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Finite-dimensional regulators for a class of infinite-dimensional systems

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We show that the 'direct approach', developed by the author for stabilization of certain classes of distributed parameter systems, can be extended to cover regulation problems as well. An iterative design algorithm is presented, together with proof that the algorithm will converge after a finite number of steps. The procedure is illustrated with an example of a constant disturbance acting on a delay system.

Keywords: Distributed parameter systems, Regulation, Finite-order controllers, Design methods.

1. Introduction

In [1], a so-called 'direct approach' has been developed to deal with the stabilization problem for distributed parameter systems. In this approach, the intricacies of the 'spillover' phenomenon [2] are avoided by working directly with the infinite-dimensional model as given, and a special design method is used to obtain controllers of finite order. It is the purpose of this note to show that the 'direct approach' can be extended to regulation problems, where the issue is not only stabilization of the controlled system, but also cancellation of disturbance signals of known frequency, or tracking of certain reference signals. Such problems have been studied before in an infinite-dimensional context by Pohjolainen [3–5] and Bhat [6]. Pohjolainen obtains controllers of finite order, but he requires the open-loop system to be stable. Bhat considers possibly unstable delay equations, but his compensators are of infinite order. We shall obtain finite-dimensional controllers even for open-loop unstable systems. On the other hand, we shall not discuss the robustness issue, which is a central theme for both cited authors. A frequency-domain approach to regulation problems for a class of delay equations has been proposed in [7], but this approach does not, in general, lead to compensators of the standard finite-order type.

2. Problem setting

We shall consider linear systems described by the equations

$$\dot{x}_1(t)=A_{11}x_1(t), \quad x_1(t)\in X_1, \tag{1}$$

$$\dot{x}_2(t)=A_{21}x_1(t)+A_{22}x_2(t) + B_2u(t), \quad x_2(t)\in X_2, u(t)\in U, \tag{2}$$

$$y(t)=(C_1 \;\; C_2)\begin{pmatrix} x_1(t) \\ x_2(t)\end{pmatrix}, \quad y(t)\in Y, \tag{3}$$

$$z(t)=(D_1 \;\; D_2)\begin{pmatrix} x_1(t) \\ x_2(t)\end{pmatrix}, \quad z(t)\in Z, \tag{4}$$

which are understood as follows. The space X_1 is finite-dimensional; the variable $x_1(t)$ represents an 'external' (disturbance or reference) signal. The space X_2 is a Banach space, and the mapping

$$A_{22}: D(A_{22})\to X_2$$

is the generator of a strongly continuous semigroup

$$\{T_2(t) | t\geqslant 0\}$$

of bounded linear operators on X_2. The variable $y(t)$ is the 'observation' and is finite-dimensional, as well as the 'variable-to-be-controlled' $z(t)$ which we try to stabilize. The mappings A_{21}, B_2, C_1, C_2, D_1, and D_2 are all bounded linear mappings between their respective spaces. This set-up represents disturbance cancellation problems ($A_{21}\neq 0$, $D_1=0$) as well as reference following problems ($A_{21}=0$, $D_1\neq 0$) (cf. also [8], or [9]).

We shall need some further assumptions. These are largely the same as in [1] and we refer to this work for comments. We suppose that a number

$\omega < 0$ has been given which represents the desired 'degree of stability'. We shall call a semigroup $T(t)$ (or its generator) simply 'stable' if there is a constant $C > 0$ such that

$$\|T(t)\| \leqslant C\,e^{\omega t} \quad \text{for all } t \geqslant 0. \tag{5}$$

We assume:

(i) The spectrum of A_{22} is discrete.

(ii) There exists $\delta > 0$ such that the half-plane

$$\{\lambda \in \mathbb{C} | \operatorname{Re}\lambda > \omega - \delta\}$$

contains only finitely many eigenvalues of A_{22}.

