A Modeling Framework for Analyzing Retail Store Durations
Dekimpe, Marnik G.; Morrison, Donald G.
Journal of Retailing; Spring 1991; 67, 1; ABI/INFORM Global
pg. 68

Volume 67 Number 1 Spring 1991

A Modeling Framework for Analyzing Retail Store Durations

MARNIK G. DEKIMPE
and
DONALD G. MORRISON
University of California, Los Angeles

A rich data set on 4,600 franchised retail stores belonging to a single chain is analyzed. Some of these stores are still in operation, so that basic statistics such as the mean and/or median duration of the sample cannot be computed directly. A stochastic mixture model is developed that allows these important characteristics to be estimated. Moreover, our formal modeling approach allows us to make comparisons and draw managerial implications that could not have been done using conventional methods. Finally, we make a case for using percentiles (e.g., medians) of the formal distribution since the "fat-tailed" behavior of the typical duration distributions renders the usual moments, e.g. means and variances, either misleading or possessing the preposterous value of infinity.

INTRODUCTION

Now widely accepted is the notion that many retail firms, especially small stores, fail during their early years. Yet there is little hard information on the life expectancy of small retail firms. This may be due both to a lack of adequate data (Marcus 1967; Hemenway 1977) and to difficulties in modeling analytically life-time or duration phenomena (Morrison and Schmittlein 1980). Information on the life expectancies of retail stores, and perhaps more importantly, on the factors that affect these expectancies, is crucial in designing programs to decrease retailers' failure rates. Such

The authors acknowledge Sunil Gupta and David C. Schmittlein for making available the programs used in the estimation part. We also wish to thank the management of the retail chain for the provided information.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

knowledge could for example help prospective entrepreneurs to improve their chances of survival in the marketplace. Similarly, franchisors could use this knowledge to maximize the life expectancies of individual franchisees joining the system. Dickinson (1981), for example, suggests improved training of store managers and better initial selection procedures as two strategies a franchisor might adopt to reduce the failure rate among his franchisees. Although these suggestions are intuitively appealing, they may not be equally effective in all situations. It is difficult to prescribe effective procedures for improving the survival probabilities of a group of stores without a systematic knowledge of the determinants of their failure rates and/or life expectancies.

The purpose of this paper is to present a systematic procedure for modeling the life expectancies (or store durations) of individual retail stores. In general, we propose a stochastic model of store durations that considers how the expected failure rate of a given store changes over time, as well as how failure rates vary among stores. The hazard functions based on alternative mixture distributions are derived and empirically estimated on data from a convenience-store franchise. A nonparametric approach, that makes no assumption about the functional form of the distribution of life times, is also considered. In each case we discuss the underlying assumptions of the model and illustrate its empirical application. We estimate these models for the entire sample of stores operated by the franchise as well as for some subgroups of stores formed on the basis of their country of origin and their time of coming into the market. These data sets differ markedly in terms of the proportions of stores that have gone out of business. Our model will allow us to make "apple to apple" comparisons across these diverse data sets, even though the usual summary statistics such as the sample mean or median could not be computed.

CHARACTERISTICS OF DURATION-TYPE DATA

Research on the life expectancy of small retail stores has been impeded not only by the lack of adequate data. Formal modeling approaches are also made more difficult by the data structure normally found in duration problems.

The data at hand typically consist of a longitudinal record of when events happened to a sample of individuals, stores, etc. (Allison 1984). In the case at hand, a typical data set could consist of:

- the starting and (for those who have failed already) closing dates of a number of stores
- a dummy variable to indicate whether the store is still in operation

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

- the values of a number of covariates, which may be time-independent (e.g., the country of origin) or time-varying (e.g., yearly sales).

The data-gathering process may therefore be conceptualized as follows. Each time period an observation is made on each store to determine whether the store is still in existence. When it is still in operation, the values for the time-varying covariates are recorded (e.g., last year's sales figures). When the store fails, the closing date is recorded, the value of the corresponding dummy variable is changed, and no further observations are made on that store. The objective then is to model a store's propensity to fail over time and/or as a function of some of these covariates. Here, we propose a stochastic model of store durations to consider (1) how a given store's propensity to fail evolves over time, and (2) how these failure rates vary among stores. The impact of covariates is considered indirectly, as the proposed framework is applied not only to the total sample but also to some relevant subsamples. No covariates are included directly in the model, as the data set used in the empirical part of the paper contained only one time-invariant covariate (i.e., the country of origin) and no time-varying covariates.

Ideally, all stores in the sample would have the same starting date and would have a completed duration (i.e., a closing time would have been observed for every store). In most practical applications however, the data set will contain right-censored and/or non-cohort data.

Right-Censored Data

Right-censored data are present when some firms are still in the market at the end of the observation period. The presence of this kind of data makes it impossible to compute the mean life-time of the observed sample, or even (when more than half of them are still in operation) to assess their median life-time. In the data set at hand, for example, more than 80 percent of the stores located in France were still in operation at the end of the observation period. Ignoring stores that are still in operation, a practice noticed by one of the authors some years ago in an analysis of job-duration data, would not only result in the omission of most of the data but also in a strong *under*-estimate of the true survival rate of this group. A procedure to incorporate information on the surviving stores in the estimation of some relevant summary statistics will be outlined in the following sections. This will allow us to gain more accurate insights in the true failure behavior of our group of stores and to make more meaningful comparisons between diverse subgroups of stores.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

Non-cohort Data

Non-cohort data are due to the fact that not all stores in operation at a certain point in time have come into business simultaneously. In the empirical example discussed in a later section, the starting dates ranged from July 1957 to December 1987, which is also the end of the observation period. The proposed modeling framework makes it possible to cope with this phenomenon in such a way that information on stores who have only very recently entered the market is also taken into account.

