

NBER WORKING PAPER SERIES

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AMONG SECTORS AND AMONG INDIVIDUALS:  
A PORTFOLIO APPROACH

Joel Slemrod

Working Paper No. 951

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge MA 02138

August 1982

I am grateful to Roger Gordon for conversations which clarified my thinking on a number of points, to Craig Swan and Yolanda Henderson for comments on an earlier draft, to the participants at workshops at the University of Minnesota, Harvard University, and Bell Laboratories for stimulating discussions, and to Leonard Burman for able research assistance. An earlier version of this paper was presented at the joint NIA-TIA and AEA session of "Taxes and the Allocation of Capital" held in Washington, D.C., on December 29, 1981. The research reported here is part of the NBER's research program in Taxation. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

Tax Effects on the Allocation of Capital Among Sectors and  
Among Individuals: A Portfolio Approach

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ABSTRACT

This paper deals with the allocational effects and implications for efficiency of a tax system in which the rate of tax on capital income differs depending on the recipient of the income and on the type of capital producing the income. It suggests that, in their attempts to measure the distortionary effect of the U.S. capital income tax system, economists may have been looking in the wrong places. In the presence of uncertainty, the intersectoral distortion may be much less than had previously been imagined. However, the tax system distorts at two other margins which have not received much attention. It distorts the inter-household allocation of the housing stock, since the after-tax rate of interest is one component of the opportunity cost of owner-occupied housing. It also distorts the inter-household allocation of risk-bearing. Calculations using a simple computable general equilibrium model suggest that the excess burden from these latter two distortions are significant components of the total distortionary impact of the tax system.

Joel Slemrod  
Department of Economics  
University of Minnesota  
1035 Management and Economics  
271 19th Avenue South  
Minneapolis, MN 55455  
(612) 373-3607

## I. Introduction

This paper deals with the allocational effects and the implications for efficiency of a tax system in which the rate of capital income tax differs depending on the recipient of the income and on the type of capital producing the income. It argues that in the presence of uncertainty the standard intersectoral misallocation problem may not be as large as previously thought, but suggests that there are other sources of inefficiency in the current capital income tax system which must be considered in addition to the intersectoral inefficiency. In particular, there is an inter-household misallocation of the housing stock, due to the differences in the opportunity cost of housing, and there is an inter-household misallocation of risk bearing. Calculations using a simple computable general equilibrium model indicate that the excess burden caused by these latter two distortions are significant components of the total static distortionary impact of the capital income tax system.

Two decades ago, Harberger showed that the magnitude of the allocational shift caused by differential factor taxation and the size of the accompanying excess burden depend on the degree of substitutability in the production sectors, the degree of substitutability among the final products, and the relative factor intensities of production. Harberger's insights have most often been applied to the allocational and efficiency implications of the extra taxation that corporate capital income bears compared to non-corporate capital income. Using a range of plausible estimates for the relevant parameters, Harberger (1966) concluded that the pattern of capital income taxation prevailing in the U.S. during the mid-1950's reduced the capital stock in the corporate sector by between one-sixth and one-third, and imposed an efficiency cost of between \$1 billion and \$3 billion. Shoven (1976) corrected Harberger's original estimates and, using the same

parameters, found the deadweight loss to be approximately 38 percent lower than Harberger had calculated it to be. The corrected efficiency cost was between 0.2 and 0.6 percent of gross national product, though between six and fifteen percent of the revenue raised by the excess taxation that corporate capital income bears. More recent models in this tradition (e.g., Fullerton et al (1981)) have improved the estimates by adding to the detail of the stylized economy.

The mechanics and implications of the Harberger model have been discussed at length elsewhere.<sup>1/</sup> However, for our purposes here it will be useful to mention three characteristics of the model structure.

1. There is no uncertainty. This assumption implies that in equilibrium the after-tax rate of return on capital must be equal for all sectors.

2. Relative tax rates on different kinds of capital income are not household-specific.<sup>2/</sup> Without this assumption, for any given vector of before-tax returns for some households the after-tax returns on some assets differ. Thus, households could arbitrage by going short in the lower-yielding asset and holding a long position in the higher-yielding asset.<sup>3/</sup> If there are constraints on borrowing, the absence of this assumption implies household specialization in assets. With this assumption the household portfolios are indeterminate, since assets are perfect substitutes for each other, each earning the same after-tax return with certainty.

3. All households face the same price vector for final goods.

None of these three characteristics accurately represents the U.S. economy. In this paper we show that these three unempirical assumptions are critical to the model's predictions about the allocational and efficiency effects of the U.S. system of taxing capital income. When

the model is modified in such a way as to allow more realistic assumptions, the model's implications may look quite different.

The remainder of the paper contains three sections. Section II presents a simple general equilibrium model under certainty where the opportunity cost of housing differs for different individuals. The resulting exchange inefficiency is discussed and compared to the inter-sectoral inefficiency caused by the tax system. In Section III a general equilibrium model under uncertainty is presented. When considered in this environment the intersectoral inefficiency may be much lower than had been thought, though the exchange inefficiency remains. This section then considers a third source of tax-induced inefficiency in a stochastic economy, distorted allocation of risk-bearing. The total welfare cost of taxation is then reconsidered using a modified version of the general equilibrium model of Section II. Section IV offers some concluding remarks.

## II. A. A Certainty Model

We will first focus our attention on the third assumption of the Harberger-type models, that all individuals face the same price vector for final goods. Even in the absence of explicit household-specific taxes, this condition will be violated whenever some of the goods are durable. One component of the opportunity cost of a durable good is the after-tax rate of interest.<sup>4/</sup> When tax rates differ, then households will necessarily face different net prices for the durable good.

Housing is by far the most significant durable good in the U.S. In Harberger's original classification of the economy into corporate and non-

corporate sectors, housing comprised 67.5 percent of the non-corporate sector.<sup>5/</sup> Since the service return from owner-occupied housing is not subject to taxation, the opportunity cost of owner-occupied housing is lower for households in higher tax brackets. In order to assess the implications of differential prices arising from differences in marginal tax rates, we present a model where the only two vehicles for holding wealth are owner-occupied housing and non-housing capital, the income from which is assumed to be taxable.

