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How Robustness can Lower the Cost of Discretion

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Abstract

Model uncertainty has the potential to change importantly how monetary policy is conducted, making it an issue that central banks cannot ignore. Using a standard new Keynesian business cycle model, this paper analyzes the behavior of a central bank that conducts policy under discretion while fearing that its model is misspecified. The main results are as follows. First, policy performance can be improved if the discretionary central bank implements a robust policy. This important result is obtained because the central bank's desire for robustness directs it to assertively stabilize inflation, thereby mitigating the stabilization bias associated with discretionary policymaking. Second, the central bank's fear of model misspecification leads it to forecast future outcomes under the belief that inflation (in particular) will be persistent and have large unconditional variance, raising the probability of extreme outcomes. Private agents, however, anticipating the policy response, make decisions under the belief that inflation will be more closely stabilized, that is, more tightly distributed, than under rational expectations. Finally, as a technical contribution, the paper shows how to solve with robustness an important class of linear-quadratic decision problems.

Keywords: *Model uncertainty, robustness, uncertainty aversion, time-consistency.*

JEL Classification: E52, E62, C61.

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1 Introduction

It is the nature of models to simplify reality. Unfortunately, this simplification goes hand-in-hand with model misspecification and model uncertainty; it weakens the foundations supporting model-based policy design and poses important challenges for central banks. To what extent does achieving robustness to model uncertainty require a sacrifice in policy performance? How does model uncertainty shape the beliefs that the central bank and private agents hold about future economic outcomes? Does a central bank's concern for model misspecification have a material effect on policy outcomes? These questions have important implications for monetary policy, and although central banks have always had to grapple with them, if not always explicitly, there is relatively little consensus about their answers.

This paper investigates these questions in the context of a standard new Keynesian business cycle model in which households, firms, and a central bank reside. The model is typical of those used to analyze monetary policy (Clarida, Galí, and Gertler, 1999) and is similar in spirit, if somewhat simpler, than Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003). To introduce model uncertainty, we follow Hansen and Sargent (2008) and assume that the central bank is skeptical of its model, fearing that it may be distorted by specification errors. Thus, the central bank in this economy designs policy while seeking robustness to unstructured perturbations about its approximating model. Importantly, in this analysis the central bank formulates its robust policy while taking into account that the distortions it fears also affect how private agents form expectations, similar to Woodford (2010).

An important result that emerges from the analysis is that robustness need not entail a decline in policy performance. To the contrary, a central bank that implements a robust policy may actually improve policy performance, and not just in extreme, low-probability states of nature, but on average. Although this result may seem surprising on the surface, it has a clear and intuitive explanation. When expectations are rational, time-inconsistency leads to a welfare-lowering stabilization bias in which inflation is understabilized and output is overstabilized relative to the commitment policy (Dennis and Söderström, 2006). To the extent that a fear of model uncertainty directs the discretionary central bank to stabilize inflation more tightly, the desire for robustness can mitigate the stabilization bias and potentially raise welfare. In effect, the central bank's fear of model uncertainty can act similarly to a commitment mechanism.

Ordinarily, a mechanism like a concern for reputation (Barro and Gordon, 1983), an optimal contract (Walsh, 1995), the appointment of an optimally conservative central banker (Rogoff, 1985), or the strategic delegation of policy objectives by a *benevolent* authority (Walsh, 2003) is required to improve on discretionary policymaking. But with robustness it is a *malevolent* planner that strategically designs the model (not the policy objectives), and the actions of the *malevolent* planner arise endogenously to reflect not the central bank's desire to raise welfare, but rather its fear of model misspecification. Because a fear of model misspecification can lower the cost of discretion, absent a commitment technology, a country may well benefit from appointing as head of its central bank someone who is suitably pessimistic about the inflationary consequences of shocks.

When analyzing the central bank's robust policy, we take advantage of results in Hansen, Sargent, and Tallarini (1999) and Hansen, Sargent, Turmuhambetova, and Williams (2006) that relate the multiple models in robust control to the multiple priors in uncertainty aversion (Gilboa and Schmeidler, 1989) to reinterpret the solution to the robust control problem. A connection between robust control and uncertainty aversion arises because the worst-case specification errors that emerge in the solution to the robust control problem manifest themselves in the form of worst-case shock processes. These worst-case shock processes can be interpreted as a set of worst-case beliefs, or a worst-case prior over future states, that is distorted relative to rational expectations. From the uncertainty aversion perspective, the analysis illustrates how a central bank's fear of misspecification can distort importantly—and asymmetrically—its beliefs about likely future economic outcomes and the beliefs that private agents hold. Thus, where the central bank's worst-case beliefs emphasize the possibility that inflation may be persistent and have a large unconditional variance, anticipating the policy response, private agents' beliefs emphasize that inflation will be more closely stabilized, and more tightly distributed, than under rational expectations. In addition, because the central bank's worst-case beliefs assign greater probability to the tails of the inflation and consumption distributions than rational expectations do, and because outcomes in the tails of these distributions come at a disproportionately high cost, the robust policy responds more forcefully to shocks than the nonrobust policy and generates greater interest rate volatility as a consequence. For this reason, the central bank's fear of model misspecification can have important effects on policy outcomes.

Relatively few papers use robust control to analyze optimal monetary policy, and even fewer focus on discretionary policymaking. Leitemo and Söderström (2008) ask whether a greater desire for robustness makes a discretionary central bank respond more aggressively to shocks and argue that the answer depends on the type of shock and the source of misspecification. In terms of its methodological contribution, this paper is most closely related to Giordani and Söderlind (2004), Dennis (2008), and Hansen and Sargent (2008, chapter 16). In contrast to Giordani and Söderlind (2004), who, as part of their analysis, also consider the solution of linear-quadratic robust control problems when policy is conducted under discretion, we make use of the fact that all agents necessarily reside within the same model. Thus, when designing its robust policy, we assume that the policymaker fears that private sector expectations are distorted by model misspecification. In contrast to Dennis (2008) and Hansen and Sargent (2008, chapter 16), this paper analyzes robust time-consistent policies, whereas they analyze robust commitment policies.

Although robust control provides an organized framework for studying how agents respond to model uncertainty, a framework that has connections to risk-sensitive preferences (Whittle, 1990), H-infinity control (Başar and Bernhard, 2008), and uncertainty aversion, it is not the only framework for analyzing model uncertainty and nor is it the only minmax-based framework. Other minmax-based approaches, some of which allow for multiple approximating models, are employed by Levin, Wieland, and Williams (1999, 2003), Tetlow and von zur Muehlin (2001), Onatski and Stock (2002), Giannoni (2002), Levin and Williams (2003), Onatski and Williams (2003), and Brock, Durlauf, and West (2007).[1] A prominent alternative to robust control is to formulate the decision problem from a Bayesian perspective, as per Batini, Justiniano, Levine, and Pearlman (2006). The Bayesian approach allows both model and parameter uncertainty to be analyzed within a unified decision-theoretic framework, but, unlike robust control which focuses on worst-case environments, it requires that priors be supplied and placed over fully articulated parameter and model spaces. Blake and Zampolli (2006) and Svensson and Williams (2007) approach robust policy design within a Markov-Jump-Linear-Quadratic framework. Kuester and Wieland (2010) study robust policy design in a Euro-area model using both Bayesian and minmax methods; they also consider uncertainty aversion.

The remainder of the paper is organized as follows. Section 2 summarizes the new Keynesian business cycle model that is used as a laboratory to study the effects of robustness. Section 3 formulates the robust Markov-perfect Stackelberg problem and presents its solution. Section 4 describes the connection between the robust Markov-perfect Stackelberg problem and uncertainty aversion. Section 5 demonstrates how the central bank's desire for robustness dis-

[1] Rustem and Howe (2002) analyze a wide-array of minimax-based robust decision environments.

torts its expectation's operator and thereby influences monetary policy. Section 6 establishes the key result that a central bank's desire for robustness can act like a commitment mechanism and lower the cost of discretion. Section 7 analyzes some factors that influence whether robustness improves policy performance and demonstrates a connection between the cost of robustness and the size of the stabilization bias. Section 8 concludes.

