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Abstract

We present a dynamic microstructure model where a dealer market (DM) and a crossing network (CN) interact. Sequentially arriving agents with different valuations for an asset maximize their profits either by trading at a DM or by submitting an order for (possibly) uncertain execution at a CN. We develop the analysis for three different informational settings: transparency, "complete" opaqueness of all order flow, and "partial" opaqueness (with observable DM trades). A key result is that the interaction of trading systems generates systematic patterns in order flow for the transparency and partial opaqueness settings. The precise nature of these patterns depends on the degree of transparency at the CN. While unambiguous with a transparent CN, they may reverse in direction if the CN is opaque. Moreover, common to the three informational settings, we find that a CN and a DM cater for different types of traders. Investors with a high willingness to trade are more likely to prefer a DM. The introduction of a CN next to a DM also affects welfare as it increases total order flow by attracting traders who would otherwise not submit orders ("order creation"); in addition, it diverts trades from the DM ("trade diversion"). We find that the coexistence of a CN and DM produces greater trader welfare than a DM in isolation. Also, more transparent markets lead to greater *trader* welfare but may reduce *overall* welfare.

JEL Codes: G10, G20

Keywords: Alternative Trading Systems, Crossing Network, Dealer Market, Order Flow, Transparency, Welfare

1 Introduction

An open issue in market microstructure is how investors behave when an asset trades simultaneously on several markets that may show a different degree of transparency. The topics of competition between markets and the optimal degree of transparency have become even more relevant in recent years with the emergence of Alternative Trading Systems (ATSs). These ATSs operate next to traditional exchanges and exhibit distinct institutional characteristics. Therefore, when submitting an order, traders face the decision where to trade, taking into account the advantages and disadvantages of each trading system.

In this paper we deal with Crossing Networks (CNs) which are one type of ATS. CNs are defined by the SEC (1998) as "systems that allow participants to enter unpriced orders to buy and sell securities. Orders are crossed at a specified time at a price derived from another market" (i.e. the continuous market). A pioneering CN is ITG's POSIT but also the NYSE has introduced post-close crossing sessions already in 1990.[1] Currently, other traditional markets are adding crossing facilities into their market structure, see e.g. Deutsche Börse's Xetra XXL in September 2001 or the Nasdaq Crossing Network in May 2007. Recently, also investment banks opt to pool institutional order flow into CNs as a response to new regulatory initiatives like Regulation NMS in the US and MiFID in Europe (see e.g. the Block Interest Discovery Service initiative (BIDS)). Despite the prevalence of CNs next to continuous markets, the dynamic aspects of the coexistence of these systems have not been well explored yet. In this paper we investigate the interaction of a CN and a continuous dealer market (DM) by analyzing the impact of different degrees of transparency on the composition and the dynamics of the order flow on both systems. In this way, we address long-standing questions within the market microstructure literature: where do investors trade when there are multiple trading venues for a single asset and what are the welfare implications of different degrees of transparency?

Our model for studying traders' trading venue decisions starts from that of Parlour (1998). While she models a limit order market, we deal with sequentially arriving traders having the choice between a CN and a DM. When both trading systems coexist, traders can obtain guaranteed execution in the DM, opt for cheaper but (possibly) uncertain execution on the CN, or refrain from trading. An important feature of the competition between CNs and traditional

[1] ITG's POSIT is used by approximately 550 major institutions and broker/dealers and crossed about 35 million US shares per day in November 2005 according to Towergroup. The total market volume amounted up to 98 million shares per day in the same period as compared to almost 1.8 billion for the NYSE.

markets is that they offer a different degree of transparency (see e.g. Bloomfield and O'Hara (2000)). Whereas traditional markets may vary in their degree of mandated transparency, Regulations ATS and NMS in the US and MiFID in Europe do not require CNs to provide information on their order book. We therefore investigate how different degrees of transparency at both markets influence traders' order submission strategies and determine welfare in the economy. More specifically, we develop the analysis for three different informational settings: transparency, "complete" opaqueness, and "partial" opaqueness. The transparency case occurs when traders are fully informed about past order flow at both markets and hence observe the prevailing CN order book before determining their order choice. In reality, however, CNs are rather opaque. We incorporate this informational environment by analyzing partial opaqueness: traders only observe previous trades at the DM. Complete opaqueness implies that both markets are opaque such that traders are uninformed on past CN and DM order flow.

Common to the three informational settings, we find that an increase in the DM's relative spread augments the CN's order flow. Moreover, the existence of a CN results in "order creation": investors with a relatively lower willingness to trade submit orders to a CN whereas they would never trade at a DM. This order creation effect induced by the CN is confirmed empirically in Gresse (2006). We also find a "trade diversion" effect, since the introduction of a CN causes some trades to be diverted away from the DM to the new trading venue. A key result of our paper is that the transparency and partial opaqueness settings generate systematic patterns in order flow. Although this result is reminiscent of the findings in Parlour (1998) for a limit order market, two major differences exist. First, in Parlour's model market and limit orders each have implications for future order flow. In our model, by contrast, only CN orders produce systematic patterns in order flow, as only the (expected) imbalance between the queue of buy and sell orders at the CN matters. Clearly, DM trades do not influence this imbalance. Secondly, we show that the transparency of a CN matters for the nature of the order flow patterns: compared to a transparent CN, order flow patterns may invert when an opaque CN operates next to a transparent DM. The result that order flow is informative about execution probabilities is novel to the market microstructure literature. The intuition for this informativeness of order flow is that, when markets are partially opaque, observing no order flow relative to a DM-trade may be good news for a successive CN order as it increases the potential for good counterparties; no order may also be bad news when it entails the preemption of a successive CN order. Overall, these theoretical insights point to time-varying order flow at a CN and trade flow at a DM, even in the absence of asymmetric information. Hence, it is important to take the interaction between trading systems, as well as their individual microstructure, into account when measuring "normal" order flow. For example, successive DM buys caused by a persistently unfavorable imbalance of the CN-book could wrongly be attributed to the activity of informed buyers. Further, our dynamic model displays two externalities as documented in the static model of Hendershott and Mendelson (2000). On the one hand, the CN is characterized by a positive (liquidity) externality as adding a CN buy (sell) order is beneficial to future sellers (buyers). On the other hand, a CN exhibits a negative (crowding) externality as early arriving investors with a low willingness to trade may preempt those with a higher willingness to trade who arrive later on the same side of the market. Finally, a CN and a DM are shown to cater for different types of traders: unless the CN offers certainty of execution, investors with a higher willingness to trade are more inclined to trade at a DM.

Furthermore, we formally define and compare welfare for the CN in isolation, DM in isolation, and coexistence of markets, to analyze the impact of different degrees of opaqueness on welfare. Our welfare analysis builds on previous work that studies welfare and the optimal degree of transparency (see e.g. Pagano and Roëll (1996), Glosten (1998), Bloomfield and O'Hara (2000), Viswanathan and Wang (2002), Parlour and Seppi (2003), Goettler, Parlour and Rajan (2005), and Rindi (2007)). Our paper complements this literature by considering the impact of transparency on welfare in a setting where trading systems compete for uninformed order flow. We employ two complementary welfare measures: "overall welfare" which measures the gains from trade of all parties involved (including dealers), and "trader welfare" which takes the traders' perspective only. Our welfare results can be summarized as follows. First, when comparing markets in isolation, we find that a DM offers both greater trader and overall welfare than a CN when the execution probability at the CN and the relative spread are low. That is when the time to a cross is short and the value of the asset is high. Second, coexistence leads to greater trader welfare but may decrease overall welfare compared to a DM in isolation. Third, more transparency unambiguously increases trader welfare as traders anticipate their orders are revealed to potential counterparties. The impact of the degree of transparency on overall welfare depends on the relative spread. When the relative spread is high, transparency ranks better than both opaqueness settings. This result reverses when the relative spread is quite low. Our welfare results for coexistence stem from two forces. The first force adds to welfare due to order creation: some low willingness to trade investors now may trade whereas they would not with a DM in isolation. The welfare contribution of order creation hinges on the execution probability of these orders and increases in the degree of transparency. The second force stems from trade diversion, i.e. traders that opt for the CN while they would choose for the DM in the isolation case. In general, more trade diversion increases trader welfare as these traders then prefer the possibly uncertain execution at the CN above a DM trade. Overall welfare, however, may be lowered by trade diversion, as it harms the dealer. Our analysis thus shows that increasing the transparency level might be beneficial but that the ultimate answer hinges on the exact welfare criterion that is employed.

Our paper is further related to two recent strands of research. A first line of work develops dynamic microstructure models for a limit order market, while focusing on different aspects: trading decisions of a variety of trader types submitting small orders (Harris (1998)); the impact of the limit order's risk of being picked off on traders' order submission strategies (Foucault (1999)); order flow persistence even in the absence of changes in the consensus value of the asset (Goettler, Parlour and Rajan (2005)); the resiliency of the limit order market (Foucault, Kadan and Kandel (2005)); and endogenous undercutting and strategic cancellation of limit orders (Rosu (2005)).[2] The limit order market model in Parlour (1998) describing traders' order placement strategies at the inside quotes is positioned closest to ours. However, a number of important differences exist between both models. First, we analyze the optimal order submission strategies and welfare consequences of traders who are confronted with the choice between two trading venues with different institutional characteristics, whereas Parlour (1998) considers the choice between market and limit orders within a single market. Second, as the cross in the CN occurs at the DM midquote, our model allows for submitting orders "within the spread". Third, while Parlour (1998) deals with transparency (which is the case for most limit order markets), we also consider two opaqueness settings. Finally, the models' resulting dynamics feature some important differences. In our model only a CN order generates systematic patterns in order flow, whereas in Parlour (1998) both market and limit orders have an impact. In general, our paper contributes to this line of research as we introduce a dynamic microstructure model to study (partly, at least) endogenous liquidity supply by looking at the competition between two different trading venues. This is in contrast

[2] Note that static equilibrium models of the limit order book are much more common. Examples include Glosten (1994), Chakravarty and Holden (1995), Rock (1996) and Seppi (1997).

to the previously mentioned papers which restrict themselves to only one market, i.e. a limit order market.

A second line of recent work models competition between financial markets when assets trade at multiple markets. The seminal contribution is provided by Glosten (1994) who considers the design of pure limit order markets to analyze their competitive viability. Parlour and Seppi (2003) extend this model by focusing on competition between a pure limit order market and a hybrid market. Foucault and Menkveld (2007) deal with order submission at two pure limit order markets when a fraction of brokers applies Smart Order Routing Technologies (SORT).[3] Recently, a few papers explicitly study the interaction between a CN and a DM. Existing models, however, consider a static environment to analyze this competition. Hendershott and Mendelson (2000) develop a model where informed and uninformed traders simultaneously decide to submit orders to one of both markets in order to analyze the effect of the introduction of a CN to a DM setting. Expanding on this paper, Dönges and Heinemann (2006) focus on game-theoretic refinements to accommodate the multiplicity of equilibria in the coordination game. We contribute to this line of work as we explicitly introduce dynamics into the analysis. These dynamics are important: a typical characteristic of a CN is that it "matches" orders at a specified time during the trading day, while the other market simultaneously operates in a continuous fashion. In particular, traders arrive sequentially, and their submission strategy is determined both by the current CN order book (when transparent) and by their expectations of the behavior of future traders until the time of the cross.

There is by now a substantial number of empirical papers analyzing the interaction between trading systems (for an overview see Biais, Glosten and Spatt (2005)). However, papers empirically investigating the impact of a CN on other trading systems are still rather scarce. Gresse (2006) studies the impact of the ITG's POSIT on the DM segment of the London Stock Exchange. She finds that POSIT has a share of total trading volume of about one to two percent in these stocks, but that its probability of execution is still low (2-4%). Moreover, she reports that activity at POSIT does not have a detrimental effect on the liquidity at the considered DM. Conrad, Johnson and Wahal (2003) use proprietary data of US institutional investors who choose between trading platforms. They find that realized execution costs are generally lower on alternative trading systems (including CNs). Næs and Ødegaard (2006)

[3] Other work on the competition between trading systems includes Glosten (1998), Santos and Scheinkman (2001), Di Noia (2001), Viswanathan and Wang (2002), Chemmanur and Fulghieri (2006) and Foucault and Parlour (2004).

focus on orders from the Norwegian Government Petroleum fund that are first sent to a CN and then, in the case of non-execution, to brokers. They also find lower CN trading costs but argue that these may be fully offset by the non-trading costs due to adverse selection, which are implicitly present at the CN. Næs and Skjeltorp (2003) find that competition from CNs is concentrated in the most liquid stocks. The significant differences in liquidity between both markets are partly related to the presence of informed trading in the non-executed CN stocks (as in Næs and Ødegaard (2006)). Finally, Fong, Madhavan and Swan (2004) focus on the impact of block trades on different trading venues – a limit order book, a CN and an upstairs market. They find that competition from the two latter markets imposes no adverse effect on the liquidity of the limit order book.

Our paper proceeds as follows. Section 2 presents the setup of the model. Section 3 provides an analysis of the transparency case. We first deal with the markets in isolation, and then study their interaction. In Section 4, we consider two degrees of opaqueness, i.e. partial and complete opaqueness. Section 5 offers a discussion of the welfare implications of our model, and Section 6 concludes. All proofs are relegated to the Appendix.

2 Setup of the Model

The model we develop is based on the setup in Parlour (1998). While her model discusses the traders' choice between market and limit orders in a continuous limit order market, we adapt it to analyze competition between two trading systems.

In our economy, there are two days. Agents decide upon consumption on day 1 and day 2, denoted by C_1 and C_2. Agents are risk neutral and differ in their preferences over consumption on these two days. These preferences are given by the following utility function:

$$U(C_1, C_2; \beta) = C_1 + \beta C_2 \tag{1}$$

with β being the subjective preference or type of the agent reflecting her personal trade-off between current and future consumption. Next to these two "goods" C_1 and C_2, an asset exists that pays out V units of C_2 per share on day 2. As we investigate the short-term interactions between both markets, the assumption of no uncertainty in V is a reasonable starting point. During the first day, the trading day, claims to the asset can be exchanged for C_1. Prices in the market are exchange ratios C_1/C_2. Agents can then construct their preferred consumption path by trading claims to this asset. The trading day consists of T periods, indexed by $t = 1, ..., T$. Each period exactly one agent (also referred to as trader) arrives in the market, and each agent arrives at most once. The arriving agent at time t is characterized by two elements. First, her initial endowments determine her trading orientation. With probability π_B, she is a buyer and has one unit of the asset she can buy in exchange for C_1, which we denote by 1. With probability $\pi_S = 1 - \pi_B$, she is a seller and has one unit of the asset she can sell, -1. Second, the agent arriving at t has a type β_t, which is drawn from an i.i.d. continuous distribution $F(.)$ with corresponding density function $f(.)$ and support $\left[\underline{\beta}, \bar{\beta}\right]$, where we assume $0 \leq \underline{\beta} \leq 1 \leq \bar{\beta}$. This β_t, which is taken from the utility function above, could also be seen as a reflection of a trader's willingness to trade.[4] In particular, if the trader is a buyer, she will be more eager to buy if she has a high beta. Conversely, a seller will be more eager to sell if she has a low β_t. In order to see this, assume that the arriving trader is a buyer. Buying the asset yields $\beta_t V$. She compares this value with the price in the market and performs the buy if the price is lower than the value she attaches to the asset. If β_t is high, she attaches more weight to consumption on the second day and hence will be more eager to trade than if β_t is low. The reasoning is that the trading gains are higher in the former case. Similarly, a seller with a low beta will be more eager to sell since she prefers consumption on the first day.

Traders can choose between submitting an order to a dealer market (DM) or to a crossing network (CN), or not to submit an order at all. We assume competition between dealers on the DM to be sufficiently intense such that the spread is one tick, that is $A - B = 1$, with B the bid price and A the ask price. This assumption allows us to focus on the interaction between markets, abstracting from strategic interactions between dealers. At the same time, a one-tick spread represents the most competitive position for the DM when competing with a CN.[5] Dealer bid and ask quotes do not move during the trading day. The implication is that buyers can always buy at a price A, the price at which a dealer is willing to sell. Sellers looking for immediacy in the DM obtain B.

[4] Alternatively, Parlour (1998) argues that β_t can be interpreted as a subjective valuation of the asset, or a prior over the next day's asset value V. Hence, the market is rendered a private values auction, as in Glosten and Milgrom (1985).

[5] Bessembinder (2003) finds average (volume-weighted) quoted spreads on NASDAQ equal to 1.77 cents (with tick size being 1 cent), which is relatively close to our one-tick assumption. More generally, this "one tick" should not necessarily be interpreted literally, but rather as a metaphor for the most competitive situation where competition between dealers has driven the inside spread to its minimum level. As will be shown later, it is the relative spread that matters for submission strategies of agents. For example, saving the half spread in the CN is more valuable when the bid is \$1 and the ask \$1.01, than if they are \$100 and \$100.01 respectively (assuming tick size is one cent).

