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Fixed cost messages *

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The occurrence and impact of fixed cost messages are analyzed by means of an asymmetric information game and related to the sender's stake in persuading the receiver, and the cost of a message.

1. Introduction

The typical base for the occurrence of informative signals of messages in asymmetric information games are *differential* exogenous signalling costs, meaning that the sender's signalling costs vary with his private information and/or with the 'content' of the signal [see, for example, the class of games in Cho and Sobel (1990)]. In the present paper it is shown that informative messages can also occur if there are no differential signalling costs, but if, alternatively, there is a *fixed* exogenous cost of sending a message (in addition to the endogenous cost/benefit of a signal due to the impact on the receiver's action).

Our model is close to Crawford and Sobel's (1982), in which it is shown that information transfer can occur in equilibrium even if messages bear *no* exogenous cost to the sender. A necessary condition for this to be possible is that the sender's and receiver's preference orderings are 'close' (in a well-defined sense). Such closeness of preferences will not be assumed in the present paper. Specifically, it will be assumed that the sender's preference ordering over actions, contrary to the receiver's, is independent of the state variable which is private information of the sender.

We think the model has relevance for asymmetric information situations in which signalling is mainly an informational but not costless activity, such as informational lobbying or advertizing. On the one hand, there may be a substantial difference of preferences over the receiver's action set, and it may be difficult for the receiver to gather the relevant information herself. On the other hand, there is no direct link between the cost of a message and the content of the message or the private information of the sender. In order words, a message bears a cost to the sender, but this cost is not related to what he says or to what he knows.

2. The model

There are two players, a sender S and a receiver R. The sender has private information on the value of some (state of the world) variable t, drawn by nature from the compact interval $T = [t_1, t_2]$

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according to a probability distribution $F(t)$ with continuous density function $f(t)$. After learning the value of t, S decides whether or not to send a message $s \varepsilon M$ to R, where M is the (measurable) set of feasible messages. [As will be seen in section 3, the specification of M is immaterial to the equilibrium outcomes as long as it is non-empty.] A sender having private information t will be referred to as an S of type t. If S decides not to send a message this will be denoted by $s = n$. After having received a particular 'signal' s – i.e., $s \in M$ or $s = n$ – R chooses an action $x \in IR$ which affects the payoffs of both R and S. All aspects of the game except t, which is private information of S, are common knowledge.

Let $u(x, t)$ denote R's twice continuously differentiable utility function over the consequences of action-state pairs. It is assumed that $u_{11}(\cdot) < 0$, $u_{12}(\cdot) > 0$, and the $u_1(x, t) = 0$ for some x. It follows that $x(t) := \text{argmax}_{x \in IR} u(x, t)$ is increasing in t and that R cannot be induced to take an action outside the set $X := [x(t_1), x(t_2)]$. Let $v(x, t)$ denote S's utility function, with $v_1(\cdot) > 0$ for all $t \in T$ and $x \in X$. Hence, the strategic incentive structure of the game is such that every type of S has an interest in persuading R that the value of t is (likely to be) 'high' since that will induce R to take 'large' action. In addition, it is assumed that $v_{12}(\cdot) > 0$, meaning that higher types have a larger stake in persuading R. It is easy to show that no information transmission can take place if $v_{12} < 0$.

It is assumed that sending a message bears a cost to S but not to R. [1] The *central assumption* in our model is that this cost is independent of both the 'content' of the message (i.e., the particular element of M) and the private information t of S. Sending no message bears no exogenous cost. Hence, $c(s) = 0$ if $s = n$ and $c(s) = c > 0$ if $s \in M$.

The game will be solved by means of the Perfect Bayesian equilibrium concept. The sender's signalling rule is denoted by $\sigma(s \mid t)$ and gives the probability that S type t sends signal s, with $s \in M \cup \{n\}$. The receiver's action rule is denoted by $\rho(s)$ and gives R's action in response to S's signal. [Note that, due to $u_{11} < 0$, R will never play a mixed strategy.] Finally, $g(t \mid s)$ denotes R's posterior beliefs. Formally, ρ and σ constitute a Perfect Bayesian Equilibrium (PBE) if:

(1) for each $t \in T$, $\sigma(n \mid t) + \int_M \sigma(m \mid t)\, \mathrm{d}m = 1$ and if $\sigma(s \mid t) > 0$ then s solves $\max_{s \in M \cup \{n\}} [v(\rho(s), t) - c(s)]$,
(2) for each s, $\rho(s)$ solves $\max_{x \in X} \int_T u(x, t) g(t \mid s)\, \mathrm{d}t$,
(3) $g(t \mid s) = \sigma(s \mid t) f(t) / \int_T \sigma(s \mid t) f(t)\, \mathrm{d}t$, whenever $\int_T \sigma(s \mid t) f(t)\, \mathrm{d}t > 0$.

Condition (1) requires that the sender's signalling rule is a best reply against the receiver's action rule. Condition (2) says that the receiver's action rule is optimal given its posterior beliefs about t after having received signal s and (3) requires the receiver's posterior beliefs to be Bayesian-consistent with its prior beliefs $f(t)$ and S's signalling strategy.

