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Several Approaches Discussed

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Abstract

An aspect of casino games that in general leads to discussions among both participants and spectators, is the relative extent to which a player can positively influence his results by making appropriate strategic choices. This question is closely related to the issue of how to distinguish between games of skill and games of chance. This is an issue that is interesting from a juridical point of view too, since in many countries the legitimacy of exploiting a specific game depends on the category to which it belongs.

This paper summarizes recent developments concerning the measurement of skill in games. It points out the elements in the definitions that need closer attention, it illustrates the analysis with examples and it discusses further possibilities.

Keywords: games of skill; games of chance.
JEL code: C72.

1 Introduction

How should one define skill in games? The definition of skill that one finds in the dictionary, is "the special ability to do something well, especially as gained by learning and practice". To be able to use such a broad definition within the context of games, it should be refined. Larkey *et al.* (1997) made an attempt and defined skill as "the extent to which a player, properly motivated, can perform the mandated cognitive and/or physical behaviours for success in a specific game". Whereas this definition concerns the player, we are interested in defining the skill level of the game he participates in. To make the definition of skill applicable to games instead of players, we modify it such that it expresses how *useful* the player's abilities can be for him in the game. To give a simple example, a perfect memory may not help you in roulette, but in poker it does. As the articles of Larkey *et al.* (1997) and Reep *et al.* (1971) indicate, the notion of skill can be defined for a large class of games, including various ball games as well as card games and thinking sports. The current paper concentrates on games for which the outcome can be expressed in terms of money and in which players can be identified by their strategies. We will refer to this class of games as casino games.

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For casino games, we will define skill as the relative extent to which the outcome of a game is influenced by the players, compared to the extent to which the outcome depends on the random elements involved. For random elements one can think of the spinning of a roulette wheel or the dealing of cards. The larger the influence of the players on the game outcome, the higher the skill level. Games without random effects, in which only the players have influence on the outcome, are called pure games of skill, whereas games in which the players cannot affect the outcome at all are pure games of chance. A game as chess belongs to the first category, while roulette is intuitively classified as a pure game of chance.[1] Although the classification is easy for these two games, there is a large number of games in which the two sources of influence are combined and for which the skill level lies somewhere in the grey area between the pure games of skill and the pure games of chance.

From a juridical perspective, it is important that one can determine for these games in the grey area if the players are sufficiently influential in a game to classify it as a game of skill or not. According to the Dutch gaming act, a license is required to exploit a game of chance, whereas anyone is allowed to offer a game of skill. Similar laws apply in other European countries, as well as in many states in the USA. It is not difficult to imagine that the proprietor of a game and the legislator have different opinions about the role of chance in a game. In the first place judging the role of chance is rather subjective and in the second place the exploitation of games of chance is a lucrative business, since these games are appealing to a large audience, as Caillois (1979) argues:

> "[Games of chance] promise the lucky player a more modest fortune than he expects, but the very thought of it is sufficient to dazzle him. Anyone can win. This illusory expectation encourages the lowly to be more tolerant of a mediocre status that they have no practical means of improving. Extraordinary luck—a miracle—would be needed. It is the function of *alea* to always hold out hope of such a miracle. That is why games of chance continue to prosper. The state itself even profits from this. Despite the protest of moralists, it establishes official lotteries, thus benefiting from a source of revenue that for once is accepted enthusiastically by the public." (Caillois 1979, p. 115)

The observation that the state itself profits from the appeal of games of chance is also true for the Netherlands. In practice, the government only grants the required license to the Holland Casino's foundation, a state-owned company. The government has both the control and the profits of this market.

Borm and Van der Genugten (1998, 2001) presented the basics of a method that can be used to determine whether a game can be classified as a game of skill or not. This method is based on the Dutch gaming act. The main goal of the current paper is to give an overview of the

[1] Even pure games of chance are not always classified as such by the participants. The way the game is presented may lead to misperceived skillful influence over non-controllable events, as Wohl and Enzle (2002) show.

relevant aspects of this method, but we also discuss some related practical issues. We describe the general framework in section 2. The sections 3 to 5 are devoted to the description of the details of the analysis.

Whereas the skill measure is meant to determine the skill involved in the game as a whole, it is in general interesting to study the skill level of individual players as well. In sports, for example, player skill levels can be recognized in the form of handicaps assigned to golf players. Within the class of games we focus on, one can think of the ratings of chess players that determine their position on the world ranking. We will spend section 6 on some discussion around this topic.

In section 7 the concepts discussed in sections 2 to 6 will be illustrated by means of a two-person poker game. In this example, which forms an addition to the study of Dreef *et al.* (2003), we will describe and explain the aspects that are relevant for the skill analysis. Section 8 sketches some possibilities to investigate the skill level of games using empirical data.

