Towards a decision support system for hierarchically allocating marketing resources across and within product groups

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In this paper a marketing planning system for the hierarchical allocation of marketing resources to product groups and products within groups is discussed and illustrated. Linkage between hierarchical decision levels is achieved through constraint co-ordination.

1. Introduction

In the last ten to twenty years, a large number of marketing models have been developed. Most of these models deal with resource allocation decisions concerning single products or brands. Yet, a majority of companies are of the multiproduct type, and do not treat marketing decisions for single products in isolation. The more descriptive marketing literature has since long paid attention to integration of single product strategies in a multiproduct context. The number of analytical approaches treating this problem in an operational way is, however, very limited.[1]

In this paper we briefly describe the essence of what could be a multiproduct marketing decision support system. It concerns a planning system that essentially deals with the allocation of a global marketing communication budget to different product groups and products within each group.

As a starting point for the analysis, a brief description of the existing planning system is provided. Next, the newly developed approach is outlined. To preserve confidentiality, the discussion must sometimes be kept rather vague; and financial figures will be expressed in K Belgian francs, where the proportionality factor K is not specified any further.

2. The existing planning system

In the company studied, the annual planning cycle starts at the beginning of April and ends in August or September. Roughly, the process can be outlined as follows.

In a first step the marketing director – after consultation with the product managers – decides which products will deserve special attention during the next year. Normally these are 'new' products, or products which have been on the market for at most about two years.

In the industry in which the company operates, new product introduction must be backed by very heavy marketing spending. Accordingly, new products absorb about 80% (sometimes up to 90%) of overall marketing resources. These

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[1] A few examples are Larréché and Srinivasan [6, 7], Lodish [9] and Gijsbrechts and Naert [4, 5].

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are mainly used for personal selling, but also for advertising, sales support brochures and other materials.

For the immediate future the rate of new product introduction is expected to drop. As a result more resources will be released for support of existing products.

As far as these existing products are concerned, resource allocation decisions are based in part on the following relationship between market share (m), communication share (A) and number of years since introduction of the product (t'):[2]

$$\frac{m_g^i(t)}{A_g^i(t)} = \alpha_g t_i' . \tag{1}$$

This model is based on the following reasoning: for a new product the amount of marketing resources needed to obtain a certain level of market share is considerably higher as compared to the budget required for an 'existing' product, which has already gained market share as well as some goodwill.

The company assumes that the proportionality constant α_g is not product-specific, but valid for the product group as a whole.

To enable a clear comparison with the new system, equation (1) is rewritten as

$$m_g^i(t) = \alpha_g t_i' A_g^i(t) + \varepsilon_t^i , \tag{2}$$

where an error term ε_t^i has been added. The parameter α_g in equation (2) can be estimated using ordinary least squares regression (OLS). Group 1 ($g = 1$) is used as illustration. Five annual data points are available for each of the eight products belonging to that group. Using the total of 40 observations for estimation we obtain for the parameter α_1 an estimated value $\hat{\alpha}_1 = 0.015$. The t-statistic corresponding to the zero-value – null hypothesis equals $t_{\hat{\alpha}_1} = 8.81$. The coefficient of determination is $R^2 = 0.477$.[3]

[2] For the detailed definition of the various symbols, see the glossary in the appendix.

[3] R^2 is defined as $R^2 = (1 - \Sigma_{i,t} (\hat{\varepsilon}_t^i)^2 / \Sigma_{i,t} (m_g^i(t) - \bar{m}_g)^2)$, with $\hat{\varepsilon}_t^i = m_g^i(t) - \hat{m}_g^i(t)$, $\bar{m}_g = \Sigma_{i,t} m_g^i(t)/n_g$. T and $\hat{m}_g^i(t)$ the estimated value of market share. Since equation (2) does not contain a constant term, it would be appropriate to define explanatory power as $R^2 = (1 - \Sigma_{i,t} (\hat{\varepsilon}_i^t)^2) / \Sigma_{i,t} (m_g^i(t)^2)$.

Next, every product manager is asked to specify a market share objective $m_g^i(p)$ for each year p in the planning period. From equation (1) the communication share $A_g^i(p)$ needed to reach this objective is then computed for each year.

Total communication expenditures in the product group $a_{g,\text{tot}}(p)$, for subsequent years, are predicted using simple extrapolation methods. These predicted figures then allow for computation of the marketing resources needed by the company for each product and period:

$$a_g^i(p) = A_g^i(p) a_{g,\text{tot}}(p) .$$

This figure is of course not arrived at in a purely mechanistic way. But deviations must be explainable by product management. If not, significant deviations between resources demanded by product management and those calculated following the procedure described above, will lead to a rejection of the budget proposal by the marketing planning manager.

Usually, the sum of the submitted budgets $\Sigma_{i,g}\, a_g^i(p)$ will exceed $a(p)$, the global communication budget corporate headquarters is willing to spend in the country.

However, when the local management then has to adjust its budget proposals downwards, it is somewhat at a loss, because the adjustments are made without explicit reference to market share response functions. The tendency is then quite often to apply the same percentage reduction for each of the products.

Once a budget $a_g^i(p)$ is determined, it is split up over different marketing communication instruments: personal selling, advertising and sales promotion. This allocation is based on average figures for the industry, independent of the product or product group studied. The expenditures ultimately realised may somewhat deviate from the average figures for practical reasons, such as the fact that the number of salesmen must be an integer.

This is a brief description of the basic elements of the existing planning system. Recently the marketing director and the marketing planning manager have felt a need to improve these existing practices or – as they call it – to introduce some marketing science into the planning process. This is to be the starting point for the development of a new planning system.

While we were not actually involved in designing and implementing a new system, the matching of our interests with those of the company management provided the basis for a mutually useful collaboration: we obtained access to a practical problem, management time and relevant data, whereas management benefited from our reflections and analysis. What follows in the remainder of this paper is the way we felt the planning system could be improved.

3. Towards an improved planning system

The complete planning system can be schematically outlined as in Fig. 1. Ultimately the marketing director wants to achieve marketing science in every part of the planning process: starting from a systematic treatment of new product introduction decisions (module I), up to an improved budget allocation to separate marketing instruments for each product (module IV).

It was decided that in the first stage of analysis special attention would be paid to the estimation of product specific market share response functions (module II), and use of these functions for judicious marketing allocation to the different products (module III). The description will therefore be limited to a further analysis of these two modules.

3.1. Module II: Estimation of market share response functions

The aim of module II is to assess, for each of the company's products, a market share response function that is an improvement over equation (2). Fig. 2 outlines the basic steps of the module. For new products it is still possible to reserve certain budgets without explicit consideration of response functions (arrow 2 in Fig. 1). However, the new planning system also allows for estimation of market share relationships for these new products, if one so desires. Such estimation could be based on objective information, if the product has been on the market for a few months. Indeed, the company's marketing research department possesses monthly data on competitive and company products, such that the diffusion of a new product in the market can be closely monitored. If objective data are too small in number or do not yield managerially or statistically meaningful information, subjective data can be generated. For this purpose one could, for example, let product managers answer the following type of question: Given a market share of $x\%$ in the previous period, and if for the next period a $y\%$ communication share is planned, what level of market share do you expect to reach in this next period? Our first experience with subjective estimation in that company has been that managers – even those responsible for

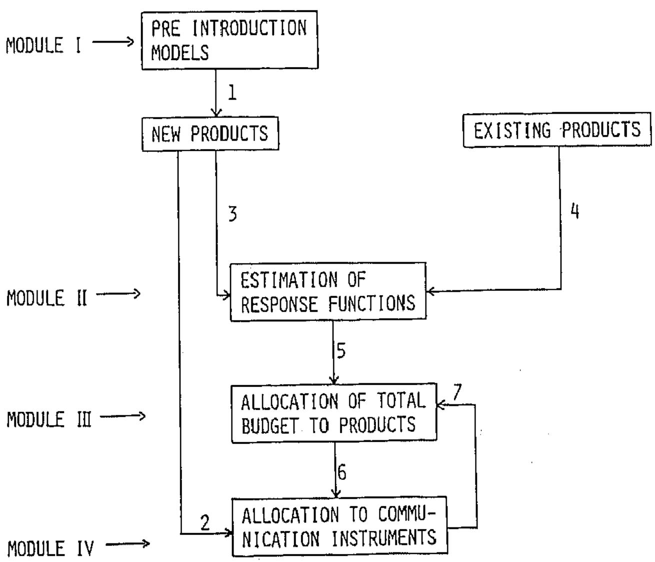

Fig. 1. Elements of the improved planning system.

