CentER

Discussion Paper

No. 2008–21

ON THE EFFECTS OF THE DEGREE OF DISCRETION IN REPORTING MANAGERIAL PERFORMANCE

By Anja De Waegenaere, Jacco L. Wielhouwer

February 2008

ISSN 0924-7815

On the effects of the degree of discretion in reporting managerial performance

Anja De Waegenaere* Jacco L. Wielhouwer†

February 4, 2008

Abstract

We consider a principal-agent setting in which a manager's compensation depends on a noisy performance signal, and the manager is granted the right to choose an (accounting) method to determine the value of the performance signal. We study the effect of the *degree* of such reporting discretion, measured by the number of acceptable methods, on the optimal contract, the expected cost of compensation and the manager's expected utility. We find that while an increase in reporting discretion never harms the manager, the effect on the expected cost of compensation is more subtle. We identify three main effects of increased reporting discretion and characterize the conditions under which the aggregate of these three effects will lead to a higher or lower cost of compensation.

JEL Code: D82, D86, M41.

Keywords: managerial compensation, reporting flexibility.

*Corresponding author: Department of Accountancy and Department of Econometrics and OR, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, The Netherlands. Tel: ++31-13-4662913. Fax: ++31-13-4663280. Email: a.m.b.dewaegenaere@uvt.nl

†VU University Amsterdam, Department of Accounting, Faculty of Economics, Room 2E-31, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands. Email: jwielhouwer@feweb.vu.nl

1 Introduction

We consider a principal who contracts with a risk- and effort-averse manager in order to motivate him to deliver the desired effort level. Since the effort provided by the manager is not directly observable, the principal contracts on the basis of a noisy signal, e.g. based on accounting numbers. Accounting standards such as Generally Accepted Accounting Principles, however, usually offer a variety of acceptable accounting methods (e.g. LIFO vs FIFO, accelerated vs straight line depreciation, etc.). It has been demonstrated in several settings that it may be optimal to grant a manager the discretion to choose an accounting method, even when his compensation depends on performance measures derived from reported accounting numbers. Demski et al. (1984) show that when accounting method choice is *verifiable*, delegating the choice to the manager may be optimal because by motivating the manager to use a different accounting method for different realizations of his private information, the manager's information rent is reduced. Verrecchia (1986) considers a setting where accounting method choice is *partially unverifiable*, and shows that even when the principal has the option to implicitly eliminate reporting flexibility by affecting the attractiveness of the acceptable reporting alternatives, it is in general not optimal to do so. Ozbilgin and Penno (2006) consider a principal-agent model with a set of ex-ante equivalent performance measurement methods, and find that delegating the choice of measurement method to the manager is optimal if he is sufficiently risk averse.

Given these various conditions under which delegating accounting method choice (or, more generally, performance measurement method) to the manager is optimal, and given the ongoing debate on the "desired" degree of flexibility in GAAP, it is clearly relevant and important to investigate the effect of the *degree of reporting flexibility* on the internal agency problem. Prior literature shows that risk aversion plays a crucial role in understanding the effect of increased reporting flexibility on the expected cost of compensation. Demski (1998) considers a multi-period model where the manager has private information and can manipulate earnings numbers. He shows that the expected cost of compensation when the manager is motivated to manipulate earnings numbers can be lower than in a situation where he has no private information, so that results can only be reported truthfully. The underlying reason is that the manager can only manipulate the performance signal in case the desirable effort level is delivered, and allowing for manipulation reduces the manager's risk. Ozbilgin and Penno (2006) show that when the manager has the discretion to choose the performance measurement method, increased reporting flexibility (as measured by the number of acceptable performance measurement methods) decreases the expected cost of compensation if the manager is sufficiently risk averse. Their setting has no information asymmetry other than the manager's action and measurement choices. These results show that more reporting flexibility for the manager, either through diversity in acceptable measurement methods (as in Ozbilgin and Penno, 2006) or through allowed earnings manipulation (as in Demski, 1998) can be beneficial to the principal since it reduces the manager's compensation risk.[1]

In this paper we take a principal-agent approach similar to Ozbilgin and Penno (2006), in which the manager's compensation depends on a noisy performance signal, and the manager is granted the right to choose an (accounting) method to determine the value of the performance signal. We study the effect of the *degree of reporting flexibility*, measured by the number of acceptable measurement methods, on the expected cost of compensation and on the manager's expected utility.[2] Our results complement and extend theirs in several directions. First, the setting in Ozbilgin and Penno (2006) is such that the manager always earns a limited liability rent. In contrast, whether the limited liability constraint is binding in our setting is endogenous and depends on the degree of reporting flexibility. This has important implications for the effect of the level of reporting flexibility on the expected cost of compensation. Second, we distinguish two critical

[1] Penno (2005) considers a manager who can choose between N performance measurement signals that are i.i.d. exponentially distributed, and shows that the expected cost of compensation is independent of N. This remarkable result is due to the nature of the exponential distribution.

[2] Ozbilgin and Penno (2006) distinguish settings in which the discretion to choose the method rests with the principal and settings where it rests with the manager. The focus in our paper is on the latter.

values of the degree of reporting flexibility, both of which are increasing in the degree of risk aversion of the manager. The first critical value determines whether the manager will earn a limited liability rent. The second critical value determines whether increased reporting flexibility makes it easier or more difficult to prevent shirking, i.e. whether a higher bonus is required to motivate high effort. As long as the degree of flexibility does not exceed either of these two critical values, the limited liability constraint will not be binding and increased reporting flexibility allows for a lower bonus. As a consequence, higher reporting flexibility then yields a lower expected cost of compensation, even though it does not affect the manager's expected utility. Above the two threshold values, both the size of the bonus and the limited liability rent are strictly increasing in the degree of reporting flexibility. Increased reporting flexibility is then strictly beneficial to the manager, but harmful to the principal. For intermediate degrees of reporting flexibility, the effect is ambiguous. We show that increased reporting flexibility may then be socially optimal in the sense that it makes *both* the principal and the manager strictly better off.

Finally, we show that a minimal level of reporting flexibility may be necessary for the existence of an optimal contract, i.e. if incentive problems cannot be resolved at finite cost, an increase in the degree of reporting flexibility can be sufficient to solve this problem.

Although related, the problem studied in this paper differs in several ways from the literature on equilibrium earnings management when the Revelation Principle fails to hold due to, e.g., restricted communication, lack of commitment, or contracting restrictions. There, the focus is on settings where the manager has private information and may be able to manage earnings in a way that would not be accepted if detected by an audit system. The issue is then whether motivating rejection of earnings management is optimal. It has been demonstrated that allowing for, and motivating, manipulation of performance measures may be beneficial to the principal in situations where manipulation requires costly effort (Demski et al. 2004, Liang 2004) or when there is limited commitment (Arya et al. 1998). In our setting, there is no private information (other than the action and measurement method choice) and all available measurement methods are equally acceptable. The issue is therefore not whether the manager should be motivated to choose a particular method. Rather, the focus is on the effect of diversity in measurement methods on the expected cost of compensation, given that the manager can strategically choose any method from the set of acceptable methods.

The rest of the paper is organized as follows. In section 2, we present the model. In section 3, we derive the optimal contract, and in section 4 we study the effect of the number of alternative measurement methods on the optimal contract, the expected cost of compensation and on the manager's expected utility. Section 5 discusses the implications of reporting flexibility for both the principal and the manager. Section 6 concludes. All proofs are deferred to the Appendix.

2 The model

We consider a principal who contracts with a risk- and effort-averse manager in order to motivate him to deliver the desired effort level. Since the effort provided by the manager is not directly observable, the principal contracts on the basis of a noisy signal.

