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GATED-TYPE POLLING SYSTEMS WITH WALKING AND SWITCH-IN TIMES

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ABSTRACT

We consider models of polling systems where switching times between channels are composed of two parts: walking times required to move from one channel (station) to another, and switch-in times that are incurred only when the server enters a station to render service.

We analyze three Gated-type systems: (i) Cyclic polling with Gated regime, (ii) Cyclic polling with Globally-Gated regime, and (iii) Elevator-type polling with Globally-Gated regime. In all systems, the server visits station i if and only if the number of customers

(jobs) present there at the gating instant is greater than or equal to a given threshold $K_i \geq 0$.
For all schemes we derive formulae for the multi-dimensional generating functions of the number of jobs in the various queues at polling instants, and derive bounds on mean waiting times, mean cycle times and mean number of jobs present in the system. For the Elevator scheme we further show that if switch-in times, as well as arrival rates, are equal among channels, then jobs' mean waiting times in all stations are equal.

1 Introduction

We consider polling systems with both walking and switch-in times. That is, when the server moves from one station (channel) to another, the switching times are composed of two parts: walking times required to move to the next station, and switch-in times that are incurred only if the server actually visits the channel. Such systems may also model access procedures of a reader-head to a hard disk in computers (see e.g., section 2 of Smith and Barnes [12]): the reader moves through the different tracks, and whenever an information is to be fetched, the head has to slow down before it gets into the track.

We assume that when the server polls (arrives at) a station it acquires the knowledge of how many jobs (customers) are present in the station's queue before it decides whether to switch in (visit) or not, and we are especially interested in service disciplines by which a station is not visited if it is empty. This is a special case of polling systems with threshold service disciplines, where the server visits a station to render service only if the number of jobs present there is larger than a prespecified threshold. Such disciplines are of interest since it may occasionally be advantageous to skip service to a station with only a small number of awaiting jobs in order to save switch-in times. Thus, threshold service disciplines, although difficult to analyze, may prove practical and more efficient.

We study the steady state behaviour of several gated-type polling schemes distinguished by their polling procedures, by their gating regimes and by their service disciplines. The polling procedures considered are the Cyclic and the Elevator-type (scan). The gating regimes are the Gated and Globally-Gated. In each queue, the service discipline is FIFO with threshold. A combination of a polling procedure, gating regime and a service discipline yields a specific polling scheme. We define a cycle as the time to complete a Hamiltonian tour through the stations, and analyze three schemes:

(i) a (Threshold) Gated regime with cyclic polling, where the server visits channel i if and only if the number of jobs present there is greater than or equal to a given threshold $K_i \geq 0$, $(i = 1, ..., N)$. If a station is visited, then only jobs that were present at the polling instant to that station are to be served during that visit.

(ii) the recently introduced (Boxma, Levy and Yechiali [4]) cyclic polling with Globally-Gated regime: a global gate is closed simultaneously on all stations at the beginning (gating instant) of each new cycle (i.e., upon the arrival of the server to station 1). The service discipline in each queue is FIFO with threshold $K_i \geq 0$, $(i = 1, ..., N)$. Hence station i will be visited if and only if the number of jobs present there at the (global) gating instant is at least K_i. If a station is visited, then only jobs that were present in that station at the beginning of the current cycle are to be served.

(iii) an Elevator type polling procedure in which instead of moving cyclically through the stations, the server first moves through the stations in the order 1,2,...,N-1,N ('up' cycle), and then moves in the opposite direction ('down' cycle), i.e., in the order N,N-1,...,2,1. The server then changes direction again and so on. This type of polling mechanism is encountered in many applications. For example, it models a common scheme of addressing a hard disk for writing (or reading) information on (or from) different tracks (see Tanenbaum [14] pp. 143-146, for a brief discussion of various techniques for head movement in disks). We consider again a FIFO threshold discipline with a Globally-Gated regime, where a new (global) gating instant is recorded at the beginning of each up or down cycle. (This model, without switch-in times and with $K_i = 0$, $(i = 1, ..., N)$, was introduced and analyzed by Altman, Khamisy and Yechiali [3]).

The cyclic polling with Gated regime and with thresholds $K_i \geq 0$ is studied in Section 2. We first obtain bounds on mean cycle time, $E[C]$, and on mean number of jobs, $E[X_i^j]$, present in queue j at a polling instant to queue i. We then obtain implicit equations for the joint generating functions of the number of jobs found in different stations at polling instants. This leads to expressions for the Laplace-Stieltjes Transform (LST) and first moment of the waiting times in the various stations, for which we derive various upper and lower bounds.

For the Cyclic polling, Globally-Gated regime (with thresholds $K_i = 1$) we obtain in Section 3 an implicit equation for the LST of the cycle duration, and derive formulae for the expected waiting times in the different stations. Several bounds are derived on moments of X_1^j and C, and extensions for $K_i > 1$, $(i = 1, ..., N)$, are discussed.

In Section 4 we study the Elevator polling procedure under the Globally-Gated regime, and show that the distribution of the cycle duration does not depend on the direction of the server. We then consider $K_i = 1$ for all stations. The LST of the cycle duration is derived by using the expressions obtained for the scheme with Cyclic polling and Globally-Gated regime. We calculate the expected waiting times for this polling scheme, and show that if the switch-in times as well as the arrival rates to all queues are equal, then the expected waiting times in all stations are equal. This extends the fairness result obtained in [3] for the fully non-symmetric case with only walking, but no switch-in, times.

Model and Notations

We consider a polling system with N independent channels, where channel i ($i = 1, 2, ..., N$) is modeled as an $M/G/1-$type queueing system. The arrival stream to station i is Poisson with rate λ_i, and service times are distributed as B_i, having LST $b_i^*(s)$ and first and second moments b_i and $b_i^{(2)}$, respectively. We denote by $\rho_i = \lambda_i b_i$ and by $\rho = \sum_{i=1}^N \rho_i$ the traffic offered to channel i, and to the system at large, respectively.