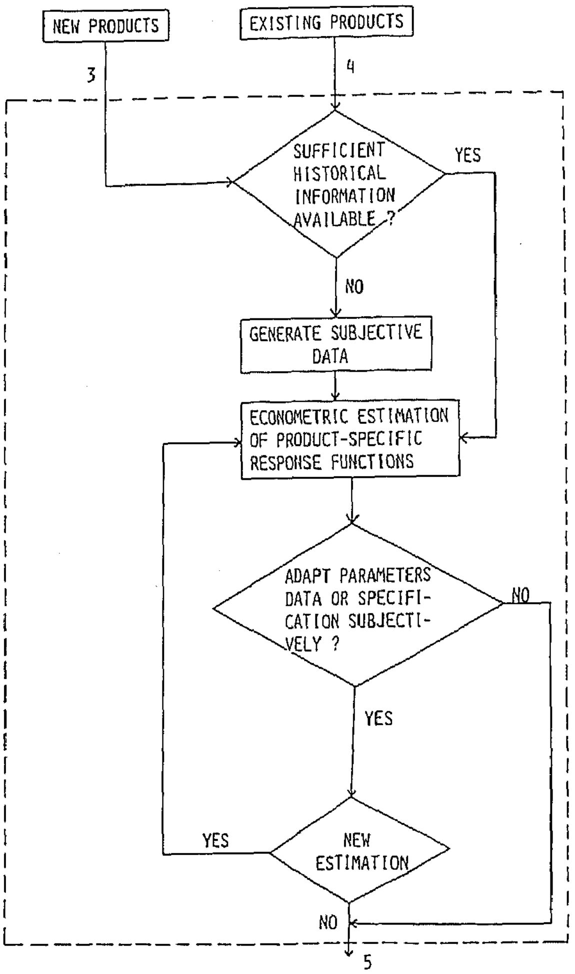

Fig. 2. Submodule II: estimation of market share functions.

marketing planning – have a hard time to separate short-term from long-term effects.[4]

Subjective expectations or objective data constitute in module II the inputs for econometric estimation of market share response functions, for both new and existing products. As indicated previously, these functions should be an improvement over the market share model used in the past (equations (1) and (2)). Indeed, the latter model exhibits important gaps, such as the fact that market share in a period p does not depend on share reached in the previous period: using the number of years since introduction as explanatory variable is indeed a very rough way of accounting for lagged effects. Furthermore, for a given value of t_i', market share is assumed proportional to communication share, a supposition which over the years has been countered by empirical evidence.

On the other hand, from an implementation point of view, it is desirable that newly proposed market share functions do not deviate too much from the presently used model, at least not in the early stages of decision support system development. Equation (3) answers this requirement:

$$m_g^i(t) = \alpha_g + \lambda_g m_g^i(t-1) + \beta_g A_g^i(t) + \varepsilon_t^i \,, \qquad (3)$$

For product group 1 this model yields the following estimation results:

$$\hat{\alpha}_1 = 0.0089\,, \quad t_{\hat{\alpha}_1} = 1.28\,,$$
$$\hat{\lambda}_1 = 0.8506\,, \quad t_{\hat{\lambda}_1} = 32.81\,,$$
$$\hat{\beta}_1 = 0.0926\,, \quad t^{\beta}{}_1 = 1.44\,,$$
$$R^2 = 0.9788\,.$$

Though the explanatory power has clearly increased compared to model (2), analysis of the residuals in Fig. 3 reveals that product 3 significantly differs from the rest of the group.

After inclusion of a separate constant term for product 3, α_1^3, the following results are obtained:

$$\hat{\alpha}_1 = 0.0022\,, \quad t_{\hat{\alpha}_1} = 0.08\,,$$
$$\hat{\alpha}_1^3 = 0.0520\,, \quad t_{\hat{\alpha}_1^3} = 10.72\,,$$
$$\hat{\lambda}_1 = 0.8572\,, \quad t_{\hat{\lambda}_1} = 73.30\,,$$
$$\hat{\beta}_1 = 0.0830\,, \quad t_{\hat{\beta}_1} = 2.86\,,$$
$$R^2 = 0.9958\,.$$

For the product group considered, it certainly pays to take into account the unique characteristics of one of the products, not in the first place in terms of explanatory power, but primarily in view of the increased reliability of the estimated parameters.

Of course, equation (3) is only one of many possible forms of market share function. Given its simplicity and high descriptive validity, it is

[4] On the problem of the capacity of people to provide reliable judgmental estimates see, for example, Chakravarti, Mitchell and Staelin [1], and Little and Lodish [8].

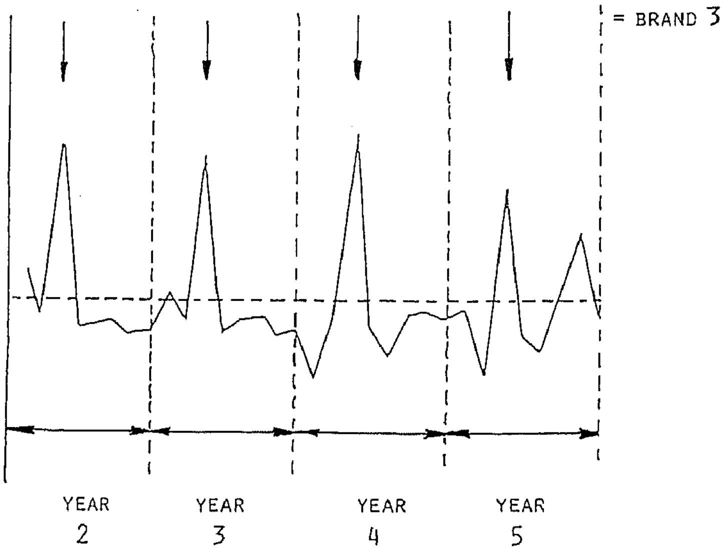

Fig. 3. Difference between real and estimated market share.

certainly very appealing. Keeping in mind, however, that the estimated market share relationships will have to serve as a basis for allocation in module III, it is interesting to examine some additional model characteristics.

First, we note that, since the estimated parameters are not product, but group-specific (except for α_1^3), and given its additive form, equation (3) implies equal gains in market share from increased communication share for every product in the group. Secondly, for fixed levels of $a_{g,\mathrm{tot}}(t)$, market share for a product becomes a linear function of communication expenditures for the product.[5]

It should be clear that the specification is not imposed, but comes about in consultation with the management, based on empirical findings, theoretical considerations, and the desire not to make models more complex than is indicated by the ultimate objectives management has in using them.

One of the managers had more confidence in S-like market share relations, for which the following specification was proposed:

$$m_g^i(t) = \gamma_g m_g^i(t-1) + \beta_g(1 - \alpha_g m_g^i(t-1)) \times \frac{A_g^i(t)^{\delta_g}}{\xi_g + A_g^i(t)^{\delta_g}} + \varepsilon_t \,. \tag{4}$$

Within the class of logistic specifications, model (4) is only one of many possible ways of expressing the relation between market share, lagged share and communication share. Some of its desirable characteristics are the following:

(i) The interaction effect between $m_g^i(t-1)$ and $A_g^i(t)$ is modelled in a flexible way by introducing the parameters β_g and α_g. Such flexibility is needed because the interaction consists of several components: first, there is a negative impact in the sense that 'loyal loyal' buyers (i.e. the fraction $\gamma_g m_g^i(t-1)$) disappear from the potential market of persons to be convinced through $A_g^i(t)$. Second, there may be a positive impact, as persons who have bought the product in the previous period but are not 'loyal loyal' (i.e. the fraction $(1-\gamma_g)(m_g^i(t-1))$) can be more easily convinced to use the product again if $A_g^i(t)$ is high. Also, the effect of $A_g^i(t)$ can serve as reinforcement to word-of-mouth communication on previous non-users (i.e. the fraction $1 - m_g^i(t-1)$). The net interaction effect may thus

[5]This is not as undesirable a characteristic as it may seem, since keeping $a_{g,\mathrm{tot}}(t)$ constant implies that giving increased communication support to one product is compensated by decreased support for other products. It is unlikely that such will systematically be the case.

be either positive or negative. A priori, however, a model describing the impact of $m_g^i(t-1)$ and $A_g^i(t)$ on $m_g^i(t)$ should allow for both positive and negative interdependence. Adding the two parameters β_g and α_g provided the necessary flexibility.

(ii) The data requirements are not dramatically increased as compared to equation (3), because the model parameters are group-specific. At the same time, equation (4) allows for varying communication effectiveness across products in a given group, since it now depends on a product's previous market share.

(iii) It is easy to see that for given $\alpha_{g,\mathrm{tot}}(p)$, the specification remains non-linear in communication budgets. If δ_g is larger than 1, the function will be S-shaped, whereas for δ_g between 0 and 1, it shows decreasing returns over the whole range of communication budgets.

