Special Repo Rates: An Introduction

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The market for repurchase agreements involving Treasury securities (known as the repo market) plays a central role in the Federal Reserve's implementation of monetary policy. Transactions involving repurchase agreements (known as repos and reverses) are used to manage the quantity of reserves in the banking system on a short-term basis. By undertaking such transactions with *primary dealers*, the Fed, through the actions of the open market desk at the Federal Reserve Bank of New York, can temporarily increase or decrease bank reserves.

The focus of this article, however, is not monetary policy but, rather, the repo market itself, especially the role the market plays in the financing and hedging activities of primary dealers. The main goal of the article is to provide a coherent explanation of the close relation between the price premium that newly auctioned Treasury securities command and the special repo rates on those securities. The next two paragraphs outline this relationship and introduce some basic terminology that will be used throughout the article. (Also see the box for a glossary of terms.)[1]

Dealers' hedging activities create a link between the repo market and the auction cycle for newly issued (on-the-run) Treasury securities. In particular, there is a close relation between the liquidity premium for an on-the-run security and the expected future overnight repo spreads for that security (the spread between the general collateral rate and the repo rate specific to the on-the-run security). Dealers sell short on-the-run Treasuries in order to hedge the interest rate risk in other securities. Having sold short, the dealers must acquire the securities via reverse repurchase agreements and deliver them to the purchasers. Thus, an increase in hedging demand by dealers translates into an increase in the demand to acquire the on-the-run security (that is, specific collateral) in the repo market.

The supply of specific collateral to the repo market is not perfectly elastic; consequently, as the demand for the collateral increases, the repo rate falls to induce additional supply and equilibrate the market. The lower repo rate constitutes a rent (in the form of lower financing costs), which is capitalized into the value of the on-the-run security. The price of the on-the-run security increases so that the equilibrium return is unchanged. The rent can be captured by reinvesting the borrowed funds at the higher general collateral repo rate, thereby earning a repo dividend. When an on-the-run security is first issued, all of the expected earnings from repo dividends are capitalized into the security's price, producing the liquidity premium. Over the course of the auction cycle, the repo dividends are "paid" and the liquidity premium declines; by the end of the cycle, when the security goes off-the-run (and the potential for additional repo dividend earnings is substantially reduced), the premium has largely disappeared.

BOX

Glossary

Announcement date: The date on which the Treasury announces the particulars of a new security to be auctioned. When-issued (that is, forward) trading begins on the announcement date.

Auction date: The date on which a security is auctioned, typically one week after the announcement date and one week before the settlement date.

Fedwire: The electronic network used to transfer funds and wirable securities such as Treasury securities.

Forward contract: A contract to deliver something in the future on the delivery date at a prespecified price, the forward price.

Forward premium: The difference between the expected future spot price and the forward price.

Forward price: The agreed-upon price for delivery in a forward contract.

General collateral: The broad class of Treasury securities.

General collateral rate: The repo rate on general collateral.

Haircut: Margin. For example, a 1 percent haircut would allow one to borrow $99 per $100 of a bond's price.

Matched book: Paired repo and reverse trades on the same underlying collateral, perhaps mismatched in maturity.

Off the run: A Treasury security that is no longer on the run (see below).

Old, old-old, etc.: When a security is no longer on the run, it becomes the old security. When a security is no longer the old security, it becomes the old-old security, and so on.

On special: The condition of a repo rate when it is below the general collateral rate (when $R < r$).

On the run: The most recently issued Treasury security of a given original term to maturity—for example, the on-the-run ten-year Treasury note.

Reopening: A Treasury sale of an existing bond that increases the amount outstanding.

Repo: A repurchase agreement transaction that involves using a security as collateral for a loan. At the inception of the transaction, the dealer lends the security and borrows funds. When the transaction matures, the loan is repaid and the security is returned.

Repo dividend: The repo spread times the value of the security: $\delta = ps = p(r - R)$.

Repo rate: The rate of interest to be paid on a repo loan, R.

Repo spread: The difference between the general collateral rate and the specific collateral rate, $s = r - R$, where $s \geq 0$.

Repo squeeze: A condition that occurs when the holder of a substantial position in a bond finances a portion directly in the repo market and the remainder with "unfriendly financing" such as in a triparty repo.

Reverse: A repo from the perspective of the counterparty; a transaction that involves receiving a security as collateral for a loan.

Settlement date: The date on which a new security is issued (the issue date).

Short squeeze: See repo squeeze.

Specific collateral: Collateral that is specified—for example, an on-the-run bond instead of some other bond.

Specific collateral rate: The repo rate on specific collateral.

Term repo: Any repo transaction with an initial maturity longer than one business day.

Triparty repo: An arrangement for facilitating an ongoing repo relationship between a dealer and a customer, where the third party is a clearing bank that provides useful services.

When-issued trading: Forward trading in a security that has not yet been issued.

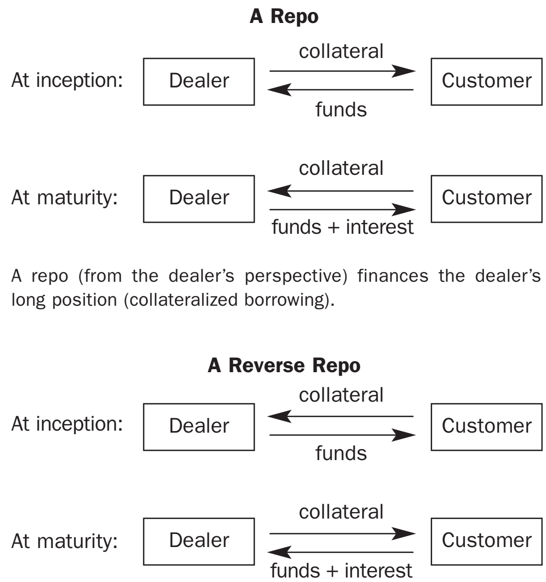
Zero-coupon bond: A bond that makes a single payment when it matures.

The next section describes what repos and reverses are, describes the difference between on-the-run and older securities, and discusses the ways dealers use repos to finance and hedge. The article then explains the difference between general and specific collateral, defines the repo spread and dividend, presents a framework for determining the equilibrium repo spread, and describes the average pattern of overnight repo spreads over the auction cycle.

The central analytical point of the article is that the rents that can be earned from special repo rates are capitalized into the price of the underlying bond so as to keep the equilibrium rate of return unchanged. The analysis derives an expression for the price premium in terms of expected future repo spreads and then computes the premium over the auction cycle from the average pattern of overnight repo spreads. Some implications of this analysis are then discussed. Finally, the article presents an analysis of a repo squeeze, in which a repo trader with market power chooses the optimal mix of funding via a triparty repo and funding directly in the repo market. Two appendixes provide additional analysis on the term structure of repo spreads and on how repo rates affect the computation of forward prices and tests of the expectations hypothesis.

CHART 1

A Repo and a Reverse Repo



A repo (from the dealer's perspective) finances the dealer's long position (collateralized borrowing).

A reverse repo (from the dealer's perspective) finances the dealer's short position (collateralized lending).

Repos and Dealers

A repurchase agreement, or repo, can be thought of as a collateralized loan. In this article, the collateral will be Treasury securities (that is, Treasury bills, notes, and bonds).[2] At the inception of the agreement, the borrower turns over the collateral to the lender in exchange for funds. When the loan matures, the funds are returned to the lender along with interest at the previously agreed-upon repo rate, and the collateral is returned to the borrower. Repo agreements can have any maturity, but most are for one business day, referred to as overnight. From the perspective of the owner of the security and the borrower of funds, the transaction is referred to as a repo while from the lender's perspective the same transaction is referred to as a reverse repo, or simply a reverse.

