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Abstract

In this paper we present a model in which a systematic banking collapse is possible in a perfect foresight, general equilibrium context. Our aim is to determine conditions under which a collapse will eventually occur and the timing of such a collapse. The collapse can occur endogenously, driven by market fundamentals. Alternatively, it can be caused by a mass hysteria which generates itself in reality. We also compare the assumptions and implications of our model to the observable phenomena of the 1930's.

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In this paper we will present a model in which a banking collapse is possible in a general equilibrium context. Our aim is to determine conditions under which a collapse will eventually occur and the timing of such a collapse. The methodology will be similar to that used in Salant and Henderson (1978), Krugman (1979) and Flood and Garber (1980); a banking collapse is a discontinuous change in agents' asset holdings which must occur in order to prevent discontinuous price jumps when an institutional regime terminates.

The model explored here assumes perfect foresight. Obviously, elements of uncertainty would complicate agents' decisions and allow discrete price jumps as new information arises. For instance, a stochastic environment is necessary to address questions about excess reserve holdings, the ratio of cash to demand deposits, or the level of bank equity (see e.g. Gorton 1980). Eventually, we will embed this model in a stochastic setting. However, it appears to us that an essential aspect of banking collapses can be captured in a perfect foresight model.

In section I we set up a model for the behavior of bank owners and produce a sufficient condition for setting off a systematic bank collapse; the condition amounts to a floor on the nominal interest rate. In section II we choose some particular forms for asset and goods demand functions to serve as an explicit example, which we solve, for the timing of a bank collapse. The model which we use is essentially Patinkin's (1965, Chapters 12.5-12.6) full-employment macroeconomic model with a mechanical banking system attached. The differences are that expectations are of the perfect foresight variety and that bank owners are explicitly profit maximizers within a constrained environment.

Of course, in constructing a model of systematic banking collapse, any researcher, at least half-seriously, probably is attempting an explanation of the enormous collapse of the great depression. Since we are not innocent of such ambitions, we describe in section III both our model's points of coincidence with the observable phenomena of the 1930's and its points of divergence.

I) A Behavioral Model for Bank Owners

In our model we will assume a real economy which is perfectly insulated, except for wealth distribution, from the financial sector. In particular, we assume that real output $\overline{Y}$, the product of labor and capital, is fixed, as are the inputs of labor and capital required to produce $\overline{Y}$. The nominal income of labor will be $(1 - \theta)P(t)\overline{Y}$ where $(1 - \theta)$ is labor's share and $P(t)$ is the nominal price level. Capital receives $\theta P(t)\overline{Y}$.

The construction of the machines which spew out $\overline{Y}$ was financed in the dim past partly through equity sales and partly through consol sales; each consol promises to pay \$1/period forever. From the nominal earnings of capital, consol owners are paid first; any remainder is paid to equity holders. In case of default on the coupon payments, we assume that ownership of the machines reverts instantly to the bond holders; thus, the original equity holders are wiped out and the bonds are converted to equities.

The number of consols in existence is $\overline{B}$, and part of $\overline{B}$ is held by the public with the remainder in the portfolios of banks. The consols are not perfectly secure; if, for example, $P(t)$ declines sufficiently $\theta P(t)\overline{Y} < \overline{B}$. Then equity holders cannot meet their nominal obligations out of current income; if this situation is expected to persist, equity holders default with bondholders assuming ownership of the entire capital stock. For the real private economy that is the end of the story.

Since banks are part of this system, we must consider bank behavior in order to describe fully the asset markets. Throughout the paper we will assume that the monetary authority is producing a deflation by destroying high powered money.[1] The state of the banking system can be summarized by the balance sheet:

Assets	Liabilities
$R(t)$	$D(t)$
$P_B(t)B_b(t)$	$K(t)$

$R(t)$ is the amount of high powered money set aside as bank reserves, $P_B(t)$ is the nominal price of a consol, $B_b(t)$ is the number of consols held by the banking sector, $D(t)$ is the amount of demand deposits, and $K(t)$ is the amount of bank capital.

The banking sector is constrained by the rules of the game to pay on demand \$1 of high-powered money for \$1 of its demand deposit liability. It can do this either by drawing down its reserves or by liquidating a bond. While a bank can default on this obligation only when it has no futher assets, the bank owners can choose to liquidate the bank and pay off the depositors at any time.

Since exit from the industry is costless, bank owners will never take less then the real rate of return on their capital. We will assume that there are barriers to entry and controls to interest payments on demand deposits so that bank owners may make a premium rate of return from the bank business.[2]

To see how bank owners would behave in a deflationary situation let us partition the banks' bonds into $B_b^*(t)$, that quantity of bonds which equals $K(t)$ in value, and the remainder $\hat{B}_b(t)$, i.e. $B_b(t) = B_b^*(t) + \hat{B}_b(t)$.

Then the balance sheet is

Assets	Liabilities
$R(t)$	$D(t)$
$P_B(t)\hat{B}_b(t)$	
$P_B(t)B_b^*(t)$	$K(t)$

where $P_B(t)B_b^*(t) = K(t)$ and $D(t) = R(t) + P_B(t)\hat{B}_b(t)$.

For the purposes of the operation of the bank, $K(t)$ is superfluous; bank owners could remove it all from the bank by liquidating $B_b^*(t)$, paying themselves the proceeds, and purchasing $B_b^*(t)$ for direct inclusion in their portfolios. The rate of return accruing to the bank owners would be identical in either case. Such an action need not precipitate a removal of demand deposits as long as depositors are confident that bank owners will make up any capital loss on $\hat{B}_b(t)$. Since the continued functioning of the bank depends only on the owners' willingness to guarantee the nominal value of its liabilities and not on $K(t)$, $K(t)$ is not determinate in this model; so we will assume $K(t) = 0$ without loss of generality.

In the course of a steady deflation, the owners of equity in machines will ultimately default on their nominal obligations; all earnings from machines will accrue to consol owners. Therefore, real transfers will be made to consol holders; in particular, banks, whose assets are primarily consols, for a while will earn increasing real profits. However, from the moment of the consol default, the consols become equity whose nominal price also declines as the price level falls.

In addition to its reserves, a bank's assets after the bond default consist of a number of claims to the earnings of the real capital stock of the economy $\hat{B}_b(t)$, provided that the bank undertakes to maintain the nominal value of these claims plus reserves. If the bank finds it unprofitable to maintain these nominal values it will either go out of business or convert $\hat{B}_b(t)$ into cash and hold 100% reserves. What is the nature of the bank's assets in terms of the returns accruing to the bank owners? After bond default, a real payment of $\theta\overline{Y}/\overline{B}$ is made to each consol, so the total nominal payments accruing to the bank are $\frac{\theta P(t)\overline{Y}}{\overline{B}}\hat{B}_b(t)$. However, the banks assets suffer a capital loss of $\dot{P}_B(t)\hat{B}_b(t)$ in nominal terms which the bank owners must make good by acquiring new bonds and adding them to the bank's assets. Denoting the new bonds $\dot{\hat{B}}_b(t)$, we must require $\dot{P}_B(t)\hat{B}_b(t) = - P_B(t)\dot{\hat{B}}_b(t)$ in order to carry out the bank's committment and maintain its operation. The total nominal earnings accruing to bank owners who desire to continue their bank's lending operations is

$$\frac{\theta P(t)\overline{Y}}{\overline{B}}\;\hat{B}_b(t) + \dot{P}_b(t)\hat{B}_b(t).$$

As long as these earnings are greater than the costs involved in operating the bank's portfolio, the bank owners will maintain both the bank's position as a lender and the nominal value of the bank's assets. Letting T(t) be the bank's transaction cost per bond, we define $\varepsilon \equiv T(t)/P_B(t)$ and assume that ε is constant. Dividing both sides of the inequality by $\hat{B}_b(t)\cdot P_B(t)$, we find that

$$\frac{\theta P(t)\overline{Y}}{\overline{B}P_B(t)} + \frac{\dot{P}_B(t)}{P_B(t)} > \varepsilon$$

is the criterion for continued lending by the bank. Note that the quantity on the left side of the inequality is the nominal interest rate, so as long as the nominal interest rate is greater than ε, the bank will hold consols. In effect bank owners are holding a nominal asset because of their committments to depositors; as long as it pays a sufficient interest rate they will hold the asset. Note that the only other private nominal debt instruments are held by depositors; so if the banks cease to hold consols, there will be no private, interest-bearing nominal assets remaining in the system, a phenomenon which we expect to observe when the nominal interest rate declines sufficiently.

If at any time the nominal interest rate reaches ε, bank owners become unwilling to maintain their nominal bond assets. To preserve the nominal value of these assets bank owners would have either to levy charges on depositors or to add infusions of capital from their private holdings. Since any bank which chooses to hold 100 percent reserves can avoid charging depositors in order to subsidize these capital losses, depositors will remove their funds from any bank which imposes such charges. Since maintaining banks' nominal bond values from bankers' private assets would entail accepting less than the real interest rate on their capital, bank owners will not follow this course.

Therefore, with the passage of the nominal interest rate through ε, one of two possible events occurs. Depositors, faced with incipient capital losses, run the bank and convert their deposits into direct holdings of bonds and high-powered money, thereby destroying the banks.[3] Alternatively, bank owners run the bond market, converting their bonds into high-powered money and extinguishing demand deposit claims. In this case, banks remain open but the remaining demand deposits are backed by 100 percent reserves. In both cases demand deposits fall and the money stock collapses to the quantity of high-powered money. Also, in both cases, bank runs are triggered by the profit maximizing decisions of bank owners in a deflationary situation.[4]

II) How to Determine the Time of a Banking Collapse

In this section we will construct a simple, linear model of the goods and money markets of an economy, including a process to generate high-powered money which is sufficient to provoke a banking collapse. Our goal is to derive an explicit solution for the time of a banking collapse as a function of the parameters of the model; so at least in some instances a banking collapse can be viewed as a predictable phenomenon rather than as a sudden outburst of mass hysteria.

