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A Simple Approach to Estimate Recovery Rates with APR Violation from Debt Spreads

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[1] We are grateful for the comments we received from participants at the Bank Structure Conference at the Federal Reserve bank of Chicago, May 1998 and VIII Tor Vergata Financial Conference, University of Rome, December 1999.

Abstract

This paper proposes a simple approach to estimate the implied recovery rates embedded in the prices of the debt securities of a firm that differ in priority at time of default. The approach allows for a complex capital structure setting assuming that the absolute priority rule (APR) can be violated. The paper demonstrates that a new statistic, the adjusted relative spread, captures recovery information in debt prices. Model implied recovery rates from corporate bond prices are observed to be consistent with the findings of Altman and Kishore (1996). Interest rates and firm tangible assets are shown to be significant determinants of recovery rates.

1 Introduction

The price of a defaultable debt instrument reflects the valuation of the default arrival (timing) risk and the default-conditional recovery risk. Much of the default literature, however, models the timing risk assuming a constant aggregate recovery rate by the security holders that do not differ in priority.[1] This is an oversimplifying assumption because firms issue debt-securities differing in priority, and in the event of default, the holders of these debt securities face an uncertain outcome regarding recovery.[2] Hence, default conditional aggregate recovery is uncertain and should be modeled as a random variable rather than an exogenously given constant.[3]In addition, empirical evidence shows that in corporate reorganizations the outcome of the bargaining process among the claimants may include violation of absolute priority rule (APR) which adds to the uncertainty.[4]

This paper proposes a simple approach to extract the mean and variance of default conditional recovery rates implicit in the prices of senior and junior debt instruments of a firm. The underlying assumption in this approach is that default timing risk and recovery risks are independent and that both senior and junior debt holders face the same default arrival risk but differ in the recovery rates at time of default. We show that the difference between the prices of senior to junior debt of a firm over the price difference between default-free bond and junior debt (termed the *relative spread*) is positively related with the expected aggregate payout rate to debt holders, and negatively related with the payout volatility.[5] We obtain this result by showing that the relative spread adjusted for the firm's share of senior debt in the capital structure (termed the *adjusted relative spread*) mirrors senior recovery relative to the excess recovery by Treasury over junior debt. Because the adjusted relative spread equation is free of default timing risk we term this equation the *pure recovery model.*

[1] A recent survey of this literature appears in Madan (2001).

[2] There exists a bargaining process among the claimants during bankruptcy and a number of papers have modelled this feature of the default event (Anderson and Sundaresan (1996), Mella and Perraudin (1993), and Leland (1994)).

[3] As underscored in CreditMetrics, "recovery rates are best characterized, not by the predictability of their mean, but by their consistently wide uncertainty."

[4] Franks and Torous (1989), Eberhart, Moore, and Rosenfeldt (1989), Weiss (1990) and Eberhart and Senbet (1993).

[5] Jarrow (2000) recently uses debt and equity prices to estimate simultaneously the arrival risk model and the assumed constant recovery level. In contrast, our approach follows Madan and Unal (1998) and focuses directly on just the characteristics of recovery in greater detail, employing debt securities differentiated by priority.

To parameterize the pure recovery model we propose a valuation expression for the default conditional payout risk embedded in junior debt prices. Toward this end we follow Black and Cox (1976) and Stulz and Johnson (1985), and express the payout rate to junior debt holders in terms of the payoff to a call option written on the aggregate default conditional payout rate. We extend their approach by valuing this call option assuming APR violation.[6] Hence, we are able to express the adjusted relative spread in terms of the mean and variance of the aggregate recovery rate and parameters that capture APR violations. We show that an empirical model can be developed expressing the mean aggregate recovery rate in terms of macro and firm specific variables. The model implies that adjusted relative spreads are positively related to factors related to aggregate mean recovery and negatively related to its volatility.

Using the Lehman Brothers Fixed Income Data base we calculate adjusted relative spreads for 28 firms identified from 10 different industries. To evaluate whether or not the cross-sectional variation in the adjusted relative spreads are related to the variation of actual recovery rates observed in practice we utilize Altman and Kishore (1996) estimates. They report the estimates of the recovery rates defaulted bonds stratified by Standard Industrial Classification sector. We show that the cross-sectional and time series variation in adjusted relative spreads are significantly related to Altman-Kishore's estimated recovery rates. This finding provides supporting evidence that adjusted relative spreads can capture recovery information embedded in debt prices.

We next examine the empirical implications of the pure recovery model. In this exercise we specify the mean aggregate recovery rate as a function of risk-free interest rates and the level of the tangible assets of the firm. We estimate the pure recovery model for 11 firms using time series data. As implied by the model, adjusted relative spreads are significantly related to interest rates and firm tangible assets. Parameter estimates reflecting the APR violation vary significantly across firms indicating that APR violation is not expected uniformly for all firms.

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[6] As indicated by Weiss (1990) such violation is plausible because bankruptcy law gives junior creditors the ability to delay final resolution. Hence, senior debt-holders will be willing to violate priority not to incur any additional costs by the delay of the bankruptcy resolution.

Of course, there exists another layer of complexity in violation of priority of claims. Senior debt-holders negotiate with the equity-holders as well. As shown below, we parameterize the level at which senior creditors are willing to violate priority. This level is applicable to the equity holders as well.

The paper is organized as follows: Section 2 develops the pure recovery model. Section 3 provides evidence that the adjusted relative spreads are related to the observed recovery rates in defaulted bonds. Section 4 proposes an empirical specification for the pure recovery model and provides the estimates. Section 5 concludes the paper.

