THE TRANSITIONAL DYNAMICS OF FISCAL POLICY IN SMALL OPEN ECONOMIES

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Abstract

The paper studies the dynamic macroeconomic effects of fiscal shocks of various duration (permanent and temporary) under different financing methods (lump-sum tax and government debt). To this end, we develop an intertemporal macroeconomic model for a small open economy, featuring monopolistic competition in the intermediate goods market, endogenous (intertemporal) labor supply, and finitely lived households. Endogenous labor supply is crucial in generating cyclical adjustment paths and yields faster convergence to the new steady state compared with exogenous labor supply. The quantitative output effects and transitional dynamics of fiscal policy differ substantially from those of an infinitely lived representative agent model. In addition, government debt is key in making the timing of shocks matter, thus yielding permanent output effects of temporary fiscal shocks.

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1 Introduction

In the wake of the New Open Economy Macroeconomics—for which Obstfeld and Rogoff (1995) laid the foundations with their 'Redux' model—there has been a revival of interest in analyzing the macroeconomic effects of fiscal policy in open economies. In contrast to the traditional open economy models, the new models offer a rigorous micro-founded framework, which incorporates monopolistic competition, endogenous labor supply, and some form of stickiness in wages or prices. The new approach puts a lot of emphasis on the transitional dynamics of the current account and the nominal exchange rate. Because of its focus on nominal variables, money plays a central role.

The present paper analyzes the dynamic macroeconomic effects of fiscal spending shocks in a small open economy. We develop an intertemporal optimization model, which features monopolistic competition, endogenous intertemporal labor supply, and finitely lived households. But our approach differs from the New Keynesian sticky-price approach by assuming fully flexible prices and wages. Indeed, money does not feature in our analysis because we are interested in the effects of fiscal policy on relative prices. We thus focus on real rather than nominal variables. Our small open economy is embedded in a world of a homogeneous final good, which is supplied under perfect competition. The final goods sector employs differentiated intermediate inputs that are produced under monopolistic competition using labor and capital as primary inputs.[1] In keeping with the literature, there is an internationally traded bond, ensuring that households can use the current account to smooth private consumption. To avoid trivial capital dynamics, we postulate adjustment costs of investment at the level of the portfolio investor.

The analysis of fiscal policy in open economy models has received little attention compared with monetary policy. The vast majority of micro-founded literature on fiscal policy assumes perfect competition. Early contributions are those by Buiter (1981) and Cardia (1993).[2] The latter, however, is the only one that specifically analyzes public spending policy in a small open economy. Contributions that introduce some form of imperfect competition in goods or labor markets (without imposing explicit price stickiness) are small in number and are primarily focused on the case of a closed economy.[3] A notable

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[1]Alternatively, we could have considered a two-sector model consisting of tradables and nontradables (as in Bruno, 1976).

[2]Giovannini (1988), Sen and Turnovsky (1989, 1990), and Bovenberg (1993) also employ small open economy models but focus on tax or tariff policy rather than expenditure policy. Frenkel and Razin (1987), Buiter (1988), and Buiter and Kletzer (1991) study public spending policy but deal with large, two-country, open economy models.

[3]Rotemberg and Woodford (1995), Devereux *et al.* (1996), Heijdra (1998), and Heijdra and Ligthart (2007) focus on the closed economy case. The first two papers take a stochastic real business cycle approach whereas the latter two assume a deterministic setting.

exception is the open economy model of Coto-Martinez and Dixon (2003) to which our work is most closely related. They study the case of an infinitely lived representative agent (RA)—a specification which imposes Ricardian equivalence—and therefore cannot meaningfully differentiate between financing methods.

The first aim of our study is to characterize analytically the transition paths induced by a fiscal impulse. To this end, we employ the Laplace transform technique (Judd, 1982). Numerical examples are used to illustrate the transition paths at business cycle frequencies. Our numerical results show that a rise in public spending financed by lump-sum taxes yields positive output multipliers both in the short and long run. Long-run multipliers are well above unity if labor supply is sufficiently elastic[4] and exceed those in the short run. Furthermore, a fiscal impulse induces a short-run trade balance deficit, which swings into surplus in the new steady state. Because the stable eigenvalues of the linearized model can have complex values, our transition paths feature endogenously determined (dampened) cycles. Elastic intertemporal labor supply in an overlapping generations (OLG) setting is the key factor in generating these cycles.[5] Due to this cyclical feature, time periods in which private consumption multipliers and output multipliers move together are followed by periods in which they move in opposite directions.

A second objective is to consider the output implications of a debt-financed fiscal impulse.[6] We model finitely lived households in the Blanchard (1985)-Yaari (1965) tradition, which gives rise to the failure of Ricardian equivalence. In this context, debt financing differs from lump-sum tax financing in its effect on output and the other macroeconomic variables. In addition, it makes the timing of taxes and public spending matter and thus affects the transitional dynamics.[7] We study shocks of various duration (temporary and permanent), types of financing (pure lump-sum taxes versus public debt) and impulse sizes (moderate and drastic). Because of debt financing, temporary shocks have permanent effects on macroeconomic variables. Not surprisingly, output multipliers of temporary fiscal shocks are shown to be smaller than those of permanent shocks.

Besides providing a meaningful role to debt financing, finitely lived households yield an endogenously determined, stationary steady state. It is well known that in RA models of a small open economy the steady state is hysteretic. The dynamic system contains a zero root if the fixed world rate of interest equals the pure rate of time preference. This knife-edge condition should hold for a steady state to exist. If the rate of interest ex-

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[4]Elastic labor supply is needed to generate non-zero output effects in the steady state.

[5]In a closed-economy OLG setting, however, Heijdra and Ligthart (2007) derive monotonic transition paths.

[6]Ganelli (2006) also considers debt financing, but employs a variant of the sticky-price Redux model, which does not yield cyclical dynamics.

[7]The output multipliers are 'balanced budget' under lump-sum taxation. Bond financing permits temporary budget imbalances as long as the government's solvency condition is met.

ceeds (falls short of) the pure rate of time preference, households permanently accumulate (deplete) foreign assets. Various other authors have employed OLG as a stationarity inducing device.[8] None of these authors, however, has compared the comparative dynamic properties of OLG models with those of RA models. In a closed economy context, output multipliers of fiscal policy in OLG and RA models do not differ much (see Heijdra and Ligthart, 2007). Moreover, the business cycle properties of both models are similar in a stochastic setting (see Rios-Rull, 1996 and Gomme *et al.*, 2005). In a small open economy context, results are dramatically different; the long-run output effects of fiscal policy in the RA model are substantially larger than in the OLG model. Furthermore, the benchmark OLG model yields cyclical transitional dynamics against a monotonic transition in the RA model. Introducing OLG in open economy models is not just a stationarity-inducing device, but also affects the quantitative and cyclical properties of fiscal shocks.[9]

Unlike the vast majority of new open economy models—which assumes a fixed number of firms—we allow for free entry and exit of firms.[10] Endogenizing the number of firms in the intermediate goods sector gives rise to policy induced Ethier-productivity effects, which increase the size of Keynesian output multipliers. Our numerical results show that more elastic labor supply magnifies the long-run output effects of a rise in public spending. In addition, the speed of convergence to the new steady state rises if labor supply becomes more elastic and the preference for diversity increases.

The paper is structured as follows. Section 2 sets out the extended Blanchard-Yaari model for a small open economy. Section 3 studies model stability, the analytical transition paths, and calibration issues. Section 4 analyzes the dynamics of both a permanent and temporary increase in public consumption under various financing methods. Section 5 studies the robustness of the results to alternative parameterizations. In addition, we compare our results with the RA case, which is the standard in the literature. Section 6 summarizes and concludes.

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[8]Authors use various specifications of OLG. Giovannini (1988), Bovenberg (1993), and Cardia (1993) also use the Blanchard-Yaari framework. Buiter (1981) and Buiter and Kletzer (1991) employ a life-cycle OLG model. Cavallo and Ghironi (2002) employ the Weil specification, which allows for OLG of infinitely lived agents (featuring population growth). See Schmitt-Grohe and Uribe (2003) for an overview alternative stationarity-inducing devices.

[9]Note that the OLG framework, in contrast to the RA model, allows for an analysis of the intergenerational distribution effects of policies. Such an analysis is beyond the scope of this paper.

[10]The papers by Ghironi and Melitz (2005), who analyze the effects of firm entry and exit on trading patterns, and Coto-Martinez and Dixon (2003) are notable exceptions.

2 The Model of Perpetual Youth

In this section, we develop a dynamic, micro-founded, macroeconomic model for a small open economy, which features forward-looking agents. Subsequently, we discuss decision making by households, firms, and the government.

2.1 Households

The household section of the model builds on Blanchard (1985) and the extension to endogenous intertemporal labor supply by Heijdra and Ligthart (2007). The model features a fixed population of agents (normalized to unity) each facing a constant probability of death ($\beta \geq 0$), which equals the rate at which new agents are born. Labor is assumed to be immobile internationally and is supplied in a perfectly competitive labor market. Households do not leave bequests—implying that generations are disconnected—and do not face liquidity constraints.

Consumption Decision   During their entire life agents have a time endowment of unity, which they allocate to labor and leisure. The utility functional at time t of the representative agent born at time v is assumed to be weakly separable in private consumption, $C(v, t)$, and leisure, $1 - L(v, t)$:

$$\Lambda(v, t) \equiv \int_t^\infty \left[\varepsilon_C \ln C(v, \tau) + (1 - \varepsilon_C) \ln(1 - L(v, \tau)) \right] e^{(\alpha + \beta)(t - \tau)} d\tau, \ \ \alpha > 0, \qquad (1)$$

where α is the pure rate of time preference and ε_C is the share of private consumption in utility (where $0 < \varepsilon_C < 1$). The agent's budget identity is:

$$\dot{A}(v, t) = (r + \beta)A(v, t) + w(t)L(v, t) - T(t) - C(v, t), \qquad (2)$$

where an overdot indicates a time derivative, $A(v, t)$ are financial assets, r is the fixed world rate of interest, $w(t)$ is the (age-independent) wage rate,[11] and $T(t)$ are *net* lump-sum taxes (all denoted in real terms).

The household chooses a time profile for $C(v, t)$ and $L(v, t)$ to maximize $\Lambda(v, t)$ subject to its budget identity (2) and a no-Ponzi-game (NPG) solvency condition. A household's optimal time profile of private consumption is described by the Euler equation:

$$\frac{\dot{C}(v, t)}{C(v, t)} = r - \alpha. \qquad (3)$$

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[11]Despite the constant rate of interest, wages are flexible, reflecting adjustment costs in investments (see below).

In the general case, we study a patient nation, that is, $r > \alpha$, which yields rising individual consumption profiles.[12] Labor supply is negatively linked to private consumption (i.e., the wealth effect) and positively associated with wages:

$$L(v,t) = 1 - \frac{(1 - \varepsilon_C)C(v,t)}{\varepsilon_C w(t)}. \tag{4}$$

Aggregate variables can be calculated as the weighted sum of the values for different generations. Aggregate financial wealth is, for example, $A(t) \equiv \int_{-\infty}^{t} A(v,t)\beta e^{\beta(v-t)}dv$. The aggregate values of the other variables can be derived in a similar fashion. By aggregating (3), we arrive at the *aggregate* Euler equation:

$$\frac{\dot{C}(t)}{C(t)} = r - \alpha - \beta\varepsilon_C(\alpha + \beta)\frac{A(t)}{C(t)} = \frac{\dot{C}(v,t)}{C(v,t)} - \beta\varepsilon_C\left(\frac{C(t) - C(t,t)}{C(t)}\right). \tag{5}$$

Equation (5) has the same form as the Euler equation for individual households (3), except for a correction term, which represents the wealth redistribution caused by the turnover of generations. Optimal individual consumption *growth* is the same for all generations since they face the same rate of interest. But old generations have a higher consumption *level* than young generations because they are wealthier. Since existing generations are continually being replaced by newborns, who are born without financial wealth, aggregate consumption growth falls short of individual consumption growth. The correction term appearing in (5) thus represents the difference in average consumption, $C(t)$, and consumption by newborns, $C(t,t)$.[13]

Investment Decision   There are three assets in the economy, that is, claims on domestic capital goods, $V(t)$, domestic government bonds, $B(t)$, and *net* foreign assets, $F(t)$, which are all measured in real terms. By assuming assets to be perfect substitutes in the household's portfolio, they earn the same real rate of return.