Next to a DM, we also introduce a CN. We assume that the matching of orders (the "cross") takes place at the end of the trading day, that is after the action of the agent arriving in period T. The price of the cross is derived from the bid and ask in the DM and equals the midquote $(A+B)/2$. Given our assumptions, orders at the CN face no price uncertainty. Orders submitted to the CN are stored in the book c_t, which is a pair $\left(c_t^b, c_t^s\right)$ where $c_t^b > 0$ ($c_t^s < 0$) represents the cumulative amount of buy (sell) orders at the CN before the order at time t. After the action of the trader at time t, there are three possible evolutions of the CN's order book:

$$\left(c_{t+1}^b, c_{t+1}^s\right) = \begin{vmatrix} \left(c_t^b+1, c_t^s\right) & \text{trader } t \text{ submits a buy order to CN} \\ \left(c_t^b, c_t^s-1\right) & \text{trader } t \text{ submits a sell order to CN} \\ \left(c_t^b, c_t^s\right) & \text{trader } t \text{ submits no order to CN} \end{vmatrix} . \tag{2}$$

The first two evolutions describe a buy and sell order, respectively. The last case, where the CN's order book remains unchanged, stems from a trade at a dealer or from not trading at all. Once submitted, orders cannot be modified or cancelled. This means that orders remain in the CN's order book until the cross. Order execution is determined by the imbalance between the queue of buy orders and the queue of sell orders. If $c_T^b = |c_T^s|$, meaning no imbalance, then all orders are executed. If $c_T^b < |c_T^s|$, some sell orders cannot be executed. We assume time priority such that the first c_T^b sell orders submitted are executed. If $c_T^b > |c_T^s|$, the first $|c_T^s|$ buy orders are executed. It goes without saying that time priority influences the order submission strategies of the traders. In practice, some CNs indeed implement a time priority rule. Examples include the Crossing Session I at the NYSE (rule 904 of SR-NYSE-90-52), ITG's POSIT-Now which offers a continuous intraday CN (implicitly granting time priority), and Xetra XXL employing a volume/time priority rule. Other CNs are often reluctant to share information on their matching procedure and may use different matching procedures like pro-rata systems.

The above details of the model are common knowledge to traders in the transparency case we discuss in Section 3. In Section 4, however, we reduce traders' information sets, introducing an opaque CN-book and different assumptions on the observability of past DM trades (i.e. partial and complete opaqueness).

3 Equilibrium under Transparency

In this section, we characterize the equilibrium order submission strategies when the CN's order book is fully transparent. As a first step, we consider successively a DM and a CN in isolation. This approach allows us to gain insight into the model and the structure and functioning of each market. Subsequently, we determine the equilibrium when both markets coexist. The methodology is identical in all cases. For a trader arriving at time t we calculate a cutoff β_t at which she is indifferent between two actions, rationally anticipating the impact of her order on execution probabilities. Furthermore, we develop empirical predictions on order flow dynamics.

3.1 Dealer Market in Isolation

Suppose for now that there is only a DM. In this case, a trader can choose between submitting an order to the DM or not trading at all. She will trade at the DM as long as this yields a positive profit; otherwise she prefers not to trade. When the profit is zero, she is indifferent. Before deciding, she observes the bid and ask in the market, the distribution of β and her personal β_t. The profit of a buy order to the dealer is the difference between the valuation of the trader $\beta_t V$ and the price paid A, i.e. the profit is $\beta_t V - A$. Similarly, for a sell order, the profit is $B - \beta_t V$. From these profits, the cutoff values for β_t are computed:

$$\begin{aligned} \beta_t^{b,DM} &= \frac{A}{V} \\ \beta_t^{s,DM} &= \frac{B}{V}, \end{aligned} \tag{3}$$

where the first superscript refers to buy (b) or sell (s), and the second to the considered market. This can be interpreted in the following way. A buyer arriving at time t who has a β_t higher than A/V will buy at the DM; all the others will not. When the trader at time t is a seller, she will sell at the DM if her β_t is smaller than B/V. The order submission strategies are depicted in Figure 1. Note that traders having a β_t between B/V and A/V will never submit an order, regardless of their individual trading orientation.

Please insert Figure 1 around here.

3.2 Crossing Network in Isolation

In this subsection, only a CN is considered (and no DM). To compare the different settings, we assume that the price at which a cross will take place is the midquote as if a DM existed: $(A+B)/2$. When arriving at time t, next to this midquote, a trader also observes the overall distribution of β, her own β_t and the CN order book. She will submit a CN order as long as this results in a positive *expected* profit. Now we need to consider expected profits, since in contrast with an order to a DM, the execution of a CN order may not be certain. If the order executes, the profit for the trader is the difference between the trader's valuation and the price paid (the midquote). When taking into account the uncertainty about execution, the expected profit of a CN buy order is $p_t^{b,CN}(\beta_t V - (A+B)/2)$, where $p_t^{b,CN}$ denotes the expected probability of execution of a CN buy order submitted at time t. For a CN sell order, the expected profit is $p_t^{s,CN}((A+B)/2 - \beta_t V)$, with $p_t^{s,CN}$ the probability of execution of a sell order submitted at time t. These probabilities depend on the book in the CN, and the time left until the end of the trading day: $p_t^{b,CN}(c_t, T-t)$ and $p_t^{s,CN}(c_t, T-t)$, but for notational convenience we suppress this dependence.[6] The reasoning for this dependence is that if a trader joins the longer queue, enough future orders need to be submitted to the shorter side of the CN's order book to obtain execution. This is more likely earlier on the trading day, when there are still a lot of periods to come. Finally, when the expected profit of a CN order is negative, the trader chooses to abstain, resulting in zero profits.

A trader's strategy whether or not to submit a CN order is determined by the expected profits of this action. Solving for β_t, we find the following cutoff value for a buyer and a seller respectively:

$$\begin{aligned} \beta_t^{b,CN} &= \frac{\frac{A+B}{2}}{V} \\ \beta_t^{s,CN} &= \frac{\frac{A+B}{2}}{V}. \end{aligned} \tag{4}$$

Hence, a trader arriving at t will submit a CN buy (sell) order if her β_t is higher (lower) than $\beta_t^{b,CN}$ $\left(\beta_t^{s,CN}\right)$. To be complete, these cutoff values hold if the execution probability is strictly positive. If it is zero, a trader is always indifferent between a CN order and no order, since both yield zero profit. If this occurs, we assume that traders prefer to abstain.

[6] Note that, next to the state variables c_t and $T-t$, execution probabilities also depend on $F(.)$, π_B and π_S.

The order submission strategies are summarized in Figure 2. Note that in contrast with a DM in isolation, there is no "gap", i.e. there is no range of betas where neither a buyer nor a seller submits an order. The reasoning is that a CN does not have a spread whereas a DM is characterized by a one-tick spread.

Please insert Figure 2 around here.

3.3 Coexistence of CN and DM

Having discussed the two trading systems in isolation, we now turn to the full model and characterize the choice problem faced by a trader arriving in the market at time t. Upon arrival, she knows her trading orientation (buyer or seller) and her own β_t. Moreover she observes the bid and ask price of the dealer, the CN order book c_t, the distribution of β, the distribution of buyers and sellers and the time remaining to the cross. Recall that the CN crosses at the midprice of the dealer's bid and ask. Based on this information, she chooses between three possible strategies. First, she can initiate a trade at the dealer; such an order has a guaranteed, immediate execution. Second, she can opt for submitting an order to the CN. This would yield a better price as it allows the trader to save the half-spread, which in our model is equal to half a tick. With such an order, however, she might face the risk of non-execution. Execution is certain when upon arrival she is able to join the shorter queue (due to time priority in the CN); in all other cases, the execution probability is lower than one. Third, she can refrain from trading when it yields a negative (expected) profit.

Denote the strategy of a buyer arriving at time t under transparency (tr) by $\phi^b_{t,tr}(c_t, \beta_t)$ and of a seller by $\phi^s_{t,tr}(c_t, \beta_t)$ where the notation stresses that the strategy depends on the time t CN's order book, c_t, and the trader's type β_t. The strategies depend on time and are non-stationary. The setup of this model can be seen as a stochastic sequential game. Moreover, due to the recursive nature of the game, an equilibrium is guaranteed to exist and this equilibrium is unique (since traders are indifferent between choices with zero probability). Applying the approach introduced above to solve the trader's choice problem, i.e. computing cutoff values for β_t where traders are indifferent between two strategies, Proposition 1 states the equilibrium strategies of a trader arriving at t.

Proposition 1 *If the time t trader is a buyer, there exist cutoff values such that*

$$\beta_t \in \left| \begin{array}{lll} \left[\underline{\beta}, \underline{\beta}^b_{t,tr}\left(p^b_{t,tr}\right)\right] & \phi^b_{t,tr}\left(c_t, \beta_t\right) = 0 & \textit{(no order)} \\ \left(\underline{\beta}^b_{t,tr}\left(p^b_{t,tr}\right), \bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right)\right] & \phi^b_{t,tr}\left(c_t, \beta_t\right) = 1^{CN} & \textit{(buy order to CN)} \\ \left(\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right), \bar{\beta}\right] & \phi^b_{t,tr}\left(c_t, \beta_t\right) = 1^{DM} & \textit{(buy at DM)} \end{array} \right. . \tag{5}$$

Similarly, if the time t trader is a seller, there exist cutoff values such that

$$\beta_t \in \left| \begin{array}{lll} \left[\underline{\beta}, \underline{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)\right) & \phi^s_{t,tr}\left(c_t, \beta_t\right) = -1^{DM} & \textit{(sell at DM)} \\ \left[\underline{\beta}^s_{t,tr}\left(p^s_{t,tr}\right), \bar{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)\right) & \phi^s_{t,tr}\left(c_t, \beta_t\right) = -1^{CN} & \textit{(sell order to CN)} \\ \left[\bar{\beta}^s_{t,tr}\left(p^s_{t,tr}\right), \bar{\beta}\right] & \phi^s_{t,tr}\left(c_t, \beta_t\right) = 0 & \textit{(no order)} \end{array} \right. . \tag{6}$$

Proof. See the Appendix ■

In the proposition, "1^{DM}" denotes a buy at the DM (which transacts at the ask), and "-1^{DM}" a sell at the DM (transacting at the bid). Similarly, "1^{CN}" and "-1^{CN}" stand for a buy and sell order to the CN, respectively. Employing our one-tick spread assumption, $A - B = 1$, we find that the value β_t of a buyer who is indifferent between an order to the CN and an order to the DM, $\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right)$, is given by:

$$\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right) = \min\left[\frac{\frac{A+B}{2}}{V} + \frac{1/2}{V\left(1-p^b_{t,tr}\right)}, \bar{\beta}\right]. \tag{7}$$

Furthermore, $\underline{\beta}^b_{t,tr}\left(p^b_{t,tr}\right)$, the β_t at which a trader is indifferent between a CN buy order and no order, is equal to:

$$\underline{\beta}^b_{t,tr}\left(p^b_{t,tr}\right) = \left| \begin{array}{ll} \frac{\frac{A+B}{2}}{V} & \text{if } p^b_{t,tr} > 0 \\ \frac{A}{V} & \text{otherwise} \end{array} \right. . \tag{8}$$

Similarly, $\underline{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)$ is the β_t of a seller who is indifferent between a CN order and an order to the dealer:

$$\underline{\beta}^s_{t,tr}\left(p^s_{t,tr}\right) = \max\left[\frac{\frac{A+B}{2}}{V} - \frac{1/2}{V\left(1-p^s_{t,tr}\right)}, \underline{\beta}\right], \tag{9}$$

whereas $\bar{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)$ holds for a seller at t who is indifferent between a CN order and no order, with

$$\bar{\beta}^{s}_{t,tr}\left(p^{s}_{t,tr}\right) = \begin{vmatrix} \frac{\frac{A+B}{2}}{V} & \text{if } p^{s}_{t,tr} > 0 \\ \frac{B}{V} & \text{otherwise} \end{vmatrix} . \tag{10}$$

The equilibrium order submission strategies are summarized in Figure 3. Comparing this graph with Figures 1 and 2, there are some notable differences. The most important one is that due to altering execution probabilities the cutoff values become *dynamic* and change every period t. For the markets in isolation this was not the case. Moreover, although the range of β's at which no buy or sell order is submitted is the same, the ranges at which DM and CN orders are submitted are, in general, different from the isolation cases. Compared to the DM in isolation, *order creation* may occur: traders with intermediate β's now submit orders to the CN; this allows them to avoid paying the half-spread. Such order creation effect, induced by the CN, is confirmed empirically by Gresse (2006). The CN also introduces competition for the DM as it may *divert trades* away from the DM.[7] The welfare implications of both effects will be discussed in Section 5.

Please insert Figure 3 around here.

It is clear that if the execution probability at the CN increases, an arriving trader is more likely to opt for a CN order. This execution probability is a crucial element in the choice between a CN order and a DM trade as it determines expected profits. When trader t submits a CN order, she changes the imbalance in the CN. This affects the execution probabilities of future CN orders and hence also the strategies chosen by future traders. When determining her optimal strategy, trader t must take these effects of her order into account. Proposition 2 shows how the length of the queues (and the imbalance) influences execution probabilities.

Proposition 2 *In equilibrium, at any time t, if the CN's order book at the buy side is one unit thicker, then the probability of execution of a buy (sell) order will be lower (higher). If the CN's order book at the sell side is one unit thicker, then the probability of execution of a buy (sell) order will be higher (lower). If the book is one unit thicker at the buy side and one unit thicker at the sell side, then the probability of execution for both order types remains*

[7] Note that this could lead to overall trade creation but also to overall trade reduction. The reasoning for a potential trade reduction is that some of the investors choosing to trade at the DM, if it were to operate in isolation, might now opt for the CN at which their order may remain unfilled.

constant. Hence, $\forall\ c_t,\ t,$

$$
\begin{array}{llll}
\textit{(i)} & p^b_{t,tr}\left(c^b_t, c^s_t\right) \leq p^b_{t,tr}\left(c^b_t - 1, c^s_t\right) & \bar{\beta}^b_{t,tr}\left(c^b_t, c^s_t\right) \leq \bar{\beta}^b_{t,tr}\left(c^b_t - 1, c^s_t\right) & \\
\textit{(ii)} & p^b_{t,tr}\left(c^b_t, c^s_t\right) \leq p^b_{t,tr}\left(c^b_t, c^s_t - 1\right) & \bar{\beta}^b_{t,tr}\left(c^b_t, c^s_t\right) \leq \bar{\beta}^b_{t,tr}\left(c^b_t, c^s_t - 1\right) & \\
\textit{(iii)} & p^b_{t,tr}\left(c^b_t, c^s_t\right) = p^b_{t,tr}\left(c^b_t + 1, c^s_t - 1\right) & \bar{\beta}^b_{t,tr}\left(c^b_t, c^s_t\right) = \bar{\beta}^b_{t,tr}\left(c^b_t + 1, c^s_t - 1\right) & \\
\textit{(iv)} & p^s_{t,tr}\left(c^b_t, c^s_t\right) \leq p^s_{t,tr}\left(c^b_t, c^s_t + 1\right) & \underline{\beta}^s_{t,tr}\left(c^b_t, c^s_t\right) \geq \underline{\beta}^s_{t,tr}\left(c^b_t, c^s_t + 1\right) & (11) \\
\textit{(v)} & p^s_{t,tr}\left(c^b_t, c^s_t\right) \leq p^s_{t,tr}\left(c^b_t + 1, c^s_t\right) & \underline{\beta}^s_{t,tr}\left(c^b_t, c^s_t\right) \geq \underline{\beta}^s_{t,tr}\left(c^b_t + 1, c^s_t\right) & \\
\textit{(vi)} & p^s_{t,tr}\left(c^b_t, c^s_t\right) = p^s_{t,tr}\left(c^b_t + 1, c^s_t - 1\right) & \underline{\beta}^s_{t,tr}\left(c^b_t, c^s_t\right) = \underline{\beta}^s_{t,tr}\left(c^b_t + 1, c^s_t - 1\right) &
\end{array}
$$

(both formulations, in terms of probabilities and in terms of betas, are equivalent).

Proof. See the Appendix ■

Intuitively, Proposition 2 argues that when the queue at one side of the market is longer when a trader arrives on that side of the market, the execution probabilities of a CN order are lower relative to when the queue is shorter (parts (i), (iv), and by symmetry (ii) and (v)). That is, traders face intertemporal competition with traders of their own type. The reasoning is as follows. Suppose that a buyer arrives at time τ and $c^b_\tau \geq |c^s_\tau|$.[8] Then if the book is one unit thicker at the buy side, an additional CN order at the sell side must arrive in order to obtain execution. This lowers the execution probability compared to the case when the buy queue is one unit shorter (meaning also a smaller imbalance). Only when both queues are one unit thicker (parts (iii) and (vi)) are execution probabilities not affected since the imbalance remains the same. Hence, this order book is conceptually identical to the original $\left(c^b_t + 0, c^s_t - 0\right)$-book in terms of execution probabilities and betas. This is in contrast with a limit order market as in Parlour (1998). In such a market, execution probabilities are influenced even when both queues become one unit longer. This proves that in a CN the imbalance between both queues matters, while in a limit order market the individual length of both queues is relevant.