Consider an economy where there are only two commodities, housing services (H) and a composite non-housing good (C). It is assumed that the only way an individual can consume housing services is by owning housing capital<sup>6/</sup>; one unit of housing capital produces one unit of housing services per period, with no labor input required. Then we can write the individual's problem as

$$\begin{aligned}
 (1) \quad & \text{Max}_{C_i, H_i} \quad U(C_i, H_i) \\
 & \text{subject to } C_i = wL_i + r(1 - t_i)(K_i - H_i) - \delta H_i + T_i \\
 & \quad \text{or } C_i + [r(1 - t_i) + \delta]H_i = wL_i + r(1 - t_i)K_i + T_i
 \end{aligned}$$

Here  $w$  represents the wage rate,  $r$  is the rate of return to holding non-housing capital,  $t_i$  the flat-rate tax on capital income,  $K_i$  is the capital endowment, (which may be divided between housing and non-housing capital),  $\delta$  is the rate of depreciation on housing capital, and  $T_i$  is the transfer received from the government. Since the imputed income from housing capital is not taxed, the tax structure is equivalent to one with an income tax at rate  $t_i$  applied to all capital income, including housing services, and a subsidy of  $t_i r$  to the consumption of housing services. Notice that two types of distortions result from this tax system. The first is that the relative price of housing services and other goods faced

by consumers in equilibrium will not equal the marginal rate of transformation between the two goods, resulting in an inefficient provision of housing services. The second distortion stems from the fact that the relative price of housing differs for different individuals. This implies that, for a given stock of housing, the allocation of housing among individuals is inefficient. In other words, reallocation of the housing stock, accompanied by some set of transfer payments, could improve the welfare of everyone.

By extending the Hotelling/Harberger measure of excess burden, we can analyze the contribution of the exchange inefficiency to the welfare cost. The standard expression, applicable when there is one representative household and when producer prices are fixed, is

$$(2) \quad L = -\frac{1}{2} \sum_i \sum_j S_{ij} T_i T_j,$$

where  $i$  and  $j$  index goods,  $S_{ij}$  is the compensated substitution term, and  $T_i$  and  $T_j$  are the rates of tax. When there are many households with different rates of tax, the appropriate expression is

$$(3) \quad L = -\frac{1}{2} \sum_h \sum_i \sum_j S_{ij}^h T_i^h T_j^h,$$

where  $h$  indexes households. If the substitution terms do not differ by household, then this expression simplifies to

$$(4) \quad L = -\frac{1}{2} N \sum_i \sum_j (S_{ij} (\bar{T}_i \bar{T}_j + \text{cov}(T_i, T_j))),$$

where  $N$  is the total number of households and  $\bar{T}_i$  refers to the average household value of  $T_i$ . In the special case where there is

a distorting tax levied on only one commodity, (for example, housing (H)) expression (4) becomes

$$(5) \quad L = -\frac{1}{2} N S_{HH} (\bar{T}_H^2 + \text{var}(T_H)) .$$

Computing the welfare loss using the average tax rate thus underestimates the actual welfare loss by the square of the coefficient of variation.

There are good reasons to believe, though, that the household tax rates will not be independent of the substitution term. Since the marginal tax rate is positively correlated with income, even if all households had identical price elasticities of demand, the substitution terms would be negatively associated with tax rates. Even with identical incomes and utility functions the slope of the compensated demand function may be different at different after-tax prices.

In this case, expression (4) may be rewritten, ignoring a higher-order term, as

$$(6) \quad L = -\frac{1}{2} N \sum_i \sum_j (\bar{S}_{ij} \bar{T}_i \bar{T}_j + \bar{S}_{ij} \text{cov}(T_i, T_j) + \bar{T}_i \text{cov}(S_{ij}, T_j) + \bar{T}_j \text{cov}(S_{ij}, T_i)) .$$

In the special case where there is only one tax, (on housing services, e.g.) this reduces to

$$(7) \quad L = -\frac{1}{2} N (\bar{S}_{HH} (\bar{T}_H^2 + \text{var}(T_H)) + 2\bar{T}_H \text{cov}(S_{HH}, T_H)) .$$

Clearly to the extent that the tax rate is positively correlated with the absolute value of the substitution term there is an amount of welfare loss not captured by the earlier expression. On the other hand if  $|S_{HH}|$  and  $T_H$  are negatively correlated the earlier measure of welfare loss is an overestimate.



These expressions overestimate the welfare loss when there is an upward-sloping supply curve of housing services. It can be shown that in this case when only housing services are taxed the welfare loss is equal to

$$(8) \quad L = -\frac{1}{2} \left[ \sum_h S_{HH}^h (T_H^h)^2 + \frac{(\sum_h S_{HH}^h T_H^h)^2}{\frac{dH^S}{dr} - \sum_h S_{HH}^h} \right],$$

where  $\frac{dH^S}{dr}$  is the slope of the supply curve of housing services. When all household's  $S_{HH}^h$  are equal,  $L$  is equal to

$$(9) \quad -\frac{1}{2} NS_{HH} \left[ \bar{T}_H^2 \left( \frac{\frac{dH^S}{dr}}{\frac{dH^S}{dr} - NS_{HH}} \right) + \text{var} (T_H) \right],$$

which is equivalent to expression (5) with the addition of a factor multiplying  $\bar{T}_H^2$ . When the supply curve is horizontal, then  $\frac{dH^S}{dr}$  is infinite, and (9) reduces to (5). When the supply curve is perfectly inelastic ( $\frac{dH^S}{dr} = 0$ ), the excess burden is proportional to the variance of the  $T_H$ . Thus even in the case where the total stock of housing is inelastic, there is exchange inefficiency due to household-specific prices for housing services.

## II.B. A General Equilibrium Illustration

The inefficiency caused by the existence of differential prices of housing services may be of the same order of magnitude as the inefficiency caused by the existence of the subsidy to housing services. To illustrate this, we consider a simple computable general equilibrium model of a two-person, two-good economy of the type described above. Housing services are produced using only capital with a constant-returns-to-scale technology,

and the non-housing good is produced with a Cobb-Douglas technology (the share of capital income in the net product is 0.2). The two individuals have identical endowments of capital and labor, but may face different tax rates on non-housing capital income. All the government revenue is returned to individuals through lump-sum transfers which are proportional to their (fixed) labor endowment. (A complete description of the system is provided in Appendix A.)