The household's cash flow from investing in physical capital, $K(t)$, which is given by:

$$V(t) \equiv \int_{t}^{\infty} \left[r^K(\tau)K(\tau) - I(\tau) \right] e^{r(t-\tau)}d\tau, \tag{6}$$

where $r^K(t)$ is the rental rate on capital and $I(t)$ denotes *gross* investment. We follow Uzawa (1969) by postulating a concave accumulation function, $\Psi(\cdot)$, which links net capital

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[12]Rising individual consumption profiles imply a positive stock of financial assets in the initial equilibrium. By using (5) in steady state, we arrive at $(r - \alpha)\hat{C} = \beta\varepsilon_C(\alpha + \beta)\hat{A}$, where hats indicate steady-state values of variables. For $\beta > 0$ and $r - \alpha > 0$, we find $\hat{A} > 0$. As a special case, which we refer to as modified OLG, we study flat individual consumption profiles (i.e., $r - \alpha = 0$), which implies $\hat{A} = 0$. See Footnote 27.

[13]We use the fact that $C(t) = \varepsilon_C(\alpha + \beta)[A(t) + H(t)]$ and $C(t,t) = \varepsilon_C(\alpha + \beta)H(t)$, where $H(t)$ is 'full' human wealth, that is, the after-tax value of the household's time endowment: $H(t) \equiv \int_{t}^{\infty} [w(\tau) - T(\tau)] e^{-(r+\beta)\tau}d\tau$.

accumulation to gross investment:

$$\dot{K}(t) = \left[\Psi \left(\frac{I(t)}{K(t)} \right) - \delta \right] K(t), \qquad \Psi(0) = 0, \quad \Psi'(\cdot) > 0, \quad \Psi''(\cdot) < 0, \tag{7}$$

where $\delta > 0$ is the constant rate of capital depreciation.

The household-investor chooses paths for gross investment and the capital stock to maximize (6) subject to (7) and taking as given the initial capital stock, $K(0) = 0$, and the path of the rental rate, $r^K(t)$. The first-order conditions are:

$$1 = q(t)\Psi'\left(\frac{I(t)}{K(t)} \right), \tag{8}$$

$$\dot{q}(t) = \left[r + \delta - \Psi\left(\frac{I(t)}{K(t)} \right) \right] q(t) - r^K(t) + \frac{I(t)}{K(t)}, \tag{9}$$

where $q(t)$ denotes Tobin's q, which measures the market value of capital relative to its replacement costs. The degree of physical capital mobility is given by $\sigma \equiv -(I/K)(\Psi''/\Psi') > 0$, where a small σ characterizes a high degree of capital mobility. Without adjustment costs (i.e., $\sigma = 0$) we have $\Psi(\cdot) = I(t)/K(t)$, which implies $q = 1$ (from (8)). In this case, $q(t)$ and $K(t)$ adjust instantaneously to their steady-state levels, reflecting an infinite rate of investment in an infinitesimal small time period. Consequently, (9) reduces to $r^K = r + \delta$, which is the familiar rental rate derived in a static framework.

2.2 Firms

Following Hornstein (1993), the firm sector consists of: (i) monopolistically competitive firms, each of which produces a unique variety of an intermediate input; and (ii) perfectly competitive firms, producing a homogeneous final output using intermediate goods.

Homogeneous Final Goods Technology in the final goods sector can be described by a Dixit-Stiglitz (1977) specification:

$$Y(t) = N(t)^{\eta - \mu} \left[\int_0^{N(t)} Z_i(t)^{1/\mu} di \right]^{\mu}, \ \eta \geq 1, \ \mu > 1, \tag{10}$$

where $Y(t)$ denotes aggregate output of final goods, $Z_i(t)$ is the quantity of variety i of the differentiated intermediate good, and $N(t)$ is the number of input varieties. The parameter η regulates the productivity effect of increased input variety and μ is the markup set by firms in the intermediate goods sector.[14] Equation (10) shows the production function of the final goods sector, which implies *external* economies of scale for $\eta > 1$, owing to

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[14]In contrast to most of the literature, we parameterize separately the diversity effect, η, and the price elasticity of input demand, $\mu/(1-\mu)$. See Ethier (1982) and Bénassy (1996).

increasing input diversity (see Ethier, 1982). External economies of scale thus only become effective if the number of firms is allowed to change.

The representative producer in the final goods sector minimizes the cost of producing a given quantity of final goods by choosing the optimal mix of input varieties. Input demand functions feature a constant elasticity of demand:

$$Z_i(t) = N(t)^{(\eta-\mu)/(\mu-1)} Y(t) \left(\frac{p_i(t)}{p(t)} \right)^{\mu/(1-\mu)},$$

(11)

where $p_i(t)$ is the price of input variety i and $p(t)$ is the 'ideal price index' corresponding to (10):

$$p(t) \equiv N(t)^{\mu-\eta} \left[\int_0^{N(t)} p_i(t)^{1/(1-\mu)} di \right]^{1-\mu}.$$

(12)

Differentiated Intermediate Goods   The intermediate goods sector features $N(t)$ monopolistically competitive firms, each of which produces a single differentiated input. Each firm i rents capital and labor from the household sector to produce *gross* output according to:

$$Z_i(t) + f \equiv L_i(t)^{\varepsilon_L} K_i(t)^{1-\varepsilon_L}, \qquad 0 < \varepsilon_L < 1,$$

(13)

where f are fixed costs modeled in terms of the output of firm i. The firm's cost function is:

$$\Gamma_i(t) \equiv \gamma_0 w(t)^{\varepsilon_L} \left[r^K(t) \right]^{1-\varepsilon_L} p(t)(Z_i(t) + f),$$

(14)

where $\gamma_0 \equiv \left[\varepsilon_L{}^{\varepsilon_L}(1 - \varepsilon_L)^{1-\varepsilon_L} \right]^{-1} > 0$. Firms maximize profits by choosing their price and factor demands subject to (11) and (14). As a result, the price of input variety i is set equal to a constant markup over marginal cost:

$$p_i(t) = \mu \left(\frac{1}{\rho_i(t)} \right) \left(\frac{\Gamma_i(t)}{Z_i(t)} \right), \quad \rho_i(t) \equiv \frac{Z_i(t) + f}{Z_i(t)} > 1,$$

(15)

where $\rho_i(t)$ measures (local) internal increasing returns to scale due to the existence of fixed costs. Furthermore, the factor demands by firm i are determined by the usual marginal productivity conditions for labor and capital, $\partial Z_i(t)/\partial L_i(t) = \mu w(t)$ and $\partial Z_i(t)/\partial K_i(t) = \mu r^K(t)$, which both feature the firm's markup.

We assume Chamberlinian monopolistic competition, implying that the instantaneous entry and exit of firms eliminates all excess profits. Accordingly, the intermediate input price equals average cost:

$$p_i(t) = \frac{\Gamma_i(t)}{Z_i(t)}.$$

(16)

By combining (15) and (16), we obtain $\mu = \rho_i$, which implies a constant equilibrium firm size in the intermediate goods sector of $Z_i(t) = \bar{Z} \equiv \left(\frac{f}{\mu - 1} \right)$, where $\mu > 1$ for the equilibrium to exist. If $\mu \to 1$ and $f \to 0$ the model converges to a perfectly competitive economy.

2.3 Government and Foreign Sector

Government spending neither yields utility to individuals nor is it productive. Public consumption is financed by lump-sum taxes or a combination of lump-sum taxes and debt. The government's periodic budget identity is given by:

$$\dot{B}(t) = rB(t) + G(t) - T(t), \tag{17}$$

where $G(t) - T(t)$ denotes the primary fiscal deficit. Because the government must remain solvent, the government budget identity can be integrated forward—while using its NPG condition—to derive the government's intertemporal budget restriction,

$$B(t) = \int_t^\infty \left[T(\tau) - G(\tau) \right] e^{r(t-\tau)} d\tau. \tag{18}$$

Equation (18) says that the present value of current and future primary surpluses must equal the public debt level.

In the non-degenerate case, households use the current account to smooth consumption (and thus acquire *net* foreign assets, $F(t)$). Foreign financial capital is perfectly mobile. The change in net foreign assets equals the current account balance:

$$\dot{F}(t) = rF(t) + \left[Y(t) - C(t) - I(t) - G(t) \right], \tag{19}$$

where the term in square brackets is the trade account, showing that domestic output less domestic absorption, $C(t) + I(t) + G(t)$, equals net exports, $X(t)$. National solvency requires: $F(t) = - \int_t^\infty X(\tau) e^{r(t-\tau)} d\tau$, showing that the pre-existing level of net foreign assets (debt) should equal the present value of trade balance deficits (surpluses).

2.4 Symmetric Perfect Foresight Equilibrium

The supply side of the model is symmetric and can thus be expressed in aggregate terms. All existing firms in the intermediate goods sector are of equal size, $\bar{Z}$, and thus charge the same price and demand the same amounts of capital and labor, that is, $K_i(t) = \bar{K}(t)$ and $L_i(t) = \bar{L}(t)$. Equation (10) yields an expression for an aggregate quantity index for production in the final goods sector, that is, $Y(t) = N(t)^\eta \bar{Z}$. Hence, aggregate output of final goods is an iso-elastic function of the number of input varieties, $N(t)$. Production of intermediate goods can be expressed in aggregate terms, using $K(t) \equiv N(t)\bar{K}(t)$ and

$L(t) \equiv N(t)\bar{L}(t)$, and subsequently be used to derive aggregate final output, $Y(t) = \Omega_0 L(t)^{\eta \varepsilon_L} K(t)^{\eta(1-\varepsilon_L)}$, where Ω_0 is a productivity parameter and η serves as the returns to scale parameter.

The stock market value of the firm, $V(t)$, equals $q(t)K(t)$. Accordingly, portfolio equilibrium amounts to $A(t) = V(t) + B(t) + F(t)$. In case of $r > \alpha$, we assume that there are no domestic government liabilities and no net foreign assets in the initial steady state (i.e., $B(0) = F(0) = 0$).

3 Solving the Loglinearized System

This section solves the model, derives the analytical impulse responses to fiscal shocks, and discusses calibration issues. Both the general case of OLG and the special case of RA are analyzed.