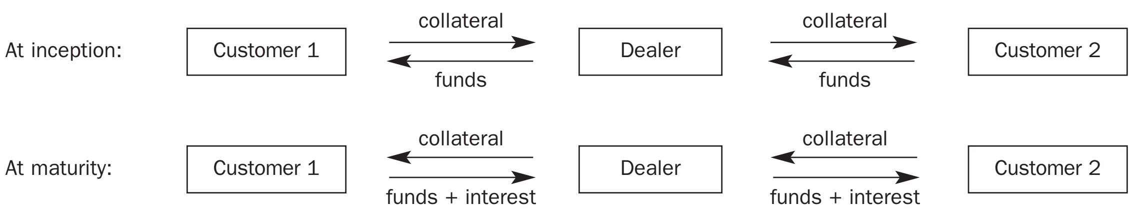
For concreteness, the discussion will refer to the two counterparties as the dealer and the customer even though a substantial fraction of repo transactions are among dealers themselves or between dealers and the Fed. Unless otherwise indicated, the article will adopt the dealer's perspective in characterizing the transaction. Repo and reverse repo transactions are illustrated in Chart 1, which can be summarized by a simple mantra that expresses what happens to the collateral at inception from the dealer's perspective: "repo out, reverse in."

Since dealers are involved with customers on both sides of transactions, it is natural for dealers to play a purely intermediary role. Chart 2 depicts a *matched book* transaction. In fact, the dealer may mismatch the maturities of the two transactions, borrowing funds short-term and lending them long-term (that is, reversing in collateral for a week or a month from customer 1 and repoing it out overnight first to customer 2 and then perhaps to another customer).

1. A number of sources provide additional material for anyone interested in reading more about the repo market. To read about how the repo market fits into monetary policy, see Federal Reserve Bank of New York (1998 and n.d.). For institutional details, see Federal Reserve Bank of Richmond (1993) and Stigum (1989). For some empirical results, see Cornell and Shapiro (1989), Jordan and Jordan (1997), Keane (1996), and Krishnamurthy (forthcoming). Duffie (1989) provides some theory as well as some institutional details and empirical results.
2. There is also an active repo market for other securities that primary dealers make markets in, such as mortgage-backed securities and *agency securities* (issued by government-sponsored enterprises such as Freddie Mac, Fannie Mae, and the Federal Home Loan Banks). In the equities markets, what is known as securities borrowing and lending plays a role analogous to the role played by repo markets, and as such much of the analysis of repo markets presented here is applicable to equities.

CHART 2

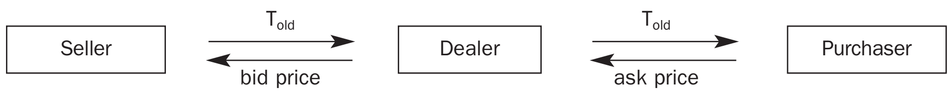
A Dealer's Matched Book Transaction



A dealer's matched book transaction involves simultaneous offsetting repo and reverse transactions. From customer 1's perspective the transaction is a repo while from customer 2's perspective the transaction is a reverse. The dealer collects a fee for the intermediation service by keeping some of the interest that customer 1 pays.

CHART 3

Making a Market I



A dealer purchases an old Treasury security (T_{old}) and immediately finds a buyer, earning a bid-ask spread.

Typically, customer 1 is seeking financing for a leveraged position while customer 2 is seeking a safe short-term investment.

On-the-run securities. The distinction between on-the-run securities and older securities is important. For example, the Treasury typically issues a new ten-year note every three months. The most recently issued ten-year Treasury security is referred to as the *on-the-run* issue. Once the Treasury issues another (newer) ten-year note, the previously issued note is referred to as the *old* ten-year note. (And the one issued before that is the *old-old* note, etc.) Similar nomenclature applies to other Treasury securities of a given original maturity, such as the three-year note and the thirty-year bond. Importantly, the on-the-run security is typically more actively traded than the old security in that both the number of trades per day and the average size of trades are greater for the on-the-run security. In this sense, the on-the-run security is more liquid than the old security.[3]

Financing and hedging. A dealer must finance, or fund, every long position and every short position it maintains. For Treasury securities, this means repoing out the long positions and reversing in the short positions. In addition to financing, the dealer must decide to what extent it will hedge the risk it is exposed to by those positions. For many positions, if not most, the dealer will want to hedge away all or most of its positions' risk exposure. The example that follows illustrates what is involved in financing and hedging a position that is generated in making a market in Treasury securities.

Suppose a dealer purchases from a customer an old (or older) Treasury security. The dealer may be able to immediately resell the security at a slightly higher price, thereby earning a bid-ask spread (see Chart 3). On the other hand, since older Treasury securities are less actively traded, the dealer may have to wait some time before an appropriate purchaser arrives. In the meantime, the dealer must (1) raise the funds to pay the seller and (2) hedge the security to reduce, if not eliminate, the risk of holding the security. The funds can be raised by repoing out the security. An important way that dealers hedge such positions is by short selling an on-the-run Treasury security with a similar maturity. The price of such an on-the-run security will tend to move up and down with the old security; consequently, if the price of the old security falls, generating a loss, the price of the on-the-run security will also fall, generating an offsetting gain. Assuming the dealer does in fact sell the on-the-run security short, the dealer now has an additional short position that generates cash (from the buyer) but requires delivery of the security. The dealer uses the cash (from the short sale) to acquire the security as collateral in a reverse repurchase agreement, which is then delivered on the short sale (see Chart 4).

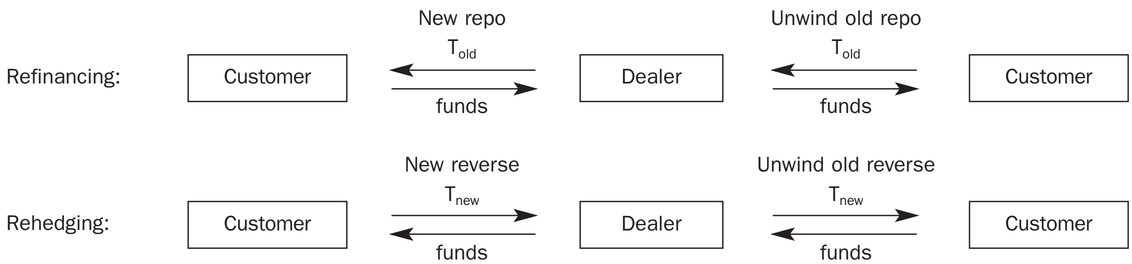
CHART 4

Making a Market II

A dealer purchases an old Treasury from a seller but has no immediate buyer.

CHART 5

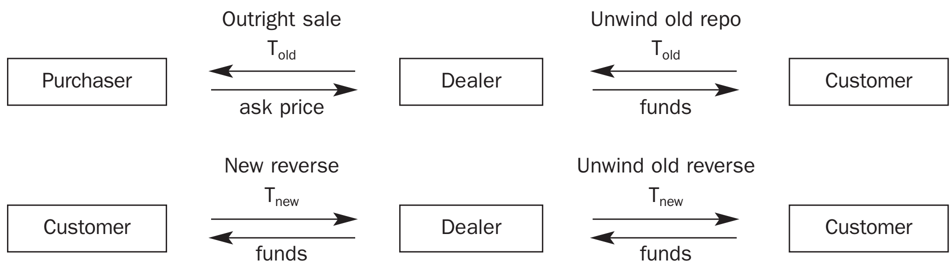
Making a Market III



If no purchaser arrives (the next day), the dealer refinances and rehedges.

CHART 6

Making a Market IV



When a purchaser arrives, the dealer sells the old Treasury (to the purchaser) and buys the on-the-run Treasury to close the short position.