2 The Pure Recovery Model

2.1 Relative Spread

Consider a frictionless economy where two classes of zero-coupon bonds are traded: default-free and defaultable. Default-free bond price with maturity τ is given by $P(\tau)$. In the case of defaultable bond, bondholders receive the promised unit face at maturity if the firm survives till maturity. The survival probability of the firm is denoted by $G(t,T)$. Default occurs at a random time and debt holders are paid a reduced value of the face. Expected value of this recovery is denoted by $E[y]$. Assuming the default arrival and the recovery processes to be independent the standard framework to express the price of defaultable bond is[7]:

$$v(\tau) = P(\tau)G(\tau) + P(\tau)(1 - G(\tau))E[y]. \tag{1}$$

To extend this framework to value defaultable senior and junior debt issues of the firm requires an explicit description of the payout structure of the debt securities facing identical default arrival risk but different conditional recovery. Toward this end, let $\overline{S}(\tau)$ and $\overline{J}(\tau)$ denote the promised face to senior and junior debt with maturity τ, respectively. Further let $\overline{S}$ and $\overline{J}$ denote the sum of the promised face across all maturities to senior and junior debt. Hence, total promised face of all debt outstanding is $\overline{P} = \overline{S} + \overline{J}$. At time of default, firm defaults on all its outstanding debt obligations. In this case, payment to the outstanding senior and junior debt with maturity τ can be expressed as

$$S = \int_0^T S(\tau)d\tau \tag{2}$$

$$J = \int_0^T J(\tau)d\tau \tag{3}$$

[7] Madan and Unal (2000) provide a detailed analysis of the assumptions behind this framework.

Thus, total payment to all debt claimants at time of default is $P = S + J$. This payoff structure can also be expressed in terms of payout rates. Denoting the aggregate payout rate to all outstanding debt by y we obtain:

$$y = \frac{P}{\overline{P}} = \frac{\overline{S}}{\overline{S} + \overline{J}} y^S + \frac{\overline{J}}{\overline{S} + \overline{J}} y^J. \tag{4}$$

or

$$y = p^S y^S + (1 - p^S) y^J. \tag{5}$$

In equations (4) and (5) $y^S = \frac{S}{\overline{S}}$ and $y^J = \frac{J}{\overline{J}}$ are the average payout rates to senior and junior debt holders, respectively. We assume that $y^S = \frac{S}{\overline{S}} = \frac{S(\tau)}{\overline{S}(\tau)}$ and $y^J = \frac{J}{\overline{J}} = \frac{J(\tau)}{\overline{J}(\tau)}$. This assumption implies that at time of default the payout rate y^S and y^J are applicable to senior and junior debt claimants regardless of maturity. Hence, utilizing the framework of equation (1) we can express the prices of zero-coupon senior $v_s(\tau)$ and junior $v_j(\tau)$ unit face debt instruments of a firm with maturity τ as follows:

$$v_S(\tau) = \left(G(\tau) + (1 - G(\tau))E[y^S]\right) P(\tau) \tag{6}$$

$$v_J(\tau) = \left(G(\tau) + (1 - G(\tau))E[y^J]\right) P(\tau) \tag{7}$$

Note that, using equations (6) and (7), the relative spread of the prices of senior to junior debt over the spread of default-free bond to junior debt is:

$$RS = \frac{v_S(\tau) - v_J(\tau)}{P(\tau) - v_J(\tau)} = \frac{E(y^S) - E(y^J)}{1 - E(y^J)} \tag{8}$$

The relative spread expression, RS, can be viewed as the *pure recovery model* because the timing risk $(G(t, T))$ does not appear in equation (8). The attractiveness of the RS is that it gives information regarding the market's expectation of the conditions at which default will occur. To see this we simplify the right-hand side of equation (8) such that the relative spread is expressed only in terms of the distribution of the aggregate payout rate. Note that by definition

$$y^S = \frac{y}{p_s} - \frac{(1 - p_s)}{p_s} y^J$$

and

$$y^S - y^J = \frac{y}{p_s} - \left(\frac{(1 - p_s)}{p_s} + 1\right) y^J. \tag{9}$$

Taking expectations

$$E(y^S) - E(y^J) = \frac{1}{p_s}(E[y] - E(y^J)) \tag{10}$$

and substituting equation (10) in equation (8) we obtain:

$$RS = \frac{1}{p_s}\left(\frac{E(y) - E(y^J)}{1 - E(y^J)}\right) \tag{11}$$

Hence equation (11) now expresses the pure recovery model in terms of expected aggregate payout rate and expected recovery by the junior debt holders.

2.2 Specification for the Expected Payout Rate for Junior Debt

The next step is to express the payout to the junior claimant as a contingent claim on the aggregate payout rate y, and once this is done, the right hand side of equation (11) involves expectations of option type payoffs with respect to the aggregate payout density $f(y)$. The resulting equation (11) then forms the basis for a model permitting estimation of the payout density and the payoff structure to the claimants from data on the relative spread RS. Toward this end, we first relate y^J to the average payout, y, by the function $y^J = J(y)$. Next, a specific density, $f(y)$, is proposed for the default conditional average payout. This results in:

$$E(y^J) = \int_0^1 J(y)f(y)dy, \tag{12}$$

Hence, integrating equation (12) yields the expected value of payout to junior debt-holders, $E(y^J)$, that is expressed in terms of the parameters of the density $f(y)$.

To specify the payout function $J(y)$, note that in terms of equation (5), under strict APR, junior debt-holders receive payments only after senior debt-holders are fully paid ($y = p_s$). In this case, the function $J(y)$ can be obtained utilizing Black and Cox (1976). They show that under strict APR, $J(y)$ represents the payoff to a long position on a call option written on the default-conditional payout.[8] In Figure 1, the payoff to senior and junior debt-holders are shown by the bold lines and the proportion of outstanding

[8] In the same manner, $S(y)$ represents the payoff to a default-free bond and a short position on a put option written on the firm's default-conditional payout.

senior-debt (p_s) is 50 percent. Junior debt-holders receive payments only after the aggregate payout rate to all debt claimants is above 50 percent .