3.1 Stability Analysis

The local stability of the model is analyzed by loglinearizing it around an initial steady state. A tilde (˜) denotes a relative change, for example, $\tilde{C}(t) \equiv dC(t)/C$, for most variables (Appendix A.1). Appendix Table 1 provides a summary of the loglinearized model. The dynamics of the model can be summarized by two predetermined variables (i.e., the physical capital stock and financial assets) and two non-predetermined variables (i.e., Tobin's q and private consumption):[15]

$$
\begin{bmatrix} \dot{\tilde{K}}(t) \\ \dot{\tilde{q}}(t) \\ \dot{\tilde{C}}(t) \\ \dot{\tilde{A}}(t) \end{bmatrix} = \Delta \begin{bmatrix} \tilde{K}(t) \\ \tilde{q}(t) \\ \tilde{C}(t) \\ \tilde{A}(t) \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ \gamma_A(t) \end{bmatrix}. \tag{20}
$$

The Jacobian matrix is denoted by Δ and its typical element by δ_{ij}:

$$
\Delta \equiv \begin{bmatrix} 0 & \frac{\bar{y}\omega_I}{\sigma} & 0 & 0 \\ (1-\varepsilon_L)\bar{y}[1-\eta\phi(1-\varepsilon_L)] & r & (1-\varepsilon_L)\bar{y}(\phi-1) & 0 \\ 0 & 0 & r-\alpha & -\frac{r-\alpha}{\omega_A} \\ r\varepsilon_L\eta\phi(1-\varepsilon_L) & 0 & -r(\omega_C+\varepsilon_L(\phi-1)) & r \end{bmatrix},
$$

where $\bar{y} \equiv Y/qK$, $\omega_A \equiv r/\bar{y}$, $\omega_C \equiv C/Y$, $\omega_I \equiv I/Y$, $\omega_T \equiv T/Y$, $\gamma_A(t) = r\omega_T\tilde{T}(t)$ is the exogenous policy shock (see (23) below), and ϕ parameterizes the *inter*temporal labor supply effect:

$$
\phi \equiv \frac{1+\theta_L}{1+\theta_L(1-\eta\varepsilon_L)} \geq 1, \tag{21}
$$

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[15]The government bond path will be discussed in Section 3.2.

where $\theta_L \equiv (1-L)/L \geq 0$ is the ratio of leisure to labor (which equals the intertemporal elasticity of substitution in labor supply). Assumption 1 guarantees a positive denominator of (21), $1 + \theta_L(1 - \eta\varepsilon_L) > 0$, so that ϕ is positive and greater than unity for endogenous labor supply. If $\eta\varepsilon_L > 1$, labor demand is upward sloping[16] and θ_L has a vertical asymptote at $1/(\eta\varepsilon_L - 1)$. Intuitively, Assumption 1 ensures that the labor supply curve is steeper (with respect to the wage axis) than the labor demand curve.

Assumption 1 *If $\eta\varepsilon_L > 1$, we assume that $0 \leq \theta_L < \hat{\phi} \equiv 1/(\eta\varepsilon_L - 1)$.*

Depending on the magnitude of ϕ, three labor supply cases can be distinguished of which the first two are consistent with saddle-point stability: (i) inelastic, $\phi = 1$; (ii) moderately elastic, $1 < \phi < \bar{\phi} \equiv 1/(\eta(1 - \varepsilon_L))$; and (iii) highly elastic, $\phi > \bar{\phi}$.

The dynamics of the OLG system depends crucially on the intertemporal labor supply elasticity. For $\theta_L = 0$, equation (21) yields $\phi = 1$, implying exogenous labor supply. In that case, $\delta_{23} = 0$ (from (20)), which yields a recursive *investment system* $(\tilde{q}(t), \tilde{K}(t))$ that can be solved completely independently from the *savings system* $(\tilde{C}(t), \tilde{A}(t))$. The investment and savings systems are saddle-path stable, giving rise to a monotonic transition to the new steady state after a fiscal policy shock. For $\theta_L > 0$ and thus $\phi > 1$ the investment system is non-recursive because $\delta_{23} > 0$. Two negative and two positive roots (potentially complex valued) result if $1 < \phi < \bar{\phi} \equiv 1/(\eta(1 - \varepsilon_L))$ whereas all roots are complex if $\phi > \bar{\phi}$.[17] In the complex case, the analytical solution for the transition paths of the variables includes cosine and sine terms (Appendix A.2 and Section 3.3), which give rise to endogenously determined cycles. Proposition 1 summarizes the local stability properties of the system.

Proposition 1 *If $\phi \in [0, \bar{\phi})$, the OLG system ($\beta > 0$) has a unique and saddle-path stable steady state, which features four characteristic roots with two negative real parts and two positive real parts.*

Proof *See Appendix A.2 and Heijdra and Ligthart (2006).*

Our model nests the RA case in which $\beta = 0$ so that $r = \alpha$ is required for a hysteretic steady state to exist. The model does not have a steady state unless $r = \alpha$, which yields perfect consumption smoothing. Notice that the economy would keep accumulating assets

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[16]This can be seen by substituting (AT1.10) from Appendix Table 1 into (AT1.6), which yields an expression for the wage rate as a function of employment and the capital stock: $\tilde{w}(t) = (\eta\varepsilon_L - 1)\tilde{L}(t) + \eta(1 - \varepsilon_L)\tilde{K}(t)$.

[17]The case of $\phi = \bar{\phi}$ yields an investment system with one zero root and one positive root. If intertemporal labor supply is sufficiently elastic (i.e., θ_L is large and thus $\phi > \bar{\phi}$), all the real parts of complex roots may plausibly turn positive, thus yielding an unstable solution. See Appendix A.2.

if $r > \alpha$[18] or be depleting assets if $r < \alpha$. In the RA case, the rate of growth of aggregate consumption does not depend on the holdings of financial assets. Mathematically, in terms of the Jacobian matrix, we have $\delta_{33} = \delta_{34} = 0$ (i.e., the third row of Δ consists of zeros), yielding a singular Jacobian matrix.[19] Thus, the RA model introduces a zero root in private consumption and labor supply, making the steady state dependent on the initial stock of foreign assets. Provided labor supply is elastic ($\phi > 1$ so that $\delta_{23} > 0$) there is also hysteresis in the physical capital stock and all variables dependent on it.[20]

Proposition 2 *The RA model (imposing $\beta = 0$ and $r = \alpha$) features a hysteretic steady state. To guarantee saddle-point stability, it is required that $\eta\phi(1 - \varepsilon_L) < 1$. The four characteristic roots are real: $h_1^* = 0$, $-h_2^* = (r - \sqrt{r^2 + 4\delta_{12}\delta_{21}})/2$, $r_1^* = r$, and $r_2^* = (r + \sqrt{r^2 + 4\delta_{12}\delta_{21}})/2$.*

Proof *See Heijdra and Ligthart (2006).*

3.2 Fiscal Policy Paths

The path of the public spending impulse in its most general form can be postulated as follows:

$$\tilde{G}(t) = \tilde{G}e^{-\xi_G t}, \quad \xi_G \geq 0, \tag{22}$$

where ξ_G denotes the adjustment speed of public consumption. A *permanent* fiscal change is represented by $\xi_G = 0$ and $0 < \xi_G \ll \infty$ models a *temporary* fiscal shock. Larger values of ξ_G generate a lower degree of shock persistence.

Under pure lump-sum tax financing of a fiscal impulse, we set $\tilde{T}(t) = \tilde{G}(t)$ for all $t \geq 0$. For deficit financing, however, the paths for $\tilde{G}(t)$ and $\tilde{T}(t)$ do not coincide and we postulate:

$$\tilde{T}(t) = -\tilde{T}_0 + \tilde{T}_\infty \left(1 - e^{-\xi_T t}\right), \quad \xi_T > 0, \tag{23}$$

where ξ_T is the adjustment speed of lump-sum taxes and $\tilde{T}_0 > 0$ if there is an initial tax cut. Using the government budget identity (17), the government solvency condition can be written as $\mathcal{L}\{\tilde{T}, r\} = \mathcal{L}\{\tilde{G}, r\}$, where $\mathcal{L}$ denotes the Laplace transform operator.[21]

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[18]In an RA model, $r > \alpha$ is not a viable interior solution because the economy would cease being small in world capital markets.

[19]The steady state is well defined because the number of non-predetermined variables equals the number of eigenvalues with strictly positive real parts (Giavazzi and Wyplosz, 1985).

[20]In their analysis of the current account effects of tariff policy, Sen and Turnovsky (1990) also find hysteresis in the capital stock.

[21]$\mathcal{L}\{G, s\}$ is the Laplace transformation of $G(t)$ evaluated at s, which is given by $\mathcal{L}\{G, s\} \equiv \int_0^\infty G(t)e^{-st}\mathrm{d}t$. Intuitively, $\mathcal{L}\{G, s\}$ represents the present value of $G(t)$ using s as the discount rate. See Heijdra and Ligthart (2006) for the derivation of equations (24)-(27).

Government solvency implies that the long-run increase in lump-sum taxes is $\tilde{T}(\infty) = \tilde{T}_\infty - \tilde{T}_0$ (from (23)), where:

$$\tilde{T}_\infty = \left(\frac{r + \xi_T}{\xi_T} \right) \left[\left(\frac{r}{r + \xi_G} \right) \tilde{G} + \tilde{T}_0 \right], \quad \xi_T > 0. \tag{24}$$

Long-run lump-sum taxes must rise to cover the additional government spending on goods plus the interest payments on the public debt that is accumulated during the transition period. Accordingly, future generations face a larger lump-sum tax burden than present generations.

The path for government debt can be written:

$$\tilde{B}(t) = \omega_G \left(\frac{r}{r + \xi_G} \right) \left[1 - e^{-\xi_G t} + \left(\frac{r}{\xi_T} \right) (1 - e^{-\xi_T t}) \right] \tilde{G} + \frac{r \omega_G}{\xi_T} \left(1 - e^{-\xi_T t} \right) \tilde{T}_0, \tag{25}$$

where ω_G is the output share of public spending. Using (23) and (24), the corresponding path for lump-sum taxes can be derived:

$$\tilde{T}(t) = \pi_0 + \pi_1 e^{-\xi_T t}, \tag{26}$$

where the policy parameters π_0 and π_1 are defined as:

$$\pi_0 \equiv \frac{r}{\xi_T} \left[\tilde{T}_0 + \left(\frac{r + \xi_T}{r + \xi_G} \right) \tilde{G} \right], \quad \pi_1 \equiv - \left(\frac{r + \xi_T}{\xi_T} \right) \left[\tilde{T}_0 + \left(\frac{r}{r + \xi_G} \right) \tilde{G} \right], \tag{27}$$

which depend on the parameters $\tilde{G}$, $\tilde{T}_0$, ξ_G, and ξ_T and the fixed rate of interest.

3.3 Analytical Impulse Responses

A key contribution of our work is that we characterize analytically the transition paths induced by a fiscal shock.[22] For this purpose, we have employed the Laplace transform technique. We focus on the case of complex roots, which are defined as follows. The two stable characteristic roots come in conjugate pairs and are denoted by $\nu = -h^* + \theta_\nu i$ and $\bar{\nu} = -h^* - \theta_\nu i$, where $-h^* < 0$ is the negative real part, θ_k is the imaginary part for $k = \{\nu, \lambda\}$, and $i \equiv \sqrt{-1}$ is the imaginary unit. The two unstable roots are $\lambda = r^* + \theta_\lambda i$ and $\bar{\lambda} = r^* - \theta_\lambda i$, where $r^* > 0$ is the positive real part.

The transition path for the capital stock is:

$$\tilde{K}(t) = \delta_{12} \tilde{q}(0) \mathbf{T}_1 (h^*, \theta_\nu, t) - \delta_{12} \delta_{23} \delta_{34} r \omega_G \left[\frac{\pi_0}{\lambda \bar{\lambda}} \mathbf{A}(h^*, 0, \theta_\nu, t) \right.$$
$$\left. + \frac{\pi_1}{(\lambda + \xi_T)(\bar{\lambda} + \xi_T)} \mathbf{A}(h^*, \xi_T, \theta_\nu, t) \right], \tag{28}$$

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[22] Appendix A.3 sets out the solution procedure and Appendix A.4 shows the transition paths for the remaining variables.

where π_0 and π_1 are defined in (27), δ_{ij} is defined in (20), and $\mathbf{T}_1\left(h^*, \theta_\nu, t\right)$ and $\mathbf{A}\left(h^*, \xi_T, \theta_\nu, t\right)$ are a temporary transition term (Definition 1) and a general adjustment term (Definition 2), respectively. Definitions 1 and 2 show that the transition terms consist of exponential functions weighted by functions that generate periodic cycles.

