasury+ 41 bp) on $25 million in exchange for 6-month LIBOR flat, the dealer can hedge by short selling 6-month T. Bills of $25 million face value and using the proceeds to buy 5-year Treasury. Any basis risk between T.Bill and Eurodollar exposures can be hedged by taking a position in TED (T. Bill over Eurodollar) futures. Once the match is found, the dealer can lift the hedge.

It should also be mentioned that while LIBOR is the most popular floating rate index, other interest rates (e.g., T.Bill yield) are also used as the floating rate index in fixed-for-floating swaps. In any case, the yields, explicit or implied, of the hedging vehicles (cash and/or futures) relevant to dollar-denominated interest rate swaps are intimately related to the basic US interest rates, namely the Treasury zero coupon yields or term structure of interest rates. As the term structure changes, dealer's bid and ask prices will change irrespective of which specific variant of arbitrage valuation is used to set the swap *midrate.* Since we intend to analyze how market conditions affect the value and risk of existing swap positions, it seems reasonable to directly (rather than indirectly) link the bid and ask prices to the term structure of interest rates.

---

34 For some examples, see Bautista and Mahabir (1994) and Marshall and Kapner (1993, pp. 147-154).

Appendix B: Industry Practice of Marking to Market Swap Positions

The industry practice of marking to market existing swap positions is based upon a derived yield curve for zero coupon swaps. A *long* position in a zero coupon swap involves a single fixed rate payment at swap maturity against periodic floating rate receipts. As illustrated by Marshall and Kappner (1993, pp. 147-154), dealers first calculate the future value of a dollar for various maturities assuming repeated reinvestment at the implied Eurostrip rates and then use these future values to calculate the implied zero coupon swap rates (fixed rates of *par value* or at-market zero coupon swaps) for various maturities. The swap midrate for a given maturity is then derived as the coupon rate that equates the notional principal to the value of the coupons and the notional principal discounted at the implied zero coupon swap rates. Thus the so-called zero coupon swaps curve implied by the par value swaps curve takes us back to the zero coupon swap rates which are used to calculate the swap midrates.

First, if the zero coupon swap rates are applied to discount the incremental cashflow stream to the dealer from taking over an existing swap position, as is the case with the industry practice of marking to market, it is being assumed that the comparable opportunity for the dealer is to invest the lumpsum (the value of the existing swap position) on a rollover basis at the Eurostrip rates. Barring credit risk, the incremental cashflows are certain and there is no apparent reason why the dealer and the swap participants should prefer Eurodollar deposits (and the accompanying Eurodollar futures positions to lock in future rates) over the Treasury zeros.

Second, as of now, Eurodollar futures contracts are available up to 5 years maturity although the liquidity for longer maturity contracts is not as high as for the shorter maturity contracts. Thus longer maturity swaps cannot be priced using Eurostrip rates. One also has to take into account the marking to market implications. Further, Eurostrip rates may not be quite appropriate for swap contracts with floating rate index tied to T. Bill yield or some other U.S. interest rate.

Third, the implied zero coupon swap rates are not directly investible rates. If dealers do not use the Euro-strip-based zero coupon swap rates in calculating the swap midrates, it is not clear what type of comparable investment opportunity they mean for the dealer or the swap counterparties.

Fourth, if there is variation in how the dealers arrive at the swap midrates, their implied zero coupon swap rates may vary even if their midrates are the same. Thus it is possible that a swap counterparty may get different estimates of the market value of her position from different dealers. This confusion can be avoided if the observable yields on Treasury zeros are used to mark to market the swap positions.

Lastly, when it comes to the evaluation of the market risk of swap positions, it is rather convenient to use the zero coupon bonds than some implied zero coupon swap rates, as we shall see later in this paper. The zero coupon bond approach thus provides a relatively direct, coherent, and easy-to-implement unified framework for the evaluation of the market value and risk of existing swap positions.

Figure 1

The market value, *LS(m,t),* of an existing *long* swap position with a notional principal of \$100 and maturity *m* at time *t,* as a function of the spot rate, *r(t).* The parameter values assumed are: $\kappa=0.10$, $\theta=0.04$, $\lambda=0.0$, $\sigma=0.05$, $i_b=0.05$, and $d_b(m,t)=d_b=0.0012$.

