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Macro-economic version of a classical formula in risk theory

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In this paper an extension of Panjer's recursive formula is computed, taking into account macro-economic elements and a delay in the claim settlement. This formula will be used to calculate the cumulative distribution function of the liabilities of an insurer up to some fixed time t.

Keywords: Interest, Inflation, Discounting factor, Handling delay, Payment delay, Risk processes, Liability process, Recursive formula.

1. Introduction

In risk theory compound distributions, such as the compound Poisson distribution, are often used to model the distribution of the total claim amount of claims that have occurred in a fixed period of time.

Panjer (1981) developed a recursive definition for the distribution of the total claim amount for a certain family of claim number distributions and arbitrary claim amount distributions. The claim number distribution has to fulfill the following condition:

$$p_n = p_{n-1}(a + b/n), \qquad n = 1, 2, 3, \ldots, \tag{1.1}$$

where p_n denotes the probability that exactly n claims occur in the fixed time interval.

This paper will be devoted to the extension of Panjer's formula in case a delay in the claim settlement is taken into account.

We shall assume that a period of time may go by between the moment a claim occurs (called claim time) and the moment of settlement (payment) of that claim.

The period of time between these two events is called the settling delay.

We divide the settling delay into two parts: the handling delay and the payment delay. The handling delay is the time period between the claim time and the time both the insurer and the insured agree upon the eventual size of the claim. The payment delay is the time going by between the agreement upon the claim amount and the compensation of the claim by the insurer.

Furthermore we shall assume that the claim amount may vary with the evolution of inflation during the handling delay, but remains independent of further possible fluctuations of inflation during the payment delay. For more details we refer to Boogaert and Haezendonck (1989).

As usual the claims are counted by a claim number process $\{N_t\colon t \in \mathbb{R}_+\}$. The moments of occurrence of the successive claims (claim times) are represented by T_n $(n \in \mathbb{N}_0)$ with $T_0 \equiv 0$. We also make the assumption that the claim number distribution fulfills condition (1.1)

Furthermore we introduce three sequences of real-valued and positive random variables: $\{X_n\colon n \in \mathbb{N}\}$, $\{H_n\colon n \in \mathbb{N}\}$ and $\{V_n\colon n \in \mathbb{N}\}$. We assume that X_n is the size, that H_n is the handling delay and that V_n is the payment delay of the nth claim. The random variables $H_n + V_n (n \in \mathbb{N})$ represent the successive settling delays. All the random variables and random processes considered in this paper are defined on some fixed probability space $(\Omega, \mathscr{A}, P)$. Mathematically it is supposed that the sequence of random variables

$$\{(X_n, H_n, V_n)\colon n \in \mathbb{N}\}$$

is independent of the claim number process $\{N_t: t \in \mathbb{R}_+\}$ and that the random vectors (X_n, H_n, V_n) $(n \in \mathbb{N})$ are mutually independent and identically distributed. This assumption implies that $\{X_n: n \in \mathbb{N}\}$, $\{H_n: n \in \mathbb{N}\}$ and $\{V_n: n \in \mathbb{N}\}$ are sequences of i.i.d. random variables. Furthermore we make the additional assumption that the handling delay H_n and the payment delay V_n are conditionally independent given the claim size X_n.

We also need the following macro-economic elements:

$i(t)$ is the force of interest at time $t (t \in \mathbb{R}_+)$,
$\delta(t)$ is the force of inflation at time $t (t \in \mathbb{R}_+)$.

We suppose that these mappings are continuous and that $j(t) = i(t) - \delta(t)$, called the force of real interest, is positive. Eventually

$$f(t) = \exp\left\{-\int_0^t j(u)\,\mathrm{d}u\right\} \qquad (t \in \mathbb{R}_+) \tag{1.2}$$

represents the discounting factor at time t. [See Delbaen and Haezendonck (1987).] The present value of the liabilities of the insurer is then defined as follows:

$$L_t = \sum_{n=1}^{N_t} \exp\left\{-\int_{T_n+H_n}^{T_n+H_n nd+V_n} i(r)\,\mathrm{d}r\right\} f(T_n + H_n) X_n \qquad (t \in \mathbb{R}_+). \tag{1.3}$$

In this formula

$$f(T_n + H_n) X_n$$

represents the present value of the nth claim amount, and

$$\exp\left\{-\int_{T_n+H_n}^{T_n+H_n+V_n} i(r)\,\mathrm{d}r\right\}$$

reflects the fact that the insurer has still income from interest on the claim amount during the payment delay. The random process $\{L_t: t \in \mathbb{R}_+\}$ is called the liability process.