Definition 1 *The temporary transition term is given by:*

$$\mathbf{T}_1\left(h^*, \theta_\nu, t\right) \equiv \frac{1}{\theta_\nu} e^{-h^* t} \sin \theta_\nu t,$$

which has properties: (i) $\mathbf{T}_1\left(h^*, \theta_\nu, 0\right) = 0$; and (ii) $\lim_{t \to \infty} \mathbf{T}_1\left(h^*, \theta_\nu, t\right) = 0$.

Definition 2 *The general adjustment term is given by:*

$$\mathbf{A}\left(h^*, \xi_T, \theta_\nu, t\right) \equiv \frac{1}{\left(\xi_T - h^*\right)^2 + \theta_\nu^2} \left[e^{-\xi_T t} - e^{-h^* t} \left(\cos \theta_\nu t + \frac{h^* - \xi_T}{\theta_\nu} \sin \theta_\nu t \right) \right],$$

which has properties: (i) $\mathbf{A}\left(h^*, \xi_T, \theta_\nu, 0\right) = 0$; (ii) $\lim_{t \to \infty} \mathbf{A}\left(h^*, 0, \theta_\nu, t\right) = 1/\left[\left(h^*\right)^2 + \theta_\nu^2\right]$; and (iii) $\lim_{t \to \infty} \mathbf{A}\left(h^*, \xi_T, \theta_\nu, t\right) = 0$ for $\xi_T > 0$.

The bracketed term in (28) drops out for exogenous labor supply (i.e., $\delta_{23} = 0$) or for infinitely lived representative households (i.e., $\delta_{34} = 0$) or both. In addition, the cosine and sine terms disappear from the transition terms for these cases.[23] In this context, the adjustment speed to the new steady state is driven by h^*. More generally, the cyclical part of the OLG model with elastic labor supply is quantitatively dominated by the exponential function unless $h^* \to 0$ (Definitions 1 and 2). In view of this, there is no hope that the periodicity of cycles would match evidence from business cycle research (see Cooley and Prescott, 1995).

3.4 Calibration

To study the quantitative significance of the comparative dynamics, a numerical treatment is pursued here. The model is calibrated for a plausible set of parameters.

In the benchmark model of moderately elastic labor supply (i.e., $\theta_L = 2$ so that $\phi = 2.27 < \bar{\phi} = 2.78$) the parameters are the following. The world rate of interest is set to 4 percent a year. By assuming a probability of death (β) of 2 percent, agents have an expected life span of 50 time periods. The output share of labor income (ε_L) is set equal to 0.70, which corresponds roughly to the value found for EU countries. Preference for diversity (η) is 1.2, giving rise to $\eta \varepsilon_L = 0.84$, implying diminishing returns to aggregate

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[23]The cyclical terms also drop out from the OLG model if θ_L is sufficiently low such that the stable roots are real and distinct. See Heijdra and Ligthart (2006) for a derivation of the expressions.

capital accumulation. The rate of capital depreciation (δ) is assumed to be 7 percent a year. We have chosen a logarithmic specification for the installation function:[24]

$$\Psi(x) \equiv \bar{z} \ln \left(\frac{x + \bar{z}}{\bar{z}} \right), \tag{29}$$

where $\bar{z}$ is an exogenous constant and $x = I/K = 0.08$. From (29), we can derive, $\sigma = x/(x + \bar{z})$, which features an asymptote at $x = -\bar{z}$. We have set $\bar{z} = 0.268$, implying steady-state adjustment costs of 11.2 percent. Government spending as share of GDP (ω_G) is 20 percent, which equals the lump-sum tax-to-GDP share.

Once the parameters are set, all other information on output shares, Tobin's q, and the investment-capital ratio can be derived.[25] The pure rate of time preference (α) is used as a calibration parameter, which—given the fixed rate of interest—yields rising individual consumption profiles. In the benchmark case, the four characteristic roots are complex valued. The two stable roots are $\nu, \bar{\nu} = -0.0410 \pm 0.0216i$ and the unstable roots are given by $\lambda, \bar{\lambda} = 0.0818 \pm 0.0222i$.

4 The Macroeconomic Effects of Fiscal Policy

This section studies numerically—using the expressions for the impulse response functions of the previous section—unanticipated shocks in fiscal spending (i.e., $\tilde{G} = 0.10$) under various fiscal regimes: (i) lump-sum tax financing versus bond financing; and (ii) permanent changes versus temporary changes. The benchmark parameter values are employed for the OLG case. The next section studies alternative parameterizations and the RA case. Table 1 summarizes the numerical results for the impact effect (recorded at $t = 0$) and the long-run effect (taken at $t \to \infty$), which will be discussed below.

4.1 Lump-Sum Tax Financing

Permanent Shocks   Under pure lump-sum tax financing of a permanent and unanticipated increase in public spending, we set $\xi_G = 0$ in (22), and $\tilde{T}(t) = \tilde{G}$ for all $t \geq 0$. The impulse-response diagrams (Figure 1)—plotted for 200 time periods—show non-monotonic transition paths for the key variables, reflecting the cyclical dynamics induced by endogenous labor supply. The key macroeconomic linkages over time are as follows. On impact, private consumption is crowded out by public consumption owing to the rise in lump-sum taxes. Consequently, households supply more labor (via the negative wealth effect in labor

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[24]It is easy to see that $\lim_{\bar{z} \to \infty} \Psi(x) = x$, that is, the installation function is linear (and adjustment costs are zero) for large $\bar{z}$.

[25]Equation (7) and (8) are solved, using (29), to yield $(\hat{I}/\hat{K}) = \bar{z} [\exp(\delta/\bar{z}) - 1]$ and $\hat{q} = \exp(\delta/\bar{z})$.

supply), which pushes down wages in the short run. Given the constant capital stock in the initial period, the capital-labor ratio falls and output rises.

The upward jump in Tobin's q induces private investment, which increases the physical capital stock over time and pushes up the capital-labor ratio. Close to period 45, the change in Tobin's q turns negative, causing a fall in the increment of the capital stock. Subsequently, Tobin's q slowly increases to the new steady state, where it is back at its old equilibrium value. Because capital and employment are modeled as cooperative factors of production, the path of employment mimics that of the capital stock. In the new steady state, wages have risen, reflecting the rise in the capital-labor ratio.[26] The long-run output effect of the fiscal impulse is clearly positive and exceeds its short-run impact. Relative changes in output and private consumption move together in the medium run, but move in opposite directions in the very short run and beyond period 40.

What is the effect of the fiscal shock on the foreign sector? The combined increase in investment and public spending exceeds the fall in private consumption thereby boosting domestic absorption. The latter exceeds the output increase in the short run, turning the trade account into deficit. Accordingly, net foreign debt is accumulated—and thus leads to interest payments to the rest of the world—which reaches its maximum increment in period 30 after which somewhat of a recovery materializes. Because the trade account swings into surplus (around period 15), foreign debt accumulation slows down. In the new steady state, the current account is balanced again, reflecting a trade balance surplus that offsets interest payments on foreign debt.

Temporary Shocks   To generate a temporary fiscal shock, we set $\tilde{G} = \tilde{T}$ and $\xi_G = 0.10$ (see (22)), which generates a substantial degree of persistence in public spending. In the short run, permanent and temporary fiscal shocks have qualitatively similar effects, but differ quantitatively. Column (3) of Table 1 presents numerical evidence. The impact effects on output and the trade balance fall short of those induced by a permanent fiscal impulse (compare columns (3) and (4)). It is well know that temporary fiscal shocks cannot have permanent effects on any of the macroeconomic variables under lump-sum taxation. This does not mean that nothing happens after the impact period. Simulations over the entire frequency domain—not shown in the table—indicate that the initial rise in output is gradually offset, causing output to return to its old steady state. Similarly, the trade account swings into deficit before it balances at its initial equilibrium level. Thus, the steady-state stock of foreign assets is unaffected.

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[26]The capital-labor ratio also shows a cycle. Without Ethier effects, however, the long-run capital-labor ratio would be unaffected.

4.2 Bond Financing

Under bond financing of a rise in public spending, we set $\xi_T = 0.10$ in (26). Besides a scenario of moderate fiscal policy, we analyze drastic fiscal policy, in which initial lump-sum taxes are cut together with $\tilde{G} > 0$.

Permanent Shocks Let us first consider a scenario of moderate fiscal policy, in which initial lump-sum taxes are not changed (i.e., $\tilde{T}_0 = 0$). Compared with pure lump-sum tax financing, the qualitative effects on the new steady state are very similar (compare columns (4) and (7)). Quantitatively, there are important differences. The long-run output multipliers under debt financing are larger than in the lump-sum tax funding regime, because debt and the associated debt-service payments shift the lump-sum tax burden from present to future generations. Intuitively, the higher tax burden faced by future generations induces them to work harder to make up for the fall in human wealth. Indeed, we see a larger fall in long-run private consumption in the debt-financing scenario. In addition, investment is much higher, reflecting capital accumulation induced by the expansion of employment. Clearly, bond financing induces bigger swings in the trade account; larger initial trade deficits—caused by demand by the investment sector and the government—are compensated by larger future surpluses. In the short run, however, output multipliers under lump-sum tax financing exceed those under debt financing, reflecting the tax shift to the future.

Drastic fiscal policy combines a fiscal impulse with an initial cut in lump-sum taxes (i.e., $\tilde{T}_0 = 0.10$). Column (8) of Table 1 shows that long-run output multipliers in this scenario are much larger than under moderate fiscal policy (compare columns (7) and (8)). Drastic fiscal policy tilts the slope of the transition path such that the initial output effect is smaller and the final output change is larger than under moderate fiscal policy. Intuitively, the cut in initial lump-sum taxes moderates the *net* rise in lump-sum taxes and thus its short-run labor supply effect. In the long run, a larger amount of debt has to be redeemed, which raises labor supply by more. Because the fall in short-run private consumption is smaller under bond financing, the initial trade account deficit is also larger. In the long run, a larger trade account surplus is needed to meet the national solvency condition.

Temporary Shocks Figure 2 shows the impulse responses of a temporary debt-financed spending impulse. It is evident from the path of output that a temporary fiscal shock has a permanent effect on long-run output. In contrast, under lump-sum tax financing, temporary shocks do not affect output permanently (compare columns (3) and (5)). Permanent effects materialize themselves through the government's intertemporal budget constraint.

The government issues debt to balance its budget, which shifts lump-sum tax payments—and their corresponding wealth effect on labor supply—beyond the time frame of the fiscal policy shock. Note that the relationship between output and consumption multipliers is negative across the short- and medium-run time profile.

Just like under permanent shocks, the size of the long-run output change is sensitive to the size of the fiscal impulse; the debt-cum-tax-cut package yields the largest output gain in the long run. It can even generate a long-run output multiplier exceeding unity. As we saw before, a fiscal impulse combined with an initial lump-sum tax cut produces a smaller short-run output gain than without a tax cut.

5 Alternative Parameterizations

In this section we study the sensitivity of the allocation results and convergence speed to parameter changes. To keep matters simple, we consider unanticipated changes in public spending financed by lump-sum taxes.

5.1 Representative Agents ($\beta = 0$)

For purposes of comparison with the literature, we study the case of a zero birth rate, which yields an infinitely lived RA model. In an RA framework, Ricardian equivalence holds, implying that the difference between lump-sum tax and debt financing is immaterial. In such framework, the knife-edge condition $r = \alpha$ yields flat private consumption profiles. Consequently, the steady state is hysteretic, implying that it depends on the initial conditions. To guarantee saddle-point stability, $\eta\phi(1 - \varepsilon_L) < 1$ (Proposition 2), requiring that the degree of monopolistic competition and the intertemporal labor supply effect cannot be too large. Real Business Cycle (RBC) theorists, however, purposefully set θ_L large to be able to generate business cycles. See, for example, Prescott (2006).