The next Lemma shows that all (sent) messages induce the same action.

Lemma 1. All messages $m \in M$ which are sent with positive probability induce the same action.

All types of S have the sames preference ordering over actions. Hence, the messages which induce a less favourable action by R will not be sent in equilibrium. It is not the content of a message as such that discloses information but merely that fact that a message is being received (or not).

[1] To assume that a message imposes an exogenous cost on R as well, would not affect the equilibrium *strategies*, provided that a message has no direct impact on R's preferences over x but only an indirect impact via the posterior beliefs. Of course, R's equilibrium *payoff* will be affected.

Consequently, nothing is lost – in terms of R's equilibrium actions – if we henceforth assume that M contains only one element, m say.

Before characterizing the equilibria, we introduce some useful notation. Let $x(p, q)$ denote R's best reply given that $p \leq t \leq q$, that is, $x(p, q) := \text{argmax}_{x \in X} \int_p^q u(x, t) f(t)\, dt$ if $p < q$, and $x(p, p) := x(p)$. Let $G(t)$ denote the utility gain for type t from pooling with all higher types instead of all lower types, that is, $G(t) := v(x(t, t_2), t) - v(x(t_1, t), t)(> 0)$. The next proposition establishes the possibility of an informative non-pooling equilibrium and demonstrates that it is a partition equilibrium in which the S types seperate in two groups.

Proposition. *If for some type p, $G(p) = c$, then $\sigma(n \mid t) = 1$ for all $t < p$, $\sigma(m \mid t) = 1$ for all $t > p$, $\rho(n) = x(t_1, p)$ and $\rho(m) = x(p, t_2)$ is a PBE.*

The higher types ($t > p$) send a costly message, whereas the lower types ($t < p$) do not. Type p is indifferent between sending a message or not. It follows that $\rho(n) < x(t_1, t_2) < \rho(m)$; both a message ($m$) and silence ($n$) make a difference relative to the prior beliefs. Moreover, both $\rho(n)$ and $\rho(m)$ increase with p, that is, decrease as the set of types that send a message becomes larger.

The proof of the Proposition follows by simple verification of the equilibrium conditions. Moreover, it is straightforward to proof that any non-pooling equilibrium must be a partition equilibrium of size two. However, a type p with $G(p) = c$ need not exist. It follows from the continuity assumptions that $G(t_1) < c < G(t_2)$ is a sufficient conditions for existence. However, it is easily checked that with $G(t_2) < c$, it is a PBE for no type to send a message, $\sigma(n \mid t) = 1$ for all t, and for R to respond with $\rho(n) = x(t_1, t_2)$ and $\rho(m) = x(t_2)$. In this case the cost of a message is prohibitive. Even the highest type – the type with the 'good' information – prefers to be disguised rather than to send a revealing but costly message. And with $G(t_1) > c$, it is a PBE for all types to send a message, $\sigma(m \mid t) = 1$ for all t, and for R to respond with $\rho(m) = x(t_1, t_2)$ and $\rho(n) = x(t_1)$. In this case even the lowest type prefers to send a costly but concealing message rather than to disclose its information by being silent. All types send a message which, therefore, has no impact on R's beliefs relative to the prior beliefs. In a sense, a message is 'too cheap talk'.

Finally, we note that even if a non-pooling PBE exists it need not be the unique PBE. Of course, the shape of $G(t)$ is crucial in this respect, and without any additional restrictions on the utility function(s) and prior beliefs, nothing can be said about it in general. A sufficient condition for uniqueness is that $G'(t) > 0$ for all t. For instance, with $u(x, t) = u(x - t)$ and $F(t)$ uniform, we always have uniqueness [cf. Crawford and Sobel (1982)]. We will come back to this in the next section.

3. Examples and comparative statics

Unambiguous comparative statics results cannot be derived for the general case. As the next, somewhat contrived, example shows, it may even be the case the more types send a message if the exogenous cost of sending a message increases!

Example 1. Let R's and S's utility function be given by $u(x, t) = -(x - t)^2$ and $v(x, t) = (t + 2)x$, respectively, and let the prior beliefs be $F(t) = t^2$, with $t \in [0, 1]$. Note that $u_{11} < 0$, $u_{12} > 0$, $v_1 > 0$, and $v_{12} > 0$, are required. Simple calculation reveals that $x(0, t) = \frac{2}{3}t$ and $x(t, 1) = \frac{2}{3}(t^2 + t + 1)/(t + 1)$. Hence, $x(t, 1) - x(0, t) = \frac{2}{3}(t + 1)^{-1}$ and $G(t) = \frac{2}{3}(t + 2)(t + 1)^{-1}$. It follows that there is a $p \in (0, 1)$ with $G(p) = c$, if $1 < c < \frac{4}{3}$. In this case, $\sigma(n) = 1$, for $t < p$, $\sigma(m) = 1$, for $t > p$, $\rho(n) = x(0, p)$ and $\rho(m) = x(p, 1)$, with $p = (\frac{4}{3} - c)(c - \frac{2}{3})^{-1}$, is a PBE. Moreover, it follows that

$dp/dc < 0$. Hence, more types will send a message if the cost of doing so increases. However, since $G(t_2) = G(1) = 1 < c$, it follows that $\sigma(n \mid t) = 1$ for all t, and $\rho(n) = x(0, 1) = 2/3$, $\rho(m) = x(t_2)$ is also a PBE.