The model is similar to the models in Penno (2005) and Ozbilgin and Penno (2006). Specifically, there is managerial reporting flexibility in the sense that there are a number of different noisy performance signals, each resulting from equally acceptable measurement methods. The manager has the discretion to choose a measurement method, and report the corresponding signal to the principal.[3] The choice occurs ex post, i.e. after all the signals have realized. Since it is assumed that verification of the signals is costly for the principal, only the reported signal will be verified and used for contracting.

There are two effort levels $a \in \{a_H, a_L\}$, and a set of N equally acceptable measurement methods. Each method yields a signal that can take two values $y \in \{y_H, y_L\}$,

[3] Ozbilgin and Penno (2006) distinguish settings in which the discretion to choose the method rests with the principal and settings where it rests with the manager, and show that delegating the choice to the manager is optimal if he is sufficiently risk averse.

with $y_H > y_L$. The signals resulting from the N different measurement methods are independent and identically distributed random variables $y^i, i = 1, ..., N$, for which the probability distribution is determined by the action chosen by the manager in the following way:

$$\begin{aligned} &P\{y^i = y_H | a = a_H\} = 1 - p, \\ &P\{y^i = y_L | a = a_H\} = p, \end{aligned} \tag{1}$$

$$\begin{aligned} &P\{y^i = y_H | a = a_L\} = 1 - q, \\ &P\{y^i = y_L | a = a_L\} = q. \end{aligned} \tag{2}$$

Without loss of generality we assume that $q > p$, i.e. the probability of outcome y_L is higher under a_L than under a_H. This implies that the monotone likelihood ratio property (MLRP) holds, i.e. if a_H is the desirable action, y_H is a "good" signal, and y_L is a "bad" signal. The principal is risk neutral; the manager is a risk averse expected utility maximizer with utility function $u(x) = -e^{-\rho x}$, where $\rho > 0$ represents the degree of risk aversion. The manager is effort-averse, and the cost of providing effort a_H (a_L) equals c_H (c_L), with $c_H > c_L$.

The timeline is as follows:

- Date 0: The principal specifies the level of compensation that will be paid to the manager in case y_H, respectively y_L, is reported. The manager decides to accept or reject the contract. If the manager accepts the contract, he then chooses his effort level $a \in \{a_H, a_L\}$.

- Date 1: The manager determines the value $y^i \in \{y_H, y_L\}$ of the signal resulting from the i^{th} acceptable performance measurement method, for $i = 1, \cdots, N$, and reports one signal $\widehat{y} \in \{y^i; i = 1, ..., N\}$ to the principal. Compensation is paid and the game ends.

3 The optimal contract

Let us denote $s(y)$ for the compensation received in case y is reported. Without loss of generality, we focus on the compensation scheme needed to motivate the manager to take action a_H.[4] Then, similarly to Dye and Magee (1991), Arya et al. (1992), and Ozbilgin and Penno (2006), the principal needs to minimize the expected cost of inducing the agent to choose action a_H, taking into account his self-interested behavior with respect to his action and reporting choices. Specifically, $s(y_H)$ and $s(y_L)$ need to be determined such that the expected cost of compensation is minimized, under the constraints that: i) the manager reports the most favorable signal (i.e. the one that maximizes his compensation) among the set of N acceptable signals y^i, $i = 1, ..., N$, ii) providing high effort yields a higher expected utility than providing low effort (incentive compatibility), iii) staying with the firm and accepting the contract is preferable to the first best alternative (individual rationality), and, iv) compensation is nonnegative (limited liability).

Let $u(M)$ denote the manager's reservation utility. Then, the following optimization problem needs to be solved:

$$\begin{array}{ll} \min\limits_{s(\cdot)} & E\left[s(\widehat{y}) \middle| a = a_H \right] \\ \text{s.t.} & \widehat{y} \in \arg \max\limits_{y \in \{y^1, \ldots, y^N\}} s(y) \\ & E\left[u(s(\widehat{y}) - c_H) \middle| a = a_H \right] \geqslant u(M) \\ & E\left[u(s(\widehat{y}) - c_H) \middle| a = a_H \right] \geqslant E\left[u(s(\widehat{y}) - c_L) \middle| a = a_L \right] \\ & s(\widehat{y}) \geqslant 0. \end{array} \tag{3}$$

Clearly,

$$\begin{aligned} s(y_H) &\geqslant s(y_L) \Longrightarrow \widehat{y} = \max\{y^i; i = 1, ..., N\}, \\ s(y_H) &\leqslant s(y_L) \Longrightarrow \widehat{y} = \min\{y^i; i = 1, ..., N\}. \end{aligned}$$

[4] It is easily verified that, due to the MLRP, the cost minimizing compensation scheme that motivates the manager to take action a_L is given by $s_L = s_H = M + c_L$, for all $N \geqslant 1$.

However, it is easy to verify that due to the MLRP, the optimal contract when $\widehat{y} = \min\{y^i; i = 1,...,N\}$ satisfies $s(y_H) > s(y_L)$. Therefore, $\widehat{y} = \max\{y^i; i = 1,...,N\}$, i.e. the reported signal $\widehat{y}$ equals y_H if for at least one measurement method it holds that $y^i = y_H$, and equals y_L otherwise. Consequently, as in Penno (2005) and Ozbilgin and Penno (2006), the probability distribution of the reported signal under high effort depends on the number of acceptable measurement alternatives in the following way:

$$\begin{aligned} P\{\widehat{y} = y_L | a = a_H\} &= P\left(\max\left\{y^i; i \in \{1,...,N\}\right\} = y_L | a = a_H\right) \\ &= P\left(y^1 = y_L, y^2 = y_L, ..., y^N = y_L | a = a_H\right) \\ &= P\left(y^1 = y_L | a = a_H\right) P\left(y^2 = y_L | a = a_H\right) \cdots P\left(y^N = y_L | a = a_H\right) \\ &= p^N, \end{aligned} \tag{4}$$

and,

$$P\{\widehat{y} = y_H | a = a_H\} = 1 - p^N. \tag{5}$$

Similarly, for low effort:

$$P\{\widehat{y} = y_H | a = a_L\} = 1 - q^N, \tag{6}$$

$$P\{\widehat{y} = y_L | a = a_L\} = q^N. \tag{7}$$

Let us denote $s(y_H) = s_H$ and $s(y_L) = s_L$. Then, (4)-(7) imply that optimization problem (3) is equivalent to:

$$\begin{aligned} \min \quad & p^N s_L + (1 - p^N) s_H \\ s.t. \quad & p^N u(s_L - c_H) + (1 - p^N) u(s_H - c_H) \geqslant q^N u(s_L - c_L) + (1 - q^N) u(s_H - c_L) \\ & p^N u(s_L - c_H) + (1 - p^N) u(s_H - c_H) \geqslant u(M) \\ & s_L \geqslant 0,\ s_H \geqslant 0. \end{aligned}$$

Our goal is to study the effect of the degree of reporting flexibility, N, on the optimal incentive contract, on the expected cost of compensation, and on the manager's expected utility.

4 The effect of increased reporting flexibility

In this section we first determine the optimal contract for any given value of N. This will then allow us to determine the effect of an increase in the level of reporting flexibility on the level of compensation and the size of the bonus (subsection 4.1), on the manager's expected utility (subsection 4.2), and on the expected cost of compensation (subsection 4.3).

The following theorem shows that, in contrast to Ozbilgin and Penno (2006), the structure of the optimal compensation contract depends crucially on whether N exceeds a threshold value, N^*, and yields the optimal compensation levels for both cases. Moreover, it is shown that an optimal contract only exists if there are sufficiently many acceptable measurement methods.

