# On the Interaction Between Vertical and Horizontal Product Differentiation: An Application to Banking

Hans Degryse

# ON THE INTERACTION BETWEEN VERTICAL AND HORIZONTAL PRODUCT DIFFERENTIATION: AN APPLICATION TO BANKING*

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We study the conditions under which banks offer remote access. Note there exists interaction between location and taste for remote access. Offering remote access is an instrument to (partially) segment depositors according to their taste for that technology. The interaction between location and taste for remote access enhances this effect. Different equilibria emerge as the result of two effects. First, introducing remote access steals depositors from the opponent as the product specification becomes more appealing. Second, deposit rate competition is affected as remote access determines the substitutability of banks.

## I. INTRODUCTION

DEVELOPMENTS in technology have a powerful impact on the mode of delivering financial services. Remote access services (postal and telephonic delivery systems) become more important. Many customers rely heavily on manual telephone and postal liaison with their bank in order to arrange payment facilities and obtain account information. Remote access offers depositors the possibility to process financial transactions without visiting a bank's branch. In addition, it enables them to perform transactions outside and to evade queues within office hours. This evolution in banking is likely to continue. Anne Perlman writes in The Banker [January 1995, p. 67]; "Ernst & Young, a leading financial analyst in the US, has carried out a survey of technology in banking. It projects that bank transactions at traditional 'brick and mortar' branches will drop from 61% to 44% by 1997, replaced by non-branch transactions conducted over a digital media network." Remote access facilities came into operation in several countries. According to BIS [1993] and BEUC [1992], virtually all major banks offer remote access services such as home- and phonebanking in Belgium, France, Germany, Sweden and the UK. Remote access is less available in Italy, Ireland and the Netherlands. This paper studies the impact of the introduction of remote access, improving the accessibility of funds, on banking competition. More specifically, we analyze how the strength of competition between banks influences the decision to introduce these new communication technologies. We also consider the impact of these technologies on intermediation margins.

Competition in banking was until recently regarded with suspicion. It would induce too much risk taking and generate financial instability (Mayer and Vives [1993]). Deregulation and European integration, however, induced banks to compete for deposits using several dimensions. These include deposit rates, accessibility and the quality of financial services. This paper examines multi-dimensional banking competition. Financial products, in the model developed in the paper, are characterized by one feature of variety (location) and one feature of quality (remote access). The novelty of this model is that banks cannot become vertically differentiated without negatively affecting the degree of horizontal differentiation between them.

In the traditional one-dimensional product differentiation literature, two models prevail, namely horizontal (Hotelling [1929]) and vertical (Gabszewicz and Thisse [1979], Shaked and Sutton [1982]) differentiation. Products are *horizontally* differentiated when there is no consensus of ranking among consumers based on their willingness-to-pay. Products are *vertically* differentiated when there is such a ranking at equal prices. However, most products embody more than one characteristic and both types of differentiation. In the recent literature (Neven and Thisse [1990], Caplin and Nalebuff [1991] and Anderson et al. [1992]), those one-dimensional models are extended towards multi-dimensional product differentiation. Some papers analyze product specifications in two characteristics (Neven and Thisse [1990], Economides [1989] and [1993], Tabuchi [1994]). In those papers, both characteristics are assumed to be independent. Our paper differs by having *negative interaction* between types of characteristics.

Financial products such as deposit accounts embody several characteristics, both horizontal and vertical. Horizontal differentiation arises with the location of a bank's branch. Vertical differentiation, in this paper, occurs whenever one bank offers remote access and the other does not. Remote access, in addition, implies *a negative interaction*[1] between transportation rate and taste for quality: depositors with a higher taste for remote access face a lower transportation rate if that technology is available. In other words, the introduction of vertical differentiation between banks negatively affects the degree of horizontal differentiation between them.

Competition is modelled as a two-stage game. In the first stage banks simultaneously choose whether they will offer remote access or not. In the second stage, they compete in deposit rates. Within the subgame where only one bank offers remote access, two mutually exclusive cases exist. First, horizontal dominance[2] arises when banks attract a positive market share for *all types* of

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[1] The computer sector delivers another example of interaction between characteristics. Some stores sell their products with the opportunity to receive phone support while others not. Clumsy computer users located further away from the store value phone support more than their clumsy colleagues at a shorter distance.

[2] This terminology was first used by Neven and Thisse [1990]. The terminology, however, does not coincide perfectly as they use it in a model without interaction term.

taste for remote access. This occurs whenever the transportation rate overwhelms the quality difference and the interaction term. Second, vertical dominance arises when the bank (not) offering remote access obtains the entire market for depositors characterised by the *highest* (*lowest*) taste for remote access. This occurs if the quality difference and the interaction term overwhelm the transportation rate. A model where depositors face different linear transportation costs arises as both banks offer remote access. A depositor with a high (low) taste for remote access has low (high) linear transportation costs.

Different equilibria emerge as the result of two effects. On the one hand, introducing remote access steals depositors from your opponent because the product specification becomes more appealing (direct effect). On the other hand, banks become closer substitutes (indirect effect). First, banks become closer substitutes as the impact of linear transportation costs decreases. Second, deposit rate competition is affected by the size of the quality difference. These two effects, "stealing" depositors versus "substitutability" between banks, determines the equilibrium. For low and high values of the ratio quality difference to transportation rate, only one bank offers remote access (specialization). Intermediate (very low) values of the ratio quality difference to transportation costs yield universal (no) remote access.

A closely related paper in the banking competition literature is Matutues and Padilla [1994]. They discuss the effects of ATM compatibility on banking competition. The similarities are the following. We both investigate and introduce non-price competition elements in banking competition. In both papers, banks cannot introduce new payment technologies without negatively affecting horizontal differentiation. Our paper, however, presents several distinctive features. First, in our model, depositors *differ* in their marginal willingness-to-pay for the new payment technology. This implies that banks face extra incentives to differentiate themselves in the quality range. Second, related to the first, the degree of negative interaction between the horizontal and vertical characteristic differs among depositors. In other words, the negative effect on horizontal differentiation of becoming vertically differentiated is partly counterbalanced as one attracts the depositors with a high taste for remote access more easily. Third, each bank individually decides whether to introduce remote access or not. In Matutes and Padilla [1994], banks propose compatibility agreements. Therefore, a compatibility decision is taken by at least two banks. Another related paper is Bouckaert and Degryse [1995] who discuss phonebanking. In that model, all depositors have the same taste for using the phone option. In our model, depositors *differ* in taste for remote access. Heterogeneity in tastes arise as the frequency of using that technology differs. For instance, businesses face different needs for payment services than individuals. Therefore, the taste for remote access depends on the type of depositor. Finally, Matutes and Vives [1992] present a model where banks differ with respect to location and perceived failure probability (reputation). Vertical and horizontal differentiation are unrelated, and all depositors have the same willingness-to-pay for reputation.