In Section 2 we will compute an extension of Panjer's recursive definition for the distribution of the liabilities in a fixed period of time.

In Section 3 some numerical illustrations will be given.

2. Recursion formula

The following lemma is useful for computations involving the probability distribution P_{L_t} of L_t.

Lemma 2.1. *Let $t > 0$ be fixed and consider a sequence $\{Y_n^{(t)}: n \in \mathbb{N}\}$ of i.i.d. real valued random variables which are uniformly distributed on the interval $[0, t]$. Assume that $\{Y_n^{(t)}: n \in \mathbb{N}\}$ is independent of $\{(N_t, (X_n, H_n, V_n)): t \in \mathbb{R}_+, n \in \mathbb{N}\}$. Define*

$$U_t = \sum_{n=1}^{N_t} \exp\left[-\int_{Y_n^{(t)}+H_n}^{Y_n^{(t)}+H_n+V_n} i(r)\,\mathrm{d}r\right] f\left(Y_n^{(t)} + H_n\right) X_n \qquad (t \in \mathbb{R}_+). \tag{2.1}$$

Then L_t and U_t have the same probability distribution.

Proof. See Boogaert and Haezendonck (1989). □

From now on we will use the following notations:

- g is the density function of the claim amount X_1.
- $\bar{g}$ is the density function of the expression $\exp\{-iV_1 - jH_1 - jY_1^{(t)}\} X_1$.

- $\bar{G}$ is the cumulative distribution function of the expression stated above.
- $f_{H_1 \mid X_1}$ is the density function of the handling delay H_1 conditionally on the claim size X_1.
- $f_{V_1 \mid X_1}$ is the density function of the payment delay V_1 conditionally on the claim size X_1.

We suppose that these functions exist, whenever we use them.

Throughout this and the following chapter we suppose that the force of interest i and the force of inflation δ are constant. From (1.3) it then follows that

$$L_t = \sum_{n=1}^{N_t} \exp\{-iV_n - jH_n - jT_n\} X_n \qquad (t \in \mathbb{R}_+). \tag{2.2}$$

Take now $t(>0)$ fixed and A belonging to $\mathscr{R}_+$. Using Lemma 2.1. we successively find that

$$\begin{aligned} P_{L_t}(A) &= \mathrm{E}[1_A(L_t)] \\ &= \mathrm{E}[1_A(U_t)] \\ &= \sum_{n \geq 0} \mathrm{E}\left[1_A\left(\sum_{m=1}^{n} \exp\{-iV_m - jH_m - jY_m^{(t)}\} X_m\right) 1_{(N_t = n)}\right] \\ &= p_0 \delta_{\{0\}}(A) + \sum_{n \geq 1} p_n \mathrm{E}\left[1_A\left(\sum_{m=1}^{n} \exp\{-iV_m - jH_m - jY_m^{(t)}\} X_m\right)\right] \\ &= p_0 \delta_{\{0\}}(A) + \int_A \left(\sum_{n \geq 1} p_n \bar{g}^{*n}(x)\right) \mathrm{d}x. \end{aligned} \tag{2.3}$$

Now we take $z \geq 0$ and we will calculate $\bar{G}(z)$. We have

$$\begin{aligned} \bar{G}(zx) &= \mathrm{E}\left[1_{[0,z]}\left(\exp\{-iV_1 - jH_1 - jY_1^{(t)}\} X_1\right)\right] \\ &= \frac{1}{t} \int_0^t \int 1_{[0,z]}(\exp\{-iV_1 - jH_1 - jy\} X_1) \,\mathrm{d}P \,\mathrm{d}y \\ &= \frac{1}{t} \int_0^t \int 1_{[0,z]}(\exp\{-iv - jh - jy\} x) f_{V_1 \mid X_1}(v \mid x) f_{H_1 \mid X_1}(h \mid x) g(x) \,\mathrm{d}v \,\mathrm{d}h \,\mathrm{d}x \,\mathrm{d}y. \end{aligned} \tag{2.4}$$

In case $z < 0$, we have that $\bar{G}(z) = 0$.