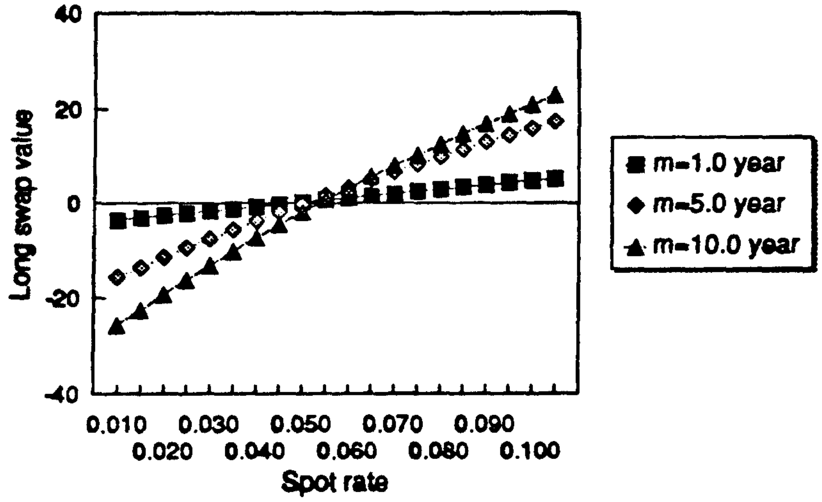

Figure 2

The market value, *LS(m,t),* of an existing *long* swap position with a notional principal of \$100, at time *t,* as a function of the swap maturity, *m,* for alternative levels of the spot rate, *r(t)*. The paramater values assumed are: $\kappa=0.10$, $\theta=0.04$, $\lambda=0.0$, $\sigma=0.05$, $i_b=0.05$, and $d_b(m,t)=d_b=0.0012$.

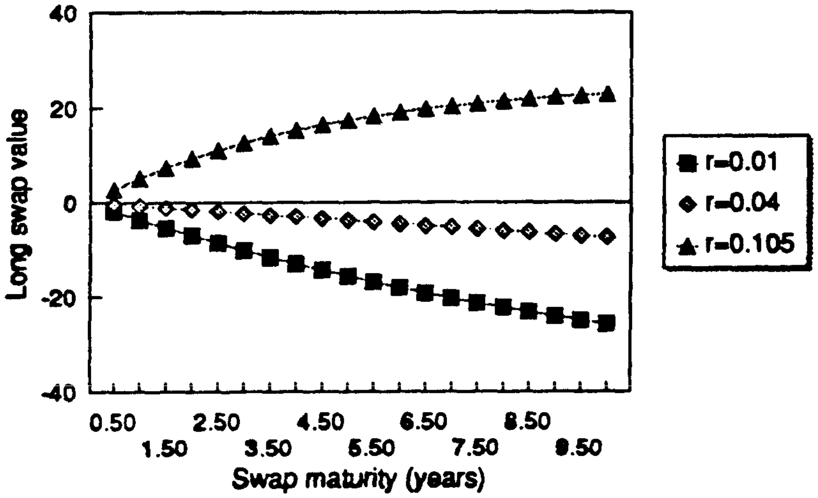

Figure 3

The market value, *LS(m,t),* of an existing *long* swap position with a notional principal of \$100, at time *t*, as a function of the swap maturity, *m*, for alternative levels of the spot rate, *r(t)*. The paramater values assumed are: κ=*0.10*, θ =*0.04*, λ= *-0.3*, σ =*0.05*, i_o =*0.05*, and $d_b(m,t)=d_b-0.0012$.

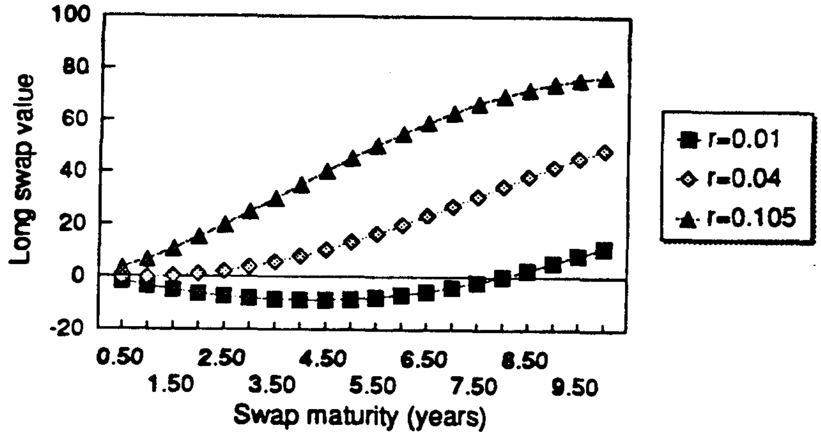

Figure 4

The market value, *LS(m,t),* of an existing *long* swap position with a notional principal of \$100, at time *t*, as a function of the swap maturity, *m*, for alternative levels of the spot rate, *r(t)*. The paramater values assumed are: κ=*0.10*, θ =*0.04*, λ=*0.2*, σ =*0.05*, i_o=*0.05*, and $d_b(m,t)=d_b=0.0012$.

