Environmental and Resource Economics **12:** 167–184, 1998.

The Acid Rain Differential Game[1]

KARL-GÖRAN MÄLER[1] and AART DE ZEEUW[2,*]
[1] *The Beijer International Institute of Ecological Economics;* [2] *Department of Economics and CentER, Tilburg University, 5000 LE Tilburg, The Netherlands (*corresponding author, e-mail: adezeeuw@kub.nl)*

Accepted 2 September 1997

Abstract. This paper considers an acid rain differential game. Countries emit sulphur which is partly transferred to other countries. Depositions above critical loads ultimately destroy the soil. Countries face a trade-off between the costs of emission reductions and the damage to the soil due to the depletion of the acid buffers. Because of the transboundary externalities the outcome will depend on whether the countries cooperate or not. This paper presents the cooperative outcome and the open-loop and Markov-perfect Nash equilibria of the acid rain differential game. It will be shown that the depositions always converge to the critical loads but the steady-state levels of the buffer stocks differ. The theory is used to analyse the acid rain differential game for sulphur between Great Britain and Ireland. Finally, some results are given for the whole of Europe.

Key words: acid rain, critical loads, differential games

JEL classification: 722, 026

1. Introduction

During the last decade, politicians and natural scientists became more and more interested in the concept of critical loads for environmental problems. A critical load is usually defined as the maximal exposure to some pollutant an ecological system can adjust to without suffering long term damage (Nilsson 1986). Today the depositions of sulphur and nitrogen oxides exceed the critical loads in many countries. Emission reductions, however, imply economic costs and, therefore, economists point at the trade-off between these costs and the damage to the ecological system. Depositions above critical loads are harmful, but also in that case the environment has some resilience. The more and the longer depositions exceed critical loads the higher the damage. This can be modelled with a buffer stock which is depleted as long as the critical loads are not respected and where damage is a function of the depletion. A quick adjustment to the critical loads does less harm but is more costly than a slow adjustment. These aspects introduce dynamics into the decision problem.

Emissions of sulphur and nitrogen oxides are not only deposited in the emitting country but winds take a large part of the pollutants across borders. These transboundary external costs turn the respective decision problems into a game, which is usually called the acid rain game (Mäler 1989; Newbery 1990). More precisely, since the decision problems of the countries involved are dynamic optimization problems with a stock variable, the game is a differential game (Başar and Olsder 1982; de Zeeuw and van der Ploeg 1991).

In this paper the full cooperative outcome and two non-cooperative Nash equilibria for this acid rain differential game are derived. In the open-loop Nash equilibrium the policies of the countries are only time-dependent, whereas in the feedback Nash equilibrium these policies are contingent on the buffer stocks so that the countries can indirectly react to each other's policies. The feedback Nash equilibrium is also denoted as the Markov-perfect equilibrium because of this contingency and because it is derived in a dynamic programming framework. The cooperative outcome and these two Nash equilibria all have the property that in each country the depositions converge to the critical loads in the long run (see also Mäler 1992). However, the steady-state levels of the buffer stocks differ and these indicate how serious the damage to the environment is and whether it is reversible or not. Therefore, regardless of the mode of behaviour critical loads will be respected in the long run, but total damage and the risk for the ecological system strongly depend on it. It will be clear that, in general, cooperative behaviour leads to less damage but suffers from the prisoners' dilemma. In this paper it is also shown that the steady-state levels of the buffer stocks in the feedback Nash equilibrium, which is the more realistic equilibrium concept, are lower than in the open-loop Nash equilibrium which implies higher depletion levels and, therefore, more damage to the environment. This result strengthens the case for international coordination of environmental policy.

Our approach to the problem in a way reconciles economists, who always stress trade-offs, and ecologists, who always stress the hardness of ecological constraints. Ecologists point at the normative aspect of critical loads which returns in the economic model, outlined above, as the long-run level of depositions. The acceptance of the critical load concept, however, does not imply, in an economic model, that the issue of cooperation disappears, because the adjustment process strongly depends on the mode of behaviour.

The availability of data on the transport of sulphur and nitrogen oxides in Europe (EMEP 1992) and on the critical loads (Downing et al. 1993) allows some empirical analyses to be performed within the framework described above. For a full empirical analysis estimates of the cost and damage functions are also needed. Damage functions are in general very difficult to estimate and any set of estimated damage functions for acid depositions for the whole of Europe does not exist. Cost functions, however, could be based on the RAINS project (Alcamo et al. 1990) but in this paper both cost and damage functions are chosen to be simple parametrized functional forms.

In this context the acid rain differential game for sulphur dioxide between Great Britain and Ireland is analysed. Furthermore, some first results are given for the whole of Europe but it will be shown that this gives rise to new and difficult theoretical problems, which are the subject of further research. The empirical analysis in this paper is rather specific and mainly meant to illustrate the theoretical results. An extensive literature exists on different aspects of the sulphur emission problem in Europe (e.g. Klaassen 1995).

In Section 2 the acid rain differential game is formulated and in Section 3 the steady states for the cooperative and two non-cooperative solution concepts are derived. It will be shown that the steady-state emissions only depend on the empirical data on the transport matrix and the critical loads. The steady-state depletion levels of the acid buffer stocks, however, also depend on the economic variables such as the parameters in the cost and damage functions and the discount rate. As a first illustration of the theoretical results of this paper, Section 4 presents the game between Great Britain and Ireland. This is the only group of two countries in Europe within our model for which the background depositions from all other countries and the sea do not exceed the critical loads already. Section 5 contains some first results for the whole of Europe and Section 6 concludes the paper.

2. The Acid Rain Differential Game

Suppose there is a group of n countries emitting e_i, i = 1, 2, . . . , n, tons of sulphur or nitrogen oxides. The matrix A is the transport matrix where the element a_{ij} denotes the fraction of country j's emissions e_j that is deposited in country i, so that the vector $A\mathbf{e}$ is the vector of depositions in the n countries as a consequence of the emissions in the same countries. In addition to that each of the countries receives the so-called background depositions from the countries outside the group as well as from the sea.