Theorem 1 *A minimal level of reporting flexibility is necessary for the existence of an optimal contract. Specifically, an optimal compensation contract exists iff*

$$N \geqslant \frac{\rho(c_H - c_L)}{\ln q - \ln p}.$$

Then, the limited liability constraint is binding iff $N > N^*$, where

$$\begin{aligned} N^* &= \max\left\{N \in \mathbb{N}; \tfrac{1-q^N}{1-p^N} \leqslant \tfrac{1-e^{-\rho(M+c_L)}}{1-e^{-\rho(M+c_H)}}\right\}, && \text{if } q < 1, \\ &= \infty, && \text{if } q = 1. \end{aligned} \tag{8}$$

Moreover:

i) If $N \leqslant N^*$, the optimal compensation scheme is given by

$$s_L = -\frac{1}{\rho} \ln \frac{(1-p^N)e^{-\rho(M+c_L)} - (1-q^N)e^{-\rho(M+c_H)}}{q^N - p^N}, \tag{9}$$

$$s_H = -\frac{1}{\rho} \ln \frac{q^N e^{-\rho(M+c_H)} - p^N e^{-\rho(M+c_L)}}{q^N - p^N}. \tag{10}$$

ii) If $N > N^*$, the optimal compensation scheme is given by

$$s_L = 0, \tag{11}$$
$$s_H = -\frac{1}{\rho} \ln \frac{q^N e^{-\rho(M+c_H)} - p^N e^{-\rho(M+c_L)}}{(1-p^N)e^{-\rho(M+c_L)} - (1-q^N)e^{-\rho(M+c_H)}}. \tag{12}$$

Note that the theorem shows that a minimal degree of reporting flexibility may actually be necessary to be able to resolve incentive conflicts at finite cost. Suppose that the manager's degree of risk aversion and cost of effort are such that $\frac{\rho(c_H - c_L)}{\ln q - \ln p} > 1$. Then, there does not exist an optimal contract if the manager is constrained to using any given measurement method. If he can choose amongst at least $\overline{N} = \frac{\rho(c_H - c_L)}{\ln q - \ln p}$ performance measurement methods, there does exist a contract that resolves incentive problems.

It is clear from Theorem 1 that the way in which the optimal contract is affected by the level of reporting flexibility depends crucially on whether the threshold value N^* is exceeded, or, equivalently whether the limited liability constraint is binding. It is therefore intuitively clear that this threshold value can also play a dominant role in the effect of reporting flexibility on the expected cost of compensation.

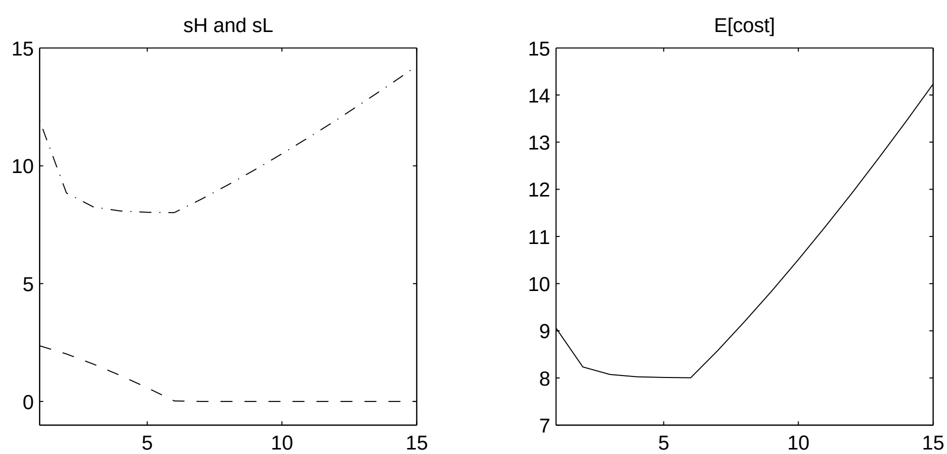

Figure 1: Optimal values of s_H (dashed-dotted) and s_L (dashed) (left panel) and expected cost (right panel), as a function of N for $\rho = 0.1$, $c_L = 0$, $c_H = 5$, $M = 3$, $p = 0.3$, $q = 0.9$.

This is illustrated in Figure 1. It shows the payoffs s_L and s_H and the expected cost of the optimal contract, as a function of the number of alternative reports. We see that the optimal payoffs as well as the expected cost of compensation first decrease and then increase as N increases.

Although in the setting in Figure 1 the limited liability constraint becomes binding at $N^* = 6$, and the expected cost of compensation decreases (increases) in N for $N < 6$ $(N \geqslant 6)$, we will show in the sequel that this is not the general pattern.

4.1 The effect on the optimal contract

For the sake of intuition, we view the compensation package as consisting of a level of compensation s_L, to which a bonus $s_H - s_L$ is added in case of a high report. Let us use the following notation:

$$\alpha = e^{-\rho(c_H - c_L)}. \tag{13}$$

The parameter α reflects the severity of the incentive problem. A lower value of α, e.g. due to a higher degree of risk aversion and/or a bigger difference between the cost of high effort and low effort, ceteris paribus, implies that compensation will be more costly. We also introduce a second threshold value $\widetilde{N}$, which is defined as follows:[5]

$$\begin{aligned} \widetilde{N} &= \frac{\ln\left(\frac{\ln p}{\ln q}\right) - \ln(\alpha)}{\ln q - \ln p}, && \text{if } 0 < p < q < 1, \\ &= \infty, && \text{if } q = 1, \\ &= 0, && \text{if } p = 0. \end{aligned} \tag{14}$$

In the following proposition we first determine the effect of the level of reporting flexibility (N) on the two levels of compensation s_L and s_H, as well as on the size of the bonus (the difference between the two levels of compensation), where the critical values N^* and $\widetilde{N}$ are as defined in (8) and (14), respectively.

Proposition 2 *For the optimal compensation contract, the following holds:*

[5]Since the solution of the optimization problem is trivial when $p = 0$ and $q = 1$, we can assume without loss of generality that $p > 0$ or $q < 1$.

i) s_H *is decreasing in* N *for* $N \leqslant \max\{N^*, \widetilde{N}\}$, *and increasing in* N *for* $N > \max\{N^*, \widetilde{N}\}$,

ii) s_L *is decreasing in* N *for* $N \leqslant N^*$, *and* $s_L = 0$ *for* $N > N^*$.

iii) $s_H - s_L$ *is decreasing in* N *for* $N \leqslant \widetilde{N}$, *and increasing in* N *for* $N > \widetilde{N}$.

First, the critical level $\widetilde{N}$ determines whether an increase in N makes incentive problems more severe, or equivalently, whether a higher bonus is required to motivate high effort. As long as $N \leqslant \widetilde{N}$, an increase in the degree of reporting flexibility makes incentive problems less severe, so that the size of the bonus can be decreased. The opposite holds when N is higher than the critical level $\widetilde{N}$. Second, the critical level N^* determines whether the limited liability constraint is binding. When the degree of reporting flexibility is lower than the threshold value N^*, the limited liability constraint is not binding. The fact that the likelihood ratios of the low and the high outcome both increase when a higher level of discretion is allowed, then implies that both levels of compensation can be decreased. When the lowerbound on compensation becomes binding ($N > N^*$), the compensation for low outcome needs to be fixed at its minimal level. Consequently, the size of the bonus can only be affected by changing the level of the compensation in case of high outcome. It needs to be increased when $N > \widetilde{N}$, but can be decreased when $N \leqslant \widetilde{N}$.

4.2 The effect on the manager's expected utility

In this section we study the effect of an increase in the degree of reporting flexibility on the manager's expected utility. Let us therefore denote $CE(N)$ for the manager's certain equivalent as a function of the number of alternative measurement methods, N, i.e.

$$CE(N) = u^{-1}\left(\, (1-p^N)u(s_H) + p^N u(s_L) \, \right),$$

where s_H and s_L are as defined in Theorem 1. The following proposition determines the effect of N on the manager's certain equivalent.

