Relationship Banking, Loan Specialization and Competition*

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Abstract

While competition constrains the ability of banks to extract informational rents from lending relationships, their informational monopoly also curtails competition through the threat of adverse selection. To analyze an intermediary's optimal strategic response to these opposing effects we specify a model where the severity of asymmetric information between banks and borrowers increases with informational distance. Intermediaries acquire expertise in a specific sector and exert effort in building lending relationship beyond their core business. They then compete with each other in transaction and relationship loan markets where they differentiate their loan offers in terms of informational location. As increased competition endogenously erodes informational rents intermediaries shift more resources to building relationships in their core markets. This retrenchment from peripheral loan segments permits banks to fend off the competitive threat to their captive market. Outside their core segment they offer transactional loans. In equilibrium, both forms of debt compete with each other but intermediaries specialize in a core market with relationship banking.

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1 Introduction

Two broad trends characterize the recent evolution of commercial banking. While the industry has consolidated both within and across economies at a rapid pace, competition has also sharply increased. Indeed, one of the most cited driving forces behind the rapid consolidation in banking are the competitive pressures exerted by products and services that are close substitutes. Investment banks, mutual funds, insurance companies, and private investment vehicles have all started to compete for the core business of commercial banks: making loans and collecting deposits. These trends beg the question of how increased competition affects the nature of financial intermediation and, in particular, the relationship between banks and borrowers. Although the emerging financial conglomerates have become more distant from their customers in their pursuit of economies of scale or scope, intermediaries also have an incentive to seek closer ties with borrowers to fend off the competition. In this paper we investigate how competition and informational asymmetries interact to shape financial intermediation and loan markets.

Financial intermediaries arise from the need to overcome the consequences of informational asymmetries between lenders and borrowers.[1] In their attempt to do so, they specialize in information production that allows them to appropriate part or all of the gains from informed intermediation.[2] Furthermore, such information about borrower quality is often relation-specific so that certain banks enjoy an information-induced competitive advantage. While competition for borrowers tends to erode informational rents and relationship value, it is hindered by an adverse selection problem, which is at the root of the monopolistic nature of financial intermediation. In the sequel, we seek to clarify how banks respond to increasing competitive pressures in terms of relationship building, what lending strategies they follow, how they differentiate markets according to lending modes,[3] and how they allocate investments between core and peripheral markets.

Our analysis starts from the observation that the importance of informational asymmetries between banks and borrowers depends on the nature of the lending relationship and the bank's expertise in the collection and processing of information. In our model, banks compete for borrowers

[1] Diamond (1984), Ramakrishnan and Thakor (1984) and Allen (1990) emphasize different aspects of monitoring and screening while Diamond and Dybvig (1983) focus on intermediation as liquidity transformation.

[2] Rajan (1992), Sharpe (1990) or von Thadden (1994) focus on the informational advantages to the inside bank, while Boot and Thakor (1999) analyze the benefits of the relationship as they accrue to the borrower.

[3] By lending mode, we mean whether banks choose to lend on a relationship or arm's length basis.

in a three stage process. Upon entering a loan market intermediaries invest in relationship building technology that generates borrower specific information. They then decide whether to build a lending relationship with borrowers who are located around them. In order to capture the varying degrees of informational expertise and relationship building present in modern banking we take the information generation success to be a function of the informational distance between bank and borrower. The closer a borrower is located to a bank in informational distance the more informative the relationship becomes and the better a bank can assess a borrower's credit worthiness. Given the location-dependent screening and relationship building choice, banks offer a particular type of loan and interest rate to borrowers who then choose the bank with the best quote.

We find that banks to engage both in transactional and relationship lending. As relationship rents attract more intermediaries into the industry, increased competition translates into more direct competition between transactional and relationship lending. In a financial sector consisting only of a few intermediaries, a bank faces three distinct loan market segments: a purely transactional market where it competes with all other banks at arm's length, a captive market in which it builds lending relationships with informationally captured borrowers and a contested market where it competes with its nearest neighbors. Entry shrinks the purely transactional segment so that, in the end, only the local market between a bank and its nearest competitor remain. In this market each bank informationally captures some of the borrowers and faces transactional competition from its nearest competitors. To fend off competitive threats to its relationship rents banks shift resources from peripheral markets to their core segment.

In contrast to traditional models of financial markets on the circle à la Salop (1979),[4] the locational differentiation impacts the bank (supplier) through information decay and not its customers (borrowers) through transportation costs. This approach allows us to cast varying degrees of lending expertise and sector specialization in terms of differentiated asymmetric information.[5] As a consequence, the strength of a relationship depends on the quality of a bank's information about a borrower's credit worthiness. There is no reason to assume that banks have equal access to information *ex ante*, so that information differentiation captures the degree of specialization in relationship building stressed, among others, by Boot and Thakor (1999). Our main contribution is to show how information dispersion within the banking industry shapes the nature of competition and financial intermediation. As banks move away from their core markets, competition from transactional lending and increasing informational asymmetries erode their specialized lending expertise. Hence, they have an incentive to retrench from peripheral markets and concentrate their relationship building effort in their principal market segment as competition grows.

Our analysis complements the results of Boot and Thakor (1999), who study the incidence of

[4]See, for instance, Chiappori, Perez-Castrillo, and Verdier (1995).

[5]Almazan (1999) analyzes a related model in which a bank's expertise for monitoring a loan is a decreasing function of the distance between borrower and bank. His focus, however, is on the role of bank capital as a way of providing incentives to monitor, and not on the generation of rents through information or on the organization of the industry.

increased competition on a bank's choice between different modes of lending and specialization. However, in our framework we explicitly derive relationship rents from informational asymmetries and show how the latter's differentiated nature leads to entry, competition, loan market segmentation, and rent erosion. These results contrast with Dell'Ariccia, Friedman and Marquez (1999) where adverse selection in lending actually blocks entry. Similar in spirit to our work, Sharpe (1990) studies borrowing under adverse selection, and shows that if relationship building generates inside information, competition in the refinancing stage will be constrained. Rajan (1992) emphasizes the disadvantages of informed (relationship) vs. uninformed ("arm's length") debt, but does not make clear how the distribution of information affects loan market equilibria and competition. Broecker (1990) and Riordan (1993) both analyze the effects of competition on loan markets under independent loan screening, and study market equilibria but do not consider the choice between different lending forms nor the incentives to invest in relationship building.

The paper is organized as follows. The next section presents a formal model of financial intermediation, information acquisition, and relationship building in the context of a locationally differentiated market. Section 3 derives the loan market equilibria in the local lending market between two banks. Next, we characterize relationship building and lending in such local market. Section 5 analyzes the choice of informational investment, loan specialization and entry in the intermediation game's overall equilibrium. Section 5.2 discusses implications of the results. Proofs are mostly relegated to the Appendix.

2 A Model of Locationally Differentiated Credit Assessment

Banking relationships take time to build and involve costly effort, especially on the bank's side. Typically, banks will make a first loan that serves as a loss leader in order to learn more about the borrower and better assess the potential for future lending. Over time, a mutual commitment to the business relationship develops and both parties accumulate private information about the other. The ensuing information duopoly benefits a bank by allowing it to extract information rents from its borrowers. In building a lending relationship banks incur costs that stem from credit analysis, refinancing (corporate rescues) and write-downs on bad loans. Hence, a convenient way to capture the informational aspects of relationship banking is to cast the analysis in terms of costly credit assessment. At the same time, the required up-front investment in intermediation technology and competitive pressures force banks to specialize in particular loan market segments. To model a bank's sector specialization we assume that the quality of its private information decays with informational distance to borrowers.

Specifically, let there be a continuum of borrowers uniformly located on a circle with circumference 1. Each potential borrower has an investment project that requires an initial outlay of \$1 and generates a terminal cash flow ξ. This cash flow can be an amount R with probability p_θ and 0 with probability $1 - p_\theta$, where $\theta \in \{l, h\}$ denotes the firm's type. We assume that the success probability for the firm with the better investment opportunity is higher: $p_h > p_l$. Final cash flows are observable and contractible, but borrower type θ is unknown to either borrower or lender.[6] The likelihood of finding a good firm h is q and this distribution of borrower types is common knowledge. We also assume that borrowers have no private resources, and that $p_l R < 1 < p_h R$, so that it is efficient to finance good borrowers but not bad ones. Moreover, letting $\overline{p} \equiv qp_h + (1-q)p_l$ denote the average success probability, we assume that $\overline{p}R > 1$, so that it is ex ante efficient to grant a loan.

N banks compete for these borrowers in three stages. First, banks decide whether or not to enter the loan market and how to invest in a relationship building technology ϕ that generates borrower-specific information. We assume that if they enter, they will locate equi-distantly around the circle. Relationship banking requires costly investments along two dimensions. Banks invest in a core competency I, i.e., by acquiring expertise in a geographic market, financial product, or borrower group, and exert an effort α to extend this expertise to other market segments (α can be thought of as the transferability of the bank's expertise). Lending relationships lead to better credit assessments that provide banks with an informative signal about a borrower's type. However, credit assessments are not perfect and depend on the distance of the borrower to the screening bank, denoted by x. In particular, the banking relationship yields a signal $\eta \in \{l, h\}$ whose quality depends on the informational distance x. The following distributional assumptions capture the idea of locationally differentiated asymmetric information:[7]

$$\begin{aligned} P^x \{\eta = h \,|\theta = h\} &= \phi(x) = P^x \{\eta = l \,|\theta = l\} \\ P^x \{\eta = h \,|\theta = l\} &= 1 - \phi(x) = P^x \{\eta = l \,|\theta = h\} \end{aligned} \tag{1}$$

where the probability of successful screening $\phi(x)$ decreases with distance, but increases in both the investment in core competency, I, and in the effort to transfer this expertise to other market segments, α. We also assume that relationship banking is informative so that $\phi(x) \geq \frac{1}{2}$.