The time it takes to move from station i to the next is called the ith walking time, and is denoted by D_i. We assume that the walking times are independent, with LST $d_i^*(s)$ and with first and second moments d_i and $d_i^{(2)}$, respectively. Let $D = \sum_{i=1}^N D_i$ be the total walking time in a cycle, and denote by d, $d^{(2)}$ and $d^*(s)$ the expectation, second moment and LST of D, respectively

The time it takes from the moment the server arrives at (polls) the ith station till service can be started to jobs in that station is called the ith switch-in time and is denoted by R_i. We assume that the switch-in times are independent, with LST denoted by $r_i^*(s)$ and first and second moments by r_i and $r_i^{(2)}$, respectively. Define $r = \sum_{i=1}^N r_i$. These times, the walking times, the inter-arrival times and the service durations are mutually independent. Let X_i^j denote the number of jobs in station j at a polling instant to station i, and let $B_i(n)$ represent the total service time of n jobs in station i. Let $A_j(T)$ denote the number of arrivals to station j during a time interval of length T. Hence $A_j(B_i(X_i^i))$, $A_j(D_i)$, and $A_j(R_i)$ denote, respectively, the number of arrivals to station j during the service of, the walking time from, and the switch-in time to, station i.

2 Cyclic Polling with Gated Regime and Threshold Discipline

The Threshold–Gated service regime is a generalization of the regular Gated discipline: when the server arrives at (polls) station i and finds at least $K_i \geq 0$ jobs, then all X_i^i jobs that are present at that polling instant will be served (after a switch-in duration of length R_i). Jobs that arrive to the station after the polling instant will have to wait for the next cycle. If $X_i^i < K_i$, then the server moves on to the next station.

The evolution of the state of the system is described by

$$X_{i+1}^j = X_i^j + 1\{X_i^i \geq K_i\} \left[A_j(R_i) + A_j(B_i(X_i^i))\right] + A_j(D_i) \; ; \; j \neq i, \tag{1}$$

$$X_{i+1}^i = \begin{cases} X_i^i + A_i(D_i) & X_i^i < K_i \\ A_i(R_i) + A_i(B_i(X_i^i)) + A_i(D_i) & X_i^i \geq K_i \end{cases}, \tag{2}$$

where $1\{\cdot\}$ denotes the indicator function.

2.1 Mean Cycle time and $E[X_i^i]$

Let C_i be a random variable distributed as the duration of a cycle (in steady state) that starts at a polling instant to station i. Observe that $E[C_i] = E[C]$, for $i = 1, ..., N$ does not depend on i (whereas higher moments do).

To compute the mean cycle duration, $E[C]$, we note that the expected period that the server is not busy during a cycle (in steady state) is given by $\sum_{i=1}^N d_i + \sum_{i=1}^N r_i P(X_i^i \geq K_i)$. Since the fraction of time that the server is busy in a cycle is given by ρ, we obtain:

$$E[C] = \frac{\sum_{i=1}^N d_i + \sum_{i=1}^N r_i P(X_i^i \geq K_i)}{1-\rho}. \tag{3}$$

As the number of jobs present in a station at a polling instant is equal to the number of jobs that have arrived there during the last cycle plus those who were not served at the previous cycle (in case that there were less jobs than the threshold) we have:

$$E[X_i^i] = \lambda_i E[C] + \sum_{m=1}^{K_i - 1} m P(X_i^i = m). \tag{4}$$

2.2 Computable bounds on the moments of C and X_i^j

From (3) and (4) it follows that the complete calculation of $E[C]$ and $E[X_i^j]$ requires knowledge of the steady state probabilities of X_i^j. One may try to compute the latter from the generating functions of X_i^j, which are derived in the sequel (Section 2.3). However, these generating functions are given as the solution of a set of implicit equations, and it seems that the complexity of numerically solving these equations grows exponentially with the number of channels. This leads us to search for computable approximations or bounds.

We first note that $E[C]$ can be trivially bounded by using (3),

$$\frac{d}{1-\rho} \le E[C] \le \frac{d+r}{1-\rho}. \tag{5}$$

Bounds for $E[X_i^j]$ are trivially obtained by using (4) and (5).

A lower bound on $E[(X_i^j)^2]$ is obtained by noting that $X_i^j \ge_{st} A_i(C_i)$, where $A \le_{st} B$ means that A is stochastically smaller than B. Stochastic ordering of the form $A \le_{st} B$ between two random variables (or vectors) is equivalent to the fact that for any nondecreasing function f,

$$E[f(A)] \le E[f(B)]. \tag{6}$$

Thus,

$$E[(X_i^j)^2] \ge E[(A_i(C))^2] = \lambda_i^2 E[C^2] + \lambda_i E[C] \ge \lambda_i^2 (E[C])^2 + \lambda_i E[C]. \tag{7}$$

Any lower bound on $E[C]$ (e.g. (5)) can now be used in (7).

Better bounds on the first moment of the cycle time, as well as bounds on other moments of C and of X_i^j, can be derived as follows.

Upper bound on $E[C]$ using a Markov-type inequality

By using an idea similar in form to the Markov inequality (see [10] Vol. **I**, p. 388), it readily follows from (4) that

$$\lambda_i E[C] = E[X_i^j] - \sum_{m=1}^{K_i-1} mP(X_i^j = m) = \sum_{m=K_i}^{\infty} mP(X_i^j = m) \ge K_i P(X_i^j \ge K_i).$$

Hence

$$P(X_i^i \geq K_i) \leq \frac{\lambda_i E[C]}{K_i}.$$

Substituting into (3) yields

$$E[C] \leq \frac{d + \sum_{i=1}^N r_i \frac{\lambda_i E[C]}{K_i}}{1-\rho}.$$

Assuming now that $\rho + \sum_{i=1}^N r_i \lambda_i / K_i < 1$, we finally obtain

$$E[C] \leq \frac{d}{1 - [\rho + \sum_{i=1}^N r_i \lambda_i / K_i]}. \tag{8}$$

The upper bound on $E[C]$ is taken as the smallest between (5) and (8), when (8) applies.

Bounds based on Jensen's inequality

Consider the case $K_i = 1$ for some $i = 1, ..., N$. Then X_i^i is distributed as the number of Poisson arrivals (with rate λ_i) during a cycle time C_i. By using Jensen's inequality we get

$$\begin{aligned} P(X_i^i \geq 1) &= E\left[\sum_{j=1}^{\infty} e^{-\lambda_i C_i} \frac{(\lambda_i C_i)^j}{j!}\right] \\ &\geq \sum_{j=1}^{\infty} e^{-\lambda_i E[C]} \frac{(\lambda_i E[C])^j}{j!} \geq \sum_{j=1}^{\infty} e^{-\lambda_i C_{up}} \frac{(\lambda_i C_{low})^j}{j!}, \end{aligned} \tag{9}$$

where C_{up} and C_{low} are any upper and lower bounds on $E[C]$. One can use in particular the bounds in (5) and in (8). A lower bound on $E[C]$ is now obtained by substituting (9) into (3).