3.4 Empirical Predictions on Order Flow Dynamics

Having determined and characterized the equilibrium order submission strategies of traders, we now investigate order flow patterns in transaction and order flow data resulting from these strategies. We do so in four propositions. In all cases, we start from a given book in the CN,

[8] If $c^b_\tau - 1 < |c^s_\tau|$, the execution probabilities with a book $\left(c^b_\tau, c^s_\tau\right)$ and $\left(c^b_\tau + 1, c^s_\tau\right)$ both equal one.

c_t, and from a specific order, a DM trade or a CN order. We investigate the effect on the order flow to the DM and the CN in the subsequent period.

We start by assuming that the previous order (at time t) was a DM trade and investigate the patterns in subsequent order submissions. Proposition 3 then states that the probability of observing a DM buy does not hinge on whether the previous transaction was a DM buy or a DM sell.

Proposition 3 *The probability of a DM buy at time $t+1$ is independent of whether the order at time t was a DM buy or a DM sell:*

$$\Pr\left[\phi^b_{t+1,tr}\left(c_{t+1},\beta_{t+1}\right) = 1^{DM} | \phi^b_{t,tr}\left(c_t,\beta_t\right) = 1^{DM}, c_t\right] \tag{12}$$
$$= \Pr\left[\phi^b_{t+1,tr}\left(c_{t+1},\beta_{t+1}\right) = 1^{DM} | \phi^s_{t,tr}\left(c_t,\beta_t\right) = -1^{DM}, c_t\right].$$

A symmetric result holds for the other side of the market.

Proof. Contained in the discussion below. ■

A similar result holds for CN orders following a DM trade. Proposition 4 shows that the probability that the current order is a CN order (buy or sell depending on the trader who arrives) is independent of whether the previous order was a DM sell or a DM buy.

Proposition 4 *The probability of a CN buy order at time $t+1$ is equal, whether the order at time t was a DM buy or a DM sell:*

$$\Pr\left[\phi^b_{t+1,tr}\left(c_{t+1},\beta_{t+1}\right) = 1^{CN} | \phi^b_{t,tr}\left(c_t,\beta_t\right) = 1^{DM}, c_t\right] \tag{13}$$
$$= \Pr\left[\phi^b_{t+1,tr}\left(c_{t+1},\beta_{t+1}\right) = 1^{CN} | \phi^s_{t,tr}\left(c_t,\beta_t\right) = -1^{DM}, c_t\right].$$

A symmetric result holds for the other side of the market.

Proof. Contained in the discussion below. ■

The results in Propositions 3 and 4 are driven by the same intuition. If the previous order was a DM trade, none of the elements determining the current trader's strategy – such as her beta, the CN-book or the execution probability – are influenced differently whether the order was a DM buy or a DM sell. These results are in sharp contrast with models of limit order markets, such as Parlour (1998). In her model, market orders (comparable to our DM trades) do influence the probabilities of subsequent orders, as they change the depth in the limit order book and hence the execution probabilities of subsequent limit orders.

The conclusions alter, however, when we assume that the order at time t was a CN order instead of a DM trade. In this case, we obtain systematic patterns in order flow despite the fact that buyers and sellers arrive randomly. Proposition 5 shows that when the trader at time t has chosen a CN buy order, it is less likely that a buyer at $t+1$ will do the same, compared to when trader t did not submit a CN buy order.

Proposition 5 *The probability of a CN buy order at time $t+1$ is smaller if the order at time t was a CN buy order than if it was a DM trade (buy or sell). This in turn is smaller than the probability of observing a CN buy order, conditional upon the previous order being a CN sell order:*

$$\begin{aligned} &\Pr\left[\phi^b_{t+1,tr}\left(c_{t+1},\beta_{t+1}\right)=1^{CN}|\phi^b_{t,tr}\left(c_t,\beta_t\right)=1^{CN},c_t\right] \\ &\leq \Pr\left[\phi^b_{t+1,tr}\left(c_{t+1},\beta_{t+1}\right)=1^{CN}|\phi^b_{t,tr}\left(c_t,\beta_t\right)=1^{DM}\ \textit{or}\ \phi^s_{t,tr}\left(c_t,\beta_t\right)=-1^{DM},c_t\right] \\ &\leq \Pr\left[\phi^b_{t+1,tr}\left(c_{t+1},\beta_{t+1}\right)=1^{CN}|\phi^s_{t,tr}\left(c_t,\beta_t\right)=-1^{CN},c_t\right]. \end{aligned} \tag{14}$$

A symmetric result holds for the other side of the market.

Proof. See the Appendix ∎

Complementary to Proposition 5, Proposition 6 shows that it becomes more likely that the current buyer submits a DM buy if the previous order was a CN buy order, than if it was another type of order.

Proposition 6 *The probability of a DM buy at time $t+1$ is greater if the order at time t was a CN buy order than if it was a DM trade (buy or sell). This in turn is greater than the probability of observing a DM buy, conditional upon the previous order being a CN sell order:*

$$\begin{aligned} &\Pr\left[\phi^b_{t+1,tr}\left(c_{t+1},\beta_{t+1}\right)=1^{DM}|\phi^b_{t,tr}\left(c_t,\beta_t\right)=1^{CN},c_t\right] \\ &\geq \Pr\left[\phi^b_{t+1,tr}\left(c_{t+1},\beta_{t+1}\right)=1^{DM}|\phi^b_{t,tr}\left(c_t,\beta_t\right)=1^{DM}\ \textit{or}\ \phi^s_{t,tr}\left(c_t,\beta_t\right)=-1^{DM},c_t\right] \\ &\geq \Pr\left[\phi^b_{t+1,tr}\left(c_{t+1},\beta_{t+1}\right)=1^{DM}|\phi^s_{t,tr}\left(c_t,\beta_t\right)=-1^{CN},c_t\right]. \end{aligned} \tag{15}$$

A symmetric result holds for the other side of the market.

Proof. See Appendix ∎

The intuition behind Propositions 5 and 6 is as follows. Assume that the time $t+1$ trader is a buyer. If the queue of buy orders at time $t+1$ is shorter than the sell queue, the equality sign applies, since the execution probability of a submitted CN buy order equals one. In this case, the type of the previous order is irrelevant for the current order flow. In contrast, if after the order of the trader at t the buy queue is longer than the sell queue, i.e. when there is an unfavorable imbalance for the time $t+1$ buyer, the type of the previous order does matter. Given her β_{t+1}, the current trader will be more likely to submit a CN buy order if the previous order increased the execution probability. This is the case when the unfavorable imbalance in the CN's order book decreases, which happens after a CN sell order in the previous period. A DM trade (be it buy or sell) does not alter the imbalance, while a CN buy order at t even increases the imbalance. Symmetrically, if trader $t+1$ is less likely to submit a CN order, she will be more likely to opt for an order to the dealer.

Propositions 5 and 6 demonstrate the existence of systematic patterns in order flow. This finding is of importance to empirical researchers. In general, the literature tends to attribute such patterns to informed trading, whereas our model shows that these can also stem from the interaction between two trading venues. For example, a series of consecutive buy trades at the DM need not imply that some traders have private information; it might result from an unfavorable imbalance in the CN-book for the buyer. Thus, empirical research focusing on patterns in DM order flow (while neglecting the CN) might point to wrong conclusions. An interesting empirical application of our model would therefore be to determine the importance of this interaction effect in explaining order flow patterns relative to other factors.

Furthermore, it is worth stressing that although the patterns outlined in Propositions 5 and 6 are similar to the case of a limit order market in Parlour (1998), the underlying dynamics are very different. As argued before, in the case of a limit order market, the length of the queues at bid and ask are important and both market and limit orders have an effect. In our model, with a DM and a CN, it is the imbalance between buy and sell queues in the CN that is relevant, and this imbalance is influenced only by CN orders, not by DM trades.

Finally, note that the CN in our model also exhibits two opposing externalities as in Hendershott and Mendelson (2000). On the one hand, a positive (liquidity) externality prevails on the CN as adding an order is beneficial to traders arriving in the future; hence increasing liquidity attracts additional liquidity. On the other hand, there is a negative (crowding) externality as early arriving low liquidity value traders may preempt higher liquidity value traders arriving later in the trading day. Hence, these externalities as identified by Hendershott and Mendelson also hold in a dynamic context with sequentially instead of simultaneously arriving traders.

4 Equilibrium under Opaqueness

In Proposition 1, traders condition their strategies on past order flow and the resulting visible CN-book.[9] As argued in the introduction however, most CNs are rather opaque and do not actively disseminate information on their order book. In this section, we adapt our model to capture opaqueness. We deal with two degrees of opaqueness, "complete" and "partial", and contrast these to the transparency case. Other models of CNs such as Hendershott and Mendelson (2000) or Dönges and Heinemann (2006) cannot compare different informational settings, as they deal with simultaneous order submissions.

4.1 Complete Opaqueness

Complete opaqueness implies that a trader no longer observes past DM trades and past CN order flow. However, she still knows her β_t and trading orientation, the time of the cross T, the distribution $F(\beta)$, π_B and π_S. In order to condition her strategy on the CN-book, c_t, she needs to form expectations about it. She does so in two steps. First, she is able to solve any past trader's optimization problem. Combining this with the common knowledge on the distributions for $F(\beta)$, π_B and π_S in a second step allows her to compute the expected CN-book at t. Formally, let $\Im_{t,co}$ denote the time t trader's information set, where the subscript "*co*" refers to complete opaqueness, and let $E(c_t|\Im_{t,co})$ be the expected CN-book. Based on this expected CN-book and the time left until the cross, she computes the expected execution probability of a CN order, which we denote by $p^b_{t,co}(E(c_t|\Im_{t,co}), T-t)$ if she is a buyer, and by $p^s_{t,co}(E(c_t|\Im_{t,co}), T-t)$ if she is a seller.[10] As in Section 3, we suppress the dependence for notational convenience and denote them in short by $p^b_{t,co}$ and $p^s_{t,co}$, respectively. Based on these calculations, she determines her personal cutoff betas: i.e. $\underline{\beta}^b_{t,co}(p^b_{t,co})$, $\overline{\beta}^b_{t,co}(p^b_{t,co})$ for a buyer and $\underline{\beta}^s_{t,co}(p^s_{t,co})$, $\overline{\beta}^s_{t,co}(p^s_{t,co})$ for a seller.[11] Now denote the optimal strategy of a buyer

[9] Other elements in the information set are the distribution of β, their individual β_t, the time left until the cross, traders' distributions at both market sides and their own trading orientation.

[10] Clearly, to derive these probabilities, as in the transparency case, she needs to solve future arriving traders' choice problems.

[11] Note that the notation of these cutoff values is analogous to that in Section 3.

arriving at t as $\phi^b_{t,co}\left(E\left(c_t|\Im_{t,co}\right),\beta_t\right)$, and that of a seller by $\phi^s_{t,co}\left(E\left(c_t|\Im_{t,co}\right),\beta_t\right)$. It is then possible to reformulate Proposition 1 for the complete opaqueness case by replacing the cutoff betas and strategies with their respective counterparts, defined in the current section. This modified Proposition 1 characterizes the equilibrium order submission strategies of traders in a complete opaqueness setting.

With complete opaqueness, later arriving traders do not observe previous traders' strategies and also anticipate their own decision is not revealed to subsequent traders. Traders' decisions then become independent from past orders, and each arriving trader decides only using information on her β_t, on general predictions about past and future traders' behavior and the resulting expected CN order book. Therefore, and in contrast to the transparency case, empirical work that ex-post observes the different decisions of investors should not find path-dependency.

4.2 Partial Opaqueness

Under "partial" opaqueness, traders do observe previous DM trades, but do neither observe CN order flow, nor do they have information on the CN-book. This informational environment corresponds closest to reality as DMs are in general transparent while CNs are rather opaque. A time t trader thus observes in each past period either a DM buy, a DM sell, or no trade at the DM. In the latter case, she does not know whether a CN buy, a CN sell or no order was submitted. Nevertheless, her information set is now clearly richer than under complete opaqueness. Let $\Im_{t,po}$ denote this extended information set, where the subscript "*po*" refers to partial opaqueness. $\Im_{t,po}$ is thus equal to $\Im_{t,co}$ plus observed DM trades. Based on $\Im_{t,po}$, she is now able to form more precise expectations about the time t CN-book $E\left(c_t|\Im_{t,po}\right)$ or in short $E_{t,po}\left(c_t\right)$. In turn, the determination of this expected CN-book allows her to compute the expected execution probability of a CN order, which we denote by $p^b_{t,po}\left(E_{t,po}\left(c_t\right),T-t\right)$ for a buy order, and by $p^s_{t,po}\left(E_{t,po}\left(c_t\right),T-t\right)$ for a sell order. Again, she can determine her cutoff betas: $\underline{\beta}^b_{t,po}\left(p^b_{t,po}\right)$ and $\bar{\beta}^b_{t,po}\left(p^b_{t,po}\right)$ if she is a buyer, or $\underline{\beta}^s_{t,po}\left(p^s_{t,po}\right)$ and $\bar{\beta}^s_{t,po}\left(p^b_{t,po}\right)$ if she is a seller. Note that, in contrast to the complete opaqueness case, these cutoff betas exhibit path-dependency.[12] In other words, they hinge on past traders' decisions which are now partly observed. The resulting optimal strategy for the time t trader is denoted by $\phi^b_{t,po}\left(E_{t,po}\left(c_t\right),\beta_t\right)$ for a buyer and $\phi^s_{t,po}\left(E_{t,po}\left(c_t\right),\beta_t\right)$ for a seller. Reformulating Proposition

[12] Clearly, however, the cutoff betas under transparency and partial opaqueness are in general not equal, because the information sets of the trader at time t differ in both cases.

1 for this partial opaqueness case characterizes the traders' equilibrium order submission strategies within this setting.

Propositions 5 and 6 demonstrated the existence of systematic patterns in order flow under transparency. The question arises whether these patterns extend to the partial opaqueness case. Propositions 7 and 8 reveal that the answer is ambiguous. Systematic patterns do also arise under partial opaqueness, but their nature is different from those under transparency. More specifically, Proposition 7 shows that the probability of a CN buy order after observing a DM trade can be smaller, equal or larger than after observing no order. As shown in the proofs, the resulting patterns depend on the probabilities the trader at $t+1$ assigns to a CN buy, CN sell or no order at t and on her expectations of the CN book, which in turn depend on the distribution of types $F(.)$ and the fraction of buyers and sellers in the market π_S and π_B.

Proposition 7 *The probability of a CN buy order at $t+1$ if the order at t was an observed DM trade (buy or sell) can be smaller, equal or larger compared to if no order was observed at t (i.e. $\phi^s_{t,po}(E_{t,po}(c_t), \beta_t) = -1^{CN}$ or $\phi^b_{t,po}(E_{t,po}(c_t), \beta_t) = 1^{CN}$ or $\phi^b_{t,po}(E_{t,po}(c_t), \beta_t) = 0$ or $\phi^s_{t,po}(E_{t,po}(c_t), \beta_t) = 0$):*

$$\Pr\Big[\phi^b_{t+1,po}\left(E_{t+1,po}(c_{t+1}), \beta_{t+1}\right) = 1^{CN} | \phi^b_{t,po}(E_{t,po}(c_t), \beta_t) = 1^{DM}$$

$$or\ \phi^s_{t,po}(E_{t,po}(c_t), \beta_t) = -1^{DM}, E_{t,po}(c_t)\Big]$$

$$<\ or\ =\ or\ >$$

$$\Pr\Big[\phi^b_{t+1,po}\left(E_{t+1,po}(c_{t+1}), \beta_{t+1}\right) = 1^{CN} | no\ order\ observed, E_{t,po}(c_t)\Big]. \tag{16}$$

A symmetric result holds for the other side of the market.

Proof. See the Appendix ■

Complementary to Proposition 7, Proposition 8 reveals that it can become more, equally or less likely that the current buyer submits a DM buy if the previous order was a DM trade, than if it was another type of order.

Proposition 8 *The probability of a DM buy at $t+1$ if a DM trade was observed at t can be smaller, equal or larger compared to if no order was observed at t (i.e. $\phi^s_{t,po}(E_{t,po}(c_t), \beta_t) =$*

-1^{CN} *or* $\phi^b_{t,po}(E_{t,po}(c_t), \beta_t) = 1^{CN}$ *or* $\phi^b_{t,po}(E_{t,po}(c_t), \beta_t) = 0$ *or* $\phi^s_{t,po}(E_{t,po}(c_t), \beta_t) = 0$*):*

$$\Pr\left[\phi^b_{t+1,po}\left(E_{t+1,po}(c_{t+1}), \beta_{t+1}\right) = 1^{DM} | \phi^b_{t,po}(E_{t,po}(c_t), \beta_t) = 1^{DM}\right.$$
$$or\ \phi^s_{t,po}(E_{t,po}(c_t), \beta_t) = -1^{DM}, E_{t,po}(c_t)]$$
$$<\ or\ =\ or\ >$$
$$\Pr\left[\phi^b_{t+1,po}\left(E_{t+1,po}(c_{t+1}), \beta_{t+1}\right) = 1^{DM} | no\ order\ observed, E_{t,po}(c_t)\right]. \tag{17}$$

A symmetric result holds for the other side of the market.