THE CONCEPT OF INERTIA

The Hazard Function

As the presence of right-censored and non-cohort data makes the computation of some commonly used statistics such as the mean and/or median difficult or impossible, one may resort to the derivation of the hazard function to summarize the quitting behavior of a group of stores. The latter function, also sometimes referred to as the empirical quitting rate,[1] can be defined as the ratio of the number of stores discontinuing business in a certain duration interval (t_1,t_2) to the number of stores having attained a duration of t_1. A decreasing quitting rate can then be interpreted as indicating that the longer a store has been open, the less likely it is to close down in the next month or year, a phenomenon also called *inertia*. Increasing quitting rates on the other hand imply that the longer stores have been in business, the more likely they are to discontinue their operation in the coming period, reflecting a negative inertia.

Empirical quitting rates can be computed both from the data reported by Star and Massel (1981), who examined the survival rates of a number of retail stores started in 1974, and from the estimated discontinuance percentages of the Department of Commerce (Dickinson 1981), and are presented in Table 1. Both types of hazard functions, i.e. first increasing and then decreasing on the one hand and monotonically decreasing on the other hand, are observed in the literature (see Schmittlein and Morrison 1983).

[1] The term "quitting rate" is somewhat more general than "failure rate," as a store can finish its operations for a variety of reasons (inability to pay creditors, retirement of the store owner, and so forth). In the current paper no distinction is made between the different causes for discontinuing one's operations, and the terms failure rate and quitting rate will be used interchangeably.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

TABLE 1

Estimates of the Aggregate Quitting Rate from Published Data

Starr and Massel (1981)-data	
year	estimated aggregate quitting rate
1	0.186
2	0.256
3	0.195
4	0.170
5	0.131
Department of Commerce estimates[a]	
year	estimated aggregate quitting rate
1	0.333
2	0.269
3	0.143
4	0.120
5	0.108

[a] derived from figures in Dickinson (1981)

It will be pointed out in the following sections that very different assumptions can produce identical aggregate hazard functions.

"Real" Versus "Spurious" Inertia

The quitting rates given in Table 1 are aggregate figures. Observed inertia effects in the aggregate data are due both to the quitting probabilities at the individual store level and to the fact that one is aggregating over a population with differing mean-duration times. Decreasing quitting rates can thus be "real," i.e., a characteristic of the individual stores, or can be "spuriously" caused by the aggregation over stores with different (mean) quitting rates. The latter case shows that even when each individual store has a constant probability of closing down in each period, the aggregate quitting rate would still display inertia.

A Hypothetical Example

Consider for example two *distinct* groups of stores. Each group has an observed (aggregate) quitting rate that is decreasing over time. For the first group however, the quitting rate of each *individual* store is assumed to be constant, implying that all observed inertia is due to the aggregation effect. Stores of the second type on the other hand are assumed to come from a

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

homogeneous population where all stores are characterized by the same mean quitting rate, and all individual quitting probabilities are assumed to be decreasing over time. All inertia effects in the latter case are thus real.

A franchisor owning the second group of stores should *not* focus on improved selection procedures to enhance the survival chances of his stores, as there is no heterogeneity in the initial failure probabilities that could be reduced by such a policy. The evolution of the individual quitting probabilities on the other hand can be interpreted as an indication of the amount of learning that is going on. The hazard function of stores in the first group is independent of the time they are already in operation. No experience effect is therefore observed in this group. The decreasing individual quitting probabilities in the second group, on the other hand, indicate the presence of a positive learning effect on the part of those store managers.

The previous discussion described only two possible situations. A more detailed discussion of the policy implications of these and other scenarios will be given after a description of the methodology. The empirical results will then be discussed in some detail and, finally, some areas for future research will be indicated.

A STOCHASTIC MODEL OF STORE DURATIONS

Two different assumptions with respect to the individual store's quitting possibilities will be examined.

Time-Independent Quitting

In a first model, the individual store's quitting rate is assumed to remain constant over time. This implies that the store has an equal probability of closing down in the next period, irrespective of how long it has already been open. The continuous distribution associated with a constant quitting rate is the exponential one, so that T, the random variable denoting the store duration time, has a probability density function (p.d.f.), cumulative distribution function (c.d.f.), and hazard function (u) that are given respectively by

$$f(t;\lambda) = \lambda e^{-\lambda t} \qquad t > 0;\ \lambda > 0 \qquad (1)$$

$$F(t;\lambda) = 1 - e^{-\lambda t} \qquad t > 0 \qquad (2)$$

$$u(t;\lambda) = \lambda \qquad t > 0 \qquad (3)$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