However, if we allow for APR violation, junior debt-holders receive payments before senior debt-holders are fully paid. Hence, in general we would have a third region where sharing occurs. We capture such sharing by introducing the parameter λ which reflects the argument that junior debt-holders receive nothing ($J(y) = 0$) as long as $y \leqq \lambda p_s$ (region 1) and start sharing by receiving payments ($J(y) > 0$) in the region ($y > \lambda p_s$) (region 2). Figure 1 demonstrates such a sharing. We assume $\lambda = .50$. As shown, violation of APR effectively makes the junior debt-holders better off by reducing the strike-price of the call option they are holding and makes the senior-debt-holders worse off. In the region, $y \leqq \lambda p_s$, $S(y)$ can be determined by the product of $(\frac{\lambda}{1-(1-\lambda p_s)})$ and y which effectively equals $\frac{y}{p_s}$. For example, when $y = \lambda p_s$, senior-debt holders will be paid only 25 percent of their promised amount and junior debt-holders will receive no payment. However, any improvement in y above λ will not totally accrue to the senior debt-holders but will be shared with the junior debt-holders. In region 2, ($y > \lambda p_s$), $J(y)$ is determined by the product of, $(\frac{1}{1-\lambda p_s})$ and the increment of y over λp_s. However, in region 2**,** we suppose that the payout rate to the senior claimant $1/p_s$ is reduced by a constant θ for a value of $\theta < 1$. The specific payout to the senior claimant in this region starts at λ and increases at the rate θ/p_s and is

$$S(y) = \lambda + \frac{\theta}{p_s}(y - \lambda p_s). \tag{13}$$

Note on this pattern the senior claimant is fully paid off at the aggregate recovery level y^*,

$$y^* = \lambda p_s + \frac{(1-\lambda)p_s}{\theta} \tag{14}$$

To ensure that $y^* \leqq 1$ we must have

$$\theta \geq \frac{p_s - \lambda p_s}{1 - \lambda p_s}. \tag{15}$$

Hence, the payout to the junior must be adjusted as $\frac{(1-\theta)(y-\lambda p_s)}{1-p_s}$. In the region $y > y^*$ (region 3) we clearly have that $S(y) = 1$ and $J(y) = \frac{y-p_s}{1-p_s}$. In summary, the payments to the junior claimant in the three regions are given by

$$J(y) = \begin{cases} 0 & y \leqq \lambda p_s \\ \frac{(1-\theta)(y-\lambda p_s)}{1-p_s} & \lambda p_s < y \leqq y^* \\ \frac{y-p_s}{1-p_s} & y^* < y \leqq 1 \end{cases} \tag{16}$$

Alternatively,

$$J(y) = \frac{1-\theta}{1-p_s}\ Max(y-\lambda p_s, 0) + \frac{\theta}{1-p_s}\ Max(y-y^*, 0)\ . \qquad (17)$$

As can be observed, for $\lambda = 1$ (APR enforced), we obtain the Black and Cox (1976) characterization of junior debt-holders holding a call option and acting like equity-holders. With APR violation, ($\lambda < 1$), the value of the call options increase making senior debt-holders worse off and the junior debt-holders better off. Hence, equation (17) show that the junior debt-holders' payoff function can be expressed in terms of two call options written on the firm's expected default-conditional average payout rate, with strikes λp_s and y^*. They are holding $\frac{1-\theta}{1-p_s}$ units of the first and $\frac{\theta}{1-p_s}$ of the second call option.

The second component to be evaluated in equation (12) is $f(y)$. From this density one can determine the probability of the call options given in equation (17) to be in the money once default occurs. Hence, the integral in equation (12), for example, represents the value of the call option held by the junior debt-holder.

A straightforward assumption would be to assume that y is normally distributed. However, such an assumption violates two important characteristics of the average payout rate. First, because y is the ratio of payout to the promised payments to debt claimants at any default time it lies between 0 and 1. Second, the mean and variance of y are related because as the mean approaches unity (100 percent payout rate) or zero the variance of y becomes zero. Hence, we propose that the average payout rate is related to a normal random variable x by the logit transformation $y = \frac{e^x}{1+e^x}$. Further, we assume that the variable x which is the logarithm of the conditional payout to loss ratio $\left(x = \ln\left(\frac{y}{1-y}\right)\right)$, is normally distributed with mean μ and variance σ^2. It follows that the conditional density for the average payout rate, $f(y)$, is:

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}y(1-y)} \exp\left(-\frac{1}{2\sigma^2}\left(\ln\left(\frac{y}{1-y}\right)-\mu\right)^2\right) \qquad 0 < y < 1 \qquad (18)$$

The characteristics of the payout rate are captured in the density given in equation (18). Figure (2) shows the density for the recovery level for $\mu = \pm.5$ and $\sigma = .2$,and .5. We observe that the density may be positioned at various points on the unit interval and it may be widely or narrowly spread out.

Given this density, the value of the call option can be expressed as follows:

Proposition 1 *The call option written on the firm's average payout rate with strike k, pays the following conditional on default:*

$$C(k;\mu,\sigma^2) = 1 - k - \int_k^1 N\left(\frac{\ln\frac{y}{1-y} - \mu}{\sigma}\right) dy. \tag{19}$$

Proof in the Appendix.

The integral of N in equation (19) is easily evaluated numerically. Hence, the expected payout rate for the junior debt allowing for APR violation is given by:

$$\begin{aligned} E(y^J) &= \frac{1-\theta}{1-p_s} C(\lambda p_s;\mu,\sigma^2) + \frac{\theta}{1-p_s} C(y^*;\mu,\sigma^2), \\ y^* &= \lambda p_s + \frac{(1-\lambda)p_s}{\theta}. \end{aligned} \tag{20}$$

Substituting equation (20) in equation (11) and adjusting the relative spread for the level of the share of senior debt, p_s,we obtain:

$$ARS = p_s RS = \left(\frac{C(0;\mu,\sigma^2) - \frac{1-\theta}{1-p_s} C(\lambda p_s;\mu,\sigma^2) - \frac{\theta}{1-p_s} C(y^*;\mu,\sigma^2)}{1 - \frac{1-\theta}{1-p_s} C(\lambda p_s;\mu,\sigma^2) - \frac{\theta}{1-p_s} C(y^*;\mu,\sigma^2)} \right). \tag{21}$$

Hence, the pure recovery model is fully expressed in terms of option type payoffs with mean μ and variance σ^2, which are related to the mean and variance of the expected aggregate payout rate.

2.3 Parameter Sensitivity of Adjusted Relative Spread

This section evaluates the sensitivity of the pure recovery model to parameters λ, θ, σ,and μ and obtains empirically testable implications. Figure 3 assumes $\lambda = 0.50$, $\theta = 0.50$ and $p_s = 0.50$ and examines the behavior of the ARS as μ and σ vary. First, we observe that ARS is an increasing function of μ. Higher levels of ARS implies higher aggregate recovery.