The Asset and Goods Markets

Our model consists of two markets, the goods market and the money market.

$$y^s = \bar{y} \tag{1}$$

$$y^d = \eta_0 - \eta_1(i-\dot{p}) + \eta_2\bar{y};\ \eta_1,\ \eta_2 > 0 \tag{2}$$

$$y^d = y^s \tag{3}$$

$$m^d - p = \gamma_0 - \gamma_1 i + \gamma_2\bar{y};\ \gamma_1;\ \gamma_2 > 0 \tag{4}$$

$$m^s = \mu + h;\quad \mu > 0 \tag{5}$$

$$m^s = m^d \tag{6}$$

Equations (1) - (3) describe the goods market. Equation (1) states that the logarithm of output supply, y^s, is the constant $\bar{y}$. Equation (2) describes the logarithm of the demand for output, y^d, which depends on $\bar{y}$ and on the real rate of interest $i-\dot{p}$. Here i is the nominal interest rate, p is the logarithm of the price level, and $\dot{p}$ is the time derivative of p. Equation (3) clears the goods market.

Equation (4) represents the demand for real balances, where m^d is the logarithm of nominal money demand. Equation (5) states that the logarithm of money supply equals the logarithm of the money multiplier, μ, plus the logarithm of high-powered money, h. μ is equal to the logarithm of $\frac{1+\Omega}{\phi+\Omega}$, where Ω is the currency/deposit ratio and ϕ is the fractional reserve/deposit ratio. Both the money multiplier and the path of high powered money will be assumed to shift at the instant of a banking collapse in a manner described later. Equation (6) clears the money market.

We use equations (1), (2) and (3) to obtain

$$i = \frac{\eta_0+(\eta_2-1)\bar{y}}{\eta_1} + \dot{p}. \tag{7}$$

Substituting from equations (7), (5), and (4) into (6) we derive[5]

$$\mu(t) + h(t) = p(t) + \beta - \alpha\dot{p}(t) \tag{8}$$

where $\beta \equiv \gamma_0 - \gamma_1[-\frac{\eta_0 + (\eta_2-1)\bar{y}}{\eta_1}] + \gamma_2\bar{y}$ and $\alpha \equiv \gamma_1$.

Equation (8) has the solution

$$p(t) = \frac{1}{\alpha} \exp(\frac{1}{\alpha} t)\int_t^{\infty}(\mu(\tau) + h(\tau) - \beta)\exp(- \frac{1}{\alpha}\tau)d\tau. \tag{9}$$

The solution in (9) excludes the possibility of banking collapses which generate themselves purely through expectations. For now we wish to consider only banking collapses which are driven by market fundamentals (Flood and Garber (1980)). For the development of a predictable banking collapse generated purely by "mass hysteria" see below.

The Timing of a Banking Collapse

According to the discussion in section I, banks collapse, perhaps not surprisingly, when they cease to be profitable and begin to make losses. In our linear model, the condition for a bank's unprofitability is

$$i(t) - \varepsilon = 0 \tag{10}$$

or, using (7),

$$\dot{p}(t) = \varepsilon - \frac{\eta_0 + (\eta_2 - 1)\bar{y}}{\eta_1} \equiv \bar{\pi}. \tag{11}$$

We would like our example to emulate closely the major banking collapse in the U.S. in 1933. To accomplish this we will first discuss some important features of that episode so that they can be incorporated in our model.

According to Friedman and Schwartz (1963), at the beginning of 1933, 17,800 commercial banks were operating in the U.S. The collapse was evident when, on March 6, President Roosevelt declared a three day bank holiday. During the initial holiday, which was declared under a 1917 wartime measure, the Emergency Banking Act was passed (March 9, 1933) allowing Roosevelt to issue a proclamation continuing the holiday. Further, under the authority of the Emergency Banking Act the President issued an executive order (March 10) empowering the Secretary of the Treasury to license Federal Reserve Banks to reopen. The reopening of licensed banks occurred from March 13 through March 15.

The reopened licensed banks faced a different environment following the holiday than they had faced entering the holiday. Fewer than 12,000 banks reopened with more than 5,000 unlicensed banks being unable to open immediately and 2,000 of those never opening. The 3,000 banks that eventually reopened were licensed gradually during the next year. The losses to depositors during the collapse, which amounted to \$2.15 per \$100 of deposits, had contributed to an increased desired currency-deposit ratio on the part of the non-bank public and, perhaps because of licensing, banks adopted a higher ratio of liquid assets to loans than they had previously. Finally, the stock of high-powered money began a rapid growth during 1933 and extending into the 1940's.

We recognize that it would have been implausible for agents in the early 1930's to have foreseen exactly all of the monetary changes of 1933-4. However, given that previous banking collapses had resulted in radical restructurings of the banking system, it would have been quite plausible for agents to predict a discontinous change in the banking environment, conditional on the advent of a banking collapse. Therefore, it is interesting to ask how our model responds to these structural alterations, given that they are forseen by agents. Ironically, it appears that such environmental changes, conditional on a banking collapse, are exactly what is required to trigger such a collapse. In the absence of such restructurings, the assumption that a banking collapse occurs produces a logical contradiction, at least in this model. See Appendix A for the development of this point.

While we focus here on three elements of the post-collapse structural changes, any one of them alone would have been sufficient to trigger the collapse. Specifically, to determine the timing of the collapse, we examine the changes in $\emptyset$ and Ω, the change in the time path of high-powered money, and the gradual re-opening of the unlicensed banks. In the main text we ignore the effects of the banking holiday itself; these we develop in Appendix B.

After the collapse both $\emptyset$ and Ω rise above their previous levels to $\emptyset^* > \emptyset$ and $\Omega^* > \Omega$. Therefore, the money multiplier falls:

$$\mu_1 \equiv \log[(1+\Omega^*)/(\Omega^*+\phi^*)] < \mu_0 \equiv \log[(1+\Omega)/(\Omega+\phi)].$$

Prior to the collapse, which takes place at time z, we assume that h(t) follows

$$h(t) = \delta_0 - \delta_1 e^{\lambda t}; \qquad \delta_1 > 0 . \tag{12}$$

After the collapse, we assume that h(t) follows

$$h(t) = w + \delta_0 - \delta_1 e^{\lambda t} + \delta_2(e^{\theta(t-z)} - 1), \ t > z, \ \ \delta_2 > 0 \tag{13}$$

where w is the logarithm of the fraction of high-powered money in banks which remain open at z, the time of collapse. The final term in (13) may reflect either new central bank injections after the collapse or the remonetization of high-powered money in the closed banks as they are gradually allowed to reopen. The high-powered money process described in (13) is basically the pre-collapse process with an additional growth term attached to it. In addition, the presence of w in (13) indicates the extent to which money is frozen in closed banks. Thus, (13) is general enough to encompass cases in which the central bank makes no response to the collapse ($\delta_2 = 0$) and in which no banks are closed ($w = 0$). Equation (14) has the property that the rate of change of h(t) in the instant after the collapse depends on the time z of the collapse. Such a property is sufficient for a determinate solution for z, as we will further explain below.

The money supply path extending through the collapse is then

$$m(t) = \mu_0 + \delta_0 - \delta_1 e^{\lambda t}, \quad t \leq z \tag{14}$$

$$m(t) = \mu_1 + w + \delta_0 - \delta_1 e^{\lambda t} + \delta_2 (e^{\theta(t-z)} - 1), \quad t > z \tag{15}$$

To solve for the time of collapse, we substitute the money path given by (14) and (15) into the price level solution (9) and employ the condition for banks' unprofitability. The solution for z should be interpreted as the time span between the initial moment of the model, i.e. when h(0) is set, and the moment of the collapse.

At any time $t \leq z$, the solution for price is

$$p(t) = -\beta + \mu_0 + \delta_0 + \frac{\delta_1}{\alpha\lambda - 1} \exp\{\lambda t\} + (w + \mu_1 - \mu_0 - \frac{\delta_2 \alpha\theta}{\alpha\theta - 1}) \exp\{\tfrac{1}{\alpha}(t-z)\} \tag{16}$$

Equation (16) results from straightforward, though tedious, integration. Finite solutions require that $\lambda < \frac{1}{\alpha}$ and $\theta < \frac{1}{\alpha}$.

The time derivative of (16) evaluated at t = z is

$$\dot{p}(z) = \bar{\pi} = \frac{w + \mu_1 - \mu_0}{\alpha} - \frac{\delta_2 \theta}{\alpha\theta - 1} + \frac{\delta_1 \lambda}{\alpha\lambda - 1} e^{\lambda z}. \tag{17}$$

Equation (17) has the solution

$$z = \frac{\ln\Sigma}{\lambda} \tag{18}$$

where $\Sigma \equiv \dfrac{[\bar{\pi} - \frac{w+\mu_1-\mu_0}{\alpha} + \frac{\delta_2\theta}{\alpha\theta-1}](\alpha\lambda-1)}{\delta_1\lambda}$.

In order for the model to be interesting, we require $\Sigma > 1$. Since $(\alpha\lambda-1) < 0$ and $\delta_1\lambda > 0$, this requirement implies that $[\bar{\pi} - \frac{w+\mu_1-\mu_0}{\alpha} + \frac{\delta_2\theta}{\alpha\theta-1}] < 0$. Since $\bar{\pi} < 0$ and $\alpha\theta-1 < 0$, the positive term $\frac{-(w+\mu_1-\mu_0)}{\alpha}$ cannot be sufficiently large to offset the two negative terms in brackets. In addition, given the other parameters, λ cannot be "too large." Violation of these requirements would imply a deflation which is sufficient to collapse the system prior to the beginning of the analysis.