Following the same approach, an alternative upper bound on $E[C]$ can be obtained as follows.

$$\begin{aligned} P(X_i^i \geq 1) &= 1 - P(X_i^i = 0) = 1 - E[P(X_i^i = 0 | C_i)] \\ &= 1 - E\left[e^{-\lambda_i C_i}\right] \leq 1 - e^{-\lambda_i E[C]} \\ &\leq 1 - e^{-\lambda_i C_{up}}. \end{aligned} \tag{10}$$

By (3), (9) and (10) we finally have

$$\frac{d + \sum_{i=1}^N r_i \sum_{j=1}^{\infty} e^{-\lambda_i C_{up}} \frac{(\lambda_i C_{low})^j}{j!}}{1-\rho} \leq E[C] \leq \frac{d + \sum_{i=1}^N r_i \left(1 - e^{-\lambda_i C_{up}}\right)}{1-\rho}. \tag{11}$$

Note that the bounds in (11) can be again substituted (iteratively) in (11) in order to further improve the bounds. Such a process yields a (strictly) monotone decreasing series of upper bounds (all bounded by $d/(1-\rho)$). Hence, this series converges to a fixed point which gives a least upper bound. A similar situation exists with respect to the lower bound.

Bounds based on stochastic ordering

We present two kinds of bounds on X_i^j. Consider the following four systems.

System (i): Identical to the original polling system, except that switch-in times are always set to zero. Note that when $K_i = 1$ for all i, this system coincides with the standard gated model (e.g. Takagi [13]) with only walking times between stations.

System (ii): Also behaves like the original one, with the difference being that switch-in times are always incurred (even if a station is not visited). Note that for $K_i = 1$ ($i = 1, ..., N$), this system does not coincide with the standard gated model, since the gating in each station does not occur immediately before service starts there. The gating occurs before the switch-in time to that station. However, for $K_i = 1$, $i = 1, ..., N$ this system can be seen as a special case of the systems analyzed in [9]. (One has to add N dummy "father" stations, for which the arrival rate is zero).

System (iii): Differs from the original one in the following: (1) switch-in times are always incurred; (2) all jobs found in a station upon the arrival of the server are served (thus the threshold is set to zero), and (3) for each station i, $i = 1, ..., N$, if $K_i > 0$ then, in addition to the Poisson arrival of rate λ_i to that station, when the server leaves station i, $K_i - 1$ extra jobs appear in that station.

System (iv): This system differs from system (iii) in that (3) is replaced by:

(3a) For each station i, the walking times from that station requires an additional time that is equal to the sum of $(K_i - 1)$ i.i.d. service times, each distributed like B_i. Note that this system too does not coincide with the standard gated model, since the gating in each station does not occur immediately before service starts, but rather before the switch-in time to that station occurs. However, this system again is a special case of the systems analyzed in [9].

The notation $\underline{A}$ ($\bar{A}$, $\hat{A}$, $\tilde{A}$) will correspond to a quantity in system (i), (system (ii), (iii) and (iv), respectively).

Proposition 1 *The following holds:*

$$(\underline{X_i^1}, \underline{X_i^2}, ..., \underline{X_i^N}) \leq_{st} (X_i^1, X_i^2, ..., X_i^N) \leq_{st} (\overline{X_i^1}, \overline{X_i^2}, ..., \overline{X_i^N}), \tag{12}$$

$$\underline{C}_i \leq_{st} C_i \leq_{st} \overline{C}_i, \tag{13}$$

$$(X_i^1, X_i^2, ..., X_i^N) \leq_{st} (\hat{X}_i^1, \hat{X}_i^2, ..., \hat{X}_i^N), \tag{14}$$

$$C_i \leq_{st} \hat{C}_i, \tag{15}$$

$$C_i \leq_{st} \tilde{C}_i, \tag{16}$$

Proof: Inequalities (12) and (13) follow from [1] Section 4. Inequalities (14) and (15) follow from arguments similar to those in [1] Section 4. Coupling between the original system and system (iii), assuming that at time zero the same station is polled in both systems, one shows iteratively that if at time zero the number of jobs in each queue in the original system is not greater than the number of jobs in each queue in system (iii), then the number of jobs in each queue in the original system is less than or equal to the number of jobs in each queue in system (iii) at the nth time that a station is polled for all $n = 1, 2, ...$. This implies (14) and (15). In order to establish (16) one compares and couples the station times (see [6, 7, 8]) between the original system and system (iv). Since the distribution of the cycle times in steady-state does not depend on the initial distribution, we may assume without loss of generality that the N first station times in the original system are less than or equal to the N first station times in system (iv). Then, by an appropriate coupling, one can show inductively that all station times in the original system are less than or equal to those of system (iv) (sample-wise). This implies (16). ■

Proposition 1 implies in particular that, for any $k > 0$ and any $i, j = 1, ..., N$,

$$E[(\underline{X}_i^j)^k] \leq E[(X_i^j)^k] \leq E[(\overline{X}_i^j)^k],$$

$$E[(X_i^j)^k] \leq E[(\hat{X}_i^j)^k], \tag{17}$$

$$E[(\underline{C}_i)^k] \leq E[(C_i)^k] \leq E[(\overline{C}_i)^k], \qquad E[(C_i)^k] \leq E[(\hat{C}_i)^k],$$

and

$$E[(C_i)^k] \leq E[(\tilde{C}_i)^k]. \tag{18}$$

The above readily yields computable bounds for $E[X_i^j]$ and $E[X_i^j X_i^l]$, $i, j, l = 1, ..., N$, for the case $K_i = 1$. Indeed, for $K_i = 1$, the expressions for $E[\underline{X}_i^j]$ and $E[\underline{X}_i^j \underline{X}_i^l]$ are obtained by solving the sets of linear equations in Takagi [13], p. 106 (since $r_i = 0$ for all i). The expressions for $E[\overline{X}_i^j]$ and $E[\overline{X}_i^j \overline{X}_i^l]$ ($K_i = 1$) are obtained by solving sets of linear equations which are quite similar to those in Takagi [13], p. 106, or by using the solution in [9].