The equilibrium of this economy can be computed exactly, and depends on the behavioral and technological parameters, the tax rates, and the government transfer policy. Given the equilibrium allocations, we can calculate the utility level for each individual. Comparative static analysis of alternative tax structures is accomplished by altering the tax parameters. Several measures of the resulting change in welfare are possible. Our procedure is to calculate the change in national income valued at prices before and after the policy change, and find the geometric mean of these figures.<sup>7/</sup>

We consider three alternative tax structures. The first features no taxation of the capital income of either individual, so that  $t_1 = t_2 = 0$ . The second structure (the "one distortion" case) taxes non-housing capital income, but at equal rates for both individuals, so that  $t_1 = t_2 = 0.3$ . Finally, the third scheme (the "two distortion" case) features a progressive tax system with rates that average to 0.3, but where rates differ so that  $t_1 = 0.1$  and  $t_2 = 0.5$ .<sup>8/</sup>

The theory of the second best tells us that, generally speaking, we cannot conclude that an economy with distortions at two margins is more

inefficient than if only one distortion existed. However, in this example which uses standard assumptions about the structure of preferences and technology the two-distortion case does have a greater excess burden than the one-distortion case, even though the total revenue raised is about seven percent lower in the two-distortion case. The details of the equilibrium solutions are presented in Table 1 of Appendix A. Compared to the no tax equilibrium, the one-distortion tax structure causes an efficiency loss of 0.92 percent of NNP, and decreases the non-housing capital stock by 9.6 percent. These results are broadly consistent with those reported by Harberger and Shoven.<sup>9/</sup> The two-distortion equilibrium, where one household faces a much higher tax rate than the other, has an excess burden of slightly more than 1.2 percent of NNP. The reduction of the non-housing capital stock compared to the no-tax equilibrium is 10.6 percent.

The additional efficiency loss in the two distortion case, as compared to the one distortion case, has two components. The first is that, due to the structure of preferences, increasing  $t_2$  from 0.3 to 0.5 provides more of an incentive for household 2 to purchase housing than the decrease in  $t_1$  from 0.3 to 0.1 provides a disincentive to household 1. Thus, the total equilibrium housing stock is greater than in the one distortion case and the intersectoral misallocation is increased. The other component of the increased excess burden is the exchange inefficiency resulting from the differential user costs of housing. Starting from any given level of the total stock of housing, differential opportunity costs imply an inefficient allocation

of the housing stock among households. To illustrate this point consider the following experiment. Take an economy where the allocation of inputs to sectors has been decided, and assume that the input allocation corresponds to the equilibrium outcome in the two distortion case. Now suppose that we can reallocate housing services so that both household's marginal rate of substitution of housing services for the non-housing goods are equal; that is, so the condition for exchange efficiency is met. This reallocation will entail shifting housing services away from the highly taxed household toward the low tax household. Once this reallocation is accomplished, adjust the level of government transfers so that household 1 is just as well off as it was before the reallocation. The change in the utility of household 2 is then a measure of the welfare gain from eliminating the exchange inefficiency.

The result of performing this experiment in our simple economy is that the excess burden falls from 1.23 percent to only 1.02 percent of GNP. Loosely speaking, the exchange inefficiency is the source of about one-sixth of the total excess burden.

### III.A. Introducing Uncertainty

Up to this point the analysis has dealt only with a nonstochastic environment. In this section that assumption is dispensed with by allowing the capital income of the corporate sector to be uncertain, due perhaps to the presence of a stochastic element in the production function. This will allow us to consider the implications of dispensing with the first two characteristics of the Harberger-type models that were discussed above.

We assume that in the presence of uncertainty households act to maximize expected utility, which we assume for the sake of simplicity can be written as<sup>10/</sup>

$$(10) \quad EU_i = U_i(\bar{C}_i, \bar{H}_i) - \frac{\beta_i}{2} \left( \frac{V_i}{K_i} \right)$$

where  $\bar{C}_i$  and  $\bar{H}_i$  now represent the  $i$ th household's expected value of consumption of the corporate good and housing services, respectively,  $\beta_i$  is a risk aversion parameter, and  $V_i$  represents the variance of the after-tax income stream.

In addition to equity shares in corporate capital, which are risky, there also are riskless securities. Any net supply of these securities is provided by the federal government; in addition there may be riskless loans among individuals.

Uncertainty in an individual's income stream has two sources. The first source is non-zero holdings of corporate equity. The second source of uncertainty is the transfer payment received from the government. When part of the tax base is corporate earnings, tax revenue itself is stochastic. Unless the government can somehow "absorb" this uncertainty (perhaps by having a random debt policy and thus spreading the risk across generations), the transfer payment must then also be stochastic.

The individual's sole decision is how to allocate his wealth among the three alternative assets: owner-occupied housing (H), risky corporate capital (E), and riskless securities (B). The choice of H determines the quantity of owner-occupied housing services that will be consumed. After the wealth allocation decision is made, the state of the world is revealed, including the return to

corporate capital and the transfer received from the government. All income is then spent on the corporate good, C.

Under these circumstances we can write the individual's maximization problem as

$$(11a) \quad \text{Max}_{H_i, B_i, E_i} \quad U(\bar{C}_i, H_i) - \frac{\beta_i}{2} \left( \frac{V_i}{K_i} \right)$$

$$\text{subject to } C_i = wL_i + r_B(1-t_{Bi})B_i + r_E(1-t_{Ei})E_i - \delta H_i + T_i$$

$$H_i = K_i - (E_i + B_i)$$

$$V_i = \sigma_E^2(1-t_{Ei})^2 E_i^2 + V_{Ti} + 2V_{ETi}$$

Here  $r_B$  and  $r_E$  refer to the rate of return on the riskless and risky securities, respectively.  $V_{Ti}$  is the variance of the stochastic transfer received from the government;  $V_{ETi}$  is the covariance between the flow of equity income and the transfer payment, and  $\sigma_E^2$  is the variance of the return from one unit of corporate capital. The first two constraints can be combined into a single one as follows:

$$(11b) \quad C_i + [r_B(1-t_{Bi}) + \delta]H_i = wL_i + r_B(1-t_{Bi})K_i + [r_E(1-t_{Ei}) - r_B(1-t_{Bi})]E_i + T_i$$

In expression (11b), only  $C_i$ ,  $r_E$ , and  $T_i$  are stochastic. Thus (11b) implies that

$$(11c) \quad \bar{C}_i + [r_B(1-t_B) + \delta]H_i = wL_i + r_B(1-t_{Bi})K_i + [\bar{r}_E(1-t_{Ei}) - r_B(1-t_{Bi})]E_i + \bar{T}_i$$

The first-order conditions for this problem reduce to:

$$(12) \quad \frac{U_{H_i}}{U_{C_i}} = r_B(1 - t_{Bi}) + \delta$$

$$(13) \quad \frac{E_i}{K_i} = \frac{U_{Ci} [r_E (1 - t_{Ei}) - r_B (1 - t_{Bi})]}{\beta_i \sigma_E^2 (1 - t_{Ei})^2} - \left( \frac{1}{(1 - t_{Ei})^2 \sigma_{EK_i}^2} \right) \left( \frac{\partial V_{ETi}}{\partial E_i} \right).$$

The first condition is the standard requirement that the marginal rate of substitution of housing for the corporate good be equal to the relative price, which in this case is the opportunity cost of owner-occupied housing plus depreciation,  $r_B(1-t_{Bi}) + \delta$ . The second condition is an asset demand relationship for equity holdings. The demand for equity depends positively on the expected after-tax premium it earns over the riskless asset, inversely on its after-tax variability and the degree of risk aversion, and is inversely related to the covariance between its return and the government transfer.