The paper is organized as follows. Section II proposes a multi-dimensional banking competition model. Section III discusses the second stage deposit rate equlibrium while section IV focuses on the first stage product choice. Section V presents some concluding remarks.

## II.  THE MODEL

Consider a duopolistic deposit market. Competition for deposits is modelled as a two-stage game. At stage one, both banks simultaneously decide to offer remote access ($T$) or not ($N$). Introducing this technology is costless. In the second stage, they simultaneously choose deposit rates. Banks $A$ and $B$ are located on a circle with a circumference of 1 denoted by $C$. Their location is exogeneous given at respectively 0 and 1/2. Therefore, the focus is on the introduction of quality variation and interaction between taste for quality and location. Deposit accounts with the associated services are defined by two characteristics. First, the location of a bank determines the physical access. Second, remote access may or may not be available. Banks invest the proceeds of their deposits and obtain an idential rate of return $R$ per unit of money. They maximize the following profit function

$$\pi_i = (R - r_i)D$$

with $R$ = rate of return obtained by banks

$r_i$ = deposit rate of bank $i$

$D$ = amount of deposits attracted by bank $i$.

Each depositor invests one unit of money at one of the two banks.[3] They perform one normalized financial transaction. Depositors have two character- istics. First, each depositor has a unique location $z$ on $C$. Second, they have a taste for remote access $\theta$ with $\theta \in [\underline{\theta}, \overline{\theta}]$. Remote access ($T$) implies that depositors can manage a fraction of their financial transaction remotely. If a bank decides not to offer remote access ($N$), all transactions need a visit to that bank. A depositor uses remote access, if available, for a fraction $\theta$ of the normalized financial transaction. The fraction $\theta$ can be interpreted as the fraction of account management transactions. The complement $1-\theta$ represents the fraction of cash withdrawals. The latter clearly need a visit to the branch. Depositors differ with respect to $\theta$, but they all prefer remote access at the same deposit rate.[4] For instance, depositors vary in their payment behaviour. The taste for remote access $\theta$ and the location $z$ are assumed to be non contractible: deposit rates cannot be made conditional either on the taste for remote access or on location. As shown in Figure 1, the space of depositor's characteristics $(z, \theta)$ is the cylinder $C \times [\underline{\theta}, \overline{\theta}]$[5] (Economides [1993]). Depositors are uniformly distributed

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[3] For instance, shopping costs explain why depositors choose only one bank (Klemperer [1992]).

[4] Alternatively, depositors can differ in the number of transactions.

[5] Opening up this cylindrical space yields two rectangles, $[0, 0.5] \times [\underline{\theta}, \overline{\theta}]$ *and* $[0.5, 1] \times [\underline{\theta}, \overline{\theta}]$, respectively.
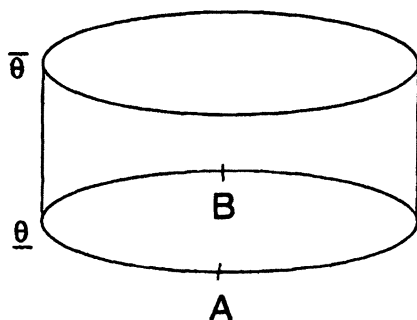
Figure 1
Depositors distributed on a cylinder

with density $1/[\bar{\theta} - \underline{\theta}]$ over the surface of the cylinder, so that their total mass is equal to one.

Using remote access has two effects. On the one hand, it implies a fixed cost $\tau$ per transaction, for instance the cost of a phone call. On the other hand, depositors save transaction costs. Firstly, they avoid the waiting cost $k$. Secondly, they save the linear transportation rate t times the distance between the bank and their location. Then, according to the presence of remote access or not, depositors have the following value for a deposit account:

$$(1) \qquad \begin{cases} V(N, z, \theta) = v + r_i - (k + tz) \\ V(T, z, \theta) = v + r_i - (k + tz) + [\theta(k - \tau)] + \theta tz \end{cases}$$

where $N$ = remote access not

$T$ = remote access offered

$\theta$ = fraction of financial transactions executed by remote access, if offered

$r_i$ = deposit rate at bank $i(i = A, B)$

$z$ = distance between depositor's location and bank

$t$ = linear transportation cost

$k$ = expected cost of waiting at a bank desk

$\tau$ = fixed cost of using remote access

$v$ = reservation value large enough such that the market is covered

This model encompasses the vertical and horizontal differentiation models. If remote access is not offered ($N$), all financial transactions need a physical visit to the bank. The quality offered equals zero. Therefore, the depositor's value is independent of $\theta$. The latter no longer holds when remote access (T) is offered. The term in square brackets in $V)T, z, \theta)$ represents the gain in fixed costs of using remote access (define $k - \tau = f$). This term is common in the vertical differentiation literature (see e.g. Gabszewicz and Thisse [1979], Tirole [1988]):

$\theta$ represents the taste for quality while $f(=k-\tau)$ describes the quality of the good. The last term in $V(t, z, \theta)$ implies non-separability in taste for quality and distance. There is *negative interaction between transportation rate and taste for quality*: depositors with a higher $\theta$ face lower transportation costs. We expect the quality difference $f$ to be high in securities and trading transactions. Speed in these markets is very important as opportunities are often short-lived. The quality difference is smaller for ordinary retail transactions.

### III.  DEPOSIT RATE EQUILIBRIUM

Given the first stage decision about the choice of remote access, three types of subgames occur: both banks do not offer remote access $(N_a, N_b)$, one bank offers remote access $(T_a, N_b)$ or both banks offer remote access $(T_a, T_b)$.

### III(i)   Subgame $(N_a, N_b)$

According to (1), the indifferent depositor is located such that

$$(2) \qquad x(\theta) = \frac{1}{t}\left[r_a - r_b + \frac{t}{2}\right]$$

with $x(\theta) =$ market share of bank $A$ for type $\theta$.

Banks are undifferentiated along the remote access dimension. This subgame is a standard model of product differentiation with linear transportation costs $t$ yielding the following deposit rate equilibrium:

$$r_i^{nn} = R - t/2 \text{ with } D_i^{nn} = \tfrac{1}{2}.$$

### III(ii)   Subgame $(T_a, T_b)$

According to (1), bank $A$'s market share for type $\theta$ equals

$$(3) \qquad x(\theta) = \frac{1}{t(1-\theta)}\left[r_a - r_b + \frac{t(1-\theta)}{2}\right]$$

Banks are undifferentiated along the remote access dimension. Notice that $A$'s market share is a function of $\theta$ due to the interaction term.[6] The demand function is derived in Appendix 1.