Proposition 3 *For the manager's certain equivalent, the following holds:*

$$
\begin{aligned}
CE(N) &= M, && \textit{for } N \leqslant N^*, \\
&= \frac{1}{\rho} \ln \frac{(1-p^N)e^{-\rho c_L} - (1-q^N)e^{-\rho c_H}}{q^N - p^N}, && \textit{for } N > N^*.
\end{aligned}
$$

The certain equivalent is strictly increasing in N for $N > N^*$.

The above proposition implies that the manager's utility is not affected by a change in the degree of reporting flexibility as long as the critical level N^* is not exceeded. The expected utility of the manager is then equal to his reservation utility. Above the critical level N^*, the manager starts earning a rent due to the fact that the limited liability constraint becomes binding. Since the rent is increasing in N, the manager strictly benefits from increased reporting flexibility.

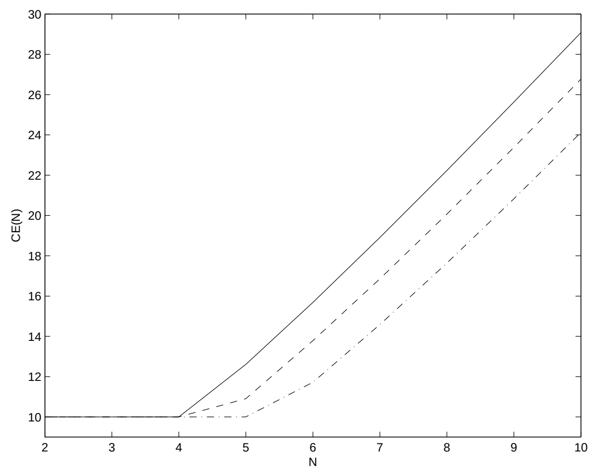

Figure 2: The manager's certain equivalent, as a function of N, for $p = 0.3$, $q = 0.7$, $\rho = 0.1$, $M = 10$, $c_L = 0$, and $c_H = 7$ (solid line), $c_H = 5$ (dashed line) and $c_H = 3.5$ (dashed-dotted line).

This is illustrated in Figure 2. It can be verified that the limited liability constraint becomes binding at $N^* = 4$, after which the size of the rent increases when the degree of reporting flexibility increases. The rate of the increase is increasing in the difference between the cost of high and low effort.

4.3 The effect on the expected cost of compensation

The analysis in the previous subsections hints at the fact that the effect of reporting flexibility on the expected cost of compensation will be driven by the following three effects:

- The *decreased risk compensation effect*: because p^N is strictly decreasing in N if $p > 0$, an increase in the degree of reporting flexibility implies that the probability that the manager will receive the higher compensation level increases. Consequently, he requires less risk compensation (Proposition 2 i) and ii)).

- The *incentive compatibility effect*: the bonus can be decreased when $N < \widetilde{N}$ (needs to be increased when $N \geqslant \widetilde{N}$) because an increase in reporting flexibility then mitigates (aggravates) incentive problems (Proposition 2 iii)).

- The *limited liability effect*: when $N \geqslant N^*$, the limited liability constraint is binding, and the manager earns a rent which increases with the level of reporting flexibility (Proposition 3).

In isolation, each of these effects is either cost increasing or cost decreasing. In the sequel, we determine under what conditions the cost increasing, respectively cost decreasing effects will be dominant. Let us start with two special cases: the case where low effort yields a low signal with certainty, i.e., $q = 1$, and the case where high effort yields a high signal with certainty, i.e., $p = 0$.

Proposition 4

- *If $q = 1$, then $N^* = \widetilde{N} = \infty$, and the expected cost of compensation is decreasing in N.*

- *If $p = 0$, then $\widetilde{N} = 0$, and*

  - *If $N \leqslant N^*$, the expected cost of compensation is independent of N.*

– *If $N > N^*$, the expected cost of compensation is increasing in N.*

If $q = 1$, an increase in the degree of reporting flexibility always mitigates incentive problems (i.e. $\widetilde{N} = \infty$). Moreover, the lowerbound on compensation never becomes binding (i.e. $N^* = \infty$). Consequently, the expected cost of compensation is monotonically decreasing in the number of alternative measurement methods. In contrast, if $p = 0$, compensation always equals s_H, so that no risk compensation is required. However, an increase in reporting flexibility always aggravates incentive problems, since $\widetilde{N} = 0$. Combined with the effect of the limited liability rent, this implies that the expected cost of compensation increases when $N > N^*$.

The results of Proposition 4 illustrate that the effect of an increase in reporting flexibility on the expected cost of compensation depends to a large extent on the parameter values: it increases the expected cost if $p = 0$ and $N > N^*$, it decreases the expected cost if $q = 1$, and it leaves the expected cost unaffected if $p = 0$ and $N \leqslant N^*$. It the sequel we characterize the conditions under which increased reporting flexibility will increase (decrease) the expected cost of compensation for all $0 < p < q < 1$. The following theorem shows that for sufficiently high values of N, the cost increasing effects of the limited liability rent and increased incentive problems are dominant.

Theorem 5 *If $N > \max\left\{N^*, \widetilde{N}\right\}$, then the expected cost of compensation is increasing in N.*

We now focus on the case where $N \leqslant \max\left\{N^*, \widetilde{N}\right\}$. The following theorem shows that whether a higher degree of reporting flexibility would increase or decrease the expected cost of compensation depends on: the probabilities p^N and q^N, whether N exceeds the threshold value N^*,[6] and the risk aversion/cost parameter α, as defined in (13).

[6]Given p^N and q^N, the number of alternative signals N affects the magnitude of the derivative of the expected cost of compensation with respect to N, but not its sign.

Theorem 6 *Let $p, q,$ and $N \leqslant \max\left\{N^*, \widetilde{N}\right\}$ be given, and denote*

$$S = \{(u, v) \in [0,1] \times [0,1] : (u - 2uv + v)\ln u - 2v(1-u)\ln v > 0\}. \tag{15}$$

Then, there exists an α^* and an α_b^* such that:[7]

i) If $N < N^*$, an increase in N

- *decreases the expected cost of compensation iff $\alpha < \alpha^*$ or $\left(p^N, q^N\right) \notin S$.*
- *increases the expected cost of compensation iff $\alpha > \alpha^*$ and $\left(p^N, q^N\right) \in S$.*

ii) If $N^* \leqslant N \leqslant \widetilde{N}$, an increase in N increases (decreases) the expected cost of compensation iff $\alpha > \alpha_b^*$ ($\alpha < \alpha_b^*$).

Figure 3 illustrates the set S.

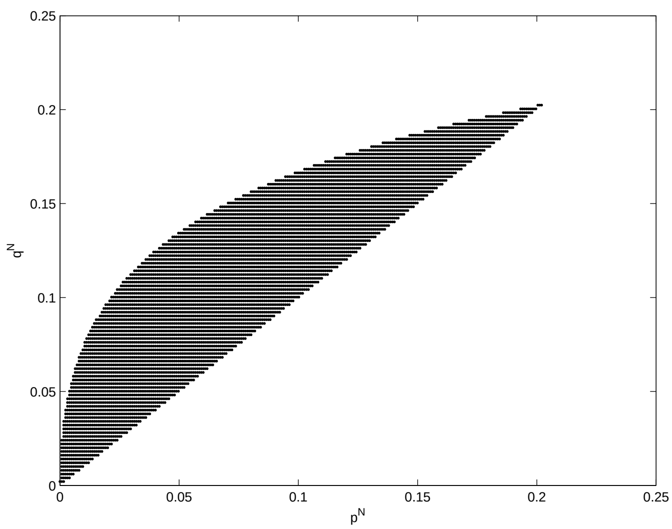

Figure 3: The combinations of p^N and q^N for which $(p^N, q^N) \in S$.