If a purchaser for the original security does not arrive the next day, the dealer will repo the security out again and, using the funds obtained from the repo, reverse in the on-the-run Treasury again (see Chart 5). When a purchaser arrives, the dealer sells the original security, uses the funds to unwind the repo on the old Treasury, and purchases the on-the-run Treasury outright and delivers it to unwind the reverse, using the funds to pay for the purchase (see Chart 6). If all goes well, the dealer earns a bid-ask spread that compensates for the cost of holding and hedging the inventory.[4]

3. This greater liquidity is reflected in smaller bid-ask spreads for the on-the-run security.
4. Implicitly, it is assumed that dealers can borrow the full value of a Treasury security. For interdealer transactions, this assumption is not unrealistic. In other transactions, dealers and/or customers face *haircuts*, which amount to margin requirements. A more accurate accounting of haircuts (larger haircuts for customers than for dealers) would complicate the story without changing the central results significantly.

Recall that the hedge is a short position in an on-the-run Treasury security. In the example, the hedged asset is another (older, less liquid) Treasury security. Dealers hedge a variety of fixed-income securities by taking short positions in on-the-run Treasuries. For example, dealers hedge mortgage-backed securities by selling short the on-the-run ten-year Treasury note. As noted above, on-the-run Treasuries are more liquid than older Treasuries; indeed, on-the-run Treasuries are perhaps the most liquid securities in the world. Liquidity is especially important for short sellers because of the possibility of being caught in a *short squeeze*. In a short squeeze, it is costly to acquire the collateral for delivery on the short positions. Because the probability of being squeezed is high for large short positions, such positions are not typically established in illiquid securities; consequently, squeezes are rarely seen in illiquid securities, which is to say the *unconditional* probability is low. The equilibrium result is that squeezes arise most often in very liquid securities (unconditionally), because the (conditional) probability of being squeezed is low.

Dealers' hedging activities create a link between the repo market and the auction cycle for newly issued (on-the-run) Treasury securities.

Repo Rates and the Repo Dividend

As noted above, repurchase agreement transactions can be thought of as collateralized loans. The loan is said to finance the collateral. For most publicly traded U.S. Treasury securities the financing rate in the repo market is the *general collateral rate* (which can be thought of as the risk-free interest rate). In contrast, for some Treasury securities—typically recently issued securities—the financing rate is lower than the general collateral rate. These securities are said to be *on special*, and their financing rates are referred to as *specific collateral rates*, also known as *special repo rates*. The difference between the general collateral rate and the specific collateral rate is the *repo spread*.

Let r denote the current one-period general collateral rate (also referred to as the risk-free rate), and let R denote the current one-period specific collateral rate, where $R \leq r$.[5] The repo spread is given by $s = r - R$. If $R < r$, then the repo spread is positive and the collateral is on special. Let p denote the value of the specific collateral.

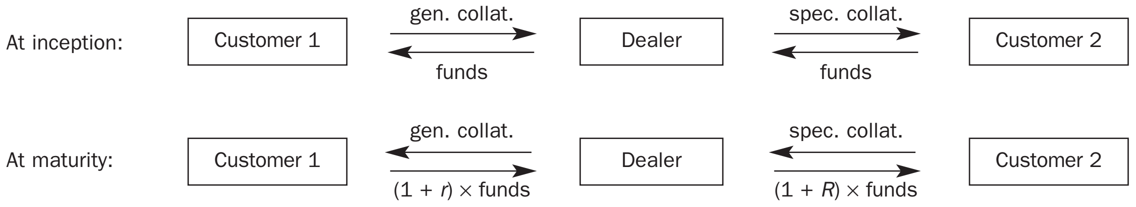
The repo spread allows the holder of the collateral to earn a *repo dividend*.[6] Let δ denote the repo dividend, which equals the repo spread times the value of the bond: $\delta = (r - R)p = sp$. A dealer holding some collateral on special (that is, for which $R < r$) can capture the repo dividend as follows (see Chart 7). The dealer repos out the specific collateral (borrows p at rate R) and simultaneously reverses in general collateral of the same value (lends p at rate r). The net cash flow is zero and the net change in risk is (effectively) zero. Next period the dealer unwinds both transactions, receiving the specific collateral back in exchange for paying $(1 + R)p$ and receiving $(1 + r)p$ in exchange for returning the general collateral. The dealer's net cash flow is the repo dividend $(r - R)p$.

Who would pay a repo dividend? The discussion has just shown how a dealer can obtain a repo dividend when a security it possesses is on special in the repo market. But what happens to the dealer's counterparty in the repo transaction? The counterparty (who may be another dealer) has just lent money at less than the risk-free rate. Why would anyone do such a thing? In other words, why would anyone pay a repo dividend?

If the counterparty (the party that is lending the money and acquiring the collateral) puts extra value on the specific collateral in question (above and beyond the value put on similar collateral), then that party will be willing to pay a fee for the privilege of obtaining the specific collateral. The dealer can package the fee as a repo dividend by having the counterparty accept a lower interest rate on the loan. In such a case, the specific collateral repo rate will be below the general collateral repo rate (below the risk-free rate). But this scenario begs the question, Why would anyone put extra value on some specific collateral? Why are other similar bonds not sufficiently close substitutes? The answer is simple: Anyone who sold that specific collateral short must deliver that bond and not some other bond. In other words, traders with short positions are willing to pay a repo dividend. These traders may well be dealers who have established short positions to hedge other securities acquired in the course of making markets. From their perspective, they are entering into reverse repos in order to acquire the collateral. By the same token, investors who do not hold short positions will be unwilling to pay the repo dividend. They place no special value on the specific collateral and accept collateral only at the general collateral rate.

CHART 7

Capturing the Repo Dividend



A dealer can capture the repo dividend by repoing out the specific collateral that is on special and simultaneously reversing in general collateral. The dealer nets (r – R) times the value of the specific collateral financed in the repo market.

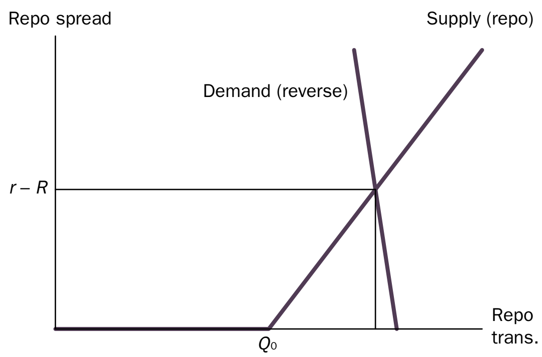
What determines the repo spread? One can adopt a simple model of supply and demand to analyze how the repo spread is determined. Chart 8 shows the demand for collateral by the shorts (those who want to do reverses) and the supply of collateral by the longs (those who want to do repos). The horizontal axis measures the amount of transactions, and the vertical axis measures the repo spread. The equilibrium repo spread and amount of transactions are determined by where supply and demand intersect. In the chart, the security is on special since the repo spread is positive. If instead the demand curve hit the horizontal axis to the left of Q_0, then the repo spread would be zero and the security would be trading at general collateral in the repo market.

Up to Q_0, the supply curve is perfectly elastic at a zero spread ($R = r$). There is a group of holders (those who hold the collateral) who will lend their collateral to the repo market at any spread greater than or equal to zero. Beyond Q_0, the supply curve slopes upward. To attract additional collateral, the marginal holders require larger and larger spreads. But why is the supply curve not infinitely elastic at all quantities? The fact that the supply curve rises at all indicates that some holders forgo repo spreads of smaller magnitudes. In fact, there are some holders who do not offer their collateral at any spread. At least for smaller spreads, transactions costs of various sorts can account for the upward slope. In addition, some holders are restricted legally or institutionally from lending their collateral.