The preceding specification captures the idea that banks enjoy an informational advantage in the market segments in which they specialize. The more they move outside their core competencies and the less effort they exert in extending their core franchise to new markets, the more severe become the information problems that they face. The investment in this screening or relationship-building technology is, however, costly. We take credit assessments with success rate $\phi(x)$ to require an initial outlay of $K(\alpha, I)$ which is increasing and convex in the expertise investment I and transfer effort α:

$$K(\alpha, I) = \frac{\alpha^2}{2} - (\ln(1-I) + I), \alpha \geq 0, I \in (0,1). \tag{2}$$

[6] Alternatively, we could assume that there are no self-selection or sorting devices such as collateral available because, e.g., the borrower is wealth-constrained and the project can not serve as collateral.

[7] All probabilities are a function of informational distance x which we suppress in the interest of notational clarity in the sequel.

Second, banks decide whether or not to engage in a relationship building effort with an applicant borrower. The screening technology[8] ϕ underlying the lending relationship allows banks to assess the borrower's type at cost c. For concreteness, suppose that for $\alpha \geq 0$ and $I \in (0,1)$, banking relationships produce information on our circle of unit circumference with location-dependent success

$$\phi(x) = \frac{1}{2} + \frac{I}{2} - \frac{x}{\alpha},\ x \in \left[0, \frac{\alpha I}{2}\right] \tag{3}$$

regardless of borrower type θ. Note that both I, the degree of specialization, and α, the inverse of the decay rate of the credit worthiness signal, are an entering bank's choice variable and that all probabilities are well defined.

If more than one bank try to establish a lending relationship with the same borrower, we assume that the banks located farther away can only gather information about the most informed bank's borrower-specific knowledge. Consider a borrower located closer to bank n who is also approached by bank $n+1$ at the screening and relationship building stage. In this case, bank $n+1$, located further away, only observes a noisy signal on the outcome of bank's n lending relationship with the borrower. This assumption captures the fact that lending relationships develop over time and are observable to outsiders. The bank that first established a business relationship not only has the most expertise in this market segment (closest in informational distance) but also can use the relationship to fend off competition by other banks for relationship lending. As a result, it will know at least as much as the less informed bank, which is precisely our noisy signal assumption.[9]

Finally, conditional on entry and the screening results (if the bank extracted information), banks compete in the third stage by simultaneously making interest rate offers. These offers can depend on the informational location of the borrower (x). Borrowers choose last by obtaining a loan from the bank quoting the lowest rate. Figure 1 summarizes the time structure of the intermediation game.

Note that our specification allows us to think of bank lending as either relationship-driven (the bank has borrower specific information) or transaction-driven (lending without screening). In our model costly screening serves as a metaphor for the time, effort and resources that it takes to build a relationship with a customer, and for the losses that a bank might incur during this period. We could equally well assume that banks first make a loan to a borrower, and learn some information about that borrower in the course of granting and maintaining the loan.[10] The focus would then

[8]The need for banks in this framework arises from their ownership of the screening technology that motivates delegating this tasks to specialized intermediaries.

[9]Specifically, we assume that the information collected by the closer bank is a sufficient statistic for any signals observed by banks further away. We make this assumption purely for tractability, so that we can model competition for borrowers as a simple auction with one informed bidder and $N-1$ uninformed bidders, and not concern ourselves with matters of information aggregation for now. Note that our specification implies that the aggregate amount of information about a borrower remains constant as a function of the number of screening banks.

[10]Our specification is completely analogous to standard relationship banking models such as that of Sharpe (1990).

Stage 1	Stage 2	Stage 3
Entry	*Screening*	*Lending Competition*
Banks decide to enter market	Borrowers apply for loans	Banks compete for borrowers
Locate equidistantly	Banks decide to screen	Borrowers choose banks and invest
Screening investment $K(\phi)$	Screening results: η	Contracts settled

Figure 1: Banking under locationally differentiated asymmetric information

be on the competition for borrowers once (some) relationships have already been established. In this context, screening permits a bank to insure a borrower against random shocks to profitability, so that even borrowers who have had bad outcomes in the past may continue to be financed if they are known (by their inside bank) to have positive NPV projects.[11] While there may be other aspects to long-term lending relationships, our focus is consistent with that of much of the recent literature (e.g. Sharpe (1990), Rajan (1992), or von Thadden (1998)).

Our specification is quite different from the usual treatment of competition with differentiated products, e.g., Salop (1979). In particular, such models assume that consumers (borrowers) have preferences for some suppliers (banks) over others, so would naturally prefer to purchase (borrow) from one particular source, all things equal.[12] Here, we place no such restriction on borrowers' preferences, and merely use the circle to model banks' expertise in evaluating some borrowers better than others. In particular, products are not differentiated *per se* so that borrowers have to choose the one that is best for them. Instead, banks differentiate the loans themselves so that, by investing in specialization, they add value to a lending relationship and generate information rents.

3 Lending Competition

As a preliminary step, we derive a potential borrower's success probability in light of the bank's credit assessment. By Bayes' rule, the probability of a project being of high or low quality given a credit assessment of $\eta = h$ or $\eta = l$ is

$$P\{\theta = h|\eta = h\} = \frac{\phi(x)q}{\phi(x)q + (1-\phi(x))(1-q)} \quad \equiv \; H(x)$$

Instead of loan screening, we could let banks initially make uninformed loan offers whose outcomes lead to an informed one in the second period. The screening cost is now simply the expected cost of bad first period loans.

[11]It is in this sense that relationships (screening) are good for both borrowers and banks. To the extent that this insurance is valuable to a borrower, relationship lending can be of value to a borrower even if it comes at the price of being somewhat locked in to their lending bank.

[12]This outcome usually results from imposing a transportation cost which is proportional to the distance between consumer and supplier. Since the cost of transportation is increasing in the distance between the consumer and the supplier, consumers have a tendency to purchase from suppliers that are nearer.

$$P\{\theta = l|\eta = l\} = \frac{\phi(x)(1-q)}{\phi(x)(1-q) + (1-\phi(x))q} \equiv L(x)$$

Note that location dependent screening success implies $H_x \equiv \frac{\partial}{\partial x}H(x) < 0$ and $L_x \equiv \frac{\partial}{\partial x}L(x) < 0$: the posterior distributions on borrower type θ deteriorate in informational distance. We obtain the project's success probability conditional on a pre-existing lending relationship and borrower location $p(\eta; x)$ as

$$p(h;x) \equiv P\{\xi = R\,|\eta = h\} = H(x)p_h + (1-H(x))p_l \in [\overline{p}, p_h]$$
$$p(l;x) \equiv P\{\xi = R\,|\eta = l\} = (1-L(x))p_h + L(x)p_l \in [p_l, \overline{p}]$$

for credit assessment outcomes $\eta = h, l$. Note that $p_x(h;x) \equiv \frac{\partial}{\partial x}p(h;x) = (p_h - p_l)H_x < 0$, since $H_x < 0$: the probability of the project being successful after a positive credit assessment ($\eta = h$) decreases for borrowers located further away. Similarly, $p_x(l;x) \equiv \frac{\partial}{\partial x}p(l;x) = (p_l - p_h)L_x > 0$ since $L_x < 0$. In other words, the probability of the project being successful after observing a negative signal from the lending relationship increases with the distance between bank and borrower. As the signal becomes less informative a bank should be less likely to believe that the borrower is, in fact, of low quality.

We now characterize the equilibrium in the lending game where informed banks compete for each others' customers. Solving the game by backward induction, we start with its last stage. The appropriate equilibrium concept is Perfect Bayesian Nash Equilibrium (PBE). Conditional on having entered the industry, each bank competes for borrowers with all competitor banks. However, the equilibrium can be fully characterized by just assuming that each bank competes only with its two nearest neighbors on either side, which is a standard feature of this class of models.[13] Hence, it suffices to study the competition for borrowers in the last stage for the case of two adjacent banks, n and $n+1$, competing for borrowers located between them. By symmetry, both banks will be informed about some borrowers and uninformed about others so that we arbitrarily label one intermediary i for informed and the other one u for uninformed. Note that the informed bank becomes the relationship bank, and the uninformed bank acts as a transaction lender.

As has been demonstrated in similar contexts (see, e.g., Broecker (1990) or von Thadden (1998)), the interest rate game between two neighboring banks does not have an equilibrium in pure strategies when one bank has superior information. However, there exists a unique equilibrium in mixed strategies which we now characterize in terms of banks' distribution functions over interest rate offers. Let $F_i(r, \eta; x)$ represent the bidding distribution by an informed bank for borrowers located at a distance x, conditional on the loan screening outcome η. Similarly, let $F_u(r; y)$ represent the bidding distribution of an uninformed bank for borrowers located at a distance y. Also, we define $\pi_i^*(\eta, x)$ as the equilibrium expected profit for an informed bank, and $\pi_u^*(y)$ as that for an

[13] For further details see, e.g., Engelbrecht-Wiggans et al. (1983).

uninformed bank.