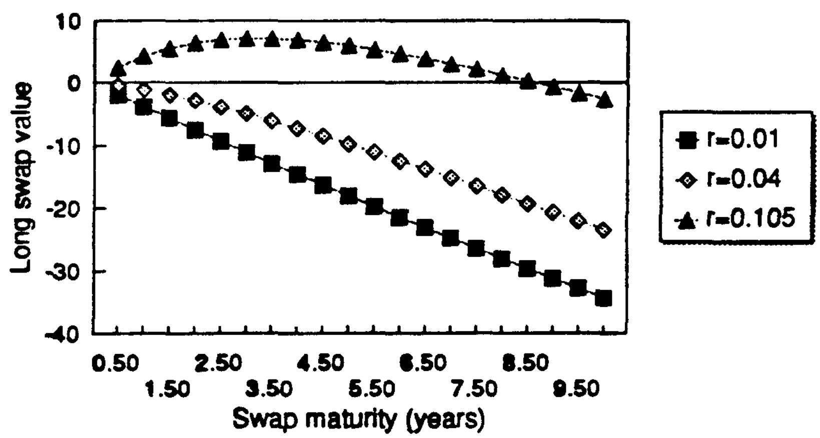

Figure 5

The adjusted stochastic duration of an existing *long* swap position with a notional principal of $100, at time *t*, and the relative change in the absolute value of the first partial (with respect to the spot rate) of long swap value minus the relative change in the absolute value of long swap, both as a function of the spot rate, *r(t)*, for alternative levels of the swap maturity, *m*. The relative changes are for a change of 0.005 in the spot rate, *r(t)*. The **paramater values assumed are:** $\kappa=0.10$, $\theta=0.04$, $\lambda=0.0$, $\sigma=0.05$, $i_o=0.05$, **and** $d_s(m,t)=d_s=0.0012$.