There is an important aspect of the repo market that is not explicitly modeled here: The amount of short interest may exceed the total quantity of the security issued by the Treasury. For example, there may be short positions totaling $20 billion in a given security even though the Treasury has issued only $5 billion of that security. In this situation, a given piece of collateral is used to satisfy more than one short position; this scenario demonstrates the velocity of collateral. In effect, the market expands to match the supply, at least to some extent. However, maintaining this velocity involves informational and technological costs. As the amount of short interest increases and more collateral needs to be reversed in, identifying holders who are willing to lend collateral becomes more difficult. Some who held collateral earlier in the day may no longer have it; others who did not have it earlier may be holders now. Overall, several features may contribute to the upward slope of the supply curve.

CHART 8

An Equilibrium Repo Spread



The supply of repos and the demand for reverse repos determine the repo spread, r – R. If the demand intersects the horizontal axis to the left of Q_0, then the repo spread will be zero.

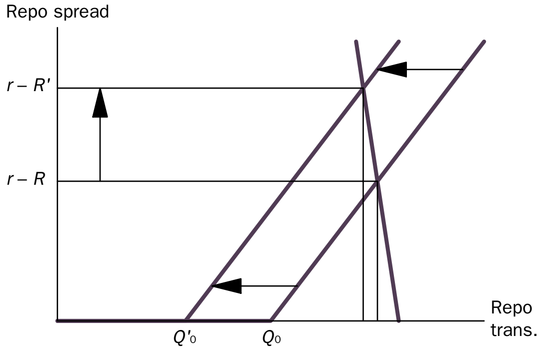
The auction cycle. The supply and demand framework can be used to illustrate the average pattern of overnight repo spreads over the course of the

5. For institutional reasons, $R \geq 0$ as well.

6. Unlike most of the technical terms in this article, the term *repo dividend* is not standard.

CHART 9

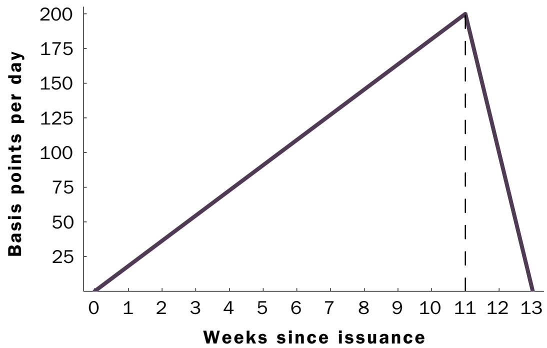
The Effect of a Decrease in the Repo Supply Curve



A decrease in the supply of collateral leads to an increase in the repo spread from $r - R$ to $r - R'$ or, equivalently, a fall in the special repo rate from R to R'.

CHART 10

The Average Pattern of Overnight Repo Spreads



The chart shows the average pattern of overnight repo spreads for an on-the-run security with a three-month (thirteen-week) auction cycle. The current on-the-run security is issued at week 0. The next security is announced at week 11 (at which point forward trading in the next security begins), auctioned at week 12, and issued at week 13. The overnight repo spreads reach a peak of 200 basis points per day at week 11. This cycle produces $0.5 \times 91 \times 200 = 9{,}100$ basis-point days of repo dividend earnings (the total area under the curve).

auction cycle. For example, the U.S. Treasury typically auctions a new ten-year Treasury note every three months (at the midquarter refunding in February, May, August, and November).[7] There are three important periodic dates in the auction cycle: the announcement date, the auction date, and the settlement (or issuance) date. On the announcement date, the Treasury announces the particulars of the upcoming auction—in particular, the amount to be auctioned—and *when-issued* trading begins.[8] The auction is held on the auction date and the security is issued on the settlement date. There is usually about one week from the announcement to the auction and one week from the auction to the issuance.

During a typical (stylized) auction cycle, the supply of collateral available to the repo market is at its highest level when the security is issued in the sense that $Q_0 \geq Q$, so that the overnight repo spread is zero. As time passes, more and more of the security is purchased by holders who do not lend their collateral to the repo market. Consequently, Q_0 declines over time, shifting the supply curve to the left and driving the repo spread up (see Chart 9). When forward trading in the next security begins on the announcement date, the holders of short positions roll out of the outstanding issue; the demand curve shifts rapidly to the left and drives the repo spread down.

Chart 10 shows how the shifts in supply and demand described above are reflected in the average pattern of overnight repo spreads for an on-the-run security with a three-month auction cycle. Actual auction cycles display a huge variance around this average. The chart shows the average overnight spread descending to (and presumably staying at) zero in week 13. This diagram is an adequate approximation for the three-year note but it is not a good approximation for the ten-year note, for which the overnight repo spread can average 25 to 50 basis points during the following auction cycle. In the next section, the analysis will demonstrate how the expected future overnight repo spreads are reflected in the price of the on-the-run bond.

Repo Dividends and the Price of the Underlying Bond

A simple rule can be used to determine what the expected payment of a repo dividend does to the price of a bond: The expected return on the bond (which includes the repo dividend) is unchanged. The expected return is simply repackaged; whatever goes into the repo dividend yield comes out of the capital gain. In other words, the spot price will rise until the expected return on the bond is exactly the risk-free rate, r, as the following analysis demonstrates.

Let p denote the current price of an n-period default-free zero-coupon bond, and let p' denote the price of that bond next period when it becomes an $(n - 1)$-period bond. Recall that r is the one-period risk-free interest rate (which is the same as the general collateral rate). In this case (assuming there is no uncertainty for the time being), the current price equals the present value of next period's price:

(1) $$p = \frac{p'}{1+r}.$$

For a one-period bond, $p' = 1$ and $p = 1/(1 + r)$. If the bond paid a dividend, δ, at the end of the period, then the current price would reflect the present value of that dividend as well:

(2) $$p = \frac{p' + \delta}{1+r}.$$

If the dividend is in fact a repo dividend, where $\delta = (r - R)p$, then

(3) $$p = \frac{p' + (r-R)p}{1+r}.$$

Because both sides of equation (3) involve the current price, p, the equation can be solved as follows:

(4) $$p = \frac{p'}{1+R}.$$

Note the similarity of equation (4) to equation (1). The value of a bond that pays a repo dividend equals next period's price discounted at its own repo rate. Equation (4) reduces to equation (1) when $R = r$.

Rearranging equation (2) produces

(5) $$\frac{p' - p}{p} + \frac{\delta}{p} = r,$$

where the first term on the left-hand side of equation (5) is the capital gain and the second term is the (repo) dividend yield ($\delta/p = r - R$). Neither the risk-free rate, r, nor next period's bond price, p', depends on the current repo dividend, δ, or the current repo rate, R. Comparing two securities with different repo rates reveals that, for the bond with the lower repo rate, (1) the repo dividend is higher, (2) the dividend yield is higher, (3) the current bond price is higher, (4) the capital gain is smaller, and (5) the expected return is the same.

Uncertainty and the forward premium. When uncertainty is introduced, risk premiums must be accounted for. Risk premiums compensate investors for bearing risk by increasing the expected return. Because repo transactions are essentially forward contracts, it is convenient to introduce risk premiums through the forward premium.

A forward contract is an agreement today to deliver something on a fixed date in the future (the delivery date) in exchange for a fixed price (the forward price). A repo establishes a forward position, and the repo rate on a bond is simply a way of quoting the forward price of the bond. An n-period default-free zero-coupon bond with a face value equal to 1, by definition, pays its owner 1 after n periods. Let p denote the current (spot) price of this bond, F denote the forward price of the bond for delivery next period, and R denote the (one-period) repo rate for the bond. A long forward position is established by buying the bond for p and financing it in the repo market for one period at rate R. (The net cash flow at purchase is zero.) In the next period, one pays $(1 + R)p$ and receives the bond.[9] Therefore, the forward price is $F = (1 + R)p$.[10] In fact, the repo rate is defined by $R = F/p - 1$.