In this subgame, depositors face different linear transportation costs. These are uniformly distributed over a continuum $[t(1-\overline{\theta}), t(1-\underline{\theta})]$. Depositors with a high (low) taste for remote access face low (high) transportation costs due to the interaction term. This situation is similar to Garella and Martinez-Giralt [1993]. There, by assumption, consumers are uniformly distributed in the space of linear transportation costs.

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[6] We end up in a situation like the $(N_a, N_b)$ case in the absence of the interaction term.

The equilibrium interest rate if $\bar{\theta} < 0.965$ equals $r_i^{tt} = R - t(\bar{\theta} - \underline{\theta})/2\ln[(1 - \theta)/(1 - \theta)]$ with $D_a^{tt} = D_b^{tt} = 1/2$ (for the derivation, see Appendix 2).

In the absence of the interaction term, profits and equilibrium deposit rates are the same as in subgame $(N_a, N_b)$. The interaction term generates higher equilibrium deposit rates and lower profits. Therefore, competition is sharpened vis-à-vis the $(N_a, N_b)$ case.

### III(iii)   Subgame $(T_a, N_b)$

Given (1), the set of depositors indifferent between bank $A$ offering remote access and bank $B$ not offering remote access are derived. For any set of depositors $\theta \in [\underline{\theta}, \bar{\theta}]$, the marginal one is located such that

$$(4) \qquad x(\theta) = \frac{2}{t(2 - \theta)}\left[r_a - r_b + \theta f + \frac{t}{2}\right]$$

with $f = k - \tau$.

Banks are differentiated along the remote access dimension. Therefore, the $x$-curve depends on the quality difference $f$ between both banks. In addition, the $x$-curve depends on the taste for remote access $\theta$. It is non-linear in $\theta$ due to the interaction term. The demand function is derived in Appendix 3. The demand function consists of a convex, a linear, and a concave segment. In this subgame, profit functions are quasi-concave. Therefore, there is a unique equilibrium.

Two mutually exclusive cases occur namely vertical and horizontal dominance. The vertical dominance case is depicted in Figure 2.

*Vertical dominance* (VD) occurs when $A$ attracts the entire market for the $\bar{\theta}$ types and has a zero market share for the $\underline{\theta}$ types. The reverse holds for bank $B$ not offering remote access. This situation is referred to as vertical dominance as it is characterised by the dominance of the quality difference and the interaction term over transportation rate. A necessary condition for VD is $f/t \geqslant$
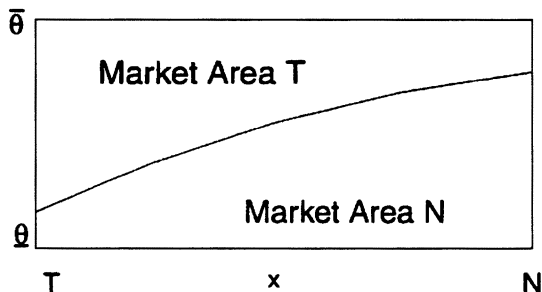


Figure 2
Vertical dominance

$[(2 - \bar{\theta})/(2(\bar{\theta} - \underline{\theta}))]$.[7] VD is more likely if the heterogeneity in taste for remote access increases. Note the concave curvature in Figure 2. This results from the interaction term. The bank offering remote access attracts depositors with a higher taste for remote access at an increasing rate. The equilibrium for VD is described in Proposition 1.

*Proposition 1* Consider the case of vertical dominance and let $z = \ln((2f + t)/(2f))$. The equilibrium interest rates, provided they belong to the linear segment of the demand function, are

$$r_a^{vd} = R - \frac{t}{6} - \frac{2f}{3} + \frac{t(\underline{\theta} - 2\bar{\theta} + 2)}{6z}$$

and

$$r_b^{vd} = R + \frac{t}{6} + \frac{2f}{3} - \frac{t(\bar{\theta} - 2\underline{\theta} + 2)}{6z}.$$

*Proof:* See Appendix 4.

Note that $r_a^{vd} \leqslant r_b^{vd}$ and $D_a^{vd} \geqslant D_b^{vd}$. We can decompose the impact of remote access on profits in two effects. The first is a direct effect; that is the effect given that the other bank sets an interest rate as in the $(N_a, N_b)$ case. The direct effect when case VD prevails is positive and equals

$$\frac{[2(2f + t)z - t(2 - \bar{\theta})]^2}{8tz(\bar{\theta} - \underline{\theta})} - \frac{t}{4} > 0 \quad \left[ \text{with } z = \ln\left(\frac{2f + t}{2f}\right) \right].$$

It is positive as remote access increases the attractibility of that bank. In other words, a bank becomes "closer" to depositors. The second is an indirect effect; that is the effect on profits of a change in the interest rate of the bank not offering remote access. It equals the difference between the equilibrium profits and the first term in the previous expression. The indirect effect is positive as the intensity of price competition is decreased. Therefore, a bank should *overinvest* in remote access for strategic purposes.

The horizontal dominance case is shown in Figure 3.

*Horizontal dominance* (HD) occurs when $A$ attracts a strictly positive market share of the $\underline{\theta}$ types while it does not serve the entire market for the $\bar{\theta}$ types. This case is characterised by the dominance of the transportation rate over the quality difference and the interaction term.

A necessary condition for HD is $f/t \leqslant [(2 - \bar{\theta})/(2(\bar{\theta} - \underline{\theta}))]$. HD is more likely the lower the heterogeneity in taste for remote access. The $\underline{\theta}$ types located at bank $A$ prefer to deposit at that bank. The $\bar{\theta}$ types located at bank $B$ not

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[7] In the absence of the interaction term, VD would arise if $f/t \geqslant 1/(\bar{\theta} - \underline{\theta})$. Therefore, the region where VD prevails increases due to the negative interaction between the taste for remote access and the transportation rate.
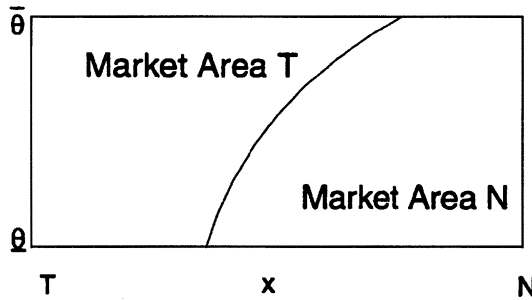
Figure 3
Horizontal dominance

offering remote access prefer to hold a deposit account at that bank. The equilibrium for horizontal dominance is described in Proposition 2.