Now we take the derivative of the right-hand side of expression (2.4.) w.r.t. z. After some tedious, but straightforward calculations we then find for $z > 0$,

$$\begin{aligned} \bar{g}(z) = -g(z) &+ \frac{1}{izt} \int_z^{z\,\mathrm{e}^{jt}} g(x) \int_0^{(1/j)\ln(x/z)} \int_0^{(1/j)\ln(x/z)-y} f_{H_1 \mid X_1}(h \mid x) \\ &\times f_{V_1 \mid X_1}\left(\frac{1}{i}\ln\frac{x}{z} - \frac{jh}{i} - \frac{jy}{i} \,\middle|\, x\right) \mathrm{d}h \,\mathrm{d}y \,\mathrm{d}x \\ &+ \frac{1}{izt} \int_{ze^{jt}}^{+\infty} g(x) \int_0^t \int_0^{(1/j)\ln(x/z)-y} f_{H_1 \mid X_1}(h \mid x) f_{V_1 \mid X_1}\left(\frac{1}{i}\ln\frac{x}{z} - \frac{jh}{i} - \frac{jy}{i} \,\middle|\, x\right) \mathrm{d}h \,\mathrm{d}y \,\mathrm{d}x. \end{aligned} \tag{2.5}$$

Lemma 2.2. *Consider a sequence $\{W_n\colon n \in \mathbb{N}\}$ of i.i.d. and real valued random variables, with common distribution function P_W. For every Borel subset A we have*

$$\int x P_W^{*n}(A - x) \,\mathrm{d}P_W(x) = \frac{1}{n+1} \int_A y \,\mathrm{d}P_W^{*(n+1)}(y). \tag{2.6}$$

Proof. We successively have

$$\int x P_W^{*n}(A-x)\,\mathrm{d}P_W(x) = \int x \mathrm{E}[1_A(W_1+W_2+\cdots+W_n+x)]\,\mathrm{d}P_W(x)$$
$$= \int W_{n+1} 1_A(W_1+\cdots+W_n+W_{n+1})\,\mathrm{d}P$$
$$= \frac{1}{n+1}\int (W_1+\cdots+W_{n+1}) 1_A(W_1+\cdots+W_{n+1})\,\mathrm{d}P$$
$$= \frac{1}{n+1}\int_A y\,\mathrm{d}P_W^{*(n+1)}(y). \qquad \square$$

Suppose that the density function (w.r.t. the Lebesgue measure) of P_W exist. We use the notation f_W. As a consequence of the lemma we then get

$$\int x\left(\int_A f_W^{*n}(z-x)\,\mathrm{d}z\right) f_W(x)\,\mathrm{d}x = \frac{1}{n+1}\int_A y f_W^{*(n+1)}(y)\,\mathrm{d}y. \tag{2.7}$$

And therefore

$$\int_A\left(\int x f_W^{*n}(z-x) f_W(x)\,\mathrm{d}x\right)\mathrm{d}z = \frac{1}{n+1}\int_A y f_W^{*(n+1)}(y)\,\mathrm{d}y. \tag{2.8}$$

Equation (2.8) implies that

$$\int x f_W^{*n}(z-x) f_W(x)\,\mathrm{d}x = \frac{z}{n+1} f_W^{*(n+1)}(z). \tag{2.9}$$

Now we put

$$\bar{\bar{g}}(x) = \sum_{n\geq 1} p_n g^{*n}(x). \tag{2.10}$$

Proposition 2.1 (Recursive formula). *For every positive x we have*

$$\bar{\bar{g}}(x) = p_1 \bar{g}(x) + \int_0^x \left(a + \frac{by}{x}\right)\bar{\bar{g}}(x-y)\bar{g}(y)\,\mathrm{d}y. \tag{2.11}$$