As indicated in the Introduction, we interprete a critical load as how much pollution a country can assimilate per year before the acid buffers of the soil in that country are affected and the soil acidification process starts (e.g. Aalbers 1993). To put it differently, a deposition above the critical load decreases the acid buffer stock of that country and the depletion d_i of this buffer stock indicates how much damage is done to the soil.[2] The different acid buffers are depleted one by one. As long as the acidification process is in the first acid buffer, the soil becomes less productive but can recover when the depositions are at or below the critical load again, so that the damage is reversible. However, when this buffer is depleted and the next buffer is activated, the soil cannot recover and will loose some of its productivity for ever. Moreover, when the last acid buffer is depleted, the soil will even become non-productive. This very rough sketch of the acidification process is sufficient for the purpose of this paper but more details can be found in the literature (e.g. Hettelingh 1989).

Critical loads are given as grams of sulphur or nitrogen oxide per year per square meter and can differ substantially from one region to the other, depending on the characteristics of the soil, the bedrock, the vegetation cover, the precipitation, etcetera. Since the analysis is performed at the aggregation level of countries, a measure is needed for a country's critical load. The data provide critical loads per region. In order to get a comparable number, the average of the regional critical loads of a country is multiplied with the surface area of that country. It is clear that this number is rough and that the conclusions should therefore be treated with care but a more detailed modelling of this aspect would not give more economic insights, which is the focus of this paper. Background depositions from the countries outside the group and from the sea are assumed to be given. Therefore, when these background depositions exceed the critical load in one of the countries, the group cannot control the acidification process in that country. In the analysis that follows it is assumed that this is not the case. Furthermore, the difference between the total depositions and the critical loads matters. Therefore, the background depositions can be substracted on both sides, so that the starting point of the analysis consists of the internal depositions $A\mathbf{e}$, on the one hand, and the critical loads minus the background depositions, denoted by the vector $\mathbf{c}$, on the other hand. Each country faces the trade-off between the costs of reducing emissions and the benefits of lower damage to the environment. Each country chooses a time path for its emissions with the objective to minimize a discounted stream of costs and damages subject to the depletion of the acid buffer stocks. Therefore, the decision problem of country i can be formulated as the following optimal control problem:

$$\min_{e_i} \int_0^\infty \mathrm{e}^{-r_i t}[C_i(e_i(t)) + D_i(d_i(t))]\mathrm{d}t, i = 1, 2, \ldots, n, \tag{1}$$

such that

$$\dot{d}(t) = Ae(t) - c,\ d(0) = d_0, \tag{2}$$

where r denotes the interest rate, C the cost function of the reduction of the emissions and D the damage function of the depletion of the acid buffer stocks. When the depositions Ae are lower than the critical loads c, this formulation implies that the buffer stocks fill up again, so that it is implicitly assumed that the soil can recover and that the damage is reversible. It is, however, not clear beforehand if the resulting steady-state levels of depletion are reversible or not. In the empirical analysis that follows these steady-state levels will simply be used as an indicator of how serious the situation is. A more complicated formulation of the optimal control problem would make the theoretical analysis intractable.

The n optimal control problems are coupled. Because of the transboundary pollution, each country influences the depletion of the acid buffer stocks of the other countries and, thus, the welfare of the other countries. The problem is a game and a game composed of optimal control problems is called a differential game (Başar and Olsder 1982; de Zeeuw and van der Ploeg 1991). The static version with flow pollution instead of stock pollution was called the acid rain game (Mäler 1989; Newbery 1990). In that tradition the game of this paper is called the acid rain differential game.

3. Solutions to the Acid Rain Differential Game

3.1. Equilibrium Concepts

It is common to start the analysis, as a benchmark, with the open-loop Nash equilibrium, in which the emissions are only a function of time. The information structure is simple and one can resort to the techniques of Pontryagin's minimum principle to solve the problem. It is, however, more realistic to derive the feedback Nash equilibrium, in which the emissions are also a function of the current state of the system, which in this case means the current levels of depletion. Moreover, by solving the game with dynamic programming a type of subgame perfectness results (de Zeeuw and van der Ploeg 1991). This means that the players are not committed to their strategies, and that only the current state matters and not the path that has led to it (bygones are bygones). The current state reflects the part of the history of the game that is relevant for current and future costs. Therefore the feedback Nash equilibrium is also called the Markov-perfect equilibrium (Maskin and Tirole 1988). The open-loop Nash game is essentially a one-shot game. At the beginning of the game the strategies are chosen for the whole time span and no new information is received or used during the course of the game. In the feedback Nash game, however, countries condition their emissions on the observed depletion levels which implies that they indirectly react to past emission choices of the other countries.[3] The strategic interaction becomes dynamic. It cannot be said in general whether the open-loop or the feedback Nash equilibrium is better for the players.

The state transition (2) is linear. If cost and damage functions are assumed to be quadratic with the usual convexity conditions, the open-loop and the linear feedback Nash equilibrium are unique and analytically tractable. However, for symmetric linear-quadratic differential games with a one-dimensional state space, it has been shown that also non-linear feedback Nash equilibria exist, which are better for the players and can even sustain the cooperative outcome (Tsutsui and Mino 1990; Dockner and Long 1993). This line of research still has to be developed for the asymmetric case with more than one state variable, which is the type of model in this paper.

The next step in the analysis is to compare the non-cooperative equilibria with outcomes that result from cooperation between the countries in order to assess the benefits of cooperation. Cooperative outcomes are vulnerable because countries generally have incentives to deviate under the assumption that the other countries stick to their cooperative emission levels. If all countries give in to these incentives, a non-cooperative equilibrium results where the players typically are worse off. This situation is usually referred to as the prisoners' dilemma. A lot of research in game theory deals with the search for non-cooperative equilibria that sustain cooperative outcomes. The main result in this area is the well-known folk theorem for two-player repeated games (Fudenberg and Maskin 1986) which states that, for a high enough discount factor, the infinite repetition of a static game allows to construct non-cooperative equilibria that sustain all outcomes that are individually rational with respect to the min-max outcome. For specific games it is even possible to construct equilibria with strong properties like renegotiation proofness (e.g. van Damme 1989).