Proposition 1 (Unique Lending Equilibrium) *The bidding game for a particular borrower located a distance x from bank i and $y = 1/N - x$ from bank u has a unique, mixed-strategy equilibrium, characterized by continuous and strictly increasing distribution functions $F_i(r, \eta; x)$, $F_u(r; y)$, such that:*

1. *the uninformed bank engaging in transactional lending breaks even ($\pi_u^*(y) = 0$);*

2. *relationship banking allows the informed bank to earn positive expected profits $\pi_i^*(h, x) > 0$ on borrowers with $\eta = h$, while it breaks even on borrowers with $\eta = l$, i.e., $\pi_i^*(l, x) = 0$.*

Proof. See Appendix. ■

The lending game has an outcome very reminiscent of Bertrand competition. Since, by definition, the uninformed transactional lender has no private information about the borrower, it is unable to obtain any rents from the loans it grants. The informed bank, however, is able to use its informational advantage over competitors to extract relationship rents on high quality borrowers (or those with a high signal). A relationship bank's ability to distinguish good from bad risks allows it to adjust its bidding and lending strategy accordingly, and subjects less informed transactional lenders to problems of adverse selection.

From the result above, we obtain an explicit characterization of the location dependent distribution functions over interest rates. For this purpose, define $\tilde{x}$ as the distance x such that $p(l; x)R = 1$.[14] It is also useful to define $r_{\overline{p}}$ as the break-even rate on an average borrower, i.e. $r_{\overline{p}} = 1/\overline{p}$.

Corollary 1 (Loan Rate Distributions) *The informed and uninformed bank make randomized loan rate offers over $[r_{\overline{p}}, R \wedge r_l(x))$ according to*

$$\begin{aligned} F_i(r, h; x) &= \frac{p(h; x) - p(l; x)}{\overline{p} - p(l; x)} \frac{\overline{p}r - 1}{p(h; x) r - 1} \\ F_u\left(r; N^{-1} - x\right) &= \frac{p(h; x)}{\overline{p}} \frac{\overline{p}r - 1}{p(h; x) r - 1} \end{aligned}$$

For values of x such that $x < \tilde{x}$, the informed bank denies credit to all $\eta = l$ borrowers, while the uninformed bank only bids with probability $\beta\left(N^{-1} - x\right) = 1 - \frac{p(h;x)\overline{p}^{-1} - 1}{p(h;x)R - 1}$. For values of x such that

[14] Since it is inefficient to lend to low quality projects it might appear that a bank should never lend to a borrower who failed the credit worthiness test ($\eta = l$). However, since the success probability conditional on a negative credit assessment is increasing in informational distance, i.e., $\frac{\partial}{\partial x} p(l; x) > 0$, there exists a location $\tilde{x}$ such that $p(l; x)R < 1$ for $x < \tilde{x}$ and $p(l; x)R > 1$ for $x > \tilde{x}$. In other words, for borrowers sufficiently far away, observing a negative credit quality signal through the lending relationship is not much more informative than not carrying out a credit assessment. Hence, a bank should still be willing to continue the business relationship and lend to such borrowers.

$x > \tilde{x}$, the informed bank offers credit to all $\eta = l$ borrowers by offering a rate of $r_l(x) = \frac{1}{p(l;x)}$, while the uninformed bank bids with probability 1 but places a positive mass on the upper bound of the support, $\overline{r} = r_l(x)$.

Proof. See the Appendix. ■

To provide some intuition for the location dependence of loan granting policies consider the simple case of two banks, i.e., $N = 2$, specialization parameter $I = 1$, and information decaying at rate $\alpha = 1/2$, so that the signal has zero information content at $x = 1/4$ ($\phi\left(\frac{1}{4}\right) = \frac{1}{2}$), which is exactly the midpoint between two banks located at opposite extremes of the circle of circumference 1. For values of x near zero, the probability that the uninformed bank bids at all is quite low, and the informed bank obtains high profits on these borrowers. As x increases, the probability of bidding by the uninformed bank increases, since the adverse selection problem it faces is decreased. In the limit, as x approaches 1/4, the distribution functions for both the informed and the uninformed bank should concentrate all mass at $r = r_{\overline{p}} = 1/\overline{p}$: $\lim_{x \to \frac{1}{4}} F_u(r;x) = \lim_{x \to \frac{1}{4}} F_i(r,h;x) = 1$ for all $r \geq r_{\overline{p}}$. In essence, at $x = \frac{1}{4}$, neither lender has any information and so compete in a symmetric way, driving all profits to zero, which is the Bertrand outcome.

Before analyzing the issues of relationship building and entry, it is useful to derive some comparative statics results on the market equilibrium. We first define a measure of the importance of relationship building and credit assessments, $\Delta_p \equiv p_h - p_l$, which represents the degree of borrower heterogeneity.

Corollary 2 (Profit Characterization) *For the equilibrium of the intermediation game's lending stage,*

1. *the profits to the informed bank on borrowers with signal $\eta = h$ are decreasing in the bank-borrower distance x: $\frac{\partial}{\partial x}\pi_i^*(h,x) < 0$;*
2. *the profits to the informed bank on borrowers with signal $\eta = h$ are increasing in Δ_p: $\frac{\partial}{\partial \Delta_p}\pi_i^*(h,x) > 0$.*

Proof. See the Appendix. ■

The first part of the proposition says that an informed bank makes lower profits on those borrowers that are farther away (in information space). The intuition for this is simply that as we move further away from a bank, the quality of information the bank has about that borrower decreases, so that its rents from information should decrease as well. The second result concerns the importance of screening by focusing on the degree of borrower heterogeneity. As borrowers become more heterogeneous, an informed bank earns higher profits on those borrowers with good signals, even if the average success probability remains constant.[15] Behind this part of the proposition lies the adverse selection problem faced by the uninformed bank, as screening becomes more important the more different are the borrowers.

4 Local Market Equilibrium

Having characterized the competition for borrowers between an informed and an uninformed bank, we now turn to the banks' decision to build a relationship with borrowers. Since, by symmetry, every bank competes both in relationship and transactional lending, we denote a given intermediary by $n \leq N$. We first verify that, in equilibrium, borrowers are not screened by multiple banks. It then follows that banks build banking relationships with borrowers on either side of their location up to a point $\hat{x}$ where expected profits from lending to that borrower equal the screening cost c (as long as this does not lead to multiple screening):

$$E\left[\pi_n^*\left(\eta,\hat{x}\right)\right] = \gamma\left(\hat{x}\right)\pi_n^*\left(h;\hat{x}\right) + \left(1-\gamma\left(\hat{x}\right)\right)\pi_n^*\left(l;\hat{x}\right) = c$$

where $\gamma\left(x\right) = P\left\{\eta = h\right\}$ is the probability of finding a borrower with a signal of high quality.

Proposition 2 (Screening Range) *Banks screen borrowers up to maximal informational distance $\hat{x}_n$ where for $A \equiv \frac{1}{p}\Delta_p q(1-q)$*

$$\hat{x}_n = \min\left\{\frac{1}{2N}, \frac{\alpha}{2}\left[I - \frac{c}{A}\right]\right\}$$

Proof. See the Appendix. ■

This proposition justifies our analysis in the previous section of one informed bank competing against a number of uninformed banks. If more than one bank had screened a borrower, competition among these banks would drive profits to zero for all but the most informed bank. Only the bank closest in informational space could possibly recoup its screening cost c, so that it would never pay to screen a borrower that is located closer to a competitor bank. The restriction that $\hat{x}$ be no greater than $\frac{1}{2N}$ is then a direct consequence of the preceding: in any symmetric equilibrium with $\hat{x}_n > \frac{1}{2N}$ there would be an interval of borrowers that are screened by both banks, so that the bank located further away would always fail to recoup its cost of screening c. Note that a bank screens more borrowers ($\hat{x}$ is greater) the larger its expertise in its core market (I), the more transferable its skill (α) and the lower the marginal screening cost c. Also, screening becomes

[15] One might suspect that this result is driven by the fact that a change in Δ_p constitutes a change in p_h, so that borrowers with a high signal should also be considered to be qualitatively better borrowers. However, a similar result is obtained by keeping p_h fixed and lowering only p_l, so that it is clear that it is exactly the severity of the information problem that drives the result.

more important as borrowers are more dissimilar. As Δ_p increases and borrower characteristics become less homogeneous, banks screen further out since creditworthiness assessments become more important. This critical informational location $\hat{x}$ then determines the size of each bank's relationship lending market.

It emerges that banks split up our informational circle into a number of captive segments where at most one bank engages in relationship lending. In particular, symmetric equilibrium with $\hat{x}_n < \frac{1}{2N}$ would imply that between any two banks there is a set of borrowers that are not screened by either bank. For these borrowers we would expect to observe a form of symmetric loan price (interest rate) competition among all banks. For all other borrowers - those less than a distance of $\hat{x}$ from some bank - competition among potential lenders will always be characterized as in the previous section, with one informed relationship bank and one, uninformed, transactional lender.