Proof. We successively find

$$\sum_{n\geq 1} p_n \mathrm{E}\left[1_A\left(\sum_{m=1}^{n}\exp\{-iV_m - jH_m - jY_m^{(t)}\}X_m\right)\right]$$
$$= p_1\mathrm{E}\left[1_A\left(\exp\{-iV_1 - jH_1 - jY_1^{(t)}\}X_1\right)\right] + \sum_{n\geq 2} p_n \mathrm{E}\left[1_A\left(\sum_{m=1}^{n}\exp\{-iV_m - jH_m - jY_m^{(t)}\}X_m\right)\right]$$
$$= p_1\mathrm{E}\left[1_A\left(\exp\{-iV_1 - jH_1 - jY_1^{(t)}\}X_1\right)\right] + \sum_{n\geq 1} p_{n+1} \mathrm{E}\left[1_A\left(\sum_{m=1}^{n+1}\exp\{-iV_m - jH_m - jY_m^{(t)}\}X_m\right)\right]$$
$$= p_1\mathrm{E}\left[1_A\left(\exp\{-iV_1 - jH_1 - jY_1^{(t)}\}X_1\right)\right]$$
$$+ a\sum_{n\geq 1} p_n \mathrm{E}\left[1_A\left(\sum_{m=1}^{n+1}\exp\{-iV_m - jH_m - jY_m^{(t)}\}X_m\right)\right]$$
$$+ b\sum_{n\geq 1} \frac{p_n}{n+1} \mathrm{E}\left[1_A\left(\sum_{m=1}^{n+1}\exp\{-iV_m - jH_m - jY_m^{(t)}\}X_m\right)\right].$$

Therefore

$$\bar{\bar{g}}(x)=p_1\bar{g}(x)+a\int\sum_{n\geq 1}p_n\bar{g}^{*n}(x-u)\bar{g}(u)\,\mathrm{d}u+b\sum_{n\geq 1}\frac{p_n}{n+1}\bar{g}^{*(n+1)}(x). \tag{2.12}$$

From equation (2.9) it then follows that

$$\bar{\bar{g}}(x)=p_1\bar{g}(x)+a\int\sum_{n\geq 1}p_n\bar{g}^{*n}(x-u)\bar{g}(u)\,\mathrm{d}u+b\int\frac{u}{x}\sum_{n\geq 1}p_n\bar{g}^{*n}(x-u)\bar{g}(u)\,\mathrm{d}u.$$

So finally we get

$$\bar{\bar{g}}(x)=p_1\bar{g}(x)+a\int\bar{\bar{g}}(x-u)\bar{g}(u)\,\mathrm{d}u+b\int\frac{u}{x}\bar{\bar{g}}(x-u)\bar{g}(u)\,\mathrm{d}u.$$

This completes the proof. □

Now we consider the special case where no handling nor payment delay is taken into account. Then we get the random process $\{\bar{S}_t\colon t\in\mathbb{R}_+\}$, where

$$\bar{S}_t=\sum_{n=1}^{N_t}f(T_n)X_n \qquad (t\in\mathbb{R}_+). \tag{2.13}$$

For more details on this random process we refer to Delbaen and Haezendonck (1987).

For $z\geq 0$ we then get

$$\bar{G}(z)=\frac{1}{t}\int_0^t\int_0^{z\,\mathrm{e}^{jy}}g(x)\,\mathrm{d}x\,\mathrm{d}y. \tag{2.14}$$

And therefore, for $z>0$,

$$\bar{g}(z)=\frac{1}{t}\int_0^t g(z\,\mathrm{e}^{jy})\,\mathrm{e}^{jy}\mathrm{d}y \tag{2.15}$$

In case the claim size is exponentially distributed with parameter $1/\mu$ ($\mu>0$), i.e.

$$g(x)=(1/\mu)\,\mathrm{e}^{-x/\mu}1_{[0,\infty[}(x),$$

we get

$$\bar{g}(z)=(1/tjz)(\mathrm{e}^{-z/\mu}-\mathrm{e}^{-z\mathrm{e}^{jt}/\mu})1_{]0,\infty[}(z). \tag{2.16}$$

3. Example

In this chapter we take $t(>0)$ fixed and we shall suppose that the claim number process $\{N_t\colon t\in\mathbb{R}_+\}$ is an homogeneous Poisson process with risk parameter λ (>0). Then

$$p_n=P(N_t=n)=\left[(\lambda t)^n/n!\right]\mathrm{e}^{-\lambda t}, \qquad n=0,1,2,\ldots. \tag{3.1}$$

Therefore condition (1.1) is fulfilled for a equal to zero and b equal to λt.