*Proposition 2* Consider the case of horizontal dominance and let $q = \ln((2 - \underline{\theta})/(2 - \bar{\theta}))$. The equilibrium interest rates, provided they belong to the linear segment of the demand function, are

$$r_a^{hd} = R - \frac{t}{6} - \frac{2f}{3} + \frac{(\bar{\theta} - \underline{\theta})}{6} \frac{(2f - t)}{q}$$

and

$$r_b^{hd} = R + \frac{t}{6} + \frac{2f}{3} - \frac{(\bar{\theta} - \underline{\theta})}{3} \frac{(f + t)}{q}.$$

*Proof:*   See Appendix 5.

Note that $r_a^{hd} \leqslant r_b^{hd}$ and $D_a^{hd} \geqslant D_b^{hd}$. Again, we can decompose the effect on profits of introducing remote access in a direct and indirect effect. The former is positive and equals

$$\frac{[(2f + t)q - f(\bar{\theta} - \underline{\theta})]^2}{2tq(\bar{\theta} - \underline{\theta})} \frac{t}{4} > 0 \quad \left[ \text{with } q = \ln\left(\frac{2 - \underline{\theta}}{2 - \bar{\theta}}\right) \right].$$

It is positive as offering remote access increases the attractibility of that bank. The latter is the difference between the equilibrium profits under HD and the first term in the previous expression. The indirect effect is negative when case HD prevails. In other words, banks should *underinvest* in new communication technologies in order to avoid an aggressive response of the competitor.

We discuss some important comparative statics before moving to the first stage product decision. The effect on profits of a change in the quality difference $t$ on the bank not offering remote access is contingent on the type of dominance. More specifically, an increase in $f$ has a positive (negative) impact on the profits under vertical (horizontal) dominance. The reason is that this change decreases

(increases) the "number of marginal depositors" under vertical (horizontal) dominance. In other words, an increase in $f$ positively (negatively) affects the incentives under horizontal (vertical) dominance to compete for the marginal depositors.

### IV. PRODUCT DECISION-MAKING

In the first stage of the game, banks simultaneously decide what product (remote access or not) they will offer. Three types of product equilibria occur: *no remote access* $(N_a, N_b)$, *specialization* $(T_a, N_b)$ or *universal remote access* $(T_a, T_b)$. The Subgame Perfect Nash Equilibria in pure strategies (SPNE) when HD prevails in subgame $(T_a, N_b)$ are presented in Proposition 3.

*Proposition 3*   Let

$$q = \ln\left(\frac{2 - \underline{\theta}}{2 - \overline{\theta}}\right), \quad w = \ln\left(\frac{1 - \underline{\theta}}{1 - \overline{\theta}}\right),$$

$$\underline{\alpha}(t) \equiv t\left[\frac{3(q(2(\overline{\theta} - \underline{\theta}))^{1/2}) - 2(\overline{\theta} - \underline{\theta}) - 2q}{4(2q - (\overline{\theta} - \underline{\theta}))}\right]$$

and

$$\overline{\alpha}(t) \equiv t\left[\frac{(\overline{\theta} - \underline{\theta})\left(4 - 3\left(\frac{2q}{w}\right)^{1/2}\right) - 2q}{4(2q - (\overline{\theta} - \underline{\theta}))}\right]$$

Given that HD is the unique equilibrium in subgame $(T_a, N_b)$, the following Subgame Perfect Nash Equilibria occur (see Figure 4):

   a) no remote access in equilibrium if $f \leqslant \underline{\alpha}(t)$ (region I).
   b) specialization in equilibrium if $\underline{\alpha}(t) \leqslant f \leqslant \overline{\alpha}(t)$ (region II).
   c) universal remote access in equilibrium if $\overline{\alpha}(t) \leqslant f$ (region III)

*Proof:*   See appendix 6.

The case where VD prevails in subgame $(T_a, N_b)$ is presented in Proposition 4.

*Proposition 4*   Given that VD is the unique equilibrium in subgame $(T_a, N_b)$, the following equilibria occur (see Figure 4):

   a) no remote access is never an equilibrium.
   b) universal remote access is an equilibrium for lower values of the ratio $f/t$ (region VII).
   c) specialization is an equilibrium for higher values of the ratio $f/t$ (region VII).
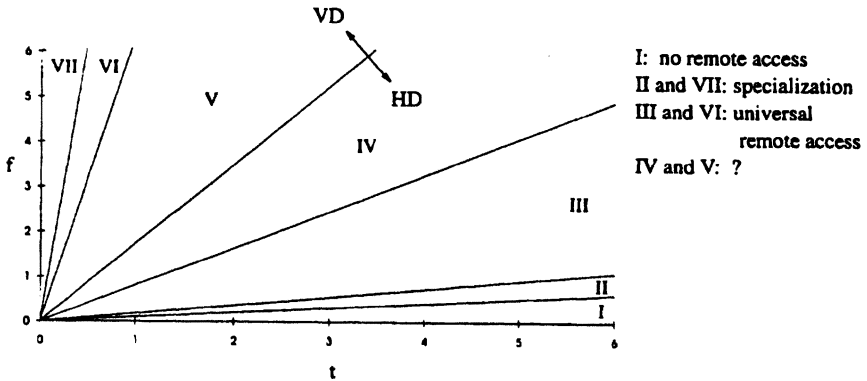
*Proof:*   See appendix 7.



Figure 4
Product decision ($\bar{\theta} = 0.6$, $\underline{\theta} = 0.2$)

Figure 4 illustrates propositions 3 and 4. The quality difference $f$ is denoted on the vertical axis and the transportation rate $t$ on the horizontal axis. The borderline between regions IV and V represents the separation between HD and VD for subgame $(T_a, N_b)$.

A necessary condition for HD to prevail in subgame $(T_a, N_b)$ is that $(f, t)$ is in regions I, II, III or IV. The functions $\underline{\alpha}(t)$ and $\bar{\alpha}(t)$ represent the borderline between regions I and II, II and III respectively. If the ratio quality difference to transportation costs is low (region I), neither bank offers remote access (*no remote access*). Offering remote access makes the deposit account more appealing (direct effect). Banks, however, become much closer substitutes since the linear transportation costs are dominant (indirect effect). Therefore, banks do not offer remote access. For intermediate values of the ratio quality difference to transportation rate (region II), exactly one bank[8] introduces remote access (*specialization*). In that region, the bank offering remote access steals sufficient depositors from its opponent since the quality difference is more important. As specialization is the equilibrium outcome in region II, horizontal dominance occurs. Each bank attracts a positive market share for each $\theta$ type. Offering remote access implies that banks partially segment their clients: the bank offering remote access attracts a higher market share of the depositors with a high taste for remote access. An increase in the quality difference $f$ implies that the profits of the bank not offering remote access decrease. If this quality difference becomes sufficiently large, both banks offer remote access (*universal*

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[8] A co-ordination problem arises as two prime strategy equilibria occur $((T_a, N_b)$ and $(N_a, T_b))$.