Given this non-linearity, parameterization of equation (4) requires the adoption of an appropriate non-linear estimation routine. The relationship is estimated for product group 1, after inclusion of a constant term α_1^3 for product 3, and under the assumption that for $A_g^i(t)$ equal to 1, long-term market share of the latter product equals 1. This assumption, which seems logical since product 3 possesses exceptional characteristics, reduces the number of parameters to be estimated by one, since it implies that β_1 can be computed as

$$\beta_1 = \frac{(1-\gamma_1-\alpha_1^3)(1+\xi_1)}{(1-\alpha_1)} .$$

The following estimation results are obtained:

$$\hat{\gamma}_1 = 0.441 , \quad t_{\hat{\gamma}_1} = 2.199 ,$$

$$\hat{\delta}_1 = 0.955 , \quad t_{\hat{\delta}_1} = 9.950 ,$$

$$\hat{\xi}_1 = 0.0281 , \quad t_{\hat{\xi}_1} = 1.308 ,$$

$$\hat{\alpha}_1^3 = 0.054 , \quad t_{\hat{\alpha}_1^3} = 18.102 ,$$

$$\hat{\alpha}_1 = -33.82 , \quad t_{\hat{\alpha}_1} = -1.891 ,$$

$$\hat{R}^2 = 0.9983 .$$

With an R^2 of 0.9983, the model's explanatory power again appears to be high.

Figs. 4a, 4b and 4c provide a graphical representation of the estimated market share evolution for products 1, 2 and 3, respectively. Given the product's current market share level, each figure depicts market share in the following year as a function of communication share in that year. Long-term (equilibrium) market share relationships are also drawn. Since δ_g is smaller than 1, each function shows decreasing returns to scale.

As indicated before, and following our specification, the long-term effectiveness of communication efforts is much higher for product 3 than for the other products. Another notable point is that the long- and short-term relationships for product 1 have no intercept: no matter how much is spent on this product, market share will always drop below the current level. This is explained by the recent introduction of a superior competitor, product 3, in the market. Returning to the estimation results, we note that the t-values – which have only asymptotic meaning here since a non-linear estimation technique was used – point to a low statistical significance of some parameters. A closer look at the data set suggests that we are confronted with a multicollinearity problem. Since this prohibits the estimation routine from making a clear-cut distinction between the effects of explanatory factors in the model, subjective adjustment of some parameters might be called for.

As indicated in Fig. 2, module II of the new planning system builds in the possibility of subjectively adapting model parameters or data. Changing model parameters is not an arbitrary process, but comes down to bringing into account additional information not present in the original data.[6] Suppose, for instance, that an important competitor is about to introduce a superior product in the market. This will probably imply a decrease in the effectiveness of our own marketing effort. Or assume that our advertising agency has developed an outstanding copy, such that our budgets will become more effective. In both cases adjustment of originally estimated parameters may, from a managerial point

[6] For a more detailed discussion about combining observed and subjected data, see, for example, Naert and Weverbergh [11].

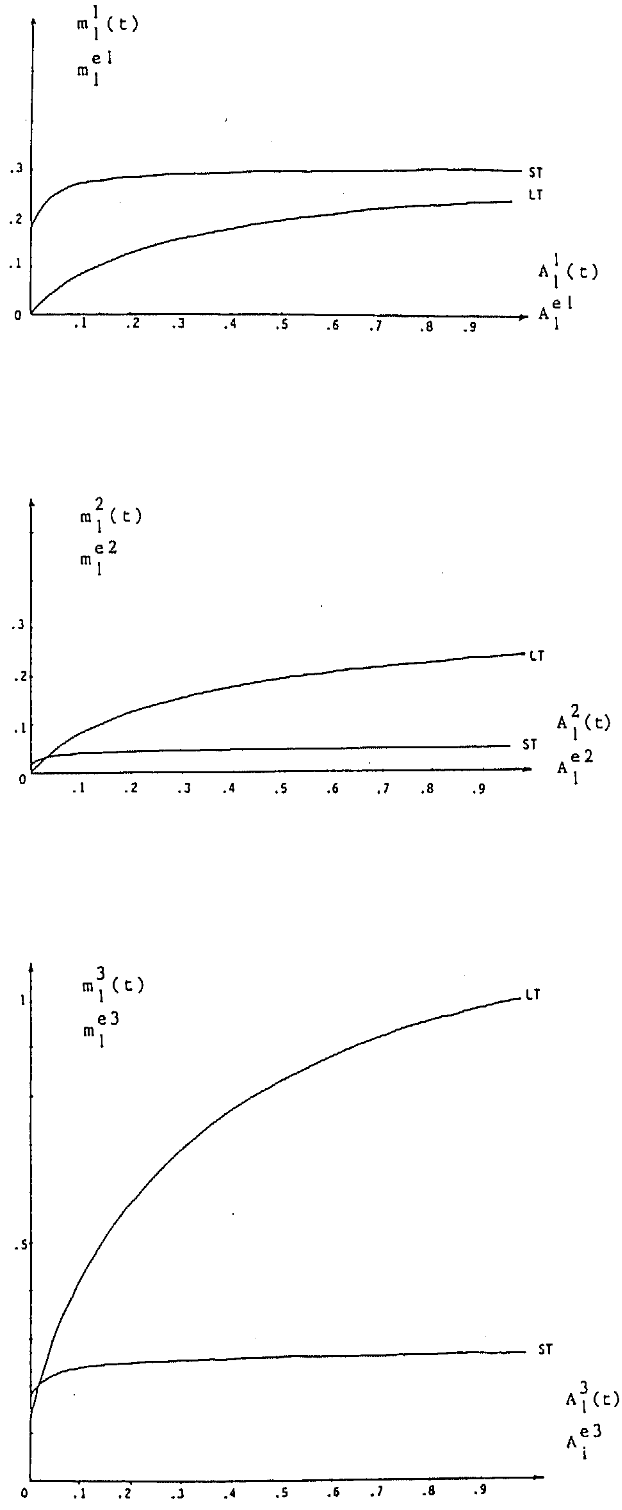

Fig. 4. (a) Short- and long-term market share for product 1, group 1; (b) short- and long-term market share for product 2, group 1; (c) short- and long-term market share for product 3, group.

of view, be desirable. The need for subjective parameter adaptation is easily explained if historical information is used for estimation. But also if parameterization is based on subjective data, adaptation may be appropriate, e.g. if the ultimate form of the estimated equation is not as expected. In the latter case, instead of changing the parameters, one could also adapt the original data and re-estimate the model. Ultimately, the output of module II should be a market share response function, for each (company) product, that forms an acceptable and adequate basis for budget allocation.

3.2. Module III: Allocating the total budget to company products

As indicated in Fig. 5, the allocation procedure proposed in module III is quite different from previous company practices.[7] Let us describe it in more detail here.

From module II, market share response functions for each product (group) enter the allocation module (arrow 5 in Figs. 1 and 2). The linear market share relationships (equation (3)) provide satisfactory estimation results for each product group. For given levels of competitive communication, $a_g^c(p)$, they further imply decreasing returns to scale, such that they appear valuable as a basis for allocating budgets to different product groups. They do, however, pose a problem for those product groups in which the company offers more than one product: as indicated previously, equation (3) is indifferent to how resources are allocated to products within the same group.

Keeping this in mind, a two-step allocation procedure is proposed. In the first step, resources will be allocated across products within the same group (submodule IIIa) on the basis of more detailed response functions (such as equation (4)). The information obtained from this allocation effort will be used to properly evaluate the communication effectiveness for the product group as a whole. It thus forms a basic input to the second step (submodule IIIb), where allocation across product groups will be handled and simpler linear functions used.

[7]As pointed out earlier, module IV has not been developed yet. That explains why we do not find a feedback from module IV to module III in Fig. 4.

The marketing planning department is essentially responsible for marketing budget allocation in the next year. It is obvious, however, that the short-term strategy should be consistent with the company's long-term planning. Consequently, it is useful to examine the effect on marketing strategy of a change in planning horizon *P*. That explains why the horizon problem will be given some attention later on.

Given current company practices, the allocation procedure proposed here may not be considered a straightforward one. In what follows, the underlying logic of the two steps and some potential implementation issues are explained in more detail.

*3.3. Module III*a: *Budget allocation within a product group*

In submodule IIIa the following basic model structure is used:

$$\begin{aligned} &\max\ \pi_g = \sum_{p=1}^{P} \sum_{i=1}^{n_g} \pi_g^i(p)(1+r_g)^{1-p} \\ &\text{s.t.}\ \sum_i a_g^i(p) = a_g(p)\,, \qquad \text{all } p\,, \qquad (5) \\ &l_g^i(p) \le a_g^i(p) \le u_g^i(p)\,, \quad \text{all } i,\ p\,. \end{aligned}$$

In the objective function, $\pi_g^i(p)$ is defined as

$$\pi_g^i(p) = m_g^i(p) M_g(p) c_g^i(p) - a_g^i(p)\,. \qquad (6)$$

The discount rate, r_g, may vary across product groups, e.g. to express differences in uncertainty about the ultimate sales level. The constraints will usually be strategic in nature. Management could, for instance, reserve a minimum communication budget for a new product, or impose an upper bound on resources spent on a product considered for withdrawal. Using the model as a basis, the company will be able to compute opportunity costs associated with some upper or lower bound.