Differential games are dynamic games. Therefore, it is to be expected that the same ideas work in that context. Indeed, one result is that if the players can condition their strategies not only on the current state but also on past states (which is called a closed-loop memory information structure) non-uniqueness results with equilibria that are better for the players than the feedback Nash equilibrium (Başar and Olsder 1982). However, a full folk theorem for differential games does not exist, except under special conditions (Gaitsgory and Nitzan 1994). A disadvantage of a closed-loop memory information structure is the loss of Markov perfectness.

In this paper non-cooperative behaviour will be characterized first by the standard open-loop Nash equilibrium and then by the more complicated but also more realistic feedback Nash or Markov-perfect equilibrium. Under the assumption of side payments, cooperative behaviour can be characterized by the full cooperative outcome, where the countries jointly minimize their total costs. In this way the benefits of cooperation can be assessed.

3.2. Full Cooperative Outcome

In the full cooperative outcome the countries jointly minimize the total sum of their objective functionals, given in (1). For simplicity, it is assumed here that all interest rates are equal. Furthermore, in order to be able to derive the feedback Nash equilibrium in Section 3.4, it is also assumed that the cost and damage functions have simple quadratic functional forms:

$$C_i(e_i) = 1/2\gamma_i(e_i - \bar{e}_i)^2; D_i(d_i) = 1/2\delta_i d_i^2, \gamma_i > 0, \delta_i > 0. \tag{3}$$

Since no full set of estimated cost and damage functions is available, on the basis of which the parameters of these functions could be calibrated, the parameters are just guessed at but also varied in order to check the robustness of the results. Note, however, that γ and δ are in fact one parameter since one of the two can be normalized to 1. The relative magnitude expresses the relative weight that is given to emission reduction costs, on the one hand, and environmental damage costs, on the other hand. One can say: the higher its δ/γ the greener the preferences of that country. Furthermore, the parameter $\bar{e}$ of the cost function must be chosen high enough in order to ensure that this cost function has a realistic form in the range of analysis, namely decreasing (which implies that the marginal emission reduction costs are increasing). If a country has already reduced emissions substantially, the marginal cost can be expected to be high. Therefore, the set of values of $\bar{e}$ will be chosen such that countries with lower per capita sulphur emissions have higher marginal emission reduction costs.

The optimality conditions for the full cooperative outcome, given by Pontryagin's minimum principle, are:

$$\dot{d}(t) = Ae(t) - c,\ d(0) = d_0, \tag{4}$$

$$\dot{p}(t) = rp(t) - \Delta d(t), \tag{5}$$

$$e_i(t) = \bar{e}_i - \frac{1}{\gamma_i} a_i^T p(t), i = 1, 2, \ldots, n, \tag{6}$$

where Δ is a diagonal matrix with the damage parameters $\delta_1,\ \delta_2,\ \ldots,\ \delta_n$ on the diagonal, a_i is the i-th column of the transport matrix A, and p is the co-state vector denoting the dynamic shadow values of the depletion of the acid buffer stocks.

Upon substitution of the optimal emission levels, given by (6), into equation (4), a system of differential equations (4)–(5) results, which has a saddlepoint equilibrium for small enough values of the interest rate r. Because of the transversality conditions for the co-state p, the optimal path jumps to the stable manifold and converges to the saddlepoint with

$$d_C = r\Delta^{-1}(A^T)^{-1}\Gamma(\bar{e} - A^{-1}c), \tag{7}$$

where Γ is a diagonal matrix with the cost parameters $\gamma_1, \gamma_2, \ldots, \gamma_n$ on the diagonal. Note that the steady-state values do not depend on the initial values of depletion but the transient path of course will depend on it.

It is easy to see that the (optimal) steady-state emission levels are given by

$$e_C = A^{-1}c, \tag{8}$$

which implies that the optimal path converges to a situation where the depositions are equal to the critical loads. It is assumed here that the values of the parameters of the problem are such that the resulting steady-state depletion and emission levels are non-negative, which is not necessarily the case, because the off-diagonal elements of the inverse of the transport matrix A (and A^T) are usually negative. We will return to this in Section 5.

3.3. Open-loop Nash Equilibrium

The difference with the full-cooperative outcome above is that the countries minimize their objective functionals separately, given the dynamic constraints and their expectations about the emission paths of the other countries. The Nash equilibrium requires consistency of the resulting optimal emission paths and these expectations.

It follows that the optimality conditions for the open-loop Nash equilibrium are:

$$\dot{d}(t) = Ae(t) - c,\ d(0) = d_0, \tag{9}$$

$$\dot{p}_i(t) = rp_i(t) - \delta_i d_i(t), i = 1, 2, \ldots, n, \tag{10}$$

$$e_i(t) = \bar{e}_i - \frac{a_{ii}}{\gamma_i} p_i(t), i = 1, 2, \ldots, n. \tag{11}$$

Note that for each country i only the i-th component of the co-state vector matters because country i is now only interested in the damage caused by its own depletion level. Therefore, equation (10) is a scalar equation, and in equation (11) only the a_{ii} element of the transport matrix A matters. In fact country i expects the other countries to choose their Nash emission paths and, therefore, treats the resulting depositions as given background depositions.