2 The model

This section describes the stylized hybrid new Keynesian business cycle model that we use as a laboratory to study how a central bank's fear of model misspecification affects policy. The model contains equations explaining inflation, π_t, and consumption, c_t, as a function of the short-term nominal interest rate, i_t, and two serially correlated shocks, s_t and d_t, and is given by

$$\pi_t = \beta(1-\delta)\,\mathrm{E}_t\pi_{t+1} + \delta\pi_{t-1} + \kappa c_t + s_t, \tag{1}$$

$$c_t = (1-\gamma)\,\mathrm{E}_t c_{t+1} + \gamma c_{t-1} - \phi\,(i_t - \mathrm{E}_t\pi_{t+1}) + d_t, \tag{2}$$

$$s_t = \rho s_{t-1} + \sigma_\varepsilon \varepsilon_t, \tag{3}$$

$$d_t = \tau d_{t-1} + \sigma_\epsilon \epsilon_t. \tag{4}$$

Equation (1) describes a hybrid new Keynesian Phillips curve in which forward-dynamics arise through sticky prices and backward-dynamics enter through inflation indexation. The parameter $\delta \in \left[0, \frac{1}{2}\right]$ governs the importance of forward-looking expectations in price-setting, equaling zero under Calvo-pricing (Calvo, 1983), $\beta \in (0,1)$ represents the subjective discount factor, and $\kappa \in (0,\infty)$, the coefficient on consumption, is a function of the share of firms that set their price optimally each period. Equation (2) summarizes consumption behavior in an environment in which consumers have external habit formation (Abel, 1990). The parameter $\gamma \in \left[0, \frac{1}{2}\right]$ regulates the importance of habits while $\phi \in (0,\infty)$, the coefficient on the ex ante real interest rate, denotes the elasticity of intertemporal substitution. Supply and demand shocks, s_t and d_t, described by equations (3) and (4), respectively, each follow first-order autoregressive processes in which $\{\rho,\tau\} \in (0,1)$ and $\{\sigma_\varepsilon,\sigma_\epsilon\} \in (0,\infty)$, with the innovations $\begin{bmatrix} \varepsilon_t \\ \epsilon_t \end{bmatrix} \sim i.i.d.\,[\mathbf{0},\mathbf{I}]$.

The short-term nominal interest rate, i_t, serves as the central bank's policy instrument. The central bank conducts policy with discretion, choosing $\{i_k\}_{k=t}^{\infty}$ to minimize its policy objective function, which is assumed to take the form

$$\mathrm{E}_t \sum_{k=t}^{\infty} \beta^{(k-t)} \left[\pi_k^2 + \lambda c_k^2 + \mu i_k^2 \right], \tag{5}$$

where $\lambda \in [0, \infty)$ and $\mu \in (0, \infty)$. Under certain circumstances, equation (5) can be viewed as a second-order accurate approximation to household welfare (Benigno and Woodford, 2006). For the purposes of this paper, however, this objective function is taken to be primal.

The monetary policy transmission mechanism largely operates as follows. Because some prices are rigid, a rise in the nominal interest rate raises the ex ante real interest rate, which lowers current period demand as households seek to defer consumption. Responding to lower demand, firms that can change their price moderate their price increase, which damps inflation. Monetary policy also operates through inflation expectations, with higher interest rates lowering inflation expectations and, hence, also current inflation.

Although the model is clearly stylized, its usefulness resides in the fact that it is simple enough to be easily understood, yet rich enough to illustrate the importance robustness plays in shaping policy and economic outcomes.

3 A discretionary Stackelberg leader's robust decision problem

This section presents a method for solving linear-quadratic decision problems in which a Stackelberg leader conducts policy under discretion while seeking robustness to model misspecification. The solution strategy is to cast the robust decision problem in a form that makes it amenable to the control methods used to solve forward-looking models with rational expectations. The section's main contribution is to show how to solve for a Markov-perfect Nash equilibrium—the approximating equilibrium—in which the leader employs a policy designed strategically to guard against model misspecification, while the followers, who do not fear model misspecification, make decisions and form expectations taking the leader's desire for robustness into account.

The material presented below is related to Giordani and Söderlind (2004), who also consider robust decision problems involving discretionary planners.[2] Unlike Giordani and Söderlind (2004), here the leader's robust decision problem is formulated on the basis that all expectations are informed by the misspecified model.

[2] Dennis (2008), Hansen and Sargent (2008, chapter 16), and Dennis, Leitemo, and Söderström (2009) all consider robust Stackelberg problems in which the leader can commit, but they do not consider discretion.

3.1 The problem

The economy consists of a Stackelberg leader, such as a central bank, a fiscal authority, or, more generally, a government, and one or more followers, such as households, firms, and other private agents. All agents in the economy are assumed to share an approximating model that they believe comes closest to describing the economy.[3] According to this approximating model, an $n \times 1$ vector of endogenous variables, $\mathbf{z}_t$, consisting of n_1 predetermined variables, $\mathbf{x}_t$, and n_2 $(n_2 = n - n_1)$ nonpredetermined variables, $\mathbf{y}_t$, evolves over time according to

$$\mathbf{x}_{t+1} = \mathbf{A}_{11}\mathbf{x}_t + \mathbf{A}_{12}\mathbf{y}_t + \mathbf{B}_1\mathbf{u}_t + \mathbf{C}_1\boldsymbol{\varepsilon}_{t+1}, \tag{6}$$

$$\mathrm{E}_t\mathbf{y}_{t+1} = \mathbf{A}_{21}\mathbf{x}_t + \mathbf{A}_{22}\mathbf{y}_t + \mathbf{B}_2\mathbf{u}_t, \tag{7}$$

where $\mathbf{u}_t$ is a $p \times 1$ vector of policy control variables, $\boldsymbol{\varepsilon}_t \sim i.i.d.\,[\mathbf{0}, \mathbf{I}_{n_s}]$ is an $n_s \times 1$ $(n_s \leq n_1)$ vector of white-noise innovations, and E_t is the private sector's mathematical expectations operator conditional upon period t information. The matrices $\mathbf{A}_{11}$, $\mathbf{A}_{12}$, $\mathbf{A}_{21}$, $\mathbf{A}_{22}$, $\mathbf{B}_1$, and $\mathbf{B}_2$ are conformable with $\mathbf{x}_t$, $\mathbf{y}_t$, and $\mathbf{u}_t$, as necessary, and contain the structural parameters that govern preferences and technology. The matrix $\mathbf{C}_1$ is determined to ensure that $\boldsymbol{\varepsilon}_t$ has the identity matrix as its variance-covariance matrix.

If the approximating model is known to be correctly specified, then the leader's problem in period t is to choose its control variables $\{\mathbf{u}_k\}_{k=t}^{\infty}$ to minimize the quadratic loss function

$$\mathrm{E}_t \sum_{k=t}^{\infty} \beta^{(k-t)} \left[\mathbf{z}_k^{\prime}\mathbf{W}\mathbf{z}_k + 2\mathbf{u}_k^{\prime}\mathbf{U}\mathbf{z}_k + \mathbf{u}_k^{\prime}\mathbf{R}\mathbf{u}_k\right], \tag{8}$$

where $\beta \in (0,1)$ is the discount factor, $\mathbf{z}_k \equiv \begin{bmatrix} \mathbf{x}_k^{\prime} & \mathbf{y}_k^{\prime} \end{bmatrix}^{\prime}$, subject to equations (6) and (7), Markov-perfection, and a known $\mathbf{x}_t$. Among other assumptions, the weighting matrices $\mathbf{W}$ and $\mathbf{R}$ are required to be symmetric and the matrix $\begin{bmatrix} \mathbf{W} & \mathbf{U} \\ \mathbf{U}^{\prime} & \mathbf{Q} \end{bmatrix}$ is required to be positive semi-definite.[4]

However, although the approximating model describes most accurately the economy's structure, the leader is skeptical of the model, fearing that it may be misspecified. To accommodate its fear, the leader introduces a vector of specification errors, $\mathbf{v}_{t+1}$, disguised

[3] This terminology follows Hansen and Sargent (2008). Elsewhere, notably Onatski and Williams (2003), Giordani and Söderlind (2004), Dennis, Leitemo, and Söderström (2006, 2009) and Tillmann (2009), the term "reference model" is used.

[4] The (nondistorted) decision problem described above is widely used to analyze discretionary policymaking by central banks. Following Oudiz and Sachs (1985), Backus and Driffill (1986), and Currie and Levine (1986), the problem is solved for a Markov-perfect Nash equilibrium by formulating it recursively and employing dynamic programming.

by the shocks that hit the model, and surrounds its approximating model with the class of distorted models

$$\mathbf{x}_{t+1} = \mathbf{A}_{11}\mathbf{x}_t + \mathbf{A}_{12}\mathbf{y}_t + \mathbf{B}_1\mathbf{u}_t + \mathbf{C}_1\left(\mathbf{v}_{t+1} + \varepsilon_{t+1}\right), \tag{9}$$

$$\mathrm{E}_t\mathbf{y}_{t+1} = \mathbf{A}_{21}\mathbf{x}_t + \mathbf{A}_{22}\mathbf{y}_t + \mathbf{B}_2\mathbf{u}_t, \tag{10}$$

where the sequence of specification errors, $\{\mathbf{v}_{k+1}\}_{k=t}^{\infty}$, is constrained by the boundedness condition

$$\mathrm{E}_t \sum_{k=t}^{\infty} \beta^{(k-t)} \mathbf{v}_{k+1}^{'} \mathbf{v}_{k+1} \leq \eta, \tag{11}$$

$\eta \in [0, \overline{\eta})$, whose satisfaction defines the sense in which the approximating model, summarized by equations (6) and (7), is a "good" one. Note that in the special case in which $\eta = 0$, the nondistorted decision problem is restored.