[7]The critical values α^* and α_b^* depend on p^N and q^N. In order to avoid overloaded notation, we do not explicitly denote this dependence, unless it is required for clarity.

5 Implications

In this section we use Proposition 3 and Theorem 6 to study the implications of increased reporting flexibility for both the principal and the manager. We will distinguish four ranges of values of N.

The case where $N \leqslant \min\{N^*, \widetilde{N}\}$.

In this case, the limited liability constraint is not binding (because $N \leqslant N^*$), and an increase in reporting flexibility mitigates incentive problems and therefore allows for a lower bonus (because $N \leqslant \widetilde{N}$). Now it can be verified numerically that:[8]

$$N \leqslant \widetilde{N} \implies \alpha < \alpha^*. \tag{16}$$

It therefore follows from Theorem 6 i) that the expected cost of compensation will be decreasing in N. Moreover, since the limited liability constraint is not binding, it follows from Proposition 3 that the manager does not earn a rent. This yields:

Implication 1: *If N is sufficiently low ($N \leqslant \min\{N^*, \widetilde{N}\}$), an increase in the degree of reporting flexibility makes the principal strictly better off, while leaving the manager's utility unaffected.*

The case where $\widetilde{N} \leqslant N \leqslant N^*$.

In this case, the limited liability constraint is still not binding (because $N \leqslant N^*$), but an increase in reporting flexibility now aggravates incentive problems and thus requires a higher bonus (since $\widetilde{N} \leqslant N$). The effect on the expected cost of compensation therefore depends on whether the cost reducing effect of reduced compensation risk outweighs the cost increasing effect of increased incentive problems. It follows from Theorem 6 i) that the aggregate effect of increased reporting flexibility on the expected cost of compensation depends on both the risk aversion/cost parameter α, as well as on $\left(p^N, q^N\right)$. Combined with the result from Proposition 3, this yields:

[8] Remember that $\widetilde{N}$ depends on α, p, and q, and that α^* depends on $\left(p^N, q^N\right)$.

Implication 2: *If* $\widetilde{N} \leqslant N \leqslant N^*$*, an increase in the degree of reporting flexibility leaves the manager's utility unaffected, and*

- *makes the principal better off if* $\alpha < \alpha^*$ *or* $\left(p^N, q^N\right) \notin S$.
- *makes the principal worse off if* $\alpha > \alpha^*$ *and* $\left(p^N, q^N\right) \in S$.

Note that since $S \subset [0, 0.2] \times [0, 0.2]$, there is a wide range of parameter values for which $\left(p^N, q^N\right) \notin S$. We illustrate this in Figure 4.

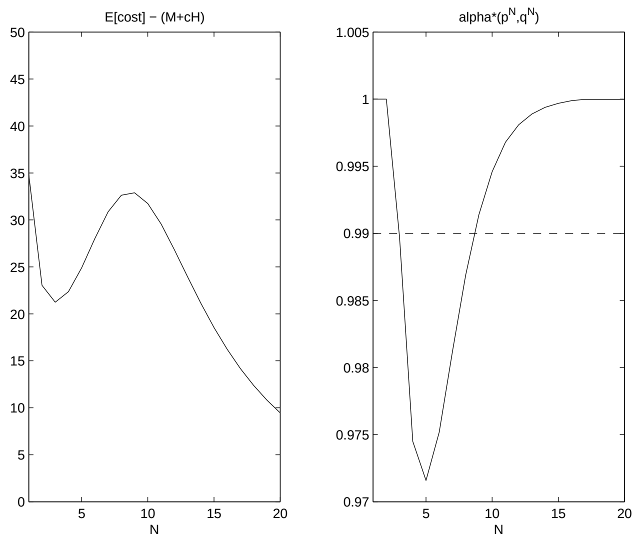

Figure 4: Left panel: Expected cost of compensation in excess of $M + c_H$ as a function of N, for $c_L = 0, c_H = 100, M = 85000, p = 0.46, q = 0.52$, and $\alpha = 0.99$. Right panel: The critical value α^* as a function of N for $p = 0.46, q = 0.52$.

For the parameter values in Figure 4, it holds that $\widetilde{N} = 2$ and $N^* = 26$. The critical value of α needed to make the increased incentive problems effect dominant, α^*, decreases for $N \leqslant 5$, and increases for $N > 5$. It can be verified that $\alpha = 0.99 > \alpha^*$ and $\left(p^N, q^N\right) \in S$ for $N \in [3, 8]$. The expected cost of compensation therefore increases over that range, but decreases outside that range.

The case where $N^* \leqslant N \leqslant \widetilde{N}$.

In this case, the limited liability constraint is binding (because $N \geqslant N^*$), but the degree of reporting flexibility is sufficiently low so that an increase in reporting flexibility allows for a lower bonus (because $N \leqslant \widetilde{N}$). It then follows from Theorem 6 ii) that the cost decreasing effects (decreased risk compensation and decreased incentive problems) dominate the limited liability effect if the manager is sufficiently risk averse, i.e. if $\alpha < \alpha_b^*$. Combined with the results of Proposition 3, this yields the following:

Implication 3: *If $N^* \leqslant N \leqslant \widetilde{N}$ and $\alpha < \alpha_b^*$, an increase in the degree of reporting flexibility is strictly beneficial to both the manager and the principal.*

We illustrate this result in Figure 5.

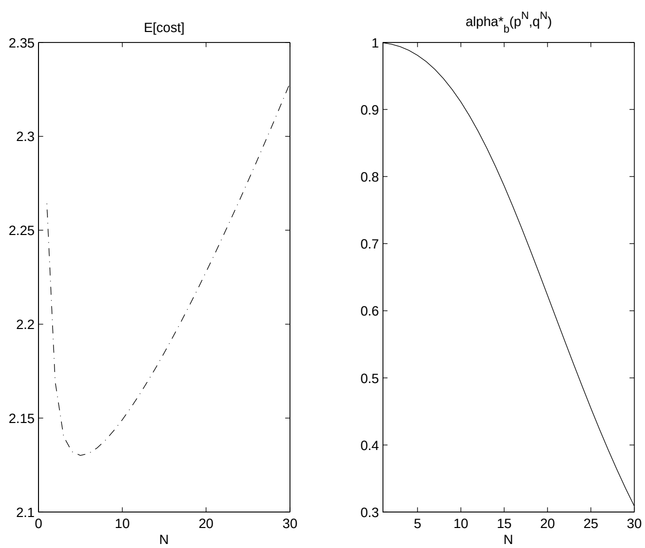

Figure 5: The left panel: the expected cost of compensation as a function of N, for $p = 0.875$, $q = 0.995$, $\rho = 0.01$, $c_L = 0$, $c_H = 2$ and $M = 0.094$, so that $\alpha = 0.98$. The right panel: α_b^* as a function of N, for $p = 0.875$ and $q = 0.995$.

For the parameter values in Figure 5, it can be verified that $N^* = 2$, and $\widetilde{N} = 26$, and that $\alpha = 0.98 < \alpha_b^*$ for all $N \leqslant 5$. Combined with Implication 1, this implies that the expected cost of compensation decreases for $N \leqslant 5$, and increases for $N > 5$.

Interestingly, for values of N between 2 and 5, the expected cost of compensation is decreasing in N, even though an increase in N strictly increases the manager's rent.

The case where $N \geqslant \max\{N^*, \widetilde{N}\}$.