In the case where all households face the same tax rates ( $t_E$ ), the expected value and variance of government revenue can be written as follows:

$$(14) \quad \bar{R} = t_B r_B B + t_E r_E E$$

$$(15) \quad V_T = t_E^2 \sigma_E^2$$

If it is assumed that the government must balance its budget and cannot absorb any risk, then the total amount of transfers must equal  $R$  and the transfers must exactly reflect the uncertainty of the revenues.

As an initial simplification, we assume that there are  $N$  identical individuals in the economy. Furthermore, the government treats each

individual identically, so each  $T$  has the same distribution. These assumptions imply that

$$(16) \quad T_i = R/N$$

$$(17) \quad V_{Ti} = t_E^2 \sigma_E^2 / N^2$$

$$(18) \quad V_{ETi} = (1 - t_E) t_E E_i \sigma_E^2 / N$$

In a large economy each individual can ignore the effect of his own demand for  $E_i$  on the aggregate value of  $E$ . Thus we can compute  $\frac{\partial V_{ETi}}{\partial E_i}$  to be

$$(19) \quad \frac{\partial V_{ETi}}{\partial E_i} = (1 - t_E) t_E E \sigma_E^2 / N$$

Substituting this expression into the asset demand equation derived earlier (expression (13)) yields

$$(20) \quad \frac{E_i}{K_i} = \frac{U_c [r_E (1 - t_E) - r_B (1 - t_B)]}{\beta_i \sigma_E^2 (1 - t_E)^2} - \frac{t_E E}{(1 - t_E) N K_i}$$

From (20) we can form an expression for  $E_i$ . Then we sum the demand functions and impose the equilibrium condition that

$$(21) \quad \sum_i E_i = E$$

Solving the aggregate equation for  $E$  then yields

$$(22) \quad \frac{E}{K} = \frac{U_c [r_E (1 - t_E) - r_B (1 - t_B)]}{\beta \sigma_E^2 (1 - t_E)}$$



We can use this equation to determine the impact of changing the taxation of corporate capital on the demand for corporate capital. For small changes, it is not unreasonable to hold  $U_c/\beta$  constant; that is, the marginal rate of substitution between corporate goods and risk per unit of capital is assumed unchanged. In this case, the partial derivative of expression (22) with respect to  $t_E$  becomes:

$$(23) \quad \frac{\partial \left( \frac{E}{K} \right)}{\partial t_E} = \frac{-U_c r_B (1 - t_B)}{\beta \alpha_E^2 (1 - t_E)^2}$$

From (23), the response of demand for corporate capital to changes in its taxation is seen to depend on the after-tax riskless rate of return. This corresponds to the result found by Gordon (1981). The intuition behind it is fairly straightforward. The expected return to corporate capital has two components, one of which is a risk premium, measured by the excess of the return over the after-tax earnings of the riskless asset. The other component is the after-tax riskless return. The tax on corporate income falls on both components of the return. However, that part of the tax that falls on the risk premium reduces the uncertainty of the return as well as the expected value of the return. In fact, when  $r_B$  is zero the reduction in the expected return is exactly the payment that individuals would be willing to pay to be rid of the uncertainty the tax eliminates. Thus the tax on the risk premium component of the return does not alter the attractiveness of the corporate equity. It is only the tax on the other component which alters its attractiveness. Thus, the bite of the tax depends on  $r_B(1 - t_B)$ , the after-tax riskless rate of return. If  $r_B$  is zero, then there is no incentive to alter

equity holdings. If  $r_B$  is non-zero, the incentive depends on the sign of  $r_B$ ; if  $r_B$  is positive, increasing  $t_E$  will decrease desired equity holdings, and if  $r_B$  is negative, increasing  $t_E$  will increase desired equity holdings.

This model easily generalizes to situations where individuals are not identical, do not face identical tax rates, and the government does not simply transfer  $1/N$  of its revenue to each individual. Consider the alternative rule that individual  $i$  receives  $s_i$  of government revenue, where  $\sum_i s_i = 1$ . Then the following relationships hold:

$$(24) \quad R = r_B \sum_i t_{Bi} B_i + r_E \sum_i t_{Ei} E_i$$

$$(25) \quad V_T = (\sum_i t_{Ei} E_i)^2 \sigma_E^2 \equiv (t_E^A)^2 \sigma_E^2$$

$$(26) \quad T_i = s_i R$$

$$(27) \quad V_{Ti} = s_i^2 V_T$$

$$(28) \quad V_{ETi} = (1 - t_{Ei}) t_{Ei}^A s_i E_i \sigma_E^2$$

The term  $t_E^A$  denotes the weighted average tax rate on equity earnings where the weights are the proportion of total equity held by the investor; thus the sum of the weights is unity. As above, we can find the optimal holding of the risky asset, which now depends on  $s_i$ ,  $t_E^A$ , and  $E$ .

$$(29) \quad \frac{E_i}{K_i} = \frac{U_{Ci} [r_E (1 - t_{Ei}) - r_B (1 - t_{Bi})]}{\beta_i \sigma_E^2 (1 - t_{Ei})^2} - \frac{t_{Ei}^A s_i E}{(1 - t_{Ei}) K_i}.$$

Of more interest than any individual's response to changes in  $t_E$  is the economy's response. To determine this we must aggregate over all individuals. After some manipulation, we can find the aggregate analogue to expression (29) to be:

$$(30) \quad \frac{E}{K} = \frac{r_E \sum_i \left( \frac{U_{ci}}{\beta_i} \right) \frac{k_i}{(1-t_{Ei})} - r_B \sum_i \left( \frac{U_{ci}}{\beta_i} \right) \frac{k_i (1-t_{Bi})}{(1-t_{Ei})^2}}{\alpha_E^2 \left( 1 + t_E^A \frac{\sum_i s_i}{i(1-t_{Ei})} \right)}$$

Here  $k_i$  denotes the share of total wealth owned by individual  $i$  ( $k_i = K_i/K$ ). Note that when  $U_{ci}/\beta_i$  is equal for all  $i$ , and  $t_{Ei}$  and  $t_{Bi}$  are identical for everyone (so that  $t_E^A = t_{Ei}$ ), then expression (30) is equivalent to expression (22). However, in the general case the response of  $E$  to changes in  $t_E$  does not reduce to a simple expression like (23).