This assumption implies, in particular, that there is a 'spectral decomposition' $X_2 = X_2^u \oplus X_2^s$ (cf. [10], p. 178), where X_2^u is the subspace spanned by the generalized eigenvectors of A_{22} that have real parts larger than ω. The following self-explaining notation will be used:

$$A = \begin{pmatrix} A_{11} & 0 & 0 \\ A_{21}^u & A_{22}^u & 0 \\ A_{21}^s & 0 & A_{22}^s \end{pmatrix},$$

$$B = \begin{pmatrix} 0 \\ B_2^u \\ B_2^s \end{pmatrix}, \qquad C = (C_1, C_2^u, C_2^s). \tag{6}$$

(iii) The (finite-dimensional) pair (A_{22}^u, B_2^u) is controllable.

(iv) The (finite-dimensional) pair

$$\left((C_1, C_2^u), \begin{pmatrix} A_{11} & 0 \\ A_{21}^u & A_{22}^u \end{pmatrix}\right)$$

is observable.

(v) The generalized eigenvectors of A_{22} are complete in X_2.

In this paper, we shall consider only compensators of the standard form; i.e. a compensator will have the form

$$\dot{w}(t) = A_c w(t) + G_c y(t),$$
$$w(t) \in W, \dim W < \infty, \tag{7}$$

$$u(t) = F_c w(t) + K y(t). \tag{8}$$

When (7)–(8) is connected to (1)–(3), one obtains a closed-loop system that has the form

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ w \end{pmatrix}(t) = \begin{pmatrix} A_{11} & 0 & 0 \\ A_{21} + B_2 K C_1 & A_{22} + B_2 K C_2 & B_2 F_c \\ G_c C_1 & G_c C_2 & A_c \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ w \end{pmatrix}(t). \tag{9}$$

Let $\tilde{A}_{22}^s$ be the 'stable' part obtained via spectral decomposition with respect to $\omega - \delta$ (δ as in (ii)), rather than ω. Let $\tilde{T}_2^s(t)$ denote the semigroup generated by $\tilde{A}_{22}^s$. Our final assumption is:

(vi) $\lim_{t\to\infty} e^{-\omega t}\|\tilde{T}_2^s(t)\| = 0.$

This assumption is only needed to guarantee that the stability of the closed-loop system (9) can be judged from the location of its eigenvalues (which is, unfortunately, not always true in infinite dimensions: [11] (p. 665), [12]). If this can be verified directly, the condition (vi) can be dispensed with.

A compensator of the form (7)–(8) will be called a *regulator* (of finite order) for the system (1)–(4), if the following conditions hold. There exists a subspace $V \subset D(A_e)$ (where A_e is the operator appearing in (9), and $D(A_e) = X_1 \oplus D(A) \oplus W$), of dimension equal to $\dim X_1$, such that $A_e V \subset V$. This subspace is also contained in Ker D_e, where

$$D_e : X_1 \oplus X_2 \oplus W \to Z$$

is defined by

$$D_e = (D_1 \;\; D_2 \;\; 0). \tag{10}$$

Finally, if $T_e(t)$ denotes the semigroup generated by A_e, the quotient semigroup induced by $T_e(t)$ on the factor space $(X \oplus W)/V$ is stable. These conditions mean that the closed-loop system is stable modulo the external signal dynamics (stabilization property), and that the variable-to-be-controlled $z(t)$ depends only on the stable part of the system (regulation property).

3. Main result

Theorem 3.1. *Let the system (1)–(4) satisfy the assumptions mentioned in the preceding section. If* *there exists a linear mapping* $S: X_1 \to X_2$ *such that*

$$\operatorname{Im} S \subset D(A_{22}), \tag{11}$$

$$\operatorname{Im}(A_{22}S - SA_{11} + A_{21}) \subset \operatorname{Im} B_2, \tag{12}$$

$$D_1 + D_2 S = 0, \tag{13}$$

then there exists a regulator of finite order for the given system.