In this case, the limited liability constraint is binding, and an increase in reporting flexibility would require a higher bonus. We know from Theorem 5 that the combination of increased incentive problems and limited liability then implies that the expected cost of compensation will increase when the level of reporting flexibility increases. Moreover, it follows from Proposition 3 that the manager's limited liability rent will also increase.

Implication 4: *If N is sufficiently high* ($N \geqslant \max\{N^*, \widetilde{N}\}$), *an increase in the degree of reporting flexibility makes the principal strictly worse off, while making the manager strictly better off.*

6 Conclusion

We identified the three main effects of an increase in the level of reporting flexibility on managerial compensation, in a setting where the manager has the discretion to choose the method. First, it reduces the manager's risk because the probability that he will be able to report a favorable signal increases. Second, the size of the bonus required to motivate the manager to provide high effort can be decreased if the current level of discretion is sufficiently low, but the opposite would happen if that level is already relatively high. Finally, the fact that the manager faces limited liability significantly affects the effect of increased reporting flexibility. The limited liability constraint will be binding if the degree of reporting flexibility is, or becomes, sufficiently high. Below the threshold value, the manager's expected utility is constant and equal to his reservation utility. Above the threshold value however, the manager earns a limited liability rent, which is increasing in the degree of reporting flexibility. The latter implies that the manager strictly benefits from increased reporting flexibility if that level is high enough.

Whether or not increased reporting flexibility would be harmful to the principal depends on the aggregate of the above described effects. For sufficiently low degrees of reporting flexibility, the cost decreasing effects are unambiguously dominant, i.e. the principal strictly benefits from a higher degree of reporting flexibility; the opposite holds for sufficiently high degrees of reporting flexibility. For intermediate values, the effect is ambiguous, and depends on the probability distributions of the signals, the manager's degree of risk aversion as well as his cost parameters. For a broad set of parameter values, increased reporting flexibility would be strictly beneficial to the principal, and would leave the manager's utility unaffected. We also identify conditions under which *both* the principal and the manager are strictly better off when more performance measurement alternatives are available.

References

[1] Arya A.,J. Glover, and S. Sunder (1998). Earnings management and the revelation principle. *Review of Accounting Studies* 3: 7:34.

[2] Arya A., R.A. Young, and P. Woodlock (1992), Managerial Reporting Discretion and the Truthfulness of Disclosures, *Economic Letters* 39, 163-168.

[3] Demski J.S. (1998), Performance Measure Manipulation, *Contemporary Accounting Research* 15, 3, 261-285.

[4] Demski J.S, H. Frimor and D.E.M. Sappington (2004), Efficient Manipulation in a Repeated Setting, *Journal of Accounting Research* 42,1, 31-49.

[5] Demski J.S, J.M. Patell and M.A. Wolfson (1984), Decentralized Choice of Monitoring Systems, *The Accounting Review* 59,1, 16-34.

[6] Dye R.A, and R.P. Magee (1991), Discretion in Reporting Managerial Performance, *Economic Letters* 35, 359-363.

[7] Dye R.A, and R.E. Verrecchia(1995), Discretion vs. Uniformity: Choices among GAAP, *The Accounting Review* 70,3, 389-415.

[8] Fishman J.F., and K.M. Hagerty (1990), The Optimal Amount of Discretion to Allow in Disclosure, *Quarterly Journal of Economics* 105, 2, 427-444.

[9] Lambert R.A. (2001), Contracting Theory and Accounting, *Journal of Accounting and Economics* 32, 3-87.

[10] Liang P.J. (2004), Equilibrium Earnings Management, Incentive Contracts, and Accounting Standards, *Contemporary Accounting Research* 21, 3, 685-717.

[11] Ozbilgin, M. and M. Penno (2006), The Assignment of Decision Rights in Formal Information Systems, *working paper* (An earlier version is available at SSRN: http://ssrn.com/abstract=605481.)

[12] PENNO M. (2005), The Contracting Value of Tainted Reports in Cost Reduction Settings, *European Economic Review* 49, 1979-1985.

[13] VERRECCHIA R.E. (1986), Managerial Discretion in the Choice among Financial Reporting Alternatives, *Journal of Accounting and Economics* 8, 175-195.

Appendix

Remarks:

1. Note that if a real-valued function $f(\cdot)$ is increasing (decreasing) over the range $[l, b]$, then $f(\cdot)$ is clearly also increasing (decreasing) over all integer values in that range. Therefore, although N can only take integer values, we can conclude that $f(N)$ is increasing (decreasing) in N over a certain range if $f'(\cdot) > 0$ (< 0) over that range.

2. The following properties will be used throughout the proofs:

$$
\begin{array}{ll}
\frac{d}{dn}(x^n) = x^n \ln(x), & \text{for all } x > 0, \\
\ln(x) < 0, & \text{for all } x \in [0,1], \\
\ln(x) \leqslant x - 1, & \text{for all } x > 0, \\
\ln(x^n) = n \ln(x), & \text{for all } x > 0 \text{ and } n \in \mathbb{N}, \\
\ln(xy) = \ln(x) + \ln(y), & \text{for all } x > 0,\ y > 0.
\end{array}
$$

Proof of Theorem 1

i) Let us first consider the optimization problem without the limited liability constraints. Then it follows immediately from the KKT-conditions that the individual rationality and the incentive compatibility constraint are both binding.

It can be verified that the solution equals $s_L = -\frac{1}{\rho}\ln(x^*)$ and $s_H = -\frac{1}{\rho}\ln(y^*)$, where

$$x^* = \frac{(1-p^N)e^{-\rho(M+c_L)} - (1-q^N)e^{-\rho(M+c_H)}}{q^N - p^N}, \tag{17}$$

$$y^* = \frac{q^N e^{-\rho(M+c_H)} - p^N e^{-\rho(M+c_L)}}{q^N - p^N}. \tag{18}$$

The resulting payment scheme is feasible (i.e., $0 \leqslant s_L < \infty$, $0 \leqslant s_H < \infty$) iff $0 < x^* \leqslant 1$ and $0 < y^* \leqslant 1$. If $x^* \leqslant 0$ or $y^* \leqslant 0$, then an optimal compensation scheme does not exist. If $x^* > 1$ or $y^* > 1$, then the limited liability constraint is violated.

Note that $y^* \leqslant x^*$ and

$$y^* \geqslant 0 \Longleftrightarrow \left(\frac{q}{p}\right)^N \geqslant e^{\rho(c_H - c_L)},$$

so that indeed an optimum exists iff $N \geqslant \frac{\rho(c_H - c_L)}{\ln q - \ln p}$.
Furthermore, the limited liability constraint is binding iff

$$x^* > 1 \Longleftrightarrow \frac{1-q^N}{1-p^N} > \frac{1-e^{-\rho(M+c_L)}}{1-e^{-\rho(M+c_H)}}.$$

Now it remains to show that

$$N \geqslant N^* \Longleftrightarrow \frac{1-q^N}{1-p^N} \geqslant \frac{1-e^{-\rho(M+c_L)}}{1-e^{-\rho(M+c_H)}}.$$

Given the definition of N^*, it is sufficient to show that $\frac{1-q^N}{1-p^N}$ is increasing in N.

$$\begin{aligned}
\frac{d}{dN}\left(\frac{1-q^N}{1-p^N}\right) &= \frac{p^N \ln p(1-q^N) - q^N \ln q(1-p^N)}{(1-p^N)^2} \\
&= \frac{1-q^N}{1-p^N}\left(\frac{p^N}{1-p^N}\ln p - \frac{q^N}{1-q^N}\ln q\right).
\end{aligned}$$