Proof. See the Appendix ■

Propositions 7 and 8 thus show that systematic patterns in order flow under partial opaqueness may change over time and work in two directions. The intuition for this finding is that observing no order relative to a DM-trade may be "good" news or "bad" news for a successive CN order. It is good news as observing no order may reveal the addition of counterparties. It is "bad" news when the observation of no order suggests that an interesting opportunity at a CN may have been preempted. This result is in contrast with the patterns in the transparency case which were unambiguously determined. In other words, we find that the CN's transparency plays an important role for order flow patterns to both the CN and the DM. Changing this institutional property of the CN may therefore impact order flow.

5 Welfare Analysis

We now introduce formal welfare definitions and characterize *ex ante* welfare for the different settings. The *ex ante* welfare measures we develop build on rational trader behavior. They are therefore identical to the "average" realized *ex post* welfare. We consider two complementary welfare measures. The first captures overall welfare, OW, and takes into account all agents' expected gains from trade. That is, the sum of all agents' expected utilities from trading (see Glosten (1998), Goettler, Parlour and Rajan (2005), or Hollifield, Miller, Sandas and Slive (2006) for a similar approach in defining gains from trade). In our setting, OW includes both trader welfare and dealer welfare. Our second measure focuses on trader welfare, TW, from an *ex ante* standpoint, as in Viswanathan and Wang (2002). TW and OW differ only because OW does take into account dealer welfare.

We first consider each of the isolation cases separately and compare them in a next step.

Changes in the degree of transparency leave welfare unaffected when considering a trading system in isolation as they do not influence order submission behavior. Finally, we turn to the coexistence of trading systems and highlight the welfare implications of different degrees of opaqueness.

5.1 Markets in Isolation

5.1.1 Dealer Market

With a DM in isolation, all submitted orders result in trades. However, an important inefficiency is that some traders refrain from submitting orders: only high valuation buyers and sellers are prepared to incur the half-spread. Overall welfare at the DM, OW^{DM}, and trader welfare at the DM, TW^{DM}, differ from each other due to dealer welfare. To evaluate dealer welfare, we procure dealers with a time-invariant β_{DM}, such that they set bid and ask prices A and B around V.[13] Formally, OW^{DM} and TW^{DM} then become:[14]

$$\begin{aligned} OW^{DM} &= T * OW_t^{DM} \\ &= T\left[\pi_B \int\limits_{A/V}^{\overline{\beta}} (\beta_t V - \beta_{DM} V)\, f(\beta_t)\, d\beta_t + (1-\pi_B) \int\limits_{\underline{\beta}}^{B/V} (\beta_{DM} V - \beta_t V)\, f(\beta_t)\, d\beta_t\right], \end{aligned}$$

$$\begin{aligned} TW^{DM} &= T * TW_t^{DM} \\ &= T\left[\pi_B \int\limits_{A/V}^{\overline{\beta}} (\beta_t V - A)\, f(\beta_t)\, d\beta_t + (1-\pi_B) \int\limits_{\underline{\beta}}^{B/V} (B - \beta_t V)\, f(\beta_t)\, d\beta_t\right], \end{aligned} \tag{18}$$

where OW_t^{DM} and TW_t^{DM} represent overall and trader welfare in period t, respectively.

Buyers with $\beta_t \leq \frac{A}{V}$ and sellers with $\beta_t \geq \frac{B}{V}$ do not submit orders (see also Figure 1). Sellers receive a price B from dealers whereas buyers pay a price A to dealers. Dealers value the stock at $\beta_{DM}V$. Prices are simple transfers between traders and dealers. They do affect, however, the participation of traders. Notice that both OW^{DM} and TW^{DM} are linear in the number of periods T, i.e. adding more periods proportionally increases both welfare measures. Hence, period t welfare, OW_t^{DM} and TW_t^{DM}, is constant over all periods t, implying it is always equal to average per-period welfare which we denote as $\overline{OW}^{DM}$ and

[13] Later on, in our explicit solutions, we will assume "symmetry" for all parameters such that $\beta_{DM} = 1$. This implies that dealers are assumed to have a median β.

[14] Strictly speaking, OW^{DM} and TW^{DM} depend also on the length of the trading day T, but we suppress this for notational convenience.

$\overline{TW}^{DM}$, i.e. $\overline{OW}^{DM} = (1/T)\sum_{t=1}^{T} OW_t^{DM}$ and $\overline{TW}^{DM} = (1/T)\sum_{t=1}^{T} TW_t^{DM}$.

5.1.2 Crossing Network

Trader and overall welfare are identical at a CN in isolation, as gains from trade go to traders only. Our two welfare measures therefore coincide and we label them as W^{CN}. All buyers with $\beta_t \geq \frac{A+B}{2V}$ and sellers with $\beta_t \leq \frac{A+B}{2V}$ submit orders (see Figure 2), but these orders may not always result in trades which represents an inefficiency of the CN. Also, trades execute at the midprice (which equals $(A+B)/2$). Again, prices simply represent transfers from one trader to another but prices determine traders' participation.

In computing welfare, we now need to take the weighted average over all states that can occur in each period t. For instance with transparency, in period 2 we have three possible states as the CN-book can be (-1,0), (0,0) or (0,1) depending on whether in period 1 a CN sell, no order, or a CN buy order was submitted, respectively. Each state has its own execution probabilities. Therefore, denote the set of all possible states in period t with a CN in isolation as Ω_t^{CN}, and an element of this set Ω_t^{CN} as ω_t^{CN}. The probability that state ω_t^{CN} occurs in period t is $\alpha_{\omega_t}^{CN}$, and the associated probabilities of execution at the CN are $p_{\omega_t}^{b,CN}$ and $p_{\omega_t}^{s,CN}$. Formally, W^{CN} can then be defined as:[15]

$$
\begin{aligned}
W^{CN} &= \sum_{t=1}^{T} W_t^{CN}, \\
\text{where} &: \\
W_t^{CN} &= \sum_{\omega_t^{CN}\in\Omega_t^{CN}} \alpha_{\omega_t}^{CN}\left[\pi_B \int_{\frac{A+B}{2V}}^{\bar{\beta}} p_{\omega_t}^{b,CN}\left(\beta_t V - \frac{A+B}{2}\right) f\left(\beta_t\right) d\beta_t \right. \\
&\quad \left. + \left(1-\pi_B\right)\int_{\underline{\beta}}^{\frac{A+B}{2V}} p_{\omega_t}^{s,CN}\left(\frac{A+B}{2} - \beta_t V\right) f\left(\beta_t\right) d\beta_t\right]. \qquad (19)
\end{aligned}
$$

Hence, we take a weighted average over all possible states and compute the state-contingent execution probabilities and welfare, which is necessary to compute expected ex ante welfare. Note that we did not need this notation in Section 3 as we there started from a given state at time t (which is then part of the information set of a trader) and analyzed cutoff betas and resulting order flow patterns conditional on this state.

It is interesting to remark that W^{CN} increases more than proportionally in the number

[15] The displayed formula for W^{CN} applies for transparency, where several states with associated probabilities are possible. Remark, however, that with a transparent CN book, the cutoff betas are equal to $\frac{A+B}{2V}$ for all possible states. When the CN book is opaque, Ω_t^{CN} reduces to a single state where the cutoff beta also equals $\frac{A+B}{2V}$. Therefore, W^{CN} is identical for transparency and opaqueness.

of periods T. The intuition for this result is that a higher number of periods T produces positive liquidity externalities on the average execution probabilities.

5.1.3 Comparing Markets in Isolation

A comparison of the two markets in isolation shows that each of them exhibits one type of inefficiency. On the one hand, a CN implies "order creation" compared to a DM: buyers with $\beta_t \in \left[\frac{A+B}{2V}, \frac{A}{V}\right]$ and sellers with $\beta_t \in \left[\frac{B}{V}, \frac{A+B}{2V}\right]$ only participate in the CN. On the other hand, the DM offers certainty of execution (but charges the half-spread for this service), while orders at a CN only execute when counterparties appear.

Comparison of the average per-period welfare $\overline{W}^{CN}$, $\overline{OW}^{DM}$ and $\overline{TW}^{DM}$ indicates that the DM is better for lower T and higher V, i.e. with shorter trading days and for assets with a higher underlying value (or lower relative spread). We illustrate this finding assuming a uniform distribution for β over $[0.8, 1.2]$, $\pi_B = 0.5$, $\frac{A+B}{2} = V$, and $\beta_{DM} = 1$. In Figure 4, we display the average per-period welfare $\overline{OW}^{DM}$, $\overline{TW}^{DM}$ and $\overline{W}^{CN}$ for different values of T (i.e. 1 to 8) as a function of V. As indicated before, both $\overline{OW}^{DM}$ and $\overline{TW}^{DM}$ are invariant to changes in T. In contrast, $\overline{W}^{CN}$ increases in T: the larger T, the larger the execution probability of submitted orders due to the liquidity externality. Indeed, we observe that $\overline{W}^{CN}$ is lower than $\overline{OW}^{DM}$ and $\overline{TW}^{DM}$ for low T and high V. The positive difference between $\overline{OW}^{DM}$ and $\overline{TW}^{DM}$ reflects dealers profits. Note that both $\overline{OW}^{DM}$ and $\overline{TW}^{DM}$ start at 0 for $V \leq 2.5$ as the spread then prices the DM out of the market.

Please insert Figure 4 around here.

As a prelude to the welfare discussion when markets coexist, we highlight that all forces driving our results when comparing markets in isolation are already at work for small T, i.e. $T \leq 4$.

Further, we also find some heterogeneous intertemporal effects, i.e. differences over periods t (results not displayed for brevity): the CN is more interesting for early arriving traders than for later arriving traders as W_t^{CN} decreases in t. This result stems from time priority. In particular, traders arriving early are better off as they "preempt" later arriving competing traders. Hence, per-period welfare decreases over time, or $W_t^{CN} \leq W_{t+i}^{CN}$, for all $i > 0$, as the execution probabilities drop in t. In contrast, per-period welfare at the DM, OW_t^{DM} and TW_t^{DM}, are $t-$invariant.

Do note that traders with different β_t may not necessarily agree on the ranking of trading systems as the ranking mentioned above holds for the "average" trader. From an ex ante perspective, traders with an "extreme" β_t tend to prefer the DM whereas traders with a "intermediate" β_t tend to prefer the CN. This will prove to be important for our welfare considerations when traders have the choice between trading systems presented in the next subsection.

5.2 Coexistence of Markets

In the remainder of this section, we analyze ex ante welfare when traders endogenously route their orders to the system maximizing their individual expected gains from trade. We also investigate how the degree of transparency at both trading systems impacts ex ante welfare. To this end, we first formally define for each informational setting ex ante overall welfare and trader welfare. In explicitly solving and illustrating the implications of a change in the informational setting, we first present the results for $T = 3$. In a subsequent robustness section, we highlight the additional insights stemming from T ranging between 2 and 4. Adding more periods only strongly complexifies the calculations. Based on our isolation cases, we expect, however, that adding more periods does not add much value as the most important insights in the isolation cases were already identified for $T \leq 4$.

We display the results for the average per-period welfare but discuss the period-specific welfare results when they bring new insights. This analysis allows us to explore whether adding a CN could improve on a DM from a trader welfare and an overall welfare perspective, i.e. whether the coexistence of a CN and a DM yields welfare gains relative to a DM in isolation, and how the degree of transparency affects ex ante welfare. Throughout the illustration, we determine ex ante welfare considering a uniform distribution for β_t with support $\left[\underline{\beta}, \bar{\beta}\right] = [0.8, 1.2]$. Further, V ranges from 2.5 to 10 and the one-tick spread is assumed to be symmetrically positioned around V (i.e. $A = V + \frac{1}{2}$ and $B = V - \frac{1}{2}$), implying that $\frac{A+B}{2} = V$.

5.2.1 Formal Definition of Welfare under Coexistence

We first proceed by presenting formal welfare definitions for the three informational settings under coexistence. Denote the set of all possible states in period t with coexistence between

CN and DM for informational setting i (with $i = tr, co, po$) as $\Omega_{t,i}$, and an element of $\Omega_{t,i}$ by $\omega_{t,i}$. The probability that state $\omega_{t,i}$ occurs in period t is $\alpha_{\omega_{t,i}}$, the cutoff betas between CN and DM in a particular state are $\overline{\beta}^b_{\omega_{t,i}}$ and $\underline{\beta}^s_{\omega_{t,i}}$, and the associated probabilities of execution at the CN, $p^b_{\omega_{t,i}}$ and $p^s_{\omega_{t,i}}$. Important to stress is that the set of states $\Omega_{t,tr}$, $\Omega_{t,co}$ and $\Omega_{t,po}$ are different from each other. This can easily be seen by noting that e.g. in period 2, under transparency we have three possible states (i.e. a CN book equal to (-1,0), (0,0) or (0,1)), while under complete opaqueness we only have one state (as nothing is observed after period 1). Finally, for partial opaqueness, two possible states exist: either a trader observed a DM order in period 1 (in this case she knows the book is (0,0)), or she observed nothing.[16]

Under coexistence of a CN and a DM, ex ante trader welfare, $TW^{CN-DM,i}$, and overall welfare $OW^{CN-DM,i}$ with $i = tr, co, po$ are then defined as follows (see also Figure 3 for $i = tr$):

$$TW^{CN-DM,i} = \sum_{t=1}^{T} TW_t^{CN-DM,i},$$

$$\begin{aligned} \text{where} &: \\ TW_t^{CN-DM,i} &= \sum_{\omega_{t,i}\in\Omega_{t,i}} \alpha_{\omega_{t,i}} * \\ &\left[\pi_B \int_{\frac{A+B}{2V}}^{\overline{\beta}^b_{\omega_{t,i}}} p^b_{\omega_{t,i}} \left(\beta_t V - \frac{A+B}{2}\right) f(\beta_t)\, d\beta_t \right. \\ &+ (1-\pi_B) \int_{\underline{\beta}^s_{\omega_{t,i}}}^{\frac{A+B}{2V}} p^s_{\omega_{t,i}} \left(\frac{A+B}{2} - \beta_t V\right) f(\beta_t)\, d\beta_t \\ &+ \pi_B \int_{\overline{\beta}^b_{\omega_{t,i}}}^{\overline{\beta}} (\beta_t V - A)\, f(\beta_t)\, d\beta \\ &\left. + (1-\pi_B) \int_{\underline{\beta}}^{\underline{\beta}^s_{\omega_{t,i}}} (B - \beta_t V)\, f(\beta_t)\, d\beta_t \right], \end{aligned} \tag{20}$$

and:

[16] Note that we did not need this more complex notation in Sections 3 and 4, when analyzing order flow patterns. In these sections, we started from a given state in period t, and computed the cutoff betas for this state. In the current Section 5, we now compute ex ante welfare and need to consider all possible states in period t. Therefore, we stress this by using $\omega_{t,i}$. In other words, e.g. $\overline{\beta}^b_{t,tr}$ in Section 3 is just short-hand notation for $\overline{\beta}^b_{\omega_{t,tr}}$, when starting from state $\omega_{t,tr}$.

$$OW^{CN-DM,i} = \sum_{t=1}^{T} OW_t^{CN-DM,i},$$

$$\begin{aligned}
\text{where} \quad &: \\
OW_t^{CN-DM,i} \quad &= \sum_{\omega_{t,i}\in\Omega_{t,i}} \alpha_{\omega_{t,i}} * \\
&\left[\pi_B \int_{\frac{A+B}{2V}}^{\overline{\beta}^b_{\omega_{t,i}}} p^b_{\omega_{t,i}} \left(\beta_t V - \frac{A+B}{2}\right) f(\beta_t)\, d\beta_t \right. \\
&+ (1-\pi_B) \int_{\underline{\beta}^s_{\omega_{t,i}}}^{\frac{A+B}{2V}} p^s_{\omega_{t,i}} \left(\frac{A+B}{2} - \beta_t V\right) f(\beta_t)\, d\beta_t \\
&+ \pi_B \int_{\overline{\beta}^b_{\omega_{t,i}}}^{\overline{\beta}} (\beta_t V - \beta_{DM} V)\, f(\beta_t)\, d\beta_t \\
&\left. + (1-\pi_B) \int_{\underline{\beta}}^{\underline{\beta}^s_{\omega_{t,i}}} (\beta_{DM} V - \beta_t V)\, f(\beta_t)\, d\beta_t \right]. \qquad (21)
\end{aligned}$$

With transparency ($i = tr$), traders rationally expect to be fully informed about previous traders' decisions when deciding where to route their order. Further, when both markets are completely opaque ($i = co$), later arriving traders anticipate they will not learn over time but know that orders by earlier arriving traders may have been submitted influencing their order submission strategies. Finally, with partial opaqueness ($i = po$), traders expect to learn over time whether orders have been submitted to the DM or not. This influences traders' order submission strategies and execution probabilities.

5.2.2 Comparing Coexistence: the Impact of Opaqueness

Figure 5 displays the impact of coexistence for the three informational settings on average per-period trader welfare and overall welfare. It considers the results for $T = 3$.

Please insert Figure 5 around here.