The bounds obtained on $E[C_i]$ following this approach coincide with those in (5). Moreover, this approach is also useful to lower-bound the second moment of the cycle times, since expressions for $E[(\underline{C}_i)^2]$ are known (see e.g. [6, 7, 8]). Expressions for $E[(\overline{C}_i)^2]$ can be obtained as the solution of the set of linear equations in [9] (which are similar to the one in [6, 7, 8]).

For the case $K_i > 1$, one can use (17) to get computable upper bounds for all moments of X_i^j. Indeed, for calculating any moment, one obtains in system (iii) a linear set of equations using the same method as in Takagi [13]. By (18), one can use [9] (an approach similar to the one in [6, 7, 8]) to calculate the moments of $\tilde{C}_i$ in system (iv), thus obtaining upper bounds for the second moments of the cycle times in the original system.

2.3 Generating Functions

We define a set of multi-dimensional joint generating functions, describing the vector-state of the system at a polling instant of queue i.

Let $F_i(\underline{z}) = E\left[\prod_{j=1}^N z_j^{X_i^j}\right]$. Let $\tilde{d}_i = d_i^*(\sum_{j=1}^N \lambda_j(1-z_j))$ and define similarly $\tilde{b}_i$ and $\tilde{r}_i$. Also, set $\tilde{d} = d^*(\sum_{j=1}^N \lambda_j(1-z_j))$ and set similarly $\tilde{b}$ and $\tilde{r}$. Using the evolution equations we obtain:

$$
\begin{aligned}
&F_{i+1}(\underline{z}) \\
&= \tilde{d}_i E\left\{ z_i^{1\{X_i^i<K_i\}X_i^i+1\{X_i^i\geq K_i\}[A_i(R_i)+A_i(B_i(X_i^i))]} \prod_{\substack{j=1\\ j\neq i}}^{N} z_j^{X_i^j+1\{X_i^i\geq K_i\}[A_j(R_i)+A_j(B_i(X_i^i))]} \right\} \\
&= \tilde{d}_i E\left\{ \prod_{\substack{j=1\\ j\neq i}}^{N} z_j^{X_i^j} E\left[\prod_{j=1}^{N} z_j^{A_j(R_i)+A_j(B_i(X_i^i))} 1\{X_i^i \geq K_i\} + z_i^{X_i^i} 1\{X_i^i < K_i\} \,\middle|\, X_i^i \right] \right\}
\end{aligned}
$$

$$= \tilde{d}_i E \left\{ \prod_{\substack{j=1 \\ j \neq i}}^{N} z_j^{X_i^j} \left[\tilde{r}_i \tilde{b}_i^{X_i^i} 1\{X_i^i \geq K_i\} + z_i^{X_i^i} 1\{X_i^i < K_i\} \right] \right\}$$

$$= \tilde{d}_i \Bigg[\tilde{r}_i \Big(F_i(z_1, z_2, ..., z_{i-1}, \tilde{b}_i, z_{i+1}, ..., z_N)$$

$$- E(z_1^{X_i^1} \cdot z_2^{X_i^2} \cdots \tilde{b}_i^{X_i^i} \cdot z_{i+1}^{X_i^{i+1}} \cdots z_N^{X_i^N} 1\{X_i^i < K_i\}) \Big)$$

$$+ E \left(z_1^{X_i^1} \cdot z_2^{X_i^2} \cdots z_i^{X_i^i} \cdot z_{i+1}^{X_i^{i+1}} \cdots z_N^{X_i^N} 1\{X_i^i < K_i\} \right) \Bigg].$$

For $K_i = 1$ we get a simpler relation:

$$F_{i+1}(\underline{z}) \tag{19}$$

$$= \tilde{d}_i \left[\tilde{r}_i F_i(z_1, z_2, ..., z_{i-1}, \tilde{b}_i, z_{i+1}, ..., z_N) + (1 - \tilde{r}_i) F_i(z_1, z_2, ..., z_{i-1}, 0, z_{i+1}, ..., z_N) \right].$$

The probabilities in (3) and (4) are given by

$$P(X_i^i = m) = \frac{1}{m!} \left. \frac{d^m F_i(1, ..., 1, z, 1, ..., 1)}{dz^m} \right|_{z=0}.$$

where z is in the ith place in $\underline{z}$.

Note that for the completely symmetric case $F_i(z_1, ..., z_N) = F_{i+1}(z_N, z_1, ..., z_{N-1})$. We may thus define $F(\underline{z}) = F_1(\underline{z})$, and obtain from equation (19),

$$F(z_1, ..., z_N) = \tilde{d}_1 \left[\tilde{r}_1 F(\tilde{b}_1, z_1, ..., z_{N-1}) + (1 - \tilde{r}_1) F(0, z_1, ..., z_{N-1}) \right]. \tag{20}$$

2.4 Waiting Times

Following Takagi [13], we define the random variables at steady-state:
$L_i(n)$ = number of jobs at station i that the nth departing job from the station (counting from the moment that the station was last polled) leaves behind it.
L_i = number of jobs at station i that an arbitrary departing job from the station leaves behind it.

Accordingly, we define the moment generating function: $Q_i(z) = E\left[z^{L_i}\right]$. As the distributions of number of jobs in the system at epochs of arrivals and epochs of departures are identical (see Kleinrock Vol. 1 [10], p. 232) then, by the well known PASTA phenomenon (Poisson Arrivals See Time Averages), $Q_i(z)$ also stands for the moment generating function of the number of jobs at station i in steady state at an arbitrary point in time. Following arguments similar to those in [13], p. 78, one gets

$$Q_i(z) = \frac{E\left(\sum_{n=1}^{X_i^i} z^{L_i(n)} 1\{X_i^i \geq K_i\}\right)}{E\left(X_i^i 1\{X_i^i \geq K_i\}\right)}.$$