Proposition 3 (Relationship Profits) *Ex-ante expected profits to a bank engaging in relationship building (gross of the screening cost c) are:*

1. *decreasing in x:* $\frac{\partial}{\partial x} E\left[\pi_n^*(\eta, x)\right] < 0$
2. *increasing in* Δ_p*:* $\frac{\partial}{\partial \Delta_p} E\left[\pi_n^*(\eta, x)\right] > 0$.

Proof. See the Appendix. ■

The proposition asserts that, before the signal is observed, expected profits are decreasing in the distance of the borrower to the bank (market specific lending expertise), but increasing in our measure of the information problem. We should therefore expect that the incentive to acquire information is lower for borrowers that are far away, but should increase as borrowers become more heterogeneous.

Proposition 4 (Loan Specialization) *The local banking market between two banks n and n + 1 is characterized by:*

1. $\frac{\partial F_i}{\partial x} > 0$*: an informed bank's bid is (stochastically) decreasing in the distance from bank to borrower;*
2. $\frac{\partial F_u}{\partial y} < 0$*: conditional on bidding, an uninformed bank's bid is (stochastically) increasing in the distance from bank to borrower.*

Proof. See the Appendix. ■

These results imply that the expected interest rate offered in both relationship and transactional lending is a decreasing function of x (where $y = N^{-1} - x$), the distance between informed bank and borrower. Relationship banks tend to be well informed about borrowers located nearby and less informed about borrowers far away. Hence, the adverse selection problem in transactional lending is greater for an uninformed bank the closer a borrower is located to a competitor bank. As the distance between the borrower and the informed bank increases, the informed bank's information advantage decreases, bringing both competitors closer to a situation of symmetric Bertrand competition. With diminishing adverse selection the uninformed bank is able to bid more aggressively. In the limit, as the informed bank's information advantage goes to zero, the expected interest rate for both banks collapses to the zero-information break-even rate, $r_{\overline{p}}$.

Corollary 3 (Competitive Intensity) *The probability of an uninformed bid decreases in distance from bank to borrower as* $\frac{\partial \beta}{\partial y} < 0$.

Proof. Since $\beta(y) = F_u(R; y)$ by Corollary 1 the preceding proposition establishes the result. ■

Once again, adverse selection is the driving force behind this result. The closer one gets to the market in which the informed bank specializes through relationship building, the more severe the adverse selection problem faced by the uninformed bank becomes. Hence, the latter needs to be careful in its transactional loan offers and will refrain more frequently from bidding for customers with established lending relationships. In fact, one can interpret the uninformed bank's bidding probability as an indicator of the local market's competitiveness, which increases in the informational distance between borrower and relationship bank. The closer borrowers are to the informed bank, the less likely they are to receive a transactional loan offer. Hence, even with increased competition banks still have an incentive to invest in relationship lending to avoid loosing high quality customers to arm's length debt, a topic we turn to next.

5 Entry and Loan Specialization

After characterizing lending and relationship building we now turn to a bank's decision to enter the loan market, its investment in screening technology and its strategic focus. Recall that the success in extracting information from lending relationships depends on banks' investments in core competencies I and their willingness to conquer a captive market as reflected in ex ante effort α. The precise mix of specialized expertise and transferability of skill determines a bank's strategy in the relationship lending market and its response to increased competition. First, we analyze the investment and intermediation strategy decision for a given number of active banks. We then investigate the free entry case where banks optimally allocate a fixed investment budget between specialized expertise and relationship building effort.

To determine the equilibrium investment K_n and its optimal allocation between specialized expertise I and effort in relationship building α, we calculate the total profits for bank n upon entering. Total gross profits for bank n, summed across all screened borrowers can be expressed for $A \equiv \frac{1}{p}\Delta_p q(1-q)$ as

$$\Pi_n = 2\int_0^{\hat{x}_n} \left(E[\pi_n(\eta, x)] - c\right) dx = 2\int_0^{\hat{x}_n} \left[A(2\phi(x) - 1) - c\right] dx.$$

Given the number of active banks N, an entering intermediary n will choose $\alpha \geq 0$ and $I \in (0,1)$ in its entry and investment decision so as to maximize net profits $V_n = \Pi_n - K_n$:

$$\max_{\alpha, I} V_n = \max_{\alpha, I} \left\{ 2\int_0^{\hat{x}_n} \left[A(2\phi(x) - 1) - c\right] dx - \left[\frac{\alpha^2}{2} - \left[\log(1-I) + I\right]\right] \right\} \quad (4)$$

Since we are only interested in the case of increasing competition, we will assume that N is sufficiently large so that $\hat{x} \leq \frac{1}{2N}$ is binding, i.e., $\hat{x} = \frac{1}{2N}$. This will allow us to look at how changes in N affect a bank's relationship building strategy.

Proposition 5 (Investment and Competition) *For a fixed value of N, the profit maximizing relationship building effort α and expertise investment I are given by:*

$$\alpha_n^* = \left[\frac{A}{4N^2}\right]^{\frac{1}{3}}$$

$$I_n^* = \frac{A}{A+N}$$

Proof. See the Appendix. ■

Once banks screen up to $(2N)^{-1}$, the entire market is "covered" by at least one bank, so that increasing the number of banks would merely shrink the share of the market screened by each bank. However, note that for this latter case a clear relationship is established between the number of banks and the choice of relationship building effort: as N increases, banks choose a lower value of α. The same is true for I : as more banks are active it becomes harder to sustain investments in specialized lending expertise. Put differently, as competition increases banks invest less in the screening technology, since K will also be lower.

One interpretation of this outcome is that, while banks may very well continue to screen all borrowers and, therefore, establish relationships with them, the resulting relationship lending resembles more and more transaction or arm's-length debt. The increased competition that banks face, while forcing them to try to maintain a hold over the largest set of borrowers possible, also decreases their incentive to invest in information acquisition. Perhaps as importantly, it follows that the aggregate amount of information in the economy decreases as the number of active intermediaries N increases, leading to more inefficient lending decisions.[16] However, as the importance

[16]Broecker (1990) illustrates a similar result using a model of independent but symmetric creditworthiness tests, and shows that as the number of banks increases, the number of bad borrowers obtaining loans increases as well. In

of the information asymmetry problem increases (Δ_p and A), banks choose a higher values of α and I, consequently investing more in the screening technology. In other words, as the risk of lending to a poor quality borrower increases, banks invest more in the screening technology, which helps them avoid making inefficient lending decisions.

5.1 Free Entry Equilibrium

In order to consider the free entry equilibrium, we first fix the relationship lending technology expenditure $K = \bar{K}$. An important part of the analysis is to investigate banks' investment strategies in response to entry into the relationship lending market. As more and more banks become active each bank's captive relationship lending market shrinks while its transactional market segment grows. The question arises how banks optimally re-allocate resources between investments in specialized expertise I and transferable skills (effort) α as competitive pressures grow.

Proposition 6 (Investment Allocation) *For fixed technology expenditure $\bar{K}$, banks increase investment in specialized expertise ($\frac{\partial I^*}{\partial N} > 0$) and cut back on non-segment specific effort ($\frac{\partial \alpha^*}{\partial N} < 0$) as the number of banks increases.*

Proof. See the Appendix. ■

A bank's upfront investment determines its scope in terms of its relationship banking market segment. The more it allocates to general relationship building effort α the more breadth it acquires in terms of lending activities. At the same time, the bank starts to venture out of its core markets and finds itself in increasing competition with banks in neighboring market segments with potentially superior expertise in those markets. As more intermediaries crowd the markets, banks are less able to appropriate the informational gains from lending relationships with borrowers outside their core expertise. Hence, they cut back on their overall scope, reducing their investment in general transferable skills or effort α. The resources freed up in this retrenchment are now invested in their core competencies, so that segment specific expertise expenditure I increases. In other words, as a result of growing competition banks become more specialized in their core markets at the expense of breadth. While each bank's captive market shrinks with entry the remaining lending relationships become more valuable to the bank. The natural outcome is then for each bank to further specialize in its core relationships in order to extract higher informational rents from these borrowers.[17]

his model, this occurs because, with N independent but noisy tests, the probability that any given borrower passes at least one test increases with the number of banks. Shaffer (1997) provides some evidence that is consistent with models of this kind: as the number of banks in a market increases, each bank's provision for loan losses increases. However, Broecker's model differs from ours in a crucial way. In Broecker, the aggregate amount of information *increases* as the number of banks increases, even if the inference problem for each bank gets worse. In our model it is not an increased winner's curse that leads to our result, but rather a reduced incentive to screen.

[17]Note that despite the increase rent extraction, this need not necessarily be bad for borrowers. As explained previously, this view of relationship lending adds an element of insurance for the borrower, and so has some value to

Given a fixed lending technology expenditure $\bar{K}$ we can now determine how the free entry number of banks depends on characteristics of the market.