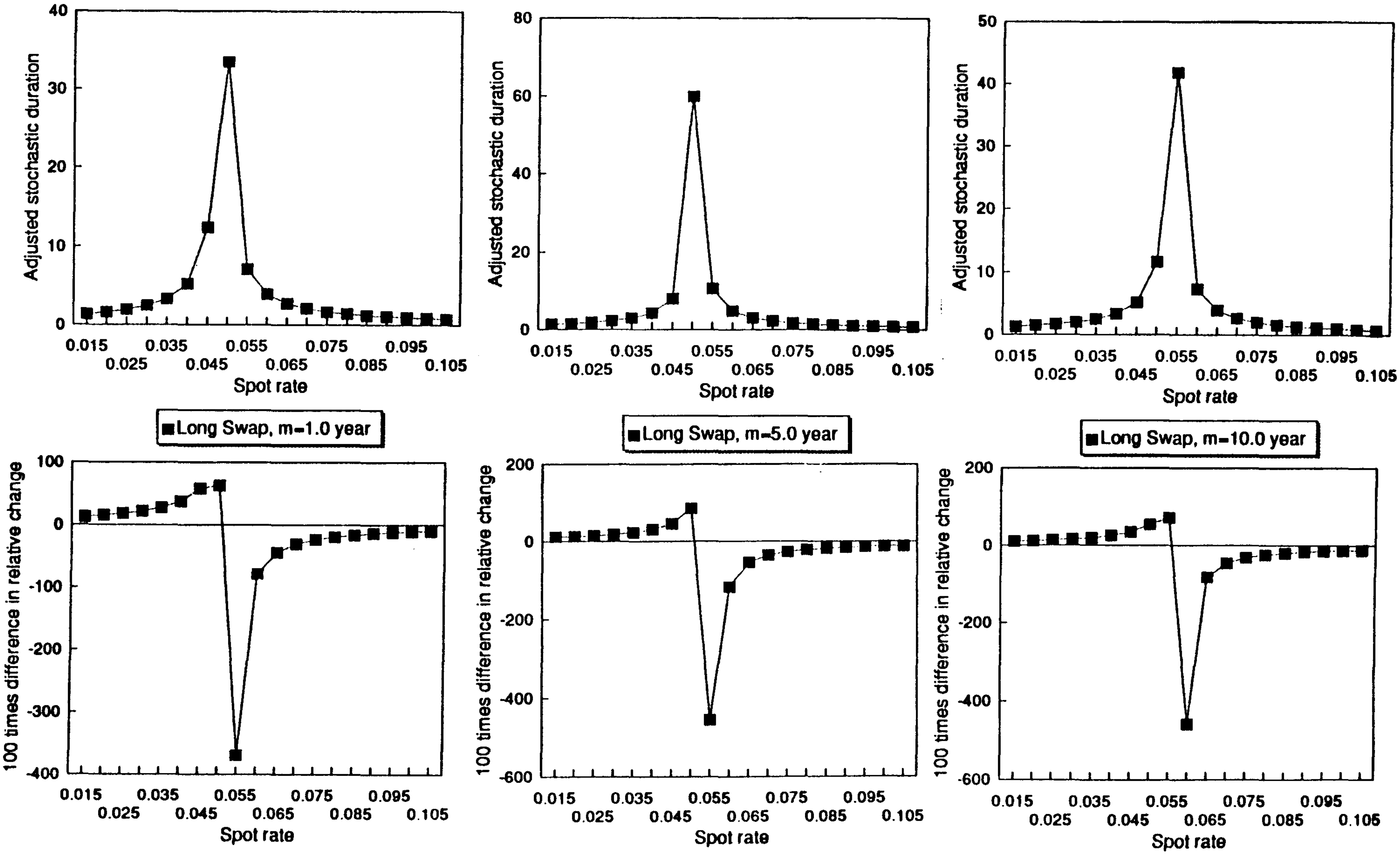

Figure 6

The adjusted stochastic duration of an existing *long* swap position with a notional principal of $100, at time *t*, and the relative change in the absolute value of the first partial (with respect to the spot rate) of long swap value minus the relative change in the absolute value of long swap, both as a function of the swap maturity, *m*, for alternative levels of the spot rate, *r(t)*. The relative changes are for a change of 0.5 year in the swap maturity, *m*. The paramater values assumed are: $\kappa=0.10$, $\theta=0.04$, $\lambda=0.0$, $\sigma=0.05$, $i_e=0.05$, and $d_b(m,t)=d_b=0.0012$.