For each country i, substitution of the Nash equilibrium emission levels, given by equation (11), into the differential equation (9) again leads to a saddlepoint equilibrium for small enough values of the interest rate r. Because of the transversality conditions for the co-states p_i, the open-loop Nash equilibrium path converges to the saddlepoint with

$$d_N = r\Delta^{-1} A_d^{-1} \Gamma(\bar{e} - A^{-1}c), \tag{12}$$

where A_d is a diagonal matrix with the diagonal from the transport matrix A. It is easy to see that the steady-state emission levels are again given by

$$e_N = A^{-1}c. \tag{13}$$

In both the full cooperative outcome and the non-cooperative open-loop Nash equilibrium, the depositions converge in the long run to the critical loads, which gives this concept some normative significance. However, there are two important differences between these modes of behaviour. The first is the speed of convergence, which is given by the stable eigenvalues of the resulting system of differential equations. The second is the steady-state value of the vector of depletion levels, which is given by equations (7) and (12) respectively. This value indicates how much damage is done to the environment before the depositions are adjusted to the critical loads, and whether this damage is reversible or not. Note that if the transport matrix A is diagonal, the steady-state values are the same. This is of course obvious because without transboundary pollution there is no game, so that cooperative and non-cooperative outcomes coincide. With transboundary pollution, one could conjecture that the steady-state depletion levels d_C in the full cooperative outcome, given by equation (7), are smaller than or equal to the steady-state depletion levels d_N in the open-loop Nash equilibrium, given by equation (12), but it can be shown that this is not generally true.

3.4. Feedback Nash Equilibrium

The Nash equilibrium of problem (1)–(3), derived in a dynamic programming framework with the depletion levels as states, is called the feedback Nash equilibrium in the theory of differential games. At each point in time for each depletion level the countries play a Nash game with the costs-to-go (the value functions) as objectives. Emissions become a function of the current depletion levels and the equilibrium is Markov-perfect. Since the constraints (2) are linear and the objectives (3) are quadratic, a unique linear equilibrium for quadratic value functions results under the usual convexity assumptions. Note, however, that also non-linear feedback Nash equilibria may exist (see Section 3.1).

The quadratic value functions, given by

$$V_i(d,t) = 1/2d^T K_i(t)d + g_i^T(t)d + h_i(t), i = 1,2,\ldots,n, \tag{14}$$

have to satisfy the Hamilton-Jacobi-Bellman equations

$$V_{it}(d,t) - rV_i(d,t) + \min_{e_i}[C_i(e_i) + D_i(d_i) + V_{id}(d,t)[Ae - c]] = 0. \tag{15}$$

The objective functionals are not time-dependent, except for the discount factors, so that the value functions are not time-dependent either. The first term in each of the Hamilton-Jacobi-Bellman equations is therefore equal to zero and the parameters K and g are constant.

The minimization leads to

$$e_i(d) = \bar{e}_i - \frac{1}{\gamma_i} a_i^T [K_i d + g_i], i = 1,2,\ldots,n. \tag{16}$$

Substitution of the Nash equilibrium emission levels, given by equation (16), and the expressions for the value functions, given by equation (14), in the Hamilton-Jacobi-Bellman equations (15) leads to quadratic equations in **d**. The coefficients of the quadratic term and the linear term have to be equal to zero, which after some tedious calculations leads to the coupled algebraic matrix Riccati equations

$$\begin{aligned} -rK_i \; + \; \Delta_i - K_i \frac{1}{\gamma_i} a_i a_i^T K_i - K_i \left[\sum_{j \neq i} \frac{1}{\gamma_j} a_j a_j^T K_j \right] \\ - \left[\sum_{j \neq i} K_j \frac{1}{\gamma_j} a_j a_j^T \right] K_i = 0 \end{aligned} \tag{17}$$

for the matrix parameters K_i, $i = 1, 2, \ldots, n$, where Δ_i is a zero matrix with only δ_i on the i-th position in the diagonal, and to the coupled algebraic tracking equations

$$\begin{aligned} -rg_i \; - \; K_i \frac{1}{\gamma_i} a_i a_i^T g_i + K_i[A\bar{e} - c] - K_i \left[\sum_{j \neq i} \frac{1}{\gamma_j} a_j a_j^T g_j \right] \\ - \left[\sum_{j \neq i} K_j \frac{1}{\gamma_j} a_j a_j^T \right] g_i = 0 \end{aligned} \tag{18}$$

for the vector parameters $\mathbf{g}_i$, i = 1, 2, ..., n.

The dynamics of the depletion levels in the feedback Nash equilibrium becomes

$$\dot{d}(t) = -\left[\sum \frac{1}{\gamma_i} a_i a_i^T K_i\right] d(t) - \sum \frac{1}{\gamma_i} a_i a_i^T g_i + A\bar{e} - c \tag{19}$$

with steady state

$$d_F = \left[\sum \frac{1}{\gamma_i} a_i a_i^T K_i\right]^{-1} \left[A\bar{e} - c - \sum \frac{1}{\gamma_i} a_i a_i^T g_i\right]. \tag{20}$$

It is not difficult to see that in the steady state the depositions are equal to the critical loads. The problem is how to find (symmetric) solutions to the coupled algebraic Riccati equations (17) that yield stable dynamics in equation (19). After adding half of the first term $-rK_i$ to each of the last two terms in equation (17), the equation fits for each i the format of the algebraic matrix Riccati equation for linear-quadratic regulator design in the Matlab Control System Toolbox. This implies that with this software we can solve equation (17) for each i, given the solutions for $j \neq i$. A simple iterative procedure then solves the set of equations, after which the check for stability can be made.[4]

Because it makes sense to assume that the emission policies are contingent on the observed depletion levels, the feedback Nash equilibrium concept is more realistic than the open-loop concept. Moreover, the dynamic programming framework yields Markov perfectness.

In a paper with a differential game model for the greenhouse effect (van der Ploeg and de Zeeuw 1992) it was shown that the steady-state level of accumulated greenhouse gases in the feedback Nash equilibrium is higher than in the open-loop Nash equilibrium. The reason is that each country expects partly offsetting reactions from the other countries in a feedback information structure and, therefore, emits more in equilibrium. Although we were not able to prove this general result for the model in this paper, it will be confirmed in the next two sections where the acid rain differential games between Great Britain and Ireland and for the whole of Europe are analysed respectively, although the effect is rather small. It implies that the benefits of cooperation are a bit higher when the conceptually more realistic equilibrium concept is used, which strengthens the need for cooperation on emission policies.