Set $\bar{b}_i = b_i^*(\lambda_i - \lambda_i z)$. As $L_i(n) = X_i^i + A_i(R_i) - n + A_i(B_i(n))$, the evaluation of the expression for $Q_i(z)$ results in:

$$Q_i(z) = \frac{\frac{\bar{b}_i}{z - \bar{b}_i}\left\{E\left[z^{X_i^i} - \bar{b}_i^{X_i^i}\right] - \sum_{j=0}^{K_i - 1} P(X_i^i = j)\left(z^j - (\bar{b}_i)^j\right)\right\}}{E\left(X_i^i 1\{X_i^i \geq K_i\}\right)} \times r_i^*(\lambda_i(1 - z)).$$

As special cases, for $K_i = 0$ (see also Takagi [13] p. 109) and for $K_i = 1$ we obtain

$$Q_i(z) = \frac{\bar{b}_i}{E(X_i^i)(z - \bar{b}_i)}\left\{E\left[z^{X_i^i} - \bar{b}_i^{X_i^i}\right]\right\} \times r_i^*(\lambda_i(1 - z)),$$

from which, by differentiation, we derive

$$E[L_i] = \rho_i + \frac{\left(E[(X_i^i)^2] - E[X_i^i]\right)(1 + \rho_i)}{2E[X_i^i]} + \lambda_i r_i. \tag{21}$$

We may obtain the quantities $E[X_i^i]$ and $E[(X_i^i)^2]$ by differentiating $F_i(\underline{z})$ at $\underline{z} = \underline{1}$. For $K_i = 1$, $F_i(\underline{z})$ may be computed by solving a set of N implicit equations given by (19). Numerical methods can be used for the calculation of these quantities (e.g. the DFT approach, [11]). However, as mentioned before, the complexity of numerically solving these equations grows exponentially with the number of queues. Therefore, when moments of X_i^j are required, it seems more practical to use the bounds introduced in Sub-section 2.2.

The LST and expectation of the waiting time W_i of an arbitrary job in queue i are obtained using the relations

$$W_i^*(\lambda_i - \lambda_i z) b_i^*(\lambda_i - \lambda_i z) = Q_i(z),$$

$$\lambda_i E[W_i] + \lambda_i b_i = E[L_i]. \tag{22}$$

2.5 Bounds on the waiting times

$E[L_i]$ and $E[W_i]$ can be bounded immediately by substituting appropriately in (21) and (22) any upper and lower bounds on the two first moments of X_i^i (see Sub-section 2.2). We thus get

$$\rho_i + \frac{(E[(X_i^i)^2]_{low} - E[X_i^i]_{up})(1+\rho_i)}{2E[X_i^i]_{up}} + \lambda_i r_i \tag{23}$$

$$\leq E[L_i] \leq \rho_i + \frac{(E[(X_i^i)^2]_{up} - E[X_i^i)]_{low})(1+\rho_i)}{2E[X_i^i)]_{low}} + \lambda_i r_i,$$

$$\frac{(E[(X_i^i))^2]_{low} - E[X_i^i]_{up})(1+\rho_i)}{2E\lambda_i[X_i^i]_{up}} + r_i \leq E[W_i] \leq \frac{(E[(X_i^i)^2]_{up} - E[X_i^i)]_{low})(1+\rho_i)}{2\lambda_i E[X_i^i]_{low}} + r_i. \tag{24}$$

Note, however, that if we define $\underline{W}_i$ and $\overline{W}_i$ according to the convention introduced prior to (12), then both inequalities below need not hold

$$E[\underline{W}_i] \leq E[W_i] \leq E[\overline{W}_i].$$

Indeed, a counter example is presented in [2], Section 5.

3 Cyclic Polling with Globally-Gated Regime

In this Section we consider an extension of the cyclic-polling Globally-Gated regime introduced by Boxma, Levy and Yechiali [4]. In that scheme there is one prespecified station (say station 1), and whenever the (cyclically moving) server arrives to that station all jobs present in the various stations are marked (global-gating instant). Upon visiting a station, the server serves only marked jobs.

The novelty of our approach is again in incorporating into the model the real phenomenon observed in many polling systems that the additional switch-in time into a station is incurred only if service is to be given to jobs in that station. This happens if the number of marked jobs is at least one. We thus focus in this section on the FIFO threshold discipline with $K_i = 1, i = 1, ..., N$.

3.1 Cycle Duration and Number of Customers at a Gating instant

Let X_j denote the number of jobs in station j at the polling instant of station 1 (i.e., at gating instant). As in section 2, consider the system in steady-state and define D as the total walking time in a cycle, and R_i as the switching time into the ith station. A cycle is defined as the time between two consecutive gating instants and its duration is denoted by C. We have:

$$C = D + \sum_{i=1}^{N} B_i(X_i) + \sum_{i=1}^{N} 1\{X_i > 0\} R_i. \tag{25}$$

Let $\gamma(s) = E\left[e^{-sC}\right]$. For any set S of stations, $S \subset \{1, 2, ..., N\}$,

$$\begin{aligned} E\left[\prod_{j\in S} z_j^{X_j}\right] &= E\left\{E\left[\prod_{j\in S} z_j^{X_j} \middle| C\right]\right\} = E\left\{\exp\left(-\sum_{j\in S}\lambda_j(1-z_j)C\right)\right\} \\ &= \gamma\left(\sum_{j\in S}\lambda_j(1-z_j)\right), \end{aligned} \tag{26}$$

where the product over an empty set equals 1. Hence the joint probability generating function of the number of jobs at a gating instant is obtained as a function of the LST of the cycle duration, which we obtain (in an implicit form) as follows:

$$\gamma(s) = E\left\{E\left[e^{-sC}\middle| X_1, ..., X_N\right]\right\} = d^*(s) E\left\{\prod_{j=1}^{N}\left(b_j^*(s)\right)^{X_j} \prod_{j:X_j>0} E\left[e^{-sR_j}\right]\right\}. \tag{27}$$