Furthermore we suppose that the claim amount X_1 is exponentially distributed with parameter $1/\mu$ ($\mu>0$), i.e.

$$g(x)=(1/\mu)\,\mathrm{e}^{-x/\mu}1_{[0,\infty[}(x). \tag{3.2}$$

To make it possible to compare the new results with the one obtained in the classical case, we will first of all consider this classical case. The liabilities of the insurer up to time t are then given by the classical risk process $\{S_t\colon t\in\mathbb{R}_+\}$, where

$$S_t=\sum_{i=1}^{N_t}X_i. \tag{3.3}$$

Table 1

x	$P(S_{0.5} \leq x)$	$P(S_1 \leq x)$
0.00	0.606531	0.367879
0.05	0.621413	0.386045
0.10	0.635747	0.403758
0.20	0.662847	0.437859
0.30	0.687978	0.472046
0.40	0.711279	0.503967
0.50	0.732880	0.530131
0.60	0.752900	0.560236
0.70	0.771452	0.583818
0.80	0.788641	0.608682
0.90	0.804563	0.632108
1.00	0.819310	0.654254

Panjer's recursive formula then gives us that

$$P(S_t \leq x) = p_0 + \int_0^x \bar{\bar{g}}(y)\,\mathrm{d}y, \qquad x \geq 0, \tag{3.4}$$

where

$$\bar{\bar{g}}(y) = p_1 g(y) + \lambda t \int_0^y \frac{x}{y} \bar{\bar{g}}(y-x) g(x)\,\mathrm{d}x, \qquad y > 0.$$

Now we calculate the cumulative distribution function of S_t, i.e. $P(S_t \leq x)$, for different values of x. The results are given in the Table 1. For these calculations we used Panjer's recursive formula [see (3.4)] and Simpson's rule.

For the calculations in the second column we took the risk parameter λ equal to 1, the mean claimsize μ equal to 1 and time t equal to 0.5. For those in the third column we took time t equal to 1. The other parameters have the same value as for the calculations in the second column.

Now we consider the special case where no handling nor payment delay is taken into account and where the force of real interest is constant and strictly positive.

Then we have [see expression (2.16)] that

$$\bar{g}(z) = (1/tjz)(\mathrm{e}^{-z/\mu} - \mathrm{e}^{-(z/\mu)\mathrm{e}^{jt}}), \qquad z > 0 \tag{3.5}$$

and

$$P(\bar{S}_t \leq x) = p_0 + \int_0^x \bar{\bar{g}}(y)\,\mathrm{d}y, \qquad x \geq 0, \tag{3.6}$$

where

$$\bar{\bar{g}}(y) = p_1 \bar{g}(y) + \lambda t \int_0^y \frac{x}{y} \bar{\bar{g}}(y-x) \bar{g}(x)\,\mathrm{d}x, \qquad y > 0.$$

Now we will compute the cumulative distribution function of $\bar{S}_t$, i.e. $P(\bar{S}_t \leq x)$, for different values of x. This will be done using the recursive formula [see (3.6)] and using Simpson's rule. For these calculations we supposed that the force of real interest was equal to 0.03. The value of the other parameters is the same as in the example stated above. (See Table 2.)

Finally we will consider an example where there is a handling and a payment delay, where the force of interest and the force of inflation are both constant and where the force of real interest is strictly positive.

Furthermore one can expect a larger claim size to give rise to a larger handling delay. Indeed, the larger the reported claim size, the more accurate the insurer will try to estimate the damage. However, the handling delay may not last longer than a specified amount of time. The same arguments apply to the payment delay.