Proposition 7 (Optimal Entry) *For fixed technology expenditure $\bar{K}$, the free entry number of banks, N^*, is increasing in the degree of borrower heterogeneity, Δ_p, and decreasing in the cost of monitoring, c.*

Proof. See the Appendix. ■

The free entry equilibrium number of banks is a function of both the choice of the information acquisition technology (I, α) as well as the cost of screening, c, and the characteristics of the borrowers, Δ_p. As should be expected, N^* is lower for higher values of c: the cost of screening borrowers lowers each bank's profitability directly, and so should lead to a lower number of banks in equilibrium. The effect of borrower heterogeneity Δ_p is more subtle because it also affects the investments in specific and non-specific expertise (I, α). However, as was demonstrated earlier, greater borrower heterogeneity has a positive effect on bank profits for a fixed screening technology. Therefore, the market can support a greater number of banks when borrowers are more heterogeneous.

5.2 Discussion

Our analysis shows how informational asymmetries concerning borrower quality lead to relationship rents, whose magnitude depends crucially on the information advantage of a lender relative to its competitors. By establishing a direct link between the degree of informational asymmetries among banks and their profitability, we are able to investigate how changes in the industry structure affect a bank's incentive to invest in its core market and informationally capture borrowers. While relationship lending confers an informational advantage on intermediaries, the ensuing informational rents attract competition from two different sources. Not only do entering banks compete for each others' established customer base, but relationship banking also finds itself under increased pressure from arm's length debt. This dual competitive threat constrains the ability of banks to extract informational rents from lending relationships. However, their informational monopoly also makes competition less effective through adverse selection hazards to competitors.

A bank's optimal strategic response to increasing competition consists in shifting more resources to its relationship lending segment in order to protect the rents obtained in that sector. Nevertheless, each bank's captive market segment shrinks so that a smaller number of borrowers becomes more captured. In equilibrium, banks offer two types of debt products. In their core market, they engage in relationship loans that are specialized in terms of borrower specific information; in all other markets they offer arm's length debt. Although we cast the analysis in terms of transactional

the borrower. Greater investment in the relationship should also lead to better insurance, even if this comes at an increased cost.

loans, they could actually comprise a much wider set of debt instruments including public debt. The key characteristic of this kind of lending is that it lacks a previous investment in information acquisition and relationship building effort.

In our model, bank profitability stems from the relationship lending market where banks are able to generate private information and extract informational rents from borrowers. For other forms of lending, competition drives rents down to zero so that banks just break even. Although banks may very well obtain rents from borrowers with which they have established a working relationship, other types of lending are little protected from competition. In particular, banks trying to poach a competitor's customers will suffer from a large adverse selection problem. Consequently, they are unable to attract a sufficiently large number of high quality borrowers so as to make positive profits.

Our results shed some light on the recent debate concerning the nature and future evolution of banking. Boot and Thakor (1999) have made the point that the changing competitive nature of the banking market can and should have an impact on banks' lending strategies. They argue that increased competition, either among banks, types of debt or from outside sources, will drive banks to invest more in relationship lending as this is the primary source of bank profits and banks are uniquely equipped to add value to borrowing firms. Our analysis highlights the role of asymmetric information and relationship rent seeking as the underlying economic forces in this process. Lending relationships are valuable to borrowers because of their implicit insurance against adverse outcomes. They are clearly also valuable for banks: borrower-specific information translates into relationship rents. Hence, banks are willing to protect these rents in the face of increased competition by specializing in a core expertise, differentiating loans in terms of the obtained information and sacrificing peripheral markets.

Growing competition has two direct effects on bank profitability. First, banks try to increase the percentage of loans they grant as relationship loans, even as each bank's overall market share shrinks. Second, while banks extract information more successfully from lending relationship, they make less effort to extend their franchise beyond their core markets. As the number of banks increases, banks retrench to their core competency by relying on relationship lending, but their relationships add less value to peripheral borrowers who provide lower rents to the bank.

This result stems from an issue that we believe is often overlooked in the literature. To the extent that most of a bank's profits come from activities where they hold a measure of market power, we would expect them to be most affected by changes in competition. Given that private information is an important determinant of bank profits, markets in which banks have no private information, e.g., transactional loan markets, should closely resemble the pure price competition ideal even with a very limited number of competitors. In these markets, increasing the number of competitors is expect to have little impact.[18] By contrast, we show that increased competition

[18]This is, once again, simply the standard Bertrand result that price competition can drive rents to zero even with just two competitors. Increasing the number of competitors has no impact on the equilibrium profits.

does have a significant impact on markets characterized by bank-borrower relationships. Banks' incentives change with the number of competitors, so that the incentive to build a relationship becomes stronger for a smaller market segment. Hence, one should not expect relationship lending to resemble more arm's length debt as competitive pressures grow.

6 Conclusion

The rapidly changing competitive landscape of financial intermediation raises questions about the industry's emerging new structure. One particular uncertainty concerns the degree to which banks will specialize in different market segments and whether they will be more or less likely to build stable business ties with their borrowers. Banking relationships revolve around a mutual commitment to engage in long-term lending. At the same time, they generate relation-specific information that permits intermediaries to informationally capture borrowers. In this analysis, we explore the consequences of increased competition between banks on banks' incentives to build lending relationships. We cast relationship lending in terms of costly credit worthiness assessments and loan related loss leaders. Intermediaries invest in core expertise and exert costly effort in transferring it to adjacent market segments. To fully take into account the importance of asymmetric information in lending decisions we incorporate an explicit model of loan specialization into our analysis in which a bank's expertise decreases for borrowers outside its core market.

As banks enter into the loan market they crowd out the purely transactional sector where all intermediaries compete on an arm's length basis. Each bank specializes in a particular market segment where it attempts to build lending relationships so as to obtain rents. At the same time, all banks continue to compete in arm's length debt outside their captive (relationship) market. Consequently, they offer both relationship and transactional loans, albeit in different market segments. Furthermore, relationship banking allows banks to differentiate their loans in terms of borrower attributes. As the quality of information extracted from lending relationships varies across borrowers, banks specialize through relationship investments to gain a competitive advantage. Informational asymmetries now exert two countervailing effects on the market equilibrium. Relationship rents attract entry, which increases competition and forces banks to concentrate their resources in a shrinking relationship lending segment. At the same time, relationship banking poses an adverse selection problem for less informed competitors. In equilibrium, banks retreat from peripheral markets in order to protect their core segment.

The benefit of a lending relationship stems from the informational advantage that it confers on a bank in fending off competition from transactional debt while its costs revolve around the informational investments it has to make. Since maintaining relationships with heterogeneous borrowers is difficult and costly, banks face retrenching decisions that lead to sector specialization with rising competitive pressures. Forming the same relationship with borrowers that are farther away from its field of expertise is more costly, so that banks keep their investment in those relationships at a minimum. Within its relationship lending market, a bank fully discriminates between its borrowers in terms of loan rates. As the quality of the information changes with borrowers that are increasingly outside its area of expertise, a bank faces increased competition from the transactional market that tempers its rent seeking and holds down loan rates. Contrary to traditional models of product differentiation, it is the bank that absorbs the cost of differentiation through investment in specialization in order to add value to the lending relationship for both parties. Hence, relationship banking can survive increased competition through specialization and loan differentiation.

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Appendix

Proof of Proposition 1: Unique Lending Equilibrium

The proof proceeds in a sequence of steps: first, we establish that there does not exist a pure equilibrium in relationship lending. Next, we show that the two competing banks offer loan rates over the same interval, then we verify that the mixed strategies are well-behaved distribution functions and, finally, we prove uniqueness by explicitly calculating the location dependent mixed strategies. Let $r_{\overline{p}} = \overline{p}^{-1}$ and $r_l(x) = p(l;x)^{-1}$.

Lemma 1 (Absence of Pure Strategy Equilibria) *There exist no pure strategy equilibria in the bidding game for borrowers between a relationship and transactional lender.*

Proof. Let pure strategies conditional on signal and borrower location be denoted by $r_i(\eta)$ and r_u for the informed and uninformed bank, respectively (we ignore the dependence on x for now). Suppose that $r_u \leq r_i(h), r_i(l)$. In order for this to be optimal for the uninformed bank, $r_u \geq \overline{p}^{-1}$. However, the informed bank could increase its profit by offering a rate $r_i(h) = r_u - \epsilon$ and lending to all $\eta = h$ borrowers. Therefore, $r_u \leq r_i(h), r_i(l)$ cannot be an equilibrium.

Suppose then that $r_i(h) \leq r_u \leq r_i(l)$. In this case, the uninformed bank only makes loans to $\eta = l$ borrowers. In order for this to be optimal for the uninformed bank, it must be that $r_u \geq p(l;x)^{-1}$. If $r_i(h) < r_u$, the informed bank would be better off charging $r_i(h) + \epsilon$. But if $r_i(h) = r_u$, the uninformed bank would be better off charging a rate $r_i(h) - \epsilon$ and lending to *all* borrowers. Therefore, $r_i(h) \leq r_u \leq r_i(l)$ cannot be an equilibrium.