If current information is available, one knows the current bond price, p; the current repo rate, R; and the current risk-free rate, r. But one does not know for sure the price of the bond next period, p' (unless it is a one-period bond, in which case $p' = 1$). Assuming that one knows the probability distribution of p', then one knows the average price (also known as the *expected* price). Let $E[p']$ denote the expected price. The actual price of the bond next period, p', equals its expected price plus a forecast error ε that is independent of everything currently known:

(6) $$p' = E[p'] + \varepsilon.$$

The forward price, F, is also known today. The forward premium, p, is defined as the difference between the expected and the forward price:

(7) $$\pi = E[p'] - F.$$

The forward premium is a risk premium. Given $p = F/(1+R)$ and the definition of the forward premium,

(8) $$p = \frac{E[p'] - \pi}{1+R}.$$

7. Occasionally, instead of issuing a new security the Treasury reopens the existing on-the-run security, selling more of the same security at the next auction. See the discussion on reopenings below.
8. When-issued trading refers to forward transactions for delivery of the next issue when it is issued.
9. A short forward position is established by selling the bond short for p and financing it in the repo market (on a reverse repurchase agreement) for one period at rate R. Next period, one receives $(1 + R)p$ and delivers the bond.
10. The forward price does *not* depend on the price of a one-period bond as is sometimes incorrectly assumed. See Appendix 2 for a discussion of how this miscalculation of the forward price can lead to a false rejection of the expectations hypothesis.

Using $R = r - \delta/p$ to eliminate R,[11] equation (8) can be reexpressed as

$$\text{(9)} \qquad \frac{E[p'] - p}{p} + \frac{\delta}{p} = r + \frac{\pi}{p},$$

which demonstrates that the expected return (capital gains plus repo dividends, both as fractions of the investment) equals the risk-free rate plus a risk premium. Equation (9) reduces to equation (5) when there is no uncertainty. The comparison following equation (5) between two bonds with different repo rates applies just as well when there is uncertainty.

Future repo rates. In order to express equation (4) in terms of future repo rates, one can assume for the moment there is no uncertainty. Recall that p is the price of an n-period zero-coupon bond. For a one-period bond, $p' = 1$, and equation (4) implies that $p = 1/(1 + R)$. For a bond with a maturity of two periods or more, let p'' denote its price two periods hence when it becomes an $(n - 2)$-period bond. Similarly, let r' and R' denote the values next period of the short-term interest rate and the repo rate. Then, following the same steps that led to equation (4),

$$\text{(10)} \qquad p' = \frac{p''}{1 + R'}.$$

Using equation (10) to eliminate p' from equation (4) yields $p = p''/(1 + R)(1 + R')$. For a two-period bond, $p'' = 1$ and $p = 1/(1 + R)(1 + R')$. An analogous expression holds for bonds of longer maturities. If p is the price of an n-period bond, then

$$\text{(11)} \qquad p = \prod_{i=0}^{n-1} \frac{1}{1 + R^{(i)}},$$

where $R^{(0)} = R$, $R^{(1)} = R'$, $R^{(2)} = R''$, etc. Equation (11) expresses the bond price as the present value of the final payment discounted at its current and future one-period repo rates.[12] As an approximation, equation (11) can be written as

$$\text{(12)} \quad p = \prod_{i=0}^{n-1} \frac{1}{1 + R^{(i)}} \approx 1 - \sum_{i=0}^{n-1} R^{(i)} = 1 + \sum_{i=0}^{n-1} s^{(i)} - \sum_{i=0}^{n-1} r^{(i)},$$

where the repo rates are expressed in terms of the risk-free (general collateral) rate and the repo spread, $R^{(i)} = r^{(i)} - s^{(i)}$. Equation (12) shows that higher repo spreads lead to higher bond prices while higher risk-free rates lead to lower bond prices.

If uncertainty is introduced into future risk-free rates and repo spreads (the current risk-free rate, r, and repo spread, s, are known, of course) and if, for expositional simplicity, all uncertainty is assumed to be resolved next period, then the price of the bond next period and its current expected price can be expressed as follows:

$$\text{(13a)} \qquad p' \approx 1 + \sum_{i=1}^{n-1} s^{(i)} - \sum_{i=1}^{n-1} r^{(i)}, \text{ and}$$

$$\text{(13b)} \qquad E[p'] \approx 1 + \sum_{i=1}^{n-1} E[s^{(i)}] - \sum_{i=1}^{n-1} E[r^{(i)}],$$

where the indexes in the sums begin at one instead of zero. Subtracting equation (13b) from (13a) yields,

$$\text{(14)} \quad p' - E[p'] \approx 1 + \sum_{i=1}^{n-1} (s^{(i)} - E[s^{(i)}]) - \sum_{i=1}^{n-1} (r^{(i)} - E[r^{(i)}]).$$

In equation (14), the uncertainty is decomposed into two components: uncertainty associated with future repo spreads and uncertainty associated with future interest rates. If a dealer is using the bond to hedge another position, then the effect of unanticipated changes in future interest rates on the bond's price is offset by the hedged position (by assumption). However, the effect of unanticipated changes in future repo spreads is not offset. The dealer faces this very real risk when using short positions in on-the-run securities to hedge other securities. If expected future repo spreads fall while the dealer's short position is open, the dealer may be forced to repurchase the bond at a significantly higher price when the hedge is removed, leading to possibly substantial losses.

The price premium and future repo spreads. The analysis next compares the price of an n-period bond that may earn repo dividends (specific collateral) with the price of a baseline n-period bond that earns no repo dividends.[13] To simplify the exposition, it is assumed there is no uncertainty.

Let the price of the specific collateral be p and the price of the baseline bond be $\bar{p}$. The price premium of the specific collateral over the baseline bond can be measured as $\psi = \log(p/\bar{p})$. The price of the baseline bond can be expressed in terms of the current and future one-period risk-free interest rates (general collateral rates) (compare equation [11]): $\bar{p} = \Pi_{i=0}^{n-1}/(1 + r^{(i)})$, where $r^{(0)} = r$, $r^{(1)}$, $r^{(2)} = r''$, and so on. Then the price premium is given by

$$\text{(15)} \quad \psi = \log\left(\frac{\prod_{i=0}^{n-1} \frac{1}{1+R^{(i)}}}{\prod_{i=0}^{n-1} \frac{1}{1+r^{(i)}}} \right) = \sum_{i=0}^{n-1} (\log(1 + r^{(i)}) - \log(1 + R^{(i)}))$$

$$\approx \sum_{i=0}^{n-1} (r^{(i)} - R^{(i)}) = \sum_{i=0}^{n-1} s^{(i)}.$$

The relative price premium equals (to a close approximation) the sum of the current and future repo spreads. A bond may have a significant price premium, even though it has no current repo dividends, as long as it has future repo dividends. This pattern can be seen in the Treasury market. When a bond is first issued, typically it has a significant price premium even though it is not on special for overnight repo transactions. Later, however, the overnight rates typically move lower than the general collateral rates, opening up a significant repo spread.

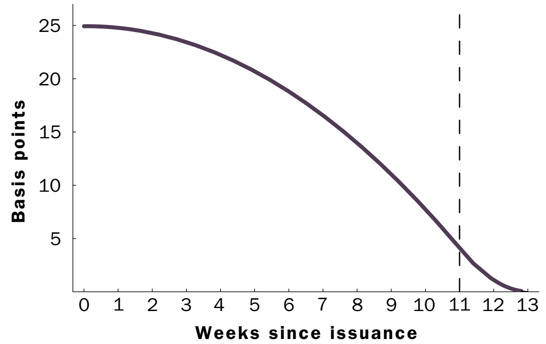
Given equation (15), the price premium can be expressed as $\psi \approx s + \psi'$, where $\psi' = \Sigma_{i=1}^{n-1} s^{(i)}$ is the price premium next period. This relation between ψ and ψ' shows that the price premium declines over time as the repo dividends are paid: $\psi' - \psi \approx -s$. The larger the repo spread, the greater the decline in the price premium; conversely, if the current repo spread is zero, then the price premium does not decline.