Proof. First, we construct a mapping

$$F = (F_1 \;\; F_2): X_1 \oplus X_2 \to U,$$

as follows. By Lemma 4.5 of [1], we can find a mapping $F_2: X_2 \to U$ such that $A_{22} + B_2F_2$ is stable and has discrete spectrum, and such that the eigenvectors of $A_{22} + B_2F_2$ are complete in X_2. Let $\{x_1, \ldots, x_r\}$ be a basis for X_1. By (12), there exist $u_1, \ldots, u_r$ such that

$$(A_{22}S - SA_{11} + A_{21})x_i = B_2 u_i \quad (i = 1, \ldots, r).$$

Determine $F_1: X_1 \to U$ by

$$F_1 x_i = -u_i - F_2 S x_i \quad (i = 1, \ldots, r).$$

Define the subspace $V_1 \subset X$ by

$$V_1 = \left\{ \begin{pmatrix} x \\ Sx \end{pmatrix} \middle| x \in X_1 \right\}. \tag{14}$$

Note that $V_1 \subset D(A)$ (by (11)) and $V_1 \subset \operatorname{Ker} D$ (by (13)). Also, V_1 is $(A + BF)$-invariant.

By Lemma 4.3 of [1], there exists a linear mapping $G: Y \to X$, and a constant $\eta > 0$, such that for every $\tilde{G}$: $Y \to X$ with $\|G - \tilde{G}\| < \eta$, the semigroup generated by $A + \tilde{G}C$ is stable. Since $X = V_1 \oplus X_2$, we can write $G = G_1 + G_2$ with $\operatorname{Im} G_1 \subset V_1$ and $\operatorname{Im} G_2 \subset V_2$. By Lemma 4.4 of [1], there exist a subspace V_2 of X_2, spanned by a finite number of generalized eigenvectors of $A_{22} + B_2F_2$, and a mapping $\tilde{G}_2: Y \to X_2$, such that $\|G_2 - \tilde{G}_2\| < \eta$ and $\operatorname{Im} \tilde{G}_2 \subset V_2$. Write $\tilde{G} = G_1 + \tilde{G}_2$, and set $V_3 = V_1 + V_2$. Since $\|G - \tilde{G}\| < \eta$, it follows that $A + \tilde{G}C$ is stable. Note that V_3 is invariant for $A + BF$ and also, because $\operatorname{Im} \tilde{G} \subset V_3$, for $A + BF + \tilde{G}C$. We now define a compensator of the form (7)–(8), in the following way. Let W be a vector space isomorphic to V_3, and let $R: V_3 \to W$ provide the isomorphism. Set

$$K = 0, \quad F_c = FR^{-1}, \quad G_c = -R\tilde{G},$$

and

$$A_c = R(A + BF + \tilde{G}C)R^{-1}.$$

It remains to show that this compensator is a regulator, i.e. that a subspace $V \subset X \oplus W$ exists having certain properties (see the end of Section 2). Define

$$V = \left\{ \begin{pmatrix} x \\ Rx \end{pmatrix} \middle| x \in V_1 \right\}. \tag{15}$$

The following facts are straightforward to verify:

$$\dim V = \dim X_1, \qquad V \subset D(A_e),$$

$$A_e V \subset V, \qquad V \subset \operatorname{Ker} D_e.$$

We are left only with the proof of the stability of the closed-loop semigroup modulo V. Let $Q: V_3 \to V_3/V_1$ denote the factor mapping, and define a mapping

$$J: (X \oplus W)/V \to X \oplus (V_3/V_1)$$

by

$$J: \left[\begin{pmatrix} x \\ w \end{pmatrix}\right] \to \begin{pmatrix} x - R^{-1}w \\ QR^{-1}w \end{pmatrix}. \tag{16}$$