Time-Dependent Quitting

A more general model allows the quitting rate to vary as a power of t, so that it can be written as[2]

$$u(t;\lambda,b) = \lambda b t^{b-1} \qquad t > 0 \tag{4}$$

"b", which can be interpreted as a shape parameter,[3] can be seen as an index of the inertia at the individual level. The individual store's quitting rate increases for b greater than one and decreases for b smaller than one. The former (latter) case implies that the longer a store has been open, the more (less) likely it is to finish its operations in the next period of time. If b equals one, (4) obviously reduces to (3) and a b-value of two implies a linearly increasing individual quitting rate. The inertia parameter can also be interpreted as being an indicator of the amount of *learning* that is currently occurring by the individual store managers. A b-value of one implies that no learning or experience effect can be observed, as the quitting probability in a certain period is found to be independent of the accumulated experience level. Stores characterized by a b-value smaller than one on the other hand are showing an experience effect, and their failure probability decreases with time. An interesting extension of the current research could therefore be to relate the magnitude of the obtained learning-parameter to covariates as the type or amount of training received. In case groups of stores trained by program A systematically have a smaller b-value than groups trained by program B, evidence for the higher efficiency of the former is provided. Finally, stores having a b-value greater than one are displaying a negative learning effect, since the more experience they have accumulated, the more likely they are to quit. This might indicate a discontentedness with some aspects of the franchise agreement. Some modifications in this agreement or improved feedback procedures might therefore be appropriate in this case.

Modeling Between-Store Differences

Although each individual store is assumed to have a store duration that is either exponentially or Weibull-distributed, differences among their mean duration times are virtually certain to exist. This heterogeneity can be modeled by allowing the scaling parameter λ to vary across the different

[2] The distribution with a hazard function defined as in (4) is the Weibull distribution whose p.d.f. and c.d.f. are given in the Appendix, Section A.1.

[3] λ, on the other hand, will in what follows be denoted as the scaling parameter of the distribution.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

stores according to a certain distribution. The aggregate distribution of store durations, which is also the one actually observed, is then obtained by weighting each $f(t;\lambda)$ or $f(t;\lambda,b)$ by the likelihood of that particular λ-value. This is illustrated in (5) for the case of exponentially distributed individual store-duration times.

$$h(t) = \int_0^\infty f(t,\lambda)g(\lambda)d\lambda \qquad (5)$$

In what follows, h and g will often be called, respectively, the observable mixture and unobservable mixing distributions. A convenient and flexible mixing distribution is the gamma distribution, whose p.d.f. is given by[4]

$$g(\lambda;r,a) = \frac{a}{\Gamma(r)}(a\lambda)^{r-1}e^{-a\lambda} \qquad \lambda > 0;\ r,a > 0 \qquad (6)$$

as it can take on a variety of J-shapes when r is less than one and unimodal shapes when r is greater than one.

As the mean and the variance of the gamma distribution are given respectively by

$$E(\lambda) = r/a$$
$$Var(\lambda) = r/a^2$$

the coefficient of variation CV, which can be interpreted as measuring the degree of heterogeneity of λ across all stores, reduces to

$$CV = SD(\lambda)/E(\lambda) = r^{-1/2} \qquad (7)$$

giving the r-parameter a straightforward interpretation. A very high r-value indicates, then, a large degree of homogeneity in the mean quitting rates. The appropriateness of using improved selection procedures to enhance the survival chances of such a group of stores can be questioned, as there is no (or little) diversity in the initial failure probabilities that could eventually be reduced by such a policy. The r-parameter is thus an indicator of the potential usefulness of improved screening procedures in reducing the quitting rate of a group of stores.

A deficiency of using the gamma mixing distribution is that it cannot take on bimodal forms. This situation occurs when quite a number of stores have a high quitting rate and many other stores are having a low rate, whereas few of them are characterized by a medium rate. One way to

[4] In this formula $\Gamma(\ )$ stands for the gamma function.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

model such a situation was proposed by Morrison and Schmittlein (1980) and involves the estimation of a two-point mixture of exponentials. This type of model will not be developed any further in this paper however, leaving the problem of modeling store durations in cases where a bimodal mixing distribution would be more appropriate as an issue for further research.

In line with the previous distinction between exponentially and Weibull-distributed individual duration times, two situations can be distinguished.

Mixtures of Exponentials

If exponentially distributed individual store duration times are mixed by a gamma distribution $g(\lambda;r,a)$, a mixture distribution known as the Pareto distribution[5] is obtained, whose hazard function is given by

$$u(t;r,a) = \frac{r}{a + t} \qquad t > 0 \tag{8}$$

It is obvious that in this case the *aggregate* hazard function u is a monotonically decreasing function of t, the time the store is already open. As the individual quitting rate of each single store is assumed to be constant in this model, this observed inertia at the aggregate level is solely due to the fact that one is aggregating across a heterogeneous population.