2 A Relative Measure

The method that Borm and Van der Genugten (1998, 2001) developed is based on the following important passage in the Dutch gaming act, which gives a qualitative characterization of the class of games for which a license is needed:

> [...] it is not allowed to: exploit games with monetary prizes if the participants in general do not have a dominant influence on the winning possibilities, unless in compliance to this act, a license is granted [...].

All games that satisfy this definition, are called *games of chance*. By definition, all other games are referred to as *games of skill*. Borm and Van der Genugten (1998) give the following three qualitative requirements which summarize the basic ideas underlying the Dutch legislation concerning the exploitation of games with chance elements.

(R1) The legislation applies exclusively to situations which involve the exploitation of games with monetary prizes.

(R2) The skill of a player should be measured as his average game result in the long run, i.e. in terms of expected result. For a game of skill it is necessary that these expected results vary among players.

(R3) The fact that there is a difference between players with respect to their expected payoffs, does not immediately imply that the underlying game is a game of skill. For a game of skill it is sufficient that the chance elements involved do not prohibit these differences to be substantial.

Using these three requirements, we are ready to give the general framework of the relative skill measure for one-person games. To take into account requirement (R1), we restrict attention in our analysis to games in which the "game-result" of a player can be expressed in terms of money.

The difference in player results that is required by (R2), can be measured by what is called the *learning effect* in the game. According to (R3), it is not the absolute size of the learning effect that determines the skill level of a game, but the relative size of this effect in relation to the restrictive possibilities within the game set by the chance elements. Therefore, one should also quantify this restrictive influence of the random moves. One can do this by investigating the possibilities of the players in the absence of these moves. This restriction by the chance elements is caught in the *random effect* of the game. Using these two effects, Borm and Van der Genugten (1998) defined

$$\text{skill level} = \frac{\text{learning effect}}{\text{learning effect} + \text{random effect}}. \tag{1}$$

Formal definitions of the learning effect and the random effect will follow later, but let us already note that these concepts will be defined such that the corresponding numbers will be be nonnegative. This implies that

$$\text{(pure games of chance)} \quad 0 \leq \quad \text{skill level} \quad \leq 1 \quad \text{(pure games of skill)}.$$

Games, in which the random effect dominates the learning effect, will have a low skill level. For games in which the learning effect dominates, the skill level will be high. For completeness, we note that a game for which both the learning effect and the random effect are equal to zero, has a skill level of zero by definition; in games of practical importance this situation does not occur.

The following sections will make clear how the concepts described above are formally defined in order to obtain a way to objectively determine the skill level of a specific casino game and to compare games with each other.

3 Player types

The jurisprudence regarding the Dutch gaming act has indicated how one should interpret the framework that we presented in the previous section in practice. Both the learning effect and the random effect should be measured by comparing two different types of players. For more details concerning the gaming act and the corresponding jurisprudence we refer to Van der Genugten and Borm (1994). In this section we will describe and briefly discuss the three player types that are used in the analysis. We will successively characterize beginners, optimal players and fictive players.

3.1 Beginners

The first of the three player types that are interesting, is the beginner. A beginner is a player who has only just familiarized himself with the rules of the game and plays a relatively simple, naive strategy.

It is not always easy to determine the behaviour of beginners in a specific game. In general, we distinguish three ways to do this. First of all, in games with a structure like Roulette, we think it is reasonable to assume that a beginner chooses randomly between all available strategies. The category of games for which this method is suitable, however, is not the most interesting category with respect to the analysis of skill. Secondly, the behaviour of beginners can be determined by means of observation. This method has two disadvantages. In the first place the collection of data could be a costly affair and in the second place this approach is only possible for games that are exploited in practice. The third way to gain insight applies to games that are not (yet) exploited in practice: have the rules and structure of the game studied by a gambling expert. This person can use his expert knowledge to formulate an idea for the beginner's strategy that satisfies a the general idea of how people act in games they are not really familiar with. In Kadane (1986) one can find a nice practical example of this process, including some discussion on the rules of thumb that might be used.

3.2 Optimal Players

The second class of players that is important for the analysis of skill is formed by the optimal players. These players completely mastered the rules of the game and exploit their knowledge maximally in their strategies. Optimal players can be seen as the refined representatives of the more natural category of advanced players. Advanced players are observed in practice in any skillful casino game that has been around for a longer period. They play a smart strategy which yields them game results close to the theoretical maximum.

The payoffs of the optimal player can be analytically computed or approximated by means of simulation. In a one-person game the optimal player just solves the underlying maximization problem, while optimal players in a two-person zero-sum game play minimax strategies.