Now,

$$\begin{aligned} & E\left\{\prod_{j=1}^{N}\left(b_j^*(s)\right)^{X_j} \prod_{j:X_j>0} E\left[e^{-sR_j}\right]\right\} \qquad (28) \\ &= E\left\{\prod_{j=1}^{N}\left(b_j^*(s)\right)^{X_j} \prod_{j:X_j>0} E\left[e^{-sR_j}\right] 1\{X_1 > 0\}\right\} \\ &\quad + E\left\{\prod_{j=1}^{N}\left(b_j^*(s)\right)^{X_j} \prod_{j:X_j>0} E\left[e^{-sR_j}\right] 1\{X_1 = 0\}\right\} \end{aligned}$$

$$= r_1^*(s) E\left\{\prod_{j=1}^{N}\left(b_j^*(s)\right)^{X_j} \prod_{\substack{j: X_j > 0 \\ j > 1}} E\left[e^{-sR_j}\right]\right\}$$

$$+ (1 - r_1^*(s)) E\left\{\prod_{j=1}^{N}\left(b_j^*(s)\right)^{X_j} \prod_{\substack{j: X_j > 0 \\ j > 1}} E\left[e^{-sR_j}\right] 1\{X_1 = 0\}\right\}$$

$$= \left(\prod_{j=1}^{N} r_j^*(s)\right) E\left\{\prod_{j=1}^{N}\left(b_j^*(s)\right)^{X_j}\right\}$$

$$+ \sum_{i_1=1}^{N}\left(\prod_{j=1}^{i_1-1} r_j^*(s)\right)\left(1 - r_{i_1}^*(s)\right) E\left\{\prod_{j=1}^{N}\left(b_j^*(s)\right)^{X_j} \prod_{\substack{j: X_j > 0 \\ j > i_1}} E\left[e^{-sR_j}\right] 1\{X_{i_1} = 0\}\right\}.$$

The expectation on the right-hand side of the last equation can be further expanded following the same procedure. We thus obtain,

$$E\left\{\prod_{j=1}^{N}\left(b_j^*(s)\right)^{X_j} \prod_{\substack{j: X_j > 0 \\ j > i_1}} E\left[e^{-sR_j}\right] 1\{X_{i_1} = 0\}\right\} \tag{29}$$

$$= \left(\prod_{j>i_1} r_j^*(s)\right) E\left\{\prod_{j=1}^{N}\left(b_j^*(s)\right)^{X_j} 1\{X_{i_1} = 0\}\right\} +$$

$$\sum_{i_2>i_1}\left(\prod_{i_1<j<i_2} r_j^*(s)\right)\left(1 - r_{i_2}^*(s)\right) E\left\{\prod_{j=1}^{N}\left(b_j^*(s)\right)^{X_j} \prod_{\substack{j: X_j > 0 \\ j > i_2}} E\left[e^{-sR_j}\right] 1\{X_{i_1} = X_{i_2} = 0\}\right\}$$

By applying the procedure in expressions (28) and in (29) repeatedly, and by using the fact that for any set $S \subset \{1, ..., N\}$,

$$E\left\{\prod_{j=1}^{N}\left(b_j^*(s)\right)^{X_j} 1\{X_j = 0,\ j \in S\}\right\} = \gamma\left(\sum_{j=1}^{N}\lambda_j - \sum_{j \notin S}\lambda_j b_j^*(s)\right),$$

we obtain from (27),

$$\gamma(s) = d^*(s)\left(\prod_{j=1}^{N} r_j^*(s)\right)\gamma\left(\sum_{j=1}^{N}\lambda_j - \sum_{j=1}^{N}\lambda_j b_j^*(s)\right) \tag{30}$$

$$+ \quad d^*(s) \sum_{i_1=1}^{N} \left(\prod_{j=1}^{i_1-1} r_j^*(s) \right) \left(1 - r_{i_1}^*(s)\right) \left[\left(\prod_{j>i_1} r_j^*(s) \right) \gamma \left(\sum_{j=1}^{N} \lambda_j - \sum_{\substack{j=1 \\ j \neq i_1}}^{N} \lambda_j b_j^*(s) \right) \right.$$

$$+ \sum_{i_2>i_1} \left(\prod_{i_1<j<i_2} r_j^*(s) \right) \left(1 - r_{i_2}^*(s)\right) \left[\left(\prod_{j>i_2} r_j^*(s) \right) \gamma \left(\sum_{j=1}^{N} \lambda_j - \sum_{\substack{j=1 \\ j \neq i_1, i_2}}^{N} \lambda_j b_j^*(s) \right) \right.$$

$$\bullet$$
$$\bullet$$
$$\bullet$$

$$\left. \left. + \sum_{i_N>i_{N-1}} \left(\prod_{i_{N-1}<j<i_N} r_j^*(s) \right) \left(1 - r_{i_N}^*(s)\right) \left[\gamma \left(\sum_{j=1}^{N} \lambda_j \right) \right] \cdots \right] \right] .$$

In order to simplify the above expression we introduce the set

$$S = \{\{i_1, ..., i_k\},\ 1 \le k \le N,\ 1 \le i_1 < i_2 < ... < i_k \le N\},$$

where the elements of S are sets denoted by S_j ($j = 1, ..., |S|$) and $S_j = \left\{i_1^j, ..., i_{k(j)}^j\right\}$, with $k(j)$ denoting the number of elements in S_j. Define $i_0^j = 0$. We then have

$$\begin{aligned} \gamma(s) \quad &= \quad d^*(s) \sum_{j=1}^{|S|} \left(\prod_{l=0}^{k(j)} \left(\prod_{i_l^j<n<i_{l+1}^j} r_n^*(s) \right) \right) \qquad (31) \\ &\times \left(\prod_{l=1}^{k(j)} (1 - r_{i_l^j}^*(s)) \right) \left(\prod_{l>i_{k(j)}^j} r_l^*(s) \right) \gamma \left(\sum_{l=1}^{N} \lambda_l - \sum_{l \notin S_j} \lambda_l b_l^*(s) \right) . \end{aligned}$$

For the 'weakly symmetrical' case in which $r_i^*(s)$, λ_i and $b_i^*(s)$ do not depend on i, but only the walking times d_i depend on i, Eq. (31) reduces to

$$\begin{aligned} \gamma(s) \quad &= \quad d^*(s) \sum_{j=1}^{|S|} \left(\prod_{l=0}^{k(j)} (r_1^*(s))^{i_{l+1}^j - i_l^j - 1} \right) \qquad (32) \\ &\times (1 - r_1^*(s))^{k(j)} \left[r_1^*(s)\right]^{N - i_{k(j)}^j} \gamma\left(N\lambda_1 - (N - k(j))\lambda_1 b_1^*(s)\right) . \end{aligned}$$