Given the above restrictions we can determine the effects of parameter changes on the timing of the collapse:

$$\frac{\partial z}{\partial \lambda} < 0 \tag{19a}$$

$$\frac{\partial z}{\partial \theta} > 0 \tag{19b}$$

$$\frac{\partial z}{\partial \bar{\pi}} < 0 \tag{19c}$$

$$\frac{\partial z}{\partial(\mu_0-\mu_1)} < 0 \tag{19d}$$

$$\frac{\partial z}{\partial \delta_2} > 0 \tag{19e}$$

$$\frac{\partial z}{\partial \delta_1} < 0 \tag{19f}$$

All of these results satisfy intuition. A rise in the rate of reduction of h(t), i.e., a rise of λ or δ_1, hastens the time of collapse. An increase in the floor value of the deflation rate $\bar{\pi}$ also hastens the moment of the collapse. An increase in the percentage discontinuity of the money stock $(\mu_0-\mu_1)$ causes the collapse to occur earlier through its effect on the deflation rate. Finally, an increase in the post-collapse high-powered money growth increases the time before the collapse.

A Problem in Solving for the Time of Collapse

In setting up the example above, we chose a special post-collapse high-powered money supply process which allowed us to solve explicitly for the time of the collapse. In this subsection we will expose the particular property of our example's h(t) process which makes the determination of z a solvable problem.

To focus our discussion let us assume that the demand for real balances assumes the simple form:

$$m(t) - p(t) = -\alpha\dot{p}(t), \quad \alpha > 0. \tag{20}$$

Again assuming that the collapse occurs at z, we know that in the instant just prior to z the rate of deflation equals $\bar{\pi}$. Then equation (20) can be written as

$$m(z^-) - p(z^-) = -\alpha\bar{\pi} \tag{21}$$

where z^- indicates the instant before z. As soon as the collapse occurs, there is a discontinuity in the nominal money stock and a change in the high-powered money creation process. Supposing that after the collapse high-powered money (and therefore the price level) grows independently of z at a constant rate θ, the equilibrium condition in the money market is

$$m(z^+) - p(z^+) = -\alpha\theta \tag{22}$$

where z^+ represents the instant after z. Since prices are continuous, $p(z^+) = p(z^-)$; so subtracting (22) from (21) we find

$$\Delta m(z) \equiv m(z^-) - m(z^+) = -\alpha(\bar{\pi}-\theta). \tag{23}$$

If neither $\Delta m(z)$, the percentage collapse of the money stock, nor θ, the constant rate of h(t) growth, depends on z, then there is not enough information to determine z. Since in our previous example we assume that $(\mu_0-\mu_1)$ is independent of z, we must assume that the rate of inflation immediately after the collapse depends on z. The money supply process in (13) is sufficient to satisfy this requirement. However, any post-collapse h(t) process which has a constant percentage growth rate must be excluded; for in this case knowledge that $p(z^-) = p(z^+)$ and that $\dot{p}(z) = \bar{\pi}$ is insufficient to determine z. We have not yet found an explanation for this difficulty; however, in a more general model in which output and the real interest rate are allowed to vary, $\bar{\pi}$ will depend on z.

A Collapse Generated by "Mass Hysteria"

We have developed above an example in which a banking collapse is driven by market fundamentals. In this section we will provide an example in which a collapse is generated purely by a belief that it will occur, i.e. a collapse caused by "mass hysteria." Alternatively stated, a collapse can be caused by the occurrence of a bubble of the type that always may arise in dynamic rational expectations models. In our example "mass hysteria" will not create a sudden, unanticipated collapse; rather, the collapse will be predictable.

We will construct this example around the simple money demand model of equation (20). We will assume that the high-powered money supply process $h(t)$ is continuous and exogenous, but otherwise arbitrary; in particular, it may even be growing at rapid rates. Before a collapse, behavior is such that the logarithm of the money multiplier is μ_0; afterwards, it is μ_1. Now at time t all agents believe that at $z > t$ the banking system will collapse. The problem is to determine how agents' behavior will cause current and time z prices to be set so that the belief will realize itself through the $\dot{p}(z^-) = \bar{\pi}$ criterion. It happens that these prices are set by the materialization of a bubble term in the p(z) solution. This bubble, reflecting agents' certainty of a time z collapse, causes the collapse at z in a consistent manner.

The general solution for price in equation (20) is

$$p(t) = \frac{1}{\alpha} \exp(\frac{1}{\alpha}t)\int_t^{\infty}(\mu + h(\tau))\exp(-\frac{1}{\alpha}\tau)d\tau + A \exp(\frac{1}{\alpha}t) \tag{24}$$

where A is an arbitrary constant and μ can be replaced by μ_0 or μ_1 depending on whether $t < z$ or $t > z$, respectively. If a collapse occurs at z, then from equation (23) we know that

$$\mu_0 - \mu_1 = -\alpha(\bar{\pi} - \dot{p}(z^+)). \tag{25}$$

However, from equation (20),

$$\dot{p}(z^+) = \frac{1}{\alpha} p(z) - \frac{1}{\alpha}(\mu_1 + h(z)). \tag{26}$$

Setting t = z in (24), substituting the result for p(z) in equation (26), and substituting the resulting $p(z^+)$ in equation (25), we can solve for the value of A for which (25) is satisfied at time z:

$$A = [\mu_0 + \alpha\bar{\pi} + h(z)]\exp(-\frac{1}{\alpha}z) - \frac{1}{\alpha}\int_z^\infty(\mu_1 + h(t))\exp(-\frac{1}{\alpha}\tau)d\tau. \tag{27}$$

Thus, agents' beliefs that a collapse will occur at z will, in general, imply that a price level bubble will exist after z. Depending on the nature of the h(t) process for $t \geq z$, A may be either positive or negative.

We have shown how a belief in a collapse at z can cause equation (23) to be satisfied through the appearance of a post-collapse bubble. Next we must demonstrate that the criterion for a collapse at z, $\dot{p}(z^-) = \bar{\pi}$, is also satisfied merely by this belief; i.e. we must show that the belief in the event generates the event. Again, from (20),

$$\dot{p}(z^-) = \frac{1}{\alpha}p(z) - \frac{1}{\alpha}(\mu_0 + h(z)). \tag{28}$$

From (24) and (27) we know what p(z) must be to satisfy the market clearing conditions just before and just after the collapse, i.e. to satisfy equation (25). Substituting this p(z) into (28), we find that $\dot{p}(z^-) = \bar{\pi}$. The bubble term of the magnitude A given in equation (27) then causes the criterion for a collapse at z to be satisfied. Finally, at any time t prior to the collapse, the solution for the price level is

$$p(t) = \frac{1}{\alpha}\exp(\frac{1}{\alpha}t)\int_t^z(\mu_0 + h(\tau))\exp(-\frac{1}{\alpha}\tau)d\tau + p(z)\exp(\frac{1}{\alpha}(t-z)). \tag{29}$$

In summary, the current price level is uniquely determined by the condition that $\dot{p}(z^-) = \bar{\pi}$. This latter condition is determined by the appearance of the post-collapse bubble which causes the market clearing conditions to be satisfied just before and just after the collapse. Real balances change discontinuously at the time of the collapse because of a postulated behavior shift (or regime switch) which occurs because banks can no longer profitably lend.

III) Relating the Model to the Great Contraction

We now wish to compare the assumptions and implications of our model to some observable phenomena associated with the Great Contraction. Since Friedman and Schwartz (1963) have written a massive volume describing these phenomena, we will not dwell on the details. Rather we will provide references to relevant sections or charts in F&S. In this section, we will compile a list of the points of coincidence and points of divergence of our model from the observable data for the depression. In some cases, we will discuss how the model may be extended to encompass phenomena which either are at variance with its implication or are not implied by the model. In other cases, we will discuss how the data, or at least interpretations of the data, may be altered so that they may encompass the model. Almost all of the cited evidence and charts are taken directly from Friedman and Schwartz.

1) Both a stock market crash and a steady decline in equity prices until the final banking panic occurred in the depression, as F&S's Chart 29 indicates. While our model contains explicitly an equities market only after the bond default, a more general model which determines simultaneously equity prices, bond prices, and the price level can readily be formulated,

though analytical solutions for z are more difficult to derive. Thus, an equities price can be constructed; and this price should fall continuously with the deflation and transfer of real wealth to the bondholders. The crash can be built into the model only if we relax the perfect foresight assumption and assume that the deflationary high-powered money policy is suddenly and unexpectedly imposed, which would certainly cause the stock market to crash. On this change of assumptions, see footnote 1.

2) The depression is a period during which there were defaults and capital losses on assets held by banks, as implied by our model. However, our model indicates that for a while before the banking panic banks earn increased real profits.

3) The nominal interest rate fell to low levels during the contraction as indicated by F&S's Charts 29 and 35. The returns on short-term U.S. securities fell continuously even after the bank collapse. The yields on Baa bonds rose to their highest levels in 1932, but these yields are conditional. Since many of the bonds were defaulted, the expected yield was probably lower.

4) From F&S's Charts 16 and 28, it appears that deflation was accelerating until the final banking collapse, though this may be in the eye of the beholder.

5) There was a switch from negative to positive rates of inflation at the end of the crisis, as indicated in F&S's Chart 37. This was associated with the simultaneous switch to positive money stock and high-powered money stock growth rates immediately after the collapse. This is indicated in Charts 34, 37, and 38.

6) There was a low overall capital loss to depositors after the collapse, so that incipient losses seem to have been enough to collapse the system.

See F&S's discussion on capital losses from pp. 437-441 and especially their use of the example of the Bank of the United States, pp. 308-313.

7) There was an increased regulation of banks immediately after the final collapse. Controls on the kinds of assets banks could hold were imposed immediately (see F&S, p. 443, note 21). The power of the Federal Reserve to alter reserve requirements was immediately enacted, though not immediately employed (F&S, p. 447).

8) There was a sudden, large-scale destruction of money at the time of the final banking panic, as indicated in Chart 16, 31 and 34. There was no comparable discontinuity in the price level at the same moment, see charts 16 and 37.

9) Real income did not remain fixed; rather, it dropped precipitously during the depression. See F&S, charts 16 and 28.

10) High-powered money was static before 1930; during 1930, it declined slightly; and after 1930, it increased. See F&S, charts 23 and 32. This removes the forcing function from our model and requires that we seek some other factor which drove the deflation.