Trader Welfare In a first step, we purely focus on *average per-period trader welfare* for the three informational settings, $\overline{TW}^{CN-DM,tr}$, $\overline{TW}^{CN-DM,po}$, and $\overline{TW}^{CN-DM,co}$, and compare those with $\overline{TW}^{DM}$. We obtain two main results. First, coexistence produces greater average trader welfare than the DM in isolation. Second, a greater degree of transparency increases average trader welfare. Formally,

$$\overline{TW}^{CN-DM,tr} \geq \overline{TW}^{CN-DM,po} \geq \overline{TW}^{CN-DM,co} \geq \overline{TW}^{DM}. \tag{22}$$

Figure 5 illustrates those two main results but remark that we do not display $\overline{TW}^{CN-DM,co}$ as the differences with $\overline{TW}^{CN-DM,po}$ are small which causes both to visually almost coincide. We, however, elaborate on the differences between complete and partial opaqueness in Figure 6.

What drives these two results on trader welfare? It is clear that coexistence induces both order creation and trade diversion. The individual magnitude of these two effects determines their joint impact on welfare relative to $\overline{TW}^{DM}$. Order creation relative to the DM-only case stems from buyers with $\beta_t \in \left[\frac{A+B}{2V}, \frac{A}{V}\right]$ and sellers with $\beta_t \in \left[\frac{B}{V}, \frac{A+B}{2V}\right]$ submitting orders which only execute when counterparties appear. Through the execution of these orders, order creation translates into trade creation which leads to an identical increase in average overall and trader welfare. We label this as the *"$\overline{TW}$ order creation effect"* or the *"$\overline{OW}$ order creation effect"*. It is important to remark that the size of this effect is different for the three informational settings as the execution probabilities of the created orders differ across these settings. As towards trade diversion, from a buyer perspective, we can show that trader welfare in a given state $\omega_{t,i}$ increases in $\bar{\beta}^b_{\omega_{t,i}}$ as long as there is no "complete" trade diversion, i.e. when $\bar{\beta}^b_{\omega_{t,i}} \in \left[\frac{A}{V}, \bar{\beta}\right[$, $\frac{\partial TW^{CN-DM,i}_{\omega_{t,i}}}{\partial \bar{\beta}^b_{\omega_{t,i}}} \geq 0$ for $i = tr, co, po$. Note, however, that when trade diversion is complete (i.e. when $\bar{\beta}^b_{\omega_{t,i}} = \bar{\beta}$), the impact on trader welfare still varies in the execution probabilities. That is, given complete trade diversion, an increase in the CN's execution probability $p^b_{\omega_{t,i}}$ improves trader welfare within state $\omega_{t,i}$. Consequently, it is impossible to provide a generally valid ranking of trader welfare that is uniquely based on the cutoff betas, $\bar{\beta}^b_{\omega_{t,i}}$ and $\underline{\beta}^s_{\omega_{t,i}}$, or on a state-weighted average of these cutoff betas. To accommodate this issue, we construct a welfare statistic that quantifies the impact of trade diversion on trader welfare which we label *"$\overline{TW}$ trade diversion effect"*. This effect measures the impact on trader welfare relative to $\overline{TW}^{DM}$ of all traders that would have gone to the DM in isolation. More specifically, the "$\overline{TW}$ trade diversion effect" is due to traders that divert from a (hypothetical) DM in isolation to a CN. Combined, the "$\overline{TW}$ order creation effect" and the "$\overline{TW}$ trade diversion effect" make up the difference between $\overline{TW}^{CN-DM,i}$ and $\overline{TW}^{DM}$ for each informational setting i.[17] Formally, both concepts are defined as follows:

[17] Note that the "$\overline{TW}$ order creation effect" in fact hinges on the degree of trade diversion. The reasoning is that the degree of trade diversion determines the expected execution probability at the CN.

$$\overline{TW}\text{ order creation effect, } i = \frac{1}{T}\sum_{t=1}^{T}\sum_{\omega_{t,i}\in\Omega_{t,i}}\alpha_{\omega_{t,i}} * \left[\pi_B \int_{\frac{A+B}{2V}}^{A/V} p^b_{\omega_{t,i}}\left(\beta_t V - \frac{A+B}{2}\right) f(\beta_t)\, d\beta_t \right.$$
$$\left. + (1-\pi_B)\int_{B/V}^{\frac{A+B}{2V}} p^s_{\omega_{t,i}}\left(\frac{A+B}{2} - \beta_t V\right) f(\beta_t)\, d\beta_t \right] \quad (23)$$

and:

$$\overline{TW}\text{ trade diversion effect, } i = \frac{1}{T}\sum_{t=1}^{T}\sum_{\omega_{t,i}\in\Omega_{t,i}}\alpha_{\omega_{t,i}} * \left[\pi_B \int_{A/V}^{\bar{\beta}^b_{\omega_{t,i}}} p^b_{\omega_{t,i}}\left(\beta_t V - \frac{A+B}{2}\right) f(\beta_t)\, d\beta_t \right.$$
$$+ (1-\pi_B)\int_{\underline{\beta}^s_{\omega_{t,i}}}^{B/V} p^s_{\omega_{t,i}}\left(\frac{A+B}{2} - \beta_t V\right) f(\beta_t)\, d\beta_t$$
$$+\pi_B \int_{\bar{\beta}^b_{\omega_{t,i}}}^{\overline{\beta}} (\beta_t V - A) f(\beta_t)\, d\beta$$
$$\left. + (1-\pi_B)\int_{\underline{\beta}}^{\underline{\beta}^s_{\omega_{t,i}}} (B - \beta_t V) f(\beta_t)\, d\beta_t \right]$$
$$-\overline{TW}^{DM} \quad (24)$$

We display the "$\overline{TW}$ order creation effect" and the "$\overline{TW}$ trade diversion effect" for the three informational settings in Figure 6. We observe that both effects are positive and largest for transparency, thus confirming the results of Figure 5. The "$\overline{TW}$ order creation effect" is considerable for low V as it entails the impact on trader welfare of a substantial set of traders. In particular, for $V = 2.5$, the "$\overline{TW}$ order creation effect" completely explains $\overline{TW}^{CN-DM,i}$ as $\overline{TW}^{DM}$ then equals zero. For higher V, the "$\overline{TW}$ order creation effect" tends towards zero as the order creation induced by the CN becomes relatively less important due to the lowering relative spread. Further, the "$\overline{TW}$ order creation effect" increases in the degree of transparency (see below for an explanation). Focusing next on the "$\overline{TW}$ trade diversion effect", we observe it increases in V at first. This is due to a substantial set of traders diverting to the CN at low levels of V. These traders' welfare gains relative to the DM in isolation increase in V within this region. For larger V, however, the "$\overline{TW}$ trade diversion effect" starts to decrease as the DM becomes more attractive and fewer traders divert to the CN. Also the "$\overline{TW}$ trade diversion effect" increases in the degree of transparency. Hence, completely transparent markets are most beneficial to both segments of traders, the CN-only traders and the traders that would trade in a DM in isolation. The reasoning is that transparency invites order flow to the CN which will be hit by counterparties leading to greater execution probabilities. This stimulates the "$\overline{TW}$ order creation effect" and the "$\overline{TW}$ trade diversion effect". As argued before, at higher values of V both effects gradually decrease due to the fact that the DM in isolation is already quite competitive by its decreasing relative spread. Still, transparency performs better than the opaqueness settings. Hence, having a transparent CN book positively affects trader welfare: it induces a higher degree of trade diversion, generates on average larger execution probabilities and thus keeps the CN competitive for a broader range of V.[18] Further, from Figure 6, we learn that partial opaqueness performs only slightly better than complete opaqueness. The reasoning for these small differences is that the degree of trade diversion is only slightly larger for partial opaqueness than for complete opaqueness, leading to quite similar execution probabilities, which also implies that the "$\overline{TW}$ order creation effect" is comparable. Even though traders obtain some information about order flow when markets are partially opaque, their welfare only increases marginally as they do not learn the "direction" of the order flow.

Please insert Figure 6 around here.

An interesting question is on which market traders generate their welfare when trading systems coexist. To highlight the merits of each market, Figure 7 splits $\overline{TW}^{CN-DM,i}$ up into welfare stemming from the CN and the DM. This figure only displays the results for partial opaqueness and transparency, as partial and complete opaqueness visually almost would coincide. As expected, we observe that trader welfare from the DM is larger than from the CN for higher V. Also trader welfare from the DM increases in V. Figure 5 showed that trader welfare, summed up over both markets, is highest with transparency. However, Figure 7 reveals that heterogeneity arises. For low values of V, trader welfare from the DM is higher with transparency. For higher V, less traders decide to go to the DM when markets are transparent. Trader welfare stemming from the CN in general is higher with transparency. Figure 7 also shows that in terms of trader welfare, the CN performs better than the DM for low V and vice versa.

[18] We also perform a per-period analysis (i.e. for $t = 1, 2$ or 3), and find that transparency produces the greatest trader welfare in every period.

Please insert Figure 7 around here.

Overall Welfare Next, we shift our focus to *average per-period overall welfare* for the three informational settings, $\overline{OW}^{CN-DM,tr}$, $\overline{OW}^{CN-DM,co}$ and $\overline{OW}^{CN-DM,po}$. The results are displayed in Figure 5, again $\overline{OW}^{CN-DM,co}$ is not included as it visually almost coincides with $\overline{OW}^{CN-DM,po}$. We observe two main results. First, overall welfare of a DM in isolation, $\overline{OW}^{DM}$, outranks all coexistence settings, except for low values of V. Second, and in contrast to trader welfare, transparency only yields higher average overall welfare than opaqueness for low values of V. Indeed, we notice in Figure 5 that $\overline{OW}^{CN-DM,po}$ and $\overline{OW}^{CN-DM,tr}$ intersect for V around 7.5.

How to explain these results? The main force driving these findings is the potentially negative impact of trade diversion on dealer welfare when markets coexist. While diverted trades imply with certainty that dealers do not earn the spread, they only yield additional trader welfare for the executed part of the CN order flow. As such, coexistence performs better than the DM in isolation case when the loss in dealer welfare (induced by trade diversion) is more than compensated by additional trader welfare (induced by trade diversion and order creation). As towards this last component, recall from our trader welfare analysis that coexistence compared to the DM in isolation case creates two effects. While diverted trades positively affect the *"$\overline{OW}$ or $\overline{TW}$ order creation effect"* (both are identical) and the *"$\overline{TW}$ trade diversion effect"*, they harm dealer welfare. The sum of the *"$\overline{TW}$ trade diversion effect"* and the loss in dealer welfare relative to a DM in isolation constitutes the *"$\overline{OW}$ trade diversion effect"*. This *"$\overline{OW}$ trade diversion effect"* is always negative and exhibits a non-linear structure as it hinges on the degree of trade diversion and the execution probabilities at the CN. To illustrate this non-linearity, note that, from a buyer perspective and for a given state, we have that the *"$\overline{OW}$ trade diversion effect"* becomes more negative in $\bar{\beta}^b_{\omega_{t,i}}$ as long as trade diversion is not complete. However, when trade diversion is complete, or $\bar{\beta}^b_{\omega_{t,i}} = \bar{\beta}$, then it is easy to see that the *"$\overline{OW}$ trade diversion effect"* actually becomes less negative in the execution probabilities at the CN. When the execution probabilities at the CN are one, the *"$\overline{OW}$ trade diversion effect"* reaches its maximum level of zero.

The sum of the positive *"$\overline{OW}$ order creation effect"* and negative (but non-linear) *"$\overline{OW}$ trade diversion effect"* determines the impact of coexistence and the degree of opaqueness on overall welfare. This drives our two main results on overall welfare identified above. Our first result is that coexistence only leads to higher welfare than a DM in isolation for low V: the *"$\overline{OW}$ order creation effect"* then dominates the modestly negative *"$\overline{OW}$* trade diversion effect" since execution probabilities at the CN are quite high. The opposite holds for higher V. The second result is that $\overline{OW}^{CN-DM,tr}$ is larger than $\overline{OW}^{CN-DM,po}$ only for low V: in this region transparency induces either complete trade diversion with high execution probabilities in certain states or no trade diversion at all in other states, whereas opaqueness tends more towards intermediate trade diversion combined with lower execution probabilities. The opposite holds for higher V, where transparency seems to induce substantial trade diversion combined with relatively low execution probabilities. Finally, we also observe in Figure 5 that $\overline{OW}^{CN-DM,po}$ exactly equals $\overline{TW}^{CN-DM,po}$ for low values of V. This implies that complete trade diversion occurs in all possible states and periods. For transparency, however, overall welfare always outranks trader welfare. In this setting, in some later periods, some states render a zero execution probability to the arriving trader implying no trade diversion and higher overall welfare.

Finally, our per-period analysis reveals some interesting heterogeneity when considering the different periods t within the trading day (results not displayed). We find that OW_3^{CN-DM} under transparency is larger than with opaqueness for all values of V as traders take fully informed decisions under transparency. In period 1, however, both opaqueness settings perform better than transparency.

Robustness checks, extensions and summary In this last subsection, we elaborate on some *robustness checks and extensions.* First, our results remain qualitatively unaffected by variations in the beta support (to e.g. $[0.7, 1.3]$ or $[0.6, 1.4]$). The impact of order creation naturally becomes less important when the distribution widens. Also, some traders exhibit a higher willingness to pay for a DM, such that coexistence adds less to welfare.

Second, Figure 8 displays the results for trader welfare for $T = 2$ to 4 and shows that our main findings as discussed for $T = 3$ continue to hold. Indeed, we observe that transparency offers the highest average per-period trader welfare for every T, which is in line with the results shown in Figure 5. In addition, we learn that average per-period trader welfare increases in T. This result is reminiscent of our findings for the welfare on the CN in isolation case. The intuition is that the execution probabilities at the CN increase in T. Further, our previously discussed results on overall welfare as depicted in Figure 5 also remain valid when considering

$T = 2$ to 4 (results not displayed for brevity).

Please insert Figure 8 around here.

Third, our analysis provides insights on the question whether CNs should opt to become more transparent or not. We will consider the answer to this question with respect to two different goals that could potentially be set by the CN. One view is that a CN aims to maximize the welfare of those trading on its system. Figure 7 shows that in general trader welfare from trading at the CN is greater when it is transparent. Another view is that a CN aims to maximize the *number of trades* on its system, for example, because it charges a fixed fee per transaction. Note that the number of trades may not be a good proxy for welfare as typically only agents with "intermediate" β trade at the CN, whereas agents with "extreme" β trade at the DM. In undisplayed exercises, we find that the number of trades at the CN is largest when the CN is transparent. While typically CNs are quite opaque, recently, we indeed observe that some CNs have become more transparent. Examples includes BIDS and ITG's POSIT-Now with BLOCKalert, which reveal the arrival of potential counterparties to the investor community.

Finally, while our analysis so far assumed that traders submit one-unit orders, in practice traders may have multiple units to trade. Consider now the setting where traders may have single or multiple units to trade. Two cases can be considered. The first is where the traders' choice of trading system is made *before* knowing their order size (as in Viswanathan and Wang (2002)). This implies that orders can not be split such that our main findings are only affected in a rather straightforward way (i.e. larger orders affect execution probabilities and therefore the choice between CN and DM). The second case is where traders that have a multiple unit order can engage in order splitting as in the extension of Parlour (1998) to multiple units. In our setting, order splitting implies choosing for different trading systems. This case is highly more complex as the state space expands dramatically as some traders may have one unit whereas others multiple units. To identify the impact of multiple units and the potential issue of order splitting, consider a stylized two-period case with all traders exposed to identical multiple-unit orders. We argue that most of our results remain unaffected for transparency and complete opaqueness as traders are risk neutral. More specifically, with transparent markets traders' order submission strategies will not change compared to our base case settings. The reasoning is that it is always optimal for a first-period buyer or seller with a given β_t to put the entire multiple-unit order either on the CN or the DM. Her decision is unaffected as the order is publicly revealed and the "good" counterparty will find it optimal to take either the entire order or no unit at all. With complete opaqueness, equivalently, the traders' strategies are unaffected by order size due to risk neutrality. Technically, the system of equations to be solved is independent from trade size. With partial opaqueness, however, traders might find it interesting to submit one order to the DM and route the remainder to the CN. In this way they "reveal" that orders were submitted to the CN. The results of partial opaqueness then could be expected become closer to those of transparency.

In *summary*, our welfare analysis shows that when comparing markets in isolation, a DM offers both greater trader and overall welfare than a CN for low T and high V. Coexistence of markets leads to greater trader welfare than a system in isolation due to the widening of traders' opportunity sets. The impact of coexistence on trader welfare is more pronounced at lower values of V. For higher values of V, though, coexistence adds less in terms of trader welfare reflecting the fact that traders opt for the DM when the "spread effect" becomes relatively less important. Coexistence, however, may decrease overall welfare compared to a DM in isolation as trade diversion decreases dealers' profits. Also, a greater degree of transparency unambiguously increases trader welfare as traders anticipate their orders will be revealed to potential counterparties. The impact of the degree of transparency on overall welfare depends on the asset value V and the associated relative spread. When V is low, transparency ranks better than both opaqueness settings. This result reverses, however, when V is high.