Finally, suppose that $r_i(l) \leq r_u \leq r_i(h)$. At r_u, the uninformed bank lends only to $\eta = h$ borrowers and makes positive expected profits if $r_i(l)$ makes non-negative profits for the informed bank. But then, as above, if $r_u < r_i(l)$, the informed bank could increase its profits strictly by lowering its bid to $r_u - \epsilon$ and lending to all good borrowers. Therefore, this also cannot be an equilibrium, and no equilibrium exists in pure strategies. ■

Lemma 2 (Common Support) *Both banks randomize their loan offers over the same interval* $[r_{\overline{p}}, r_l(x) \wedge R)$*. Moreover, the informed bank earns positive expected profits and the uninformed one breaks even.*

Proof. Let $F_i(r, \eta; x)$ represent the bidding distribution for the informed bank for a borrower located at distance x, and $F_u(r; N^{-1} - x)$ the bidding distribution for the uninformed bank, conditional on η for loan rate offers $r \in [\underline{r}_i^{\eta}, \overline{r}_i^{\eta})$ and $r \in [\underline{r}_u, \overline{r}_u)$, respectively. The expected profits for both banks from offering a rate r are for $F(r^-) := \lim_{s \uparrow r} F(r)$

$$\pi_i(r, \eta; x) := \left(1 - F_u(r^-; N^{-1} - x)\right) \{p(\eta; x) r - 1\} \tag{5}$$

$$\begin{aligned} \pi_u\left(r; N^{-1} - x\right) := {} & P\{\eta = h\} \left(1 - F_i(r^-, h; x)\right) [p(h;x) r - 1] \\ & + P\{\eta = l\} \left(1 - F_i(r^-, l; x)\right) [p(l;x) r - 1] \end{aligned} \tag{6}$$

We first consider a borrower located at $x < \tilde{x}$ from the informed bank so that $p(l;x)R < 1$.

Claim 1 *Both banks offer loan rates* $r \in \left[\overline{p}^{-1}, R\right]$ *so that the informed bank makes positive profits on high-quality borrowers and does not offer loans to low-quality ones.*

Obviously, the informed bank never bids on a $\eta = l$ borrower for so that $F_i(r, l; x) = 0$ for all $x < \tilde{x}$ and all $r \in [\underline{r}_i^l, \overline{r}_i^l)$. The uninformed bank will never bid less than $r_{\overline{p}} = \overline{p}^{-1}$ because low quality firms switch banks at any offer so that the loan pool has a success probability of at most $\overline{p}$. But then, the informed bank will never offer rates below $r_{\overline{p}}$ to its high quality customers ($\eta = h$) making positive profits on high-quality borrowers: $\pi_i (r, h; x) > 0$. Clearly, both informed and uninformed banks will never offer (gross) rates higher than R, the project's pay-off in the successful state.

Claim 2 *$F_i (r, h; x)$ is continuous on $[\underline{r}_i^h, \overline{r}_i^h)$.*

Suppose not, i.e., there exists $s \in [\underline{r}_i^h, \overline{r}_i^h)$ such that $F_i (s^-, h; x) < F_i (s, h; x)$. Since the informed bank makes positive (expected) profits on high-quality borrowers $p (h; x) r - 1 > 0$ on $[\underline{r}_i^h, \overline{r}_i^h)$ so that $\pi_u \left(s^-; N^{-1} - x\right) > \pi_u \left(s; N^{-1} - x\right)$ by (6) and the right-continuity of $F_i (r, \eta; x), \eta = h, l$; furthermore, since $\pi_u \left(r; N^{-1} - x\right)$ is also right-continuous there exists a neighborhood $[s, s + \varepsilon], \varepsilon > 0$, say, on which F_u must be constant implying $F_u(s; N^{-1} - x) = F_u(s^-; N^{-1} - x)$. Hence, by (5) $\pi_i (r, h; x)$ is continuous at s and strictly increasing on the neighborhood; but then, $F_i (r, h; x)$ can not have any mass on $[s, s + \varepsilon]$ so that $F_i (s^-, h; x) = F_i (s, h; x)$. $\Rightarrow\Leftarrow$

Claim 3 *The uninformed bank breaks even: $\pi_u \left(r; N^{-1} - x\right) = 0$.*

Since $F_i(r, l; x) = 0$ (6) simplifies to

$$\pi_u \left(r; N^{-1} - x\right) = P \{\eta = h\} (1 - F_i(r, h; x)) [p (h; x) r - 1] + P \{\eta = l\} [p (l; x) r - 1] \tag{7}$$

which is continuous on $[\underline{r}_u, \overline{r}_u)$ by continuity of $F_i (r, h; x)$. To show that the uninformed bank earns 0 expected profits by bidding F_u suppose the contrary; but $F_i (\overline{r}_u^-, h; x) = 1 = F_i (\overline{r}_u, h; x)$ so that $\pi_u \left(\overline{r}_u^-; N^{-1} - x\right) = 0$ by continuity. $\Rightarrow\Leftarrow$

Claim 4 *The lower and upper bounds of the common support are given by $\underline{r}_u = \overline{p}^{-1} = \underline{r}_i$ and $\overline{r}_i^h = \overline{r}_u = R$, respectively.*

At $\underline{r}_u$, the uninformed bank wins almost surely, and so makes a profit of $\underline{r}_u\overline{p} - 1 = 0$, where $\overline{p} = qp_H + (1 - q)p_L$ is just the average success probability, implying $\underline{r}_u = \overline{p}^{-1}$. But then, the informed bank will never bid below $\overline{p}^{-1}$ so that $\underline{r}_u = \overline{p}^{-1} = \underline{r}_i$. Also, if $\overline{r}_u < \underline{r}_i$ then $\pi_i (r, h; x) = 0$ on $(\overline{r}_u, \underline{r}_i]$ which contradicts $\pi_i (r, h; x) > 0$ so that $\underline{r}_i \leq \overline{r}_u \leq R$; since the uninformed bank never underbids the informed one on high-quality borrowers $\overline{r}_i^h = \overline{r}_u = R$. Hence, $[\underline{r}_i^h, \overline{r}_i^h) = [\underline{r}_u, \overline{r}_u) = \left[\overline{p}^{-1}, R\right)$ which does not depend on x.

We now turn to the case of borrowers located at $x > \tilde{x}$ from the informed bank.

Claim 5 *If $x > \tilde{x}$ the informed bank bids $r_i (l; x) = r_l (x) = p (l; x)^{-1}$ almost surely for $\eta = l$.*

Consider $[\underline{r}_i^l, \overline{r}_i^l)$, the support of $F_i(r, l; x)$ for $x > \tilde{x}$. Clearly $\underline{r}_i(l; x) \geq r_l (x) = \frac{1}{p(l;x)}$; otherwise the informed bank would lose money. To show that $\overline{r}_i(l; x) \leq r_l (x)$ suppose the contrary, i.e., that $\overline{r}_i(l; x) > r_l (x)$. By bidding $r_u = \overline{r}_i(l; x) - \varepsilon$ for small $\varepsilon > 0$, the uninformed bank would make strictly positive profits: while the worst expected type of borrower now is $p(l; x)$, it wins with a positive probability. Hence, it must be that $\overline{r}_u(x) < \overline{r}_i(l; x)$. However, the informed bank could similarly realize positive profits by bidding $r_i = \overline{r}_u(x) - \delta$ for small $\delta > 0$, contradicting the assumption that $\overline{r}_i(l; x) > r_l (x)$ is in the support of $F_i(r, l; x)$.[19] $\Rightarrow\Leftarrow$

[19] Note that this is analogous to a standard Bertrand undercutting argument.

The remainder of the proof closely follows the case for $x < \tilde{x}$ with minor modification to give $[\underline{r}_i^h, \overline{r}_i^h) = [\underline{r}_u, \overline{r}_u) = [\overline{p}^{-1}, r_l(x))$ so that the upper bound depends on the borrower's location. From the proof's first part one has that $[\underline{r}_i^h, \overline{r}_i^h) = [\underline{r}_u, \overline{r}_u) = [r_{\overline{p}}, r_l(x) \wedge R)$. ■

Lemma 3 (Loan Rate Distribution Functions) *For all* $x, y \in [0, N^{-1}]$, $F_i(r, h; x)$ *and* $F_u(r; y)$ *are strictly increasing, continuous distribution functions so that profits* π_i *and* π_u *are constant on* $[r_{\overline{p}}, r_l(x) \wedge R)$.