3.3 Fictive Players

The third and last category of players that we need is a fictive one, consisting of players who know in advance the realization of the random elements in the game. Here we distinguish between two kinds of random elements. In the first place, a fictive player is informed about the outcome of the *external* chance moves. External chance moves are for example the dealing of the cards and the spinning of the roulette wheel. The other sort of chance move a player can face, occurs in more-person games and is caused by his opponents. Players generate uncertainty for their opponents by playing mixed strategies. We call these random elements *internal* chance moves. Besides his information regarding the external chance moves, a fictive player can be informed about these internal chance moves of the other players too.

To be complete, we want to mention a second type of fictive player, the fictive worst player. This is a player who has the same information as a (normal) fictive player, but deliberately uses this information to reach a game result as bad as possible. The fictive worst player was used in an earlier version of the skill measure (see Van der Genugten and Borm (1994)) to create a reference point. The role that fictive players can play in analyzing the role of information in games, is currently studied by Dreef and Borm (2003).

3.4 The Use of the Player Types in the Skill Measure

To conclude this section, we will see how the player types that were defined in the previous three subsections fit in the framework that was set up in section 2. In formula (1), we have seen that the two deciding factors of the relative skill measure are the learning effect and the random effect. The learning effect is defined as the difference between the result of the optimal player and the beginner, while the random effect is defined as the difference in game result between the fictive player and the optimal player.

Regarding the random effect, one has to be careful. Two different definitions are used. Borm and Van der Genugten (1998, 2001) use the game results of the fictive player that is only informed about the external chance moves, whereas Dreef *et al.* (2001) compare the result of the optimal player to the result of the fictive player to whom also the realization of the internal chance moves is revealed.

A player type that was not mentioned above, but which certainly is of theoretical interest when one studies a casino game, is the average player. When compared to the results of the player types we just introduced, the results of the average casino visitor of a casino in a specific game could be helpful when determining the skill level of this game. Borm and Van der Genugten (1998) indeed use the average player in the development of the measure, but they also explain why this type does not make it into the final model: it is often hard, if not impossible, to reach agreement about the strategic behaviour of the average player.

4 Measuring the game result

In the preceding sections we have spoken about the learning effect, the random effect and the three player types whose game results form the basis for the numbers. However, we did not yet define exactly what determines a player's game result. As Borm and Van der Genugten (1998) already suggested, the two numbers that can be taken into account are the payoff of the player and the stakes (bets) that are needed to obtain this payoff. The two sensible definitions of a player's game result that one can come up with, using these numbers, are *(net) gains* and *returns*:

$$\text{gains} = \text{payoffs} - \text{stakes} \qquad\qquad \text{returns} = \frac{\text{gains}}{\text{stakes}}$$

Which of the two definitions should we use as an indication for the strength of the player types? One should be careful when making a selection. Implicitly, the choice of measurement implies an assumption about the goals of the players in a game. In general, a player's strategy will depend on his focus: the strategy that maximizes the expected net gain is not the same as the strategy optimizing the expected returns. In practice, in most games the players seem to aim for the highest possible gains.

However, there are games in which expected gains do not form an appropriate strategy evaluation. A practical example is the game of roulette. Intuitively, roulette is a pure game of chance. A player cannot influence his expected results by varying his strategy; i.e., if results are measured in terms of expected returns. Of course, by betting twice as much, one can double the expected gains, but the expected returns are not affected. If we define the strategy of a beginner, we have to make assumptions about the bet size he uses. For roulette we know that the optimal player will bet the minimum, since the expectation of his gains is negative. If we assume that a beginner plays a strategy that assigns a positive probability to a bet larger than the minimum, his expected gains will be smaller than those of the optimal player and, as a result, roulette will have a positive learning effect. This positive learning effect will not occur if we use expected returns to evaluate the player's strategies.[2]

This solution has some disadvantages. In the first place, the linearity of the game results is lost. This makes computations more difficult. Besides that, in more-person games we have the complication that zero-sum games are turned into games of which the payoffs are not zero-sum. There is an alternative that may seem to use the best of two worlds: one could determine the strategies in the linear, zero-sum environment and consequently compute the corresponding expected returns and use these in the relative skill measure. This possibility has a theoretical drawback. The expected gains of a beginner will be smaller than or equal to the expected gains of an optimal player and an optimal player will never have expected gains that are strictly higher than the expected gains of the fictive player. However, this logical ordering is not necessarily preserved when when we look at the expected returns that correspond to the strategies of the three player types. As a result, for some games this may lead to the undesirable result that the skill level lies outside the interval $[0, 1]$.