The presence of uncertainty complicates the situation slightly. If there is uncertainty about future repo spreads, then revisions in their expectations will also affect the change in the price premium. In this case, it is the expected change in the price premium that is approximately equal to (the negative of) the current one-period repo spread: $E[\psi'] - \psi \approx -s$.

Chart 11 shows the price premium, π, computed from the repo spreads shown in Chart 10. Equation (15) provides the link between the two graphs. The height of the curve in Chart 11 for a given week equals the sum of the remaining repo spreads, which in turn equals the area under the curve in Chart 10 to the right of that week. Thus, the premium of 25 basis points at week 0 in Chart 10 equals the total area under the curve.[14] The premium of 25 basis points is in line with that for the thirty-year bond during the late 1980s and early 1990s. During the same period the average price premium at issuance for the ten-year note was about 60 basis points while it was about 10 basis points for the three-year note. These average price premia all display the same general shape as that displayed in Chart 11. The one significant difference is that the ten-year note retained a 20 basis point premium throughout the following cycle. The huge variance around the average pattern shown in Chart 11 is consistent with the variance around the pattern of overnight repo rates shown in Chart 10.

CHART 11

The Average Price Premium



The chart shows the average price premium for an on-the-run security with a three-month (thirteen-week) auction cycle. This price premium is computed from the stylized overnight repo spreads in Chart 10. The price premium is the sum of all remaining repo spreads. When the security is issued, it has a price premium of about 25 basis points of the value of a reference bond. (The bid-ask spread for an on-the-run security is less than or equal to 1/32 per 100 = 3.125 basis points.)

Implications and Discussion

Factors that determine the total specialness. One implication of the analysis is that the size of the price premium at the auction depends on the total number of basis-point days of "specialness" that the security will generate during its life. The security's total specialness can increase either through the overnight spread increasing or by the security being on special for a longer time. For example, the main reason the price premium for the two-year note is small on average (less than 10 basis points) is that it is on a monthly cycle and therefore has only about thirty days to accumulate repo dividends versus the ninety-one days for securities with a three-month cycle. As noted above, securities that are on quarterly cycles typically have larger price premiums; the significant variation across the price premiums of such securities can be attributed to the average size of the spreads.[15]

Occasionally, instead of selling a new security at the next auction, the Treasury reopens the existing on-the-run security. Such a reopening extends the length of time the security is on the run. Since the

11. Recall that $\delta = (r - R)p$.
12. Consequently, the yield to maturity on a bond is (approximately) the average of these repo rates: $-\log(p)/n = \Sigma_{i=0}^{n-1} \log(1 + R^{(i)})/n \approx \Sigma_{i=0}^{n-1} R^{(i)}/n$.
13. There are a sufficient number of securities that can reasonably be assumed to satisfy this condition so that the value of a baseline bond can be calculated for any specific security.
14. The repo spreads in Chart 10 are quoted in basis points per day, which must be converted to basis points per year before they can be plugged into Equation (15). The total area under the curve in Chart 10 is $\frac{1}{2} \times 91 \times 200 = 9{,}100$ basis-point days, which equals approximately 25 basis-point years.
15. In addition, owing to institutional details, the repo spread cannot exceed the general collateral rate, so it is possible to have larger repo spreads when short-term rates are higher.

supply available to the repo market is replenished by the new issuance, overnight repo rates tend to follow the same pattern on average for a reopened issue. Nevertheless, if such a reopening can be forecast ahead of time, all of the repo dividends from the next auction cycle will be capitalized into the value of the current on-the-run security, raising the premium.

A number of factors that affect the total specialness came together to produce a spectacularly large price premium for one issue. In the early 1990s, the Treasury changed the auction cycle for the thirty-year bond from quarterly to semiannually. This change effectively doubled the number of days that the new thirty-year would maintain its on-the-run status. Therefore, it was reasonable to forecast that the total amount of specialness that would accrue to the bond had increased significantly, and capitalizing those increased dividends led to a price premium that was substantially larger than usual.

An increase in expected future short selling drives up the current price of a Treasury bond because future repo dividends are capitalized while the expected return on the security is unchanged.

Once the price premium reached a certain critical amount, another factor entered the picture. Just prior to the change in the auction cycle, the Treasury had committed to reopening any security for which there appeared to be a significant "shortage." A significant price premium was considered to be one symptom of a shortage. When the Treasury changed the auction cycle, there had not yet been an opportunity to demonstrate a willingness to follow through on the stated commitment, and it was widely believed that the Treasury would do so at the first opportunity. With the price premium on the thirty-year bond reaching new heights, it was reasonable to forecast that the Treasury would reopen the bond at the next auction (six months hence) with the result that the bond would remain on the run for a whole year. Given this belief, it was reasonable to forecast the amount of specialness that would accrue to the thirty-year had increased significantly yet again. As a consequence, the price premium increased even more, tending to confirm the belief that the Treasury would reopen the bond, and such a reopening is of course just what occurred.

Not all reopenings increase the length of time a security remains on the run. In the wake of the disaster in New York City on September 11, 2001, the Treasury conducted a surprise reopening of the ten-year note in the middle of the auction cycle. This reopening had the desired effect of increasing the supply available to the repo market and raising overnight repo rates significantly. The reopening apparently also had a salutory effect on other issues as market participants recognized the Treasury's willingness to undertake such reopenings as it saw fit.

Convergence trades. The price premium that the on-the-run bond commands typically disappears by the time it goes off the run; that is, the on-the-run security displays a predictable capital loss relative to the baseline security (which for practical purposes can be taken to be the old security). In other words, the price of the on-the-run security converges to the price of the baseline bond. The popular press has described a *convergence trade* that purports to profit from this price convergence. But, as discussed earlier, the systematic movement in relative prices is offset by the relative financing costs. Although individual episodes may have produced substantial profits for convergence trades, other episodes have produced substantial losses. On average such trades are not profitable.

If uninformed speculators came to dominate the short interest in the on-the-run security in a mistaken attempt to profit from convergence, such speculators would change the dynamics of the auction cycle. Convergence occurs precisely because the shorts (who ordinarily are hedgers) roll out of the current on-the-run security and into the next issue, thereby eliminating the possibility of substantial future repo dividends for the current issue. By contrast, convergence traders (who are also short) will wait until the liquidity premium disappears before they close their short positions. But if convergence traders constitute a sufficient amount of short interest, then their short positions—by themselves—will keep the liquidity premium from disappearing. At this point, other speculators who simply observed the price premium without considering the repo market might conclude that a special profit opportunity had appeared and jump into the convergence trade, further increasing the repo spread and price premium. Those who jumped in early would find the premium has diverged instead of converged.

The Repo Squeeze

The analysis thus far has assumed that the repo spread is determined in a market in which no agent has (or exercises) market power. By contrast, an agent with a sizable position to finance faces an interesting problem—how to finance at the cheapest possible rate given that the amount financed may affect the rate paid. In order to understand the tradeoffs a repo trader faces in choosing the optimal mix, it is necessary to be familiar with a triparty repo.

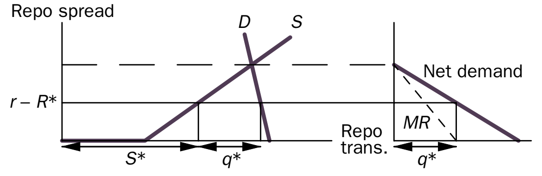
Triparty repo. To obtain reliable sources of funding, dealers quite commonly establish ongoing relationships with customers seeking safe short-term investments for their funds such as repos. To facilitate this relationship, the dealer and the customer may enter into a *triparty repo* agreement in which the third party is a clearing bank. Both the dealer and the customer must have clearing accounts with the bank. The bank provides a number of services, including verifying that the collateral posted by the dealer meets the prespecified requirements of the customer. An important aspect of a triparty repo is that the transfer of collateral and funds between the dealer and the customer occurs entirely within the books of the clearing bank and does not require access to Fedwire. This feature is convenient because it allows for repo transactions to be consummated late in the day after Fedwire is closed for securities transfers, which typically is midafternoon.