The qualitative impact and long-run output effects of permanent fiscal policy in the RA model are similar to those in the OLG model (compare columns (2) and (4) of Table 1). The quantitative effects differ, however. But Table 1 does not provide a proper basis of comparison given that the slopes of the individual consumption profiles differ (which are upward sloping in the OLG case and flat in the RA case). To address this shortcoming, we compare the two models with the restriction $r = \alpha$ imposed on the OLG model (Table 2).[27] We call the latter the modified OLG case. Because of the knife-edge property $r = \alpha$, the RA model is likely to be highly nonlinear. Indeed, we need to pick a β close to zero (for example, $\beta = 0.00001$) in the modified OLG model to generate short-run output effects

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[27]The restriction $r = \alpha$ imposed on the OLG model implies $\hat{A} = 0$ and $-\hat{F} = \hat{q}\hat{K}$ in the steady state. Equation (AT1.3) now reduces to $\dot{\hat{C}}(t) = \delta_{34}\tilde{A}(t)$, where $\delta_{34} \equiv \beta\varepsilon_C (\alpha + \beta) / (\alpha\omega_C)$.

that approximate those found in the RA model. Labor supply plays a key role in causing nonlinearities. We can illustrate this, by comparing the rows $\beta = 0$ and $\beta = 0.01$ of Table 2. For $\theta_L = 2$, the short-run output multiplier declines by 9.8 percent when β rises by 0.01 compared with only a fall in the multiplier of 0.5 percent at $\theta_L = 0.5$.

Recall that the long-run output multipliers in the OLG framework are unaffected by the probability of death, but differ from those found in an RA framework. We can see that in the benchmark RA case long-run output multipliers exceed unity[28] and are substantially larger (by 40.6 percent) than those found in the OLG framework. In contrast, Heijdra and Ligthart (2007)—employing closed-economy OLG and RA models—find that the quantitative results for the two types of models do not differ much. In sum, this casts doubts about the often assumed approximate validity of RA models for more complex OLG cases (see Bernheim, 1987), specifically if θ_L is large.

The OLG structure may also give rise to differing properties of the transition path induced by a shock. Indeed, transitional dynamics in the RA model are non-cyclical against the case of cycles in the benchmark OLG model. In fact, for the benchmark parametrization, we find a monotonic transition path. Note that cycles are parameter specific. Glancing over the OLG rows in Table 2 (for the benchmark θ_L) shows that for small and large values of β, the roots of the characteristic equation are real rather than complex, implying a monotonic transition in the capital stock. Intermediate values of β yield cycles, however. Moreover, the consumption-output correlation in the RA model is zero across the entire time profile, against a medium-run comovement of both variables in the OLG case. In a deterministic setting, the cyclical properties make a difference, which is not necessarily true in a stochastic RBC environment. For example, Rios-Rull (1996) shows that the business cycle implications—in terms of the first moments of variables and correlations between variables—of a life-cycle model calibrated for the US economy are similar to those found in a standard RA model.

Because of the hysteretic property, after a temporary fiscal shock the economy does not return to its initial steady state. Temporary shocks can thus have permanent effects even under lump-sum tax financing (see column (1) of Table 1). Long-run output multipliers of temporary shocks are smaller than unity, a result which is also found in a closed economy setting.

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[28]In contrast to our RA results, Coto-Martinez and Dixon (2003) find in their RA model long-run output multipliers that are: (i) between zero and unity; and (ii) smaller than short-run output multipliers. Their modeling framework is very different from ours by allowing for non-separable preferences over consumption and leisure and by not incorporating Ethier productivity effects.

5.2 Overlapping Generations ($\beta > 0$)

Tables 3 and 4 report the sensitivity of long-run and short-run output multipliers in the OLG model to changes in the intertemporal substitution elasticity of labor supply (measured across columns), the birth rate (measures across the rows of Table 3), and the preference for diversity parameter (measured across the rows of Table 4).

As was argued in Section 3, the type of roots (real versus complex) varies with the elasticity of labor supply, but also with other parameters. For a small θ_L and a small β (and/or a small η), the steady state features two stable real roots. For very large values of θ_L and η, the real parts of roots may turn positive, yielding an unstable solution. In many instances of elastic labor supply, the stable (negative) roots are complex. In this case, following Alvarez-Cuadrado *et al.* (2004), we employ the modulus of the stable complex root, $s_M \equiv \sqrt{(h^*)^2 + \theta_\nu^2}$, as an appropriate measure of the speed of convergence. In case of two stable (negative) real roots, the speed of convergence is a weighted average of these roots. Asymptotically, the speed of adjustment is given by the larger of the two (negative) roots.[29] The speed of convergence for $\beta = 0.05$ and $\theta_L = 1.0$ is 6.8 percent (Table 3), which falls with the length of the planning horizon. The effect of θ_L on the convergence speed depends on the model specification. In the standard OLG specification of upward-sloping individual consumption profiles (Table 3), a larger θ_L leads to faster convergence. But, in the modified OLG model (Table 2) for $\beta \in [0.05, 0.10]$, a larger θ_L reduces the convergence speed.

We first turn to elastic labor supply for which a number of results stand out. First, all long-run output multipliers are positive and increasing in θ_L.[30] But, in the short run, output multipliers are hump shaped in θ_L. Intuitively, at higher values of θ_L private consumption falls less on impact, which depresses short-run labor supply (via the wealth effect) more and thus causes output to rise by less. For the benchmark value of $\theta_L = 2$, all long-run output multipliers (generating a stable outcome) are bigger than unity even if goods markets are perfectly competitive. Low values of θ_L may generate long-run output multipliers smaller than unity. Second, long-run output multipliers always exceed those in the short run, a result which is also found by Heijdra and Ligthart (2007) in a closed economy setting. Finally, for large substitution elasticities of labor supply we find stable spiralling relationships between consumption multipliers and output multipliers, yielding alternating periods of positive and negative relationship.

For inelastic labor supply, impact and long-run output multipliers are zero, which

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[29]Over time, the weight of the smaller (more negative) eigenvalue declines, implying that the larger (less negative) root determines the asymptotic speed of convergence. See Alvarez-Cuadrado *et al.* (2004).

[30]Heijdra and Ligthart (2007) find negative long-run output multipliers if the generational turnover effects dominates the labor supply effect. In our case, capital accumulation does not affect the rate of interest in and out of steady state.

demonstrates the crucial role the intertemporal labor supply effect plays in generating non-zero long-run output effects. During transition, however, output rises, reflecting capital formation induced by the rise in Tobin's q on impact. The fall in the positive increment in Tobin's q together with capital depreciation, cause the capital stock and thus output to fall back to its old equilibrium level.

Long-run output multipliers are unaffected by agents' planning horizons (Table 3 for $\beta > 0$), owing to the fixed rate of interest, which pins down the capital-labor ratio and thus the long-run marginal product of capital. The effect of shorter planning horizons (i.e., a larger β) on short-run multipliers is negative at low values of θ_L, but shows a nonlinear pattern for higher values of θ_L.

Turning to the diversity effect, it can be seen from Table 4 that more diversity increases long-run output multipliers, but has an ambiguous effect on the size of short-run multipliers if θ_L is small. In the benchmark scenario, a large η has a negative effect on short-run output multiplier. For $\eta > 1/\varepsilon_L$, labor demand is upward sloping. In this case, a shift in the labor supply curve to the right reduces employment, which explains the negative short-run output multiplier. Combinations of a large θ_L and η yield an unstable solution. The first row of the table is isomorphic to the perfectly competitive case ($\eta = 1.0$), which yields a long-run output multiplier a little above unity in the benchmark scenario. Only for small values of θ_L do we find long-run output multipliers smaller than unity. In sum, monopolistic competition increases the size of long-run output multipliers.

6 Conclusions

The paper has developed an OLG model for a small open economy with a view to analyze the dynamic response of the macroeconomy to fiscal shocks. The OLG model is compared with an RA model. Various instruments (lump-sum taxes and debt), duration of shocks (temporary and permanent), and impulse sizes (moderate and drastic) are considered.

A number of key results can be extracted from the analysis. First, the *sign* of steady-state output multipliers of fiscal policy shocks is robust to parameter changes (within the parameter set generating a stable outcome), financing method, and the size of fiscal impulse. The *size* of output multipliers, however, is affected by the financing method and alternative parameterizations. Bond-financed long-run output multipliers are larger than those found under lump-sum tax financing. In the benchmark case, long-run output multipliers of permanent fiscal shocks exceed unity irrespective of the degree of monopolistic competition or the financing method. Smaller intertemporal elasticities of labor supply reduce output multipliers, possibly below unity, whereas a higher degree of monopolistic competition boosts output multipliers.

Second, elastic intertemporal labor supply gives rise to endogenously determined cycles, which increase in frequency with the size of the labor supply effect. Because of this cyclical feature, time periods with a negative association between private consumption multipliers and output multipliers are followed by periods with a positive association.

Third, the size of lump-sum tax financed output multipliers and the comparative dynamic properties of the OLG model differ from those of an RA model, whereas the two models generate very similar results in a closed-economy setting. For benchmark parameter values, the dynamics of a fiscal shock in an RA model imply a monotonic transition, against cyclical adjustment in the OLG case. The RA model features a hysteretic steady state, which is highly nonlinear, due to the 'knife-edge condition' (which prescribes a rate of interest equal to the pure rate of time preference) and elastic intertemporal labor supply. The difference in size between long-run output multipliers in the RA and OLG frameworks rises with the intertemporal labor supply effect. Consequently, the often assumed approximate validity of RA models is tenuous in a small open economy environment.

Finally, temporary fiscal shocks have permanent output effects if public spending is financed by government debt (or if planning horizons of agents are infinite). Output multipliers of permanent debt-financed fiscal shocks are larger than that of temporary debt-financed shocks, particularly if the rise in public spending is combined with a cut in initial lump-sum taxes.

There are of course many aspects of fiscal policy that have not been addressed here, such as the output effects of anticipated fiscal shocks and the optimal level of public spending. In addition, the model could easily be extended to allow for money in utility and sticky prices, which would provide a link with Obstfeld and Rogoff's Redux model. We leave these extensions for further research.

Appendix

A.1 Loglinearization

Loglinearizing the key expressions of the OLG model of Section 2 around an initial steady state (assuming that $B = F = 0$ initially) yields Appendix Table 1. The following notational conventions are employed. A tilde ($\tilde{}$) denotes a relative change, for example, $\tilde{C}(t) \equiv dC(t)/C$ for most variables. Financial assets (i.e., $A(t), B(t), F(t)$), however, are scaled by steady-state output and multiplied by r, for example, $\tilde{B}(t) \equiv rdB(t)/Y$. Time derivatives are defined as $\dot{\tilde{C}}(t) \equiv d\dot{C}(t)/C$, except for financial assets, for example, $\dot{\tilde{B}}(t) \equiv rd\dot{B}(t)/Y$.