The specification errors enter the distorted model nonparametrically. However, their nature and behavior are determined as part of equilibrium and since we seek a Markov-perfect equilibrium, in equilibrium the specification errors must follow a Markov process. Further, the specification errors enter the distorted model additively and are constrained by a boundedness condition that is quadratic. These assumptions are not without loss of generality, but they facilitate solution by ensuring that the robust decision problem remains linear-quadratic. As will become clear, because the specification errors enter the distorted model additively, the approach highlights uncertainty about the shock processes and allows a connection between robust control and uncertainty aversion. At the same time, with additive specification errors, the approach may be less germane if data uncertainty (Orphanides, 2003) or parameter uncertainty is the primary concern (Onatski and Williams, 2003).

The distorted model, described by equations (9) and (10), constrains the leader's decision problem. In making its decision, the leader believes that all agents reside within the distorted model. As a consequence, in this decision problem the leader and the followers are all assumed to form expectations using the distorted model. Put differently, as part of its robust decision, the leader fears that private agents use the distorted model to form expectations.

To guard against the specification errors that it fears, the leader formulates policy subject to the distorted model with the mind-set that the specification errors will be as damaging as possible, a position operationalized through the metaphor that $\{\mathbf{v}_{k+1}\}_{k=t}^{\infty}$ is chosen by a fictitious evil agent whose objectives are diametrically opposed to those of the leader. Accordingly, the leader's robust problem is to choose $\{\mathbf{u}_k\}_{k=t}^{\infty}$ to minimize equation (8) and for the evil agent to choose $\{\mathbf{v}_{k+1}\}_{k=t}^{\infty}$ to maximize equation (8), subject to equations (9), (10), and (11). The evil agent's role in this problem is simply to help the leader devise a robust policy. Following Hansen and Sargent (2008, chapter 2), the way forward is to apply the Luenberger (1969) Lagrange multiplier theorem to replace this constraint problem, involving equation (11), with an equivalent multiplier problem in which

$$\mathrm{E}_t \sum_{k=t}^{\infty} \beta^{(k-t)} \left[\mathbf{z}_k^{'} \mathbf{W} \mathbf{z}_k + 2\mathbf{u}_k^{'} \mathbf{U} \mathbf{z}_k + \mathbf{u}_k^{'} \mathbf{R} \mathbf{u}_k - \beta\theta \mathbf{v}_{k+1}^{'} \mathbf{v}_{k+1} \right], \tag{12}$$

$\theta \in [\underline{\theta}, \infty)$, is minimized with respect to $\{\mathbf{u}_k\}_{k=t}^{\infty}$ and maximized with respect to $\{\mathbf{v}_{k+1}\}_{k=t}^{\infty}$, subject to equations (9) and (10). The multiplier, or robustness parameter, θ, represents the shadow price of a marginal relaxation of the boundedness condition, equation (11). Larger values for θ, which correspond to smaller values of η, signify greater confidence in the adequacy of the approximating model. Of course, in the limit as $\theta \uparrow \infty$, the nondistorted decision problem is again restored.

The difficulty with the robust decision problem as it currently stands is that nonpredetermined variables, $\mathbf{y}_t$, enter the constraints. However, this difficulty can be overcome by exploiting the Stackelberg relationship between the leader and the followers to obtain an expression linking the nonpredetermined variables to the state variables and the leader's decision variables. Because the state vector is given by $\mathbf{x}_t$ and the decision problem is linear-quadratic, in any Markov-perfect Nash equilibrium the nonpredetermined variables, $\mathbf{y}_t$, must be a linear function of the state vector, $\mathbf{x}_t$. As a consequence, expectations of future nonpredetermined variables must satisfy

$$\mathrm{E}_t \mathbf{y}_{t+1} = \mathbf{H} \mathrm{E}_t \mathbf{x}_{t+1}, \tag{13}$$

where $\mathbf{H}$ has yet to be determined. Combining equations (13), (10), and (9), and exploiting the fact that the specification errors are measurable with respect to period t information, implying $\mathrm{E}_t \mathbf{v}_{t+1} = \mathbf{v}_{t+1}$, leads to the expressions

$$\mathbf{x}_{t+1} = \overline{\mathbf{A}} \mathbf{x}_t + \overline{\mathbf{B}} \mathbf{u}_t + \overline{\mathbf{C}} \mathbf{v}_{t+1} + \mathbf{C}_1 \varepsilon_{t+1}, \tag{14}$$

$$\mathbf{y}_t = \mathbf{J} \mathbf{x}_t + \mathbf{K} \mathbf{u}_t + \mathbf{L} \mathbf{v}_{t+1}, \tag{15}$$

where $\overline{\mathbf{A}}$, $\overline{\mathbf{B}}$, $\overline{\mathbf{C}}$, $\mathbf{J}$, $\mathbf{K}$, and $\mathbf{L}$ depend on $\mathbf{H}$. Equation (14) describes the law of motion for the state variables while equation (15) is generally interpreted as the reaction function for the aggregate private sector. Notice that this reaction function depends on $\mathbf{v}_{t+1}$ as well as $\mathbf{u}_t$, reflecting the fact that in the leader's decision problem the fictitious evil agent is also a

Stackelberg leader with respect to private agents. This reaction function can be substituted into equation (12), thereby eliminating the nonpredetermined variables from the problem.

Following these substitutions, recognizing that the value function must take the form $V(\mathbf{x}_t) = \mathbf{x}_t^{\prime}\mathbf{V}\mathbf{x}_t + d$, the robust multiplier problem can be written recursively as

$$\mathbf{x}_t^{\prime}\mathbf{V}\mathbf{x}_t + d \underset{\mathbf{u}_t\ \mathbf{v}_{t+1}}{\underset{\min\max}{=}} \mathbf{x}_t^{\prime}\overline{\mathbf{W}}\mathbf{x}_t + 2\widetilde{\mathbf{u}}_t^{\prime}\overline{\mathbf{U}}\mathbf{x}_t + \widetilde{\mathbf{u}}_t^{\prime}\overline{\mathbf{R}}\widetilde{\mathbf{u}}_t + \beta \mathrm{E}_t\left(\mathbf{x}_{t+1}^{\prime}\mathbf{V}\mathbf{x}_{t+1} + d\right), \tag{16}$$

where the robustness parameter θ enters $\overline{\mathbf{R}}$, the matrices $\overline{\mathbf{W}}$, $\overline{\mathbf{U}}$, and $\overline{\mathbf{R}}$ all depend on $\mathbf{H}$, and $\widetilde{\mathbf{u}}_t \equiv \begin{bmatrix} \mathbf{u}_t \\ \mathbf{v}_{t+1} \end{bmatrix}$, subject to equation (14). Because the objectives of the leader and the fictitious evil agent are perfectly misaligned (they play a zero-sum game), the solution to this minmax problem can be obtained by solving the simultaneous choice problem (Hansen and Sargent, 2008, chapter 7), which, for a given $\mathbf{H}$, is equivalent to the standard discounted stochastic optimal linear regulator problem, whose solution is known to have the form (Anderson, Hansen, McGrattan, and Sargent, 1996)

$$\mathbf{x}_{t+1} = \mathbf{M}\mathbf{x}_t + \mathbf{C}_1\boldsymbol{\varepsilon}_{t+1} \tag{17}$$

$$\widetilde{\mathbf{u}}_t = -\mathbf{F}\mathbf{x}_t. \tag{18}$$

Partitioning $\mathbf{F} = \begin{bmatrix} \mathbf{F}^{\mathbf{u}} \\ \mathbf{F}^{\mathbf{v}} \end{bmatrix}$ conformable with $\mathbf{u}_t$ and $\mathbf{v}_{t+1}$, an update of $\mathbf{H}$ is available from the follower's reaction function, which implies

$$\mathbf{H} = \mathbf{J} - \mathbf{K}\mathbf{F}^{\mathbf{u}} - \mathbf{L}\mathbf{F}^{\mathbf{v}}. \tag{19}$$

Thus, given a conjecture of $\mathbf{H}$, the equations above provide an iterative method for calculating numerically the worst-case equilibrium, which is the equilibrium that governs the economy's behavior according to the leader's worst-case fears. The worst-case equilibrium is of interest in its own right, but, more importantly, it is the vehicle through which the approximating equilibrium is obtained.

3.2 Approximating equilibrium

In the approximating equilibrium, although the leader employs its robust decision rule, the approximating model is not actually misspecified and the followers, who are not robust decisionmakers, naturally form their expectations using the approximating model. Given the leader's robust decision rule, it is straight-forward to recover the (Markov) relationship linking the nonpredetermined variables to the state variables in the approximating equilibrium by solving for the fixed point of

$$\widehat{\mathbf{J}} = \left[\mathbf{A}_{22} - \widehat{\mathbf{H}}\mathbf{A}_{12}\right]^{-1}\left[\widehat{\mathbf{H}}\mathbf{A}_{11} - \mathbf{A}_{21}\right], \tag{20}$$

$$\widehat{\mathbf{K}} = \left[\mathbf{A}_{22} - \widehat{\mathbf{H}}\mathbf{A}_{12}\right]^{-1}\left[\widehat{\mathbf{H}}\mathbf{B}_{1} - \mathbf{B}_{2}\right], \tag{21}$$

$$\widehat{\mathbf{H}} = \widehat{\mathbf{J}} - \widehat{\mathbf{K}}\mathbf{F}^{\mathbf{u}}. \tag{22}$$

Therefore, in the approximating equilibrium, the state variables, the nonpredetermined variables, and the leader's decision variables are given by

$$\mathbf{x}_{t+1} = \left(\mathbf{A}_{11} + \mathbf{A}_{12}\widehat{\mathbf{H}} - \mathbf{B}_1\mathbf{F}^{\mathbf{u}}\right)\mathbf{x}_t + \mathbf{C}_1\varepsilon_{t+1}, \tag{23}$$

$$\mathbf{y}_t = \widehat{\mathbf{H}}\mathbf{x}_t, \tag{24}$$

$$\mathbf{u}_t = -\mathbf{F}^{\mathbf{u}}\mathbf{x}_t, \tag{25}$$

respectively.