Proof. By construction, $F_i(r, h; x)$ and $F_u(r; x)$ satisfy the usual requirements of distribution functions on their common support. By the proof of Lemma 2 $F_i(r, h; x)$ is continuous in both cases. A similar argument establishes **continuity** of $F_u(r; x)$: suppose that there exists $s \in [\underline{r}_u, \overline{r}_u)$ such that $F_u(s^-; x) < F_u(s; x)$. Since F_u is right-continuous $\pi_i(s^-, h; x) > \pi_i(s, h; x)$ as the informed bank's expected profits for high-quality borrowers is strictly positive. Hence, there exists a neighborhood $[s, s+\varepsilon], \varepsilon > 0$, say, on which $F_i(r, h; x)$ must be constant. Since $F_i(r, h; x)$ is continuous so is $\pi_u(r; x)$ at s and strictly increasing on the neighborhood by (7); but then, $F_u(r; x)$ can not have any mass on $(s, s+\varepsilon)$. $\Rightarrow\Leftarrow$

To show **strict monotonicity**, suppose that $F_i(r, h; x)$ is constant on some interval $[\underline{s}, \overline{s}] \subset [r_{\overline{p}}, r_l(x) \wedge R)$ which we can choose without loss of generality so that $F_i(\underline{s}^-, h; x) < F_i(r, h; x) = \bar{F} < F_i(\overline{s}^-, h; x)$ for $r \in [\underline{s}, \overline{s})$. By continuity, π_u is strictly increasing on the interval so that F_u must be constant over $[\underline{s}, \overline{s})$ by the zero profit condition. But now, $\pi_i(r, h; x)$ is strictly increasing on the interval so that $F_i(r, h; x)$ can not have a mass point at $\overline{s}$. $\Rightarrow\Leftarrow$

Monotonicity of F_u is established by a completely analogous argument. ■

Lemma 4 (Uniqueness) *The mixed strategy equilibrium is unique; in particular the informed and uninformed bank make offers over* $[r_{\overline{p}}, r_l(x) \wedge R)$ *according to*

$$\begin{aligned} F_i(r, h; x) &= \frac{p(h; x) - p(l; x)}{\overline{p} - p(l; x)} \frac{\overline{p}r - 1}{p(h; x)r - 1} \\ F_u(r; N^{-1} - x) &= \frac{p(h; x)}{\overline{p}} \frac{\overline{p}r - 1}{p(h; x)r - 1}. \end{aligned}$$

Proof. Since the mixing distributions are strictly increasing by the preceding Lemma expected profits must be constant on $[r_{\overline{p}}, r_l(x) \wedge R)$. For $\pi_i(r, h; x) = \bar{\pi}$ and $\pi_u(r; x) = 0$ (5) and (7) yield the following system of equations defining the loan rate distributions:

$$\begin{aligned} \left(1 - F_u(r; N^{-1} - x)\right)\{p(h; x)r - 1\} &= \bar{\pi} \\ P\{\eta = h\}(1 - F_i(r, h; x))[p(h; x)r - 1] + P\{\eta = l\}[p(l; x)r - 1] &= 0 \end{aligned}$$

Evaluating the first equation at the lower bound of the support shows by $F_u(r_{\overline{p}}; N^{-1} - x) = 0$ that the constant is $\bar{\pi} = p(h; x) r_{\overline{p}} - 1$. Similarly, one can derive a second expression for $\gamma(x) = P\{\eta = h\}$ by evaluating the second equation at $r_{\overline{p}} = \overline{p}^{-1}$ to find $\gamma = \frac{\overline{p} - p(l;x)}{p(h;x) - p(l;x)}$ with slight abuse of notation. Solving out for F_u and F_i one finds

$$\begin{aligned} F_i(r, h; x) &= \frac{p(h; x) - p(l; x)}{\overline{p} - p(l; x)} \frac{\overline{p}r - 1}{p(h; x)r - 1} \\ F_u(r; N^{-1} - x) &= \frac{p(h; x)}{\overline{p}} \frac{\overline{p}r - 1}{p(h; x)r - 1} \end{aligned}$$

The preceding distributions represent the unique equilibrium of the competitive bidding game for a given borrower. As both banks randomize over the full support of the distribution functions they can not profitably deviate from their mixed strategies which establishes uniqueness. ■

Taken together the preceding three Lemmata prove the Proposition as well as the first part of Corollary 1. ■

Proof of Corollary 1: Loan Rate Distributions

The mixing distributions are derived in Lemma 4. To prove the second part of the corollary, we need to consider the usual cases $x < \tilde{x}$ and $x > \tilde{x}$. In the former, the informed bank randomizes over $[\overline{p}^{-1}, R)$ for high-quality borrowers without any atoms but with point mass at $R : F_i(r, h; x)$ is continuous on $[\overline{p}^{-1}, R)$ with $F_i(R^-, h; x) = \frac{p(h;x)-p(l;x)}{\overline{p}-p(l;x)} \frac{\overline{p}R-1}{p(h;x)R-1} < 1$ by $p(h;x) > p > p(l;x)$ so that

$$\mu_i(R, h; x) = 1 - \frac{p(h;x) - p(l;x)}{\overline{p} - p(l;x)} \frac{\overline{p}R - 1}{p(h;x)R - 1}.$$

Since $P\{r \neq \emptyset, r \leq R\} = F_u\left(R; N^{-1} - x\right) < 1$ one finds that the uninformed bank abstains from competing for borrowers with probability $1 - F_u\left(R; N^{-1} - x\right)$ because $\{r \leq R\}$ has full measure. Hence it competes with probability $\beta\left(N^{-1} - x\right) = P\{r \neq \emptyset\} = 1 - \frac{p(h;x)\overline{p}^{-1}-1}{p(h;x)R-1}$ for borrowers mixing according to $F_u\left(r; N^{-1} - x\right)$ over $\left[\overline{p}^{-1}, R\right]$ without any atoms.

For borrowers located at $x > \tilde{x}$ the common support becomes $[r_{\overline{p}}, r_l(x) \wedge R) = [r_{\overline{p}}, r(l;x))$ for $r(l;x) = p(l;x)^{-1}$, the break-even loan rate for low-quality borrowers, since $1 < p(l;x)R$. It is easily verified that $F_i(r(l;x)^-, h; x) = 1$ in this case so that the informed bank randomizes its offers over the whole support $\left[\overline{p}^{-1}, r(l;x)\right]$ without atoms. F_u, however, has a mass point $\mu_u\left(r(l;x)^-; x\right) = \frac{p(h;x)\overline{p}^{-1}-1}{p(h;x)r(l;x)-1}$ at $r(l;x)$ as $F_u(r(l;x)^-; x) < 1$. Finally, note that as $x \to \tilde{x}$

$$\begin{aligned} F_i(r, h; x) &\to \frac{p(h;\tilde{x}) - p(l;\tilde{x})}{\overline{p} - p(l;\tilde{x})} \frac{\overline{p}r - 1}{p(h;\tilde{x})r - 1} \\ F_i(R^-, h; \tilde{x}) &= 1 \Rightarrow \mu_i(R, h; \tilde{x}) = 0. \; ■ \end{aligned}$$

Proof of Corollary 2: Profit Characterization

By the proof of Proposition 1, we know that the equilibrium profits for both banks are the same for every interest rate offered in the support of the mixing distributions. In particular, at $\underline{r} = \overline{p}^{-1}$ the informed bank wins with probability 1, but only bids if $\eta = h$, realizing a profit of $\pi_i^*(h, x) = \pi_i(\underline{r}, h; x) = \overline{p}^{-1}p(h;x) - 1 > 0$. Differentiating this expression with respect to distance x shows that profits to the informed bank decrease in informational distance:

$$\frac{\partial}{\partial x}\pi_i(\underline{r}, h; x) = \underline{r}p_x(h;x) = \underline{r}(p_h - p_l)H_x < 0,$$

since $H_x < 0$.

To show that profits to the informed bank conditional on $\eta = h$ are increasing in Δ_p, note that the profit expression above can be written as $\pi_i^*(h, x) = \overline{p}^{-1}p(h;x) - 1 = \frac{(H(x)-q)(p_h-p_l)}{\overline{p}} =$

$\frac{(H(x)-q)\Delta_p}{\overline{p}}$. Therefore, we can differentiate the above profit expression with respect to Δ_p:

$$\frac{\partial}{\partial \Delta_p}\pi_i\left(\underline{r}, h; x\right) = \frac{\partial}{\partial \Delta_p}\frac{(H(x)-q)\Delta_p}{\overline{p}} = \frac{H(x)-q}{\overline{p}} > 0,$$

as long as $H(x) > q$ (which it is). Therefore, profits to the informed bank are increasing in Δ_p, as desired. ■

Proof of Proposition 2: Screening Range

We first show that for $c > 0$, borrowers are screened by at most one bank. Suppose there are two banks, n and m, and that they both screen a borrower located at a distance x from bank n and y from bank m. By assumption, if $x < y$, bank n's signal is a sufficient statistic for bank m's. By an argument similar to that used in Proposition 1, profits in the subsequent competition stage would be zero for bank m. This occurs because, even though it is informed, its information is a subset of bank n's information set, and so the usual zero-profit result holds (see Engelbrecht-Wiggans et al (1983) for a discussion of this point). If $c > 0$, bank m would not then recoup its cost of screening, and so, anticipating that bank n will screen, will not itself screen. Note that this also implies that a bank will never screen a borrower that is closer to another bank.