Another option is to model the bet size as a *pre-game decision* of how many unit games to play at the same time, where the *unit game* is the game with fixed, normalized bet size. For example, in roulette, deciding to bet 10 euro on black, is equivalent to deciding to play 10 games of "unit roulette" simultaneously, in which you bet 1 euro (the fixed, normalized bet size) on black. A similar decomposition is possible for instance for trajectory games like golden-ten, but also for blackjack played with an automatic card-shuffling machine. We think that what one really wants to know if one asks for the skill involved in a game, is the skill level of the unit game. When playing multiple instances of a game simultaneously, one has the same relative influence on the expected result as in one instance of the game. In defining our three player types, we

[2]To be complete, we note that there is a difference in expected returns between simple strategies (e.g. red or black, even or odd) and non-simple strategies (e.g. single numbers). However, the learning effect will always be small compared to the random effect that is caused by the fictive player who always bets maximally on the correct number.

can therefore restrict ourselves to defining the strategies they use in the unit game. Measuring expected gains is then equivalent to measuring expected returns and the ordering problem will not occur anymore.

We can use this last way of modelling if the following conditions are satisfied:

(C1) the size of the bet that is chosen does not affect the course of the game;

(C2) at the moment the bet size is chosen, no information about the outcome of the chance move is available yet;

(C3) the structure of the payoff function is such that the expected gains with a bet size b are of the form cb, where c is a positive constant.

Within the class of one-person games we find games that satisfy the three conditions above. However, when we move from one-person games to more-person games, the first of the conditions at the end of the previous section is no longer satisfied. E.g., in a two-person game where the players do not move simultaneously and where the second player is informed about the amount bet by the first player, different bets of the first player lead to different information sets of the second player. This type of bet of the first player is an example of a *strategic bet*, whereas the bets that satisfy the conditions above are called *non-strategic bets*. In a game that contains strategic bets a reduction to the analysis of a unit game is not possible. This is not a problem, since for more-person games there is no need for an alternative definition of player strength; expected gains can serve this purpose very well. The only assumption we have to make in the skill analysis of more-person games, is that all participants have sufficiently large resources. In this way, buying out an opponent by means of extraordinarily large (bluffing) bets is not possible and, as a consequence, the analysis only takes into account the "real" strategic features of the game.

5 Definition of opposition

The framework for the skill analysis that was introduced in section 2 is not only applicable to one-person games, but also to games with more players. Although in one-person games the game results for the three player types are unambiguously determined by the strategies chosen by the players, in more-person games the payoff of a player clearly depends on the way the opposition acts.

In the analysis of skill two approaches are used to model the opposition of the beginners, the optimal players and the fictive players. Borm and Van der Genugten (1998, 2001) compute what would be (jointly) optimal for the opponent(s) against an optimal player. Next, the three player types are evaluated against this resulting optimal (joint) strategy of the opposition. Dreef *et al.* (2001) use a different approach. They assume that the opponents play in such a way that they offer maximal opposition to the player type under consideration.

Whereas this direct opposition is clear in two-person zero-sum games, in games with more participants it is not. In a game with at three or more players the mutual competition is of a more indirect and complex nature. Although money still is only reallocated in an n-person zero-sum game, two particular participants cannot be viewed as direct adversaries in the sense that they should (or could) act such that they oppose each other as strongly as possible. The solution that is chosen for this problem in the skill analysis is the following. In an n-person game the $n-1$ opponents of a specific player are assumed to act as one. In terms of cooperative game theory these $n-1$ players form a coalition. By defining the payoff of the coalition as the sum of the individual member payoffs, we obtain a two-person zero-sum game again. Thus, we can find the optimal opposition in the familiar way.

However, one should be careful following this approach of creating large coalitions to play against the player (type) one wants to evaluate. Especially, as the number of participants grows, this becomes less realistic. How realistic is the cooperation of six players in a Seven Card Stud Poker game between seven players? Perhaps an alternative option is to investigate all possible divisions of the player set into two coalitions. One-person coalitions are part of the model then, but they do not get as much weight as they do in the current model. Such a method would take into account the fact that the coalitions that might form in practice might not even be constant during one instance of the game.

6 Player skill vs game skill

Before we turn to an example in section 7, we would like to spend a few words on the relation between player skill and game skill. As the first paragraph of the introduction already indicates, there is a distinction between the two concepts. The previously mentioned article of Larkey *et al.* (1997) focusses on skill differences between players. Their ideas are presented by means of a large example, in which twelve different player types play against each other a simplified version of stud poker. Each player is described by means of a complete, algorithmically described strategy. Skill differences are created by carefully varying certain characteristics over the twelve strategies. These players play a complete tournament and in the end the table with game results is used to draw conclusions about skill differences between players and about different types of skill that can be useful in the poker game. Their results make clear that a player's performance strongly depends on the opposition he faces. They can, given the opposition, distinguish between more and less skillful player types. However, it is not directly clear how one could use their results to say something about the skill level of the poker variant itself.