Conditional on the state variables and the policy shocks, the static part of the model (given by (AT1.6)-(AT1.12)) can be condensed to the following quasi-reduced form expressions:

$$\tilde{Y}(t) = \eta\phi\left(1 - \varepsilon_L\right)\tilde{K}(t) - (\phi - 1)\tilde{C}(t), \tag{A.1}$$

$$\eta\varepsilon_L\tilde{L}(t) = \tilde{Y}(t) - \eta(1 - \varepsilon_L)\tilde{K}(t), \tag{A.2}$$

$$\eta\varepsilon_L\tilde{w}(t) = (\eta\varepsilon_L - 1)\tilde{Y}(t) + \eta(1 - \varepsilon_L)\tilde{K}(t). \tag{A.3}$$

A.2 Stability

Using (A.1)-(A.2) and the expressions in Appendix Table 1, the system can be written as (20) in the main text, which is a four dimensional system of first-order differential equations for the physical capital stock, Tobin's q, private consumption, and financial assets. The determinant of Δ is:

$$\det(\Delta) = \left(\frac{r}{\omega_A}\right)\frac{\bar{y}^2\omega_I}{\sigma}\left(1 - \varepsilon_L\right)(r - \alpha)\left[\omega_G\left(\phi - 1\right) + \phi\chi\left(\omega_C - \omega_A\right)\right] > 0, \tag{A.4}$$

where $\chi \equiv 1 - \eta(1 - \varepsilon_L) > 0$ and $\det(\Delta) \equiv \nu\bar{\nu}\lambda\bar{\lambda}$, where $\bar{\nu}$ and $\bar{\lambda}$ denote the roots of the investment system $(\tilde{q}(t), \tilde{K}(t))$ and ν and λ are the roots of the savings system $(\tilde{C}(t), \tilde{A}(t))$. The trace of Δ is:

$$\text{tr}(\Delta) = \nu + \bar{\nu} + \lambda + \bar{\lambda} = 2(r^* - h^*) = 3r - \alpha > 0, \tag{A.5}$$

where h^* and r^* are both positive. The system of four first-order differential equations gives rise to a characteristic polynomial of the fourth order:

$$h(s) \equiv |s\text{I} - \Delta| = 0 \quad \Leftrightarrow \quad (s - \nu)(s - \bar{\nu})(s - \lambda)(s - \bar{\lambda}) = 0, \tag{A.6}$$

where the characteristic roots are in general terms:

$$\nu \equiv -h^* + \theta_\nu i, \quad \bar{\nu} \equiv -h^* - \theta_\nu i, \quad \lambda \equiv r^* + \theta_\lambda i, \quad \bar{\lambda} \equiv r^* - \theta_\lambda i, \tag{A.7}$$

where an overbar denotes its complex conjugate and i is the imaginary unit. Recall that the exponential form of any complex number is:

$$e^{(a \pm \theta_k i)t} = e^{at} \left[\cos \theta_k t \pm i \sin \theta_k t \right], \tag{A.8}$$

where we have used Euler's formula: $e^{i\theta_k t} = \cos \theta_k t + i \sin \theta_k t$. It follows that the sign of the *real part* of (A.8) (denoted by a) dictates stability.

For inelastic and moderately elastic labor supply, that is, $1 \leq \phi < \bar{\phi} \equiv 1/(\eta(1-\varepsilon_L))$,[31] it can be shown that the OLG system has a unique and saddle-path stable steady state. The determinant of the investment system is given by:

$$|\Delta_I| = -\delta_{12}\delta_{21} < 0, \tag{A.9}$$

and its trace $\text{tr}(\Delta_I) = \delta_{22} = r > 0$. Hence, it features a saddle-path equilibrium. The $\dot{\tilde{K}}(t) = 0$ locus is horizontal and the $\dot{\tilde{q}}(t) = 0$ locus is downward sloping in the $(\tilde{q}(t), \tilde{K}(t))$ space. For $\phi = \bar{\phi}$ we get $|\Delta_I| = 0$ and for $\phi > \bar{\phi}$ we find $|\Delta_I| > 0$. The savings system is always saddle-path stable:

$$|\Delta_S| = r(r-\alpha) \left[1 - \frac{\omega_C + \varepsilon_L(\phi-1)}{\omega_A} \right] < 0, \tag{A.10}$$

where $\text{tr}(\Delta_S) = 2r - \alpha > 0$.

A.3 Solving the Model

The Laplace transform method of Judd (1982) is used to solve the model. By taking the Laplace transform of (20), and noting that $\tilde{K}(0) = 0$ and $\tilde{A}(0) = \omega_A \tilde{q}(0)$, we obtain:

$$\Lambda(s) \begin{bmatrix} \mathcal{L}\{\tilde{K}, s\} \\ \mathcal{L}\{\tilde{q}, s\} \\ \mathcal{L}\{\tilde{C}, s\} \\ \mathcal{L}\{\tilde{A}, s\} \end{bmatrix} = \begin{bmatrix} 0 \\ \tilde{q}(0) \\ \tilde{C}(0) \\ \omega_A \tilde{q}(0) - \mathcal{L}\{\gamma_A, s\} \end{bmatrix}, \tag{A.11}$$

where $\Lambda(s) \equiv s\mathrm{I} - \Delta$, where I is the identity matrix. By pre-multiplying both sides of (A.11) by

$$\Lambda(s)^{-1} \equiv \frac{1}{(s-\nu)(s-\bar{\nu})(s-\lambda)(s-\bar{\lambda})} \text{adj}\, \Lambda(s), \tag{A.12}$$

---

[31]For the special case of exogenous intertemporal labor supply (i.e., $\phi = 1$ so that $\delta_{23} = 0$), the investment system decouples from the savings system so that it can be solved recursively.

and rearranging we obtain the following expression in Laplace transforms:

$$(s-\nu)(s-\bar{\nu})\begin{bmatrix} \mathcal{L}\{\tilde{K},s\} \\ \mathcal{L}\{\tilde{q},s\} \\ \mathcal{L}\{\tilde{C},s\} \\ \mathcal{L}\{\tilde{A},s\} \end{bmatrix} = \frac{\operatorname{adj}\Lambda(s)\begin{bmatrix} 0 \\ \tilde{q}(0) \\ \tilde{C}(0) \\ \omega_A\tilde{q}(0)-\mathcal{L}\{\gamma_A,s\} \end{bmatrix}}{(s-\lambda)(s-\bar{\lambda})}. \tag{A.13}$$

The adjoint matrix is equal to:

$$\operatorname{adj}\Lambda(s) \equiv \begin{bmatrix} (s-\delta_{22})\,\varphi\,(s) & \delta_{12}\varphi\,(s) & \delta_{12}\delta_{23}\,(s-\delta_{22}) & \delta_{12}\delta_{23}\delta_{34} \\ \delta_{21}\varphi\,(s)+\delta_{23}\delta_{34}\delta_{41} & s\varphi\,(s) & \delta_{23}s\,(s-\delta_{22}) & \delta_{23}\delta_{34}s \\ \delta_{34}\delta_{41}\,(s-\delta_{22}) & \delta_{12}\delta_{34}\delta_{41} & (s-\delta_{22})\,\psi\,(s) & \delta_{34}\psi\,(s) \\ \delta_{41}\,(s-\delta_{22})\,(s-\delta_{33}) & \delta_{12}\delta_{41}\,(s-\delta_{33}) & \delta_{43}\psi\,(s)+\delta_{12}\delta_{23}\delta_{41} & (s-\delta_{33})\,\psi\,(s) \end{bmatrix},$$

where $\psi\,(s)$ and $\varphi\,(s)$ are defined as:

$$\begin{aligned} \psi\,(s) &\equiv s\,(s-\delta_{22})-\delta_{12}\delta_{21}, \\ \varphi\,(s) &\equiv (s-\delta_{33})\,(s-\delta_{22})-\delta_{34}\delta_{43}, \end{aligned}$$

which represent the characteristic equations of the investment system and savings system, respectively.

A.4   Analytical Impulse Responses

This section derives analytical impulse response functions of fiscal shocks. The formulas presented, cover the case of stable complex roots, which is generated by the benchmark parameter values. We can easily show that the impact and long-run results are still valid even if the stable roots are real and distinct (the unstable roots can be complex or real). In that case, the transition terms differ from those under complex roots because the cyclical terms disappear. The details of the derivations can be found in Heijdra and Ligthart (2006).

A.4.1   Impact Effects

The jumps in $\tilde{C}(0)$ and $\tilde{q}(0)$ can be derived from (A.13). Because the rank of $\operatorname{adj}\Lambda(s)$ equals 1 (for $s=\lambda,\bar{\lambda}$) either row of the matrix can be used. Using the first row of $\operatorname{adj}\Lambda(s)$, for example, we get a system of two equations in $\tilde{C}(0)$ and $\tilde{q}(0)$, which can be solved to yield:

$$\begin{bmatrix} \tilde{q}(0) \\ \tilde{C}(0) \end{bmatrix} = \delta_{23}\delta_{34}\begin{bmatrix} \varphi\,(\lambda)+\delta_{23}\delta_{34}\omega_A & \delta_{23}\,(\lambda-\delta_{22}) \\ \varphi\,(\bar{\lambda})+\delta_{23}\delta_{34}\omega_A & \delta_{23}\,(\bar{\lambda}-\delta_{22}) \end{bmatrix}^{-1}\begin{bmatrix} \mathcal{L}\{\gamma_A,\lambda\} \\ \mathcal{L}\{\gamma_A,\bar{\lambda}\} \end{bmatrix}. \tag{A.14}$$

A.4.2 Transition Paths

We first study the investment system. The path for the capital stock is given in the main text (see (28)), which is derived by taking the inverse Laplace transform of the first row of (A.13). Similarly, we can derive the path for Tobin's q:

$$
\begin{aligned}
\tilde{q}(t) \;=\;& \tilde{q}(0)\,\mathbf{T}_2\left(h^*,\theta_\nu,t\right) \\
& + \left[\left(\lambda + \bar{\lambda} - \delta_{22} - \delta_{33}\right)\tilde{q}(0) + \delta_{23}\tilde{C}(0)\right]\mathbf{T}_1\left(h^*,\theta_\nu,t\right) \\
& + \frac{\delta_{23}\delta_{34}\xi_T r\omega_G\pi_1}{\left(\lambda+\xi_T\right)\left(\bar{\lambda}+\xi_T\right)}\mathbf{A}\left(h^*,\xi_T,\theta_\nu,t\right),
\end{aligned}
\tag{A.15}
$$

where the general transition term, $\mathbf{A}\left(h^*,\xi_T,\theta_\nu,t\right)$, and the first temporary transition term, $\mathbf{T}_1\left(h^*,\theta_\nu,t\right)$, are given in the main text and the second temporary transition term is defined as:

Definition 3 *The temporary transition term* $\mathbf{T}_2\left(h^*,\theta_\nu,t\right)$:

$$
\mathbf{T}_2\left(h^*,\theta_\nu,t\right) \equiv e^{-h^*t}\left[\cos\theta_\nu t - \frac{h^*}{\theta_\nu}\sin\theta_\nu t\right],
$$

which has properties: (i) $\mathbf{T}_2\left(h^*,\theta_\nu,0\right)=1$*; and (ii)* $\lim_{t\to\infty}\mathbf{T}_2\left(h^*,\theta_\nu,t\right)=0$,

We now turn to the savings system. The paths for private consumption and financial capital are:

$$
\begin{aligned}
\tilde{C}(t) \;=\;& \left[\delta_{34}\omega_A\tilde{q}(0) + \left(\lambda + \bar{\lambda} - 2\delta_{22}\right)\tilde{C}(0)\right]\mathbf{T}_1\left(h^*,\theta_\nu,t\right) + \\
& + \tilde{C}(0)\,\mathbf{T}_2\left(h^*,\theta_\nu,t\right) + \frac{\delta_{34}\delta_{12}\delta_{21}r\omega_G\pi_0}{\lambda\bar{\lambda}}\mathbf{A}\left(h^*,0,\theta_\nu,t\right) \\
& - \frac{\delta_{34}\left[\xi_T\left(\xi_T+\delta_{22}\right)-\delta_{12}\delta_{21}\right]r\omega_G\pi_1}{\left(\lambda+\xi_T\right)\left(\bar{\lambda}+\xi_T\right)}\mathbf{A}\left(h^*,\xi_T,\theta_\nu,t\right).
\end{aligned}
\tag{A.16}
$$

$$
\begin{aligned}
\tilde{A}(t) \;=\;& \left[\omega_A\left(\lambda+\bar{\lambda}-\delta_{22}-\delta_{33}\right)\tilde{q}(0) + \delta_{43}\tilde{C}(0) - r\omega_G\left(\pi_0+\pi_1\right)\right]\mathbf{T}_1\left(h^*,\theta_\nu,t\right) \\
& + \omega_A\tilde{q}(0)\,\mathbf{T}_2\left(h^*,\theta_\nu,t\right) - \frac{\delta_{33}\delta_{12}\delta_{21}r\omega_G\pi_0}{\lambda\bar{\lambda}}\mathbf{A}\left(h^*,0,\theta_\nu,t\right) \\
& - \frac{\left(\xi_T+\delta_{33}\right)\left[\delta_{12}\delta_{21}-\xi_T\left(\xi_T+\delta_{22}\right)\right]r\omega_G\pi_1}{\left(\lambda+\xi_T\right)\left(\bar{\lambda}+\xi_T\right)}\mathbf{A}\left(h^*,\xi_T,\theta_\nu,t\right).
\end{aligned}
\tag{A.17}
$$

Note that equations (A.1)-(A.3) can be used to derive the transition paths for $Y(t)$, $L(t)$, and $w(t)$. The paths for $B(t)$, $F(t)$, and $I(t)$, follow from (25), (AT1.12), and (AT1.8), respectively.