The repo squeeze. Suppose a repo trader has a sizable (long) position in a Treasury security to finance. The position may have been acquired outright by the dealer's Treasury desk or it may have been acquired by the repo trader himself via reverse repos for some term to maturity. The collateral can be financed either directly in the market at rate R or via a triparty repo at the general collateral rate, r.[16] What makes this choice interesting is that the amount the trader finances directly in the market may affect the repo rate itself. If the trader's position is substantial, then as more and more collateral is lent directly in the repo market, the special repo rate will rise. In this case, the traders must take care to compute the financing mix that minimizes the total financing cost.

Let Q denote the total amount of collateral to be financed and q denote the amount financed directly in the market so that $Q - q$ is the amount financed via a triparty repo. Therefore, the cost of financing the collateral is $Rq + r(Q - q)$. This financing cost can be rewritten as $rQ - (r - R)q$, which expresses the financing cost as the general collateral rate

CHART 12

Maximizing the Repo Dividend



In the left panel, the demand curve for collateral by the shorts is labeled *D,* and the supply curve of collateral by others is labeled *S.* In the right panel, the difference between *D* and *S* is the net demand facing the trader, and the diagonal dashed line is marginal revenue. The arrow indicates the profit-maximizing (or cost-minimizing) quantity of collateral to supply directly to the market, q^*. The area of the rectangle is the maximized repo dividend, $(r - R^*)q^*$. If the trader's total amount to be financed, Q, is greater than q^*, then the difference, $Q - q^*$, is financed via a triparty repo. The amount supplied by others is S^*.

applied to the total amount of collateral, rQ, less the repo dividend on the amount financed directly in the market, $(r - R)q$. Thus, minimizing the finance cost is the same as maximizing the repo dividend. The problem for a trader with a large position is that an increase in q leads to an increase in R, decreasing the spread, $r - R$. Whether the repo dividend goes up or down when q increases depends on just how responsive the special repo rate is to the amount of collateral lent directly to the market.

In effect, the trader has the same problem as a monopolist: The amount "produced" (q) affects the price ($r - R$). The trader faces a downward-sloping demand curve. In this case, the demand curve facing the trader is a *net demand curve*, in which the supply of collateral by others is subtracted from the demand for collateral by the holders of short positions.[17] This situation is depicted in Chart 12. The quantity that maximizes the repo dividend (q^*) is determined by the condition that marginal revenue be zero. If $Q \leq q^*$, then all the collateral is financed directly in the market. On the other hand, if $Q > q^*$, then q^* is financed directly in the market and $Q - q^*$ is financed via a triparty repo at the higher rate, r.

This situation (that is, when $Q > q^*$) is known as a *repo squeeze*. There are two essential ingredients for a repo squeeze. First, there must be outstanding short positions; otherwise, the security could not go on special. Second, the trader must have possession of the collateral, by having acquired it outright or

16. The dealer's counterparty on a triparty agreement has no interest in whether any of the collateral in its triparty repo account at its clearing bank is on special, and it will not accept less than the general collateral rate on its loans to the dealer secured by that collateral.

17. The trader plays the role of the dominant firm among a competitive fringe of other suppliers.

via term reverse repos. (For collateral acquired via reverse repo, the term of the repo limits the duration of the repo squeeze; when the reverses mature, the collateral must be returned.) One should recognize that the "profits" from a repo squeeze come from driving the repo spread up and earning a larger repo dividend than otherwise. While it is true that a repo squeeze drives the price of the security higher than it otherwise would be, this feature is a side effect. Since a squeeze can be maintained only by one who controls the collateral, selling the security is counterproductive.

Note that if the repo squeeze is fully anticipated, the shorts bear no cost since they establish their short positions at appropriately high prices. By the same token, the trader earns no profits from a fully anticipated squeeze since the prices at which he acquired the security fully reflected his actions. Thus, a repo squeeze is profitable only if it is not fully anticipated.

Conclusion

This article has presented the somewhat surprising proposition that an increase in expected future short selling drives up the current price of a Treasury bond because future repo dividends are capitalized while the expected return on the security is unchanged. The repo dividends arise when a bond goes on special—that is, when the bond's repo rate falls below the risk-free rate. The liquidity premium for an on-the-run Treasury security can be attributed to this effect.

The premium goes away when the bond goes off the run because the holders of short positions roll out of the current issue and into the new issue, thereby eliminating the possibility of significant future repo earnings. The on-the-run security's predictable capital loss relative to other bonds is offset by its financing cost relative to other bonds. Consequently, there are no profits to be made from so-called convergence trades on average.

APPENDIX 1

Term Structure of Repo Spreads

Thus far, the discussion has considered only one-period repo transactions, that is, those in which the securities have been repoed (or reversed) for one period only. However, term repo transactions are quite common. In a term repo, a single, fixed repo rate is agreed to at the inception of the transaction. Perhaps the best way to think about term repo rates is as a way of quoting forward prices for delivery more than one period in the future.

Consider an n-period bond that has a current price of p. To establish a long forward position in the bond for delivery in two periods, one buys the bond and repos it in the term repo market for two periods at the rate of R_2 per period. The current cash flow is zero. At time two, one pays $(1 + R_2)^2 p$ and receives the bond. Thus, $F_2 = (1 + R_2)^2 p$, where F_2 is the forward price of a given bond for delivery in two periods. In general, the forward price for delivery in m periods is $F_m = (1 + R_m)^m p$, where R_m is the m-period repo rate (per period) and F_m is the forward price for delivery in m periods.

Compare the cost of financing a bond for m periods with a term repo versus rolling over one-period financing. In the first case the cost is $(1 + R_m)^m$ while in the second case the cost is $\Pi_{i=0}^{m-1}(1 + R^{(1)})$. When there is no uncertainty, these two costs must be the same, implying an expectations hypothesis for repo rates: $R_m \approx \Sigma_{i=0}^{m-1} R^{(i)}/m$.

Consider again the price premium, $\psi = \log(p/\overline{p})$. For the baseline n-period bond, $\overline{p} = 1/(1 + r_n)^n$, where r_n is the yield to maturity of the baseline bond (which earns no repo dividends). The price premium can be reexpressed as

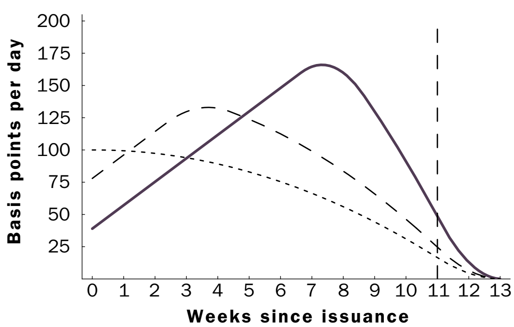
$$\text{(A1.1)} \quad \psi = \log\left(\frac{1/(1+R_n)^n}{1/(1+r_n)^n}\right) \approx n(r_n - R_n) = ns_n,$$

where $s_n = r_n - R_n$ is the term repo spread. Comparing the expression for ψ in equation (A1.1) with that in equation (15) yields $s_n \approx \Sigma_{i=0}^{n-1} s^{(i)}/n$, which shows that the term repo spread is (approximately) the average of the one-period repo spreads. Moreover, the term structure of repo spreads can be used to forecast the dynamics of the price premium. In particular, the n-period change in the price premium is approximately equal to (the negative of) the n-period term repo spread, $\psi^{(n)} - \psi \approx -s_n$, where $\psi^{(n)}$ is the price premium n periods later. Uncertainty, of course, complicates matters a bit: $E[\psi^{(n)}] - \psi \approx -s_n$.