*remote access*) (region III). The bank offering remote access steals a lot of depositors away from the opponent. This direct effect overwhelms the indirect effect of banks becoming closer substitutes. If universal remote access prevails, banks become less differentiated and competition is sharpened. The equilibrium is not on the linear segment of the demand function in region IV. Therefore, we cannot investigate the overall SPNE for that region.

A necessary condition for VD to prevail in subgame $(T_a, N_b)$ is that $(f, t)$ is in regions V, VI or VII. In region V, the equilibrium is not on the linear segment of the demand function. Therefore, we cannot investigate the overall SPNE for that region. The functions separating regions V, VI and VII are implicit in $f$ and $t$. No remote access is never a SPNE. Offering remote access yields a higher intermediation margin as well as market share. Universal remote access is the unique equilibrium in region VI. Not offering remote access decreases substantially the attractiveness of that bank. If this decrease in attractiveness is not compensated by a decrease in substitutability, universal remote access is the unique SPNE. Banks are less horizontally differentiated when universal remote access prevails. The profits of a bank not offering remote access in the $(T_a, N_b)$ case are positively affected by an increase in $f$ because this softens deposit rate competition. In other words, an increase in $f$ reduces the substitutability between banks. Therefore, specialization arises for high values of the ratio quality difference to transportation rate (region VII). In that region, the bank (not) offering remote access attracts all high (low) taste for remote access types.

The introduction of remote access by at least one bank becomes more likely the lower the transportation costs. In other words, the attractiveness of introducing remote access increases if banking competition is already very strong.

Solving the game *without the interaction term* yields a different outcome. In the case where HD prevails both banks introduce the technology.[9] Banks steal depositors from their opponent. However, the substitutability is not affected as linear transportation costs do not decrease. Therefore, a different outcome results. If VD prevails, the region where specialization occurs decreases. Offering the technology has lower competitive effects as there is no negative effect on horizontal differentiation.

## V.  CONCLUDING REMARKS

In our framework, the disutility of the transportation rate depends on the presence of remote access. This explains the negative interaction between transportation rate and taste for remote access. Therefore, the degree of horizontal differentiation between banks is negatively affected by the introduction of remote access.

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[9] Without the interaction term, one cannot speak of remote access since depositors need to visit the bank: depositors only save waiting costs.

A range of equilibria emerge as the result of two effects. On the one hand, introducing remote access steals depositors from your opponent because the product specification becomes more appealing (direct effect). On the other hand, remote access affects the substitutability of banks for two reasons (indirect effect). First, banks become closer substitutes as the impact of linear transportation costs decreases. Second, deposit rate competition is affected by the size of the quality difference. For low and high values of the ratio quality difference to transportation cost, only one bank offers remote access (specialization). Intermediate (very low) values of the ratio quality difference to transportation costs yield universal (no) remote access.

If specialization arises, offering remote access is an instrument to (partially) segment clients. Vertical dominance occurs whenever the bank (not) offering remote access supplies the entire market of depositors with a high (low) taste for remote access. A necessary condition is that the ratio quality difference to transportation costs is sufficiently large. Horizontal dominance occurs if that ratio is sufficiently low. Then, both banks serve part of all taste for remote access types. Under both types of dominance, the bank offering remote access pays a lower deposit rate and faces a higher demand.

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APPENDIX 1

*Demand Function $(T_a, T_b)$*

Aggregate demand for bank A is derived by intergrating (3) over $[\underline{\theta}, \overline{\theta}]$. The demand function has five segments.

$$D_a^1 = 0 \text{ iff } r_a \leqslant r_b - \frac{t(1-\underline{\theta})}{2} \equiv r_a^{\min}$$

$$D_a^2 = \frac{1}{(\overline{\theta} - \underline{\theta})} \int_{\underline{\theta}}^{\frac{2}{t}[r_a - r_b + \frac{1}{2}]} (3) d\theta$$

$$= \frac{1}{(\overline{\theta} - \underline{\theta})} \left[ \frac{(r_b - r_a)}{t} \ln\left( \frac{2(r_a - r_b)}{t(\underline{\theta} - 1)} \right) + \frac{1}{t}\left[ r_a - r_b + \frac{t}{2} \right] - \frac{\underline{\theta}}{2} \right] \quad \text{if}$$

$$r_a^{\min} \leqslant r_a \leqslant r_b - \frac{t(1-\overline{\theta})}{2} \equiv \tilde{r}_a$$

$$D_a^3 = \frac{1}{(\overline{\theta} - \underline{\theta})} \int_{\underline{\theta}}^{\overline{\theta}} (3) d\theta$$

$$= \frac{1}{(\overline{\theta} - \underline{\theta})} \left[ \frac{(r_a - r_b)}{t} \ln\left[ \frac{\overline{\theta} - 1}{\underline{\theta} - 1} \right] + \frac{1}{2}(\overline{\theta} - \underline{\theta}) \right] \quad \text{if } \tilde{r}_a \leqslant r_a \leqslant r_b + \frac{t(1-\overline{\theta})}{2} \equiv \hat{r}_a$$

$$D_a^4 = \frac{1}{(\overline{\theta} - \underline{\theta})} \left[ \int_{\underline{\theta}}^{\theta'} (3) d\theta + \int_{\theta'}^{\overline{\theta}} d\theta \right] \left( \text{with } \theta' = \frac{2}{t}\left[ r_b - r_a + \frac{t}{2} \right] \right)$$

$$= \frac{1}{(\overline{\theta} - \underline{\theta})} \left[ \frac{(r_b - r_a)}{t} \ln\left( \frac{2(r_a - r_b)}{t(1 - \underline{\theta})} \right) - \frac{1}{t}\left( r_b - r_a + \frac{1}{2} \right) + \overline{\theta} - \frac{\underline{\theta}}{2} \right] \quad \text{if}$$

$$\hat{r}_a \leqslant r_a \leqslant r_b + \frac{t(1-\underline{\theta})}{2} \equiv r_a^{\max}.$$

$$D_a^5 = 1 \text{ if } r_a \geqslant r_a^{\max}.$$

## APPENDIX 2

*Demand Function* $(T_a, N_b)$

Aggregate demand is derived by integrating (4) over $[\underline{\theta}, \overline{\theta}]$. The demand function has five segments. The shape of the third segment depends on the slope of (4)

$$D_a^1 = 0 \text{ if } r_a \leqslant r_b - \overline{\theta}f - \frac{t}{2} \equiv r_a^{\min}$$

$$D_a^2 = \frac{1}{(\overline{\theta} - \underline{\theta})} \int_{\frac{2r_b - 2r_a - t}{2f}}^{\overline{\theta}} (4)d\theta$$

$$= \frac{1}{\overline{\theta} - \underline{\theta}} \left[ -\frac{2r_a - 2r_b + t + 4f)}{t} \ln \left( \frac{2f(2 - \overline{\theta})}{2r_a - 2r_b + t + 4f} \right) \right.$$

$$\left. - \frac{2r_a - 2r_b + t + 2f\overline{\theta}}{t} \right]$$

if $r_a^{\min} \leqslant r_a \leqslant r_b - \overline{\theta}f + t(1 - \overline{\theta})/2 \equiv \tilde{r}_a^{vd}$ and attracts no $\theta$ types

$$(r_a^{\min} \leqslant r_a \leqslant r_b - \underline{\theta}f - t/2 \equiv \tilde{r}_a^{hd}).$$