It is not trivial to specify conditions for the utility function(s) and prior beliefs which ensure that the occurence of a message is less likely if the cost of a message increases. At a non-pooling equilibrium it holds that $G(p) = c$, and hence, that $dp/dc = [G'(p)]^{-1}$. Consequently, $dp/dc > 0$ if and only if $G'(p) = v_2(x(p, t_2), p)[dx(p, t_2)/dp] - v_2(x(t_1, p), p)[d(x(t_1, p)/dp] > 0$. Since $v_{12} = v_{21} > 0$, a sufficient condition for this inequality to hold – and for uniqueness of the PBE – is that $\Delta'(t) \geq 0$, for all $t \in T$, where $\Delta(t) := x(t, t_2) - x(t_1, t)$. In turn, a sufficient condition for the latter inequality to hold is that $u(x, t) = u(x - t)$ and $F(t)$ is uniform [cf. Crawford and Sobel (1982)]. Moreover, in this case it follows that $(d\rho(m)/dp)\, dp/dc > 0$, and $\Delta'(p)\, dp/dc > 0$, meaning that the impact of a message relative to prior beliefs and relative to silence, increases as the cost of a message increases.

Example 2. Let $F(t)$ be uniform on $T = [0, 1]$ and let R's and S's utility function be $u(x, t) = -(x - t)^2$, and $v(x, t) = \beta tx$, with $\beta > 0$. Now, $x(p_1, p_2) = \frac{1}{2}(p_1 + p_2)$ and $G(t) = \frac{1}{2}\beta t$. The definition of p, $G(p) = c$, implies that $p = 2c/\beta$. Hence, existence and uniqueness of a non-pooling equilibrium are ensured if $0 < c < \frac{1}{2}\beta$. Moreover, we see that more types will send a message if c (the cost of a message) decreases or if β (the mariginal utility of x to S) increases, but at the same time, since $\rho(m) = x(p, 1) = \frac{1}{2}(p + 1) = c/\beta + \frac{1}{2}$, such changes lower the impact of a message.

The result that more types send a message if S's stake in persuading R increases ($dp/d\beta < 0$) can easily be generalized. Introduce a parameter β, such that $v_{13}(x, t;\beta) := \partial v_1(\cdot)/\partial\beta > 0$. By $(\partial G/\partial p)\, dp + (\partial G/\partial\beta)\, d\beta = 0$ and $\beta G/\partial p > 0$, it follows that sign $(dp/d\beta) = -\text{sign}(\partial G/\partial\beta) = -\text{sign}(v_3(x(p, t_2), p; \beta) - v_3(x(t_1, p), p; \beta))$. Since $v_{31} = v_{13} > 0$ and $x(p, t_2) > x(t_1, p)$, we have $dp/d\beta < 0$. A message is more likely to be sent if there is more at stake for S. At the same time, since $d\rho(m)/d\beta < 0$, R's response to message is negatively related to S's stake in persuading. This result is supported by the experimental observation [see, e.g., Tedeshi et al. (1973, Ch. 4)] that the persuasiveness of messages decreases with the 'source's' stake in persuading the 'target'. Furthermore, it is consistent with the last part of Gross's (1971, p. 269) assertion that communications may be influential 'particularly when there is confidence in the wisdom or disinterestedness of the proposers or advisers'.

Moreover, the result that the impact of a message is increasing in c, is in line with the observation that the cost or trouble of writing letters is positively related to their impact on legislators [Bauer et al. (1963, p. 439)]. Further tentative support for this result can be found in Van der Putten (1980). After a thorough investigation of the realization of some important policies in the Netherlands, he concluded inter alia that reports from official advisory councils had a negligible impact on the policy process, whereas messages and reports from unofficial advizing agents did have a substantial impact on the policy process, especially in those cases where they were not invited by policymakers to give advice. Since official advisory councils have easy access to policymakers and are often even invited to send a report, they bear a lower cost of sending a message than the agents (interest groups) which have to take own initiative in getting their uninvited messages accross to policymakers. Therefore, the higher costs of the latter messages could be (part of) an explanation for their greater impact.

Finally, we note that the receiver's ex ante expected utility, $E[u] := \int_T \Sigma_{\{n,m\}} u(\rho(s), t)\sigma(s \mid t) f(t)\, dt$ is a function of p, and consequently, of the incentive structure of the game, in particular, of c. In Example 2, $E[u] = F(p)\text{Var}[t \mid t < p] + (1 - F(p))\text{Var}[t \mid t > p]$ is maximized by $p = \frac{1}{2}$, that is, if $c = \frac{1}{4}\beta$. Hence, if R could influence or manipulate c then R will generally have an incentive to do so. In particular, R could try to extract an 'optimal amount' of information by being more accessible or hospitable (a low c) for messages from less interested senders (a low β).

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