The story which we will tell concerns the holding of U.S. currency by foreigners. If currency substitution were a strong force during this period and if foreigners were increasingly demanding U.S. currency, then equation (12) would represent foreign demand by the term $\delta_1 e^{\lambda t}$, which is subtracted from the static high-powered money supply, δ_0. Equation (12) can then be interpreted as the U.S. economy's supply of high-powered dollars.

This story avoids the problem of a static, measured high-powered money stock, but how plausible is it? Reference to F&S's Chart 32 indicates that foreign demand provides at least a possible forcing function for the deflation.

While total high-powered money is probably measured accurately, there is no way to determine the whereabouts of its components other than of reserves and vault cash. High-powered money was essentially constant from 1925-1931; of the $7 billion in high-powered money, $4-1/2 billion consisted of Federal Reserve notes, Treasury currency, and gold coin and certificates. Foreigners were certainly holding some of this cash abroad, but the magnitudes cannot be determined.

However, during the hyperinflationary period of the early 1920's, agents in the hyperinflating countries directly held foreign currencies on a large scale, the dollar being a principal among them. Some estimates of the extent of this foreign currency holding were made. Bresciani-Turroni (1937), (p. 345), mentions that 1.5-4 billion goldmarks ($357-$952 million) in foreign currencies and exchange were held in Germany in the autumn of 1923. Graham (1930) reports (p. 73) an estimate of 2-3 billion goldmarks ($476-$714 million) in foreign currencies in Germany. Walre' des Bordes (1924) cites (p. 192) a League of Nations estimate that there were 1 billion Swiss francs ($189 million) worth of foreign currency in Austria in May, 1921. If half of these currencies were dollars, then at the outside some $500 million dollars of U.S. currency were held in Germany and Austria. In addition there were simultaneously hyperinflations in Hungary, Poland and Russia; presumably foreign currencies were also held there. However, the currencies held in Germany and Austria alone may have comprised as much as 1/13 of the $6.5 billion in U.S. high-powered money and 1/9 of the cash in circulation in 1923.

This discussion indicates that there is some loose evidence that foreigners had demanded large quantities of U.S. high-powered money as recently as 6 years before the onset of the depression. Possibly, the turbulent state of European currencies in the late 1920's caused them to demand dollars on a large scale once again.

That data do not exist to refute or support this conjecture may make it more or less believable. However, it does provide a monetary impetus which, when combined with the fixed supply policy of the Federal Reserve, is sufficient to set off a banking collapse.

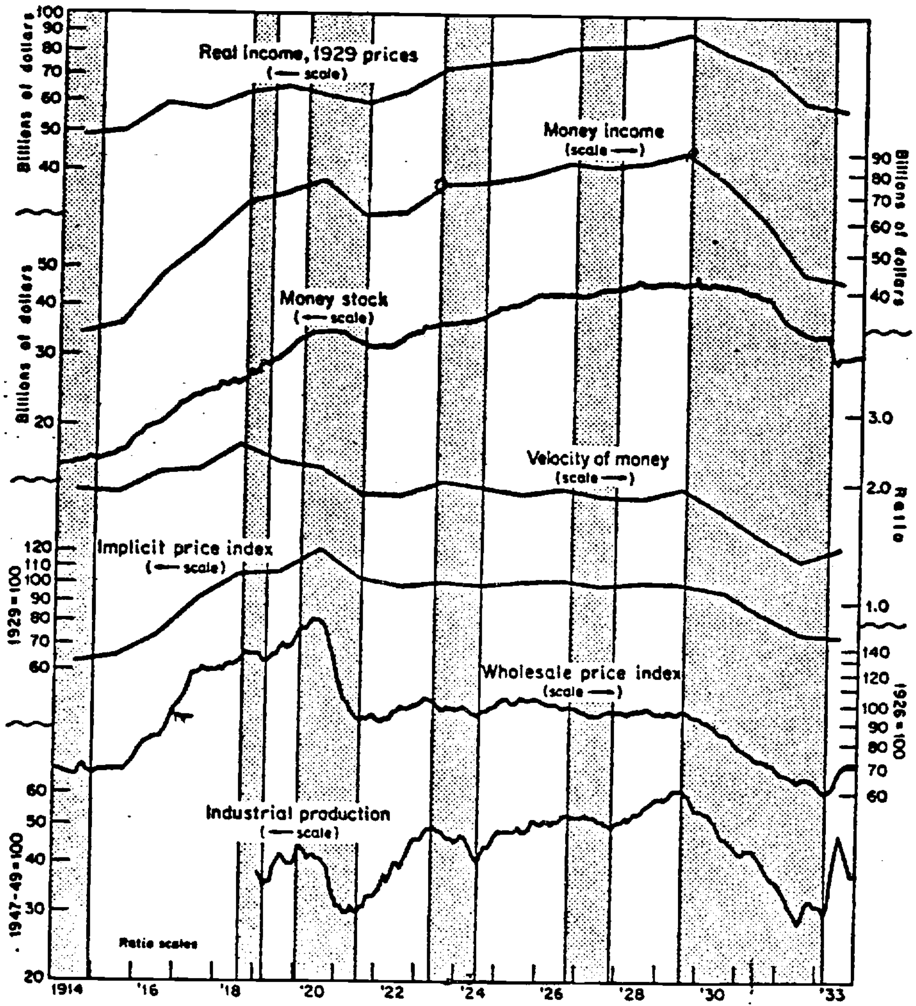
11) While the deposit-currency ratio and the deposit-reserve ratio both fell after each banking crisis, the changes were not obviously sudden. Rather these ratios both fell steadily both before and after each crisis. See F&S's Chart 31. After the final collapse, the deposit-reserve ratio continued to fall, but the deposit-currency ratio started to rise. See F&S, Chart 38.

12) Banks paid interest on deposits and, at the end of the 1930's made charges on them. See F&S, pp. 443-445, p.504. It is difficult to determine the extent of the interest payments from F&S's discussion, which seems to be concerned with inter-bank deposits. In any case interest payments to deposits can be worked into our model fairly easily.

13) There was not one banking crisis; rather there was a series of domestic banking panics culminating in the Banking Holiday of 1933. See for example F&S's Chart 31 for the time intervals between these crises. In addition, Summer, 1931 brought the central European banking collapses, followed by runs on the Reichsmark and the British pound which forced Germany and Britain from the gold standard.

If we assume that the deflationary high-powered money policy remains in effect until the final collapse, our model can encompass such cascading bank collapses. Each collapse in the sequence requires a discontinuous change in the money stock; such discontinuities will arise if, conditional on a collapse, the money multiplier declines discontinuously at the moment of the collapse. For instance, if, after each collapse in the sequence, agents' behavior causes the deposit-reserve ratio and the deposit-currency ratio to jump to even lower levels, then the economic system will surely generate the sequence of collapses. The post-banking crises movements of the deposit-currency and deposit-reserve ratios depicted in F&S's Chart 31 are consistent with this explanation.

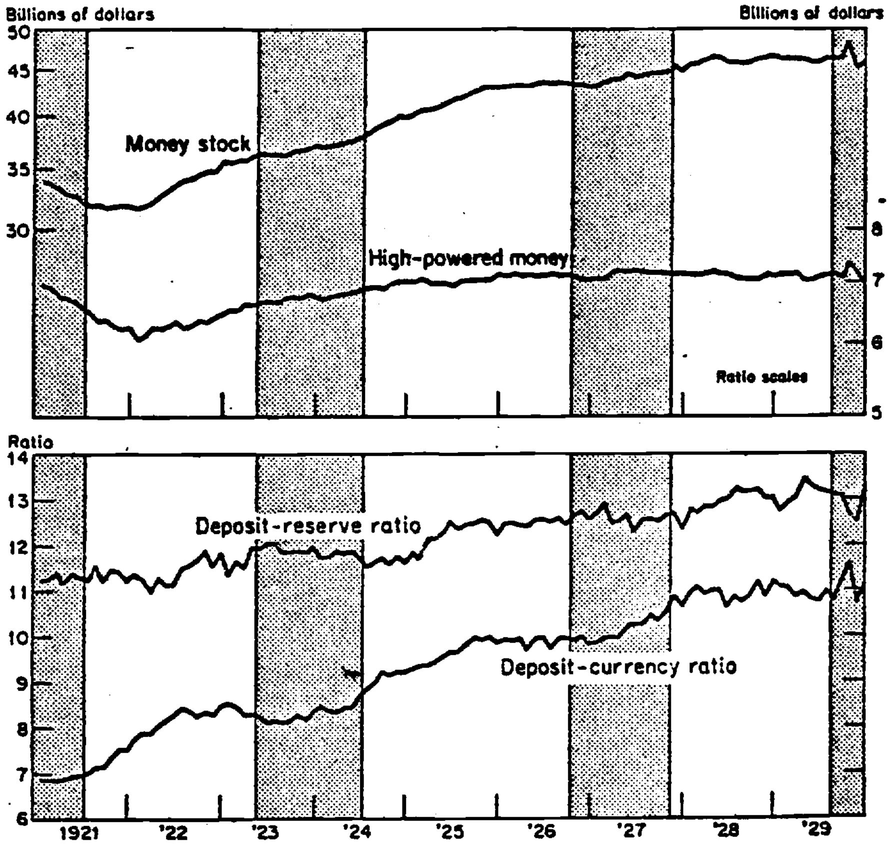
CHART 16
Money Stock, Income, Prices, and Velocity, in Reference Cycle Expansions and Contractions, 1914–33



NOTE: Shaded areas represent business contractions; unshaded areas, business expansions.
SOURCE: Industrial production, seasonally adjusted, from *Industrial Production, 1959 Revision,* Board of Governors of the Federal Reserve System, 1960, p. S-151 (manufacturing and mining production only). Other data, same as for Chart 62.