The important message here is that the magnitude of the inter-sector distortionary effect of the tax system may be much less than has previously been thought. Much of what has been considered to be distortionary taxation is more properly thought of as payment (at the market price) for the government's participation in risk sharing. Does this mean that our system of taxing capital income is less distortionary than we had thought? The answer is uncertain, for three reasons. The first is that the extent to which corporate capital income taxation is merely payment for risk sharing has not been precisely measured, and would appear to be sensitive to a number of features of the tax system.<sup>11/</sup>

The second reason is that even when the presence of uncertainty is recognized the user cost of housing differs dramatically among individuals, and is therefore a source of exchange inefficiency. Our simple example in the riskless case suggested that this distortion may amount to a non-trivial fraction of the inter-sector distortion; its significance is not diminished when uncertainty is introduced. Finally, under uncertainty the tax system causes an additional distortion due to the inefficient allocation of risk-bearing among the economy's individuals. This source of excess burden must be considered in a complete assessment of the tax system. This is our next topic.

### III. B. The Welfare Cost of Inefficient Risk Bearing

To see that the tax system may also cause an inefficient bearing of the economy's risk, consider the market for the claim to a flow of income which has an expected value of zero, which has unit variance, and which is perfectly correlated with the stochastic income from the corporate sector. At the consumer optimum, the marginal rate of substitution of this risky claim for an expected unit of  $C$  is

$$(31) \quad MRS_i = \frac{-\beta_i \sigma_E}{K_i U_{ci}} [E_i(1 - t_{Ei}) + t_{Ei}^A s_i E].$$

If expression (29) is used to substitute for  $E_i$ , this reduces to

$$(32) \quad MRS_i = - \frac{r_E(1 - t_{Ei}) = r_B(1 - t_{Bi})}{\sigma_E(1 - t_{Ei})}$$

In order to eliminate the negative sign, we can consider the marginal rate of substitution of the elimination of one unit of the risky claim for an expected unit of good  $C$ . That is just the negative of expression (32), or

$$(33) \quad MRS_i = \frac{r_E(1 - t_{Ei}) - r_B(1 - t_{Bi})}{\sigma_E(1 - t_{Ei})} .$$

Notice that this expression does not depend on how much risk the government returns to the individual. The independence property obtains because individuals can undo (or augment) any uncertainty from the transfer by selling or buying risky (and perfectly correlated) corporate shares.

That the MRS need not be the same for all individuals is clear from expression (33). However, the presence of a progressive tax system does not by itself assure the existence of differences. As long as the safe asset and the risky asset attract the same tax rate, then the MRS reduces to  $(r_E - r_B)/\sigma_E$  for all individuals. However, in the U.S. corporate capital income is first subject to a flat-rate corporate income tax, and then the after-corporate-tax income is subject to the individual income tax, at the standard rate for that part of earnings which is paid out as dividends and at the capital gains rate for that part of earnings which is retained within the corporation. Income from government debt is taxed at the individual income tax rate. If the personal tax rate is denoted  $t_B$ , then  $t_E = \tau + (1 - \tau)(dt_B + (1 - d)\gamma t_B)$  where  $\tau$  is the corporate income tax rate,  $d$  is the dividend payout rate, and  $\gamma$  is the ratio of the effective accrual-equivalent capital gains tax rate to the ordinary income tax rate. If this expression for  $t_E$  is substituted into expression (33), it is clear that the MRS will vary among individuals depending on  $t_B$ . By calculating the derivative of expression (33) with respect to  $t_B$ , we can ascertain the direction of influence. After some manipulation, we can find

$$(34) \quad \frac{d(\text{MRS})}{d(t_B)} = \frac{(1 - \tau)(1 + d + (1 - d)\gamma)r_B}{\sigma_E(1 - t_{Ei})^2}$$

Since all terms other than  $r_B$  are strictly positive, the direction of influence depends on the sign of  $r_B$ . As long as  $r_B$  is positive, the marginal rate of substitution of a unit of risk reduction for an expected unit of good  $C$  increases with the tax rate. In other words, the inefficiency in risk bearing takes the form of the higher-taxed individuals having too much risk and the lower-taxed individuals having too little risk. There exists a reallocation of risk and expected consumption (with the higher-taxed individuals bearing less risk) such that all individuals are better off.

Thus it is the combination of a progressive individual income tax system and the current system of corporate capital income taxation that causes the inefficiency. Complete integration of the individual and corporate income tax systems, so that  $t_E$  equals  $t_B$  for all individuals, would eliminate the distortion. More generally, though, any feature of the tax system which differentially taxed the return of the risky asset and safe asset would be subject to this kind of inefficiency.

Even in the presence of a completely integrated tax system, there is reason to believe that there will still be inefficiency in risk-bearing. This is because inefficiency may result whenever the average tax rate on risky capital is different than the rate which is applied to deviations from the expected return, that is when the total tax rate,  $t$ , is equal to  $t_E \overline{r_E} + t_R(r_E - \overline{r_E})$  where  $\overline{r_E}$  is the expected return and  $t_R$  is the tax rate applicable to deviations from  $\overline{r_E}$ . There are many features

of the current system of taxing corporate capital income which cause  $t_R$  to diverge from  $t_E$ . First of all, there are a number of tax incentives, such as the investment tax credit, which serve to reduce  $t_E$  but which do not reduce the marginal tax on earnings. Working in the other direction is corporations' observed tendency to keep dividends relatively constant while absorbing income swings in variations in retained earnings. Since the effective tax on retained earnings, the capital gains tax rate, is lower than the tax rate on dividends, this kind of financial policy causes  $t_R$  to be lower than  $t_E$ .

Although there are forces operating in both directions, there is no reason to expect that  $t_E$  and  $t_R$  will be equal. The import of this inequality is that it is  $t_R$  and not  $t_E$  which belongs in the denominator of expression (33). In this case even though  $t_B$  and  $t_E$  are equal, unless the ratio of  $(1-t_E)$  to  $(1-t_R)$  is the same for everyone, there will be inefficiency in risk-bearing. When  $t_E$  and  $t_B$  are not identical, then the additional divergence of  $t_R$  and  $t_E$  may either increase or decrease the inefficiency of bearing risk.