6 Conclusion

This paper presents a dynamic microstructure model to study the interaction between two trading systems. We study the competition between a crossing network (CN) and a dealer market (DM) within three different informational environments. In particular, a transparency setting where agents have full information on the CN's order book and DM trades is contrasted to two opaqueness settings. Under "complete" opaqueness, traders do not have any information on the CN's order book, nor on DM trades, while with "partial" opaqueness they observe only DM trades, but not order flow to the CN. In practice, CNs are indeed quite opaque as they often prevent traders from observing the CN's order book.

We find that introducing a CN next to a DM generates two effects on order flow. First, it leads to "*order creation*" as the CN attracts investors who would refrain from trading in the absence of a CN. The reasoning is that traders can save the half-spread when trading at the CN, making the submission of a CN order profitable for these investors. Second, some orders by relatively low willingness to trade agents trading on the DM are now diverted to the CN. This "*trade diversion*" induces competition for the DM.

We also show that the execution probability at a CN is endogenous. It depends on the state of the CN's order book (if transparent), the observed order flow, and the expectation of past and future orders. Thus, although we start from dealers willing to provide liquidity at exogenously given bid and ask prices, we partly endogenize liquidity supply and demand by looking at traders submitting orders for potential execution at a CN. Our dynamic model displays two externalities on the CN as documented in Hendershott and Mendelson (2000), who consider competition between a DM and a CN in a static game with simultaneous order submissions. On the one hand, the CN is characterized by a positive (liquidity) externality as adding a CN buy (sell) order is beneficial to future CN sellers (buyers). On the other hand, a CN exhibits a negative (crowding) externality as investors with a low willingness to trade who arrive early in the trading day may preempt investors with a higher willingness to trade who arrive later.

Our welfare results can be summarized as follows. First, when comparing markets in isolation, we find that a DM offers both greater trader and overall welfare than a CN when the execution probability at the CN and the relative spread are low. Second, order creation and trade diversion determine the impact of coexistence of trading systems and the degree of transparency on welfare. Coexistence of trading systems leads to greater trader welfare but may decrease overall welfare compared to a DM in isolation. A greater degree of transparency unambiguously increases trader welfare as traders base their order submission strategies on a larger information set. The impact of the degree of transparency on overall welfare depends on the relative spread in the DM. When the relative spread is high, transparency ranks better than both opaqueness settings. The positive contribution to welfare of order creation is then substantial whereas the negative impact of trade diversion is limited. This result reverses when the relative spread is low. In sum, our welfare analysis shows that greater transparency might be beneficial but that the ultimate answer hinges on the exact welfare criterion that is employed. Regulators therefore should stimulate transparency when considering trader welfare. The answer is less clearcut when considering overall welfare.

Finally, our model offers a number of empirical predictions. In particular, we find systematic patterns in order flow under transparency and partial opaqueness. These patterns stem from changes in the imbalance at the CN's order book. With transparency we find that the probability that the next order is a CN order at the same side of the market is smaller after such an order than after any other order. Also, the probability of a DM sell decreases and the probability of a DM buy increases when the previous order was a CN buy order. Only CN orders generate time-varying order flow on both trading systems as DM trades leave the CN's order book unaffected. Systematic patterns also arise with partial opaqueness. Although traders now only observe past DM trades and no CN orders, they use this information to form expectations on the CN's order book imbalance and to determine their trading strategy. We show that the degree of transparency at the CN has important implications for order flow. More specifically, compared to a transparent CN, order flow patterns may reverse when the CN is opaque. In general, our empirical predictions demonstrate that it is important to take the interaction between trading systems, as well as their institutional characteristics, into account when measuring "normal" order flow. Some order or trade flow sequences, when analyzed in individual markets, could wrongly be interpreted as being driven by informed trading, whereas they are actually caused by the interaction of trading systems. An interesting empirical application of our model would therefore be to determine the importance of this "interaction effect" in explaining observed order flow patterns relative to other factors such as private information or dealers' inventory management.

Appendix: Proofs

Proof of Proposition 1. Suppose first that the trader arriving at time t is a buyer. She selects her strategy to maximize her profits, i.e. $\max\left[\beta_t V - A, p^b_{t,tr}\left(\beta_t V - (A+B)/2\right), 0\right]$. Now define

$$\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right) = \left| \begin{array}{ll} \bar{\beta} & \text{if } p^b_{t,tr} \geq \frac{\bar{\beta}V - A}{\bar{\beta}V - (A+B)/2} \\ \text{solves } p^b_{t,tr}\left(\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right)V - (A+B)/2\right) = \bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right)V - A & \text{otherwise} \end{array} \right. .$$

This implies that $\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right)$ is an upper bound on CN buying because in the second case $p^b_{t,tr}\left((A+B)/2 - \bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right)V\right)$ increases in β at rate $p^b_{t,tr}V$, whereas $\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right)V - A$ increases at rate V. The condition

$$p^b_{t,tr} \geq \frac{\bar{\beta}V - A}{\bar{\beta}V - (A+B)/2}$$

can be interpreted as follows. If this condition is fulfilled, then $p^b_{t,tr}\left(\bar{\beta}V - (A+B)/2\right) \geq \bar{\beta}V - A$, implying that even for $\bar{\beta}$ the profit of an order to the CN is higher than the profit of a DM trade. In that case, traders always choose to submit a CN order and the region of β's for which investors submit DM trades is empty. After solving and some rewriting, we find that

$$\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right) = \min\left[\frac{\frac{A+B}{2}}{V} + \frac{1/2}{V\left(1 - p^b_{t,tr}\right)}, \bar{\beta}\right].$$

The cutoff β's between submitting an order to the CN and remaining out of the market are determined by how large the trader's valuation of the asset is, relative to its price. The lowest β-type who would buy at the CN is the one who values the asset at $\frac{(A+B)/2}{V}$. Hence, define

$$\underline{\beta}^b_{t,tr}\left(p^b_{t,tr}\right) = \left| \begin{array}{ll} \frac{\frac{A+B}{2}}{V} & \text{if } p^b_{t,tr} > 0 \\ \frac{A}{V} & \text{otherwise} \end{array} \right. .$$

Consequently, also $\bar{\beta}^b_{t,tr}\left(p^b_{t,tr}\right) \geq \underline{\beta}^b_{t,tr}\left(p^b_{t,tr}\right)$.

Suppose that the trader arriving at time t is a seller. She chooses her strategy to $\max\left[B - \beta_t V, p^s_{t,tr}\left((A+B)/2 - \beta_t V\right), 0\right]$. Define

$$\underline{\beta}^s_{t,tr}\left(p^s_{t,tr}\right) = \left| \begin{array}{ll} \underline{\beta} & \text{if } p^s_{t,tr} \geq \frac{B - \underline{\beta}V}{(A+B)/2 - \underline{\beta}V} \\ \text{solves } p^s_{t,tr}\left((A+B)/2 - \underline{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)V\right) = B - \underline{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)V & \text{otherwise} \end{array} \right. .$$

$\underline{\beta}^s_{t,tr}$ is a lower bound on CN selling because in the second case $p^s_{t,tr}\left(\left(A+B\right)/2-\underline{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)V\right)$ decreases in β at rate $p^s_{t,tr}V$, whereas $B-\underline{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)V$ decreases at rate V.

The condition

$$p^s_{t,tr}\geq\frac{B-\underline{\beta}V}{\left(A+B\right)/2-\underline{\beta}V}$$

can be understood as follows. If this condition is fulfilled, then $p^s_{t,tr}\left(\left(A+B\right)/2-\underline{\beta}V\right)\geq B-\underline{\beta}V$, meaning that even for $\underline{\beta}$ the profit of an order to the CN is higher than the profit of a DM trade. In that case, traders will always choose to submit a CN order and the region of β's for which investors submit DM trades is empty. Rewriting the cutoff value gives:

$$\underline{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)=\max\left[\frac{\frac{A+B}{2}}{V}-\frac{1/2}{V\left(1-p^s_{t,tr}\right)},\underline{\beta}\right].$$

The cutoff β's between submitting an order to the CN and remaining out of the market are determined by how large the trader's valuation of the asset is, relative to its price. The highest β-type who would CN sell is the one who values the asset at $\frac{(A+B)/2}{V}$. Define

$$\bar{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)=\left|\begin{array}{ll}\frac{(A+B)/2}{V} & \text{if } p^s_{t,tr}>0\\ \frac{B}{V} & \text{otherwise}\end{array}\right.,$$

so we find that $\underline{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)\leq\bar{\beta}^s_{t,tr}\left(p^s_{t,tr}\right)$. ∎

Proof of Proposition 2. We prove the proposition in a recursive way and by contradiction. As a starting point, it can be seen that the proposition holds for the terminal period T. At time T, the execution probability of a CN order is either one (if a trader can join the strictly shortest queue in the CN) or zero otherwise. Then the proposition holds since:

$$\begin{array}{lll}\bar{\beta}^b_{T,tr}=\underline{\beta}^b_{T,tr}=\frac{\frac{A+B}{2}}{V} & \text{or} & \bar{\beta}^b_{T,tr}=\underline{\beta}^b_{T,tr}=\frac{A}{V}\\ \underline{\beta}^s_{T,tr}=\bar{\beta}^s_{T,tr}=\frac{\frac{A+B}{2}}{V} & \text{or} & \underline{\beta}^s_{T,tr}=\bar{\beta}^s_{T,tr}=\frac{B}{V}.\end{array}$$

Suppose now that the proposition is false. However, since it is true at T, there must exist a period τ such that for $t>\tau$, all parts of the proposition hold, but at τ at least one part does not hold.

Suppose that (iv) does not hold at τ. It must then be the case that:

$$\underline{\beta}^s_{\tau,tr}\left(c^b_\tau, c^s_\tau\right) < \underline{\beta}^s_{\tau,tr}\left(c^b_\tau, c^s_\tau + 1\right).$$

This means that a seller having a $\beta_{\tau,tr} \in \left[\underline{\beta}^s_{\tau,tr}\left(c^b_\tau, c^s_\tau\right), \underline{\beta}^s_{\tau,tr}\left(c^b_\tau, c^s_\tau + 1\right)\right)$ will submit a DM order when the CN's order book is $\left(c^b_\tau, c^s_\tau + 1\right)$ and a CN order when the CN's order book is $\left(c^b_\tau, c^s_\tau\right)$. In contrast, suppose that the trader would opt for a CN order in the former case (i.e. when the CN's order book is $\left(c^b_\tau, c^s_\tau + 1\right)$), the CN's order book at $\tau + 1$ then becomes $\left(c^b_\tau, c^s_\tau\right)$. If the CN's order book at τ is $\left(c^b_\tau, c^s_\tau\right)$, the trader submits a CN sell order, resulting in the CN's order book $\tau + 1$ being $\left(c^b_\tau, c^s_\tau - 1\right)$. The next trader, arriving at time $\tau + 1$, can be either a buyer or a seller.

case a: A seller arrives at $\tau + 1$.

In this case, we know that by assumption (iv) holds for all periods $t > \tau$. Therefore:

$$p^s_{\tau+1,tr}\left(c^b_\tau, c^s_\tau - 1\right) \le p^s_{\tau+1,tr}\left(c^b_\tau, c^s_\tau\right).$$

Moreover, due to time priority at the CN, an order that has been submitted in $\tau + 1$ will only be executed if the previous order in the queue has been executed. This means that conditional on a seller arriving at $\tau + 1$:

$$p^s_{\tau,tr}\left(\left(c^b_\tau, c^s_\tau\right) |\text{seller arrives at } \tau + 1\right) \le p^s_{\tau,tr}\left(\left(c^b_\tau, c^s_\tau + 1\right) |\text{seller arrives at } \tau + 1\right).$$

Then it follows that

$$\begin{aligned}
&\left(\frac{A+B}{2} - \beta_\tau V\right) p^s_{\tau,tr}\left(\left(c^b_\tau, c^s_\tau + 1\right) |\text{seller arrives at } \tau + 1\right) \\
&\ge \left(\frac{A+B}{2} - \beta_\tau V\right) p^s_{\tau,tr}\left(\left(c^b_\tau, c^s_\tau\right) |\text{seller arrives at } \tau + 1\right) \\
&\ge B - \beta_\tau V.
\end{aligned}$$

Hence, if the trader at time $\tau + 1$ is a seller, the payoff of a CN sell order is higher when the sell side of the CN's order book in the CN is thinner in period τ. Hence, it cannot be optimal for an investor to submit a DM trade when the queue at the sell side is shorter.

case b: A buyer arrives at $\tau + 1$.

We know that by assumption (ii) is true at $\tau+1$. This means that either traders do not change their behavior or traders with:

$$\beta_{\tau+1} \in \left[\overline{\beta}^b_{\tau+1,tr}\left(c^b_\tau, c^s_\tau\right), \overline{\beta}^b_{\tau+1,tr}\left(c^b_\tau, c^s_\tau - 1\right)\right]$$

submit a DM trade when the CN's order book is $\left(c^b_\tau, c^s_\tau\right)$, which results in a CN's order book at time $\tau+2$ of $\left(c^b_\tau, c^s_\tau\right)$, and submit a CN order when the CN's order book is $\left(c^b_\tau, c^s_\tau - 1\right)$ giving a book at $\tau+2$ of $\left(c^b_\tau + 1, c^s_\tau - 1\right)$. Since (vi) holds at $\tau+2$ (vi), for a seller arriving in this period:

$$p^s_{\tau+2,tr}\left(c^b_\tau, c^s_\tau\right) = p^s_{\tau+2,tr}\left(c^b_\tau + 1, c^s_\tau - 1\right).$$

Similarly, because (iv) is true:

$$p^s_{\tau+2,tr}\left(c^b_\tau, c^s_\tau + 1\right) \geq p^s_{\tau+2,tr}\left(c^b_\tau, c^s_\tau\right).$$

Since an order submitted at $\tau+2$ can only be executed if an order submitted at time τ has been executed, it follows that:

$$\begin{aligned}
&\left(\frac{A+B}{2} - \beta_\tau V\right) p^s_{\tau,tr}\left(\left(c^b_\tau, c^s_\tau + 1\right) |\text{buyer arrives at } \tau+1\right) \\
&\geq \left(\frac{A+B}{2} - \beta_\tau V\right) p^s_{\tau,tr}\left(\left(c^b_\tau, c^s_\tau\right) |\text{buyer arrives at } \tau+1\right) \\
&\geq B - \beta_\tau V.
\end{aligned}$$

Hence, conditional upon a buyer arriving at time $\tau+1$, there is a contradiction.

Statement (iv) is therefore true.

A symmetric proof can be constructed for (i). Along the same lines as above, the other parts of the proposition can be proven. ■

Proof of Proposition 5. The time t probability of observing a CN buy at time $t+1$ is

$$\Pr\left[\phi^b_{t+1,tr}\left(c_{t+1}, \beta_{t+1}\right) = 1^{CN}\right] = \pi_B\left[F\left(\overline{\beta}^b_{t+1,tr}\left(c_{t+1}\right)\right) - F\left(\underline{\beta}^b_{t+1,tr}\left(c_{t+1}\right)\right)\right].$$

Suppose that the order imbalance in the CN $c^b_t - |c^s_t| < T - t - 1$, such that the probability of execution of a CN buy is not zero. Then $\underline{\beta}^b_{t+1,tr}\left(c_{t+1}\right)$ is independent of the CN's order book. If the order at t was a CN buy order, then the CN-book at $t+1$ is $\left(c_t^b+1, c_t^s\right)$, if it was a CN sell the CN-book becomes $\left(c_t^b, c_t^s-1\right)$ and if the order was a DM trade, the CN-book does not change: $c_t = c_{t+1}$. From Proposition 2, we know that

$$\bar{\beta}_{t+1,tr}^b\left(c_t^b+1, c_t^s\right) \leq \bar{\beta}_{t+1,tr}^b\left(c_t^b, c_t^s\right) \leq \bar{\beta}_{t+1,tr}^b\left(c_t^b, c_t^s-1\right).$$

Since $F(.)$ is monotonically nondecreasing in β, the result follows.

Suppose now that $c_t^b-|c_t^s| \geq T-t-1$, this means that either no CN orders are submitted, in which case the proposition holds trivially, or at time t the extra CN order submitted changes the execution probability to zero. In this case $\bar{\beta}_{t+1,tr}^b\left(c_t^b+1, c_t^s\right) = A/V$; hence the result follows since also $\underline{\beta}_{t+1,tr}^b\left(c_{t+1}\right) = A/V$.

A similar proof can be constructed for CN sell orders. ■

Proof of Proposition 6. The time t probability of observing a CN buy at $t+1$ is

$$\Pr\left[\phi_{t+1,tr}^b\left(c_{t+1}, \beta_{t+1}\right) = 1^{CN}\right] = \pi_B\left[F\left(\bar{\beta}\right) - F\left(\underline{\beta}_{t+1,tr}^b\left(c_{t+1}\right)\right)\right].$$

$F\left(\bar{\beta}\right)$ is fixed and independent of the CN's order book. If the order at t was a CN buy order, then the CN's order book at $t+1$ is $\left(c_t^b+1, c_t^s\right)$, if it was a CN sell the CN's order book becomes $\left(c_t^b, c_t^s-1\right)$ and if the order was a DM trade, the CN's order book does not change: $c_t = c_{t+1}$. From Proposition 2, we know that

$$\bar{\beta}_{t+1,tr}^b\left(c_t^b+1, c_t^s\right) \leq \bar{\beta}_{t+1,tr}^b\left(c_t^b, c_t^s\right) \leq \bar{\beta}_{t+1,tr}^b\left(c_t^b, c_t^s-1\right).$$

Since $F(.)$ is monotonically nondecreasing in β, the result follows.