" The net national product figures shown for 1917–19 modify Kuznets' esti-

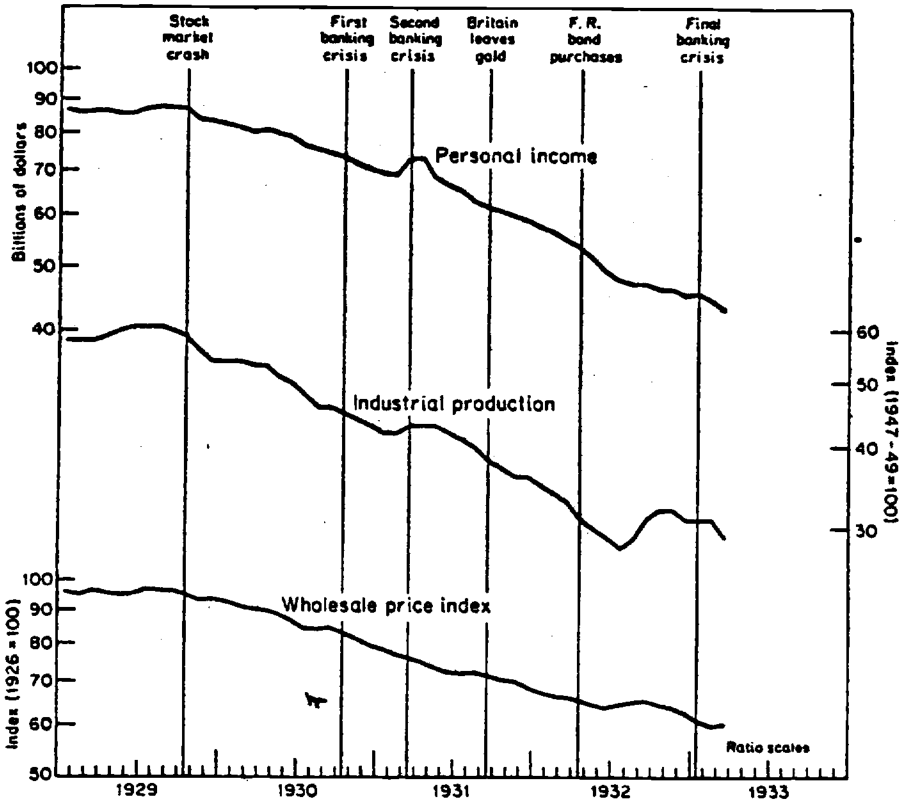
CHART 23
The Stock of Money and Its Proximate Determinants, 1921–29



NOTE: Shaded areas represent business contractions; unshaded areas, business expansions.
SOURCE: Tables A-1 (col. 8) and B-3.

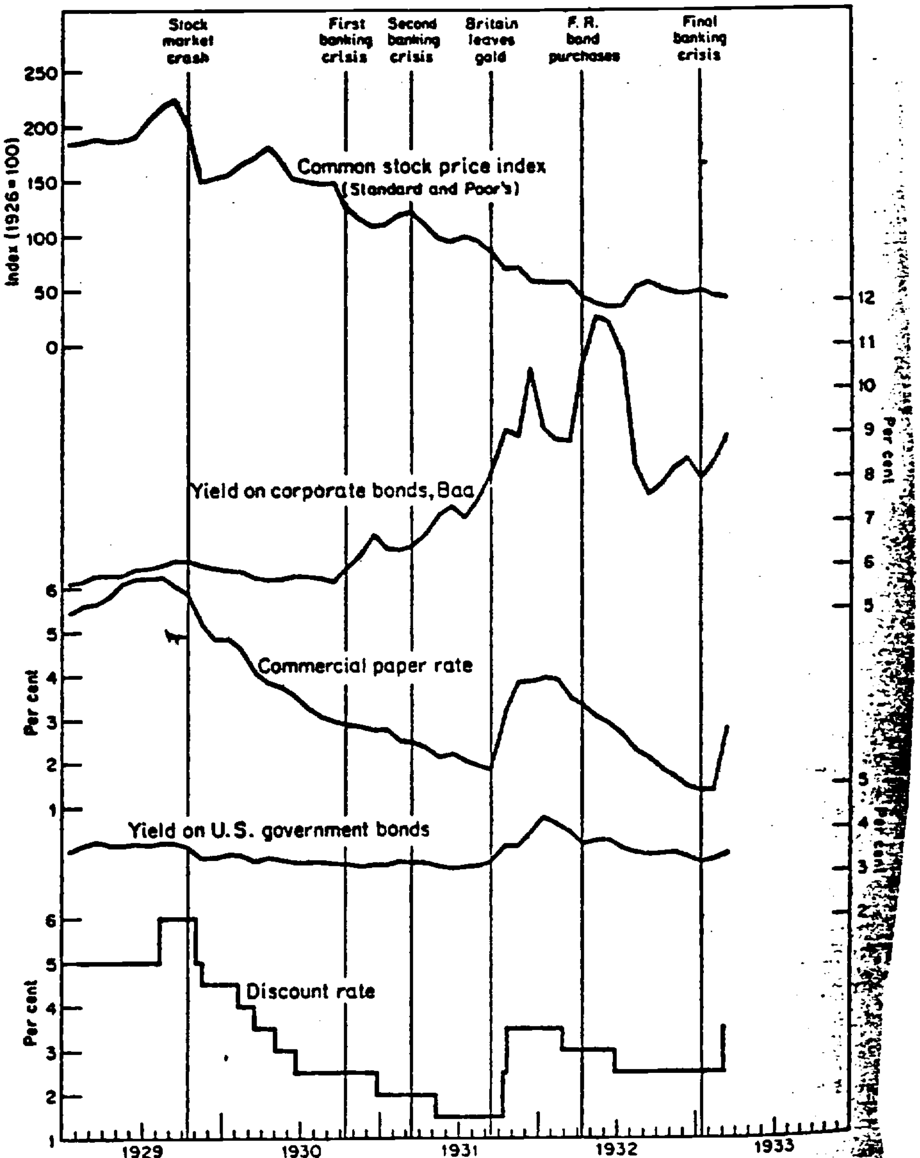
CHART 28

Prices, Personal Income, and Industrial Production, Monthly, 1929–March 1933



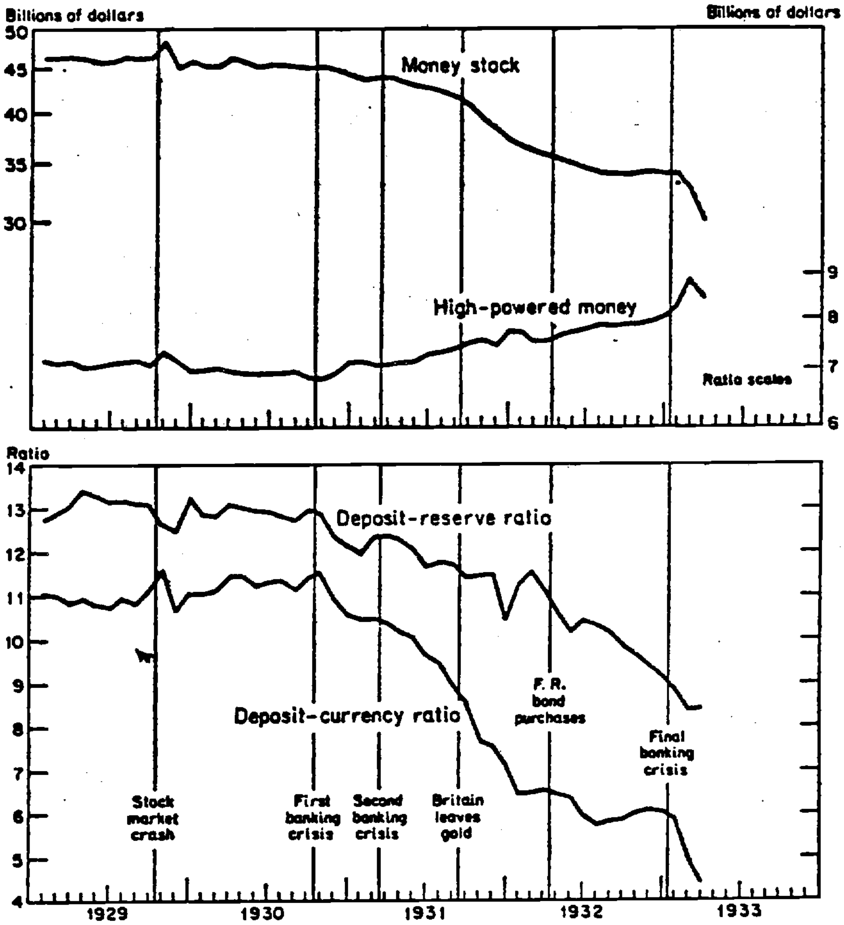
SOURCE: Industrial production, same as for Chart 16. Wholesale price index, same as for Chart 62. Personal income, *Business Cycle Indicators* (Princeton for NBER, G. H. Moore, ed., 1961), Vol. II, p. 139.

CHART 29
Common Stock Prices, Interest Yields, and Discount Rates of Federal Reserve Bank of New York, Monthly, 1929–March 1933



SOURCE: Common stock price index, Standard and Poor's, as published in *Common-Stock Indexes, 1871–1937* (Cowles Commission for Research in Economics, Bloomington, Ind., Principia Press, 1938), p. 67. Discount rates, *Banking and Monetary Statistics*, p. 441. Other data, same as for Chart 35.