The second important observation is that ARS reflects the payout to senior debt. This is expected because note that the numerator of equation (21) represents the difference between the aggregate payout and the payout to junior debt holders. This difference is nothing but the payout to senior debt holders. Hence, ARS can be seen as a statistic capturing the payout to senior debt holders deflated by the premium of the junior debt over the risk-free debt. As σ approaches zero the ARS curve begins reflecting the payoff structure described in Figure 1. This is plausible because a low volatility implies the mean recovery will be realized with certainty. Hence, the curve represents recovery for the senior debt at various levels of mean recovery as depicted by the three different payout regions. Junior debtholders receive nothing in region 1. Sharing occurs in region 2, that starts after $y = \lambda p_s = 0.25$. For $y \geq y^* = 0.75$ (region 3) senior debt becomes risk-free and ARS becomes 0.5. However, ARS decreases with increased uncertainty of the aggregate recovery rate which is negative news for the senior debt holders. Hence ARS curve shifts down.

Figure 4 displays the impact of APR violation parameter λ on ARS. As λ increases, sharing between senior and junior debtholders starts after a higher portion of senior debt is paid. Hence, senior debt will be more valuable, and ARS will increase. This is what we observe in Figure 4 and the ARS curve shifts up as λ increases. Similarly, senior debt holders benefit as the rate of increase in λ, the θ parameter, increases. This is because a higher θ indicates less of the recovered face value is shared with junior debtholders. Therefore, senior debt is paid out more quickly and is more valuable, which benefits the senior debt holders.

3 Adjusted Relative Spreads and Recovery Rates

Our empirical analysis proceed in two layers. First, we provide evidence that the ARSs are indeed related to the recovery rates observed in bond defaults. Once we provide supporting evidence toward this end the second layer includes the estimation of the pure recovery model.

3.1 Data

Corporate bond data are obtained from the Lehman Brothers Fixed Income Data Base. The database provides end-of-month bid price, coupon rate, yield-to-maturity, industry classification and other important information for the bonds constituting the Lehman Bond Index. Putable bonds, non-regular bonds and bonds with sinking fund features are excluded from the sample. We further remove bond observations with more than 10 years and less than 6 months of maturity. Firms with only senior or only junior bonds are also deleted from the sample. We include those callable bonds where we could identify junior and senior bond issue of a firm that are both callable.

Majority of the corporate bond issues are coupon paying bonds and restricting the sample to zero-coupon bonds would have caused very few observations. However, identifying senior and junior debt issue of a firm with identical coupon structure is also very difficult. To include coupon bonds in the study we follow the following matching strategy. For each date, we match a junior bond to another senior bond issued by the same firm with closest possible duration and coupon rate. Our decision criteria for this match is defined by two numbers, $\delta_1 = \dfrac{abs(d_S - d_J)}{(d_S + d_J)/2}$ and $\delta_2 = abs(C_S - C_J)$, where C_S and C_J (d_S and d_J) are the coupon rates (Macaulay durations) of senior and junior bonds, respectively. If $\delta_1 \leqq 0.3$ and $\delta_2 \leqq 0.03$ we accept the match, otherwise relative spread will be missing for that junior bond at this date. Then, we calculate zero coupon senior, junior and Treasury bond prices $v_S(\tau)$,$v_J(\tau)$ and $P(\tau)$ by discounting a \$100 face value with the available yield-to-maturity at $\tau = d_J$.

The resulting sample consists of 33 ARS statistics for 28 companies. The companies are reported in Table 1 together with the industry they represent. The table also reports starting and ending dates of the observations. As can be observed in three cases we are able to determine the ARS statistic using more than one pairings of the bond.

3.2 Cross-sectional and Time-series Variation in Adjusted Relative Spreads

Unfortunately, not much research exist that reports evidence regarding actual recovery rates. One important exception is the study by Altman and Kishore (1996). They document recovery rates in bond defaults classified by Standard Industrial Classification (SIC) sectors. We utilize this study and contrast the industry estimates reported in Altman and Kishore (1996) with the ARSs reported in Table 1.

Table 2 reports the comparison. We observe that the relationship between ARSs and the recovery rates are remarkably close. Public utilities and chemical and petroleum companies have the highest ARSs, which is consistent with the recovery rates estimated for these industries by Altman and Kishore. Furthermore, the correlation between ARSs at the firm level and the recovery rates of the industry the firm belongs is 0.73 and is significant at the 1 percent level.

To gain further insight, we group firms into high recovery, medium recovery, and low recovery industries using the Altman and Kishore industry recovery estimates. Industries where Altman and Kishore recovery rate estimates exceed 45% are defined as high recovery group, industries with recovery rates below 35% constitute the low recovery group. Hence, industries 1-3, 4-7 and 8-10 in Table 2, constitute the High, Medium and Low recovery groups, respectively. Next we assign firms reported in Table 1 to one of these three portfolios and obtain monthly average of ARS for each portfolio. Figure 5 plots the time series pattern of ARSs for the three portfolios. Consistent with our expectations there is a pecking order going from the ARS curve of the high recovery group toward the low recovery group. This difference also persists over time.

Hence, the evidence presented strongly supports the argument that ARSs are indeed related to recovery rates. A high level of ARS is associated with higher recovery level and this prediction holds for cross-sectional as well time series behavior of adjusted relative spreads.

4 Estimating the Pure Recovery Model

4.1 Empirical Specification

The relative spread model of equation (21) may be adapted to analyze the conjectured dependence of recovery rates on the business cycle and on appropriate firm specific information. For such an exercise we denote by x_t a time series on a vector of macro and firm specific variables that are presumed to affect recovery levels. We then consider the model

$$\mu_t = \beta_0 + \beta' x_t \tag{22}$$

and summarize the model of equation (21) by the relation

$$ARS_t = \Phi(\lambda, \theta, \mu_t, \sigma, p_s) + \varepsilon_t \tag{23}$$

where it is supposed that the error term represents uncorrelated statistical noise.

Equation (23) in conjunction with equation (22) constitutes a potentially estimable econometric model permitting estimation of the recovery model of equation (22) together with the APR violation parameters, λ, θ and the volatility of the log payout to loss ratio, σ.