It is easily verified that J is well defined and that it gives a bijection between

$$(X \oplus W)/V \quad \text{and} \quad X \oplus (V_3/V_1).$$

Assuming, without loss of generality, that $R: V_3 \to W$ is an isometry, we can make the following estimates for any $x_1 \in V_1$, $w \in W$, and $x \in X$:

$$\|x - R^{-1}w\| \leqslant \|x - x_1\| + \|R^{-1}w - x_1\|$$
$$= \|x - x_1\| + \|w - Rx_1\|, \tag{17}$$

$$\|QR^{-1}w\| \leqslant \|R^{-1}w - x_1\| = \|w - Rx_1\|. \tag{18}$$

Using the norm

$$\left\|\begin{pmatrix} x \\ w \end{pmatrix}\right\| = \max(\|x\|, \|w\|) \tag{19}$$

on $X \oplus W$, we obtain

$$\left\|\begin{pmatrix} x - R^{-1}w \\ QR^{-1}w \end{pmatrix}\right\| \leqslant 2\left\|\begin{pmatrix} x \\ w \end{pmatrix} - \begin{pmatrix} x_1 \\ Rx_1 \end{pmatrix}\right\|. \tag{20}$$

Since this holds for all $x_1 \in V_1$, we get

$$\left\|\begin{pmatrix} x - R^{-1}w \\ QR^{-1}w \end{pmatrix}\right\| \leqslant 2\left\|\left[\begin{pmatrix} x \\ w \end{pmatrix}\right]\right\| \tag{21}$$

or, J is bounded. By the Banach open mapping theorem ([13], p. 212–213), J^{-1} is also bounded. So J is a Banach space isomorphism between the spaces

$$(X \oplus W)/V \quad \text{and} \quad X \oplus (V_3/V_1).$$

Let us write $\overline{A_e}$ for the mapping induced by A_e on $(X \oplus W)/V$, and $\overline{A+BF}$ for the mapping induced by $A+BF$ on V_3/V_1. It is straightforward to verify that the following relation holds for all $x \in X$ and $w \in W$:

$$J\overline{A_e}\left[\begin{pmatrix} x \\ w \end{pmatrix}\right] = \begin{pmatrix} A+\tilde{G}C & 0 \\ -Q\tilde{G}C & \overline{A+BF} \end{pmatrix} J\left[\begin{pmatrix} x \\ w \end{pmatrix}\right]. \quad (22)$$

By construction, both $A+\tilde{G}C$ and $\overline{A+BF}$ are stable. It follows that the semigroup generated by $\overline{A_e}$ is stable as well.

A sufficient condition for the existence of a mapping S that satisfies (11)–(13) is given by the following proposition.

Proposition 3.2. *Consider the system* (1)–(4). *If* A_{11} *is diagonalizable, if the spectra of* A_{11} *and* A_{22} *do not overlap, and if the matrix*

$$W(\lambda) = D_2(\lambda - A_{22})^{-1}B_2 \quad : U \to Z \quad (\lambda \in \sigma(A_{22})) \quad (23)$$

has a right inverse $W^+(\lambda)$ *for each* $\lambda \in \sigma(A_{11})$, *then there is a mapping* $S: X_1 \to X_2$ *such that* (11)–(13) *hold.*

Proof. Let $\{x_1, \dots, x_r\}$ be a basis for X_1 consisting of eigenvectors of A_{11}, and let $\{\lambda_1, \dots, \lambda_r\}$ be the corresponding eigenvalues. Define $u_i \in U$ $(i = 1, \dots, r)$ by

$$u_i = W^+(\lambda_i)\big(D_2(\lambda_i - A_{22})^{-1}A_{21} + D_1\big)x_i. \quad (24)$$

Define S: $X_1 \to X_2$ by

$$Sx_i = (\lambda_i - A_{22})^{-1}(A_{21}x_i - B_2u_i) \quad (i = 1, \dots, r). \quad (25)$$

By straightforward calculation, one verifies that S satisfies (11)–(13).

The matrix $W(\cdot)$ is, of course, the transfer function from the control input u to the variable-to-be-controlled z; it can only be right invertible if

$$\dim U \geqslant \dim Z,$$

i.e. the variety of controls must be at least as large as the variety of goals. More general results on the equations (11)–(13) appear in [14], p. 316, be it still under the assumption of disjoint spectra of A_{11} and A_{22}. There are simple examples in which the disjointness does not hold, but still a mapping S satisfying (11)–(13) can easily be found (cf. [15], p. 150).