Mixtures of Weibulls

Considering the more general case where Weibull-distributed individual durations are mixed by a gamma distribution, a mixture distribution known as the Burr distribution is obtained, where the hazard function is given by

$$u(t;r,a,b) = \frac{rbt^{b-1}}{a + t^b} \qquad t > 0 \tag{9}$$

Two different situations arise, depending on whether b is greater or less than one. If b is less than one, each individual store displays a decreasing quitting rate, and consequently the aggregate rate will show the same pattern, as both forces (the individual-level inertia and the mixing across a heterogeneous population) are working in the same direction. If b is greater than one, each individual store's quitting rate increases over time. As is

[5] The reader is referred to the Appendix (Sections A.2 and A.3) for the expressions of the p.d.f. and c.d.f. of, respectively, the Pareto and Burr distributions.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

derived in Morrison and Schmittlein (1980), the aggregate quitting rate then increases monotonically up to

$$t^* = [a(b - 1)]^{1/b} \tag{10}$$

after which it decreases monotonically. Inertia will then be displayed at the aggregate level after t^*, although not a single store is showing inertia at the individual level. An intuitively appealing Bayesian interpretation of this phenomenon is given by the same authors, stating that for small values of t little information has been collected on the mean duration of an observed store. All individual stores are characterized by an increasing quitting rate, and little additional information on the observed store is available. Thus the aggregate rate is also seen to increase. Having observed stores for a duration $t > t^*$ without having seen them go out of the market, the belief that we are observing a store with a long mean duration becomes stronger than the knowledge that each store is in fact showing an increasing quitting rate, resulting in a decreasing aggregate hazard function.

A Nonparametric Approach

A nonparametric approach also will be considered. As opposed to the method described in the previous sections, no assumptions will be made about the functional form of the underlying duration distribution. Denoting by $\hat{u}(t_{i-1}, t_i)$ the empirical quitting rate in the period (t_{i-1}, t_i), which is the number of stores closing down in that duration interval after having been in operation for at least t_{i-1} time units, an estimate of the observable mixture distribution function $\hat{H}(t_i)$ can be obtained from

$$1 - \hat{H}(t_i) = \prod_{j=1}^{i} [1 - \hat{u}(t_{j-1}, t_j)]. \tag{11}$$

Each factor at the right-hand side indicates the empirical probability that a store will *not* close down in the period $(j - 1, j)$, *given* that this has not yet happened in a previous period. The product of all these factors results then in an estimate of the empirical aggregate survival function.[6] In what follows a smoothed version of (11) will be used to obtain nonparametric

[6] When the t_i are chosen in such a way that they coincide with the observed closing times of the individual stores in the sample, the nonparametric estimate $\hat{H}(t_i)$ is often referred to as the Kaplan-Meier estimate of the cumulative distribution function (Kaplan and Meier 1958).

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

estimates of a number of relevant quantiles, so that a comparison with the corresponding parametric estimates will be possible. The reader is referred to Schmittlein and Morrison (1983) for a detailed discussion on the implementation of this smoothing procedure.

EMPIRICAL RESULTS

Description of the Data

The duration of over 4,600 retail stores belonging to a large convenience chain with locations geographically dispersed all over the world will be analyzed. The data set consists of the opening dates of those 4,600 stores, and, for those who have finished their operation, the closing date. A store duration will be defined as the number of months elapsed between the opening date and either the closing date (for the non-censored observations) or the end of the observation period (for the censored observations). No distinction is made in this analysis between the different possible causes of a store's discontinuance.

We are confronted with non-cohort data, as stores have started their operation at different times between July 1957 and December 1987. A lot of data are also right-censored, since by the end of the observation period (December 1987) 75 percent of the stores have not yet closed down. The complete data set will be analyzed, as well as subgroups of stores formed on the basis of:

- country of origin
- time of coming into the market.

Analysis of the Total Sample

The observed hazard function, as well as the estimated parameters and some summary statistics for the respective models are given in Table 2 (refer to the Appendix, Section B for details on the estimation procedure).

The Empirical Quitting Rate

The empirical quitting rate is reported on a per-year basis for successive periods of one year. Each figure is computed on at least ten closings in order to obtain a greater stability of the estimates. As a consequence, some quitting rates had to be calculated over more than one year, and those cases

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

TABLE 2

The Empirical Hazard Function, Parameter Estimates and Summary Statistics: Total Sample and According to the Country of Origin

Time (years)	Total Sample	Benelux	Germany	France	Japan
0–1	0.0393	0.0277	0.0214	0.0279	
					{0.0200}
1–2	0.0472	0.0280	0.0328	0.0469	
2–3	0.0330	0.0431	0.0248	0.0265	0.0308
3–4	0.0286	0.0274		0.0214	
			{0.0097}		{0.0128}
4–5	0.0185				
		{0.0174}		{0.0120}	
5–6	0.0207		0.0148		
					{0.0133}
6–7	0.0204				
				{0.0120}	
7–8	0.0158				
		{0.0096}	{0.0043}		{0.0206}
8–9	0.0184				
9–10	0.0226				
				{0.0121}	
10–11	0.0194				
11–12	0.0192				
		{0.0145}	{0.0074}	{0.0140}	{0.0143}
12–13	0.0218				
13–14	0.0134				—
14–15	0.0224				—
			{0.0128}		
15–16	0.0157				—
		{0.0247}		{0.0012}	—
16–17	0.0269				—
17–18					—
	{0.0166}				
18–19				—	—
			{0.0078}		
19–20	0.0276			—	—
20–21	0.0461		—	—	—
		{0.0203}			
21–22	0.0388		—	—	—