A similar proof can be constructed for CN sell orders ■

Before starting the proof of Proposition 7, we first state the following corollary:

Corollary 1 *In equilibrium, at any time t, if the expected CN's order book at the buy side is one unit thicker, then the probability of execution of a buy (sell) order will be lower (higher). If the CN's expected order book at the sell side is one unit thicker, then the probability of execution of a buy (sell) order will be higher (lower). If the expected book is one unit thicker at the buy side and one unit thicker at the sell side, then the probability of execution for both* *order types remains constant. Hence,* $\forall\ c_t$, t,

$$
\begin{array}{ll}
\textit{(i)} & p^b_{t,po}\left(E_{t,po}\left(c^b_t\right), E_{t,po}\left(c^s_t\right)\right) \leq p^b_{t,po}\left(E_{t,po}\left(c^b_t - 1\right), E_{t,po}\left(c^s_t\right)\right) \\
& \textit{or}: \bar{\beta}^b_{t,po}\left(E_{t,po}\left(c^b_t\right), E_{t,po}\left(c^s_t\right)\right) \leq \bar{\beta}^b_{t,po}\left(E_{t,po}\left(c^b_t - 1\right), E_{t,po}\left(c^s_t\right)\right) \\
\textit{(ii)} & p^b_{t,po}\left(E_{t,po}\left(c^b_t\right), E_{t,po}\left(c^s_t\right)\right) \leq p^b_{t,po}\left(E_{t,po}\left(c^b_t\right), E_{t,po}\left(c^s_t\right) - 1\right) \\
& \textit{or}: \bar{\beta}^b_{t,po}\left(E_{t,po}\left(c^b_t\right), E_{t,po}\left(c^s_t\right)\right) \leq \bar{\beta}^b_{t,po}\left(E_{t,po}\left(c^b_t\right), E_{t,po}\left(c^s_t\right) - 1\right) \\
\textit{(iii)} & p^b_{t,po}\left(E_{t,po}\left(c^b_t\right), E_{t,po}\left(c^s_t\right)\right) = p^b_{t,po}\left(E_{t,po}\left(c^b_t\right) + 1, E_{t,po}\left(c^s_t\right) - 1\right) \\
& \textit{or}: \bar{\beta}^b_{t,po}\left(E_{t,po}\left(c^b_t\right), E_{t,po}\left(c^s_t\right)\right) = \bar{\beta}^b_{t,po}\left(E_{t,po}\left(c^b_t\right) + 1, E_{t,po}\left(c^s_t\right) - 1\right) \\
\textit{(iv)} & p^s_{t,po}\left(E_{t,po}\left(c^b_t\right), E_{t,po}\left(c^s_t\right)\right) \leq p^s_{t,po}\left(E_{t,po}\left(c^b_t\right), E_{t,po}\left(c^s_t\right) + 1\right) \\
& \textit{or}: \underline{\beta}^s_{t,po}\left(E_{t,po}\left(c^b_t\right), E_{t,po}\left(c^s_t\right)\right) \geq \underline{\beta}^s_{t,po}\left(E_{t,po}\left(c^b_t\right), E_{t,po}\left(c^s_t\right) + 1\right) \\
\textit{(v)} & p^s_{t,po}\left(E_{t,po}\left(c^b_t\right), E_{t,po}\left(c^s_t\right)\right) \leq p^s_{t,po}\left(E_{t,po}\left(c^b_t\right) + 1, E_{t,po}\left(c^s_t\right)\right) \\
& \textit{or}: \underline{\beta}^s_{t,po}\left(E_{t,po}\left(c^b_t\right), E_{t,po}\left(c^s_t\right)\right) \geq \underline{\beta}^s_{t,po}\left(E_{t,po}\left(c^b_t\right) + 1, E_{t,po}\left(c^s_t\right)\right) \\
\textit{(vi)} & p^s_{t,po}\left(E_{t,po}\left(c^b_t\right), E_{t,po}\left(c^s_t\right)\right) = p^s_{t,po}\left(E_{t,po}\left(c^b_t\right) + 1, E_{t,po}\left(c^s_t\right) - 1\right) \\
& \textit{or}: \underline{\beta}^s_{t,po}\left(E_{t,po}\left(c^b_t\right), E_{t,po}\left(c^s_t\right)\right) = \underline{\beta}^s_{t,po}\left(E_{t,po}\left(c^b_t\right) + 1, E_{t,po}\left(c^s_t\right) - 1\right)
\end{array}
\tag{25}
$$

(both formulations, in terms of probabilities and in terms of betas, are equivalent).

Proof of Corollary 1. The proof proceeds along the same lines as the one of Proposition 2 and is omitted for brevity. ■

Proof of Proposition 7. The time t probability of observing a CN buy at $t+1$ is

$$
\begin{aligned}
\Pr\left[\phi^b_{t+1,po}\left(E_{t+1,po}\left(c_{t+1}\right), \beta_{t+1}\right) = 1^{CN}\right] \quad = \quad & \left[\pi_B F\left(\bar{\beta}^b_{t+1,po}\left(E_{t+1,po}\left(c_{t+1}\right)\right)\right)\right. \\
& \left. - F\left(\underline{\beta}^b_{t+1,po}\left(E_{t+1,po}\left(c_{t+1}\right)\right)\right)\right].
\end{aligned}
$$

Suppose that the expected order imbalance in the CN is $E_{t,po}\left(c^b_t - |c^s_t|\right) < T - t - 1$, such that the expected probability of execution of a CN buy is not zero. Then $\underline{\beta}^b_{t+1,po}\left(E_{t+1,po}\left(c_{t+1}\right)\right)$ is independent of the CN's order book.

Suppose at t a DM trade is observed, then the expected CN book does not change: since $c_t = c_{t+1}$, then $E_{t+1,po}\left(c_t\right) = E_{t+1,po}\left(c_{t+1}\right) = E_{t,po}\left(c_{t+1}\right)$.

If no order is observed at t, then the expected CN-book at $t+1$ is one of three following cases $\left(E_{t+1,po}\left(c^b_t\right) + 1, E_{t+1,po}\left(c^s_t\right)\right)$ or $\left(E_{t+1,po}\left(c^b_t\right), E_{t+1,po}\left(c^s_t\right) - 1\right)$ or $\left(E_{t+1,po}\left(c^b_t\right), E_{t+1,po}\left(c^s_t\right)\right)$, depending on whether at t a CN buy, CN sell or no order was submitted. The probabilities of occurrence of each case can be computed from $F\left(.\right), \pi_S, \pi_B$ and the expected book at t.

Attach to these three possibilities the probabilities π_1, π_2 and π_3, respectively. Then the expected book is:

$$\begin{aligned} & \pi_1\left(E_{t+1,po}\left(c_t^b\right)+1, E_{t+1,po}\left(c_t^s\right)\right)+\pi_2\left(E_{t+1,po}\left(c_t^b\right), E_{t+1,po}\left(c_t^s\right)-1\right) \\ & +\pi_3\left(E_{t+1,po}\left(c_t^b\right), E_{t+1,po}\left(c_t^s\right)\right) \\ = & \left(E_{t+1,po}\left(c_t^b\right)+\pi_1, E_{t+1,po}\left(c_t^s\right)-\pi_2\right) \end{aligned}$$

since $\pi_1+\pi_2+\pi_3=1$.

Hence, comparing the expected book after observing a DM trade and observing nothing, we get:

$$\left(E_{t+1,po}\left(c_t^b\right), E_{t+1,po}\left(c_t^s\right)\right) \text{ versus } \left(E_{t+1,po}\left(c_t^b\right)+\pi_1, E_{t+1,po}\left(c_t^s\right)-\pi_2\right)$$

Three cases can occur:

1. $\pi_1<\pi_2$: from Corollary 1 it then follows that:

$$\bar{\beta}_{t+1,po}^b\left(E_{t+1,po}\left(c_t^b\right), E_{t+1,po}\left(c_t^s\right)\right)<\bar{\beta}_{t+1,po}^b\left(E_{t+1,po}\left(c_t^b\right)+\pi_1, E_{t+1,po}\left(c_t^s\right)-\pi_2\right)$$

such that, given that $F(.)$ is monotonically nondecreasing in β:

$$\begin{aligned} & \Pr\left[\phi_{t+1,po}^b\left(E_{t+1,po}\left(c_{t+1}\right), \beta_{t+1}\right)=1^{CN} \mid \phi_{t,po}^b\left(E_{t,po}\left(c_t\right), \beta_t\right)=1^{DM}\right. \\ & \left.\text{or } \phi_{t,po}^s\left(E_{t,po}\left(c_t\right), \beta_t\right)=-1^{DM}, E_{t,po}\left(c_t\right)\right] \\ & < \\ & \Pr\left[\phi_{t+1,po}^b\left(E_{t+1,po}\left(c_{t+1}\right), \beta_{t+1}\right)=1^{CN} \mid \phi_{t,po}^s\left(E_{t,po}\left(c_t\right), \beta_t\right)=-1^{CN}\right. \\ & \left. or \phi_{t,po}^b\left(E_{t,po}\left(c_t\right), \beta_t\right)=1^{CN} \text{ or } \phi_{t,po}^s\left(E_{t,po}\left(c_t\right), \beta_t\right)=0 \text{ or } \phi_{t,po}^b\left(E_{t,po}\left(c_t\right), \beta_t\right)=0, E_{t,po}\left(c_t\right)\right]. \end{aligned} \tag{26}$$

2. $\pi_1>\pi_2$: from Corollary 1 it then follows that:

$$\bar{\beta}_{t+1,po}^b\left(E_{t+1,po}\left(c_t^b\right), E_{t+1,po}\left(c_t^s\right)\right)>\bar{\beta}_{t+1,po}^b\left(E_{t+1,po}\left(c_t^b\right)+\pi_1, E_{t+1,po}\left(c_t^s\right)-\pi_2\right)$$

such that:

$$\Pr\left[\phi^b_{t+1,po}\left(E_{t+1,po}\left(c_{t+1}\right),\beta_{t+1}\right)=1^{CN}|\phi^b_{t,po}\left(E_{t,po}\left(c_t\right),\beta_t\right)=1^{DM}\right.$$
$$\left.\text{or } \phi^s_{t,po}\left(E_{t,po}\left(c_t\right),\beta_t\right)=-1^{DM},E_{t,po}\left(c_t\right)\right]$$
$$>$$
$$\Pr\left[\phi^b_{t+1,po}\left(E_{t+1,po}\left(c_{t+1}\right),\beta_{t+1}\right)=1^{CN}|\phi^s_{t,po}\left(E_{t,po}\left(c_t\right),\beta_t\right)=-1^{CN}\right.$$
$$\left.\text{or } \phi^b_{t,po}\left(E_{t,po}\left(c_t\right),\beta_t\right)=1^{CN} \text{ or } \phi^s_{t,po}\left(E_{t,po}\left(c_t\right),\beta_t\right)=0 \text{ or } \phi^b_{t,po}\left(E_{t,po}\left(c_t\right),\beta_t\right)=0,E_{t,po}\left(c_t\right)\right]. \tag{27}$$

3. $\pi_1=\pi_2$: from Corollary 1 it then follows that:

$$\bar{\beta}^b_{t+1,po}\left(E_{t+1,po}\left(c^b_t\right),E_{t+1,po}\left(c^s_t\right)\right)=\bar{\beta}^b_{t+1,po}\left(E_{t+1,po}\left(c^b_t\right)+\pi_1,E_{t+1,po}\left(c^s_t\right)-\pi_2\right)$$

such that:

$$\Pr\left[\phi^b_{t+1,po}\left(E_{t+1,po}\left(c_{t+1}\right),\beta_{t+1}\right)=1^{CN}|\phi^b_{t,po}\left(E_{t,po}\left(c_t\right),\beta_t\right)=1^{DM}\right.$$
$$\left.\text{or } \phi^s_{t,po}\left(E_{t,po}\left(c_t\right),\beta_t\right)=-1^{DM},E_{t,po}\left(c_t\right)\right]$$
$$=$$
$$\Pr\left[\phi^b_{t+1,po}\left(E_{t+1,po}\left(c_{t+1}\right),\beta_{t+1}\right)=1^{CN}|\phi^s_{t,po}\left(E_{t,po}\left(c_t\right),\beta_t\right)=-1^{CN}\right.$$
$$\left.\text{or } \phi^b_{t,po}\left(E_{t,po}\left(c_t\right),\beta_t\right)=1^{CN} \text{ or } \phi^s_{t,po}\left(E_{t,po}\left(c_t\right),\beta_t\right)=0 \text{ or } \phi^b_{t,po}\left(E_{t,po}\left(c_t\right),\beta_t\right)=0,E_{t,po}\left(c_t\right)\right]. \tag{28}$$

Suppose that the expected order imbalance in the CN is $E_{t,po}\left(c^b_t-|c^s_t|\right)>T-t-1$. This means that either no CN orders are expected to have been submitted, in which case the proposition holds trivially, or at time t the extra CN order submitted changes the expected CN book such that the execution probability becomes zero. In this case $\bar{\beta}^b_{t+1,po}\left(E_{t+1,po}\left(c^b_t\right)+1,E_{t+1,po}\left(c^s_t\right)\right)=A/V$; hence the result follows since also $\underline{\beta}^b_{t+1,po}\left(E_{t,po}\left(c_{t+1}\right)\right)=A/V$. ■

Proof of Proposition 8. This proof proceeds along the same line as the proof of Proposition 7 and is omitted for brevity. ■

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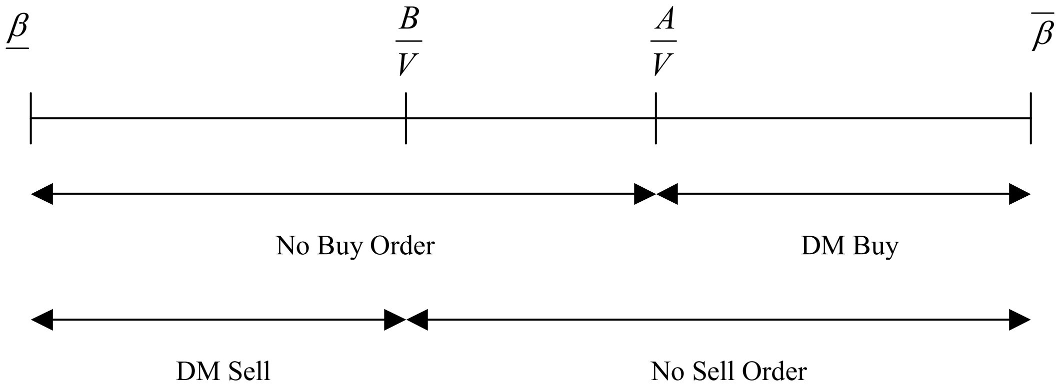
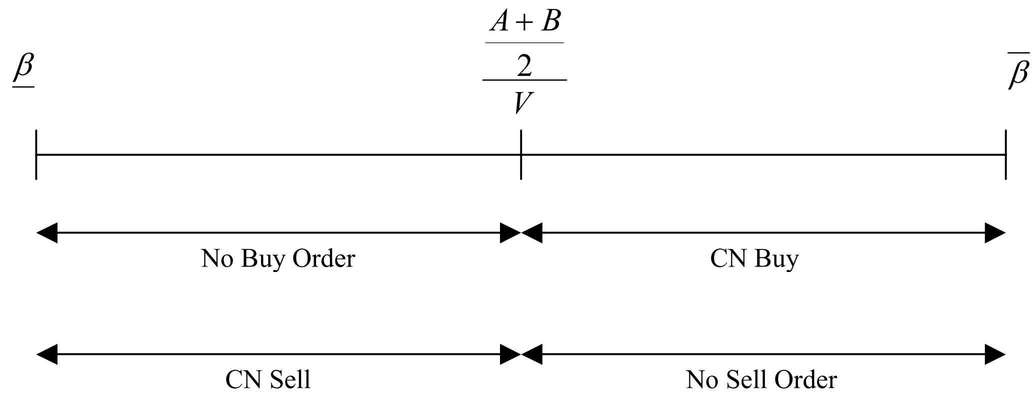


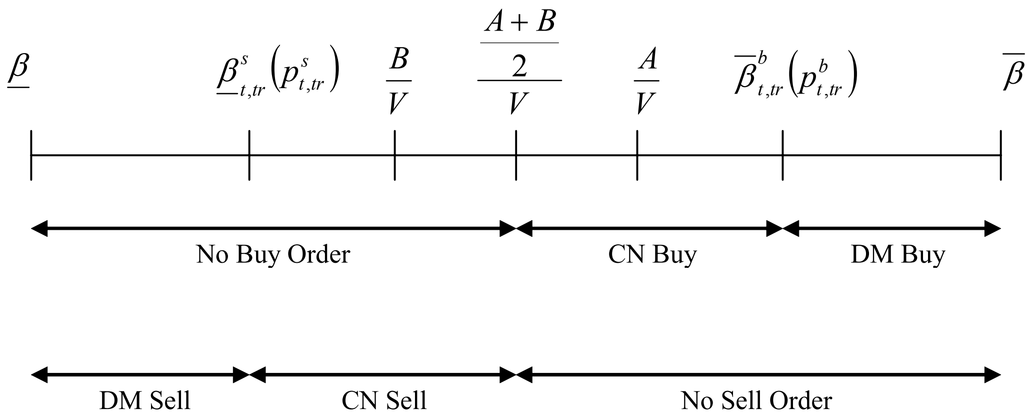
Note: This figure depicts the equilibrium of our model with only a dealer market. The optimal strategies of agents are drawn, conditional upon their β and trading orientation.