4. Design procedure

The proof of the theorem is constructive, at least in principle. An outline of an actual design algorithm could be the following.

Step 1. Find a mapping S which satisfies (11)–(13).
Step 2. Determine F: $X \to U$ as in the proof of the theorem.
Step 3. Select G such that $A + GC$ is amply stable.
Step 4. Compute V_1 (as defined in (14)), and write $G = G_1 + G_2$ with $\operatorname{Im} G_1 \subset V_1$ and $\operatorname{Im} G_2 \subset V_2$.
Step 5. Select k eigenvectors of $A_{22} + B_2F_2$, and find an approximation $\tilde{G}_1$ to G_1, with $\operatorname{Im} \tilde{G}_1$ contained in the subspace spanned by the selected eigenvectors.
Step 6. Find out, by a direct method (for instance 'Weinstein–Aronszajn', cf. [1]), if $A + \tilde{G}C$ is stable. If this is so, construct the k-th order compensator as in the proof of the theorem. If not, repeat step 5 with an enlarged value of k.

This algorithm avoids the calculation of the constant η that appeared in the proof of the theorem. The algorithm is, therefore, iterative, and Theorem 3.1 takes the form of a convergence result, guaranteeing that success will be obtained after a finite number of steps, without recourse to any particular smartness on the part of the designer.

5. Examples

Consider the following delay system, on which a constant disturbance acts:

$$\dot{x}_1(t) = 0, \quad (26)$$

$$\dot{x}_2(t) = -\tfrac{1}{2}\pi x_2(t-1) + x_1(t) + u(t), \quad (27)$$

$$z(t) = y(t) = x_2(t). \tag{28}$$

Even without the disturbance $x_1(t) \equiv x_1(0)$, the open-loop equation (27) is unstable, with oscillatory eigenvalues at $\pm\frac{1}{2}\pi i$. So we want both to stabilize the system and to reject the constant disturbance.

The system can be brought into the standard form (1)–(4) by making the following definitions. Let the spaces X_1, U, Y, and Z all be equal to $\mathbb{R}$, and set

$$X_2 = M_2(-1, 0) = \mathbb{R} \times L_2(-1, 0).$$

Elements of $M_2(-1, 0)$ are written as row vectors (ϕ_0, ϕ) with $\phi_0 \in \mathbb{R}$ and $\phi \in L_2(-1, 0)$. Let $H^1[-1, 0]$ be the Sobolev space of functions on $[-1, 0]$ whose first derivative is in $L_2(-1, 0)$ ([16], p. 44). Define various mappings by

$$A_{11} = 0, \tag{29}$$

$$D(A_{22}) = \{(\phi_0, \phi) \in M_2(-1, 0) | \phi \in H^1[-1, 0], \phi(0) = \phi_0\}, \tag{30}$$

$$A_{22}(\phi_0, \phi) = (-\tfrac{1}{2}\pi\phi(-1), \dot{\phi}),$$

$$A_{21}: 1 \to (1, 0), \tag{31}$$

$$B_2: 1 \to (1, 0), \tag{32}$$

$$C\begin{pmatrix} \alpha \\ (\phi_0, \phi) \end{pmatrix} = D\begin{pmatrix} \alpha \\ (\phi_0, \phi) \end{pmatrix} = \phi_0$$
$$(\alpha, \phi_0 \in \mathbb{R}; \phi \in L_2(-1, 0)). \tag{33}$$

Under these definitions, the system (1)–(4) is a representation of (26)–(28) (cf. [17]). Let us define the notion 'stable' with respect to a growth constant $\omega = -1$ (cf. (5)). Using standard results on delay equations ([17–24], [6]), one can prove that all conditions of Section 2 hold. So we can start the design procedure.