Appendix Table 1: The Loglinearized Model

$$\dot{\tilde{K}}(t) = \bar{y}\omega_I[\tilde{I}(t) - \tilde{K}(t)] \tag{AT1.1}$$

$$\dot{\tilde{q}}(t) = r\tilde{q}(t) - (1 - \varepsilon_L)\,\bar{y}\left(\tilde{Y}(t) - \tilde{K}(t)\right) \tag{AT1.2}$$

$$\dot{\tilde{C}}(t) = (r - \alpha)\left[\tilde{C}(t) - (1/\omega_A)\tilde{A}(t)\right] \tag{AT1.3}$$

$$\dot{\tilde{A}}(t) = r\left[\tilde{A}(t) + \varepsilon_L(\tilde{w}(t) + \tilde{L}(t)) - \omega_T\tilde{T}(t) - \omega_C\tilde{C}(t)\right] \tag{AT1.4}$$

$$\dot{\tilde{B}}(t) = r\left[\omega_G\tilde{G}(t) + \tilde{B}(t) - \omega_T\tilde{T}(t)\right] \tag{AT1.5}$$

$$\tilde{L}(t) = \tilde{Y}(t) - \tilde{w}(t) \tag{AT1.6}$$

$$\tilde{r}^K(t) = \tilde{Y}(t) - \tilde{K}(t) \tag{AT1.7}$$

$$\tilde{q}(t) = \sigma[\tilde{I}(t) - \tilde{K}(t)] \tag{AT1.8}$$

$$\tilde{L}(t) = \theta_L\left[\tilde{w}(t) - \tilde{C}(t)\right] \tag{AT1.9}$$

$$\tilde{Y}(t) = \eta\left[\varepsilon_L\tilde{L}(t) + (1 - \varepsilon_L)\tilde{K}(t)\right] \tag{AT1.10}$$

$$\left(\frac{\eta - 1}{\eta}\right)\tilde{Y}(t) = \varepsilon_L\tilde{w}(t) + (1 - \varepsilon_L)\tilde{r}^K(t) \tag{AT1.11}$$

$$\tilde{A}(t) = \omega_A(\tilde{q}(t) + \tilde{K}(t)) + \tilde{B}(t) + \tilde{F}(t) \tag{AT1.12}$$

Table 1: Multipliers of Key Macroeconomic Variables in the Benchmark Model

	RA Model		OLG Model					
	Lump-Sum Tax		Lump-Sum Tax		Public Debt			
					Temporary		Permanent	
	Temporary	Permanent	Temporary	Permanent	Moderate	Drastic	Moderate	Drastic
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\frac{dY(0)}{dG}$	0.0918	0.3214	0.1572	0.3600	0.0811	0.0050	0.2838	0.2077
$\frac{dY(\infty)}{dG}$	0.5050	1.7676	0	1.2963	0.5185	1.0370	1.8148	2.3333
$\frac{dC(0)}{dG}$	−0.0446	−0.1561	−0.0764	−0.1748	−0.0394	−0.0024	−0.1379	−0.1009
$\frac{dC(\infty)}{dG}$	−0.0446	−0.1561	0	−0.1145	−0.0458	−0.0916	−0.1603	−0.2060
$\frac{dI(0)}{dG}$	0.0792	0.2773	0.1180	0.3044	0.0746	0.0312	0.2610	0.2176
$\frac{dI(\infty)}{dG}$	0.0919	0.3215	0	0.2358	0.0943	0.1886	0.3301	0.4244
$\frac{dX(0)}{dG}$	−0.9428	−0.7998	−0.8844	−0.7696	−0.9541	−1.0238	−0.8393	−0.9090
$\frac{dX(\infty)}{dG}$	0.4578	0.6022	0	0.1750	0.4700	0.9400	0.6450	1.1150
$\frac{\tilde{L}(0)}{G}$	0.0219	0.0765	0.0374	0.0857	0.0193	0.0012	0.0676	0.0495
$\frac{\tilde{L}(\infty)}{G}$	0.0770	0.2694	0	0.1975	0.0790	0.1580	0.2765	0.3556
$\frac{\tilde{K}(0)}{G}$	0	0	0	0	0	0	0	0
$\frac{\tilde{K}(\infty)}{G}$	0.1010	0.3535	0	0.2593	0.1037	0.2074	0.3630	0.4667
$\frac{\tilde{w}(0)}{G}$	−0.0035	−0.0122	−0.0060	−0.0137	−0.0031	0.0002	−0.0108	−0.0079
$\frac{\tilde{w}(\infty)}{G}$	0.0240	−0.0842	0	0.0617	0.0247	0.0494	0.0864	0.1111

Notes: Permanent shock. Parameters are set at their benchmark values.
The RA model sets $r = \alpha$ whereas $r > \alpha$ holds in the OLG model.

Table 2: Sensitivity of Output Multipliers to Changes in θ_L and β ($r = \alpha$ case)

| | | Value of θ_L | | | | |
		0	0.5	1.0	2.0	4.0
$\beta = 0$	$\frac{dY(0)}{dG}$	0	0.3891	0.4420	0.3313	—
	$\frac{dY(\infty)}{dG}$	0	0.7728	1.1654	1.8220	—
	$\nu, \bar\nu$	$0, -0.1135$	$0, -0.0983$	$0, -0.0835$	$0, -0.0531$	us
	s_M	—	—	—	—	—
$\beta = 0.01$	$\frac{dY(0)}{dG}$	0	0.3912	0.4511	0.3639	0.0411
	$\frac{dY(\infty)}{dG}$	0	0.7447	1.0396	1.2963	1.4789
	$\nu, \bar\nu$	$-0.0100, -0.1135$	$-0.0120, -0.0976$	$-0.0146, -0.0817$	$-0.0338, -0.0368$	$-0.0068 \pm 0.0387i$
	s_M	—	—	—	—	0.0387
$\beta = 0.02$	$\frac{dY(0)}{dG}$	0	0.3924	0.4573	0.3851	0.1065
	$\frac{dY(\infty)}{dG}$	0	0.7447	1.0396	1.2963	1.4789
	$\nu, \bar\nu$	$-0.0200, -0.1135$	$-0.0239, -0.0965$	$-0.0294, -0.0784$	$-0.0421 \pm 0.0233i$	$-0.0190 \pm 0.0458i$
	s_M	—	—	—	0.0481	0.0496
$\beta = 0.05$	$\frac{dY(0)}{dG}$	0	0.3927	0.4664	0.4228	0.2120
	$\frac{dY(\infty)}{dG}$	0	0.7447	1.0396	1.2963	1.4789
	$\nu, \bar\nu$	$-0.0500, -0.1135$	$-0.0666, -0.0848$	$-0.0700 \pm 0.0257i$	$-0.0597 \pm 0.0427i$	$-0.0415 \pm 0.0583i$
	s_M	—	—	0.0746	0.0734	0.0715
$\beta = 0.10$	$\frac{dY(0)}{dG}$	0	0.3864	0.4639	0.4443	0.2921
	$\frac{dY(\infty)}{dG}$	0	0.7447	1.0396	1.2963	1.4789
	$\nu, \bar\nu$	$-0.1000, -0.1135$	$-0.1008 \pm 0.0323i$	$-0.0953 \pm 0.0432i$	$-0.0855 \pm 0.0559i$	$-0.0692 \pm 0.0692i$
	s_M	—	0.1059	0.1047	0.1021	0.0978
$\beta = 0.50$	$\frac{dY(0)}{dG}$	0	0.2595	0.2517	0.1997	0.1370
	$\frac{dY(\infty)}{dG}$	0	0.7447	1.0396	1.2963	1.4789
	$\nu, \bar\nu$	$-0.1135, -0.5000$	$-0.1186, -0.4704$	$-0.1215, -0.4465$	$-0.1259, -0.4067$	$-0.1340, -0.3443$
	s_M	—	—	—	—	—

Notes: A solution in which the roots have positive real parts is identified by 'us' and is unstable.
Results are for permanent fiscal spending shocks financed by lump-sum taxes. We have set $r = \alpha$ for all cases.

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Table 3: Sensitivity of Output Multipliers to Changes in θ_L and β ($r > \alpha$ in OLG case)

		Value of θ_L				
		0	0.5	1.0	2.0	4.0
$\beta = 0$	$\frac{dY(0)}{dG}$	0	0.3480	0.4123	0.3214	—
	$\frac{dY(\infty)}{dG}$	0	0.6959	1.0870	1.7676	—
	$\nu, \bar{\nu}$	$0, -0.1135$	$0, -0.0983$	$0, -0.0835$	$0, -0.0531$	us
	s_M	—	—	—	—	—
$\beta = 0.01$	$\frac{dY(0)}{dG}$	0	0.3393	0.4106	0.3459	0.0374
	$\frac{dY(\infty)}{dG}$	0	0.7447	1.0396	1.2963	1.4789
	$\nu, \bar{\nu}$	$-0.0077, -0.1135$	$-0.0102, -0.0977$	$-0.0130, -0.0819$	$-0.0281, -0.0413$	$-0.0061 \pm 0.0384i$
	s_M	—	—	—	—	0.0384
$\beta = 0.02$	$\frac{dY(0)}{dG}$	0	0.3306	0.4071	0.3600	0.1003
	$\frac{dY(\infty)}{dG}$	0	0.7447	1.0396	1.2963	1.4789
	$\nu, \bar{\nu}$	$-0.0148, -0.1135$	$-0.0198, -0.0969$	$-0.0255, -0.0794$	$-0.0410 \pm 0.0216i$	$-0.0180 \pm 0.0450i$
	s_M	—	—	—	0.0463	0.0485
$\beta = 0.05$	$\frac{dY(0)}{dG}$	0	0.3054	0.3905	0.3779	0.1959
	$\frac{dY(\infty)}{dG}$	0	0.7447	1.0396	1.2963	1.4789
	$\nu, \bar{\nu}$	$-0.0326, -0.1135$	$-0.0473, -0.0919$	$-0.0653 \pm 0.0190i$	$-0.0562 \pm 0.0405i$	$-0.0392 \pm 0.0576i$
	s_M	—	—	0.0680	0.0693	0.0697
$\beta = 0.10$	$\frac{dY(0)}{dG}$	0	0.3393	0.3545	0.3713	0.2582
	$\frac{dY(\infty)}{dG}$	0	0.7447	1.0396	1.2963	1.4789
	$\nu, \bar{\nu}$	$-0.0547, -0.1135$	$-0.0977, -0.0102$	$-0.0824 \pm 0.0388i$	$-0.0764 \pm 0.0541i$	$-0.0633 \pm 0.0688i$
	s_M	—	—	0.0911	0.0936	0.0935
$\beta = 0.50$	$\frac{dY(0)}{dG}$	0	0.1064	0.1312	0.1260	0.0904
	$\frac{dY(\infty)}{dG}$	0	0.7447	1.0396	1.2963	1.4789
	$\nu, \bar{\nu}$	$-0.1135, -0.1194$	$-0.1357 \pm 0.0350i$	$-0.1479 \pm 0.0422i$	$-0.1610 \pm 0.0490i$	$-0.1673 \pm 0.0603i$
	s_M	—	0.1401	0.1538	0.1683	0.1779

Notes: A solution in which the roots have positive real parts is identified by 'us' and is unstable.