Now, let us introduce the function

$$g(x) = \frac{x \ln x}{1-x}. \tag{19}$$

Then

$$\begin{aligned}
g'(x) &= \frac{(\ln x + 1)(1-x) + x \ln x}{(1-x)^2}, \\
&= \frac{\ln x - x\ln x + 1 - x + x \ln x}{(1-x)^2}, \\
&= \frac{\ln x + 1 - x}{(1-x)^2} \leqslant 0.
\end{aligned}$$

The last inequality follows from the fact that $\ln x \leqslant x - 1$. Therefore,

$$\begin{aligned}
\frac{d}{dN}\left(\frac{1-q^N}{1-p^N}\right) &= \frac{1-q^N}{1-p^N}\left(g(p^N) - g(q^N)\right) \cdot \frac{1}{N}, \\
&\geqslant 0.
\end{aligned}$$

Moreover,

$$\left(\frac{1-q^N}{1-p^N}\right)_{N=1} = \frac{1-q}{1-p} < 1,$$

and

$$\lim_{N\to\infty} \frac{1-q^N}{1-p^N} = 1.$$

Therefore,

$$\frac{1-q^N}{1-p^N} \leqslant \frac{1-e^{-\rho(M+c_L)}}{1-e^{-\rho(M+c_H)}} \;\Leftrightarrow\; N \leqslant N^*.$$

ii) It follows from the proof of i) that

$$\frac{1-q^N}{1-p^N} > \frac{1-e^{-\rho(M+c_L)}}{1-e^{-\rho(M+c_H)}}$$

implies that $x^* > 1$, so that the limited liability constraint $s_L \geqslant 0$ is binding. It then follows that the optimal compensation under y_H satisfies

$$\begin{aligned} s_H = \min\; & -\tfrac{1}{\rho}\ln(y) \\ \text{s.t.}\quad & e^{\rho c_H}[p^N x + (1-p^N)y] \leqslant e^{\rho c_L}[q^N x + (1-q^N)y] \\ & e^{\rho c_H}[p^N x + (1-p^N)y] \leqslant e^{-\rho M} \\ & 0 < y \leqslant 1 \\ & x = 1 \end{aligned}$$

It can be verified that the incentive compatibility constraint is binding, and

$$s_H = -\frac{1}{\rho}\ln(y_c^*),$$

where

$$y_c^* = \frac{q^N e^{-\rho(M+c_H)} - p^N e^{-\rho(M+c_L)}}{(1-p^N)\,e^{-\rho(M+c_L)} - (1-q^N)\,e^{-\rho(M+c_H)}}. \tag{20}$$

Proof of Proposition 2

Let us introduce the following notation:

$$\widetilde{x} = \frac{q^N\alpha - p^N + (1-\alpha)}{q^N - p^N},$$
$$\widetilde{y} = \frac{q^N\alpha - p^N}{q^N - p^N},$$
$$\widetilde{y}_c = \frac{q^N\alpha - p^N}{q^N\alpha - p^N + (1-\alpha)}.$$

Then, for $N \leqslant N^*$,

$$s_L = M + c_L - \frac{1}{\rho}\ln\left(\widetilde{x}\right),$$
$$s_H = M + c_L - \frac{1}{\rho}\ln\left(\widetilde{y}\right),$$

and for $N > N^*$

$$s_H = -\frac{1}{\rho}\ln\left(\widetilde{y}_c\right).$$

Moreover, for all N,

$$s_H - s_L = -\frac{1}{\rho}\ln\widetilde{y}_c.$$

Therefore, it is sufficient to show that: i) $\widetilde{x}$ is increasing in N, ii) $\widetilde{y}$ is increasing in N, and iii) $\widetilde{y}_c$ is increasing in N for $N \leqslant \widetilde{N}$, and decreasing in N for $N > \widetilde{N}$.

i)

$$\begin{aligned}\frac{d\widetilde{x}}{dN} &= \frac{\left(q^N \cdot \ln q \cdot \alpha - p^N \cdot \ln p\right) * \left(q^N - p^N\right)}{(q^N - p^N)^2} \\ &\quad - \frac{\left(q^N \cdot \ln q - p^N \cdot \ln p\right) * \left(q^N\alpha - p^N\right)}{(q^N - p^N)^2}, \\ &= \frac{1-\alpha}{(q^N - p^N)^2} \cdot (\ln p \cdot p^N(1-q^N) - \ln q \cdot q^N(1-p^N)).\end{aligned}$$

Therefore,

$$\frac{d\widetilde{x}}{dN} = \frac{(1-\alpha)(1-p^N)(1-q^N)}{N(q^N - p^N)^2}\left(g(p^N) - g(q^N)\right) \geqslant 0, \tag{21}$$

where the function $g(\cdot)$ is as defined in (19).

ii) Since

$$\widetilde{y} = \widetilde{x} - \frac{1-\alpha}{q^N - p^N},$$

it follows that

$$\begin{aligned}
\frac{d\widetilde{y}}{dN} &= \frac{d\widetilde{x}}{dN} - (1-\alpha)\frac{d}{dN}\left(\frac{1}{q^N - p^N}\right), \\
&= \frac{(1-\alpha)}{(q^N - p^N)^2}\left[\ln p \cdot p^N(1-q^N) - \ln q \cdot q^N(1-p^N) - \ln p \cdot p^N + \ln q \cdot q^N\right], \\
&= \frac{(1-\alpha)}{(q^N - p^N)^2} p^N q^N \left[\ln q - \ln p\right], \qquad (22) \\
&\geqslant 0.
\end{aligned}$$

iii) It can be verified that

$$\frac{dy_c^*}{dN} = (1-\alpha) * \frac{\alpha q^N \ln q - p^N \ln p}{(\alpha q^N - p^N + 1 - \alpha)^2} \quad , \qquad (23)$$

which is negative iff

$$\frac{\ln q}{\ln p}\left(\frac{q}{p}\right)^N \geqslant \frac{1}{\alpha} \Longleftrightarrow N \geqslant \widetilde{N}.$$

Proof of Proposition 3

Let us denote $\widetilde{s}_L$ and $\widetilde{s}_H$ for the optimal compensation scheme when there is no limited liability constraint. Then it follows immediately from the proof of Theorem 1 that $\widetilde{s}_L$ and $\widetilde{s}_H$ are given by (9) and (10), respectively, for any given value of N.
Note that

$$\begin{aligned}
&s_L = \widetilde{s}_L, && s_H = \widetilde{s}_H, && \text{if } N < N^*, \\
&s_L = \widetilde{s}_L - \widetilde{s}_L = 0, && s_H = \widetilde{s}_H - \widetilde{s}_L, && \text{if } N \geqslant N^*.
\end{aligned} \qquad (24)$$

i.e., as a consequence of the limited liability constraint, the compensation increases with the amount $-\widetilde{s}_L$.

As can be seen from the proof of Theorem 1, the individual rationality constraint is binding when $N < N^*$. Given (24), this implies that the manager's certain equivalent is given by:

$$\begin{aligned} CE(N) &= M, && \text{if } N < N^*, \\ &= M - \widetilde{s}_L, && \text{if } N \geqslant N^*. \end{aligned}$$

Now first note that the rent $CE(N) - M$ is zero when $N < N^*$, and equals $-\widetilde{s}_L$ when $N \geqslant N^*$. It follows from the proof of Proposition 2 that $-\widetilde{s}_L$ is strictly increasing in N.

Proof of Proposition 4

First consider the case $q = 1$. Then, the expected cost of compensation equals

$$s_L + (1 - p^N)(s_H - s_L),$$

where

$$\begin{aligned} s_L &= M + c_L, \\ s_H &= M + c_L - \frac{1}{\rho} \ln \frac{\alpha - p^N}{1 - p^N}. \end{aligned}$$

Now it can be verified that

$$\begin{aligned} \frac{d}{dN} \ln \frac{\alpha - p^N}{1 - p^N} &= \left(\frac{1 - \alpha}{\alpha - p^N} - \ln \left(1 + \frac{1 - \alpha}{\alpha - p^N} \right) \right) p^N \ln p \\ &\leqslant 0, \end{aligned}$$

where the inequality follows from the fact that $\ln x \leqslant x - 1$.