Table 2

x	$P(\bar{S}_{0.5} \le x)$	$P(\bar{S}_1 \le x)$
0.00	0.606531	0.367879
0.05	0.621523	0.386316
0.10	0.635958	0.404288
0.20	0.663238	0.438864
0.30	0.688517	0.471670
0.40	0.711927	0.502758
0.50	0.733581	0.532171
0.60	0.753573	0.559934
0.70	0.771972	0.586054
0.80	0.788827	0.610512
0.90	0.804157	0.633262
1.00	0.817957	0.654225

Therefore we make the following assumptions:

$$f_{H_1 \mid X_1}(h \mid x) = [1/\beta_1(x \wedge \alpha_1)] 1_{[0,\beta_1(x \wedge \alpha_1)]}(h) \qquad (\beta_1 > 0,\ \alpha_1 > 0) \tag{3.7}$$

and

$$f_{V_1 \mid X_1}(v \mid x) = [1/\beta_2(x \wedge \alpha_2)] 1_{[0,\beta_2(x \wedge \alpha_2)]}(v) \qquad (\beta_2 > 0,\ \alpha_2 > 0), \tag{3.8}$$

where $x \wedge y = \inf\{x, y\}$ and $x \vee y = \sup\{x, y\}$.

Then, for $z > 0$, expression (2.5) becomes

$$\bar{g}(z) = (1/\mu) \exp\{-z/\mu\} + I_1(z) + I_2(z), \tag{3.9}$$

where

$$I_1(z) = \frac{1}{izt} \int_z^{z\,e^{jt}} g(z) \int_0^{(1/j)\ln(x/z)} \int_0^{(1/j)\ln(x/z)-y} f_{H_1 \mid X_1}(h \mid x) f_{V_1 \mid X_1}\left(\frac{1}{i} \ln\frac{x}{z} - \frac{jh}{i} - \frac{jy}{i} \Big| x \right) \mathrm{d}h\, \mathrm{d}y\, \mathrm{d}x$$

and

$$I_2(z) = \frac{1}{izt} \int_{z\,e^{jt}}^{+\infty} g(x) \int_0^t \int_0^{(1/j)\ln(x/z)-y} f_{H_1 \mid X_1}(h \mid x) f_{V_1 \mid X_1}\left(\frac{1}{i} \ln\frac{x}{z} - \frac{jh}{i} - \frac{jy}{i} \Big| x \right) \mathrm{d}h\, \mathrm{d}y\, \mathrm{d}x.$$

To simplify the notations we put

$$A = (1/j) \ln(x/z),$$
$$B = \beta_1(x \wedge \alpha a_1),$$

and

$$C = (i/j)\beta_2(x \wedge \alpha_2).$$

We find that

$$I_1(z) = \frac{1}{izt} \int_z^{z\,e^{jt}} \frac{1}{\beta_1(x \wedge \alpha_1)\beta_2(x \wedge \alpha_2)} g(x)\, \mathrm{d}x \times \int_{(A-B-C)\vee 0}^{A} (((A-y) \wedge B) - ((A-y-C) \vee 0))\, \mathrm{d}y. \tag{3.10}$$

Furthermore we find that

$$I_2(z) = \frac{1}{izt} \int_{z\,e^{jt}}^{+\infty} \frac{1}{\beta_1(x \wedge \alpha_1)\beta_2(x \wedge \alpha_2)} g(x)\, \mathrm{d}x \times \int_{(A-B-C)\vee 0}^{t \wedge A} (((A-y) \wedge B) - ((A-y-C) \vee 0))\, \mathrm{d}y. \tag{3.11}$$

The integration of these integrals w.r.t. y is tedious but straightforward.

Table 3

x	$P(L_{0.5} \leq x)$	$P(L_1 \leq x)$
0.00	0.606531	0.367879
0.05	0.625912	0.388253
0.10	0.637012	0.404919
0.20	0.665578	0.439685
0.30	0.691310	0.473684
0.40	0.719009	0.503017
0.50	0.735975	0.536515
0.60	0.754175	0.564181
0.70	0.777107	0.586281
0.80	0.789840	0.616813
0.90	0.809706	0.636712
1.00	0.823739	0.663711

We then calculate the cumulative distribution function of L_t, i.e. $P(L_t \leq x)$, for different values of x. We suppose that the force of interest is equal to 0.05, that β_1 is equal to 0.25, that α_1 is equal to 20, that β_2 is equal to 0.0833 and that α_2 is equal to 12. The other parameters have the same values as mentioned above. (see Table 3.)

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