All results are for permanent fiscal spending shocks financed by lump-sum taxes.

We have set $r = \alpha$ in the RA model and $r > \alpha$ in the OLG model.

Table 4: Sensitivity of Output Multipliers to Changes in θ_L and η

		Value of θ_L				
		0	0.5	1.0	2.0	4.0
$\eta = 1.0$	$\frac{dY(0)}{dG}$	0	0.2997	0.4063	0.4648	0.4506
	$\frac{dY(\infty)}{dG}$	0	0.5882	0.8333	1.0526	1.2121
	$\nu, \bar{\nu}$	$-0.0148, -0.1195$	$-0.0167, -0.1069$	$-0.0177, -0.1015$	$-0.0189, -0.0887$	$-0.0203, -0.0711$
	s_M	—	—	—	—	—
$\eta = 1.1$	$\frac{dY(0)}{dG}$	0	0.3171	0.4134	0.4325	0.3342
	$\frac{dY(\infty)}{dG}$	0	0.6644	0.9345	1.1729	1.3444
	$\nu, \bar{\nu}$	$-0.0148, -0.1165$	$-0.0180, -0.1037$	$-0.0207, -0.0919$	$-0.0269, -0.0697$	$-0.0356 \pm 0.0225i$
	s_M	—	—	—	—	0.0421
$\eta = 1.3$	$\frac{dY(0)}{dG}$	0	0.3386	0.3833	0.2269	—
	$\frac{dY(\infty)}{dG}$	0	0.8295	1.1490	1.4230	—
	$\nu, \bar{\nu}$	$-0.0148, -0.1104$	$-0.0221, -0.0890$	$-0.0374, -0.0583$	$-0.0305 \pm 0.0401i$	us
	s_M	—	—	—	0.0504	—
$\eta = 1.5$	$\frac{dY(0)}{dG}$	0	0.3335	0.2580	—	—
	$\frac{dY(\infty)}{dG}$	0	1.0145	1.3816	—	—
	$\nu, \bar{\nu}$	$-0.0148, -0.1040$	$-0.0317, -0.0663$	$-0.0350 \pm 0.0374i$	us	us
	s_M	—	—	0.0512	—	—
$\eta = 1.7$	$\frac{dY(0)}{dG}$	0	0.2885	-0.0590	—	—
	$\frac{dY(\infty)}{dG}$	0	1.2230	1.6346	—	—
	$\nu, \bar{\nu}$	$-0.0148, -0.0972$	$-0.0405 \pm 0.0254i$	$-0.0102 \pm 0.0589i$	us	us
	s_M	—	0.0478	0.0598	—	—

Notes: A solution in which the imaginary roots have positive real parts is identified by 'us' and is unstable.
All results are for permanent fiscal spending shocks financed by lump-sum taxes.
We have set $r = \alpha$ in the RA model and $r > \alpha$ in the OLG model.
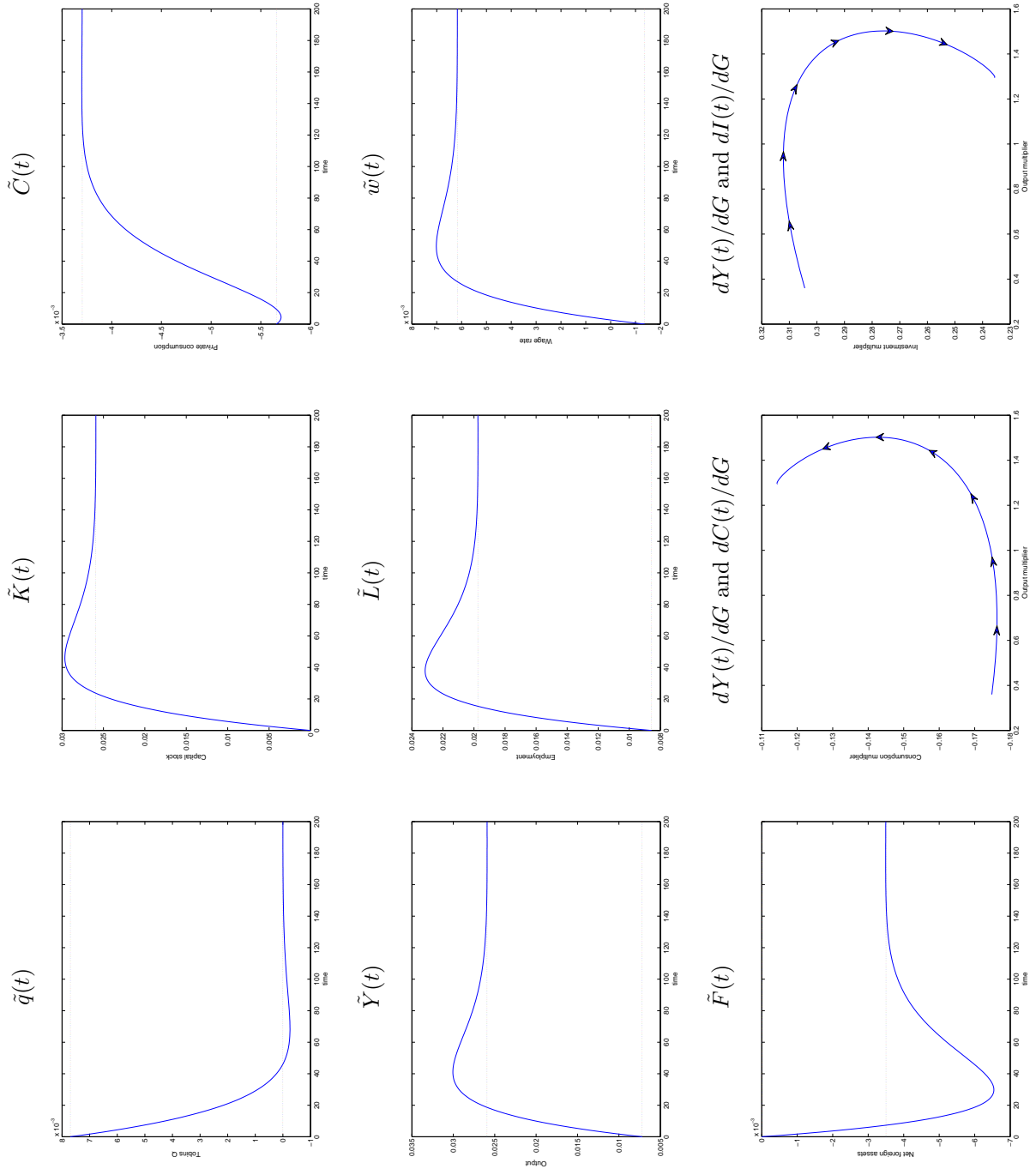
Figure 1: Permanent Spending Shock (Tax-Financed)
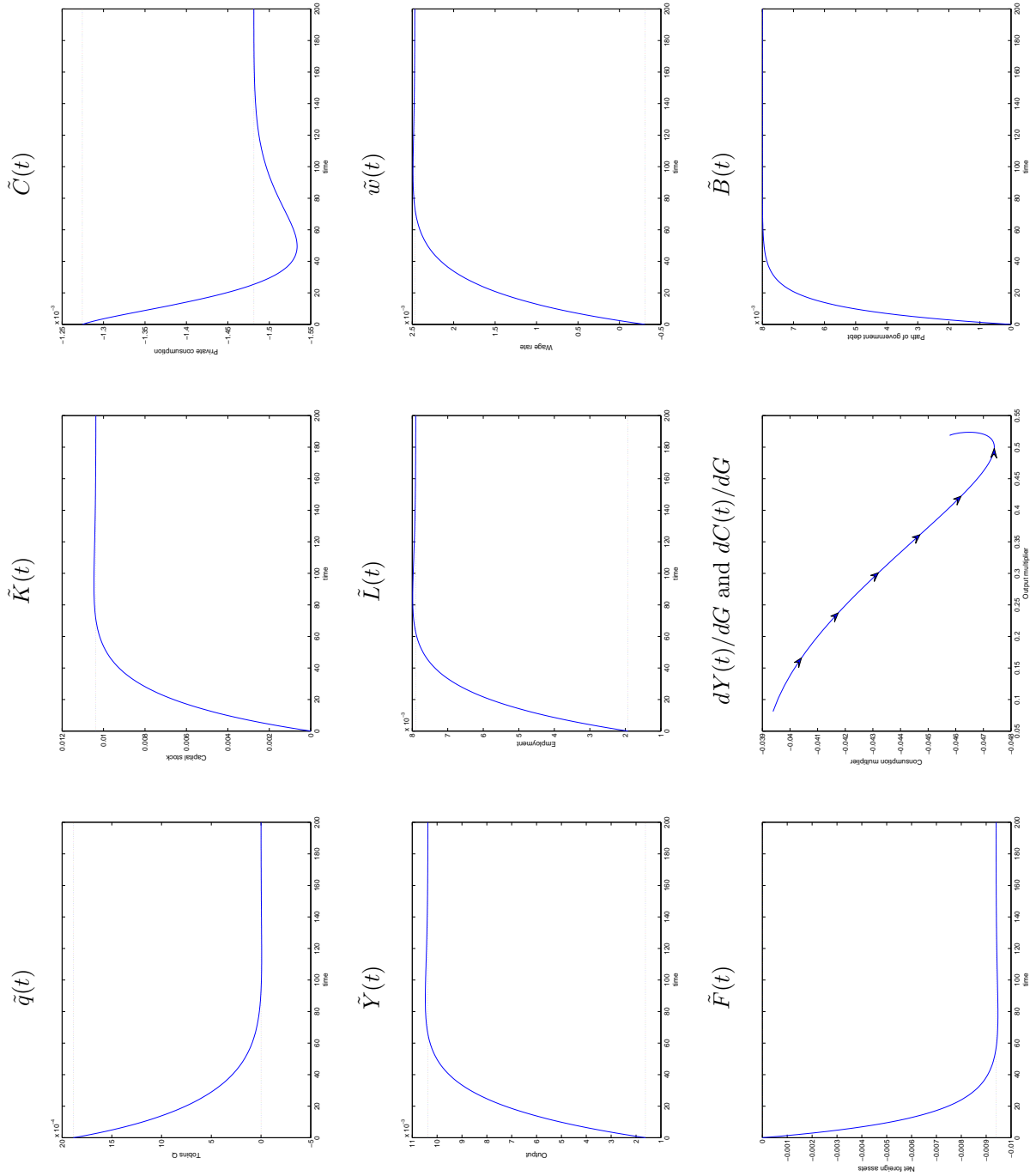
Figure 2: **Temporary Spending Shock (Bond-Financed)**

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