How does this work in our skill analysis? In section 2 we defined the notions of learning effect and random effect that are used to compute the relative influence of a player on the game result. If we consider a one-person game, we can just fill in these numbers in formula (1) to find the skill level of the game. For a one-person game, the relative influence that the player has determines the skill level of the game. For more-person games some more work is required. For each player (or player role) in the game we can compute the learning effect and the random effect. Next, there are two ways to use these numbers to draw conclusions about the skill level of the game. In the first place, we can compute the overall learning effect and random effect by taking the average over the, say, n players and use the results in formula (1) to compute the skill level. This is the approach followed by both Borm and Van der Genugten (1998, 2001) and Dreef *et al.* (2001). An alternative would be to compute the relative skill level for each player separately and take the average over these n numbers to find the skill of the game as a whole. Both methods seem to make sense, but in general they do not yield the same results.

We abbreviate the learning effect by LE and the random effect by RE, write RS for relative skill level and use a subscript to indicate if we speak about a player or the game itself, then we can write the formula for the first method as

$$RS_{\text{game}} = \frac{LE_{\text{game}}}{LE_{\text{game}} + RE_{\text{game}}} = \frac{\frac{1}{n}\sum_{i=1}^{n} LE_{\text{player } i}}{\frac{1}{n}\sum_{i=1}^{n}\left(LE_{\text{player } i} + RE_{\text{player } i}\right)}. \tag{2}$$

The second method can then be written as

$$RS_{\text{game}} = \frac{1}{n}\sum_{i=1}^{n} RS_{\text{player } i} = \frac{1}{n}\sum_{i=1}^{n} \frac{LE_{\text{player } i}}{LE_{\text{player } i} + RE_{\text{player } i}}. \tag{3}$$

An example in which the difference between two methods is illustrated, is presented by Black Jack. In principle, Black Jack is a one-person game. Although the dealer draws cards too, he has a dummy strategy and cannot make any strategic decisions. For this one-person game, we can compute the learning effect LE_{BJ}, the random effect RE_{BJ} and the resulting skill level RS_{BJ}. Next, we modify the game such that you can play it with two players. In each play one of the participants takes the role of the bank. Having the role of the bank, a player has no choices; he still has to play the dummy strategy. Therefore, a beginner and a fictive player will have the same expected game result as an optimal player. As a result, the learning effect, and thus the relative skill of this player role are zero. For the other player we already have the numbers: LE_{BJ}, RE_{BJ} and RS_{BJ}. If we use formula (2) to determine the skill level of the new game, we find that it is equal to the skill level of "standard" Black Jack, whereas the skill level turns out to be halved according to formula (3). Which of the two is better, depends on the context. It may for example depend on the type of games one wants to compare a game with and on what information regarding skill is available for the other games.

7 An Example

In this section we want to illustrate the use of the method for measuring skill. We will work out the analysis for a simple poker game for two players. This game, to which we will refer as *minipoker*, is studied in detail in Dreef *et al.* (2003). The part of that paper that discusses skill focusses on the measure that was defined by Borm and Van der Genugten (1998). We will use the abbreviation RS_{1998} for this skill measure. Here, we will pay more attention to the skill measuring approach of Dreef *et al.* (2001), shortly written as RS_{2001}. We will use some of the results derived by Dreef *et al.* (2003) and include their results for comparison. We use expected gains as the definition of game results.

7.1 Game Description

The formal description of the rules of minipoker is as follows. To begin the game, both players add an *ante* of size 1 to the stakes. Then the cards are dealt. Instead of considering the $\binom{52}{5}\binom{47}{5}$ possible hand combinations that can be dealt in a general poker game, the hands are assumed to be real numbers, drawn from the unit interval. Player 1's hand is the value u of a continuous random variable U and player 2's hand is the value v of a continuous random variable V. U and V are independently, identically distributed on $[0,1]$ according to the cumulative distribution function $F : [0,1] \to \mathbb{R}_+$. The function $f : [0,1] \to \mathbb{R}_{++}$ denotes the probability density function for this distribution and is assumed to be *positive* and *continuous* on its domain. For the skill analysis, we will consider the case where F and f correspond to the uniform distribution on $[0,1]$.

After seeing his hand, player 1 can choose between *passing* and *betting*. If he passes, a *showdown* follows immediately. In the showdown, the players compare their hands and the player with the highest hand wins the pot. Betting means adding an extra amount 1 to the stakes. After a bet by player 1, player 2 can decide to *fold* or to *call*. If he folds, he loses his ante of 1 to player 1. To call, player 2 must put an extra amount 1 in the pot. In that case a showdown follows and the player with the better hand takes the pot.

Figure 1 displays our poker model in extensive form. Two possible hands, u_1 and u_2, for player 1 are shown. To keep the picture clear, player 2 is shown receiving his hand v after player 1 has decided how to bet. From the description above and the payoffs in the picture, it is clear that the hand v is chosen such that it satisfies $u_1 < v < u_2$. Since the payoffs are zero-sum, it suffices to give the payoff for player 1.