4. Great Britain and Ireland

Great Britain and Ireland form the only group of two European countries for which all the background depositions from the countries outside the group and from the sea do not exceed the critical loads already. It should be noted that this observation is based on the very rough measures that are calculated in this paper. The larger the group of countries considered the more room these countries have to (jointly) control the acidification of their soils.

The transport matrix of sulphur is derived from the publications of the European Programme for Monitoring and Evaluation in Oslo (EMEP, Acid News 1992). The critical loads for the regions are taken from an RIVM report (Downing et al. 1993). Emission and transport data are available at the country level, but also a critical load per country is needed. As indicated in Section 2, this value is determined by multiplying the average critical load with the total area of that country. The units are 100 tons of sulphur per year.

With Ireland as country 1 and Great Britain as country 2 the transport matrix becomes:

$$A = \begin{bmatrix} 0.2143 & 0.0057 \\ 0.0786 & 0.2619 \end{bmatrix}. \tag{21}$$

Because the area of Great Britain is three to four times as big as the area of Ireland and the average critical loads are about the same, the critical load of Great Britain is three to four times as big as the critical load of Ireland: 1077 and 303 tons, respectively. The background depositions from outside Great Britain and Ireland are 711 and 137, so that the critical loads minus these background depositions are 166 for Ireland and 366 for Great Britain.

The steady-state emission levels for the full cooperative outcome and both non-cooperative equilibria are the same and yield depositions equal to the critical loads. The numbers are:

$$e = A^{-1}c = \begin{bmatrix} 4.7045 & -0.1031 \\ -1.4114 & 3.8492 \end{bmatrix} \begin{bmatrix} 166 \\ 366 \end{bmatrix} = \begin{bmatrix} 744 \\ 1174 \end{bmatrix}. \tag{22}$$

The emissions in the base year (which is the average of 1990 and 1991) are 840 for Ireland and 19160 for Great Britain. This leads to the first interesting observation. In order to meet the critical load Great Britain has to reduce emissions drastically to about 6% of the current level, which is so beneficial for Ireland that this country only has to reduce its emissions by about 11% to meet the critical load. If the countries do not realize that a game is played and consider each other's current emissions as additional background depositions, the interaction disappears and two separate pollution control problems have to be solved. Then the outcome is that Great Britain has to reduce emissions down to 1145 and Ireland down to 261 in order to meet the respective critical loads. It follows that the gaming aspect of the problem relaxes the task of Ireland considerably. Some sensitivity analysis on the data has shown that it can even happen that Ireland may increase its emissions above the current level and still meet the critical load because of this interaction! Great Britain is then forced to bring down emissions even further than in the separate pollution control case.

The parameters γ_i of the cost functions in equation (3) are normalized to 1 and the parameters δ_i of the damage functions in equation (3) are first also set equal to 1 and then varied in order to study the effect of changing preferences with respect to emission reduction costs, on the one hand, and environmental damage costs, on the other hand. The parameters $\bar{e}$ of the cost functions are chosen such that for current emission levels the product of marginal emission reduction costs and per capita emissions is constant and such that the cost function is decreasing in the range of analysis (see Section 3.2): $\bar{e}$ becomes 105690 for Ireland and 91600 for Great Britain.

The interest rate is set equal to 4%. Because the eigenvalues of the system of differential equations (4)–(6) for the full cooperative outcome are (-0.2700, -0.1735, 0.3100, 0.2135) and the eigenvalues of the system of differential equations (9)–(11) for the open-loop Nash equilibrium are (-0.2447, -0.1927, 0.2847, 0.2327), the interest rate is indeed small enough to have saddlepoint equilibria for these systems.

The coupled algebraic matrix Riccati equations (17) are solved with the algorithm described in Section 3.4. The solution to the coupled algebraic tracking equations (18) follows easily. Because the eigenvalues of the differential equation (19) are (-0.2445, -0.1925), the system is stable and the depletion levels converge to the steady state. The speed of convergence, however, is smaller than for the open-loop Nash equilibrium and this indicates already that the steady-state depletion levels in the feedback Nash equilibrium will be higher.

Finally, the steady-state depletion levels for the full cooperative outcome, given in equation (7), for the open-loop Nash equilibrium, given in equation (12), and for the feedback Nash equilibrium, given in equation (20), are calculated for the game between Great Britain and Ireland, which yields:

$$d_C = \begin{bmatrix} 14644 \\ 13489 \end{bmatrix}. \tag{23}$$

$$d_N = \begin{bmatrix} 19590 \\ 13810 \end{bmatrix}. \tag{24}$$

$$d_F = \begin{bmatrix} 19671 \\ 13856 \end{bmatrix}. \tag{25}$$

These numbers partly depend on the data on the transport matrix and the critical loads, but also partly on the assumptions on the cost and damage functions such as functional form and parameter values. Therefore, these numbers only have some relative meaning: the higher the steady-state values of the depletion levels the more damage is done to the environment in the process of adapting the emissions to the critical loads. Moreover, high values also indicate a higher chance on irreversible damage or even a break-down of the ecological system.

The full cooperative outcome is especially beneficial for Ireland. Compared with the Nash equilibria Great Britain has not much to gain from cooperation and will therefore have to be compensated to do so. The feedback Nash equilibrium has somewhat higher depletion levels than the open-loop Nash equilibrium, which means that the use of the less realistic open-loop equilibrium concept underestimates somewhat the possible benefits of cooperation in the control of transboundary pollution.

The effects of varying the parameters δ_i are as to be expected. Increasing δ_i implies that a higher weight is put on environmental damage, so that the speed of convergence in all cases goes up and the steady-state depletion levels go down. The numbers will be omitted here.

5. Europe

In this section the analysis of Section 4 is performed for the whole of Europe. The sulphur transport matrix and the initial sulphur emissions are provided by the EMEP and the critical loads are based on the RIVM report. Three countries are not included in the analysis: Luxemburg, because it is too small, and Iceland and Turkey, because no critical loads are available. Furthermore, East and West Germany are unified. However, no separate data are available for the countries that used to form Czechoslovakia, the Soviet Union (the European part) and Yugoslavia, so that these countries are still treated as one in the analysis. Finally, data on the total area and the population of the countries are taken from the Times Atlas.