Step 1. We can use Proposition 3.2; this gives

$$S: 1 \to (0, 0). \tag{34}$$

Step 2. By the construction of Lemma 4.5 in [1], there exists $F_2: X_2 \to U$ such that $A_{22} + B_2F_2$ has the same eigenvalues as A_{22}, except for those at $\pm\frac{1}{2}\pi i$, which can be relocated to, say, $-1 \pm \frac{1}{2}\pi i$. Completeness of eigenvectors is retained in this operation. There will be no need to compute F_2 explicitly. The mapping $F_1: X_1 \to U$ is found immediately as $F_1: 1 \to -1$.

Step 3. Via spectral decomposition (cf. [1]), we find that

$$G: 1 \to \begin{pmatrix} -\frac{1}{2}\pi^2 \\ (-2\pi, \psi) \end{pmatrix},$$

$$\psi(\theta) = -\pi\cos\tfrac{1}{2}\pi\theta - 2\pi\sin\tfrac{1}{2}\pi\theta - \pi \tag{35}$$

shifts the eigenvalues at 0 and $\pm\frac{1}{2}\pi i$ of A to new eigenvalues at $-\frac{1}{2}\pi$ (double) and $-\pi$ of $A + GC$, while the other eigenvalues of A remain unchanged.

Step 4. We find

$$G_1: 1 \to \begin{pmatrix} -\frac{1}{2}\pi^2 \\ (0, 0) \end{pmatrix}, \quad G_2: 1 \to \begin{pmatrix} 0 \\ (-2\pi, \psi) \end{pmatrix}. \tag{36}$$

Step 5. We select the two eigenvectors of $A_{22} + B_2F_2$ corresponding to the eigenvalues at $-1 \pm \frac{1}{2}\pi i$. A basis for the space spanned by these vectors is given by (cf. [1]) $\{(1, \psi_1), (0, \psi_2)\}$, with

$$\psi_1(\theta) = e^{-\theta}\cos\tfrac{1}{2}\pi\theta, \qquad \psi_2(\theta) = e^{-\theta}\sin\tfrac{1}{2}\pi\theta. \tag{37}$$

By computing the best $L_2(-1, 0)$ approximation of ψ by ψ_1 and ψ_2, we arrive at the following trial mapping $\tilde{G}$:

$$\tilde{G}: 1 \to \begin{pmatrix} -\frac{1}{2}\pi^2 \\ (a_1, a_1\psi_1 + a_2\psi_2) \end{pmatrix},$$
$$a_1 = -3.666, \quad a_2 = -1.990. \tag{38}$$

Step 6. By numerical computation of the rightmost eigenvalues of $A + \tilde{G}C$ (as discussed in [1]), it is verified that $A + \tilde{G}C$ is stable. So we can put our compensator together; the result is

$$\dot{w}(t) = \begin{pmatrix} -4.666 & 1.571 & 0 \\ -3.561 & -1 & 0 \\ -4.935 & 0 & 0 \end{pmatrix} w(t) + \begin{pmatrix} 3.666 \\ 1.990 \\ 4.935 \end{pmatrix} y(t), \tag{39}$$

$$u(t) = -w_1(t) - 2.699w_2(t) - w_3(t). \tag{40}$$

The stability of the closed-loop system can be verified by direct calculation. The pole at 0 of the compensator transfer function, evident from (39), will appear as a zero in the closed-loop transfer function from x_1 to x_2. This shows that the above compensation scheme does, indeed, reject the constant disturbance.

Further examples can be found in [15].

6. Conclusions

Under a certain 'regulation condition', as given in the statement of Theorem 1, we have been able to prove the existence of finite-dimensional regulators for a class of infinite-dimensional linear systems. The main features of this class are: bounded input and output mappings; finiteness of the number of unstable eigenvalues; and completeness of eigenvectors. An obvious question for further research is, to what extent these restrictions can be relaxed. For the stabilization problem, some results in this direction can be found in [25–27] (unbounded inputs and outputs) and [28] (nonlinear systems). Also, further analysis of the design algorithm is required with respect to numerical and robustness properties.

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