### III. C. A Further General Equilibrium Illustration

The foregoing analysis has indicated that, compared to the certainty case presented first, the excess burden of the capital income tax system is lower due to the fact that the tax on the risk premium is non-distortionary, but is higher due to the tax-induced inefficiency in private risk-bearing. In order to assess the net effect on the magnitude of efficiency loss, it would be useful to construct a general formula for the approximate excess burden, as we did in the certainty case. However, even in this relatively simple model, a simple expression is impossible to obtain, due to the complicated general equilibrium effects of changes in  $t_{Ei}$  and  $t_{Bi}$ . A

reconsideration of the stylized two-good, two-person economy will, though, be helpful in assessing the effects of introducing uncertainty into the model. A few additional parameters are required (e.g., measures of household risk aversion, the riskiness of corporate equity, separate tax rates for riskless debt and corporate equity) to complete the model. A complete description of the model is presented in Appendix B.

As with the certainty model, we will investigate several different tax structures and compare the equilibria and the excess burden. We include the three tax structures studied under the certainty case, where equity and riskless security income are not distinguished; both tax rates are zero in the first case, both are 0.3 in the second case, and  $t_1 = 0.1$  and  $t_2 = 0.5$  in the third case. In the additional case we consider, corporate equity and riskless security income are taxed differently. In order to approximate the total effective tax on equity income that results in a classical corporate income tax system, we set  $t_E = \tau + (1 - \tau)d t_B$  where  $\tau$  is the average corporate income tax rate and  $d$  is the dividend payment ratio. This formulation assumes a zero effective capital gains tax rate. Setting  $\tau$  to 0.4 and  $d$  to be 0.5, we obtain a value of 0.43 for  $t_{E1}$  and 0.55 for  $t_{E2}$ , given  $t_{B1}$  and  $t_{B2}$  are 0.1 and 0.5, respectively.

The results of these experiments, detailed in Table 2 of Appendix B, are revealing. Starting from the no-tax equilibrium, imposing a flat rate tax of 0.3 on both individuals for both debt and equity income causes an excess burden equal to 0.44 percent of  $NNP^{12/}$  and reduces the corporate capital stock by 6.9 percent (compared to 0.92 percent of  $NNP$  and a reduction of 9.6 percent in the riskless case). Since the riskless interest rate is positive, not all of the tax represents a market



payment for risk sharing, but the distortionary impact of the tax is substantially less than in the riskless case. Next we impose a progressive tax system, but do not discriminate between assets, so that  $t_{E1} = t_{B1} = 0.1$  and  $t_{E2} = t_{B2} = 0.5$ . This tax system marginally increases aggregate demand for housing and also causes an inefficient allocation of the housing stock. The new equilibrium has an excess burden of 0.65 percent of NNP, and a corporate capital stock of 7.6 percent lower than the no tax equilibrium. The additional inefficiency and allocational shift is of approximately the same magnitude as it was in the riskless case. However, it amounts to a larger fraction of the distortion caused in the absence of progressivity (forty-five percent compared to twenty-five percent). This is due to the fact that the inter-sector distortion is much lower when the risk-sharing nature of corporate taxation is considered. Finally, we consider the unintegrated tax structure where  $t_{E1} = 0.43$ ,  $t_{B1} = 0.1$ ,  $t_{E2} = 0.55$ , and  $t_{B2} = 0.5$ . This tax structure has two important effects: first, it raises the average rate of taxation on corporate capital income, and, second, it introduces inefficiency in the private bearing of risk. In the new equilibrium, there is an excess burden equal to 1.67 percent of NNP, while the non-housing capital stock is 13.6 percent lower than in the no-tax case.

We can perform a loose decomposition of the total inefficiency by considering two experiments of the type discussed earlier. First, for the given housing stock, we reallocate the housing until the two household's marginal rates of substitution between housing services and the non-housing good are equal. Then we readjust

transfers so as to keep one household just as well off. This elimination of the exchange inefficiency reduces the excess burden from 1.67 to 1.54 percent of NNP. Just as in the riskless model, the distorted allocation of the housing stock is a non-trivial component of the tax system's inefficiency.

The next experiment keeps all capital and final good allocations the same and reallocates the private bearing of risk until both households have the same marginal rate of substitution between a risky claim and a certain unit of income. This experiment by itself reduces the inefficiency from 1.67 to 1.48 percent of NNP, thus accounting for about one-eighth of the distortion. The riskiness of corporate capital income and the degree of risk aversion parameters of the model are such that the utility loss from bearing risk amounts, in the classical corporate income tax case, to about 4.2 percent of the utility gained from the expected value of NNP. However, the private risk is borne very inefficiently, with the higher taxed individual in debt to buy corporate capital and the lower taxed individual holding very little corporate equity. When the risk is efficiently reallocated, the same amount of total risk causes disutility equal to 4.0 percent of NNP.

When both the exchange inefficiency and the risk bearing inefficiency are eliminated, the excess burden declines from 1.67 to 1.35 percent of net national product. Thus about one-fifth of the total burden may be thought of as coming from these two usually ignored sources.

The welfare cost due to inefficient risk bearing was noted by Gordon and Malkiel (1981). They calculated that the efficiency cost from inefficient risk-bearing was approximately  $0.01 r_z^2 V$ , where  $r_z$  is the nominal risk-free rate of return and  $V$  is the total amount of risky securities outstanding. In 1975, this amounted to \$44 million, or only 0.004 percent of GNP.

There are several reasons for the divergence between the Gordon-Malkiel (G-M) estimate and the one presented here, which is approximately 0.3 percent of NNP. First, their calculation, if performed correctly, yields a welfare loss of  $0.01 r_z V$ , not  $0.01 r_z^2 V$  as they indicate.<sup>13/</sup> Using their estimate of 0.08 for  $r_z$ , their corrected measure of welfare loss is \$550 million, or 0.05 percent of GNP. Second, the G-M calculation assumes that the only source of risk is corporate equity. In the model presented here, all non-housing capital income is risky. If the fraction of total capital that is risky is 0.5 (as it approximately is in the examples used in this paper) instead of approximately one-fourth, as assumed by G-M, and all risky capital is equally risky, then their estimates of the efficiency costs of misallocating risk should be increased by a factor of two. Finally, the illustrative values chosen by G-M imply that the before personal tax risk premium of equity is only 0.048. However, calculations performed by, for example, Friend and Blume (1975) indicate that the observed historic rate-of-return premium earned by corporate shares has been on the order of 0.09. A recalculation of the efficiency loss due to inefficient risk-bearing using numbers consistent with this risk premium would yield a substantially larger magnitude.