Figure 1: **Order Submission Strategies with a Dealer Market in Isolation**



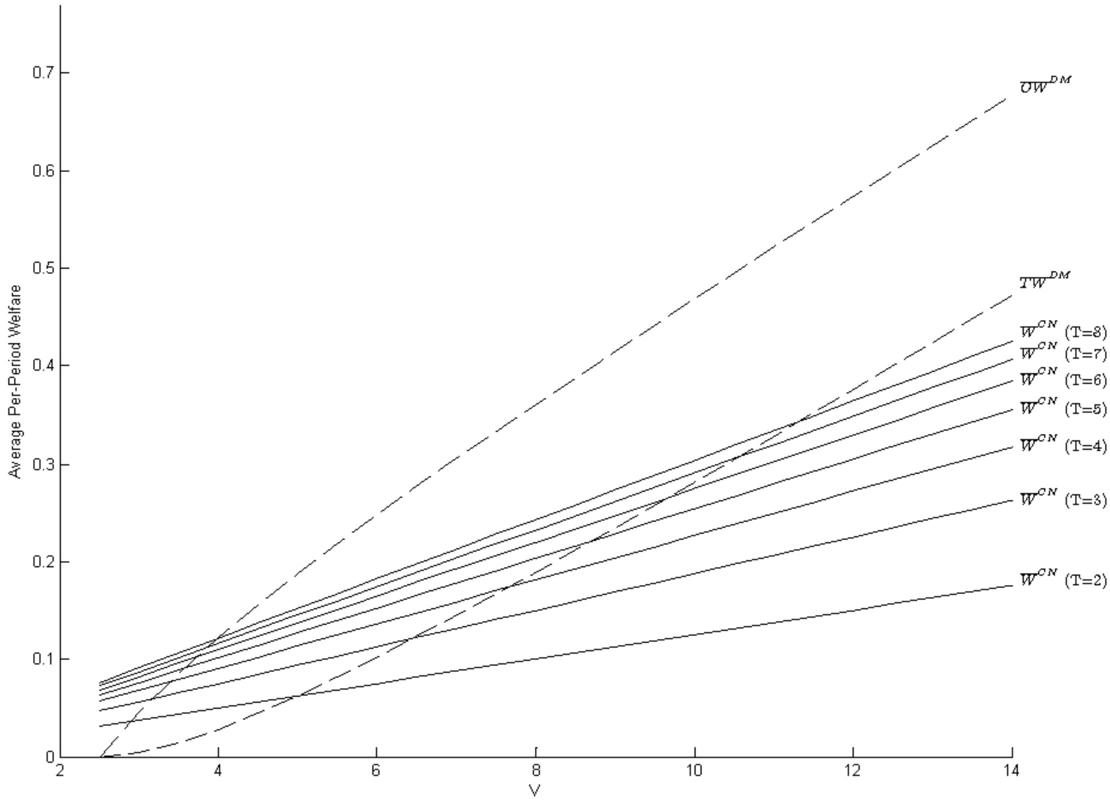
Note: This figure depicts the equilibrium of our model with only a crossing network. The optimal strategies of agents are drawn, conditional upon their β and trading orientation.

Figure 2: **Order Submission Strategies with a Crossing Network in Isolation**



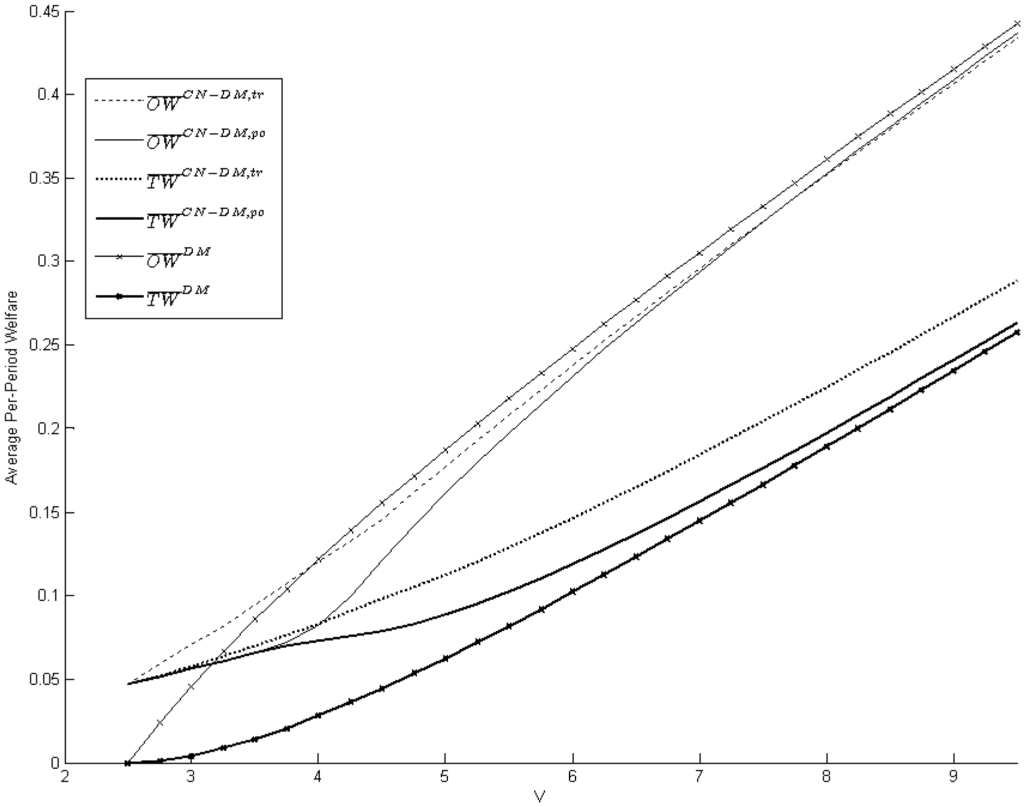
Note: This figure depicts the equilibrium of our model with a dealer market and a crossing network for transparency. The optimal strategies of agents are drawn, conditional upon their β and trading orientation.

Figure 3: **Order Submission Strategies with Dealer Market and Crossing Network**



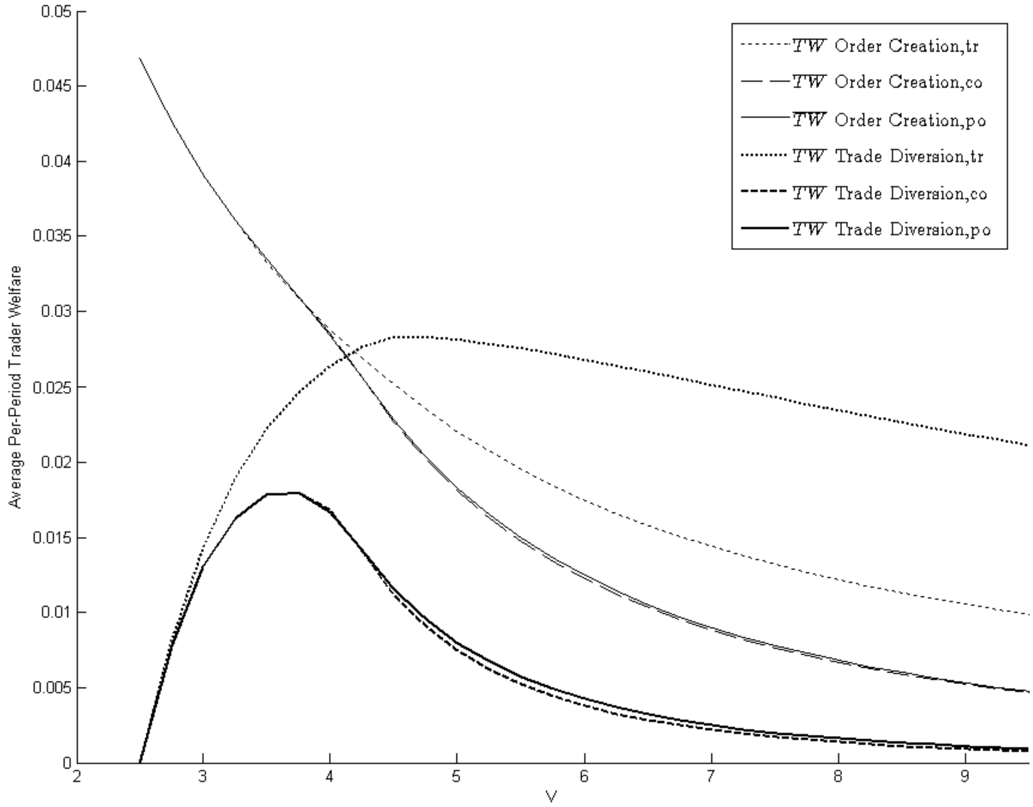
Note: This figure presents the average per-period overall welfare $\overline{OW}^{DM}$ and trader welfare $\overline{TW}^{DM}$ (upper and lower dashed lines, respectively) for the DM in isolation, as well as average per-period welfare $\overline{W}^{CN}$ for a CN in isolation (full lines), for different values of T (i.e. 2 to 8) as a function of the value of the asset V.

Figure 4: **Welfare for Markets in Isolation**



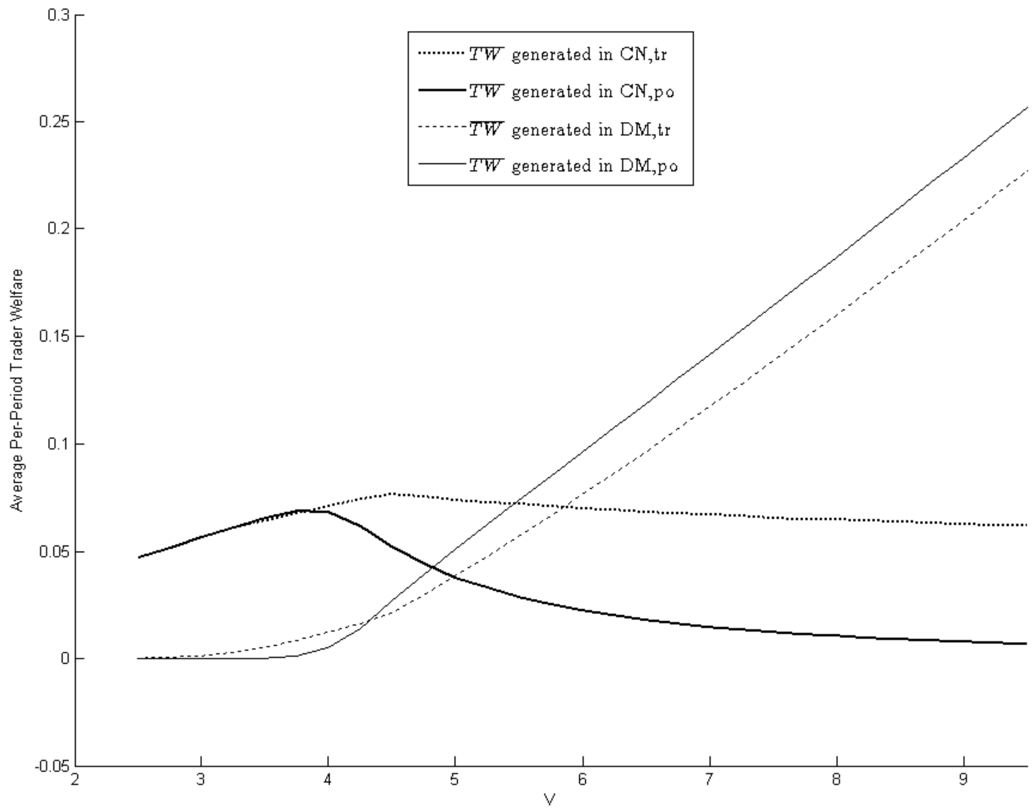
Note: This figure presents average per-period overall welfare for transparency $\overline{OW}^{CN-DM,tr}$ and partial opaqueness $\overline{OW}^{CN-DM,po}$. Transparency is presented as a thin dotted line, partial opaqueness with a thin full line. Also overall welfare for the DM in isolation $\overline{OW}^{DM}$ is included for comparison as a thin line, marked with crosses (x). Thick lines present trader welfare for transparency $\overline{TW}^{CN-DM,tr}$ and partial opaqueness $\overline{TW}^{CN-DM,po}$ (again dotted and full lines, respectively). Trader welfare in the DM in isolation $\overline{TW}^{DM}$ is shown as a thick line, marked with crosses (x). The results for complete opaqueness are very close to those of partial opaqueness and are therefore not shown.

Figure 5: **Coexistence: Average Per-Period Overall Welfare and Trader Welfare**



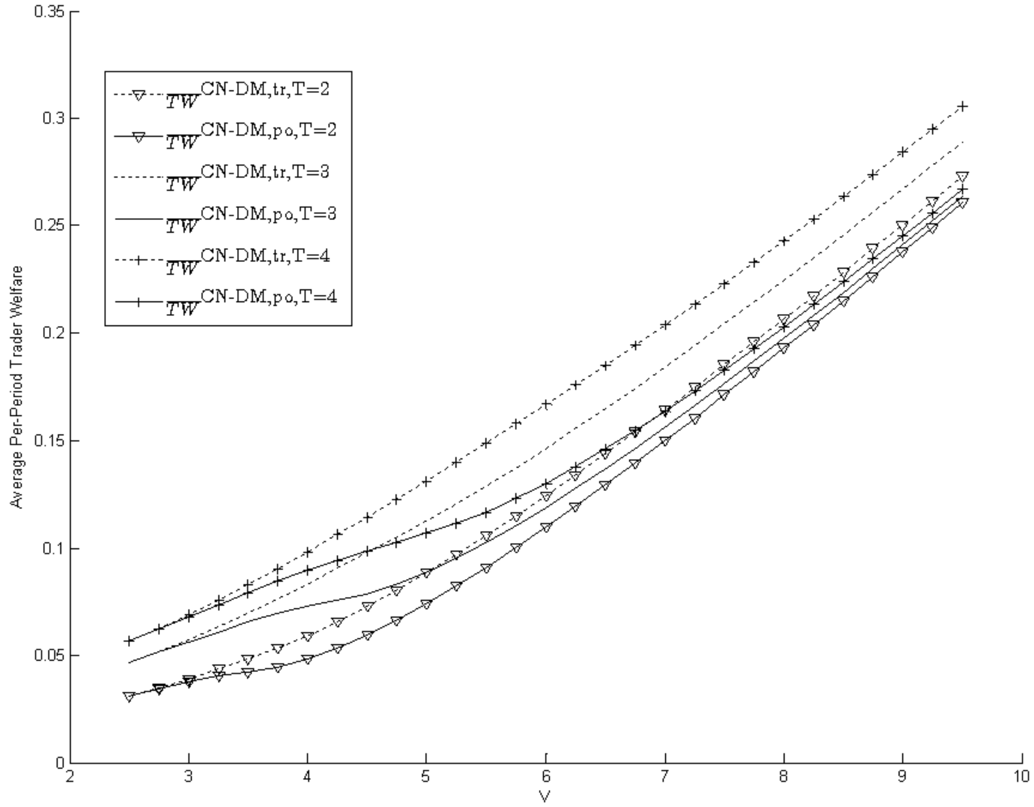
Note: This figure presents the *"$\overline{TW}$ order creation effect"* and the *"$\overline{TW}$ trade diversion effect"* for the three informational settings, $i = tr, co, po$ (dotted, dashed and full lines, respectively). Thin lines are the order creation effect, while thick lines represent the trade diversion effect. Jointly, both effects make up the difference between $\overline{TW}^{CN-DM,i}$ and $\overline{TW}^{DM}$.

Figure 6: **Coexistence: Trader Welfare Order Creation and Trade Diversion Effect**



Note: This figure presents trader welfare decomposed in the part that is generated in the CN (thick lines) and in the DM (thin lines). Transparency is presented as dotted lines and partial opaqueness with full lines. Results for complete opaqueness are very close to those for partial opaqueness and are therefore not shown.

Figure 7: **Coexistence: Decomposition of Trader Welfare generated in the DM and the CN**



Note: This figure displays $\overline{TW}$ for the transparency and partial opaqueness for $T = 2, 3$ and 4.

Figure 8: **Coexistence: Average Per-Period Trader Welfare for Various Lengths of the Trading Day**