4 Robustness and uncertainty aversion

Using a big "**X**" little "**x**" notation, the equilibrium law of motion for the state variables in the worst-case equilibrium can be written as as

$$\mathbf{x}_{t+1} = \left(\mathbf{A}_{11} + \mathbf{A}_{12}\mathbf{H} - \mathbf{B}_1\mathbf{F}^{\mathbf{u}}\right)\mathbf{x}_t + \mathbf{C}_1\left(\mathbf{F}^{\mathbf{v}}\mathbf{X}_t + \varepsilon_{t+1}\right), \tag{26}$$

$$\mathbf{X}_{t+1} = \mathbf{M}\mathbf{X}_t + \mathbf{C}_1\varepsilon_{t+1},$$

where $\mathbf{M} \equiv \mathbf{A}_{11} + \mathbf{A}_{12}\mathbf{H} - \mathbf{B}_1\mathbf{F}^{\mathbf{u}} - \mathbf{C}_1\mathbf{F}^{\mathbf{v}}$ and $\mathbf{x}_t = \mathbf{X}_t$ in equilibrium. Thus, the worst-case law of motion for the state variables is one in which the shock processes appear distorted, with their conditional mean twisted, or slanted, relative to the approximating model.

Equation (26) suggests a connection between robust control and the maxmin expected utility framework developed by Gilboa and Schmeidler (1989) to describe behavior they refer to as uncertainty aversion.[5] Gilboa and Schmeidler (1989) assume that beliefs about the likelihood of future states are so vague that they are represented by a set of prior densities rather than by a single prior density. The relationship between uncertainty aversion and robust control is considered in Hansen, Sargent, Turmuhambetova, and Williams (2006), who document conditions under which the multiple models in the robust control framework is

[5] See also Epstein and Wang (1994), who extend the Gilboa and Schmeidler (1989) analysis to intertemporal models.

behaviorally equivalent in the equilibrium to the multiple priors in the Gilboa and Schmeidler (1989) framework.

The arguments in Hansen, Sargent, Turmuhambetova, and Williams (2006) suggest that the solution to the robust control problem described in the previous section can be obtained equivalently by solving the problem in which the Stackelberg leader chooses $\{\mathbf{u}_k\}_{k=t}^{\infty}$ to minimize, and an evil agent chooses point-wise the probabilities in the probability density function, $p^*(\mathbf{x}_{k+1}|\mathbf{x}_t)$, associated with the expectations operator, E_t^*, to maximize

$$\mathrm{E}_t^* \sum_{k=t}^{\infty} \beta^{(k-t)} \left[\mathbf{z}_k^{'} \mathbf{W} \mathbf{z}_k + \mathbf{2u}_k^{'} \mathbf{U} \mathbf{z}_k + \mathbf{u}_k^{'} \mathbf{R} \mathbf{u}_k \right], \tag{27}$$

subject to

$$\mathbf{x}_{t+1} = \mathbf{A}_{11}\mathbf{x}_t + \mathbf{A}_{12}\mathbf{y}_t + \mathbf{B}_1\mathbf{u}_t + \mathbf{C}_1\boldsymbol{\varepsilon}_{t+1}, \tag{28}$$

$$\mathrm{E}_t^*\mathbf{y}_{t+1} = \mathbf{A}_{21}\mathbf{x}_t + \mathbf{A}_{22}\mathbf{y}_t + \mathbf{B}_2\mathbf{u}_t, \tag{29}$$

Markov-perfection, and a known $\mathbf{x}_t$. In addition, the difference between the distorted conditional probability density function, $p^*(\mathbf{x}_{k+1}|\mathbf{x}_t)$, and the rational expectations conditional probability density function, $p(\mathbf{x}_{k+1}|\mathbf{x}_t)$, is constrained to satisfy

$$\beta \sum_{k=t}^{\infty} \beta^{(k-t)} \left[\int_{\mathbf{x}_{k+1}} p^*(\mathbf{x}_{k+1}|\mathbf{x}_t) \ln \left(\frac{p^*(\mathbf{x}_{k+1}|\mathbf{x}_t)}{p(\mathbf{x}_{k+1}|\mathbf{x}_t)} \right) d\mathbf{x}_{k+1} \right] \leq \omega, \quad \omega \in [0, \overline{\omega}), \tag{30}$$

where ω in equation (30) plays the same role as η in equation (11).[6] Of course, it must also be the case that $\int_{\mathbf{x}_{k+1}} p^*(\mathbf{x}_{k+1}|\mathbf{x}_t)\, d\mathbf{x}_{k+1} = 1, \forall\, \mathbf{x}_t \in \Re^{n_1}$. Equation (30) is a (discounted) relative entropy condition (Kullback and Leibler, 1951), in which the expectation of a (log-) likelihood ratio is taken with respect to a distorted probability density.

The connection between robust control and uncertainty aversion suggests an alternative interpretation of the worst-case equilibrium. In an important sense, the worst-case equilibrium can be viewed as a tool, or as a vehicle, for generating the worst-case prior density, with decisions then made in view of this worst-case prior density. More generally, the connection between robust control and uncertainty aversion facilitates analyzing robust control problems in terms of the effect a fear of model uncertainty has on the beliefs held by the various

[6] In the approximating model, and hence in the worst-case equilibrium, the number of innovations, n_s, will generally be less than n_1. With the state vector, $\mathbf{x}_t$, consisting of shocks, $\mathbf{s}_t$, and predetermined variables, $\mathbf{p}_t$, $p(\mathbf{x}_{t+1}|\mathbf{x}_t)$ is given by $p(\mathbf{x}_{t+1}|\mathbf{x}_t) = |\mathbf{D}|\, p(\mathbf{s}_{t+1}|\mathbf{x}_t)$, where $\mathbf{D}$ is a Jacobian of transformation. The solution to the robust control problem provides the Jacobians relevant for the worst-case equilibrium and the approximating equilibrium.

agents residing in the model. For this reason, it is the properties of the probability density functions that underlie the rational expectations equilibrium, the worst-case equilibrium, and the approximating equilibrium that we characterize and discuss when we analyze the new Keynesian business cycle model.

5 Robust monetary policy

In this section, we explore the effect a central bank's desire for robustness can have on expectations, monetary policy, and the broader economy by applying the tools developed above to the hybrid new Keynesian model presented in Section 2. Exploiting the connection between robust control and uncertainty aversion, the analysis focuses on the probability density functions that underlie beliefs and expectation formation. The model is parameterized on the basis that the data are observed quarterly. Drawing on an array of studies, but on work by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003), in particular, in the equations for inflation and the consumption gap, we set $\beta = 0.99$, $\delta = 0.5$, $\kappa = 0.12$, $\gamma = 0.4$, and $\phi = 0.05$; in the shock processes, we set $\rho = \tau = 0.5$, and $\sigma_\varepsilon = \sigma_\epsilon = 1.0$; and in the policy objective function, we set $\lambda = 0.5$ and $\mu = 0.1$. With this parameterization, we solve the central bank's robust decision problem, examine the nature of the specification errors that it fears, document how these specification errors distort the expectation operator that the central bank uses to form expectations, and analyze the relationship between robustness and policy performance.

Before introducing robustness, it is useful to construct a benchmark by solving the nondistorted problem in which all expectations are formed rationally. For the parameterization above, the central bank's optimal discretionary policy can be described by the state-contingent decision rule[7]

$$i_t = 5.180 s_t + 6.133 d_t + 0.937 \pi_{t-1} + 0.800 c_{t-1}. \tag{31}$$

The optimal discretionary policy is to raise the nominal interest rate in response to adverse supply shocks and stimulatory demand shocks, thereby mitigating their contemporaneous impact on inflation and consumption, and to tighten policy in response to (past) higher inflation and consumption, thereby returning the economy to steady state more quickly. A notable feature of equation (31) is that its feedback coefficients are large, revealing aggressive policy

[7] It is worth noting that all of the policy rules reported in this paper are implementable. In other words, if the central bank were to implement policy according to $\mathbf{F^u}$ and private agents were allowed to reform expectations, then in the unique stable rational expectations equilibrium, private-agent decision rules are governed by $\widehat{\mathbf{H}}$.

responses even under rational expectations.[8]

5.1 Robustness

To introduce robustness a value for θ must be provided. Within the robust decision problem, θ represents the shadow price of a marginal relaxation of the constraint on the sequence of specification errors, equation (11). Intuitively, "large" values of θ indicate that the central bank is confident that the approximating model is "close" to the data-generating process and therefore that the specification errors should be "small." In this application, we follow standard practice and choose θ in order to generate a particular detection-error probability, here 0.1.[9] A detection-error probability is the probability that an econometrician observing equilibrium outcomes would make an incorrect inference about whether the approximating equilibrium or the worst-case equilibrium generated the data. The intuitive connection between θ and the probability of making a detection error is that when θ is small, greater differences between the distorted model and the reference model (more severe misspecifications) can arise, which are more easily detected.