7.2 Optimal Play

Dreef *et al.* (2003) derive that the optimal strategy for player 1, stated in terms of probabilities, is

$$Pr\{\text{bet with hand } u\} = \widetilde{p}(u) = \begin{cases} 1 & \text{if } u \leq \frac{1}{10} \text{ or } u > \frac{7}{10}, \\ 0 & \text{otherwise,} \end{cases}$$

and that it is optimal for player 2 to play

$$Pr\{\text{call with hand } v\} = \widetilde{q}(v) = \begin{cases} 0 & \text{if } v \leq \frac{2}{5}, \\ 1 & \text{otherwise.} \end{cases}$$

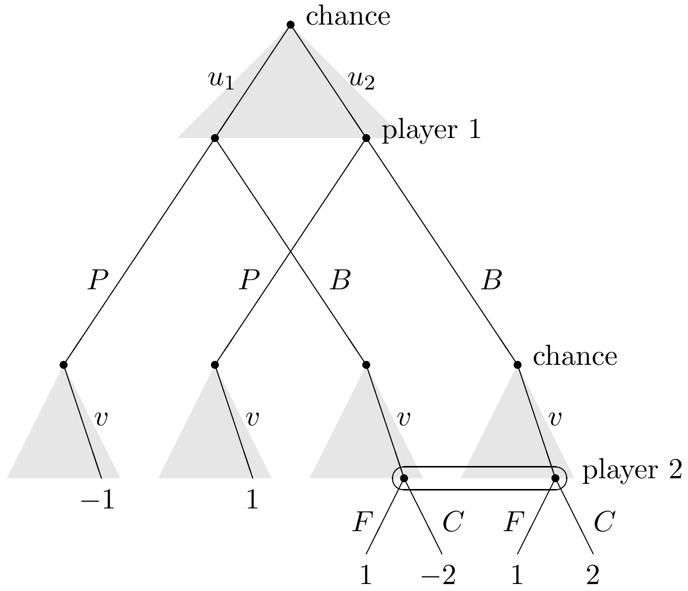


Figure 1: The extensive form of two-person minipoker ($u_1 < v < u_2$).

So player 1 should bet with high hands, bluff with very low hands and pass with intermediate hand values. Player 2 has one boundary hand value. If he has a hand that is better he calls, otherwise he folds. The value of the game is $\frac{1}{10}$. This is the first result we need for the skill analysis: if player 1 plays optimally and he faces optimal opposition, then his expected payoff will be equal to the game value. In both RS_{1998} and RS_{2001} we use this number as the game result for the optimal player 1. Obviously, the payoff for player 2 as an optimal player is equal to $-\frac{1}{10}$. We will collect all results in Table 1 in section 7.5.

7.3 Beginners

Next, we are interested in the behaviour of players who play this game for the first time, just after the rules are explained to them. We have to make assumptions about their strategic choices. Dreef *et al.* (2003) present the following reasoning. Perhaps beginners have heard about the famous video poker variant "Jacks or Better". In this game, as the name suggests, only hands with a pair of Jacks, Queens, Kings or Aces (and all hands from higher classes) have value for the player. As a result, naive players may be betting or calling with exactly these hands. Even if they do not know this game, this border seems to be a reasonable one. After all, poker players tend to like hands that look fancy; any hand with at least a pair of images surely satisfies this condition of prettiness.

What does this reasoning mean for the strategies of the beginners? Player 1 bets only if his hand is at least $(J, J, 4, 3, 2)$. For each player the total probability of receiving a hand up to $(J, J, 4, 3, 2)$ is $\frac{1189}{1498} \approx 0.7937$.[3] So we can formulate the strategy for player 1 as a beginner as

$$p_0(u) = \begin{cases} 0 & \text{if } 0 \leq u \leq 0.7937 \\ 1 & \text{if } 0.7937 < u \leq 1, \end{cases}$$

while the beginner's strategy for player 2 can be formulated as

$$q_0(v) = \begin{cases} 0 & \text{if } 0 \leq v \leq 0.7937 \\ 1 & \text{if } 0.7937 < v \leq 1. \end{cases}$$

Let $U_i(p, q)$ denote the expected gains for player i if player 1 plays strategy p and player 2 plays strategy q. Then we can write down the expected payoff of the beginners against opponents who are playing the minimax strategies:

$$U_1(p_0, \widetilde{q}) = \frac{310}{3817} \approx 0.0812 \quad \text{and} \quad U_2(\widetilde{p}, q_0) = -\frac{265}{2254} \approx -0.1176.$$

These numbers are the game results of the beginners in RS_{1998}. For the RS_{2001} analysis, we need to do some more work. Given the strategies for the beginners (p_0 and q_0), we have to find out what is the optimal response of the opponent. We will describe in detail how player 2 determines what will be his best strategy. For each possible value v of his hand, he has to decide whether calling or folding is optimal against p_0. Figure 2 displays the payoff for player 2 for each of his two actions, given a hand combination (u, v). The P and B under the horizontal axis indicate for which values of u player 1 passes or bets according to strategy p_0, while b is the boundary value 0.7937 in player 1's beginner's strategy p_0.