As in Boxma, Levy and Yechiali [4], we introduce C_P and C_R, the past and residual time, respectively, of a cycle. We have (see [4] eq. (2.11)):

$$E\left[e^{-sC_P}\right] = E\left[e^{-sC_R}\right] = \frac{1-\gamma(s)}{sE[C]}, \tag{33}$$

$$E[C_P] = E[C_R] = \frac{E[C^2]}{2E[C]}. \tag{34}$$

3.2 First two moments of C and X_i

To compute the first two moments of the cycle duration one may differentiate Eq. (31). However, a direct approach is simpler. Let $P_i(0)$ denote the probability that station i is found empty at the (global) gating instant. Taking expectation of (25) yields:

$$E[C] = d + \sum_{i=1}^{N} \rho_i E[C] + \sum_{i=1}^{N}(1-P_i(0))r_i \tag{35}$$

and

$$P_i(0) = E\left[e^{-\lambda_i C}\right] = \gamma(\lambda_i). \tag{36}$$

Thus,

$$E[C] = \frac{d + \sum_{i=1}^{N}(1-P_i(0))r_i}{1-\rho} = \frac{d + \sum_{i=1}^{N}(1-\gamma(\lambda_i))r_i}{1-\rho}. \tag{37}$$

Using again eq. (25) the second moment is given by

$$\begin{aligned} E[C^2] &= d^{(2)} + E\left[\left(\sum_{i=1}^{N} B_i(X_i)\right)^2\right] + \sum_{i=1}^{N}(1-\gamma(\lambda_i))r_i^{(2)} \\ &+ \sum_{i\neq j} r_i r_j E[1\{X_i>0\}1\{X_j>0\}] \\ &+ 2d\rho E[C] + 2d\sum_{i=1}^{N}(1-\gamma(\lambda_i))r_i + 2\sum_{i=1}^{N} r_i\left[\rho_i E[C] + \sum_{j\neq i} b_i E(X_i 1\{X_j>0\})\right]. \end{aligned} \tag{38}$$

To complete the calculation of $E[C^2]$ we use $E(X_i) = \lambda_i E[C]$ and $E[(X_i)^2] = \lambda_i E[C] + \lambda_i^2 E[C^2]$. Furthermore, for $i \neq j$,

$$\begin{aligned} E(X_i 1\{X_j>0\}) &= E(X_i) - E(X_i 1\{X_j=0\}) = \lambda_i E[C] - E(X_i 1\{X_j=0\}), \\ E(X_i 1\{X_j=0\}) &= \frac{d}{dz_i} E\left[z_i^{X_i} z_j^{X_j}\right]\Big|_{z_i=1,z_j=0} = \frac{d}{dz_i}\gamma\left(\lambda_i(1-z_i)+\lambda_j\right)\Big|_{z_i=1} \end{aligned}$$

$$
\begin{aligned}
&= \lambda_i \frac{d}{ds}\gamma(s)\Big|_{s=\lambda_j}, \\
E(1\{X_i > 0\}1\{X_j > 0\}) &= 1 - \gamma(\lambda_i) - \gamma(\lambda_j) + \gamma(\lambda_i + \lambda_j), \\
E[X_i X_j] &= \lambda_i \lambda_j E[C^2].
\end{aligned}
$$

Thus,

$$
\begin{aligned}
E\left[\left(\sum_{i=1}^{N} B_i(X_i)\right)^2\right] &= \sum_{i=1}^{N} var(B_i) E X_i + \sum_{i,j} b_i b_j E(X_i X_j) \\
&= \sum_{i=1}^{N} \lambda_i b_i^{(2)} E[C] + \sum_{i,j} \rho_i \rho_j E[C^2] \\
&= \sum_{i=1}^{N} \lambda_i b_i^{(2)} E[C] + \rho^2 E[C^2].
\end{aligned}
$$

Substituting these expressions in (38) results in

$$
E[C^2] = (1-\rho^2)^{-1}\left(d^{(2)} + 2d\rho E[C] + \sum_{i=1}^{N} \lambda_i b_i^{(2)} E[C] + \sum_{i=1}^{N}(1-\gamma(\lambda_i)) r_i^{(2)}\right.
$$

$$
\left. +2d\sum_{i=1}^{N}(1-\gamma(\lambda_i)) r_i + \sum_{i\neq j} r_i r_j \gamma(\lambda_i + \lambda_j) + 2\sum_{i=1}^{N} r_i \rho_i \left[E[C] + \sum_{j\neq i} \frac{d}{ds}\gamma(s)\Big|_{s=\lambda_j}\right]\right). \quad (39)
$$

3.3 Bounds on moments of C and X_i

Several approaches, similar to those used for the Gated regime, can be applied to obtain bounds on moments of C and X_i. We first note that (37) implies that the trivial bound (5) on the expected cycle duration holds for the Globally-Gated regime as well. Since X_i are distributed like the number of Poisson arrivals (with rate λ_i) during a cycle time C, we can apply again the approach based on Jensen inequality, which implies that (11) holds for the Globally-Gated regime too. The bounds based on the stochastic ordering (12) and (13) can be shown to hold as well. We exploit this to further improve the upper bound on C. Consider system (i) and (ii) defined prior to (12). It follows from (13) that

$$
-\underline{C} \geq_{st} -C \geq_{st} -\overline{C},
$$

so (6) now implies that (for $s \geq 0$)

$$\overline{\gamma}(s) \le \gamma(s) \le \underline{\gamma}(s).$$

Combining this with (37) finally yields

$$\frac{d + \sum_{i=1}^{N}(1 - \underline{\gamma}(\lambda_i))r_i}{1-\rho} \le E[C] \le \frac{d + \sum_{i=1}^{N}(1 - \overline{\gamma}(\lambda_i))r_i}{1-\rho}. \tag{40}$$

The advantage of using this approach for the Globally-Gated regime is that explicit expressions exist for $\underline{\gamma}(\cdot)$, and similar ones can be derived for $\overline{\gamma}(\cdot)$, see Eq. (2.7) in [4]. Clearly, since $E[X_i] = \lambda_i E[C]$, the bounds (40) result in corresponding bounds on $E[X_i]$.