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

TABLE 2 Cont'd

Time (years)	Total Sample	Benelux	Germany	France	Japan
22–23	0.0480		—	—	—
23–24			—	—	—
. . .					
N^a	4646	505	1028	779	601
Estimated quantities: Gamma mixture of Weibulls					
r	0.394	0.475	0.704	0.0050	0.146
a	138.096	390.055	239.507	293.930	307.251
b	1.073	1.280	0.853	2.145	1.434
t^* (yrs.)	0.72	3.26	0	1.26	2.53
Mean (yrs.)	∞	∞	∞	∞	∞
Median (yrs.)	35.43	22.36	93.83	738.80	123.65
10 percent q. (yrs.)	2.73	2.97	75.59	2.96	4.70
20 percent q. (yrs.)	6.38	5.91	193.725	9.39	11.07
Estimated quantities: Gamma mixture of exponentials					
r	0.557	3.037	0.214	0.232	1.448
a	158.739	995.499	105.663	73.786	798.394
Mean (yrs.)	∞	40.72	∞	∞	148.39
Median (yrs.)	32.72	21.27	216.97	115.54	40.84
10 percent q. (yrs.)	2.75	2.93	67.165	3.53	5.02
20 percent q. (yrs.)	6.52	6.32	194.097	9.94	11.09
Likelihood ratio statistic					
χ^2	1.172	1.785	0.414	11.329**	2.728
Nonparametric estimates					
10 percent q. (yrs.)	2.33	3.18	64.17	3.15	5.59
20 percent q. (yrs.)	7.42	n.a.	n.a.	n.a.	n.a.

[a] *N* stands for the number of observations in the sample, whereas all other symbols are as defined in the previous sections.

** $p < 0.01$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

are in the following tables indicated by pairs of braces. To illustrate the interpretation of the empirical hazard rate, the 0.0472 in the second column of Table 2 indicates that 4.72 percent of all stores that were still open after one year closed down in the next year.

A direct comparison between the hazard function corresponding with the estimated parameter values and the empirical quitting rate is difficult, as the latter is dependent on the window size, i.e. the length of the intervals used in the computations, which in our case was one year (Watson and Leadbetter 1964). The empirical hazard function is helpful in comparing:

- the forecasted time of maximal hazard, which is given by equation (10), with the point where the empirical hazard function reaches its maximum value
- nonparametric estimates of some relevant percentiles with their parametric counterparts.

The forecasted (zero years) and observed (within the second year) time of maximum hazard rate are lying in adjacent intervals. The usefulness of this comparison is however moderated both by the fact that a rather broad window size had to be used in the calculations and by the inherent instability in the estimation of the empirical hazard rate as illustrated by the saw-tooth pattern of the figures in Table 2. The latter phenomenon is due to the fact that the number of stores at risk in a certain period (which constitutes the denominator in the computation of the hazard rate) diminishes in successive periods.[7]

Better insights can be obtained by comparing the nonparametric estimates of the ten and twenty percentiles (2.33 and 7.42 years) with the ones resulting from the Pareto-distribution[8] (2.75 and 6.52). The latter estimates are more efficient when the underlying model approximates closely reality, whereas the former offer the advantage of a greater generalizability when that condition is not met. The obtained results in all analyses were however very similar, increasing our confidence in the appropriateness of the adopted modeling assumptions.

---

[7] This can be clarified by considering the extreme case where in a certain period the last (and only remaining) store stops its operation. An empirical quitting rate of one would then be observed, although in the previous periods the ''real'' quitting rate will probably have been higher.

[8] The Kaplan-Meier estimates are compared with the estimates resulting from the fitted Pareto distribution, as the likelihood ratio test indicated that the estimated value of b was not significantly different from one. In cases where a significant difference is found, the estimates resulting from the Burr-distribution will be used as comparison base.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

Policy Implications of the Obtained Parameter Values

The parameter values obtained for r and b respectively indicate:

- the degree of diversity in the mean duration of the stores included in the sample
- the degree of learning or experience on the part of the individual store managers.

The coefficient of variation (CV) of the unobservable gamma mixing distribution, $r^{-1/2}$, is a measure of the heterogeneity in the population. The low r-value (0.394), which gives a high CV (= 1.533) implies a great diversity in the mean duration times of the stores in our data set. Improved initial selection procedures might therefore be a highly efficient way to enhance the survival chances of this chain's stores.

The estimated b-value of 1.07 in the mixture of Weibulls is very close to one, and the likelihood ratio test also indicates no significant difference between this model and the gamma-mixture of exponentials where the b-value is one. This implies that at the individual store-level a constant quitting rate is observed. All observed inertia effects will therefore be due to an aggregation across a heterogeneous population. Another implication of this result is that as yet no learning is occurring on the part of the individual store managers, which may question the appropriateness of the currently used training programs.

The obtained parameter estimates result in an estimated mean duration of infinity, making it useless both as a summary statistic and as a means of comparing the quitting behavior across groups. This phenomenon, which is due to the skewed and fat-tailed nature of the underlying distribution, occurs frequently in analyzing duration data. We therefore propose to compute the median or other quartiles of the distribution. These are not sensitive to the tail of the distribution, and are better suited to make meaningful comparisons across different groups of stores. The subsequent sections will therefore focus on the percentiles rather than on the moments of the distribution.