To choose plausible firm specific variables we follow the study by Altman and Kishore (1996). They argue that recovery rates are related to the asset structure of firms and provide evidence that firms with more tangible and liquid assets have a higher liquidation value, and therefore higher recovery rates upon default. As a result, we employ the following two factor model to capture impact of firm specific and macroeconomic variables on mean recovery rates.

$$\mu_t = \beta_0 + \beta_1 RF_t + \beta_2 TANG_t \tag{24}$$

The model is estimated using time-series data. $TANG$ represents the tangible assets of the firm. We define tangible assets as the sum of current assets (COMPUSTAT quarterly item 40) and net plant property and equipment (COMPUSTAT quarterly item 42) divided by total assets (COMPUSTAT quarterly item 44). We predict a positive relationship between $TANG$ and implied recovery rates. RF is the risk free rate and controls for the interest rate risk environment. The risk free rates at the desired date and maturity are calculated from daily treasury bond yields that come from the H15 release of the Federal Reserve System. The yield curve is spanned with cubic spline method to find the risk free rate at any maturity.

The requirement that data availability in COMPUSTAT files for firms whose adjusted relative spreads are reported in Table 1 causes further shrinkage in our sample. We identify 11 out of 28 firms to have data in both sources and have sufficient time series data available for the ARSs.

4.2 Results

The nonlinear least squares estimate of the pure recovery model is reported in Table 3 for the sample firms. The first three columns report estimates of the hypothesized determinants of the aggregate recovery rate. The risk-free rate is positive and significant in six cases. This is plausible given that rising interest rates benefit the assets by increasing cash and earnings implying a higher recovery in case of default. The estimates relating to the tangible assets are also as expected. They are all positive and in 9 cases the t-values are significant. This confirms Altman and Kishore (1996) arguments that recovery rates are higher for firms having higher tangible assets.

Column 5 and 6 report estimates of the APR violation parameters. First we observe that the estimated values vary significantly across firms. This can be construed as evidence that ex ante there is no uniform expectation by the market participants about how APR will be violated conditional on default, across firms.

Column 7 shows the estimate of the volatility term, σ. Note that the mean and standard deviation of the logarithm of the payout to loss ratio, $x = \ln\left(\frac{y}{1-y}\right)$, are μ_t and σ. The variable x has a normal distribution, and μ_t can take any finite value between $-\infty$ and $+\infty$. To calculate the mean of the aggregate recovery rate y, given μ_t and σ we evaluate the term

$\left[1 - \int_0^1 N\left(\frac{\ln\left(\frac{y}{1-y}\right)-\mu_t}{\sigma}\right) dy\right]$. Column 8 reports the estimate of the mean of the aggregate recovery rate. They are reasonably close to the average recovery rates estimated by Altman and Kishore (1996). However, as Column 7 shows the uncertainty related to the mean recovery rate can vary significantly across firms.

The parameters λ, θ, and σ are structural and reflect variations in the functional form of the dependence of adjusted relative spreads to the data on the explanatory variables (interest rates and the level of tangible assets). The exact functional form is not identified with precision and this is reflected in high standard errors for the estimates of λ, θ, and σ. Hence, the t-statistics reported for the explanatory variables are conditional on the estimated values for λ, θ, and σ.

5 Conclusion

This paper proposes a potentially attractive way to discover the econometric determinants of recovery rates from data on the time series relative spreads and the proposed explanatory variables. This is an important advance in understanding the determinants of default spreads as there is little possibility of direct observation of the quantities of interest, given the absence of the occurrence of the event, ex ante.

The empirical experiments reported ascertain market sentiments on the recovery dimension of default. An important contribution of the paper is to demonstrate that a new statistic, the adjusted relative spread, captures recovery information embedded in debt prices. Aggregate recovery rate estimates for a sample of industrial firms confirm with Altman and Kishore (1996) estimates of recovery in bond defaults. In addition, we are able to show that recovery rate volatility vary significantly across firms. Furthermore, we show that interest rates and firm's tangible assets are significant determinants of recovery rates. Finally, the recovery level at which senior debt holders agree to share with the junior debt holders vary across firms indicating that APR violation is a firm specific risk for senior debt holders.

6 Appendix

Proof of Proposition 1*:* To obtain the pricing expression for the call option, note that for strike k we can express the price of a call option written on the underlying asset y as follows:

$$C(k) = \int_k^1 (y-k) f(y) dy, \tag{25}$$

where $f(y)$ is the probability density of y. Equation 25 is expressed as:

$$C(k) = \int_k^1 y f(y) dy - k \int_k^1 f(y) dy. \tag{26}$$

We evaluate the second integral first. Note that this term is equal to

$$\int_k^1 f(y) dy = \text{Prob}(y > k) = 1 - F(k), \tag{27}$$

where $F(y)$ is the distribution function of y. We first determine $F(y)$ in terms of the standard normal distribution function $N(\cdot)$. For any real number u, $0 \leqq u \leqq 1$,

$$\begin{aligned} F(u) &= \text{Prob}(y < u) \\ &= \text{Prob}\left(\frac{e^x}{1+e^x} < u\right) \qquad (28) \\ &= \text{Prob}\left(e^x < \frac{u}{1-u}\right) \\ &= \text{Prob}\left(x < \ln\left(\frac{u}{1-u}\right)\right) \qquad (29) \end{aligned}$$

Assuming $x \simeq N(\mu, \sigma^2)$

$$= N\left(\frac{\ln\left(\frac{u}{1-u}\right) - \mu}{\sigma}\right). \tag{30}$$

The second term in equation (26) is therefore

$$k \int_k^1 f(y) dy = k - kN\left(\frac{\ln\left(\frac{k}{1-k}\right) - \mu}{\sigma}\right) \tag{31}$$

The first term in equation (26) is obtained on integration by parts as:

$$\begin{aligned} \int_k^1 y f(y) dy &= y F(y) \Big|_k^1 - \int_k^1 F(y) dy \\ &= 1 - kF(k) - \int_k^1 F(y) dy. \end{aligned} \tag{32}$$

It follows from equation (32) that

$$\int_k^1 y f(y) dy = 1 - kN \left(\frac{\ln\left(\frac{k}{1-k}\right) - \mu}{\sigma} \right) - \int_k^1 N \left(\frac{\ln\left(\frac{y}{1-y}\right) - \mu}{\sigma} \right) dy. \tag{33}$$

Substituting equation (31) and equation (33) into equation (26) we obtain the call option valuation expression given in Proposition 1:

$$C(k; \mu, \sigma^2) = 1 - k - \int_k^1 N \left(\frac{\ln\left(\frac{y}{1-y}\right) - \mu}{\sigma} \right) dy. \tag{34}$$

Q.E.D.