As a first illustration of model use, we consider the situation in product group 1, where the company offers two different products. In this

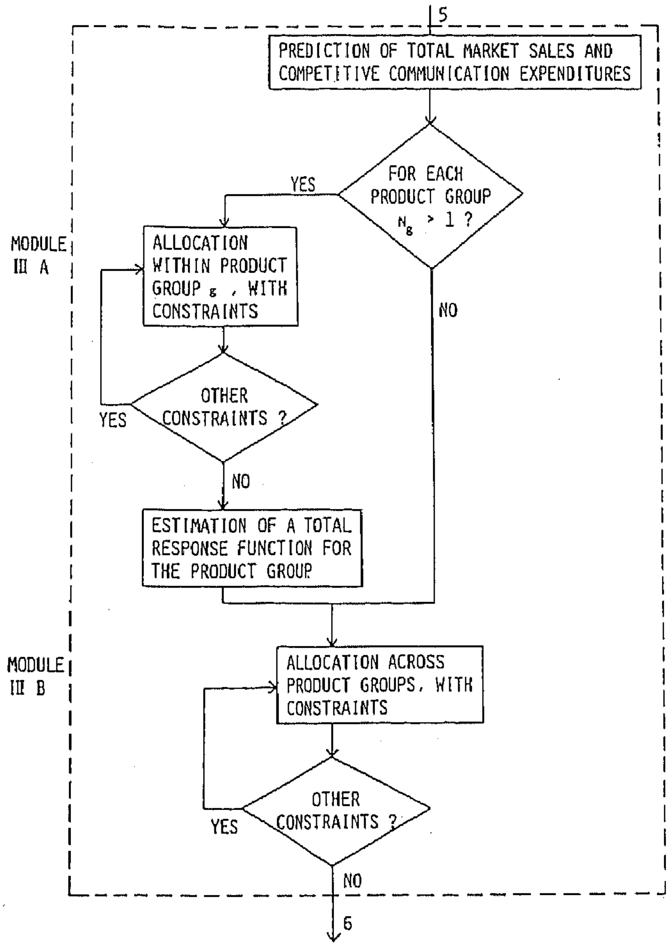

Fig. 5. The allocation module (module III).

group the current situation (time index 0) can be characterized as follows:[8]

$$c_1(0)M_1(0) = 65K, \quad a_1^c(0) = 3.8K,$$
$$a_1(0) = 0.23K,$$
$$a_1^1(0) = 0.1245K, \quad a_1^2(0) = 0.1055K,$$
$$A_1^1(0) = 0.031, \quad A_1^2(0) = 0.026,$$
$$m_1^1(0) = 0.304, \quad m_1^2(0) = 0.030.$$

Suppose, for the time being, that the following conditions will hold over the planning period ($P = 5$):

$$c_1(p)M_1(p) = 65K \quad p = 1, \ldots, 5,$$
$$a_1^c(p) = 3.8K \quad p = 1, \ldots, 5,$$
$$a_1(1) = 0.23K, \quad a_1(p) = 0.5K \quad P = 2, \ldots, 5,$$
$$r_1 = 0.2,$$

and that no additional restrictions are imposed. Furthermore, the estimated equation (4) is accepted as a valuable response function for both company products in the group. If the allocation scheme of period 0 will be accepted again in period 1, resulting profit in that period will amount to $\pi_1(1) = 17.04K$.

On the other hand, optimal allocation over the next five years – which is obtained through solution of the non-linear programming model (5) –

[8]Both products have the same profit margin.

will yield $\pi_1(1)^* = 17.5K$, or a 2.7% profit increase. Since K is large, this 2.7% increase represents a significant absolute profit gain. The associated optimal allocation of communication budgets is given by

$$a_1^1(1)^* = 0.2133\,K \quad \text{and} \quad a_1^2(1)^* = 0.0177$$

Solving model (5) for alternative levels of $a_g(p)$, we can analyse the optimal allocation of these budgets across company products in the group. Fig. 6 indicates how, from a long-term point of view (horizon $P = 5$), the company budget for group 1 in the next period can be optimally divided across products 1 and 2.

In the next step, we can study the relationship between $\pi_g^*(p)$ and $a_g(p)$, and between π_g^* and $a_g(p)$. For $p = 1$ the results are depicted in Figs. 7 and 8. Fig. 7 shows optimal profit (i.e. corresponding to an optimal within group allocation) for group 1 in period 1, as a function of communication expenditures in product group 1. Fig. 8 shows optimal profit for group 1, but now over the planning horizon.

These figures are particularly interesting, since they indicate the *optimal* (short- and long-term, respectively) profit for group 1 associated with any global budget $a_1(1)$ available for that group in the first period. Otherwise stated, the relationships implicitly assume optimal allocation of resources $a_1(1)$ across products in group 1, with reference to a planning period profit P. From Fig. 7, we derive that optimal first-period profit $\pi_1^*(1)$, reaches a maximum for $a_1(1) = 1.07K$. Fig. 8 indicates that from a longer term point of view ($P = 5$), $a_1(1)$ should optimally exceed 2 K. Given a current level of 0.23 K, it is not very likely that the company would suddenly be willing to spend that high an amount!

One must also keep in mind that the foregoing results are associated with the previously

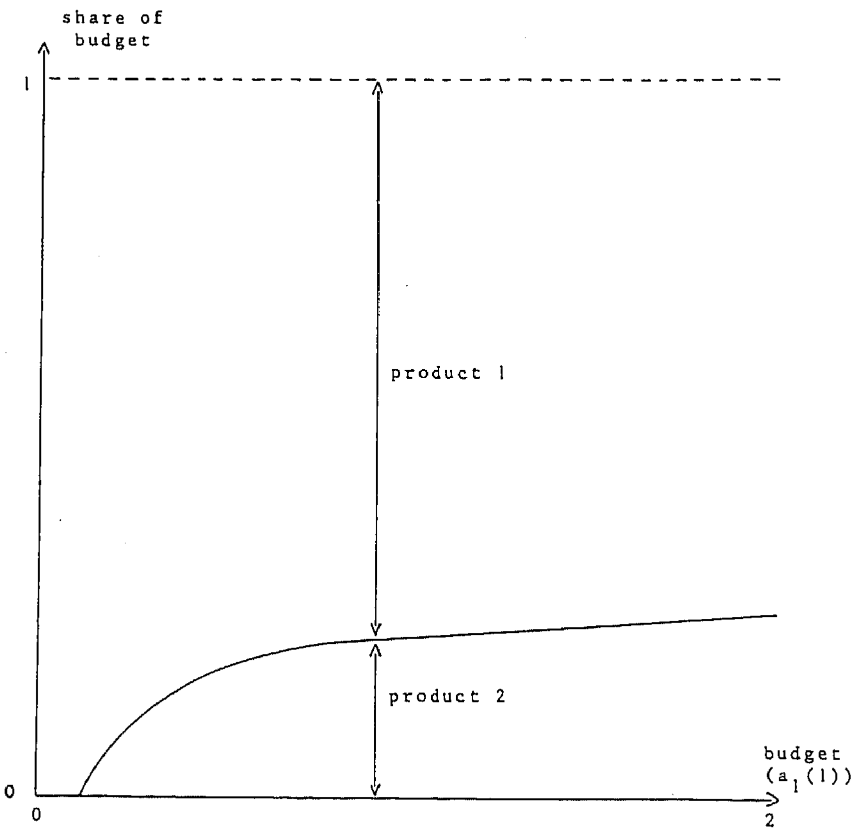

Fig. 6. Share in product group 1 budget for products 1 and 2 as a function of product group budget in period 1.

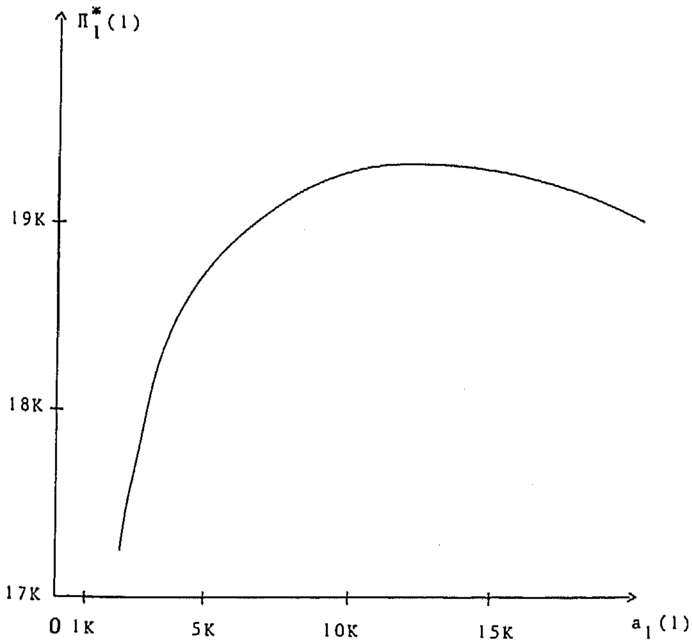

Fig. 7. Optimal first-period profit as a function of communication budget in period 1 (group 1).