Applying the solution method developed above, the worst-case shock processes are

$$s_{t+1} = 0.545 s_t + 0.038 d_t + 0.007 \pi_{t-1} + 0.004 c_{t-1} + \varepsilon_{t+1}, \quad (32)$$

$$d_{t+1} = 0.041 s_t + 0.544 d_t + 0.007 \pi_{t-1} + 0.005 c_{t-1} + \epsilon_{t+1}. \quad (33)$$

These worst-case shock processes convey information about the location and behavior of specification errors that the central bank should be concerned about. Specifically, the central bank is concerned that the demand and supply shocks may exhibit greater serial correlation than the approximating model asserts, that the demand and supply shocks might be correlated, and that the Phillips curve and the consumption Euler equation may omit terms involving lags of consumption and inflation. At the same time, these worst-case shock processes reflect how the central bank's expectation operator is twisted, or slanted, by its fear of misspecification.[10]

[8] These feedback coefficients, particularly those on the shocks, depend importantly on the interest rate stabilization parameter, μ, and would be smaller were μ larger.

[9] A technical appendix that accompanies this paper describes in detail the interpretation and calculation of detection-error probabilities. Alternative descriptions can be found in Hansen, Sargent, and Wang (2002), Giordani and Söderlind (2004), Hansen and Sargent (2008, chapter 9), Dennis (2008), Dennis, Leitemo, and Söderström (2006, 2009), and Cateau (2006).

[10] Values of θ that deliver detection-error probabilities below the 0.1 value used here serve to raise the coefficients on all of the state variables in the worst case shock processes.

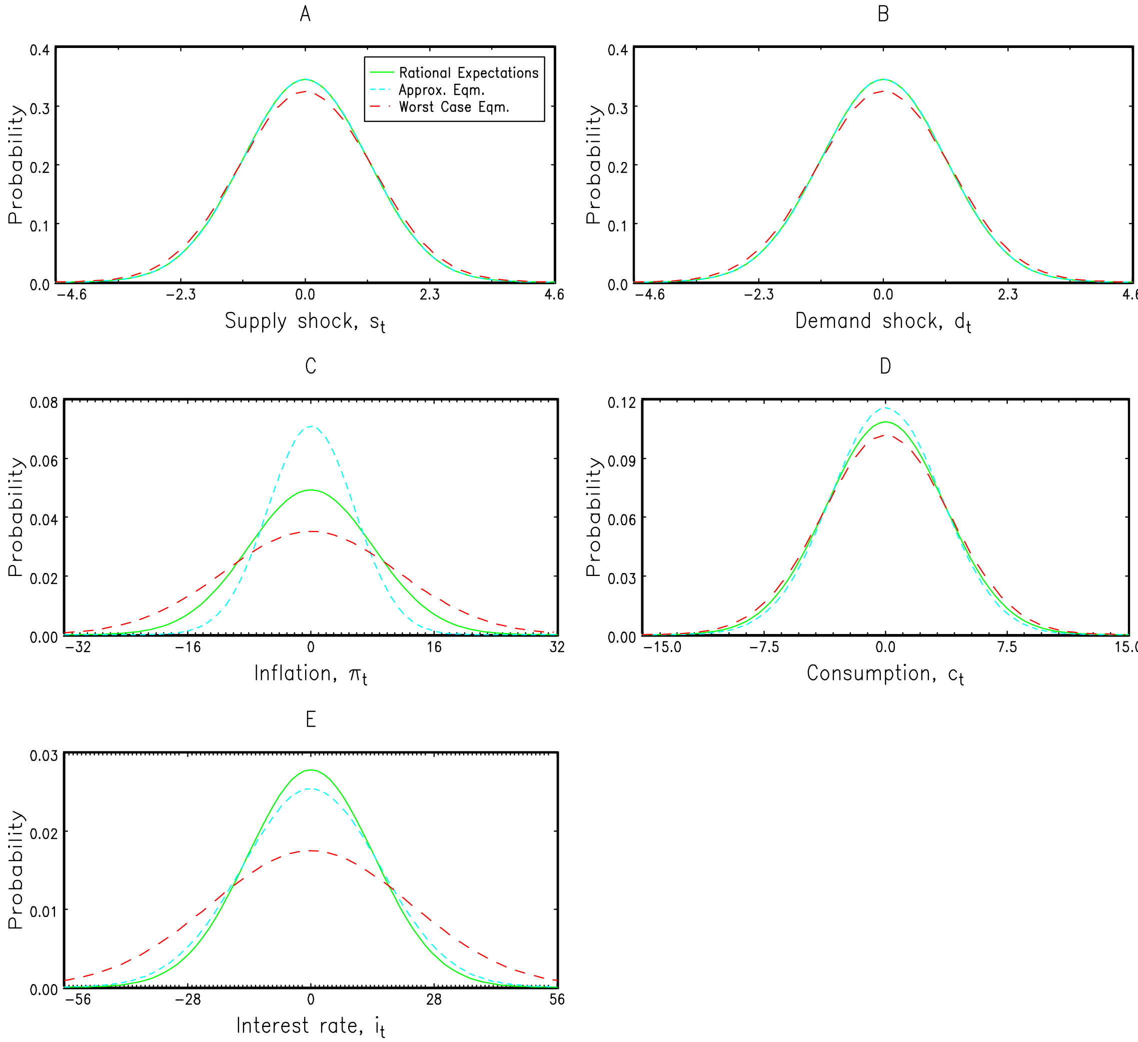

Figure 1: Distorted and nondistorted unconditional probability densities

Panels A through E display the unconditional probability density functions for the supply shock, the demand shock, inflation, consumption, and the nominal interest rate, respectively, according to the rational expectations equilibrium, the worst case equilibrium, and the approximating equilibrium. The worst case equilibrium densities characterize the pessimistic central banker's expectations. The approximating equilibrium densities characterize the private sector's expectations. The rational expectations equilibrium densities characterize the expectations of all agents when all agents believe the approximating model to be correctly specified.

Figure 1 displays the unconditional distributions of the supply shock, the demand shock, inflation, consumption, and the interest rate in the rational expectations equilibrium, the worst-case equilibrium, and the approximating equilibrium, under the illustrative assumption that the innovations, ε_t and ϵ_t, are joint $n.i.i.d.\,[\mathbf{0},\mathbf{I}]$ distributed. The probability density functions associated with the worst-case equilibrium characterize how the central bank forms expectations and how it fears that private agents will also form their expectations. By comparison, the probability density functions associated with the approximating equilibrium characterize how households and firms actually form expectations, which, due to the influence of the central bank's robust policy, differ from the rational expectations associated with the nonrobust decision problem. Note that the probability density functions that the central bank employs do not coincide with the economy's data generating process, reflecting the central bank's enduring pessimism about its model.

Relative to rational expectations, the worst-case supply shock (panel A) and demand shock (panel B) each have greater unconditional variance. Although these distortions to the supply and demand shocks appear small, they have important effects on the worst-case distributions of inflation (panel C), consumption (panel D), and the interest rate (panel E). Specifically, the central bank's fear of misspecification causes it to assign greater probability to inflation and consumption outcomes that would seem extreme under rational expectations. Similarly, with the central bank's robust policy represented by the state-contingent rule

$$i_t = 6.939s_t + 7.814d_t + 1.208\pi_{t-1} + 0.967c_{t-1}, \tag{34}$$

the interest rate's worst-case distribution also exhibits a much greater unconditional variance than the rational expectations distribution (panel E). Essentially, in terms of its unconditional expectations operator, the central bank obtains robustness by overweighting the probability it attaches to extreme inflation (in particular) and consumption outcomes, and this leads to an interest rate distribution that also assigns greater probability to extreme interest rate outcomes.

The central bank's desire to guard against extreme outcomes has important implications for the approximating equilibrium. By designing policy to guard against extreme inflation outcomes, the robust policy has a strong damping effect on the distribution of inflation (especially) and consumption in the approximating equilibrium. As shown in panel C, in the approximating equilibrium, inflation is distributed much more tightly about its unconditional mean than when expectations are rational, illustrating how the central bank's fear of misspecification leads it to "overstabilize" inflation. Similarly, the robust central bank also "overstabilizes" consumption (panel D), but at the cost of greater interest rate volatility (panel E).