Vertical dominance (VD) occurs if $\tilde{r}_a^{vd} \leqslant r_a \leqslant \tilde{r}_a^{hd}$, this is when $A$ attracts the entire market for the $\overline{\theta}$ types (first inequality) and has a zero market share for the $\underline{\theta}$ types (second inequality).

$$D_a^{3vd} = \frac{1}{(\overline{\theta} - \underline{\theta})} \left[ \int_{\frac{2r_b - 2r_a - t}{2f}}^{\frac{t - 2(r_a - r_b)}{2f + t}} (4)d\theta + \int_{\frac{t - 2(r_a - r_b)}{2f + t}}^{\overline{\theta}} d\theta \right]$$

$$= \frac{1}{(\overline{\theta} - \underline{\theta})} \left[ \frac{(2r_a - 2r_b + t + 4f)}{t} \ln \left( \frac{2f + t}{2f} \right) + \overline{\theta} - 2 \right].$$

Horizontal dominance (HD) occurs if $\tilde{r}_a^{hd} \leqslant r_a \leqslant \tilde{r}_a^{vd}$, this is when $A$ attracts for the $\underline{\theta}$ types a strict positive market share (first inequality) while it does not serve the entire market for the $\overline{\theta}$ types (second inequality).

$$D_a^{3hd} = \frac{1}{(\overline{\theta} - \underline{\theta})} \int_{\underline{\theta}}^{\overline{\theta}} (4)d\theta = \frac{1}{(\overline{\theta} - \underline{\theta})} \left[ \frac{(2r_a - 2r_b + t + 4f)}{t} \ln \left( \frac{2 - \underline{\theta}}{2 - \overline{\theta}} \right) + \frac{2f(\underline{\theta} - \overline{\theta})}{t} \right].$$

If $r_a$ is "high", $A$ attracts the entire market for $\overline{\theta}$ types $(r_a \geqslant \tilde{r}_a^{vd})$ and serves at least some $\underline{\theta}$ types $(r_a \geqslant \tilde{r}_a^{hd})$. The demand $A$ faces becomes

$$D_a^4 = \frac{1}{(\overline{\theta} - \underline{\theta})} \left[ \int_{\underline{\theta}}^{\frac{t - 2(r_a - r_b)}{2f + t}} (4)d\theta + \int_{\frac{t - 2(r_a - r_b)}{2f + t}}^{\overline{\theta}} d\theta \right]$$

$$= \frac{1}{(\overline{\theta} - \underline{\theta})} \left[ -\frac{(2r_a - 2r_b + t + 4f)}{t} \ln \left( \frac{(2r_a - 2r_b + t + 4f)}{(t + 2f)(2 - \underline{\theta})} \right) \right.$$

$$\left. + \frac{2f(2r_a - 2r_b + t(\underline{\theta} - 1) + 2\underline{\theta}f)}{t(t + 2f)} + \frac{2r_a - 2r_b + \overline{\theta}(t + 2f) - t}{t + 2f} \right]$$

$$D_a^5 = 1 \text{ if } r_a \geqslant r_b - \underline{\theta}f + \frac{t(1 - \underline{\theta})}{2} \equiv r_a^{\max}.$$

APPENDIX 3

*Equilibrium subgame* $(T_a, T_b)$

We will proceed as follows. First, it is shown that there is only an equilibrium if $r_a \in D_a^3$ and $r_b \in 1 - D_a^3$. Second, the equilibrium is computed and characterized.

a) There exists no equilibrium for $r_a \in D_a^2$ and $r_b \in 1 - D_a^2$ and its symmetric $r_a \in D_a^4$ and $r_b \in 1 - D_a^4$.

  Necessary conditions to have a maximum for $r_a \in D_a^2$ and $r_b \in 1 - D_a^2$ are respectively

(5) $\qquad \dfrac{\partial \Pi_a}{\partial r_a} = 2r_a - 2r_b + t(1 - \underline{\theta}) - 2(2r_a - r_b - R)\ln\left(\dfrac{2(r_a - r_b)}{t(\underline{\theta} - 1)}\right) = 0$

(6) $\qquad \dfrac{\partial \Pi_b}{\partial r_b} = 2(r_a - 2r_b - R)\ln\left(\dfrac{2(r_a - r_b)}{t(\underline{\theta} - 1)}\right) - (2r_a - 2r_b + t(\underline{\theta} - 2\overline{\theta} + 1)) = 0$

A necessary is that $(5) = (6)$ iff

(7) $\qquad 4r_a - 4r_b + 2t(1 - \overline{\theta}) = (6r_a - 6r_b - 4R)\ln\left(\dfrac{2(r_a - r_b)}{t(\underline{\theta} - 1)}\right)$

Since $r_a \in [r_a^{\min}, \tilde{r}_a]$, the LHS of (7) is negative while the RHS of (7) is positive yielding a contradiction.

  By symmetry, there cannot be an equilibrium for $r_a \in D_a^4$ and $r_b \in 1 - D_a^4$.

b) The first order conditions for profit maximization on $D_a^3$ and $1 - D_a^3$ have the following solution: $r_i^{tt} = R - \dfrac{t(\overline{\theta} - \underline{\theta})}{2 \ln\left(\dfrac{1 - \underline{\theta}}{1 - \overline{\theta}}\right)}$. The equilibrium exists iff $\Pi_a(r_a^{tt}, r_b^{tt}) \geqslant \Pi_a(r_a, r_b^{tt})$ and $\Pi_b(r_a^{tt}, r_b^{tt}) \geqslant \Pi_b(r_a^{tt}, r_b)$.

i) The profit function is concave for $r_a \in [\tilde{r}_a, r_a^{\max}]$ since demand is concave for those segments. Hence, if profits reach their maximum in $[\tilde{r}_a, \hat{r}_a]$, the maximum is no element of $[\tilde{r}_a, r_a^{\max}]$.

ii) If $r_a \in [r_a^{\min}, \tilde{r}_a]$, the derivative of the profit function is negative if $\ln\left(\dfrac{1 - \underline{\theta}}{1 - \overline{\theta}}\right) \leqslant \dfrac{7(\overline{\theta} - \underline{\theta})}{2(1 - \underline{\theta})}$. If $r_a \in [r_a^{\min}, \tilde{r}_a]$, then $r_a = \alpha r_a^{\min} + (1 - \alpha)\tilde{r}_a. (\alpha \in [0, 1])$.