In Table I the transport matrix A is given. The EMEP provides the matrix of contributions of sulphur from one country to the other and the total sulphur emissions for each country. The columns of this matrix are divided by the total emissions to get the fractions of the matrix A. The total sulphur emissions (the average of years 1990 and 1991) are the initial emissions of the game and can be found in Table II. This table also provides for each country the critical load (which is the average critical load per square meter times the area of the country), the background deposition, the total population (which is used to determine a value for the cost parameter by relating marginal emission reduction costs and the emissions per capita), the steady-state emissions and the steady-state depletion levels for the full cooperative outcome, the open-loop Nash equilibrium and the feedback Nash equilibrium. The eigenvalues are not reported but, as in Section 4, the systems for the full cooperative outcome and the open-loop Nash equilibrium prove to have saddlepoints and the dynamic system for the feedback Nash equilibrium proves to be stable. These eigenvalues again indicate the speed of convergence and increase when the weight on environmental damage costs is increased.

The steady-state depletion levels for the feedback Nash equilibrium are again a bit higher than for the open-loop Nash equilibrium, because the countries condition their emissions on the depletion levels which results in a dynamic interaction making the countries worse off. The explanation is as follows. The reaction to higher levels is lower emissions. Each country takes this reaction of the other countries into account, which implies that in equilibrium the countries emit more.

It is interesting to compare the results for the European game with the results for the game between Great Britain and Ireland in Section 4. Comparing the full

Table I.

	AL	AT	BE	BG	CS	DK	FI	FR	DE	GR	HU	IE
AL	0.164	0	0	0.004	0.001	0	0	0	0	0.007	0.002	0
AT	0	0.253	0.010	0	0.023	0.003	0	0.013	0.016	0	0.014	0.001
BE	0	0	0.194	0	0.001	0.001	0	0.019	0.004	0	0	0.002
BG	0.016	0.004	0.001	0.249	0.004	0.001	0	0.001	0.002	0.014	0.014	0
CS	0	0.049	0.012	0.001	0.255	0.007	0.002	0.010	0.044	0	0.059	0.002
DK	0	0	0.005	0	0.002	0.113	0.001	0.002	0.005	0	0	0.002
FI	0	0	0.003	0	0.003	0.015	0.300	0.002	0.005	0	0.001	0.002
FR	0	0.010	0.091	0	0.010	0.006	0	0.326	0.018	0	0.005	0.013
DE	0	0.039	0.132	0	0.070	0.041	0.002	0.081	0.307	0	0.009	0.014
GR	0.032	0.002	0.001	0.041	0.002	0	0	0.001	0.001	0.196	0.005	0
HU	0	0.035	0.003	0.002	0.022	0.002	0	0.003	0.007	0.001	0.264	0
IE	0	0	0.002	0	0	0	0	0.001	0.001	0	0	0.214
IT	0.008	0.037	0.010	0.003	0.013	0.003	0	0.025	0.010	0.006	0.022	0.001
NL	0	0	0.048	0	0.001	0.002	0	0.011	0.006	0	0	0.004
NO	0	0	0.010	0	0.003	0.029	0.009	0.004	0.006	0	0.001	0.011
PL	0	0.025	0.029	0.002	0.086	0.041	0.006	0.015	0.101	0	0.036	0.005
PT	0	0	0	0	0	0	0	0.001	0	0	0	0
RO	0.012	0.012	0.003	0.028	0.020	0.003	0.002	0.002	0.009	0.006	0.071	0
ES	0	0	0.007	0	0.001	0.002	0	0.018	0.003	0	0	0.005
SE	0	0.002	0.012	0	0.006	0.090	0.047	0.005	0.012	0	0.002	0.007
CH	0	0.006	0.006	0	0.003	0.002	0	0.016	0.004	0	0.002	0.001
SU	0.008	0.037	0.037	0.025	0.077	0.092	0.183	0.020	0.070	0.009	0.099	0.011
GB	0	0	0.019	0	0.002	0.005	0	0.012	0.005	0	0	0.079
YU	0.060	0.053	0.006	0.032	0.023	0.003	0.001	0.007	0.012	0.013	0.081	0.001

Source: Acid News (1992).

cooperative outcome with the Nash equilibria, Great Britain now has a lot to gain from cooperation whereas the gains for Ireland are not much higher than in the two-country game. Another interesting point is that for Ireland the steady-state emissions are now higher than the initial emissions, so that this country can increase its emissions. The same holds for Greece, Portugal, Spain and the former Soviet Union. These countries have in common that they are border states in Europe, so that much of their emissions is not a concern in the European context. Furthermore, the average critical load per square meter is very high in Greece, Portugal and the former Soviet Union, and the total area is very high in the former Soviet Union. This might explain why it is possible to have an increase in emissions in these countries. However, for most countries a very drastic reduction of emissions is needed to meet the critical loads.

The analysis in this section has one serious drawback. In Table II it can be seen that some of the steady-state emission levels and some of the full-cooperative steady-state depletion levels are negative.[5] The analysis in Section 3 was based on

Table I. (continued).