The chart displays the average pattern of three term repo spreads over the course of a three-month auction cycle. The term spreads are computed in accordance with the expectations hypothesis from the average pattern of overnight spreads shown in Chart 10. These term repo spreads are in line with those observed on the market.

CHART

The Term Structure of Repo Spreads



Term repo spreads are computed from the stylized overnight repo spreads in Chart 10: thirty-day (solid), sixty-day (dashed), and ninety-day (dotted). When the security is issued at week 0, the term structure slopes upward, anticipating the rise in overnight rates; the thirty-day spread is about 39 basis points while the ninety-day spread is about 100 basis points. By week 8, the term structure slopes steeply downward; the thirty-day spread is about 160 basis points while the ninety-day spread is about 56 basis points.

APPENDIX 2

Forward Prices and the Expectations Hypothesis

The forward rate, F, can be used to forecast the bond price next period, p'. A linear forecast has the form $\hat{p}' = \alpha + \beta F$, where $\hat{p}'$ is the forecast. The coefficients α and β are constants that can be chosen to produce unbiased forecasts and to minimize the variance of the forecast error, $p' - \hat{p}'$. The slope coefficient, β, is computed as

$$\text{(A2.1)} \qquad \beta = \frac{\mathrm{Cov}[p', F]}{\mathrm{Var}[F]},$$

where $\mathrm{Cov}[p', F]$ is the covariance between p' and F and $\mathrm{Var}[F]$ is the variance of F.[1] If the expectations hypothesis holds, then $\beta = 1$, in which case $\hat{p}' = \alpha + F$ and changes in the forecast ($\Delta\hat{p}'$) correspond to changes in the forward price (ΔF).[2] The forward risk premium plays a central role in determining whether the expectations hypothesis holds. It will be shown that if π is constant (that is, if π is the same in every period), then $\beta = 1$.

The strategy is to write both F and p' in terms of $E[p']$ by using the definition of the forward premium in equation (7) and the relation between next period's price and its current expectation in equation (6). Substituting these expressions for F and p' into equation (A2.1) produces

$$\text{(A2.2)} \qquad \beta = \frac{\mathrm{Cov}[E[p'] + \varepsilon, E[p'] - \pi]}{\mathrm{Var}[E[p'] - \pi]}.$$

In the numerator of equation (A2.2), the properties of covariances and the independence of ε imply[3]

$$\begin{aligned}\mathrm{Cov}[E[p'] + \varepsilon, E[p'] - \pi] &= \underbrace{\mathrm{Cov}[E[p'], E[p']]}_{=\mathrm{Var}[E[p']]} \\ &\quad - \mathrm{Cov}[E[p'], \pi] \\ &\quad + \underbrace{\mathrm{Cov}[\varepsilon, E[p']]}_{=0} - \underbrace{\mathrm{Cov}[\varepsilon, \pi]}_{=0} \\ &= \mathrm{Var}[E[p']] - \mathrm{Cov}[E[p'], \pi].\end{aligned}$$

The regression coefficient can now be reexpressed:

$$\text{(A2.3)} \qquad \beta = \frac{\mathrm{Var}[E[p']] - \mathrm{Cov}[E[p'], \pi]}{\mathrm{Var}[E[p'] - \pi]}.$$

Finally, if π is constant, then equation (A2.3) reduces to

$$\beta = \frac{\mathrm{Var}[E[p']]}{\mathrm{Var}[E[p']]} = 1.$$

A number of empirical studies have purported to estimate β for U.S. data. These estimates typically reject the hypothesis that $\beta = 1$ and conclude therefore that π is not constant. The following discussion illustrates how these studies may have incorrectly computed forward rates. Consequently, the slope coefficient that was estimated involves additional factors that were ignored.

Pseudo forward rates. The way of computing forward prices that has been used in many empirical studies is incorrect and can lead to spurious rejections of the expectations hypothesis.

As before, let p denote the current price of an n-period bond (where in this case $n \geq 2$). Define the *pseudo forward price* of the bond as $\tilde{F} = p/p_1$, where p_1 is the current price of a one-period bond. The price of the one-period bond can be written in terms of its own one-period repo rate: $p_1 = 1/(1 + R_1)$, where R_1 is the one-period repo rate for the one-period bond. Thus, the pseudo forward price can be expressed as $\tilde{F} = (1 + R_1)p$. Comparing this expression for $\tilde{F}$ with the expression for the true forward price, $F = (1 + R)p$, shows that $\tilde{F}$ uses the wrong repo rate. The definition of the pseudo forward price implicitly assumes one can finance the n-period bond at R_1 rather than at its own repo rate R.

Defining the pseudo forward premium, $\tilde{\pi} = E[p'] - \tilde{F}$, note that $\tilde{\pi} = (E[p'] - F) + (F - \tilde{F}) = \pi + (R - R_1)p$. In other words, the pseudo forward premium equals the true forward premium plus another term that reflects the difference between the two repo rates. Even if the true forward premium were identically zero, the pseudo forward premium would equal $(R - R_1)p$.

Suppose the pseudo forward price (instead of the true forward price) is used to forecast the price of the bond next period. Let the linear forecast be given by $\tilde{\alpha} + \tilde{\beta}\tilde{F}$, where the coefficients $\tilde{\alpha}$ and $\tilde{\beta}$ are chosen to minimize the forecast error. The slope coefficient is

$$\text{(A2.4)} \qquad \tilde{\beta} = \frac{\mathrm{Cov}[p', \tilde{F}]}{\mathrm{Var}[\tilde{F}]}.$$

APPENDIX 2 (continued)

If π is constant but $R_1 - R$ is random, then $\tilde{\beta} \neq 1$. Following the steps above that lead from equation (A2.1) to equation (A2.3), but replacing F and π with $\tilde{F}$ and $\tilde{\pi}$, leads from equation (A2.3) to

$$\text{(A2.5)} \qquad \tilde{\beta} = \frac{\text{Var}[E[p']] - \text{Cov}[E[p'], \tilde{\pi}]}{\text{Var}[E[p'] - \tilde{\pi}]}.$$

By the properties of variance,[4] equation (A2.5) can be expressed as

$$\text{(A2.6)} \qquad \tilde{\beta} = \frac{A}{A+B},$$

where $A = \text{Var}[E[p']] - \text{Cov}[E[p'], \tilde{\pi}]$ and $B = \text{Var}[\tilde{\pi}] - \text{Cov}[E[p'], \tilde{\pi}]$. If $B = 0$, then $\tilde{\beta} = 1$. If π is constant, then $\tilde{\pi} = (R - R_1)p$ and $B = \text{Var}[(R - R_1)p] - \text{Cov}[E[p'], (R - R_1)p]$. In general, $B \neq 0$.

1. Conditionally (that is, given the information available at the beginning of the current period), F and $E[p']$ are known constants. However, over time F and $E[p']$ vary from period to period. Therefore, unconditionally, they are random variables with nonzero variances and covariances.
2. Strictly speaking, by itself $\beta = 1$ characterizes the weak form of the expectations hypothesis. The strong form also requires $\alpha = 0$.
3. (i) $\text{Cov}[a+b, c+d] = \text{Cov}[a,c] + \text{Cov}[a,d] + \text{Cov}[b,c] + \text{Cov}[b,d]$ and (ii) $\text{Cov}[a,a] = \text{Var}[a]$. If a is independent of b, then $\text{Cov}[a,b] = 0$.
4. $\text{Var}[a-b] = (\text{Var}[a] - \text{Cov}[a,b]) + (\text{Var}[b] - \text{Cov}[a,b])$.

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