If all the alterations suggested in the last paragraph are made, the corrected G-M calculation of \$550 million in efficiency loss would become a figure on the order of three billion dollars (about 0.27 percent of GNP), which is similar to the estimate obtained in the model presented here.<sup>14/</sup>

#### IV. Concluding Remarks

This paper suggests that, in their attempts to measure the distortionary effect of the capital income tax system, economists may have been

looking in the wrong places. In the presence of uncertainty, the intersectoral distortion may be much less than had previously been imagined. However, it is important to recognize that the tax system also distorts the inter-household allocation of the housing stock and the inter-household allocation of risk-bearing. Calculations using a computable two-person, two-sector general equilibrium model suggest that the excess burden from these latter two distortions amount to a significant fraction of the inter-sectoral inefficiency that has been estimated in previous studies.

Several words of caution are in order here. First of all, the figures presented here are based on one particular parameterization of a stylized economy. There are several reasons to expect that a more realistic model may alter the results somewhat. The assumed unit elasticity of demand between housing services and the non-housing good is on the high side of recent econometric results. A lower elasticity would reduce the estimate of exchange inefficiency. The assumed dispersion in capital income tax rates may somewhat overstate the true dispersion. Also, the presence of tax-exempt securities puts a lower bound on the after-tax rate of interest earned by high tax bracket individuals, thus limiting the inter-household dispersion in the opportunity cost of housing services. On the other hand, though, the taxation of nominal interest rates in the presence of inflation serves to increase the dispersion. Also, the existence of rental housing as a source of housing services and as an alternative asset must be dealt with in a complete treatment of these issues. Finally, the assumption that there is a riskless asset in real terms is untenable in a world of uncertain rates of inflation. Future development of this approach should consider the case where all assets, including owner-occupied housing, earn a stochastic return.

These caveats all apply to the particular estimates of the impact of the tax system on the economy presented here. Improvements in the success of the model in representing the U.S. economy will no doubt increase our confidence in the model's predictions.<sup>15/</sup> Nevertheless, it seems clear that models which maintain the three assumptions mentioned at the beginning of this paper will likely fail to accurately assess the effects of alternative systems of taxing capital income.

Footnotes

1/ See, for example, the excellent survey by McLure (1975).

2/ More precisely, it must be true for any two assets A and B that  $\frac{1 - t_{Ai}}{1 - t_{Bi}} = \frac{1 - t_{Aj}}{1 - t_{Bj}}$  for any two individuals i and j, where  $t_{Ai}$  is the ith individual's total tax rate on capital income from asset A.

3/ For further discussion of this point, see Feldstein and Slemrod (1980).

4/ The other components are the rate of physical depreciation and the (negative of the) expected capital gain. Since physical depreciation on durable goods is not deductible from taxable income, this component of opportunity cost is the same for all households. To the extent that capital gains are taxable, the expected capital gain term will vary by household. The accrual-equivalent tax rate on capital gains on owner-occupied housing is, however, very small and this source of inter-household variation in the opportunity cost of housing will be ignored in the analysis that follows.

5/ See Harberger (1966), p. 110.

6/ In fact, sixty-five percent of all households are owner-occupiers.

7/ This procedure is similar to the one adopted in Fullerton et al (1981). Note that owner-occupied housing is valued at its net-of-tax full opportunity cost.

8/ Notice that when owner-occupied housing is one of the assets the fact that the relative rate of tax on the two assets differs does not invite arbitraging or specialization. This is because the return to owner-occupied housing is in the form of services which must be consumed by the owner and are subject to diminishing marginal utility.

9/ Shoven reports that, in the case where the two sectors have elasticities of substitution of one and zero, the reduction in the capital stock in response to a tax on gross capital income of 0.46 will amount to 13.7 percent (See Shoven (1976), p. 1270, Case 5). This result is only broadly comparable to the example in the text due to differing parameterizations.

10/ This utility function is a generalization to two goods of the standard mean-variance utility function used in portfolio choice analysis.

11/ For example, the foregoing analysis assumed that the tax system reduces the expected return and standard deviation of the return by equal factors. The validity of this assertion depends on corporate financial behavior and the specific structure of corporation income taxation. See the discussion below.

12/ In the calculation of national income, risk is valued at the bearer's marginal rate of substitution. All the excess burden terms mentioned in this section are computed as percentages of the value of national income including the (negative) value of the private risk.

13/ The factors multiplying the integrals in the expression on page 188 of G-M should be divided by  $r_z$ , as should the values on the vertical axis of Figure 3 on the preceding page.

<sup>14/</sup> Another difference between the G-M calculation and the one reported here is the presence of tax-exempt securities in the former model. In their presence, it is middle-income households who own equity, while those poorer and richer own taxable debt and tax-exempt debt, respectively. This implies that the marginal rate of substitution between risk and certain income is not monotonic with respect to tax rate, and the efficiency costs of risk-bearing are less than in the absence of tax-exempt debt. See G-M, pages 173-174.

<sup>15/</sup> See Slemrod (1982) for a more detailed general equilibrium model in the same spirit as the one presented in Section III of this paper.



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Appendix A

The following equations comprise the two-good, two-person model discussed in section II of the paper.

$$Y_i = wL_i + r(1 - t_i)K_i + T_i \quad i = 1, 2$$

$$G_i = (1 - \alpha)Y_i \quad i = 1, 2$$

$$H_i = \frac{\alpha_i Y_i}{r(1 - t_i) + \delta} \quad i = 1, 2$$

$$E_i = K_i - H_i \quad i = 1, 2$$

$$K_c = \sum_i E_i$$

$$L = \sum_i L_i$$

$$\sum_i C_i = K_c^\gamma L^{1-\gamma}$$

$$r = \gamma \left( \frac{K_c}{L} \right)^{\gamma-1}$$

$$w = (1 - \gamma) \left( \frac{K_c}{L} \right)^\gamma$$

$$R = r \sum_i t_i K_{ci}$$

$$T_i = s_i R \quad i = 1, 2$$

where the terms not defined in the text are:

$Y$  ; income including imputed service flow from house

$\alpha$  ; share of spending for housing services

$E$  ; corporate equity holdings

$K_c$  ; corporate capital

$\gamma$  ; share of corporate value added for capital income

$s$  ; share of tax revenue transferred back to individual

The parameter values used in the text examples are:

$$L_i = 50 \quad \alpha_i = 0.15 \quad K_i = 175 \quad \gamma = 0.2 \quad s_i = 0.5 \quad \delta = 0.04$$

Table 1

## Solution Details of General Equilibrium Model With No Uncertainty

|  | Case 1                  | Case 2                      | Case 3                      |
|--|-------------------------|-----------------------------|-----------------------------|
|  | $t_1 = 0 \quad t_2 = 0$ | $t_1 = 0.3 \quad t_2 = 0.3$ | $t_1 = 0.1 \quad t_2 = 0.5$ |
| $K_c$  | 220.8                   | 199.6                       | 197.5                       |
| $\Sigma H_i$                                   | 129.2                   | 150.4                       | 152.5                       |
| $E_1$  | 110.4                   | 99.8                        | 107.9                       |
| $H_1$  | 64.6                    | 75.2                        | 67.1                        |
| $C_1$  | 53.4                    | 51.4                        | 54.9                        |
| $E_2$  | 110.4                   | 99.8                        | 89.6                        |
| $H_2$  | 64.6                    | 75.2                        | 85.4                        |
| $C_2$  | 53.4                    | 51.4                        | 47.4                        |
| $r$  | 110.4                   | 0.115                       | 0.116                       |
| $w$  | 0.937                   | 0.919                       | 0.917                       |
| $R$  | 0                       | 6.89                        | 6.45                        |
| change in<br>real income                       |                         | -0.92                       | -1.23                       |
| change with<br>efficient housing<br>allocation |                         |                             | -1.02                       |

## Appendix B

The following equations comprise the two-good, two-person model with uncertainty discussed in Section III of the paper.

$$Y_i = wL_i + r_B(1 - t_{Bi})K_i + [r_E(1 - t_{Ei}) - r_B(1 - t_{Bi})]E_i + T_i \quad i = 1, 2$$

$$C_i = (1 - \alpha_i)Y_i \quad i = 1, 2$$

$$H_i = \frac{\alpha_i Y_i}{r_B(1 - t_{Bi}) + \delta} \quad i = 1, 2$$

$$\frac{E_i}{K_i} = \frac{U_{ci} [r_E(1 - t_{Ei}) - r_B(1 - t_{Bi})]}{\beta_i \sigma_E^2 (1 - t_{Ei})^2} - \frac{t_E^A s_i K_c}{(1 - t_{Ei}) K_i} \quad i = 1, 2$$

$$B_i = K_i - H_i - E_i \quad i = 1, 2$$

$$K_c = \sum_i E_i$$

$$B = \sum_i B_i$$

$$L = \sum_i L_i$$

$$\sum_i C_i = K_c^\gamma L^{1-\gamma}$$

$$r_E = \gamma \left( \frac{K_c}{L} \right)^{\gamma-1}$$

$$w = (1 - \gamma) \left( \frac{K_c}{L} \right)^\gamma$$

$$R = r_E \sum_i t_{Ei} E_i + r_B \sum_i t_{Bi} B_i$$

$$T_i = s_i R$$

$$t_E^A = \frac{\sum_i t_{Ei} E_i}{K_c} \quad i = 1, 2$$

$$U_{ci} = (1 - \alpha_i) \left( \frac{H_i}{C_i} \right)^{\alpha_i} \quad i = 1, 2$$

The parameter values used in the text example are:

$$L_i = 50 \quad \alpha_i = 0.15 \quad K_i = 175 \quad \gamma = 0.2 \quad s_i = 0.5 \quad \beta_i = 3 \quad \sigma_E^2 = 0.04 \quad \delta = 0.04$$

Table 2

## Solution Details of General Equilibrium Model With Uncertainty

|   | Case 1       |              | Case 2         |                | Case 3         |                | Case 4         |                 |
|---|--------------|--------------|----------------|----------------|----------------|----------------|----------------|-----------------|
|   | $t_{B1} = 0$ | $t_{E1} = 0$ | $t_{B1} = 0.3$ | $t_{E1} = 0.3$ | $t_{B1} = 0.1$ | $t_{E1} = 0.1$ | $t_{B1} = 0.1$ | $t_{E1} = 0.43$ |
|   | $t_{B2} = 0$ | $t_{B2} = 0$ | $t_{B2} = 0.3$ | $t_{E2} = 0.3$ | $t_{B2} = 0.5$ | $t_{B2} = 0.5$ | $t_{B2} = 0.5$ | $t_{E2} = 0.55$ |
| $K_c$   | 177.5        |              | 165.2          |                | 164.0          |                | 153.3          |                 |
| $\Sigma H_1$  | 172.5        |              | 184.8          |                | 186.0          |                | 196.7          |                 |
| $E_1$   | 88.7         |              | 82.6           |                | 55.7           |                | 23.1           |                 |
| $H_1$   | 86.3         |              | 92.4           |                | 82.7           |                | 85.2           |                 |
| $B_1$   | 0            |              | 0              |                | 36.6           |                | 66.8           |                 |
| $C_1$   | 49.2         |              | 47.9           |                | 50.2           |                | 47.4           |                 |
| $E_2$   | 88.7         |              | 82.6           |                | 108.3          |                | 130.2          |                 |
| $H_2$   | 86.3         |              | 92.4           |                | 103.3          |                | 111.5          |                 |
| $B_2$   | 0            |              | 0              |                | -36.6          |                | -66.8          |                 |
| $C_2$   | 49.2         |              | 47.9           |                | 45.3           |                | 45.7           |                 |
| $r_B$   | 0.061        |              | 0.073          |                | 0.075          |                | 0.065          |                 |
| $r_E$   | 0.126        |              | 0.126          |                | 0.135          |                | 0.142          |                 |
| $w$   | 0.897        |              | 0.884          |                | 0.883          |                | 0.871          |                 |
| $R$   | 0            |              | 6.63           |                | 6.94           |                | 9.86           |                 |
| % change in<br>real income  |              |              | -0.44          |                | -0.65          |                | -1.67          |                 |
| % change with<br>efficient housing allocation                               |              |              |                |                | -0.51          |                | -1.54          |                 |
| % change with<br>efficient risk bearing                                     |              |              |                |                |                |                | -1.48          |                 |
| % change with<br>efficient housing allocation<br>and efficient risk bearing |              |              |                |                |                |                | -1.35          |                 |