Analysis by Country

In Tables 2 and 3 the estimates are given for a grouping of the stores according to the country where they are located. Surprising findings were:

- some of the groupings are better modeled by a homogeneous distribution

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

TABLE 3

The Empirical Hazard Function, Parameter Estimates and Summary Statistics: According to the Country of Origin

Time (years)	United Kingdom and Ireland	Australia
0–1	0.0625	
		{0.0271}
1–2	0.0983	
2–3	0.0906	
		{0.0334}
3–4	0.0773	
4–5	0.0755	
		{0.0275}
5–6	0.0626	
6–7	0.0958	0.0612
7–8		
	{0.0457}	{0.0366}
8–9		
9–10	0.0129	—
10–11		—
	{0.0701}	
11–12		—
12–13		—
	{0.0568}	
13–14		—
14–15		—
15–16		—
	{0.0797}	
16–17		—
17–18		—
18–19		—
	{0.1081}	
19–20		—
21–22		—
22–23		—
	{0.1273}	
23–24		—
. . .	—	
N	432	277

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

TABLE 3 Cont'd

	United Kingdom and Ireland	Australia
Estimated quantities: homogeneous Weibull		
λ	0.00407	0.000991
b	1.129	1.230
Median (yrs.)	7.89	17.15
10 percent q. (yrs.)	1.49	3.71
20 percent q. (yrs.)	2.89	6.82
Estimated quantities: homogeneous exponential		
λ	0.00753	0.003065
Median (yrs.)	7.67	18.85
10 percent q. (yrs.)	1.17	2.86
20 percent q. (yrs.)	2.46	6.07
Likelihood ratio statistic		
χ^2	5.59*	4.93*
Nonparametric estimates		
10 percent q. (yrs.)	1.40	3.59
20 percent q. (yrs.)	2.59	n.a.

* $p < 0.05$

- some of the estimated medians are quite different despite similar initial quitting rates.

For stores located in either the United Kingdom or Ireland or in Australia, no convergence was obtained by the computer program, since the value of r increased without bound. As $r^{-1/2}$ is a measure of the homogeneity of the scale parameter λ, this would indicate that the program was actually trying to fit a homogeneous Weibull or exponential distribution. The parameters of these distributions were therefore estimated, and a very good correspondence was obtained between the resulting estimates of the ten and twenty percentiles and their nonparametric counterparts. The absence of hetero-

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

geneity in the mean quitting rates makes the use of improved initial selection procedures inappropriate as a policy to reduce the failure rates of stores located in those countries. The increasing individual-level quitting probabilities suggest however the desirability of improving some aspects in the channel relationships, as a negative learning effect is observed with those store managers.

Comparing further the Benelux Countries (Belgium, Luxembourg, and The Netherlands) with France, we note that, although their initial quitting rates are of the same order of magnitude, the estimated median duration for stores located in France is much higher than for the ones in the Benelux, a phenomenon attributable to the much larger heterogeneity in the mean life times of stores located in France.

Analysis According to the Year of Entry in the Market

Table 4 contains the results from dividing the sample in two groups according to their date of entry into the market. December 31, 1974 was selected as the dividing point, as this date coincided (approximately) with the first half of the chain's existence.

Comparing these two groups of stores, the following observations can be made:

- each of them is best described through a homogeneous distribution, although their respective parameters are very different
- the group of "early" stores is characterized by a much larger median duration
- one only sees a positive learning effect for the "late" stores.

Both subsamples are, for similar reasons as the ones given before, best described through a homogeneous Weibull distribution. This result might seem surprising: the overall sample was showing a lot of heterogeneity, whereas all early stores and all late stores are best described by a homogeneous distribution. Although the scaling parameter λ is very homogeneous within each subgroup, one notes that its value is much greater for stores opened after 1975 than for stores that started their operation before that date (0.00564 versus 0.000037, or a ratio of 152 to one). The corresponding medians are therefore also quite different, namely 14.34 versus 46.43 years. The total sample thus consists of two homogeneous but very distinct subsamples, which explains the heterogeneity observed when analyzing all stores as one group. This illustrates the usefulness of further analyses for separate subgroups, since better insights into the nature of the

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

TABLE 4

The Empirical Hazard Function, Parameter Estimates and Summary Statistics: According to the Year of Entry

Time (years)	openings year <75	openings year ≥75
0–1	0	0.0533
1–2	0	0.0667
2–3	0	0.0554
3–4	0	0.0454
4–5		0.0305
5–6		0.0368
	{0.0029}	
6–7		0.0374
7–8		0.0294
8–9	0.0083	0.0340
9–10	0.0174	0.0324
10–11	0.0144	0.0324
11–12	0.0188	
		{0.0308}
12–13	0.0218	
13–14	0.0134	—
14–15	0.0224	—
15–16	0.0157	—
16–17	0.0269	—
17–18	0.0132	—
18–19	0.0230	—
19–20	0.0276	—
21–22	0.0461	—
22–23	0.0391	—
23–24	0.0480	—
24–25		—
25–26		—
	{0.0167}	—
26–27		—
. . .		
N	1229	3417

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

TABLE 4 Cont'd

	openings year <75	openings year ≥75
Estimated quantities: homogeneous Weibull		
λ	0.000037	0.00563
b	1.556	0.935
Median (yrs.)	46.43	14.34
10 percent q. (yrs.)	13.83	1.91
20 percent q. (yrs.)	22.41	4.27
Estimated quantities: homogeneous exponential		
λ	0.001016	0.00426
Median (yrs.)	56.85	13.56
10 percent q. (yrs.)	8.64	2.06
20 percent q. (yrs.)	18.30	4.37
Likelihood ratio statistic		
χ^2	129.06**	5.34*
Nonparametric estimates		
10 percent q. (yrs.)	13.21	1.74
20 percent q. (yrs.)	19.18	4.59

* $p < 0.05$
** $p < 0.01$

underlying heterogeneity can be obtained in this way. The presence of two homogeneous subgroups further indicates that a bimodal mixing distribution might have been better in capturing the diversity in mean duration times. The use of mixing distributions other than the gamma distribution (which cannot take on bimodal forms) is however left as an area for future research.