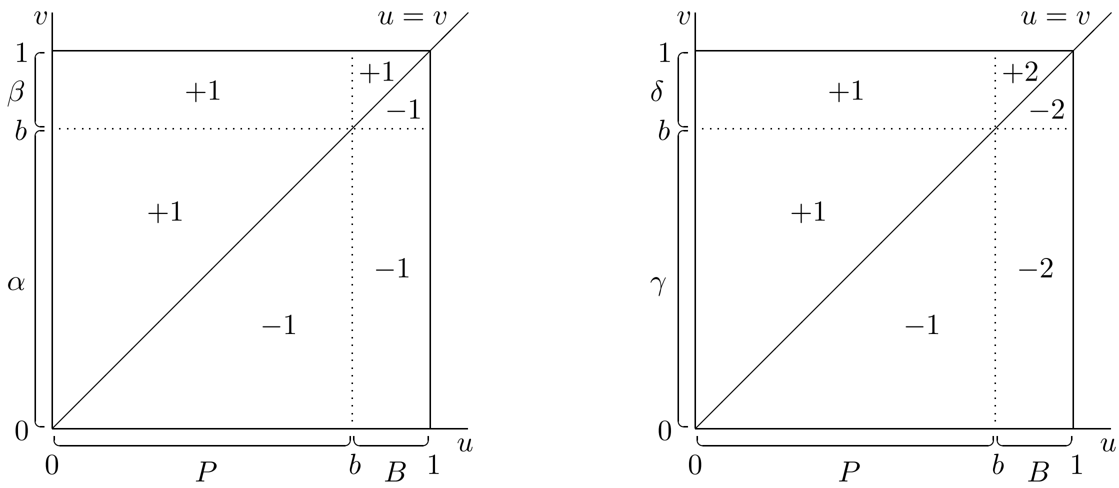
For each of the four marked intervals along the vertical axes (α, β, γ and δ) we can compute the expected payoff for player 2 for a specific hand value v.

Interval	Expected payoff for player 2 with a hand v
α	$v - (1 - v) = 2v - 1$
β	$b - (1 - b) = 2b - 1$
γ	$v - (b - v) - 2(1 - b) = 2v - 2 + b$
δ	$b + 2(v - b) - 2(1 - v) = 4v - 2 - b$

Player 2 should base his decisions on the numbers in this table. He has to compare the expected result for each v in α with the expectations for the same v in γ. If for a certain v the result in α is better than the result in γ, player 2 should fold. Otherwise he should call with this hand value. A similar comparison he should make between β and δ. We find that the optimal reply against a player playing p_0 is

$$Pr\{\text{call with hand } v\} = \widetilde{q}_0(v) = \begin{cases} 0 & \text{if } v \leq 0.8453, \\ 1 & \text{otherwise.} \end{cases}$$

[3]For details about probabilities in poker we refer to Dreef *et al.* (2003).



(a) player 2 folds

(b) player 2 calls

Figure 2: Expected payoffs for player 2 against beginner's strategy p_0 of player 1.

The boundary value of 0.8453 is rounded. We can now compute the resulting expected gains of player 1. A similar analysis leads to the payoff for player 2 when he acts as a beginner in RS_{2001}. The results are

$$U_1(p_0, \widetilde{q}_0) \approx -0.0053 \quad \text{and} \quad U_2(\widetilde{p}_0, q_0) \approx -0.4875.$$

7.4 Fictive Players

In the current section we will compute the expected payoffs of fictive players in minipoker. Fictive players have more information than normal players. According to the assumptions used for RS_{1998}, they know the outcome of the chance move in the game and they can use this information in their strategies. For minipoker this means that the fictive player can base his actions on his own hand, but also on the hand of his opponent. Given the fact that he plays against a player who uses the minimax strategy, he can decide what will be his best action for any hand combination (u, v). We call the resulting strategy p_f^{1998}. Dreef *et al.* (2003) already showed that the expected gains of player 1 are equal to

$$U_1(p_f^{1998}, \widetilde{q}) = \frac{17}{50}.$$

Analogously, we can determine the expected game result for player 2 as a fictive player:

$$U_2(\widetilde{p}, q_f^{1998}) = \frac{7}{50}.$$

We also want to compute the expected gains of the fictive players under the assumptions of RS_{2001}. Under these assumptions, fictive players are also informed about the outcome of any randomization caused by their opponents. Therefore, when we determine optimal play for an opponent, we have to consider pure strategies only; randomizing is useless against such a fictive player.