	IT	NL	NO	PL	PT	RO	ES	SE	CH	SU	GB	YU
AL	0.002	0	0	0.001	0	0.002	0	0	0	0	0	0.005
AT	0.018	0.007	0	0.008	0	0.001	0.001	0.002	0.039	0	0.002	0.012
BE	0	0.025	0	0.001	0	0	0.001	0	0	0	0.006	0
BG	0.002	0.001	0	0.004	0	0.040	0	0	0	0.003	0.001	0.018
CS	0.004	0.010	0	0.034	0	0.006	0.001	0.002	0.007	0.001	0.003	0.012
DK	0	0.008	0.006	0.002	0	0	0	0.006	0	0	0.006	0
FI	0	0.004	0.015	0.007	0	0.001	0	0.047	0	0.010	0.003	0.001
FR	0.022	0.045	0	0.005	0.009	0.001	0.029	0.002	0.052	0	0.021	0.006
DE	0.008	0.116	0.006	0.022	0.002	0.001	0.004	0.007	0.071	0.001	0.029	0.003
GR	0.003	0.001	0	0.002	0	0.011	0	0	0	0.001	0	0.007
HU	0.006	0.003	0	0.010	0	0.013	0	0.001	0.003	0.001	0.001	0.032
IE	0	0.002	0	0	0	0	0	0	0	0	0.006	0
IT	0.265	0.007	0	0.007	0.004	0.004	0.006	0.001	0.065	0.001	0.006	0.036
NL	0	0.150	0	0.001	0	0	0	0	0	0	0.011	0
NO	0	0.013	0.212	0.004	0	0.001	0.001	0.027	0	0.003	0.016	0
PL	0.003	0.030	0.006	0.330	0	0.009	0.001	0.016	0.007	0.005	0.011	0.010
PT	0	0	0	0	0.225	0	0.010	0	0	0	0	0
RO	0.005	0.003	0	0.017	0	0.314	0	0.002	0	0.009	0.001	0.051
ES	0.004	0.005	0	0.001	0.105	0	0.265	0	0.003	0	0.005	0.001
SE	0	0.019	0.079	0.010	0	0.002	0	0.275	0	0.004	0.014	0.001
CH	0.014	0.004	0	0.001	0	0	0.002	0	0.265	0	0.002	0.001
SU	0.007	0.042	0.036	0.164	0.001	0.094	0.002	0.096	0.007	0.413	0.021	0.032
GB	0	0.021	0.003	0.002	0.001	0	0.002	0.002	0	0	0.262	0
YU	0.032	0.005	0	0.013	0	0.022	0.002	0.002	0.007	0.001	0.002	0.327

the assumption that this is not the case. It is not difficult to see why it happens. Non-negative emissions result from equation (8) if and only if the vector **c** of critical loads minus background depositions is an element of the non-negative span of the columns of the transport matrix A. If A is a diagonal matrix, which means that the emissions are only deposited in the country of origin, this condition is always satisfied. However, in case of transboundary pollution, which implies off-diagonal elements in the matrix A, this is not generally true. For the game between Great Britain and Ireland in Section 4 the condition was satisfied but it broke down for the European game. The negative depletion levels can be explained in the same way with equation (7) and the matrix A^T.

Unfortunately this problem does not have an easy answer. The dynamics in the formulation of the problem (1)–(2) will change. In this paper it is implicitly assumed that critical loads will be met by $Ae = c$ but it is of course more precise to state that critical loads are met by $Ae \leqslant c$. If $Ae = c$ is not feasible for non-negative emissions the countries will partition into two groups. In the first group the steady-state depositions are equal to the critical loads with corresponding positive

Table II.

		e_0	cl	bd	pop	e	d_C	d_N	d_F
Albania	AL	250	196	69	3.25	183	733.23	792.85	798.66
Austria	AT	490	268	168	7.76	−643	578.85	627.73	641.05
Belgium	BE	2100	208	63	9.85	145	−125.02	245.32	253.66
Bulgaria	BG	5150	1109	216	9.01	1255	22.02	76.43	77.58
Czechoslovakia	CS	12820	1279	226	15.68	3039	−58.68	63.40	66.67
Denmark	DK	1330	241	74	5.14	1179	−63.28	343.20	349.67
Finland	FI	1280	809	360	4.99	−1254	82.48	133.22	135.84
France	FR	6030	3046	810	56.56	4502	185.55	289.99	297.67
Germany	DE	31220	856	620	78.50	−2282	28.45	125.52	130.73
Greece	GR	2500	1848	272	10.27	6694	165.59	201.01	202.88
Hungary	HU	5050	1116	152	10.34	2704	−4.13	81.09	83.67
Ireland	IE	840	303	92	3.52	875	117.66	195.66	196.49
Italy	IT	12030	2048	627	57.69	3834	85.72	193.70	197.53
Netherlands	NL	1190	181	76	15.02	120	645.98	841.90	872.15
Norway	NO	330	518	454	4.24	−1134	513.87	608.76	621.94
Poland	PL	16050	1751	578	38.18	1254	51.92	90.06	92.47
Portugal	PT	1060	623	93	10.53	1442	397.09	441.55	445.52
Romania	RO	9000	2850	396	23.19	4221	66.64	88.22	89.82
Spain	ES	10950	6059	521	38.99	19934	74.77	121.05	122.34
Sweden	SE	1020	720	509	8.62	−591	240.00	310.13	320.40
Switzerland	CH	310	182	88	6.71	−290	668.36	819.44	829.83
Soviet Union	SU	44650	42214	5581	83.13	86350	−6.40	4.70	4.76
Great Britain	GB	19160	1077	338	55.51	2129	32.67	136.64	137.72
Yugoslavia	YU	7400	1433	535	23.90	344	44.85	107.29	110.34

e_0: initial emission; cl: critical load; bd: background deposition; pop: population $\times 10^6$;
"steady-state" outcomes: e: emission; d_C, d_N, d_F: stocks of pollutants $\times 10^2$.
Sources e_0, cl, bd, pop: Acid News (1992), RIVM (1993), Times Atlas (1992).

steady-state depletion levels, and in the second group the steady-state depositions are smaller than the critical loads with corresponding zero steady-state depletion levels. However, in this way the differential game becomes very complex. A lot of further research is needed before a proper analysis of the acid rain differential game for the whole of Europe can be presented.

6. Conclusions

The acid rain game is a dynamic game because the depletion of acid buffer stocks matters and not so much the flow of pollutants. On the other hand, in the steady state, the flow of pollutants cannot exceed the critical load. This paper analyses a model that integrates these two aspects.

It has been argued that the issue of cooperation and non-cooperation disappears when all the countries accept the concept of critical load. This is true for steady-state emission levels but not for steady-state depletion levels and the transition to the steady state. The policy issue is not so much the critical load but the speed of convergence to the critical load and the amount of damage that is done to the ecological system in this transition process. From this point of view cooperation or non-cooperation matters a lot.