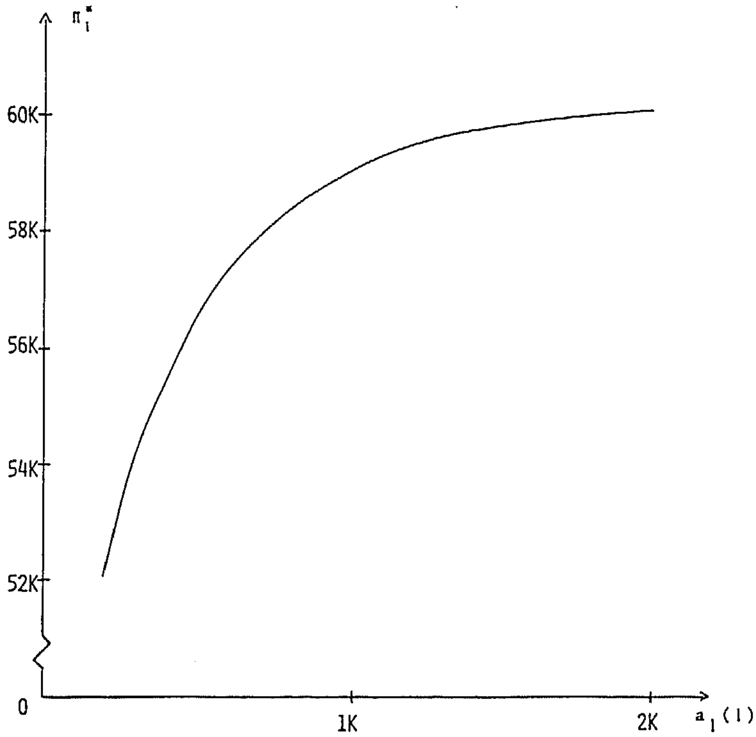

Fig. 8. Optimal profit over the planning horizon as a function of communication budget in period 1 (group 1).

specified assumption that all external variables remain constant over the planning period. Judicious model use requires some effort to formulate acceptable estimates for market sales and competitive communication effort in each product group over the planning period. As far as submodule IIIa is concerned, these estimates will have little impact on the optimal budget allocation across products (though, of course, π_g^* is clearly affected!). They will, however, be rather important in submodule IIIb, where communication budgets are assigned to product groups.

3.4. *Module IIIb: Budget allocation across product groups*

Allocation of communication resources across product groups is based on the following model:

$$\max \pi = \sum_{p=1}^{P} \sum_{g=1}^{G} \pi_g(p)(1+r_g)^{1-p}$$

$$\text{s.t.} \sum_g a_g(p) \leq a(p), \quad \text{all } p, \tag{7}$$

$$1_g(p) \leq a_g(p) \leq u_g(p), \quad \text{all } g, p,$$

where, in the objective function, we have:

$$\pi_g(p) = m_g(p) M_g(p) c_g(p) - a_g(p). \tag{8}$$

At first sight, model (7) seems almost identical to model (5). In fact, however, a big difference exists. In model (5), an increase in the budget for one product – say $a_1^1(1)$ – increases its market share – $m_1^1(1)$ – but negatively affects the market share $m_1^2(1)$ of the other product the company has in this product category. In model (7), however, market shares (of different product groups!) are mutually independent.

For product groups where $n_g = 1$, market share can be estimated using equation (3). Examples are group 2 and group 3, where the following estimation results are obtained:[9]

$$m_2(t) = \alpha_2 + \lambda_2 m_2(t-1) + \beta_2 A_2(t),$$

$$\hat{\alpha}_2 = 0.0016, \quad t_{\hat{\alpha}_2} = 0.42,$$

$$\hat{\lambda}_2 = 0.9295, \quad t_{\hat{\lambda}_2} = 49.85,$$

$$\hat{\beta}_2 = 0.0757, \quad t_{\hat{\beta}_2} = 2.24,$$

$$R^2 = 0.9888,$$

and

$$m_3(t) = \alpha_3 + \lambda_3 m_3(t-1) + \beta_3 A_3(t),$$

$$\hat{\alpha}_3 = 0.019, \quad t_{\hat{\alpha}_3} = 2.38,$$

$$\hat{\lambda}_3 = 0.6732, \quad t_{\hat{\lambda}_3} = 9.24,$$

$$\hat{\beta}_3 = 0.1034, \quad t_{\hat{\beta}_3} = 1.73,$$

$$R^2 = 0.7738.$$

If $n_g > 1$, the company's market share in the group is the total of shares reached by its different products in the group, or

$$m_g(t) = \sum_{i \in C_g} m_g^i(t).$$

As such, market share $m_g(t)$ will not only depend on the global communication budget allocated to the group, $a_g(t)$ – and thus, given $a_g^c(t)$ on $A_g(t)$ – but also on how this budget is divided across products in the group. In other words, the effectiveness of communication budgets at the group level – market shares $m_g(t)$ associated with budgets $a_g(t)$ – depends on detailed allocation practices within the group. Optimal allocation decisions at the higher company level, which are based on the effectiveness of $a_g(t)$ budgets, thus require insight into lower level decisions. On the other hand, lower level decisions – budget allocation to products in a group – must take place within the constraints (budget and other restrictions) imposed by higher levels.

Clearly, in order to determine optimal use of communication budgets throughout the company, co-ordinative action between the subsequent decision levels is needed.

In the more general context of a hierarchical solution of large scale problems, different coordination procedures have been proposed. In essence the following approaches can be distinguished. First, one has the so-called 'composition' approaches, which mainly *describe* possible

[9] The superscript i in equation (3) can be omitted in the allocation model, since the company offers only one product in groups 2 and 3.

or existing co-ordination mechanisms in a particular problem context. Next, there is a vast literature on 'decomposition' approaches which try to derive *optimal* linkage schemes for varying classes of large scale problems. Two basic types of co-ordination schemes are put forward in the decomposition literature. The first type is the so-called 'goal co-ordination' method, in which higher and lower level decisions are linked through the use of appropriate transfer prices that affect lower level goals. In the second type of approach – the 'constraint co-ordination' method – co-ordination is achieved through manipulation of lower level constraints.[10]

The linkage mechanism described here is based on the principles used in the constraint co-ordination method. This co-ordination mechanism implies that for these product groups, where $n_g > 1$, $m_g(p)$ in equation (8) is obtained as an output of module IIIa.

In particular, this linkage mechanism consists of the following steps:

Step 1. Sensitivity analysis of the lower level (module IIIa). For each product group where $n_g > 1$, the allocation model in module IIIa (model 5) is solved for alternative levels of $a_g(1)$ and $\{m_g^i(0),\ i \in C_g\}$. For each set $[a_g(1), \{m_g^i(0)\}]$, associated optimal levels for $m_g(1)$ can be computed. This repeated optimization effort thus yields a number of data points,

$$\left[m_g(1), a_g(1), m_g(0) = \sum_{i=1}^{n_g} m_g^i(0)\right],$$

on the basis of which a market share response function (similar to equation (3)) can be estimated for the product group as a whole.

Step 2. Upward information flow (module IIIa→module IIIb). For each product group where $n_g > 1$, the market share response function based on lower level sensitivity analysis, is communicated to the higher company level. (For product groups containing a single product ($n_g = 1$), market share functions directly estimated at the higher company level are incorporated in equation (8). Examples of such estimation were already given for groups 2 and 3.)

[10]For a more detailed discussion of the different approaches with particular attention to implementation aspects, see Gijsbrechts [2, 3].

Step 3. Solution of the higher level problem: formulation of final budget constraints. The higher level allocation problem (7) is solved using the market share functions obtained in Step 2, resulting in 'optimal' product group budgets $a_g(p)$.

Step 4. Downward information flow (module IIIb→module IIIa). Optimal budgets $a_g(1)$ obtained in the previous step are communicated to lower levels, where they serve as 'final' budget constraints for the allocation module IIIa (model 5).

The proposed coordination scheme is based on the constraint co-ordination method; it uses the variables $a_g(1)$ (budget constraints in the lower level problem) as co-ordinating variables; clearly results in feasible allocations; and benefits from good 'starting values' for the $a_g(1)$ variables.

However, instead of manipulating the co-ordinating variables at the higher level ($a_g(1)$), and the decision variables at the lower level ($a_g^i(p)$), in an iterative scheme involving repeated feedback between levels, only *one* upward information flow (based on lower level sensitivity analysis) and *one* downward information flow are needed to complete coordination. The co-ordinability (or 'optimality') of the proposed procedure has already been discussed in theoretical terms in Gijsbrechts [3]. To clarify its implications for this particular application, let us illustrate the procedure considering product group 1, where two products are offered.

For this group, model (5) is first solved for alternative levels of $a_1(1)$, leading to the results in Fig. 9. Note that these results also form the basis for Figs. 7 and 8. Secondly, different levels of $m_1(0)$ are considered for model solution, yielding the optimal results in Fig. 10.