We already noted that the estimated median for the early stores (46.43 years) is higher than the one estimated for the later ones (14.34 years). One should however exert some care in interpreting this last result. The figure for the early stores is estimated more from the tail of the distribution. The late stores on the other hand will be situated more in the front of the distribution. The above result should therefore not be too surprising.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

Finally, stores opened after December 1974 display an inertia at the individual level, as the estimated *b*-value for this sample is significantly lower than one. The early stores on the other hand are characterized by the fact that the longer they have been open, the more likely they are to stop their operation in the next period. A possible interpretation of this finding is that the support given to the individual store managers has improved over the years.

CONCLUSION

A general model for the analysis of store duration times has been presented. In contrast to previous analyses we took explicit account of problems caused by the presence of non-cohort and right-censored data in the sample. The proposed model also allowed us to distinguish between inertia effects due either to decreasing quitting rates at the individual store level and/or to the fact that one is aggregating across a heterogeneous population when one is observing empirical quitting rates.

The parameters of the models were shown to have a clear behavioral interpretation and direct policy implications. Alternative strategies to reduce the failure rates were conjectured to be more or less effective in different situations, depending on the parameter values of the model.

This model was applied to a data set consisting of store durations of stores belonging to one large chain, both at the total sample level and for some subsamples. The latter type of analysis indicated that not all store segments should be approached with a similar kind of strategy in order to reduce their failure rates.

Several areas for future research remain open however, as

- an explicit consideration of alternative causes for a store's discontinuance
- the incorporation of explanatory variables.

It was indicated before how the fat-tailed nature of most duration distributions leads to an estimated mean duration of infinity. Stores obviously do not stay in operation for an infinite time period. Their discontinuance can be due to a variety of reasons: inadequate profits, ill health, retirement or death of the store owner, and so on (Dickinson 1981). A more complete model would be a competing-risk model where the different underlying processes that can lead to the discontinuance of a store's operations are considered simultaneously. A duration t is then defined as

$$T = \min\{T_1, T_2, \ldots\} \tag{12}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

where

T_1 = the time till the business stops its operation due to inadequate profits, provided no other reason causes its discontinuance
T_2 = the time till the store owner retires, provided . . .

Vilcassim and Jain (1991) adopted a competing-risk framework in their analysis of individual households' purchase timing and brand choice decisions. A Markovian-type framework was adopted where the different possible outcomes (i.e., the different brands chosen) defined the event space, and where the transition probabilities were modeled in terms of hazard functions. This approach was not used in this paper for the following reasons: (1) the data set at hand did not distinguish between the alternative causes of a store's discontinuance and (2) the added complexity would not alter the spirit of our empirical results. Indeed, a competing-risk model would affect the tail of the distribution (and therefore solve the infinite mean problem), but the percentiles in the important part of the distribution would not be altered. We therefore decided to use the simpler models and ignore the estimated mean of the distribution.

The more fruitful area for future research is the incorporation of explanatory variables, so that the managerial implications would become more specific. Helsen and Schmittlein (1989) estimated a Weibull-hazard model with time-varying covariates to investigate the impact of marketing-mix variables on the interpurchase times of saltine crackers. They did not account however for the possible confounding effects of unobserved heterogeneity, which could have biased their results (Gupta 1991). Jain and Vilcassim (1991) on the other hand considered both unobserved heterogeneity and covariates in a hazard-rate model. In their framework the hazard rate is defined as

$$h(t) = h_0(t)exp(aX_t + c\theta) \qquad (13)$$

where

$h_0(t)$ = the base-line hazard with a prespecified distribution (e.g., exponential, Weibull)
X_t = the considered covariates, which can cause a proportional shift in the base-line hazard
θ = an added component with a distribution across the population to capture the unobserved heterogeneity.

The impact of covariates that are an explicit function of time (e.g., the time since the last training, time since entering the market) can however not be

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

considered in this framework, since an identification problem would result between the parameters of the base-line hazard function on the one hand and the parameters associated with these time-dependent variables on the other hand. Further research is therefore needed on how to include best time-varying covariates and unobserved heterogeneity in a hazard-type framework.

REFERENCES

Allison, Paul D. (1984), "Event History Analysis: Regression for Longitudinal Data,"*Sage University Papers Series: Quantitative Applications in the Social Sciences,* **07–046,** 1–87.

Dickinson, Roger (1981), "Business Failure Rate," *American Journal of Small Business,* **6** (October–December), 17–25.

Gupta, Sunil (1991), "Stochastic Models of Interpurchase Time with Time-Dependent Covariates," *Journal of Marketing Research,* **28** (February), 1–15.