Let us first focus at player 1 as a fictive player. What is the best thing player 2 can do if player 1 knows, besides the value u of his own hand, the value v of player 2's hand too? Suppose player 1 has bet. If player 2 calls with a specific value v, he will get $-(-v+2(1-v)) = 3v-2$. For folding he will get -1 on any hand value v. Therefore, it is optimal for player 2 to fold if $v < \frac{1}{3}$ and to call otherwise. Then player 1 should always bet and his expected gains are

$$U_1(p_f^{2001}, \widetilde{q}_f) = \frac{1}{3}.$$

To see what the expected gains of player 2 as a fictive player are, consider what player 1 gets for betting and what he gets for passing, both with a hand of value u. Whereas betting will yield him a dollar if he has the more valuable hand, he will have to pay 2 dollars if his opponent has the better hand. Passing also gives him a win of one dollar if $u > v$, but with this action he will only lose one dollar in case his opponent has the better hand. Passing, therefore, is optimal for all possible values of u. Clearly, the expected payoff for player 2 as a fictive player then is equal to

$$U_2(\widetilde{p}_f, q_f^{2001}) = 0.$$

7.5 Results of the Skill Analysis for Minipoker

We have computed all relevant numbers to complete our skill analysis. Table 1 gives an overview. We observe the following things in Table 1. The expected payoffs of the beginners and the fictive

	RS_{1998}			RS_{2001}		
	Player 1	Player 2	Game	Player 1	Player 2	Game
Beginner	0.0812	-0.1176	-0.0182	-0.0053	-0.4875	-0.2464
Optimal	0.1000	-0.1000	0.0000	0.1000	-0.1000	0.0000
Fictive	0.3400	0.1400	0.2400	0.3333	0.0000	0.1667
LE	0.0188	0.0176	0.0182	0.1053	0.3875	0.2464
RE	0.2400	0.2400	0.2400	0.2333	0.1000	0.1667
RS	0.0726	0.0682	0.0704	0.3110	0.7949	0.5965

Table 1: Results of the skill analysis.

players are lower in the RS_{2001} model. This is what we expected, since their opponents now try to make life as hard as possible for them. The results for optimal players are the same in both models. Therefore the learning effect in the RS_{2001} model is larger and the random effect is smaller than in the RS_{1998} analysis. This combination of effects leads to a higher skill level for the game. This should be a warning: never base a comparison between the skill levels of two games on two different skill measures.

A second observation is that, for both measures, $RS_{\text{game}} \neq \frac{1}{2}RS_{\text{player 1}} + \frac{1}{2}RS_{\text{player 2}}$, as we already indicated in section 6. If we compare the skill of both players within the RS_{2001} model, we see that the skill of player 2 is relatively high. This can be explained by the beginner's strategy of player 2. This is a relatively dumb one, in the sense that player 1 can really profit from his mistakes. So against a player who gives maximal opposition, the beginner in the role of player 2 does relatively bad.

8 Determining the Skill Level Using Empirical Data

In the foregoing we have described, discussed and illustrated general aspects concerning a skill analysis of casino games. In this last section we want to indicate briefly what could be the role of empirical data in determining the skill level of a game.

In the first place one could think about collecting player results in a casino and using the resulting numbers as input for the skill measure that was given in formula (1). However, one should be careful. For a one-person game, we can imagine that it is possible to collect information about the game results of beginners and advanced players, or otherwise about the average player that was mentioned in section 3.4. The expected results for the fictive player should still be computed, since this is a theoretical player type.

For more-person games, life is more difficult. Of course, it is still possible to observe and collect the game results of beginners and advanced players. However, one should now know exactly against what kind of opposition the results in this data set are obtained. In an ideal situation one should obtain detailed information about the decisions made by all players. After all, the results for the fictive players still have to be computed and for these computations information about the opposition is needed.

If information is not available in so much detail, one could come up with simple rules of thumb to deduce an idea the skill level of a game out of the data. For example, suppose one has a collection of game results and one knows which part of the data corresponds to beginners and which to advanced players. Then one could apply a sort of analysis of variance. A difference in mean between the two groups indicates that there is some skill involved. A large spread or variance within each of the two parts of the data separately might point out that there is a significant random effect involved. Clearly, the details for such an analysis have to be worked out, but it might prove worthwhile and certainly deserves attention in the future.

As a final remark we wish to mention the possibility designing experiments to collect data for a specific game. Yu and Cowan (1995) give an example of a statistical model using duplicate tournaments to deduce information about the luck-skill balance in the game. They argue, however, that it is difficult to separately estimate effects of luck and effects of a better or worse execution of strategies by a player.

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