The outcomes obtained are now used as data points for estimation of a market share response function for group 1:

$$m_1(t) = \alpha_1 + \lambda_1 m_1(t-1) + \beta_1 A_1(t)\,, \tag{9}$$

$$\hat{\alpha}_1 = -0.0042\,, \quad t_{\hat{\alpha}1} = -0.35\,,$$

$$\hat{\lambda}_1 = 0.8056\,, \quad t_{\hat{\lambda}_1} = 23.73\,,$$

$$\hat{\beta}_1 = 0.1994\,, \quad t_{\hat{\beta}_1} = 11.68\,,$$

$$R^2 = 0.9729\,.$$

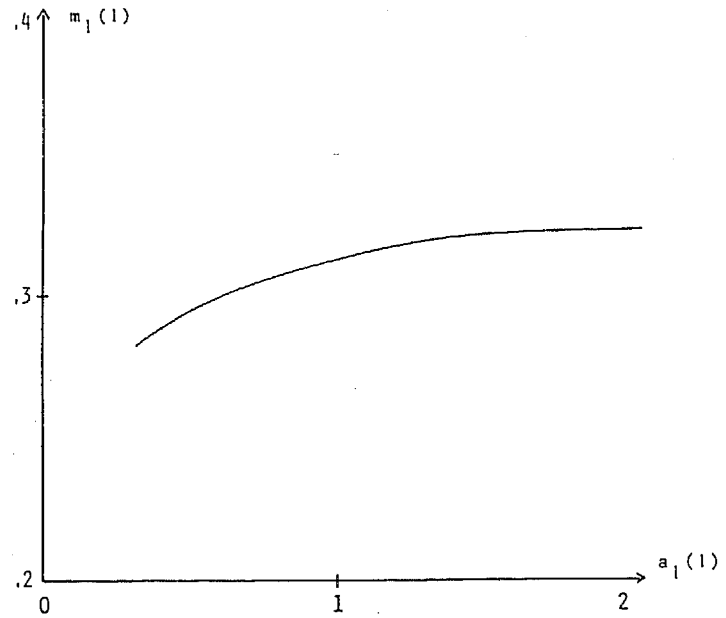

Fig. 9. Optimal market share in period 1 as a function of communication budget in period 1 (group 1).

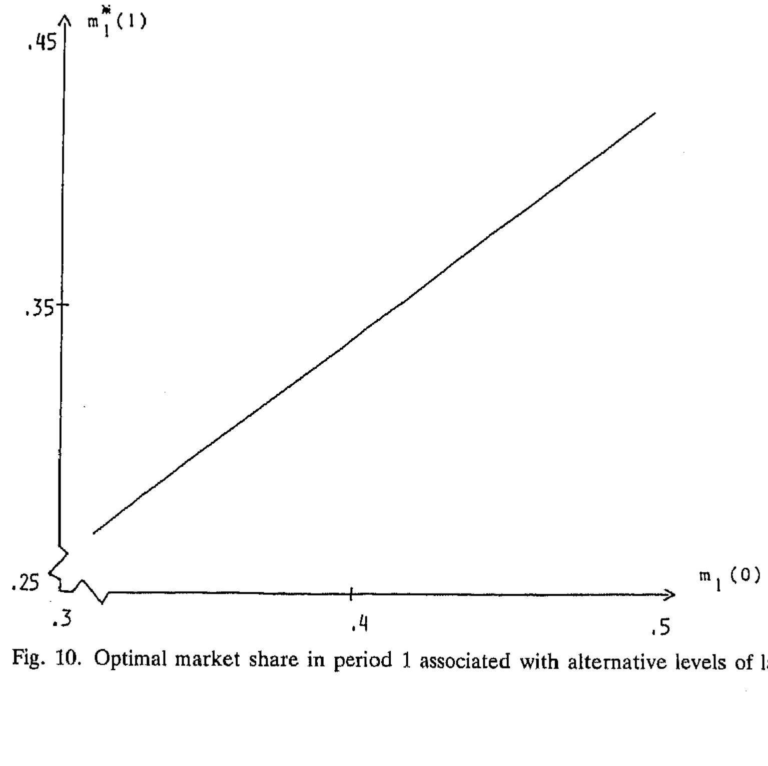

Fig. 10. Optimal market share in period 1 associated with alternative levels of lagged market share (group 1).

Hereafter we will refer to the above procedure as the 'indirect' estimation.

The indirect procedure might appear to the reader to be unduly complicated. To illustrate its potential usefulness, let us contrast it with the more straightforward 'direct' estimation practice. In estimating equation (3) we have in fact applied this direct procedure. Just to distinguish the two sets of coefficients we give the latter ones a superscript D. We thus have:

$$\hat{\alpha}_1^{\mathrm{D}} = 0.0089\,, \quad t_{\hat{\alpha}_1} = 1.28\,,$$

$$\hat{\lambda}_1^{\mathrm{D}} = 0.8506\,, \quad t_{\hat{\lambda}_1} = 32.81\,,$$

$$\hat{\beta}_1^{\mathrm{D}} = 0.0926\,, \quad t_{\hat{\beta}_1} = 1.44\,.$$

Both the direct and indirect estimates have a constant term that does not significantly differ from zero. The coefficient of lagged market share is highly significant and takes on similar values in both cases. A striking difference in estimation results is that $\hat{\beta}_1^{\mathrm{D}}$ is almost half of $\hat{\beta}_1$ or, the communication effectiveness in group 1 reported by the direct estimation procedure is less than half as large as that given by the indirect method.[11]

Whether one should use the direct or indirect estimates as an input to model (5) is therefore not a 'theoretical' or 'cosmetic' question, but might well bear on the reported optimal allocation across product groups of a given company budget. In any case, the objective function of (5) will be affected by this choice.

To explain why we favour the indirectly estimated response function, we must examine in what respect it differs from direct estimation. As explained previously, indirect estimation presupposes *optimal* budget allocation *within* the group, and thus indicates the full share potential of the group's communication resources $a_g(t)$, with respect to *the planning period*.

The response function obtained through 'direct' estimation, on the contrary, implicitly reflects *past* company practices in allocating the communication budget $a_g(t)$ across products in the group. As is illustrated for group 1, the distinction may be important, since past allocations can significantly differ from optimal ones. For the case at hand, direct estimation seems to seriously underestimate the potential effectiveness of group 1's communication budget.

But there is more. Even if within-group budget allocation would have been decided upon optimally in the past, direct estimation of the response function would yield potential communication effectiveness *under past conditions*. Since environmental conditions and constraints in the planning period may substantially differ from historical ones, the potential impact of $a_g(t)$ may again be misjudged through direct assessment. Indirect estimation, based on sensitivity analysis, allows one to anticipate future evolutions and their impact on communication effectiveness.

In short, the indirect estimation method is preferred to the direct one, because it allows for more accurate assessment of the attainable communication effectiveness in subsequent groups (where $n_g > 1$), thus leading to a better allocation of resources across groups in the planning period.

The market share functions constitute only one important input of model (8). The optimal allocation across product groups may also be affected by the evolution of market sales and competitive expenditures in the groups. For the time being, future levels of these variables are predicted on the basis of simple extrapolation techniques, eventually supplemented by subjective judgement.

Market sales evolutions in groups 1, 2 and 3 are approximated by some simple time series models, using the limited historical information available. The following evolutions are seen as reasonable by management:[12]

$$\begin{aligned} &c_1(t+1)M_1(t+1) \\ &\quad = \underset{(4.49)}{7.06}\,\underset{(4.80)}{(c_1(t)M_1(t))^{0.541}}\,, \quad R^2 = 0.92\,,\ R_\alpha^2 = 0.88 \end{aligned}$$

$$\begin{aligned} &c_2(t+1)M_2(t+1) \\ &\quad = \underset{(1.247)}{3.43}\,\underset{(2.654)}{(c_2(t)M_2(t))^{0.692}}\,, \quad R^2 = 0.78\,,\ R_a^2 = 0.67 \end{aligned}$$

$$c_3(t+1)M_3(t+1) = c_3(t)M_3(t)\,.$$

[11] We should of course be aware that on the basis of $\hat{\beta}_1 = 0.1994$ we can reject the hypothesis that, for example, $\beta_1 = 0.0926$, but the direct estimate $\hat{\beta}_1^{\mathrm{D}} = 0.0926$ does not significantly differ from $\beta_1 = 0.1994$.

[12] The figures in brackets are t-statistics.