The FOC for $r_a$ then equals

$$\frac{(\overline{\theta} - \underline{\theta})(1 - \alpha)w}{\overline{\theta} - \underline{\theta} + 2(1 - \overline{\theta} + \alpha(\overline{\theta} - \underline{\theta}))w} - \ln\left(\frac{1 - \underline{\theta}}{1 - \overline{\theta} + \alpha(\overline{\theta} - \underline{\theta})}\right) \quad \text{with } w = \ln\left(\frac{1 - \underline{\theta}}{1 - \overline{\theta}}\right).$$

If (4) $\leqslant 0$ for all $\alpha \in [0, 1]$ then $A$'s best reply is not in $[r_a^{\min}, \tilde{r}_a]$. Notice that if $\alpha = 0$, (4) becomes $2(1 - \overline{\theta})w^2 / [2(\overline{\theta} - 1)w - (\overline{\theta} - \underline{\theta})] < 0$ and if $\alpha = 1$, (4) $= 0$. If $\partial(4)/\partial\alpha \geqslant 0$ for all $\alpha \in [0, 1]$ then (4) $\leqslant 0$.

$$\frac{\partial(4)}{\partial\alpha} \geqslant 0 \text{ iff } 2[\alpha(\underline{\theta} - \overline{\theta}) + \overline{\theta} - 1][2\alpha(\underline{\theta} - \overline{\theta}) - 1 + 2\overline{\theta} - \underline{\theta}]w^2$$

$$+ 3(\underline{\theta} - \overline{\theta})[\alpha(\underline{\theta} - \overline{\theta}) + \overline{\theta} - 1]w(\underline{\theta} - \overline{\theta})^2 \geqslant 0$$

Expression (5) reaches its minimum for $\alpha^* = [3(\overline{\theta} - \underline{\theta}) + 2(\underline{\theta} - 8\overline{\theta} + 6)z]/(8z(\overline{\theta} - \underline{\theta}))$. Hence, if at $\alpha^*$, $\partial(4)/\partial\alpha \geqslant 0$, then it is positive for all $\alpha$. Plugging $\alpha^*$ into (5) yields the following condition $\ln((1 - \underline{\theta})/(1 - \overline{\theta})) \leqslant 7(\overline{\theta} - \underline{\theta})/(2(1 - \underline{\theta}))$ which is always fulfilled if $\overline{\theta} \leqslant 0.965$.

Now, (i) and (ii) imply that the profit function has a global maximum in the interior of $D_a^3$ if $\theta \leqslant 0.965$. By symmetry, the same analysis holds for $\Pi_b(r_a^{tt}, r_b^{tt}) \geqslant \Pi_b(r_a^{tt}, r_b)$.

## APPENDIX 4

*Proof of Proposition 1*

The proof will proceed as follows. First, we show that the profit functions are quasi-concave and continuous in their own interest rate. Second, the equilibrium is characterized for $r_a \in D_a^3$ and $r_b \in 1 - D_a^3$.

a) Let's focus on the profit function of bank $A$ when VD occurs. The demand function for segments $D_a^3$ and $D_a^4$ is concave implying a concave profit function in $r_a$. The demand function described by segment $D_a^2$ is convex in $r_a$. Since $\partial^3 \pi_a^2 / \partial r_a^3 < 0$ and the RHS derivative at $r_a^{min}$ equals zero, $\partial \pi_a^2 / \partial r_a = 0$ has at most one solution on the segment $D_a^2$. Notice that at $\bar{r}_a^{hd}$, the LHS and RHS derivative are equal.

Combining the concavity of $\pi_a$ on $D_a^3$ and $D_a^4$, the fact that $\partial \pi_a / \partial r_a$ equals zero at $r_a^{min}$ and $r_a^{max}$ and the results from segment $D_a^2$, we obtain that $\pi_a$ is quasi-concave in $r_a$. Since the profit function is continuous and quasi-concave, $\pi_a$ has a unique maximum with respect to $r_a$.

The same kind of reasoning applies for $\pi_b$. Hence the subgame possesses a Pure Strategy Nash Equilibrium (Dasgupta and Maskin [1986]).

b) The appropriate demand functions are given by $D_a^{3vd}$ and $1 - D_a^{3vd}$. The FOC for maximizing the profit functions have a unique solution given by $r_a^{vd}$ and $r_b^{vd}$. These deposit rates are equilibrium deposit rates iff $r_a^{vd} \in [\bar{r}_a^{vd}(r_b^{vd}), \tilde{r}_a^{hd}(r_b^{vd})]$ and $r_b^{vd} \in [\tilde{r}_b^{vd}(r_a^{vd}), \bar{r}_b^{Rd}(r_a^{vd})]$. First $r_a^{vd} \geqslant \bar{r}_a^{vd}(r_b^{vd})$ and $r_b^{vd} \leqslant \tilde{r}_b^{hd}(r_a^{vd})$ hold if $0 \geqslant [2f(4 - 3\theta)] + t(5 - 3\theta)]z - t(4 - \underline{\theta} - \overline{\theta})$ holds. Second, $r_a^{vd} \leqslant \tilde{r}_a^{hd}(r_b^{vd})$ and $r_b^{vd} \geqslant \bar{r}_b^{vd}(r_a^{vd})$ hold if $0 \leqslant [2f(4 - 3\underline{\theta}) - t]z - t(4 - \underline{\theta} - \overline{\theta})$ holds. This completes the proof.

## APPENDIX 5

*Proof of Proposition 2*

Similarly as for the vertical dominance case, we can show that profit functions are quasi-concave. Then, $r_a^{hd}$ and $r_b^{hd}$ are the equilibrium interest rates if $r_a^{hd} \in [\bar{r}_a^{hd}(r_b^{hd}), \tilde{r}_a^{vd}(r_b^{hd})]$ and $r_b hd \in [\tilde{r}_b^{hd}(r_a^{hd}), \bar{r}_b^{vd}(r_a^{hd})]$     iff     $0 \geqslant [2f(4 - 3\underline{\theta}) - t]q - (\overline{\theta} - \underline{\theta})(4f + t)$     and $0 \leqslant [2f(4 - 3\overline{\theta}k) + t(5 - 3\overline{\theta})]q - (\overline{\theta} - \underline{\theta})(4f + t)$ hold.