Although the unconditional probability densities displayed in Figure 1 reveal the relationship between the central bank's pessimism and the probability it assigns to extreme outcomes, because they are unconditional they do not reveal how model uncertainty twists, or slants, the central bank's conditional expectations operator. To this end, with the initial state, $\mathbf{x}_t$, illustratively given by $s_t = d_t = c_{t-1} = \pi_{t-1} = 1$, Figure 2 presents the marginal probability density functions associated with one-quarter-ahead forecasts.

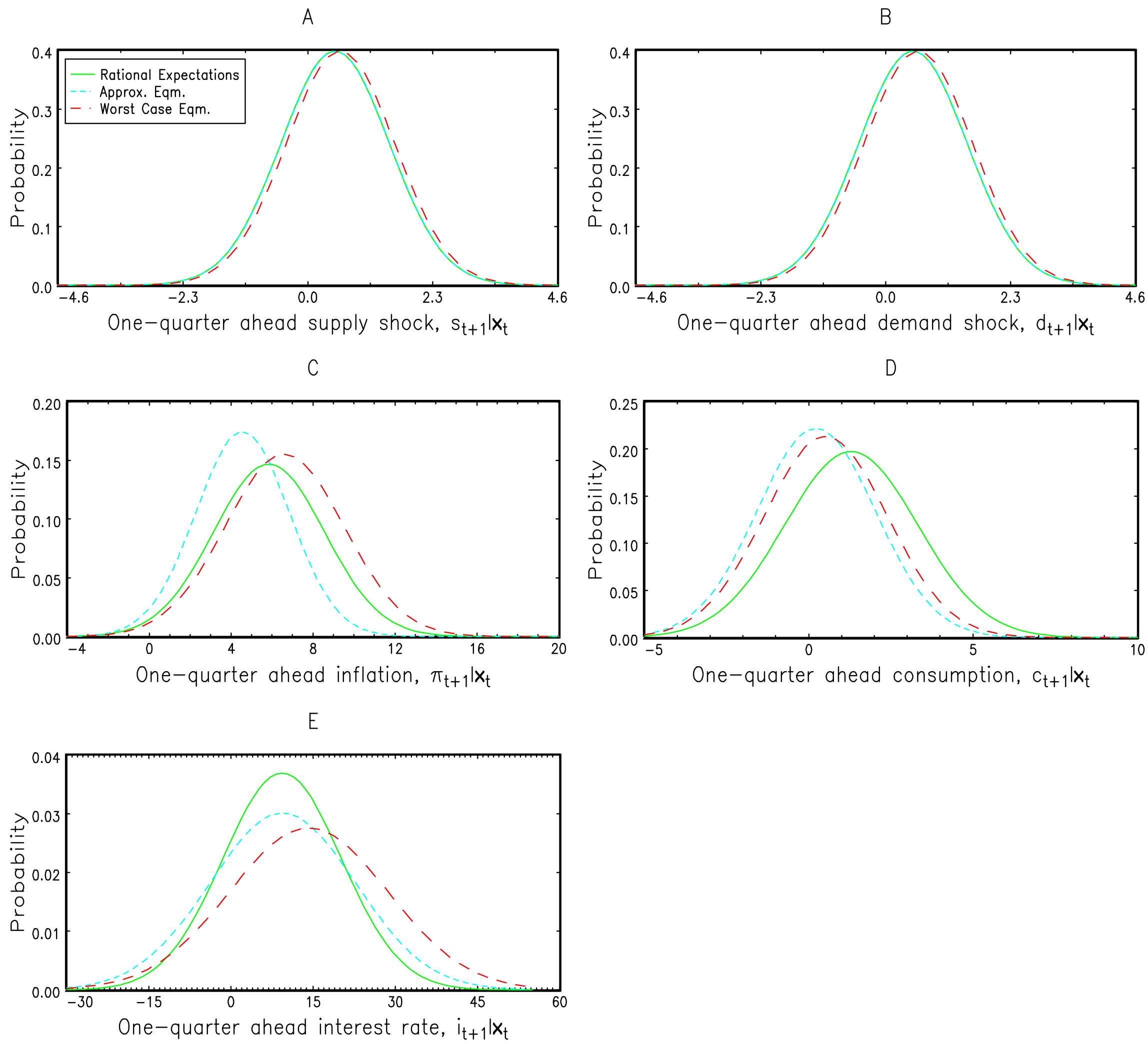

Figure 2: Distorted and nondistorted one-quarter-ahead forecast densities

Panels A through E display the one-quarter-ahead probability density functions for the supply shock, the demand shock, inflation, consumption, and the nominal interest rate, respectively, according to the rational expectations equilibrium, the worst case equilibrium, and the approximating equilibrium, conditional on the initial state: $s_t = d_t = \pi_{t-1} = c_{t-1} = 1$. The worst case equilibrium densities characterize the pessimistic central banker's expectations. The approximating equilibrium densities characterize the private sector's expectations. The rational expectations equilibrium densities characterize the expectations of all agents when all agents believe the approximating model to be correctly specified.

Figure 2 focuses on one-quarter-ahead forecast densities because the recursive nature of the robust optimization problem implies that it is these densities that are critical for the central bank's robust decision problem. Complementing Figure 1, panels A and B in Figure 2 show that, although the worst-case probability densities for the supply and the demand shocks are shifted to the right of those associated with the approximating equilibrium (which, of course, coincide with rational expectations), the distortions are reasonably modest. At the same time, these apparently small distortions to the shock distributions have a large impact on the one-quarter-ahead forecast densities for inflation (panel C), consumption (panel D) and the interest rate (panel E). Revealing a more subtle story than Figure 1, Figure 2 shows that the worst-case density for inflation is slanted to the right, with the central bank fearing higher inflation outcomes, and that the worst-case density for consumption is slanted to the left, with the central bank fearing lower consumption outcomes. Although it may seem more intuitive for the central bank to fear higher consumption outcomes, which would be inflationary, the probability densities are not unconstrained. Through the structure of the approximating model, because the central bank pessimistically expects higher inflation outcomes, it also expects higher interest rate outcomes, which leads it to expect lower consumption outcomes. Notice, however, that, unlike for consumption and the interest rate, where the distorted probability density function for future inflation is right-slanted, its counterpart in the approximating equilibrium is left-slanted.

6 Detectability and the cost of robustness

In the absence of misspecification, the optimal commitment policy is (weakly) superior to all other policies, including robust policies. It follows immediately that a desire for robustness cannot improve policy loss when the central bank can commit. However, when policy is conducted with discretion, stabilization bias provides an avenue through which robust policies can potentially improve upon nonrobust policies. To investigate the relationship between robustness and policy performance, this section examines the cost of robustness and its relationship to the robustness parameter and the probability of making a detection error.

One simple measure of the cost of robustness is

$$C = 100 \times \frac{\left(L_{ap}^{d} - L_{re}^{d}\right)}{L_{re}^{d}}, \tag{35}$$

where L_{ap}^{d} denotes policy loss in the approximating equilibrium and L_{re}^{d} denotes policy loss in the rational expectations equilibrium. According to this measure, positive values for C indicate that policy performance in the approximating equilibrium is inferior to what would be achieved in the rational expectations equilibrium.[11]

Figure 3 traces out the relationship between the robustness parameter and the probability of making a detection error (panel A), between the robustness parameter and the cost of robustness (panel B), and between the robustness parameter and the level of policy loss (panel C). Panel A reveals that the probability of making a detection error is monotonically increasing in the robustness parameter. Underlying this result is the fact that, as θ increases, greater weight is placed on the approximating model as being correct, the worst-case distortions are more tightly constrained, and the robust policy converges to the rational expectations policy. As a consequence, in the limit as $\theta \uparrow \infty$, data generated from the approximating equilibrium look increasingly like those generated from the worst case equilibrium and the probability of making a detection error converges to 0.5 (Hansen and Sargent, 2008, chapter 9). Panel B depicts the relationship between the robustness parameter and the cost of robustness. What panel B reveals is that an increase in the central bank's desire for robustness (smaller values for θ) actually cause the cost of robustness to decline, not rise. In effect, even if specification errors are absent, the central bank is better off using the robust policy than the rational expectations policy. Although this result may seem surprising at first, its genesis lies in the fact that monetary policy is conducted with discretion rather than with commitment.[12] Because private agents are forward-looking, the time-consistent policy with rational expectations is not optimal—it does not coincide with the optimal commitment policy—and other policies exist whose performance more closely approaches that of the optimal commitment policy. Related to panel B, panel C shows that the difference in policy performance between the robust and nonrobust policies can be large in an absolute sense.

[11] A technical appendix that accompanies this paper describes how the policy loss function is evaluated.

[12] Dennis and Ravenna (2008) obtain a related result from a model in which a central bank conducts policy while learning. The connection between the two results is that in each case policy is conducted with discretion and the central bank is only boundedly rational.