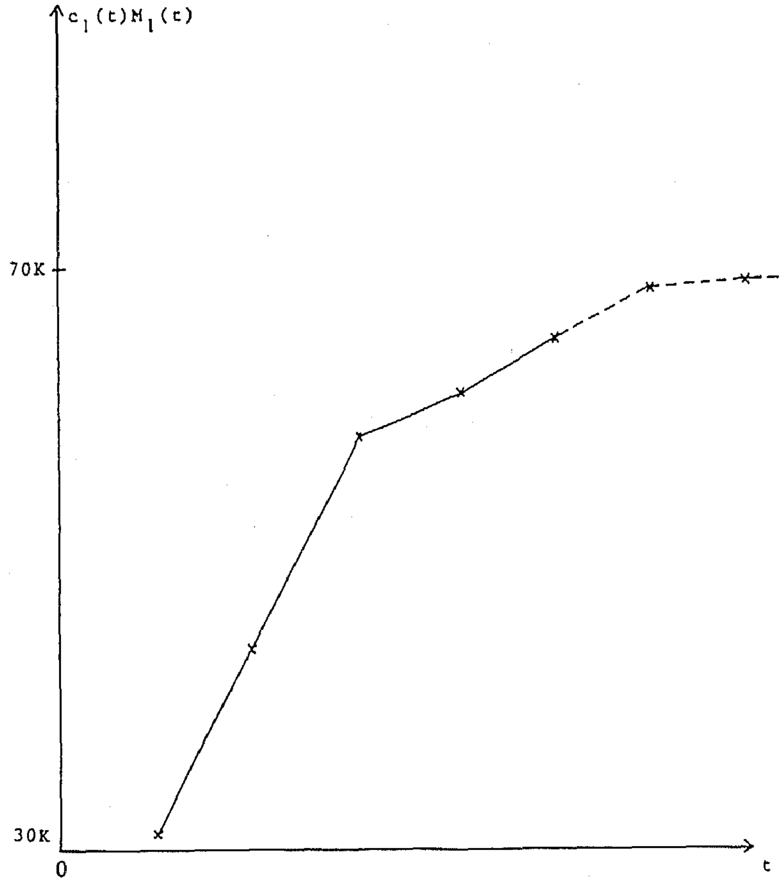

Fig. 11. Actual (———) and predicted (– – –) market sales (group 1).

The historical data on market sales and predictions based on the time series are graphically represented in Figs. 11, 12 and 13.

As far as competitive budgets for the groups are concerned, no major shifts are expected. We thus have:

$$a_1^c(p) = 3.8\ K\,,$$
$$a_2^c(p) = 2.7\ K,$$
$$a_3^c(p) = 2.8\ K, \quad \text{for } p = 0, \ldots, 3\,.$$[13]

Finally, given

$$m_1(0) = 0.335\,, \quad r_g = 0.2\,, \quad \text{all } g\,,$$
$$m_2(0) = 0.076\,,$$
$$m_3(0) = 0.092\,,$$

[13] We now take P = 3 to illustrate that the optimal budget corresponding to maximum profit in period one deviates significantly from that leading to maximum profit over the planning horizon, even if that horizon is short.

model (7) can be used to allocate given budgets $a(p)$, $p = 1, \ldots, 3$, optimally across product groups. The resources thus allocated to group 1, for which $n_g = n_1 > 1$, can then be considered as final budget restrictions for solving model (5).

Given the simple form of the response functions, and assuming that future conditions remain unchanged, one could also easily calculate the optimum (equilibrium) allocation of a period budget $a(p)$ across product groups, over an infinite horizon. Fig. 14 provides a graphical representation of this optimal allocation across groups 1, 2 and 3, for different levels of $a(p)$.

The potential benefits of model (7) do, however, extend beyond the optimal allocation of *given* budgets $a(p)$, with *given* constraints $l_g(p)$ and $u_g(p)$. Indeed, sensitivity analysis based on the model may provide valuable information for managers responsible for imposing those constraints.

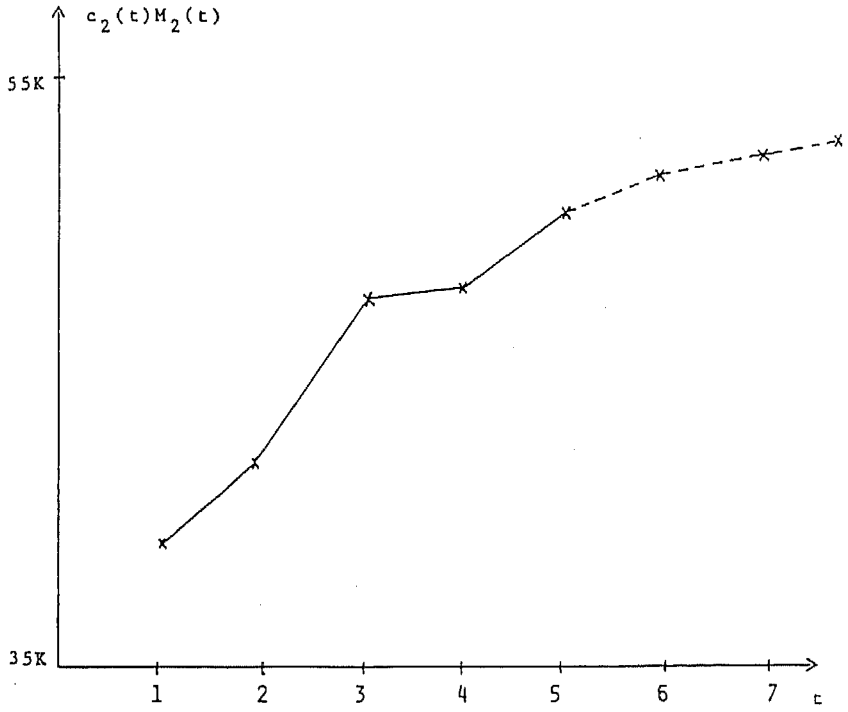

Fig. 12. Actual (———) and predicted (---) market sales (group 2).

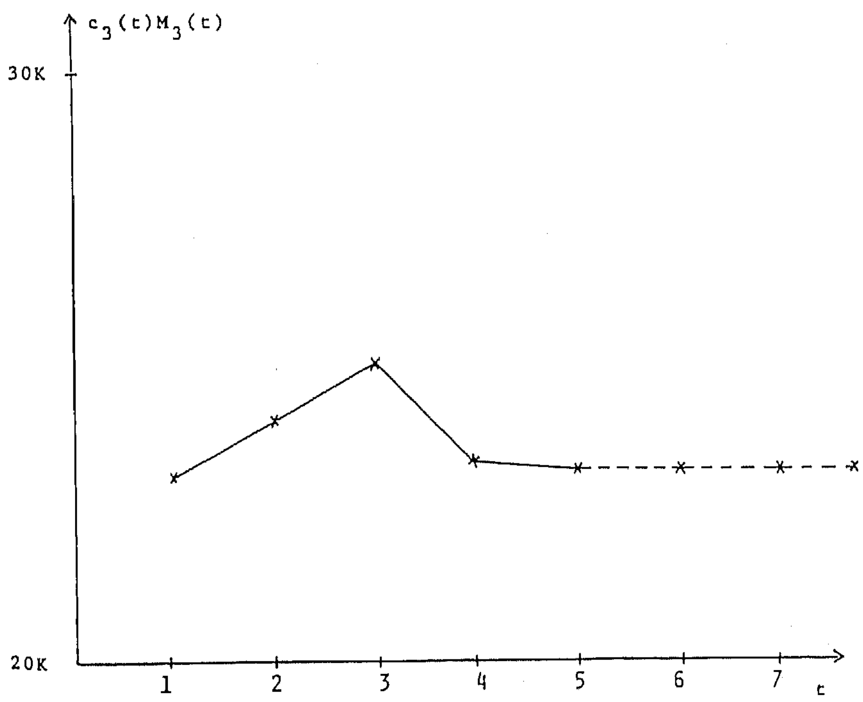

Fig. 13. Actual (———) and predicted (---) market sales (group 3).

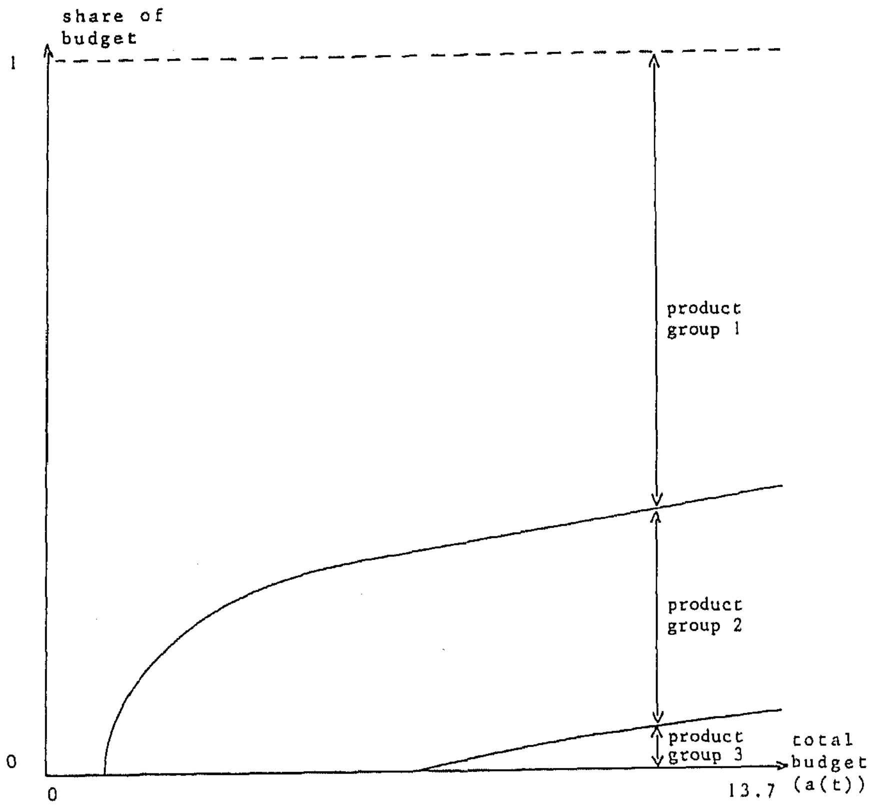

Fig. 14. Optimal shares of product group budgets as a function of total communication budget for the company (infinite horizon).