## APPENDIX 6

*Proof of Proposition 3*

Notice that $\pi_a(T_a, N_b) = \pi_a(N_a, N_b)$ iff $\alpha = \underline{\alpha}(t)$ and $\pi_b(T_a, N_b) = \pi_b(T_a, T_b)$ iff $\alpha = \overline{\alpha}(t)$. In addition, notice that $\underline{\alpha}(t) \leqslant \overline{\alpha}(t)$. Hence,
   a) $(N_a, N_b)$ in equilibrium if $f \leqslant \underline{\alpha}(t)$.
   b) $(T_a, N_b)$ in equilibrium if $\underline{\alpha}(t) \leqslant f \leqslant \overline{\alpha}(t)$.
   c) $(T_a, T_b)$ in equilibrium if $f \geqslant \overline{\alpha}(t)$.

APPENDIX 7

*Proof of Proposition 4*

$(N_a, N_b)$ is never an equilibrium since $r_a^{vd} \leqslant r_a^{nn}$ and $x_a^{vd} \geqslant x_a^{nn}$. $(T_a, T_b)$ is an equilibrium iff $\pi_b(T_a, N_b) \leqslant \pi_b(T_a, T_b)$. This inequality cannot be solved, yielding an implicit inequality. However, since $\partial \pi_b(T_a, N_b)/\partial f \geqslant 0$ and $\partial \pi_b(T_a, Tb)/\partial f = 0$, we can conclude that for higher values of $f$ (given $t$), $(T_a, N_b)$ will be the SPNE.

REFERENCES

ANDERSON, S., DE PALMA, A. and THISSE, J. F., 1992, *Discrete Choice Theory of Product Differentiation* (MIT Press, Cambridge).

BANK FOR INTERNATIONAL SETTLEMENTS, 1993, *Payment systems in the Group of Ten Countries* (BIS, Basle).

BOUCKAERT, J. and DEGRYSE, H., 1995, 'Phonebanking', *European Economic Review*, 39, 229–244.

BUREAU EUROPEÉN DES UNIONS DES CONSOMMATEURS, 1992, *Home Banking in the EC* (Brussels).

CAPLIN, A. and NALEBUFF, B., 1991, 'Aggregation and Imperfect Competition: on the Existence of Equilibrium', *Econometrica*, 59, 25–59.

DASGUPTA, P. and MASKIN, E., 1986, 'The Existence of Equilibrium in Discontinuous Economic Games, I: Theory', *Review of Economic Studies*, 53, 1–26.

ECONOMIDES, N., 1989, 'Quality Variations and Maximal Variety Differentiation', *Regional Science and Urban Economics*, 19, 21–29.

ECONOMIDES, N. 1993, 'Quality Variations in the Circular Model of Variety-Differentiated Products', *Regional Science and Urban Economics*, 23, 235–257.

GABSZEWICZ, J. and THISSE, J. F., 1979, 'Price competition, Quality and Income Disparities', *Journal of Economic Theory* 20, 340–359.

GARELLA, P. and MARTINEZ-GIRALT, X., 1993, 'After all Hotelling was (almost) Right', paper presented at the EARIE-Conference in Tel-Aviv.

HOTELLING, H., 1929, 'Stability in Competition', *Economic Journal*, 29, 41–57.

KLEMPERER, P., 1992, 'Equilibrium Product Lines: Competing Head-to-Head May Be Less Competitive', *American Economic Review*, 82, 740–755.

MATUTES, C. and PADILLA, A. J., 1994, 'Shared ATM Networks and Banking Competition', *European Economic Review*, 38, 1113–1138.

MATUTES, C. and VIVES, X., 1992, 'Competition for Deposits, Risk of Failure and Regulation in Banking', paper presented at the EEA conference in Dublin.

MAYER, C. and VIVES, X., 1993, 'Introduction', in Mayer, C. and Vives, X. (eds), Capital Markets and Financial Intermediation, Cambridge University Press.

NEVEN, D. and THISSE, J. F., 1990, 'On Quality and Variety Competition' in Gabszewicz, J. J., Richard, J. F. and Wolsey, L. A. (eds), *Economic Decision Making: Games, Econometrics and Optimisation* (Elsevier Science Publishers).

PERLMAN, A., 1995, 'A job for multimedia: squeezed profits? Strapped by the competition? Multimedia is at hand for the retail banks,' *The Banker*, 67–69.

SHAKED, A. and SUTTON, J., 1982, 'Relaxing Price Competition Through Product Differentiation', *Review of Economic Studies*, 49, 3–14.

TABUCHI, T., 1994, 'Two-stage two-dimensional spatial competition Between Two Firms', *Regional Science and Urban Economics*, 24, 207–227.

TIROLE, J., 1988, *The Theory of Industrial Organization*, (MIT Press London England).

# LINKED CITATIONS

*- Page 1 of 2 -*

**On the Interaction Between Vertical and Horizontal Product Differentiation: An Application to Banking**
Hans Degryse
*The Journal of Industrial Economics*, Vol. 44, No. 2. (Jun., 1996), pp. 169-186.
Stable URL:
http://links.jstor.org/sici?sici=0022-1821%28199606%2944%3A2%3C169%3AOTIBVA%3E2.0.CO%3B2-V

## [Footnotes]

### [3] **Equilibrium Product Lines: Competing Head-to-Head May be Less Competitive**
Paul Klemperer
*The American Economic Review*, Vol. 82, No. 4. (Sep., 1992), pp. 740-755.
Stable URL:
http://links.jstor.org/sici?sici=0002-8282%28199209%2982%3A4%3C740%3AEPLCHM%3E2.0.CO%3B2-U

## References

**Aggregation and Imperfect Competition: On the Existence of Equilibrium**
Andrew Caplin; Barry Nalebuff
*Econometrica*, Vol. 59, No. 1. (Jan., 1991), pp. 25-59.
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28199101%2959%3A1%3C25%3AAAICOT%3E2.0.CO%3B2-M

**Stability in Competition**
Harold Hotelling
*The Economic Journal*, Vol. 39, No. 153. (Mar., 1929), pp. 41-57.
Stable URL:
http://links.jstor.org/sici?sici=0013-0133%28192903%2939%3A153%3C41%3ASIC%3E2.0.CO%3B2-F

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## LINKED CITATIONS

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Paul Klemperer
*The American Economic Review*, Vol. 82, No. 4. (Sep., 1992), pp. 740-755.
Stable URL:

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