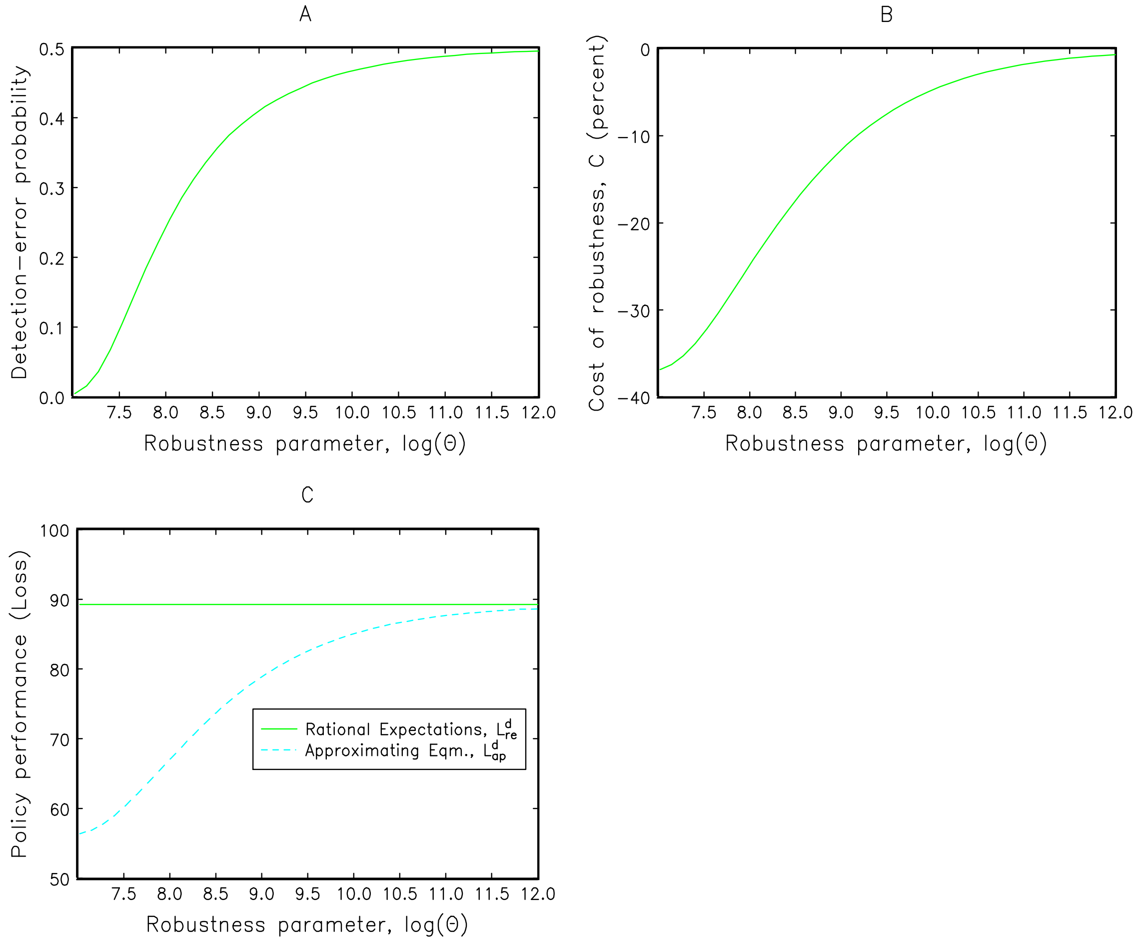

Figure 3: Detectability and the cost of robustness

> For the hybrid new Keynesian model, the panels in Figure 3 display the relationship between the robustness parameter and the probability of making a detection error (panel A), the relationship between the robustness parameter and the cost of robustness (panel B), and the relationship between the robustness parameter and policy loss in the rational expectations and approximating equilibria (panel C).

As Dennis and Söderström (2006) document, in rational expectations models discretionary policies overstabilize consumption and understabilize inflation, relative to commitment policies, giving rise to a stabilization bias. This bias can be unwound by stabilizing inflation more aggressively and stabilizing consumption less aggressively, however the absence of a commitment mechanism makes this infeasible when expectations are rational. But model uncertainty imparts a deviation from rational expectations, which causes the central bank to implement a policy that counteracts the likelihood of extreme inflation outcomes, partly mitigating the size of the stabilization bias. At the same time, as Figure 1 showed, the robust policy also stabilizes consumption more aggressively, and, as a consequence, whether robustness raises or lowers policy loss relative to the time-consistent rational expectations policy is likely to be parameter and model dependent, an issue to which we now turn.

7 Factors that influence the cost of robustness

The previous section found that the cost of robustness was negative. This implies, of course, that the discretionary central bank's desire for robustness actually improves policy performance. This section focuses on the factors that influence the cost of robustness and that determine whether it is positive or negative. We show that the cost of robustness is not always negative, but that it is negative for a wide range of parameter values, and that the cost of robustness is related to the size of the discretionary stabilization bias. Specifically, the cost of robustness tends to be more negative when the discretionary stabilization bias is large.

We begin by examining a special case of the model in which the central bank's desire for robustness is detrimental to policy performance. Let $\mu = 0$, so that the central bank assigns no weight to interest rate stabilization, $\delta = 0$, so that there is no inflation indexation in the Phillips curve, and $\rho = 0$, so that the supply shock is not longer serially correlated. With this parameterization the discretionary central bank optimally offsets demand shocks, which makes the policymaker immune to misspecification of the consumption Euler equation. The nonrobust decision problem is for the central bank to choose $\{c_k\}_{k=t}^{\infty}$ to minimize

$$\mathrm{E}_t \sum_{k=t}^{\infty} \beta^{(k-t)} \left[\pi_k^2 + \lambda c_k^2\right], \tag{36}$$

subject to

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa c_t + s_t. \tag{37}$$

After introducing a desire for robustness, and parameterizing the model according to $\beta = 0.99$, $\kappa = 0.05$, $\lambda = 0.08$, and $\sigma_\varepsilon = 1.0$ (Woodford, 2010), Figure 4 displays the relationship between the robustness parameter, θ, and the probability of making a detection error (panel A), between the robustness parameter and the cost of robustness (panel B), and between the robustness parameter and policy performance (panel C).

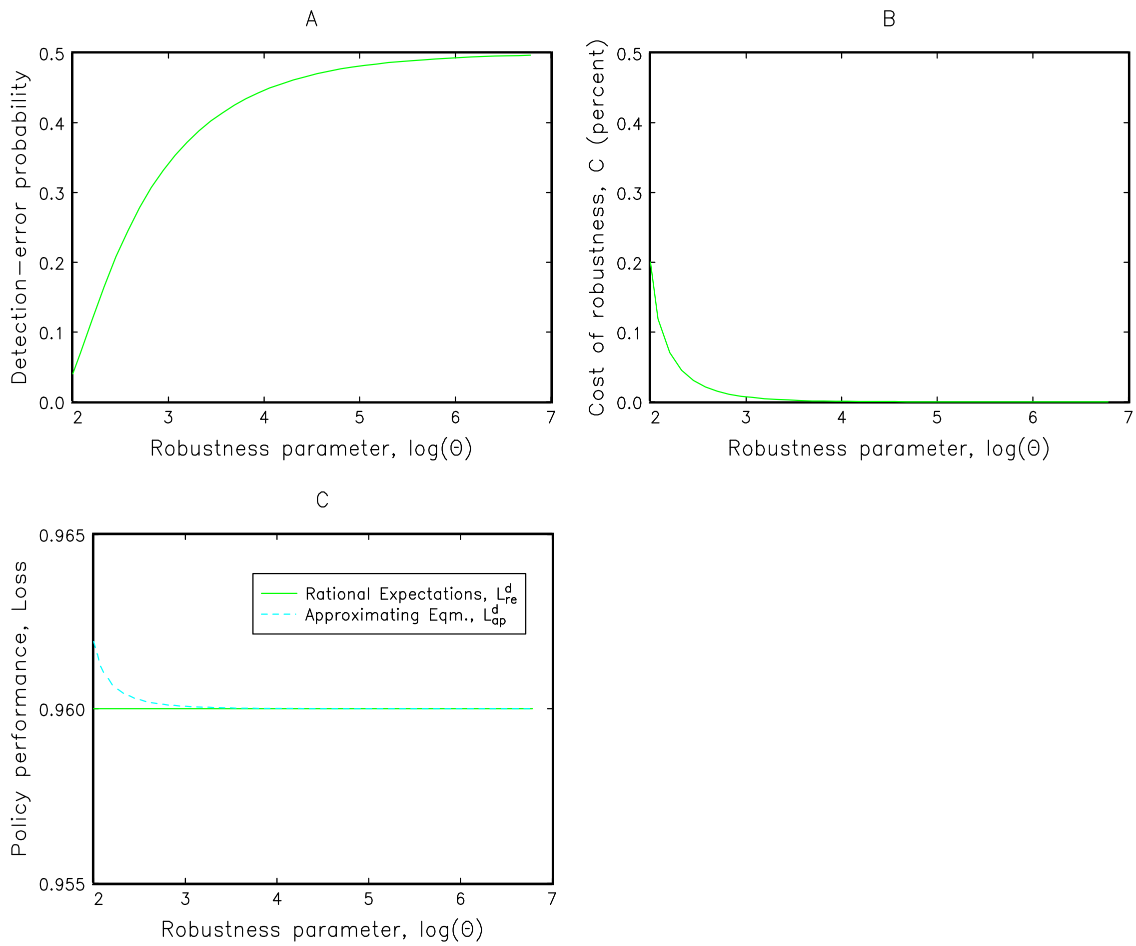

Figure 4: Detectability and the cost of robustness (Woodford model)

For the Woodford (2010) model, the panels in Figure 4 display the relationship between the robustness parameter and the probability of making a detection error (panel A), the relationship between the robustness parameter and the cost of robustness (panel B), and the relationship between the robustness parameter and policy loss in the rational expectations and approximating equilibria (panel C).