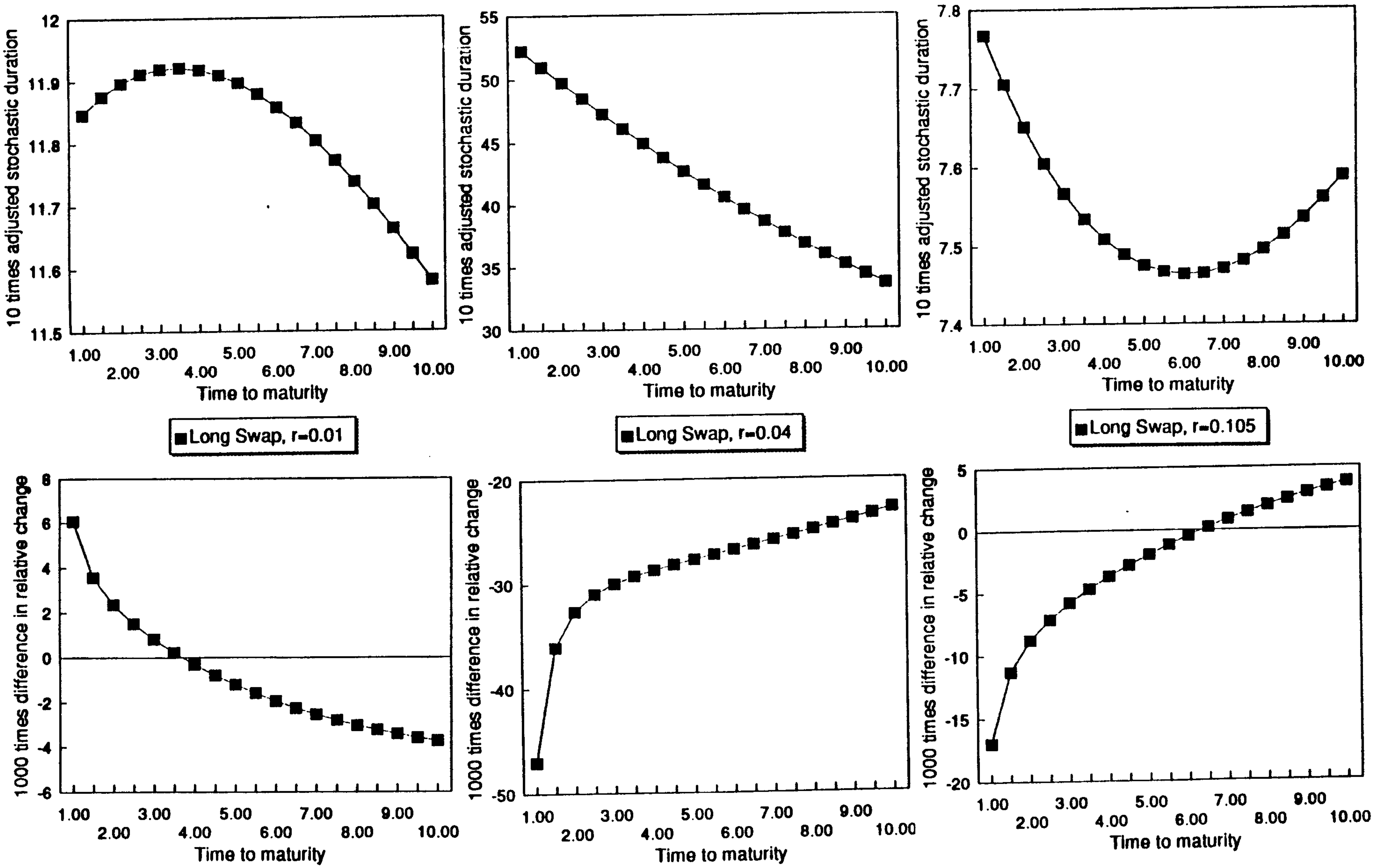

Table 1

Percentage changes in the value of a *long* swap position and the corresponding reference coupon bond for a change of 10 basis points in the spot rate from its initial level, $r(t)$. The notional principal of the *long* swap and the face value of the coupon bond are both set equal to $1. The parameter values assumed are: $\kappa=0.10$, $\theta=0.04$, $\lambda=0.0$, $\sigma=0.05$, $i_0=0.05$, and $d_s(m,t)=d_s=0.0012$.

A. Reference coupon bond

Spot rate, r	Time to maturity, m (in years) 0.5	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0.010	0.049	0.094	0.175	0.245	0.305	0.358	0.403	0.441	0.475	0.504	0.529
0.020	0.049	0.094	0.175	0.245	0.305	0.357	0.402	0.440	0.473	0.502	0.526
0.030	0.049	0.094	0.175	0.245	0.305	0.356	0.401	0.439	0.471	0.499	0.524
0.040	0.049	0.094	0.175	0.245	0.304	0.356	0.399	0.437	0.469	0.497	0.521
0.050	0.049	0.094	0.175	0.244	0.304	0.355	0.398	0.436	0.468	0.495	0.518
0.060	0.049	0.094	0.175	0.244	0.303	0.354	0.397	0.434	0.466	0.493	0.515
0.070	0.049	0.094	0.175	0.244	0.303	0.353	0.396	0.433	0.464	0.490	0.513
0.080	0.049	0.094	0.175	0.244	0.303	0.353	0.395	0.431	0.462	0.488	0.510
0.090	0.049	0.094	0.175	0.244	0.302	0.352	0.394	0.430	0.460	0.485	0.507
0.100	0.049	0.094	0.175	0.243	0.302	0.351	0.393	0.428	0.458	0.483	0.504

Table 1 Continued

Percentage changes in the value of a *long* swap position and the corresponding reference coupon bond for a change of 10 basis points in the spot rate from its initial level, $r(t)$. The notional principal of the *long* swap and the face value of the coupon bond are both set equal to $1. The parameter **values assumed are:** $\kappa=0.10$, $\theta=0.04$, $\lambda=0.0$, $\sigma=0.05$, $i_0=0.05$, **and** $d_t(m,t)=d_t=0.0012$.

B. *Long* swap

Spot rate, *r*	Time to maturity, *m* (in years) 0.5	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0.010	2.627	2.635	2.646	2.651	2.651	2.646	2.637	2.625	2.611	2.594	2.576
0.020	3.542	3.538	3.523	3.502	3.474	3.442	3.407	3.368	3.328	3.286	3.244
0.030	5.451	5.410	5.319	5.219	5.115	5.007	4.898	4.789	4.681	4.576	4.473
0.040	11.892	11.610	11.049	10.502	9.978	9.483	9.018	8.585	8.184	7.815	7.474
0.050	63.516	73.950	114.146	276.313	541.007	131.759	73.775	50.771	38.505	30.925	25.802
0.060	8.616	8.762	9.106	9.525	10.030	10.636	11.362	12.234	13.288	14.573	16.162
0.070	4.611	4.636	4.702	4.786	4.890	5.012	5.155	5.318	5.502	5.708	5.938
0.080	3.143	3.143	3.151	3.169	3.198	3.236	3.284	3.340	3.404	3.477	3.557
0.090	2.381	2.371	2.358	2.354	2.356	2.366	2.382	2.403	2.430	2.461	2.497