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Table 1: The Sample

This table reports for each firm the issuer name, sample period, number of observations (T), and sample averages of adjusted relative spread (*ARS*). *ARS* is the product of relative spread and senior debt ratio. Relative spread is defined as the price difference between senior and junior bond divided by the price difference between default-free bond and junior bond. For Kroger, Ralphs Grocery Co and Stone Container Corp more than one pairings of senior and junior bonds are used to calculate *ARS*.

2 Digit SIC Code	Company Name	Sample Period	T	Average of *ARS*
78	AMC Entertainment Inc.	9208-9601	42	0.250
80	American Medical Intn'l	9111-9503	40	0.123
49	Coastal Corporation	9002-9305	38	0.614
15	Del Webb Corp	9305-9703	30	0.262
75	Envirotest Systems	9403-9712	46	0.343
58	Family Restaurants	9402-9712	11	0.237
58	Flagstar	9309-9712	42	0.126
30	Foamex L.P.	9410-9706	24	0.284
58	Foodmaker, Inc	9206-9712	35	0.352
54	Grand Union	9207-9506	30	0.181
75	Hertz Corp	9105-9705	48	0.390
33	Kaiser Alum. and Chemical	9302-9712	47	0.140
54	Kroger I	9402-9701	36	0.148
54	Kroger II	9402-9510	21	0.105
54	Kroger III	9208-9712	28	0.162
48	Lenfest Communictions Inc.	9610-9712	15	0.092
37	Newport News Shipbuilding	9706-9712	7	0.057
76	Prime Hospitality Corp	9706-9712	7	0.141
26	Printpack Inc	9704-9712	9	0.178
54	Ralphs Grocery Co I	9506-9712	31	0.154
54	Ralphs Grocery Co II	9506-9712	31	0.185
28	Revlon Consumer Products	9308-9712	52	0.382
26	Riverwood International	9206-9606	45	0.190
54	Safeway Stores Inc.	9703-9712	8	0.033
44	Sea Containers	9412-9712	37	0.175
37	Sequa Corp	9312-9712	49	0.390
26	Stone Container Corp I	9204-9712	59	0.095
26	Stone Container Corp II	9705-9712	8	0.125
30	Sweetheart Cup	9309-9712	35	0.485
59	Thrifty Payless Holding	9404-9505	12	0.058
59	Thrifty Payless	9404-9604	25	0.087
37	UNC Inc	9611-9712	11	0.383
73	Valassis Inserts	9203-9712	60	0.143

Table 2: Variation of Implied Recovery Rates Across Industries

This table classifies the firms in the sample into ten different industry groups and presents averages of actual recoveries and adjusted relative spreads(ARS). ARS is the product of relative spread and senior debt ratio. Relative spread is defined as the price difference between senior and junior bond divided by the price difference between default-free bond and junior bond. Industry classifications, and industry average recovery rates in Column 5 are obtained from Table 3 in Altman and Kishore(1996). Industry averages of ARS are calculated by averaging ARS statistic first across time, then across firms.

Industry Number	Industry Name	2 Digit SIC Codes	Number of firms	Recovery rates by industry	Average of ARS by industry
1	Public utilities	49	1	0.705	0.614
2	Chemicals, petroleum,rubber and plastic products	28-30	3	0.627	0.383
3	Machinery, instruments and related products	35,36,38	3	0.462	0.292
4	Building materials,metals and fabricated products	32-34	1	0.388	0.140
5	Transportation and transportation equipment	37,41,42,45	4	0.384	0.251
6	Communication,broadcasting,movie production, printing and publishing	27,48,78	2	0.371	0.171
7	Construction and real estate	15,65	1	0.353	0.261
8	General merchandise stores	53-59	12	0.332	0.152
9	Wood, paper and leather products	24-26,31	4	0.298	0.147
10	Lodging, hospitals and nursing facilities	70-89	2	0.265	0.132

Table 3: Time series estimation of the pure recovery model

The pure recovery model is $ARS_t = \left(\frac{C(0;\mu,\sigma^2) - \frac{1-\theta}{1-p_s} C(\lambda p_s;\mu,\sigma^2) - \frac{\theta}{1-p_s} C(y^*;\mu,\sigma^2)}{1 - \frac{1-\theta}{1-p_s} C(\lambda p_s;\mu,\sigma^2) - \frac{\theta}{1-p_s} C(y^*;\mu,\sigma^2)} \right)$ where $y^* = \lambda p_s + \frac{(1-\lambda)p_s}{\theta}$ and $\mu_t = \beta_0 + \beta_1 RF_t + \beta_2 TANG_t$. The dependent variable ARS is the product of the senior debt ratio, p_s, and the relative spread (price difference between senior and junior bond divided by the price difference between default-free bond and junior bond). The parameters λ and θ capture APR violation. . The Treasury rate is RF and $TANG$ is the sum of current assets and net property,plant and equipment divided by total assets. The model is estimated by non-linear least squares. The Root Mean Squared Error is RMSE $\equiv \sqrt{\frac{1}{T}\sum_{t=1}^{T}(ARS_t - \widehat{ARS}_t)^2}$. Estimated mean recovery is the estimated mean of aggregate recovery, y. Industry recovery-averages in bond defaults are obtained from Altman and Kishore(1996). For each company and each parameter the first row reports parameter estimates and the second row gives conditional t-statistics for β_1 and β_2.