Although the increase is small, no larger than 0.25 of a percentage point for the values of θ considered, Figure 4, panel B, shows that for this parameterization of the model the central bank's desire for robustness raises policy loss, if only modestly (panel C), establishing that robustness does not improve policy loss for all parameterizations of the hybrid new Keynesian model. This robust decision problem is closely related to one considered by Woodford (2010), who employed this parameterization and also found that the discretionary central bank's desire for robustness led to a deteriation in policy performance.

Returning to the hybrid new Keynesian model analyzed in sections 5 and 6, we now consider independent variation in ρ, τ, δ, and γ, again holding the detection-error probability constant at 0.25[13], by varying (separately) ρ between 0.00 and 0.95, τ between 0.00 and 0.95, δ between 0.00 and 0.50, and γ between 0.30 and 0.50, keeping the parameters that are not being changed at the values reported in section 5. The results of this exercise are displayed in Figure 5 alongside a measure of the discretionary stabilization bias, which is constructed according to

$$S = 100 \times \frac{\left(L_{re}^{c} - L_{re}^{d}\right)}{L_{re}^{d}}, \tag{38}$$

where L_{re}^{c} denotes policy loss under commitment and L_{re}^{d} denotes policy loss under discretion, both in the absence of robustness. Aside from special cases in which there is no time-consistency problem, this measure of the stabilization bias, which measures as a percent the decline in loss that would be associated with the central bank being able to commit, is unambiguously negative. Because the policy loss associated with the optimal commitment policy with rational expectations cannot be surpassed, equation (38) provides a lower bound for the cost of robustness measure.

[13] To prevent the detection-error probability from changing as the model parameters are changed, θ is recalibrated for each parameterization. Note, however, that changes in θ, while important for magnitudes, do not influence whether the cost of robustness is positive or negative. The detection-error probability was set to 0.25 to ensure that results could be obtained for all parameterizations of the model.

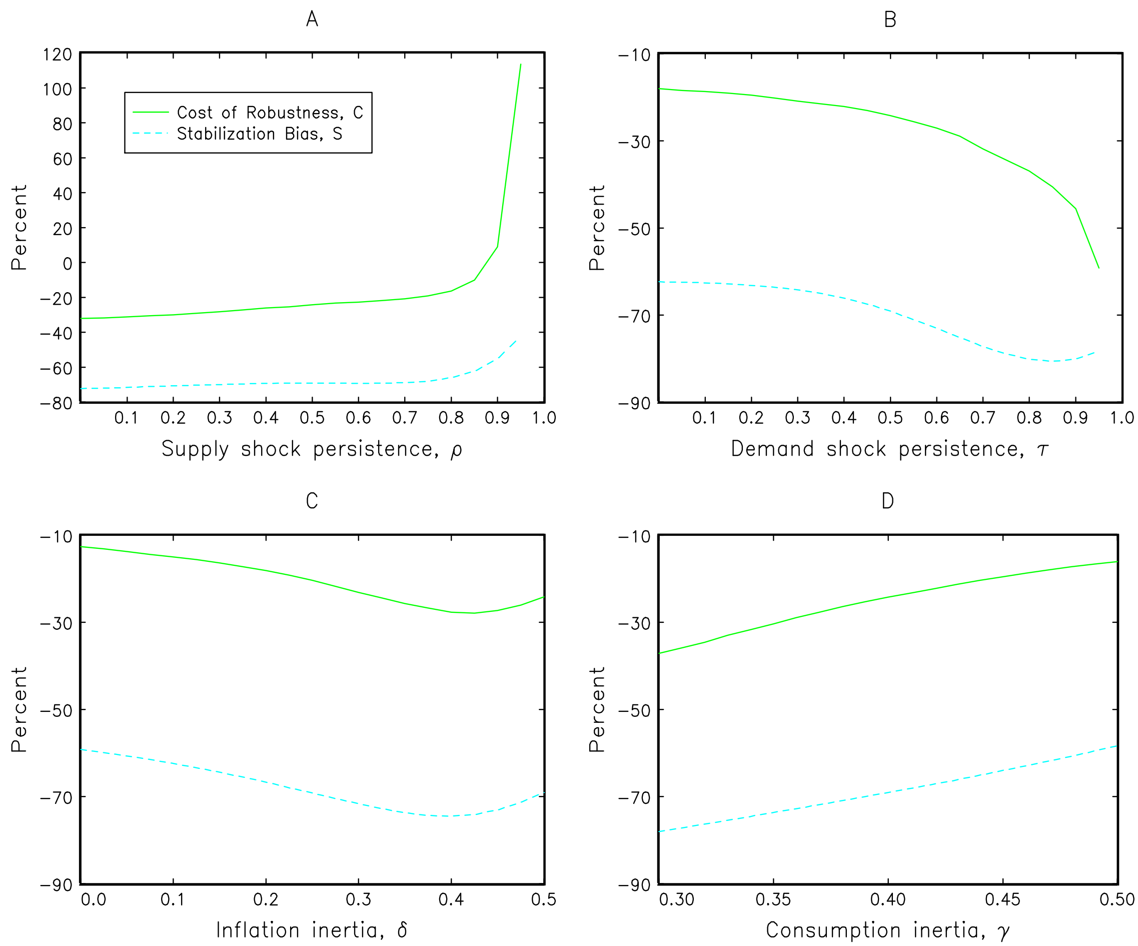

Figure 5: Stabilization bias and the cost of robustness (hybrid NK model)

For the hybrid new Keynesian model, panels A through D in Figure 5 display how the cost of robustness measure, C, and the stabilization bias measure, S, vary with changes in ρ, τ, δ, and γ, respectively.

There are three important results to take away from Figure 5. First, the cost of robustness is negative for almost all parameter combinations considered. Only in Panel A, for very large values of ρ, is the cost of robustness positive. Second, the cost of robustness and the stabilization bias are strongly correlated, particularly with respect to variation in δ and γ. Specifically, smaller values for the stabilization bias are associated with smaller values for the cost of robustness. Third, although the cost of robustness is generally negative, it generally falls well shy of the stabilization bias, which is to say that although robustness can lower policy loss it is certainly not a complete substitute for a commitment mechanism.

The results in Figure 5 suggest strongly that persistence in inflation, particularly persistence introduced through the cost-push shock, is closely associated with the finding that robustness can improve policy loss. Figure 5 also reveals that inflation persistence is a key factor governing the magnitude of the discretionary stabilization bias, consistent with Dennis and Söderström (2006). Importantly, the result that the cost of robustness is negative holds for a wide range of parameter values in this model, suggesting that it may hold more generally among new Keynesian models, particularly those for which expectations are an important policy channel and the discretionary stabilization bias is large.

8 Conclusion

This paper developed a method for solving robust Markov-perfect Stackelberg problems. This solution method was applied to a stylized hybrid new Keynesian business cycle model to examine the effect a concern for model misspecification can have on the behavior and policy decisions of a central bank that conducts policy with discretion. Although robust control methods are used to generate the relevant equilibria, a connection between robust control and uncertainty aversion was exploited to focus the analysis on the properties of the probability densities that households, firms, and the central bank use to form expectations.

The analysis revealed that a concern for model uncertainty causes the central bank to make decisions on the basis of a distorted conditional expectations operator that emphasizes the possibility that inflation (in particular) and consumption may be more persistent than their approximating model acknowledges. Because the central bank fears that shocks to inflation and consumption will persist, it implements a policy that tends to stabilize inflation and consumption more tightly than the rational expectations policy. Through their effect on inflation, robust policies can improve on nonrobust policies and lower policy loss. This result arises because the central bank's concern for robustness moves it to stabilize inflation more tightly than would be credible were expectations rational, and this greater inflation stabilization partly offsets the higher variance of inflation associated with the discretionary stabilization bias. As a consequence, in this hybrid new Keynesian model and with discretionary policymaking, some degree of robustness to model uncertainty can be attained without sacrificing policy performance. Although the result that robustness can improve policy performance in the absence of a commitment mechanism is parameter dependent, it holds for a wide range of parameter values in the hybrid new Keynesian model. In fact, the connection between the cost of robustness and the magnitude of the discretionary stabilization bias suggests that robustness is more likely to improve policy performance in models and for parameterizations where the time inconsistency problem is important and the stabilization bias is large.

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