CHART 31
The Stock of Money and Its Proximate Determinants, Monthly, 1929–March 1933

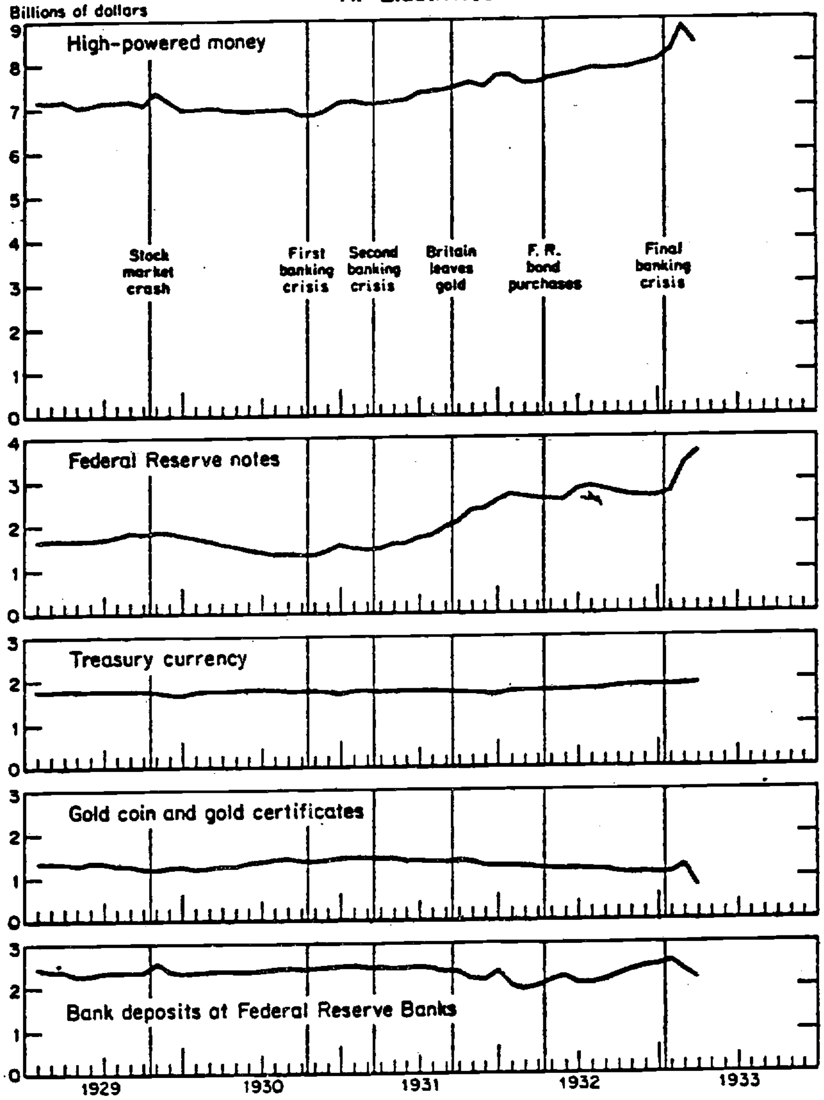


SOURCE: Tables A-1 (col. 8) and B-3.

CHART 32

High-Powered Money, by Assets and Liabilities of the Treasury and Federal Reserve Banks, Monthly, 1929–March 1933

A. Liabilities



NOTE: Federal Reserve notes, Treasury currency, and gold coin and certificates are outside the Treasury and Federal Reserve Banks.

SOURCE: Same as for Chart 19.

CHART 32 (Concluded)

B. Assets

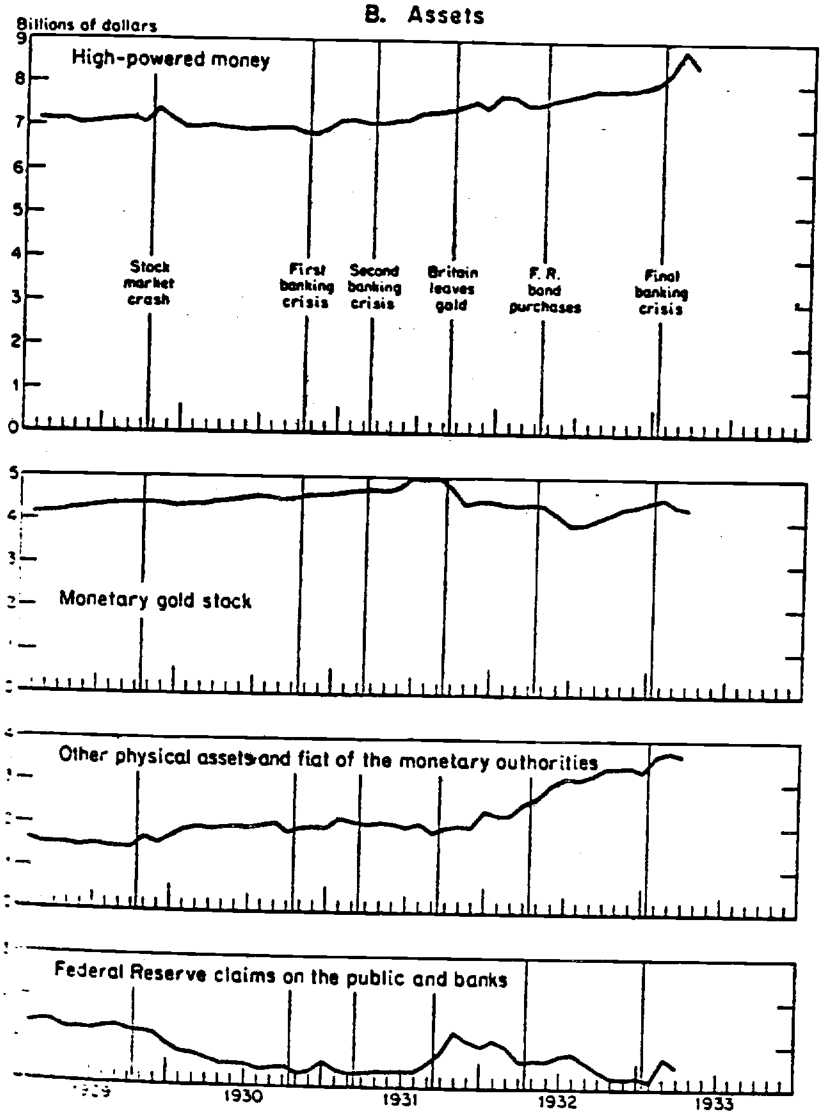
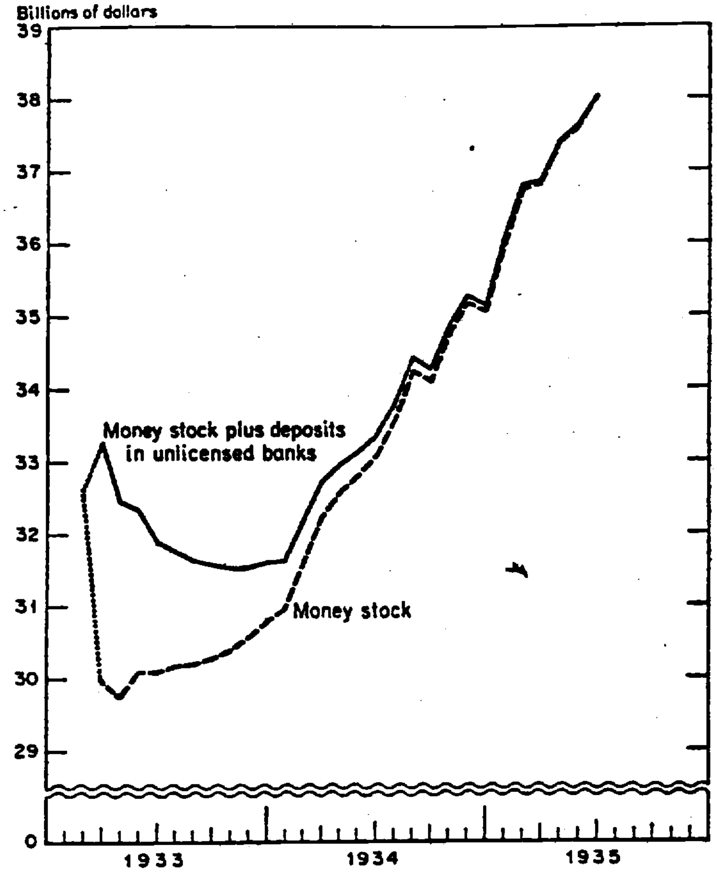
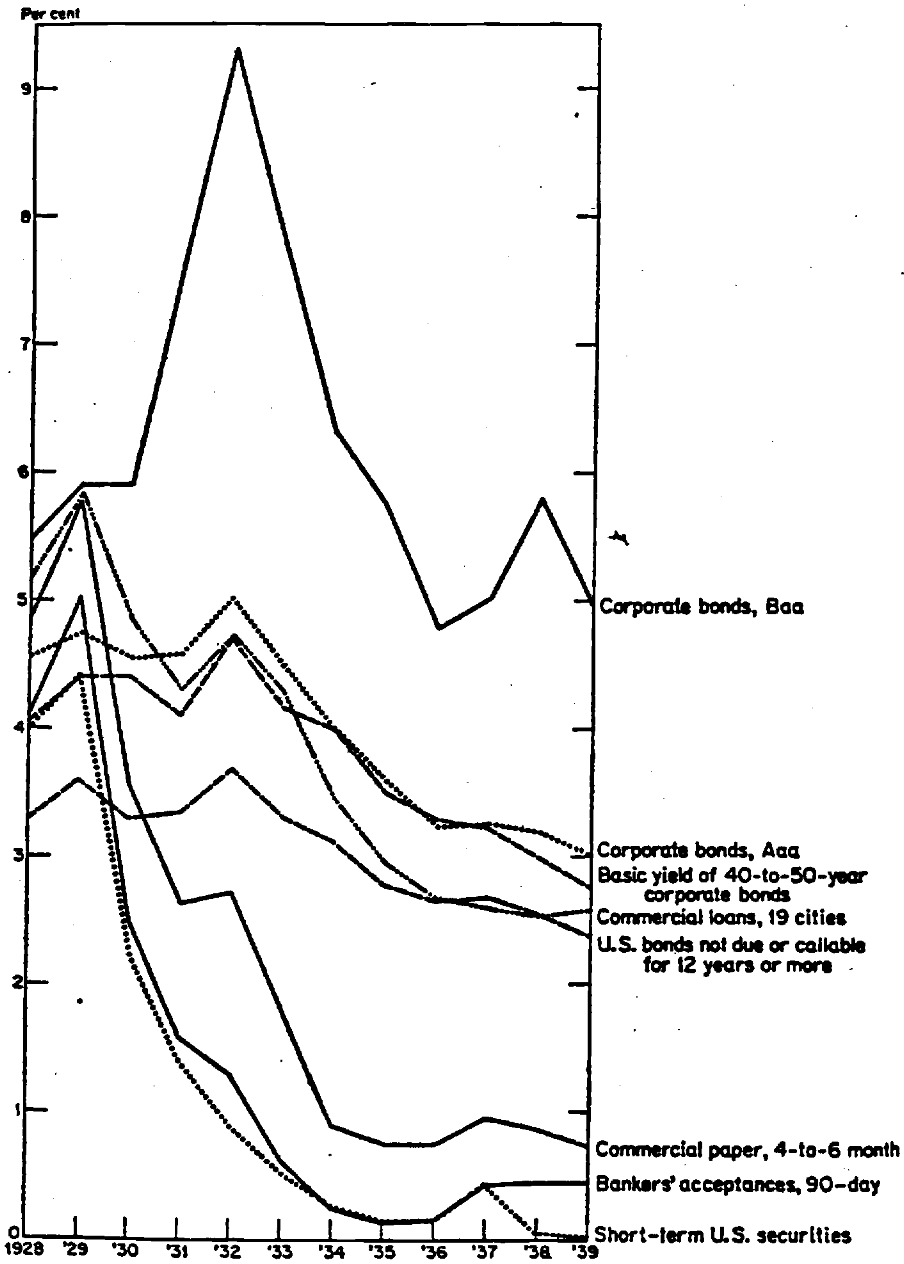