In this paper the full-cooperative outcome, the open-loop Nash equilibrium and the feedback Nash (or Markov-perfect) equilibrium for the acid rain differential game are derived. In all three cases the depositions converge to the critical loads but the speed of convergence and the steady-state depletion levels differ.

The model is used to analyse the acid rain differential game for Great Britain and Ireland as well as for the whole of Europe. It is shown that benefits of cooperation exist even when the concept of critical loads is accepted, and that these benefits are higher when non-cooperative behaviour is modelled as a feedback Nash equilibrium. Most countries have to reduce their emissions drastically in order to meet the critical loads, which can be so beneficial for some countries that they can even increase their emissions. Note that no side payments are needed for this because it also happens when all countries only act in their own interest. However, side payments may be needed to sustain the full cooperative outcome. The empirical results should be treated with care, because the critical load numbers are not very precise.

The analysis in this paper assumes that the transport matrix and the critical loads are such that the depositions can be equal to the critical loads in all countries at once. This is true for the case of Great Britain and Ireland but unfortunately not for the whole of Europe. When this assumption does not hold the analysis becomes very complex and has to be subject of further research. Another point is that the empirical part of this paper only considers a part of the acid rain problem, namely sulphur. It is, however, straightforward to apply the theory to the data on nitrogen oxides and ammonia as well. Finally, the analysis is deterministic but this does not mean that we are not aware of the many uncertainties surrounding these problems.

Notes

1. An earlier version of this paper was presented at the 5th EAERE Annual Conference in Dublin, Ireland, at a HCM workshop in Rethymnon, Crete, and a NATO workshop in Wageningen, the Netherlands, 1994. We are grateful for the comments of the participants of these meetings, especially Olli Tahvonen and Henry Tulkens. We are also grateful to Robin Mason for correcting a mistake, to Arne Jernelöf and Gerhard Freiling for advice, and to Anton Markink for programming assistance.
2. To be more precise, in the beginning of the acidification process no damage is done to the soil and even more nutrients become available for the plants, but this aspect is left out of the analysis. We are grateful to Arne Jernelöf for pointing this out to us.
3. One would like to model here that a country has better information on its own stock than on the stocks of the other countries but this is quite difficult. Therefore one has to choose from two alternatives: countries observe all stocks or just their own stock. We have chosen the first alternative.
4. No conditions are known for convergence of this algorithm. However, we found convergence to the same outcome from different starting points and others found the same for similar algorithms.

5. In the streets of Haifa the ecologist Dan Cohen argued that negative depositions can be interpreted as the use of limestone. In fact he put forward that acidification is not a problem because of the ample availability of limestone but he did not fully convince us.

References

Aalbers, R. F. T. (1993), *A Note on Acidification*, Working Paper, Tilburg University.

Acid News 5 (1992), *A Newsletter from the Swedish and Norwegian NGO Secretariats on Acid Rain*. Göteborg.

Alcamo, J., L. Hordijk and R. Shaw (1990), *The RAINS Model of Acidification: Science and Strategies in Europe*. IIASA/Kluwer.

Başar, T. and G.-J. Olsder (1982), *Dynamic Noncooperative Game Theory*. Academic Press.

van Damme, E. (1989), 'Renegotiation Proof Equilibria in Repeated Prisoner's Dilemma', *Journal of Economic Theory* **47**, 206–217.

Dockner, E. J. and N. V. Long (1993), 'International Pollution Control: Cooperative versus Noncooperative Strategies', *Journal of Environmental Economics and Management* **24**, 13–29.

Downing, R. J., J.-P. Hettelingh and P. A. M. de Smet (1993), *Calculation and Mapping of Critical Loads in Europe: Status Report 1993*. RIVM, National Institute of Public Health and Environmental Protection, Report No. 259101003, Bilthoven, the Netherlands.

EMEP MCS-W Report 1/92, Calculated budgets for airborn acidifying components in Europe, 1985, 1987, 1988, 1989, 1990 and 1991. The Norwegian Meteorological Institute, Oslo.

Fudenberg, D. and E. Maskin (1986), 'The Folk Theorem in Repeated Games with Discounting and with Incomplete Information', *Econometrica* **54**, 533–554.

Gaitsgory, V. and S. Nitzan (1994), 'A Folk Theorem for Dynamic Games', *Journal of Mathematical Economics* **23**, 167–178.

Hettelingh, J.-P. (1989), *Uncertainty in Modeling Regional Environmental Systems*, Doctoral Thesis, Free University Amsterdam.

Klaassen, G. (1995), *Trading Sulphur Emission Reduction Commitments in Europe*, Doctoral Thesis, Free University Amsterdam.

Mäler, K.-G. (1989),' The Acid Rain Game', in H. Folmer and E. van Ierland, eds., *Valuation Methods and Policy Making in Environmental Economics.* Elsevier Science Publishers.

Mäler, K.-G. (1992), 'Critical Loads and International Environmental Cooperation', in R. Pethig, ed., *Conflicts and Cooperation in Managing Environmental Resources*. Springer Verlag.

Maskin, E. and J. Tirole (1988), 'A Theory of Dynamic Oligopoly, I: Overview and Quantity Competition with Large Fixed Costs', *Econometrica* **56**, 549–569.

Newbery, D. (1990), 'Acid Rain', *Economic Policy* **5**, 297–346.

Nilsson, J. (1986), 'Critical Loads for Nitrogen and Sulphur', *Miljørapport 1986:* **11**, Nordic Council of Ministers, Copenhagen.

van der Ploeg, F. and A. J. de Zeeuw (1992), 'International Aspects of Pollution Control', *Environmental and Resource Economics* **2**, 117–139.

Tsutsui, S. and K. Mino (1990), 'Nonlinear Strategies in Dynamic Duopolistic Competition with Sticky Prices', *Journal of Economic Theory* **52**, 136–161.

de Zeeuw, A. J. and F. van der Ploeg (1991), 'Difference Games and Policy Evaluation: A Conceptual Framework', *Oxford Economic Papers* **43**, 612–636.