Company	β_0 (CONST)	β_1 (RF)	β_2 (TANG)	λ	θ	σ	Estimated Mean Recovery	Industry Average	RMSE
AMC	-36.556	82.406 3.56	38.242 22.68	0.579	0.831	2.969	52.2	37.1	0.042
American Medical	-3.185	3.948 0.54	1.607 2.02	0.800	0.800	0.500	12.5	26.5	0.037
Coastal Corp	-11.645	36.012 3.89	11.229 17.15	0.782	0.745	0.010	63.3	70.5	0.100
Envirotest Systems	-0.552	-37.983 -5.00	2.947 4.88	0.960	0.798	0.118	34.3	46.2	0.075
Flagstar	-2.174	0.926 0.11	0.008 0.01	0.786	0.806	0.713	12.7	33.2	0.045
Revlon	-35.596	19.259 2.43	46.636 66.75	1.000	0.999	0.447	40.5	62.7	0.083
Sequa Corp	-60.847	6.575 0.54	88.047 76.65	0.321	0.672	0.081	59.2	38.4	0.073
Stone Container	-17.391	-0.395 -0.02	20.466 13.79	0.008	0.979	0.113	9.6	29.8	0.082
Sweetheart Cup	-67.898	10.808 0.97	69.566 94.38	0.661	0.814	0.124	56.7	62.7	0.064
Valassis Insterts	-9.991	40.970 2.55	8.911 6.36	0.270	0.720	0.010	19.1	46.2	0.086
Del Webb Corp	-3.538	46.952 10.61	0.479 0.06	0.786	0.795	1.163	39.3	35.3	0.026

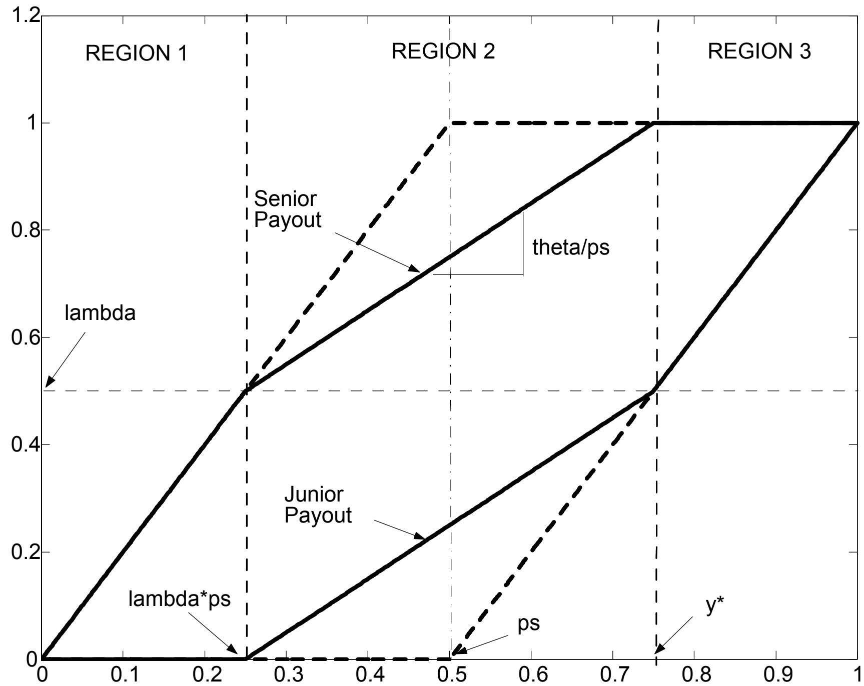

Figure 1: **Senior and junior debt payoff structure** The average payout rate to debt-holders conditional on default is y and is shown on the horizontal axis. The vertical axis gives the payout rate to senior and junior debt. Solid (dashed) lines depict the payoff structure under(without) APR violation. p_s denotes the strike price of the call option junior debt-holders are holding. $\lambda(lambda)$ represents the payout level at which absolute priority rule (APR) is violated. Hence, λp_s represents the exercise price of the call option under APR violation assumption. The senior debt holders receive payments at the rate of $\theta(theta)/p_s$ once the APR is violated. y^* represents the strike at which junior debt holders receives payment once the senior debt holders are fully paid.

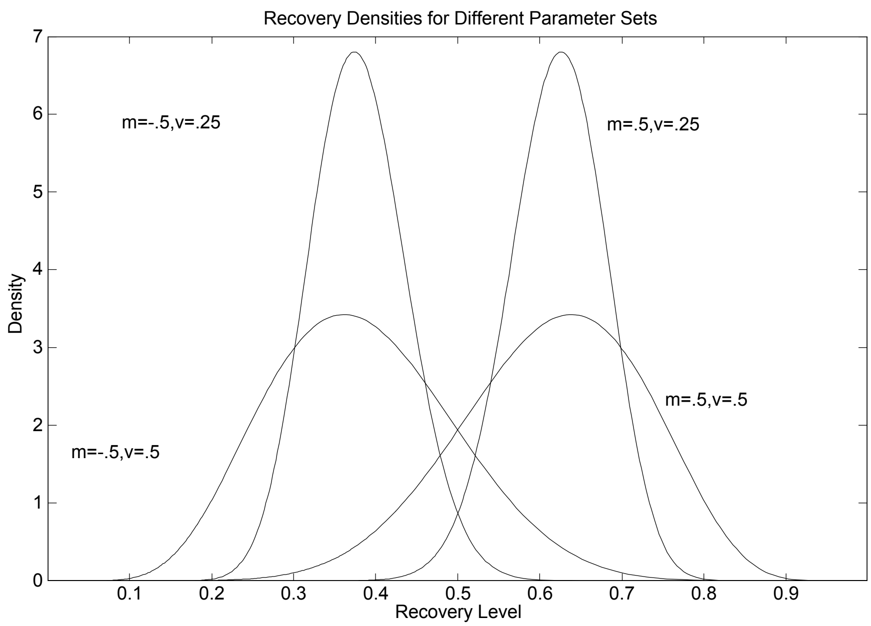

Figure 2: **Recovery density for different parameter values** The density for the for the ratio of payout to the promised payments to debt claimants at any default time lies between 0 and 1. The mean and variance of the density is denoted by m and v.