CHART 34
Alternative Money Stock Estimates, February 1933–June 1935



SOURCE: Table 15, cols. 6 and 7, and Table A-1, col. 8.

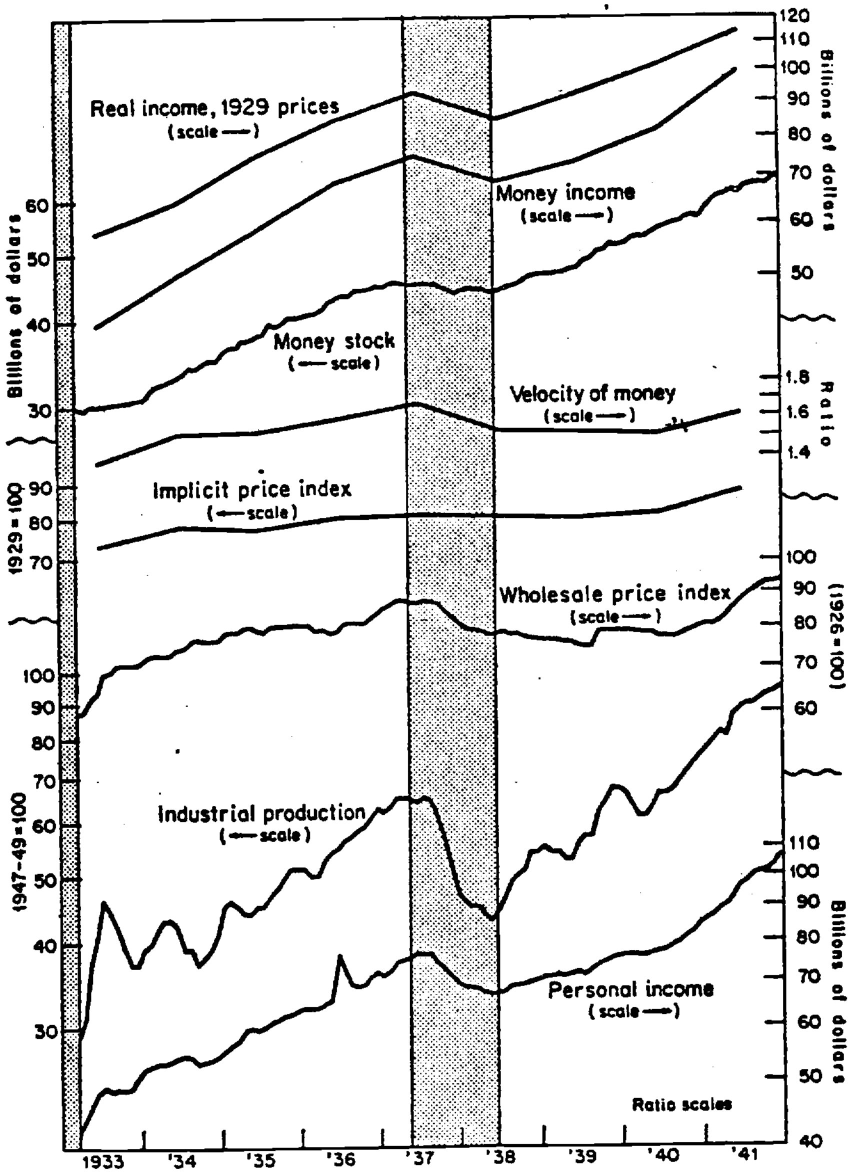
CHART 35
Changing Relations Among Interest Yields, 1928–39



SOURCE: Baa and Aaa corporate bonds, commercial loans, bankers' acceptances, *Banking and Monetary Statistics*, pp. 448, 464, 468. Commercial loan rate, annual averages; for 1939 based on Jan.-Feb. only; monthly data unavailable thereafter on a comparable basis. Basic yield of 40- to 50-year corporate bonds, *Historical Statistics*, 1960, p. 657, Series X-347. U.S. government bonds, *Federal Reserve Bulletin*, Dec. 1938, p. 1045; Feb. 1940, p. 139. Commercial paper, *Historical Statistics*, 1949, p. 346, averaged annually. Short-term U.S. government securities, *FRB*, May 1945, p. 483: 3- to 6-month certificates and notes, 1928–30; 3- to 9-month Treasury bills, 1931–42.

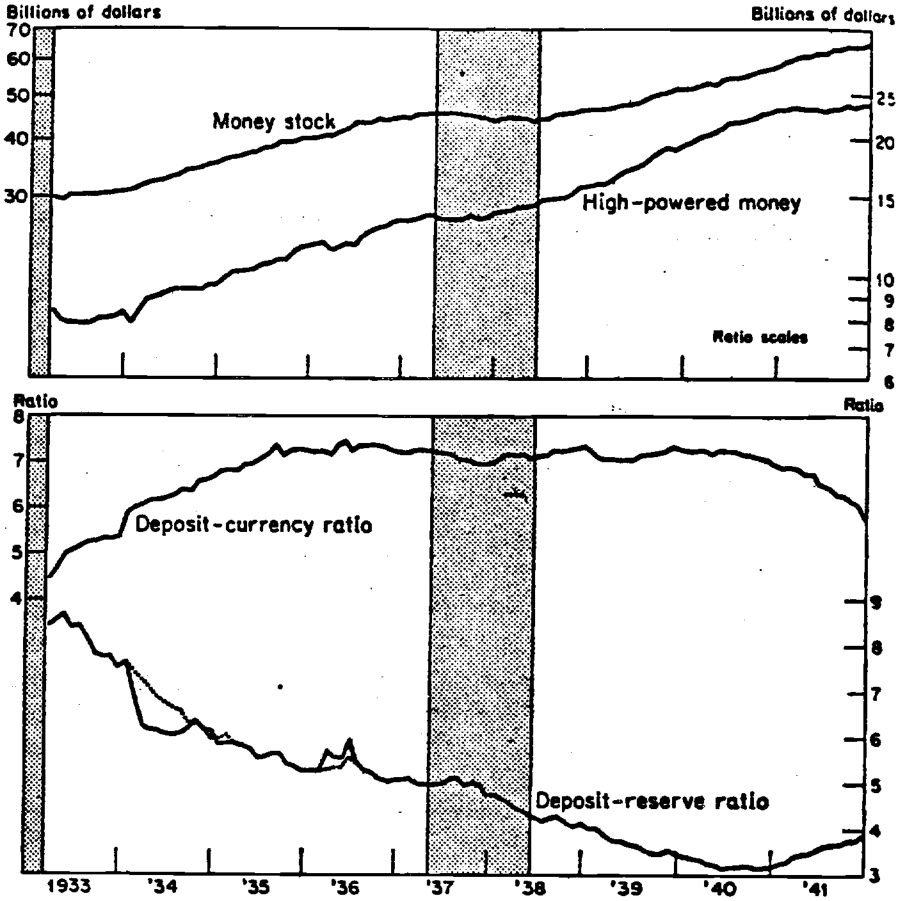
CHART 37

Money Stock, Income, Prices, and Velocity, Personal Income and Industrial Production, in Reference Cycle Expansions and Contractions, March 1933–December 1941



NOTE: Shaded areas represent business contractions; unshaded areas, business expansions.
SOURCE: Industrial production, same as for Chart 16. Personal income, same as for Chart 28. Other data, same as for Chart 62.

CHART 38
The Stock of Money and Its Proximate Determinants,
March 1933–December 1941



NOTE: Shaded areas represent business contractions; unshaded areas, business expansions.
SOURCE: Tables A-1 (col. 8) and B-3. Dotted section of deposit-reserve ratio smoothes deposits and reserves (see Chart 44 and the accompanying text).

IV) Conclusion

One of our goals in this paper is to show that banking collapses can be predictable phenomena, produced by an economy's dynamic workings when faced by a deflationary policy. We have demonstrated that most of the important events associated with banking collapses can be produced by an economy devoid of random elements and that the collapse can be predicted, perhaps far in advance.

In addition, we have attempted to breathe some empirical life into our simple model by comparing it to the events of the depression. Either our model or simple extensions of it can capture many of these events. The only aspect of our model which diverges from the depression in an important way is the deflationary forcing function; high-powered money was static rather than declining. It is difficult either to verify or to refute our suggestion that foreign demand for U.S. currency may have been sufficiently strong to produce a deflationary forcing function; the data seem to be unavailable. However, it may well be worth the investment to try to determine who was holding U.S. currency during this period.