Helsen, Kristiaan, and David C. Schmittlein (1989), "Analyzing Duration Times in Marketing Research," working paper, Wharton School, University of Pennsylvania.

Hemenway, David (1977), *Prices and Choices: Micro-economic Vignettes,* Cambridge, MA: Ballanger Publishing Company.

Himmelblau, D. M. (1972), *Applied Nonlinear Programming,* New York, NY: McGraw-Hill.

Jain, Dipak C., and Naufel J. Vilcassim (1991), "Investigating Household Purchase Timing Decisions: A Conditional Hazard Function Approach," *Marketing Science,* **10** (Winter), 1–23.

Kaplan, E. L., and P. Meier (1958), "Nonparametric Estimation from Incomplete Observations," *Journal of the American Statistical Association,* **53** 457–481.

Marcus, Matityahu (1967), "Firms' Exit Rates and Their Determinants," *Journal of Industrial Economics,* **16** (November), 10–22.

Morrison, Donald G., and David C. Schmittlein (1980), "Jobs, Strikes and Wars: Probability Models for Duration," *Organizational Behavior and Human Performance,* **25,** 224–251.

Schmittlein, David C., and Donald G. Morrison (1983), "Modelling and Estimation Using Job Duration Data," *Organizational Behavior and Human Performance,* **32,** 1–22.

Star, Alvin D., and Michael Z. Massel (1981), "Survival Rates for Retailers," *Journal of Retailing,* **55** (Summer), 87–99.

Vilcassim, Naufel J., and Dipak C. Jain (1991), "Modeling Purchase Timing and Brand Switching Behavior Incorporating Explanatory Variables and Unobserved Heterogeneity," *Journal of Marketing Research,* **28** (February), 29–41.

Watson, G. S., and M. S. Leadbetter (1964), "Hazard Analysis I," *Biometrika,* **51,** 175–184.

APPENDIX

A. Expressions for the used p.d.f.'s and c.d.f.'s

A.1 The Weibull Distribution

$$f(t;\lambda,b) = (\lambda b)t^{b-1}e^{-\lambda t^b} \qquad t > 0;\ \lambda,\ b > 0$$

$$F(t;\lambda,b) = 1 - e^{-\lambda t^b} \qquad t > 0$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

A.2 The Pareto Distribution

$$h(t;r,a) = \frac{r}{a}\frac{a^{r+1}}{(a+t)^{r+1}} \qquad t > 0;\ r,a > 0$$

$$H(t;r,a) = 1 - \frac{a^r}{(a+t)^r} \qquad t > 0$$

A.3 The Burr Distribution

$$h(t;r,a,b) = \frac{rba^r t^{b-1}}{(a+t^b)^{r+1}} \qquad t > 0;\ r,a,b > 0$$

$$H(t;r,a,b) = 1 - \frac{a^r}{(a+t^b)^r} \qquad t > 0$$

B. Derivation of the likelihood function

The parameters of the Pareto and Burr distribution are estimated by means of the method of maximum likelihood. It might be instructive to first consider the contribution of each individual store to that function. If the observed durations could be considered as a continuous variable, the likelihood of observing a store with a completed duration of length t_1 would be described by a term $h(t_1)$, where $h(\,)$ is the p.d.f. under consideration. Stores that are still in operation at the end of the observation period and that have been so already for t_2 time units, are contributing a term $1 - H(t_2)$, giving in fact the probability of observing a duration greater than t_2. Given the independence of the different observations and making the appropriate adjustments asked for by the discrete nature of the available data, the likelihood can be written as

$$L(r,a,b) = \prod_{k=1}^{K} [H(u_k) - H(u_l)]^{X_k}[(1 - H(u_k)]^{Y_k}$$

where H is the c.d.f. of the Pareto or Burr distribution as given in Section A.2 and A.3 of the Appendix. u_k and u_l denote respectively the upper and lower bounds of the discrete duration intervals that are considered. The likelihood of observing a duration of length t_k is therefore replaced by the probability of observing a duration between u_k and u_l. X_k and Y_k, finally, are counts of the number of stores with respectively a completed or right-censored duration falling in the k-th duration interval.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

The parameters of the model are estimated by maximizing this likelihood function, whereby a Hookes-Jeeves accelerated-pattern search (Himmelblau 1972) is used by the computer program. Using the invariance property of maximum likelihood estimators, these estimates will also make it possible to derive maximum likelihood estimates for the mean, median, and mode of the respective distributions.[9] As the gamma mixture of exponentials is just a special case of the gamma mixture of Weibulls, where in the former case b is restricted to be one, a likelihood ratio test will be performed to test whether b is in fact significantly different from one.

C. Formulas for the mean and median of the Pareto and Burr distribution

C.1 Pareto Distribution

$$\text{mean} \quad : \frac{a\Gamma(r-1)}{\Gamma(r)}$$

$$\text{median} : a(2^{1/r}-1)$$

C.2 Burr Distribution

$$\text{mean} \quad : \frac{a^{1/b}\Gamma(r-1/b)\Gamma(1/b)}{\Gamma(r)}$$

$$\text{median} : [a(2^{1/r}-1)]^{1/b}$$

[9] The formulas for these quantities are given in the Appendix, Section C.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.