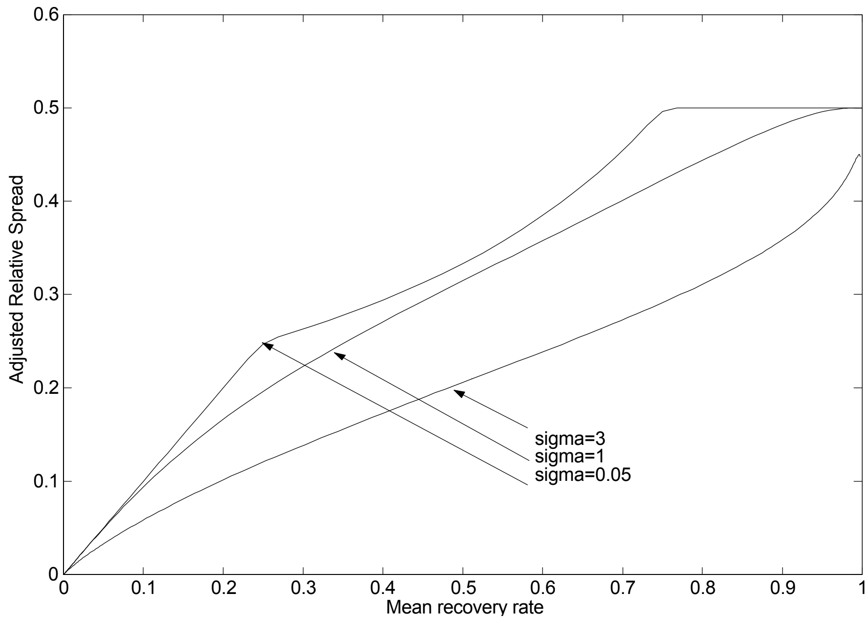

Figure 3: **Adjusted relative spread**(ARS) **sensitivity to payout volatility** σ ARS is the product of relative spread and senior debt ratio. Relative spread is defined as the price difference between senior and junior bond divided by the price difference between default-free bond and junior bond. For all three curves $\theta = 0.5$, $\lambda = 0.5$ and $p_S = 0.5$.

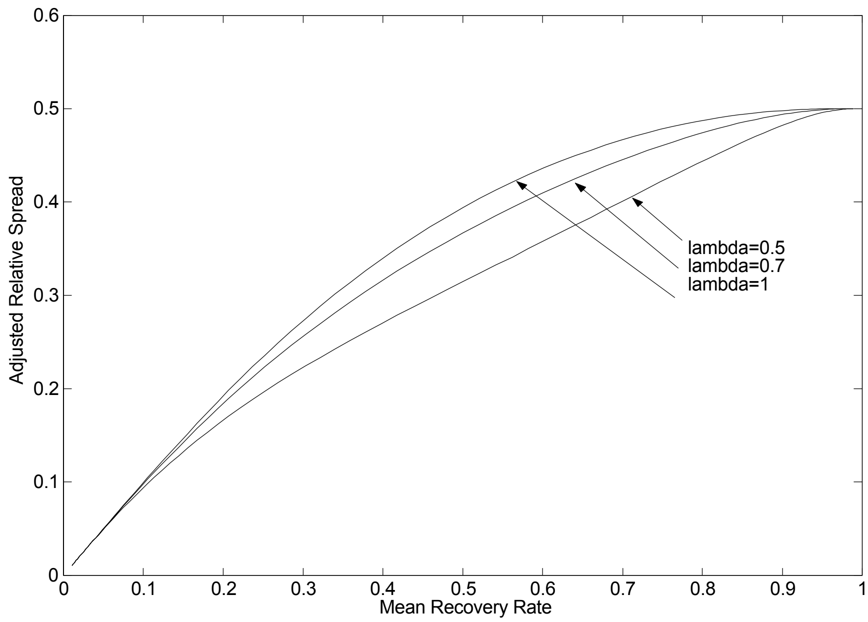

Figure 4: **Adjusted relative spread** (ARS) **sensitivity to APR violation level,** λ ARS is the product of relative spread ans senior debt ratio. Relative spread is defined as the price difference between senior and junior bond divided by the price difference between default-free bond and junior bond.For all three curves θ=0.5, p_S=0.5, σ=1.

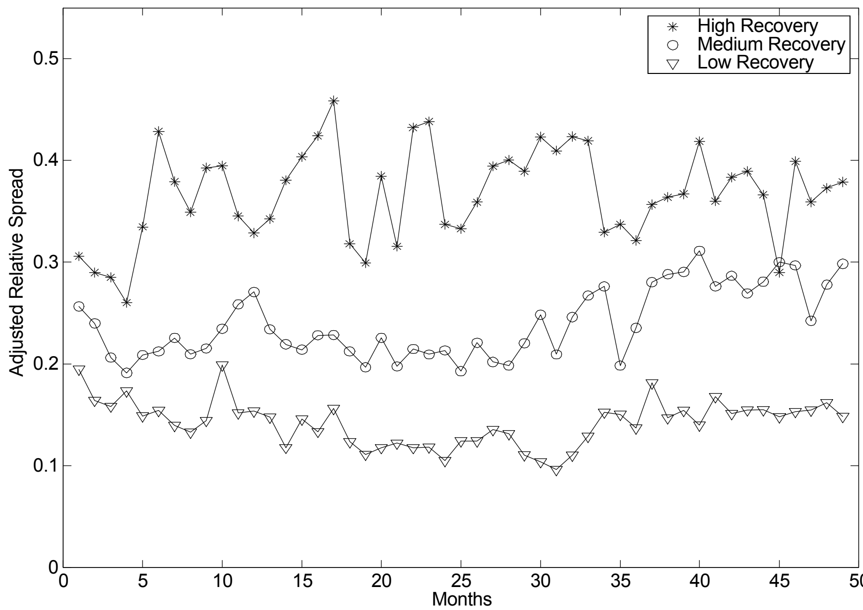

Figure 5: **Time series behavior of adjusted relative spread**(ARS) This figure plots the time series graphs of ARS for High, Medium and Low recovery groups from 93/12 to 97/12. ARS is the product of relative spread ans senior debt ratio. Relative spread is defined as the price difference between senior and junior bond divided by the price difference between default-free bond and junior bond. On the horizontal axis 93/12 (97/12) is labeled as the 1st (49th) month. Industries where Altman and Kishore recovery estimates exceed 45% are defined as High recovery group, industries with recovery rates below 35% constitute the Low recovery group. Curves are obtained by averaging ARS statistics across the firms in each recovery group.