Remark: For $K_i > 1$ it follows easily that the bounds (7), (12), (13), (14), (15) and (16) hold as well (with X_i replacing X_i^i). (12) and (13) imply (5). One can easily derive exact expressions for systems (iii)-(iv) using the same approach as in [4]. System (iv) can be seen as a special case of the standard Globally-Gated model [4], if one adds an additional dummy station, (with no arrivals), at which the global gating occurs.

3.4 Waiting Times

Consider an arbitrary job M at station k. Its waiting time is composed of (i) the residual cycle time C_R, (ii) the service times of all jobs who arrive at stations 1 to $k-1$ during the cycle in which M arrives, (iii) the walking times through stations 1 to $k-1$, (iv) the service times of all jobs who have arrived at station k during the past part C_P of the cycle in which M arrives, (v) the switch-in times that occurred in stations $1, ..., k$. The first four terms are identical to those appearing in the 'standard' Globally-Gated regime [4]. Denoting by $W_k^{(m)}$ the mth component of the waiting time of M, from equation (2.17) of Boxma, Levy and Yechiali [4], the sum of the expectations of the first four terms is given by

$$\sum_{m=1}^{4} E\left[W_k^{(m)}\right] = (1 + 2\sum_{i=1}^{k-1} \rho_i + \rho_k)E[C_R] + \sum_{i=1}^{k-1} d_i. \tag{41}$$

The expectation of the fifth term is

$$E[W_k^{(5)}] = r_k + \sum_{i=1}^{k-1} r_i(1 - P_i(0)) = r_k + \sum_{i=1}^{k-1} r_i(1 - \gamma(\lambda_i)). \tag{42}$$

Note that r_k is incurred as there is at least one job (M) in station k. Clearly, $E[W_k] = \sum_{m=1}^{5} E[W_k^{(m)}]$. It readily follows from (41) and (42) that

$$E[W_{k+1}] - E[W_k] = (\rho_{k+1} + \rho_k)E[C_R] + d_k + r_{k+1} - r_k\gamma(\lambda_k). \tag{43}$$

As expected, the difference (43) depends on the probability of no arrivals to station k during a cycle.

Finally, we note that the expected waiting times can be easily bounded by using

$$\frac{{C_{low}}^2}{2C_{up}} \leq E[C_R] \leq \frac{{C_{up}}^2}{2C_{low}},$$

where C_{low} (C_{up}) and C^2_{low} (C^2_{up}) are arbitrary lower (upper) bounds on the first and second moments of C. For bounds on $(1 - \gamma(\lambda_i))$, one can use the bounds derived in (40).

4 Elevator-Type Polling, Globally-Gated Regime

In this Section we consider an Elevator-type polling procedure in which, instead of moving cyclically through the stations, the server first visits the stations in the order 1,2,...,N, ('up' cycle), and then reverses its orientation and visits the stations in the opposite direction ('down' cycle), i.e., going through stations N,N-1,...,2,1. It then changes direction again and so on. We assume that the walking time distributions are the same in both directions, i.e., the walking time from any station j to station $j+1$ has the same distribution as the walking time from station $j+1$ to station j. Note that, compared with a cyclic polling procedure, the Elevator-type polling procedure saves the return walking time from station N to station 1. Thus, the total walking time in any direction is $D' = \sum_{i=1}^{N-1} D_i$ with d', $d'^{(2)}$ and $d'^*(s)$ denoting, respectively, the expectation, second moment and LST of D'.

We consider again a FIFO threshold discipline with a Globally-Gated regime, where a new (global) gating instant is recorded at the beginning of each up or down cycle. (This model, without switch-in times and with $K_i = 0$ was introduced and analyzed by Altman, Khamisy and Yechiali [3]). As in [3], a 'cycle' will be either an up cycle or a down cycle, and the distribution of its duration does not depend on the direction.

4.1 Cycle Duration

The discussion above is summerized in the following proposition, whose proof follows the same arguments as in Lemma 1 of [3].

Proposition 2 *The distribution of a cycle duration (in steady state) does not depend on the direction, and the expressions for the distribution and moments of the cycle duration are those given in Section 3 (see (30), (35) and (39)) where D' takes the role of D.*

4.2 Waiting Times

Consider an arbitrary job M at queue k. As the distributions of the up and down cycles are the same, with probability 0.5 it arrives during an up cycle, and with probability 0.5 it arrives during a down cycle. Thus, denoting the waiting time by W_k, we can write

$$E[W_k] = 0.5\left(E\left[W_k \middle| \begin{matrix} server \\ moves\ up \end{matrix}\right] + E\left[W_k \middle| \begin{matrix} server \\ moves\ down \end{matrix}\right]\right). \tag{44}$$

Below, we shall restrict to $K_i = 1$, $i = 1, ..., N$. The waiting time of job M, if it arrives when the server moves down, is composed of (i) the residual cycle time C_R, (ii) the service times of all jobs who arrive at queues $i < k$ during the (down) cycle in which M arrives, (iii) the walking times from queue 1 to queue k, (iv) the service times of all jobs who arrive at queue k during the past part C_P of the cycle in which M arrives, (v) the switch-in times that occurred in stations $1, ..., k$. As all cycles possess the same distribution, using (36), (41) and (42), we write

$$E[W_k|down] = (1 + 2\sum_{i=1}^{k-1} \rho_i + \rho_k)E[C_R] + \sum_{i=1}^{k-1} d_i + r_k + \sum_{i=1}^{k-1} r_i(1 - \gamma(\lambda_i)). \tag{45}$$

Similarly,

$$E[W_k|up] = (1 + 2\sum_{i=k+1}^{N} \rho_i + \rho_k)E[C_R] + \sum_{i=k}^{N-1} d_i + r_k + \sum_{i=k+1}^{N} r_i(1 - \gamma(\lambda_i)). \tag{46}$$

Combining (44), (45) and (46) we obtain

$$E[W_k] = (1 + \rho)E[C_R] + 0.5d' + r_k + 0.5\sum_{i \neq k} r_i(1 - \gamma(\lambda_i)). \tag{47}$$

$E[C_R]$ is given by (34), (37) and (39) with d' and $d'^{(2)}$ replacing d and $d^{(2)}$, respectively. It follows that whenever r_i and λ_i are equal for all channels, the expected jobs' waiting times are equal in all stations. This generalizes the 'fairness' result reported in [3]. (For further discussion on fairness, the reader is referred to Boxma [5]).

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