For instance, letting $a(2) = a(3) = 2\,K$, Fig. 15 depicts optimal profit in period 1, $\pi^*(1)$, as a function of $a(1)$. Unless $a(1)$ exceeds $7K$, the optimal allocation implies that product group 2 will receive no resources at all.

Table 1 indicates how first-period profit is affected by imposing a lower bound on group 2's budget. As in module IIIa, it can also be interesting – from a more strategic point of view – to examine the impact of changes in $a(1)$ and $l_g(1)$ on π^*, and thus assess the long-term impact of resource changes, as well as the long-term opportunity cost of the constraints $l_g(1)$ (or $u_g(1)$).

Table 1
Sensitivity of $m_2(1)$ and $\pi^*(1)$ to changes in $l_2(1)$

$l_2(1)$	$m_2(1)$	$\pi^*(1)$
$0K$	0.0809	$25.17K$
$0.05K$	0.0828	$25.15K$
$0.1K$	0.0846	$25.12K$
$0.2K$	0.0881	$25.05K$
$0.3K$	0.0913	$24.98K$

Alternatively, one may want to examine the optimal allocation and associated profit for different scenarios with respect to competitive spending or overall market sales in different groups.

Ultimately, as for model (6), the results of repeated solution of model (8) could be used to estimate a response function which implicitly reflects optimal allocation.

Considering only groups 1, 2 and 3, and accepting the market sales evolutions given in Figs. 11, 12 and 13, the following optimal profit function can be estimated:

$$\pi^*(1) = S^*(1) - a(1)\,,$$

where

$$S^*(1) = \alpha a(1)^{\beta} S(0)^{\gamma} a^c(1)^{\delta}\,,$$

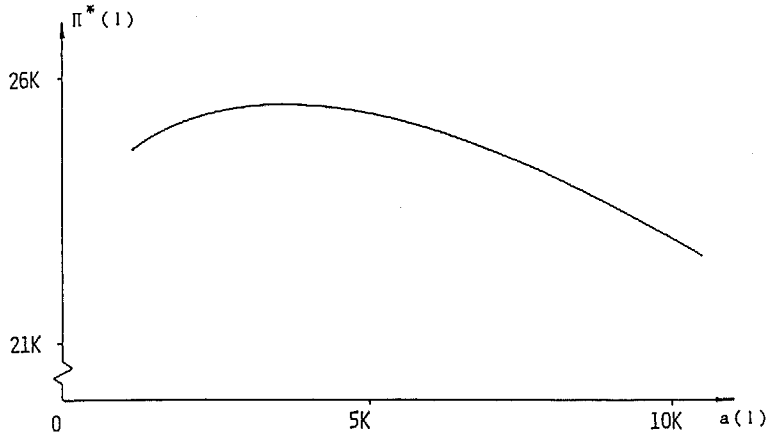

Fig. 15. Optimal first year company profit as a function of communication budget in period 1.

with

$$\hat{\alpha} = 3.6263\,,\quad t_{\hat{\alpha}} = 1.47\,,$$
$$\hat{\beta} = 0.107\,,\quad t^{\hat{\beta}} = 10.3\,,$$
$$\hat{\gamma} = -0.086\,,\quad t_{\hat{\gamma}} = -0.24\,,$$
$$\hat{\delta} = 0.6517\,,\quad t_{\hat{\delta}} = 8.55$$
$$R^2 = 0.88\,,$$
$$R_a^2 = 0.866\,.$$

This estimated relationship indicates the highest profit one can expect from a given budget $a(1)$, for a given planning period P, and with pre-specified constraints $u_g(p)$ and $l_g(p)$. It could eventually assist higher level management in allocation communication resources over time, or across national departments of the company, and thus link the problem of allocating budgets across groups, to still higher level decisions.

The results provided above are only meant to be illustrative of potential model benefits. In practical situations, some model inputs have to be more carefully considered than was done here. This is, for instance, the case for the 'planning horizon', which was set at three or five periods in our analysis. Before final decisions are made, sufficient attention must be paid to the choice of the planning horizon, since the latter may affect model outcomes. If the planning period is too short, important future evolutions are not taken into account, which results in the so-called 'horizon bias'. In our higher level model, for instance, a change in P may affect the allocation across segments in each period.[14] Final decisions on budget use in the next period require some sensitivity analysis on the effect of P. In our example, despite the importance of carryover effects, the impact of the planning horizon remains fairly limited, as indicated in Table 2.

The problem of 'horizon bias' becomes more important if – instead of working with periodic budget constraints – a total budget a has to be allocated not only across segments, but over time. In such a problem context, there is a tendency to allocate more to early periods as the model's planning horizon decreases, since the long-term profitability of budgets used at the end of the planning horizon is not included in the objective function. If the company's objective is to maximize long-run profit, it is clear that truncation of the objective function in the model

Table 2
Influence of planning horizon on budget allocated to product groups

P	$a(1)$	$a_3(1)$	$a_2(1)$	$a_1(1)$
3	2	0	0.0946	1.9054
5	2	0	0.1649	1.8351
∞	2	0	0.3256	1.6744
3	3	0	0.4312	2.5688
5	3	0	0.5094	2.4906
∞	3	0	0.6890	2.3110

[14]Similar remarks are valid for the lower level model as well.

after a limited number of periods leads to sub-optimal results.[15] Appropriate measures must then be taken to reduce the horizon bias. Several procedures are possible, such as

(i) prolonging the planning horizon,

(ii) introducing a post-planning horizon,[16] during which some equilibrium performance must be maintained,

(iii) introducing terminal constraints on performance or budgets.

An elaborate treatment of these issues would lead us too far. For our particular problem situation, some pros and cons of the different approaches are provided in Gijsbrechts [2].

4. Conclusion

In this paper we indicated how a decision support system could be developed for hierarchically allocating scarce communication resources to product groups, and further to products within a group.

We suggested how these subsequent allocation decisions could be mutually linked and integrated in a more strategic decision framework. The linkage scheme proposed for co-ordinating the decisions made at different company levels is consistent with the basic principles of general large scale solution methods, but at the same time flexible and tractable.

Of course, the analysis presented here is far from complete. In practice, alternative environmental scenarios may be explicitly compared. Also, we must keep in mind that the market share response functions are estimated and their parameters uncertain. The sensitivity of the marketing plan to parameter changes is thus worth considering.

Appendix: Glossary of symbols

$a(t)$ = total communication budget for the company (in year t)

$a_g(t)$ = communication expenditures of the company in product group g

$a_{g,\,\mathrm{tot}}(t)$ = communication expenditures in product group g (company and competitors)

$a_g^i(t)$ = communication expenditures for product i in group g

$a_g^c(t)$ = communication expenditures of competitors in group g

$a^c(t)$ = communication expenditures of competitors in all groups

$A_g^i(t) = a_g^i(t)/a_{g,\,\mathrm{tot}}(t)$ = communication share of brand i in group g

$A_g(t) = a_g(t)/a_{g,\,\mathrm{tot}}(t)$ = communication share of the company in group g

$c_g^i(t)$ = unit profit contribution for product i in group g

C_g = set of superscripts in group g denoting company products

ε_t^i = stochastic disturbance term

G = number of product groups

$m_g^i(t)$ = market share (product i in group g, year t)

m_g^{ei} = long-term (equilibrium) market share for product i in group g

$m_g(t)$ = market share when the company has only one product in group g

$M_g(t)$ = total market sales (in units) in group g

n_g = number of products in group g (in the estimation module)
= number of products of the company group g (in the allocation module)

p = year p in the planning period

P = planning horizon

$\pi_g^i(t)$ = profit in year t (product i, group g)

$\pi_g(t)$ = profit in year t in product group g

π_g = profit in group g over the planning horizon

$\pi(t)$ = total profit for the company in year t

π = total profit for the company over the planning horizon

r_g = discount factor for product group g

t_i' = number of years since introduction of product i $(=t - t_{\mathrm{intro},i})$

$t_{\mathrm{intro},i}$ = year of introduction of product i

$S(t)$ = sales revenue for the company in year t

[15] At least, if any dynamic effects exist.

[16] This is the approach taken by Larréché and Srinivasan in STRATPORT.

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