Ex ante expected profits are given for $\gamma\left(x\right) = P\left\{\eta = h\right\} = \phi_H(x)q + (1-\phi_L(x))(1-q)$ by

$$\begin{aligned} E[\pi_n\left(\eta, x\right)] &= \gamma\left(x\right)\pi_n\left(r, h; x\right) + \left(1-\gamma\left(x\right)\right)\pi_n\left(r, l; x\right) \\ &= \frac{1}{\overline{p}}(p_h - p_l)q(1-q)\left[\phi_h(x) - (1-\phi_l(x))\right] \end{aligned}$$

by $\pi_n\left(r, l; x\right) = 0$. If $\phi_h\left(x\right) = \phi_l(x) = \phi(x)$, we find that $E[\pi_n\left(\eta, x\right)] = \frac{1}{\overline{p}}(p_h - p_l)q\left(1-q\right)\left[2\phi(x)-1\right]$, which is well defined: by the informativeness restriction on screening, $\phi_h\left(x\right), \phi_l(x) \geq \frac{1}{2}$: $E[\pi_n\left(\eta, x\right)] \geq 0$. $E[\pi_n\left(\eta, x\right)] = c$ now defines $\hat{x}$ from $\frac{1}{\overline{p}}(p_h - p_l)q\left(1-q\right)\left[I - \frac{2x}{\alpha}\right] = c$ as

$$\hat{x} = \frac{\alpha}{2}\left(I - \frac{c}{A}\right),$$

for $A \equiv \frac{1}{\overline{p}}\Delta_p q(1-q)$ as long as $\frac{\alpha}{2} - \frac{\overline{p}\alpha c}{2(p_h - p_l)q(1-q)} \leq \frac{1}{2N}$. Otherwise, we would have an overlap in the screening interval (since banks are symmetric ex-ante), which would contradict the first result above. Therefore, it must be that

$$\hat{x} = \min\left\{\frac{1}{2N}, \frac{\alpha}{2}\left[I - \frac{c}{A}\right]\right\},$$

as desired. ■

Proof of Proposition 3: Relationship Profits

These results are easily obtained by noting that, as above, expected ex-ante profits (before a signal is observed) can be written as:

$$E[\pi_n^*\left(\eta, x\right)] = \frac{1}{\overline{p}}(p_h - p_l)q(1-q)\left[\phi_h(x) - (1-\phi_l(x))\right]$$

These are also decreasing in informational distance:

$$\frac{\partial}{\partial x}E[\pi_n^*(\eta,x)] = \frac{1}{\overline{p}}(p_h - p_l)q(1-q)\left[\phi_h'(x) + \phi_l'(x)\right] < 0$$

since $\phi_h'(x), \phi_l'(x) < 0$. That expected profits are increasing in $\Delta_p = (p_h - p_l)$ is clear once we recall that $\phi_h(x), \phi_l(x) \geq \frac{1}{2}$ for all x. ■

Proof of Proposition 4: Loan Specialization

Simple differentiation with respect to x and algebraic manipulations yield for the informed bank's loan rate distribution function F_i for $D_i = [\overline{p} - p\,(l;x)]\,[p\,(h;x)\,r - 1]$

$$\begin{aligned}\frac{\partial}{\partial x}F_i\,(r,h;x) = \quad & D_i^{-2}p_x\,(h;x)\,[\overline{p} - p\,(l;x)]\,[p\,(l;x)\,r - 1]\,[\overline{p}r - 1] \\ + \quad & D_i^{-2}p_x\,(l;x)\,[p\,(h;x) - \overline{p}]\,[p\,(h;x)\,r - 1]\,[\overline{p}r - 1] > 0\end{aligned}$$

since $p_x\,(h;x)\,, [p\,(l;x)\,r - 1] < 0$ and $p_x\,(l;x)\,, [p\,(h;x) - \overline{p}]\,, [\overline{p} - p\,(l;x)]\,, [\overline{p}r - 1] > 0$. Proceeding similarly for F_u observing that $y = N^{-1} - x$ we find that

$$\frac{\partial}{\partial y}F_u\,(r;y) = D_u^{-2}p_y\left(h;N^{-1} - y\right)[\overline{p}r - 1] < 0$$

by $p_x\,(h;x) < 0$ and $[\overline{p}r - 1] > 0$ where $D_u = \overline{p}\left[p\left(h;N^{-1} - y\right)r - 1\right]$. ■

Proof of Proposition 5: Investment and Competition

The result is easily obtained by differentiating equation (4) with respect to α and I appealing to the Envelope Theorem and Leibniz' rule. This gives us the following first order conditions

$$\begin{aligned}\frac{\partial V_n}{\partial \alpha} &= 0 = 2A\left(\frac{\hat{x}_n}{\alpha}\right)^2 - \alpha \\ \frac{\partial V_n}{\partial I} &= 0 = 2A\hat{x}_n - \left(\frac{1}{1-I} - 1\right)\end{aligned}$$

yielding for $\hat{x} = \frac{1}{2N}$ the desired expressions. ■

Proof of Proposition 6: Investment Allocation

Let $L \equiv \bar{K}$ so that $\bar{K} = C\,(\alpha) + C\,(I) = \frac{\alpha^2}{2} - [\log\,(1-I) + I]$ and consider the constrained maximization problem

$$\max_{I,\alpha} L_n(I,\alpha) = \max_{I,\alpha}\left\{2\int_0^{(2N)^{-1}}\left[A\left(I - \frac{2x}{\alpha}\right) - c\right]dx - \lambda\left[\frac{\alpha^2}{2} - (\ln(1-I) + I) - \overline{K}\right]\right\}$$

whose FOCs are

$$\begin{aligned}
\frac{\partial}{\partial \alpha} L_n &= 0 = 2A\left(\frac{1}{\alpha 2N}\right)^2 - \lambda\alpha \Longleftrightarrow \lambda\alpha^3 = \frac{A}{2N^2} \\
\frac{\partial}{\partial I} L_n &= 0 = \frac{A}{N} - \lambda\left[(1-I)^{-1} - 1\right] \Longleftrightarrow \lambda\left[(1-I)^{-1} - 1\right] = \frac{A}{N} \\
\frac{\partial}{\partial \lambda} L_n &= 0 = \frac{\alpha^2}{2} - \left[\log(1-I) + I\right] - \bar{K}
\end{aligned}$$

Eliminate λ to find α as a function of I and vice versa

$$\begin{aligned}
\alpha^3 &= \frac{1}{2N}\left[(1-I)^{-1} - 1\right] \\
1 - I &= \left[2N\alpha^3 + 1\right]^{-1}
\end{aligned}$$

and substitute into the constraint for α or I. Denoting optimal investment levels by (α^*, I^*) we define $G(I, N) := C(\alpha(I)) + C(I) - \bar{K}$ so that $G(I^*, N) = 0$. We can now use the Implicit Function Theorem to study the effect of N on investment levels (α^*, I^*):

$$G(I, N) = \frac{1}{2}\left[\frac{1}{2}\frac{1}{N}\right]^{\frac{2}{3}}\left[(1-I)^{-1} - 1\right]^{\frac{2}{3}} - \left[\log(1-I) + I\right] - \bar{K}$$

so that $\frac{\partial G}{\partial N}(I, N) < 0$ and

$$\frac{\partial G}{\partial I}(I, N) = \frac{1}{3}\left[\frac{1}{2}\frac{1}{N}\right]^{\frac{2}{3}}\left[(1-I)^{-1} - 1\right]^{-\frac{1}{3}}(1-I)^{-2} + (1-I)^{-1} - 1 > 0$$

by $I \in (0, 1)$ implying

$$\frac{d}{dN} I^*(N) = -\left[\frac{\partial G}{\partial I}\right]^{-1}\frac{\partial G}{\partial N} > 0.$$

Repeating the previous step for $G(\alpha, N) := C(\alpha) + C(I(\alpha)) - \bar{K}$ we have

$$G(\alpha, N) = \frac{\alpha^2}{2} + \log\left[2N\alpha^3 + 1\right] + \left[2N\alpha^3 + 1\right]^{-1} - 1 - \bar{K}$$

so that

$$\begin{aligned}
\frac{\partial G}{\partial N}(\alpha, N) &= \frac{2\alpha^3}{2N\alpha^3 + 1} - \frac{2\alpha^3}{(2N\alpha^3 + 1)^2} > 0 \\
\frac{\partial G}{\partial \alpha}(\alpha, N) &= \alpha + \frac{6\alpha^2 N}{2N\alpha^3 + 1} - \frac{6\alpha^2 N}{(2N\alpha^3 + 1)^2} > 0
\end{aligned}$$

imply

$$\frac{d}{dN}\alpha^*(N) = -\left[\frac{\partial G}{\partial \alpha}\right]^{-1}\frac{\partial G}{\partial N} < 0$$

again as desired. ■

Proof of Proposition 7: Optimal Entry

Given the choice of α and I, total profits can be expressed as:

$$V_n(\alpha, I) = \Pi_n - \bar{K} = 2\hat{x}\left[A\left(I - \frac{\hat{x}}{\alpha}\right) - c\right] - \bar{K}$$

The free entry value of N is obtained by setting this equation equal to zero and solving for N. Note that, for any given α, I, there is always a value of N, $\overline{N}$, such that $\hat{x} = \frac{1}{2N}$ for all $N \geq \overline{N}$. Therefore, as long as an equilibrium with positive profits exists for the $N = 2$ case, we can solve for the free entry value of N. Keeping in mind that the investment levels α and I are also functions of N, the following equation implicitly defines N^*, the free entry number of banks (for $\hat{x} = \frac{1}{2N}$):

$$\bar{K}N^2 - (AI - c)\,N + \frac{A}{2\alpha} = 0$$

To obtain the comparative statics results in the proposition, it suffices to focus on the first term in V_n, Π_n. It is straightforward to show that, for fixed I and α, Π_n is increasing in Δ_p (see Proposition 3). Allowing I and α to vary to their optimal values for a change in Δ_p can only weakly increase profits relative to the case where they remain fixed. Similarly, for $\hat{x} = 1/2N$, it is clear that Π_n is decreasing in N. Therefore, N^* must increase in order to satisfy the zero profit constrain when Δ_p increases.

A similar and more direct argument applies to changes in c. Increasing the cost of screening, c, reduces each bank's profits, Π_n. This then implies that N^* must decrease in order to satisfy the zero profit constrain when c increases. This establishes our results. ■