Now consider the case where $p = 0$. Then, the expected cost of compensation is given by s_H, and it follows immediately from Proposition 2 and the fact that $\widetilde{N} = 0$, that s_H is decreasing (increasing) in N for $N \leqslant N^*$ ($N > N^*$).

Proof of Theorem 5

Suppose that $N > \max\{N^*, \widetilde{N}\}$. Then, we know that $s_L = 0$ and s_H is increasing in N. Moreover, since $s_L = 0$, the expected cost equals

$$(1 - p^N) s_H.$$

The fact that $1-p^N$ is increasing in N completes the proof.

Proof of Theorem 6

First consider the case where $N \leqslant N^*$. Then the derivative with respect to N of the expected cost equals

$$D(\alpha) = -p^N \ln p(s_H - s_L) + (1-p^N)\frac{d}{dN}s_H + p^N \frac{d}{dN}s_L.$$

It follows from the proof of Proposition 2 that

$$\begin{aligned}\frac{d}{dN}s_L &= -\frac{1}{\rho}\frac{1-\alpha}{(q^N-p^N)}\frac{\ln p \cdot p^N(1-q^N) - \ln q \cdot q^N(1-p^N)}{(1-p^N)-(1-q^N)\alpha}, \\ \frac{d}{dN}s_H &= -\frac{1}{\rho}\frac{1-\alpha}{(q^N-p^N)}\frac{p^N q^N(\ln q - \ln p)}{q^N\alpha - p^N}.\end{aligned}$$

Some straightforward but tedious computations show that

$$D(\alpha) = \frac{p^N}{\rho} G\left(\frac{q^N\alpha - p^N}{1-\alpha}\right),$$

where

$$G(z) = \frac{1}{z^2+z} \cdot \frac{\ln p \cdot p^N\left(1-q^N\right) - \ln q \cdot q^N\left(1-p^N\right)}{q^N-p^N} + \ln p\left(\frac{1}{z} - \ln\left(1+\frac{1}{z}\right)\right).$$

Our goal is to determine the sign of $D(\alpha)$. Now, first notice that

$$\begin{aligned}&\lim_{z\to 0} G(z) \\ =& \lim_{z\to 0}\frac{1}{z}\left(\frac{1}{1+z}\frac{\left(\ln p \cdot p^N\left(1-q^N\right) - \ln q \cdot q^N\left(1-p^N\right)\right)}{(q^N-p^N)} + \ln p - \ln p \frac{\ln\left(1+\frac{1}{z}\right)}{\frac{1}{z}}\right), \\ =& \frac{(\ln p - \ln q)q^N(1-p^N)}{q^N-p^N}\lim_{z\to 0}\frac{1}{z}, \\ =& -\infty. \qquad (25)\end{aligned}$$

$$\lim_{z\to\infty} G(z) = 0. \qquad (26)$$

For ease of notation, we define

$$\begin{aligned}c_1 &= \frac{\ln p \cdot p^N\left(1-q^N\right) - \ln q \cdot q^N\left(1-p^N\right)}{q^N-p^N} > 0, \\ c_2 &= -\ln p > 0,\end{aligned}$$

$$
\begin{aligned}
f(z) &= \frac{1}{z+z^2}, \\
g(z) &= \frac{1}{z} - \ln\left(1+\frac{1}{z}\right).
\end{aligned}
$$

Then

$$
G(z) = c_1 f(z) - c_2 g(z).
$$

It can easily be verified that

$$
\begin{aligned}
f'(z) &= -\frac{1+2z}{z^2(1+z)^2}, \\
g'(z) &= -\frac{1}{z^2(1+z)}.
\end{aligned}
$$

Consequently,

$$
G'(z) = \frac{1}{z^2(1+z)^2}\left((c_2 - 2c_1)z + c_2 - c_1\right).
$$

Now, notice that

$$
\begin{aligned}
c_1 - c_2 &= \frac{1}{q^N - p^N} \cdot \left(\ln p \cdot p^N\left(1-q^N\right) - \ln q \cdot q^N\left(1-p^N\right) + \ln p\left(q^N - p^N\right)\right), \\
&= \frac{1}{q^N - p^N}(\ln p - \ln q) \cdot q^N\left(1-p^N\right) < 0.
\end{aligned}
$$

Now, we consider the following two situations:

- $2c_1 \leqslant c_2$

  Then since $c_1 - c_2 < 0$, it follows that $G'(z) > 0$ for all z. Combined with (25) and (26) this implies that $G(z) \leqslant 0$ for all z.

- $2c_1 > c_2$

  Then, since $c_1 - c_2 < 0$ and $2c_1 - c_2 > 0$ we know that $G'(z)$ has exactly one strictly positive root. Therefore, $G'(z) \geqslant 0$ for $z \leqslant \widetilde{z}$ and $G'(z) < 0$ for $z > \widetilde{z}$, where $\widetilde{z}$ denotes the unique positive root of $G'(z)$. Again, combined with (25) and (26), this implies that $G(z)$ has a unique positive root z^*. This implies that $D(\alpha) \leqslant 0$ for $\alpha \leqslant \alpha^*$, and $D(\alpha) > 0$ for $\alpha > \alpha^*$ where

  $$
  \alpha^* = \frac{z^* + p^N}{z^* + q^N}.
  $$

It now only remains to see that, since $\ln p = \frac{\ln p^N}{N}$ and $\ln q = \frac{\ln q^N}{N}$, it follows that $\left(p^N, q^N\right) \in S$ iff $2c_1 > c_2$.

Now, consider the case where $N > N^*$. Then, given (24), it follows that the derivative with respect to N of the expected cost equals

$$D_b\left(\alpha\right) = -p^N \ln p(s_H - s_L) + (1-p^N)\frac{d}{dN}s_H + (p^N - 1)\frac{d}{dN}s_L,$$

where s_H and s_L are as defined in (10) and (9), respectively.
Therefore,

$$D_b\left(\alpha\right) = \frac{1}{\rho}G_b\left(\frac{q^N\alpha - p^N}{1-\alpha}\right),$$

where

$$G_b\left(z\right) = \left(p^N\frac{1}{z^2+z} + \frac{1}{1+z}\right) * \frac{\ln p \cdot p^N\left(1-q^N\right) - \ln q \cdot q^N\left(1-p^N\right)}{q^N - p^N} + p^N \ln p\left(\frac{1}{z} - \ln\left(1+\frac{1}{z}\right)\right).$$

Again,

$$\lim_{z\to 0} G_b\left(z\right) = -\infty \quad (27)$$
$$\lim_{z\to\infty} G_b\left(z\right) = 0. \quad (28)$$

It can easily be verified that

$$G_b^{'}\left(z\right) = \frac{1}{z^2\left(1+z\right)^2}\left(-\frac{c_1}{p^N}z^2 + (c_2 - 2c_1)\,z + c_2 - c_1\right)$$

where

$$c_1 = p^N\frac{\ln p \cdot p^N\left(1-q^N\right) - \ln q \cdot q^N\left(1-p^N\right)}{q^N - p^N} > 0$$
$$c_2 = -p^N \ln p > 0.$$

Since $G_b^{'}\left(0\right) = +\infty$ and $-\frac{c_1}{p^N} < 0$, if follows that $G_b^{'}\left(z\right)$ has exactly one strictly positive root. This concludes the proof.