NOTES

1 The monetary authority may have maintained this deflationary policy since the beginning of time, or it may have suddenly imposed the policy. In the latter case prices can fall discontinously to the extent that the new policy was unanticipated; specifically, there may be an immediate collapse of the equities market price. However, there may be a terminological problem in referring to this situation as one of "perfect foresight". One may think of the policy switch as a probability zero event, which, once having occurred persists into the indefinite future with certainty. Alternatively, one may explicitly assume a stochastic model and state asset demands in terms of expected values of future prices. Since in a model with random future regime switching this assumption greatly increases the analytical difficulty with no increase in insights, we avoid the complication here. See Flood and Garber (1981) for an explicit analysis of such a problem.

2 If there were free entry into the banking industry, the banks should earn no more than the market real rate of return on their original capital; otherwise, more firms would enter the industry. Since bonds earn the real rate of return in the model, this would force reserves to equal the amount of demand deposits or require interest payments on demand deposits.

3 Of course, the bank could maintain its operation even if it were insolvent, as long as the public were willing to accept its checks at face value. In order to trigger the run at the moment of incipient insolvency, all that is needed is an agency which will close the bank and force liquidation if its assets decline far enough below its liabilities. Such a closure would cause a discontinuous decline in the value of deposits, thereby forcing a run to occur at the time of incipient insolvency.

4 However, for a collapse to occur in a logically consistent manner, the post-collapse banking environment must be different from the pre-collapse environment. The change is imposed in our model in the form of a different money multiplier or of a different monetary base. If a collapse occurs, agents desire higher ratios of currency to deposits, bankers require higher ratios of reserves to deposits, and the central bank destroys some monetary base.

In Appendix A, we demonstrate how a logical inconsistency arises when it is assumed that a banking collapse occurs, based on our criterion, in the absence of such a post-collapse environmental change. To understand the problem, the reader should first read section II. Ironically, the threat of a more restrictive banking environment in the presence of a collapse, together with the deflationary high-powered money policy, is sufficient to generate the collapse in a logically consistent manner.

5 Equation (2) serves little purpose other than to fix the real interest rate at some constant level. If the reader prefers a little movement in his real rate, equation (2) can be written instead in the form:

$$\bar{y} = \eta_0 - \eta_1(i-\dot{p}) + \eta_2\bar{y} + \eta_3(m-p) \tag{2'}$$

Substituting for (m-p) from (4) in equation (2') and solving for the nominal interest rate, we derive

$$i = \frac{\eta_0 + \eta_3\gamma_0 + \bar{y}(\eta_2 - 1 + \eta_3\gamma_2)}{\eta_1 + \gamma_1\eta_3} + \left(\frac{\eta_1}{\eta_1 + \gamma_1\eta_3}\right)\dot{p}. \tag{*}$$

Subtracting $\dot{p}$ from both sides of (*), we find that the real rate of interest is a linear function of the inflation rate. However, the remaining steps in the solution for the time of the banking collapse are the same as in the body of the paper.

Appendix A

In this appendix we will demonstrate a logical inconsistency which arises if we assume that a banking collapse, based on the criterion of our model, occurs with no post-collapse change in the banking environment. Thus, there will be no post-collapse shift in the high-powered money process or in the currency-deposit ratio. The reserve-deposit ratio may take any value desired by bankers above the legal minimum.

We will refer to Figures I, II, and III to make our argument. In Figure I we plot the path of the logarithm of the price level against time; in Figure II, we plot the nominal interest rate against time; and in Figure III we plot the logarithm of the money stock against time.

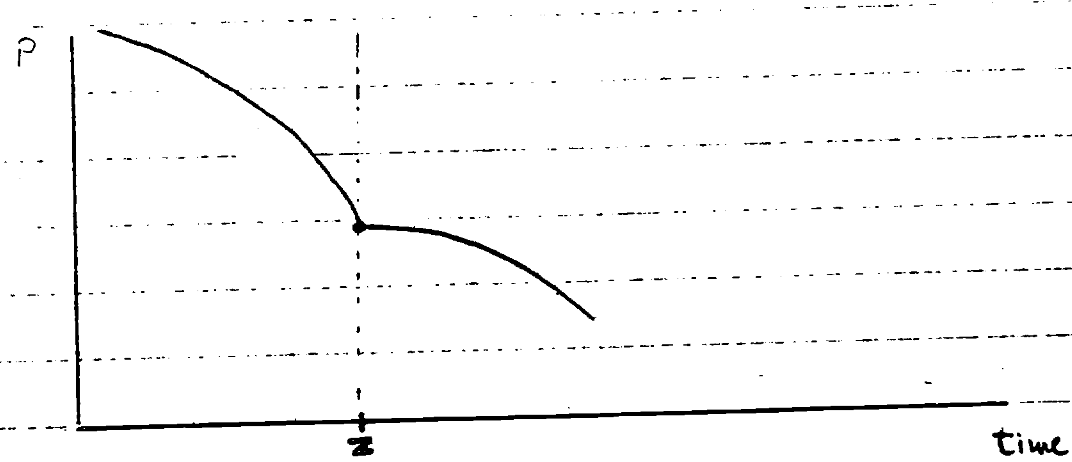


Figure I

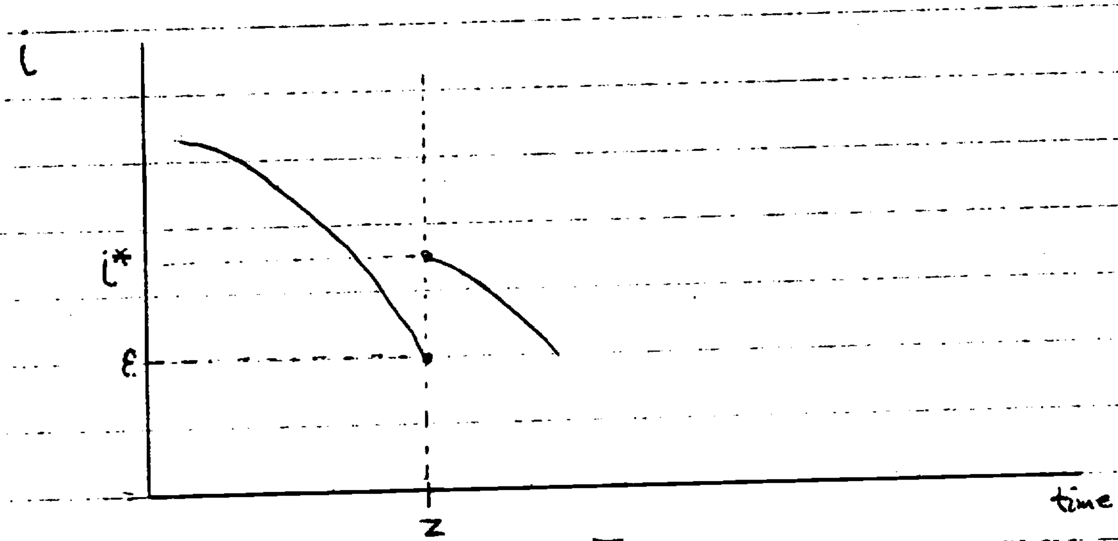


Figure II

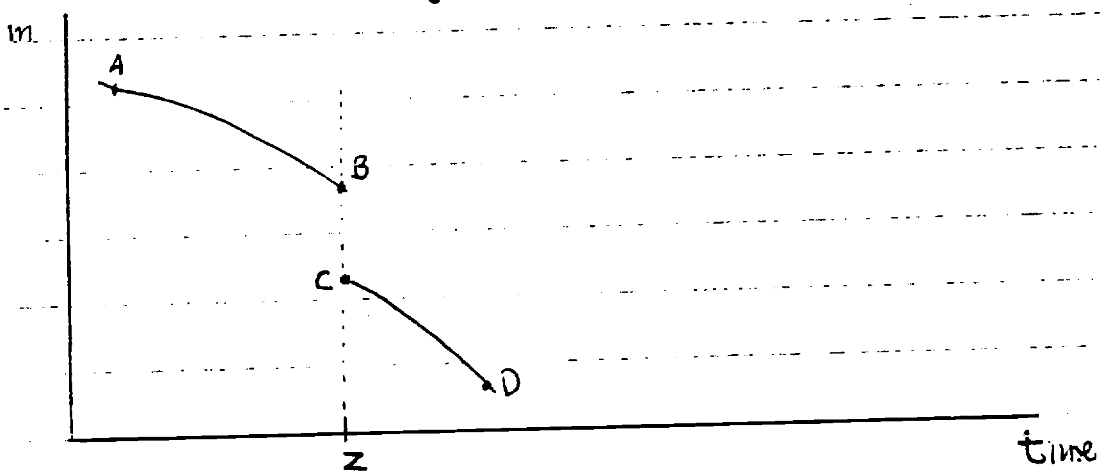


Figure III

Let us assume that the rate of high-powered money destruction is such that a banking collapse occurs at time z, i.e. the banks find it unprofitable to continue their operation as lenders and switch to a 100% reserve ratio. A collapse entails a discontinuous destruction of the money stock at the moment of the collapse, so that m would follow a path like ABCD in Figure III. Since there is a discontinuous reduction of money at z, the path of p will decline at a much more rapid rate prior to z than after z; i.e. there will be a kink in the price path at z. See Sargent and Wallace (1973) or Boyer and Hodrick (1980) for the development of this result.

Our criterion for a collapse at z requires that the nominal interest rate begins to pass through ε at time z. Since i is the constant real rate of interest plus $\dot{p}$ the left-hand time derivative of the p curve at time z equals ε minus the real interest rate.

At the instant of the collapse $\dot{p}$ jumps discontinuously upward, i.e. it becomes less negative. Therefore, the nominal interest rate jumps discontinuously above ε to i^*. But with the higher interest rate, the banks suddenly find it profitable to make loans again. Since required reserves, the currency-deposit ratio, and the monetary base are unchanged just after the collapse, banks will expand their loans and the total money stock exactly to the levels which prevailed just prior to the collapse. But then there can be no discontinuity in the path of m, no kink in the p path, and no discontinuous shift in i. Therefore, there can be no collapse at z.

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