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ABSTRACT

A famous dictum in open economy macroeconomics -- which obtains in the Mundell-Fleming world of sticky prices and perfect capital mobility -- holds that the choice of the optimal exchange rate regime should depend on the type of shock hitting the economy. If shocks are predominantly real, a flexible exchange rate is optimal, whereas if shocks are mainly monetary, a fixed exchange rate is optimal. There is no obvious reason, however, why this paradigm should be the most appropriate one to think about this important issue. Arguably, asset market frictions may be as pervasive as goods market frictions (particularly in developing countries). In this light, we show that in a model with flexible prices and asset market frictions, the Mundell-Fleming dictum is turned on its head: flexible rates are optimal in the presence of monetary shocks, whereas fixed rates are optimal in response to real shocks. We thus conclude that the choice of an optimal exchange rate regime should depend not only on the type of shock (real versus monetary) but also on the type of friction (goods versus asset market).

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1 Introduction

One of the most influential results in open economy macroeconomics – which derives from the Mundell-Fleming model – holds that the choice of the exchange rate regime should depend on the type of shock hitting the economy. If shocks are predominantly of real origin, then flexible exchange rates are optimal. Instead, if shocks are mainly monetary, fixed (or, more generally, predetermined) exchange rates are optimal. In fact, as Calvo (1999, p. 4) aptly puts it, this is "a result that every well-trained economist carries on [his/her] tongue". Calvo (1999) himself offers a simple derivation of this result in a model in which the policymaker's objective is to minimize output variability. The intuition is simple enough: in the Mundell-Fleming world of sticky prices and perfect capital mobility, real shocks require an adjustment in relative prices which, in the presence of sticky prices, can most easily be effected through changes in the nominal exchange rate. In contrast, monetary shocks require an adjustment in real money balances that can be most easily carried out through changes in nominal money balances (which happens endogenously under fixed exchange rates). By and large, this key result has remained unscathed in modern variations of the Mundell-Fleming model. For instance, Cespedes, Chang, and Velasco (2004) incorporate liability dollarization and balance sheets effects and conclude that the standard prescription in favor of flexible exchange rates in response to real shocks is not essentially affected.

But rather than tweaking at the margin with variations of the traditional Mundell-Fleming model, it could be argued that one should take issue with its most critical assumption: imperfection in goods markets (i.e., sticky prices) but undistorted capital markets (i.e., perfect capital mobility). Is this the world we necessarily live in? Far from it. In developing countries, in particular, asset market frictions appear to be equally, if not more important, than goods market frictions. In fact, a large segment of the population does not seem to have access to asset markets.[1] In this light, it seems worth revisiting the Mundell-Fleming question in a model with flexible prices but *segmented*

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[1] Mulligan and Sala-i-Martin (2000) report that even for the United States, 59 percent of the population (as of 1989) did not hold interest bearing assets. One would conjecture that this figure is even higher for developing countries.

asset markets. This type of model posits that while a fraction of the population (referred to as traders) has access to asset markets, the rest of the population (non-traders) does not. In a first paper (Lahiri, Singh, and Végh (2006a)), we examine this issue in the context of a stochastic model in which traders have access to incomplete markets. In contrast, this paper develops a much starker, perfect-foresight version of the model which, by avoiding a myriad of technical complications, allows the essential mechanisms and intuition to shine through. The paper's punchline is that – contrary to the Mundell-Fleming prescription mentioned above – if shocks are real, fixed exchange rates are optimal whereas if shocks are monetary, flexible exchange rates are optimal.

Intuitively, flexible exchange rates allow for a costless adjustment to monetary shocks by altering the real value of existing nominal money balances. In contrast, under fixed rates, asset-market segmentation prevents non-traders from rebalancing real money balances by accessing asset markets, which affects the consumption path. Under real shocks, fixed rates allow purchasing power to be transferred across periods, which results in some consumption smoothing. Under flexible rates, on the other hand, non-traders are forced to consume their current endowment.

We thus conclude that the optimal exchange rate regime should depend not only on the *type of shock* (real versus monetary) – as rightly emphasized by Mundell-Fleming models – but also on the *type of distortion* (goods markets versus asset markets frictions).[2] These ideas can be succinctly summarized in the following 2x2 matrix:

Table 1. Optimal exchange rate regime		
	Goods market friction	Asset market friction
Real shock	Flexible	Fixed
Monetary shock	Fixed	Flexible

The optimal exchange rate regime thus becomes an empirical issue that depends both on the type of shock hitting a particular economy and on the relative distortions present in goods and asset markets.

[2]It is worth noting that our results are in the spirit of an older literature that focused on the pros and cons of alternative exchange rate regimes in models with no capital mobility (see, for instance, Fischer (1977) and Lipschitz (1978)). See also Ching and Devereux (2003) for a related analysis in the context of optimal currency areas.

The paper proceeds as follows. Section 2 develops the main model – a perfect-foresight version of Lahiri, Singh, and Végh (2006a) – and solves it for the cases of both flexible and fixed exchange rates. Section 3 compares the two regimes for fluctuating output and velocity paths. Section 4 contains some brief concluding remarks. Some technical issues are relegated to appendices.

2 The model

Consider a discrete-time model of a small open economy perfectly integrated with the rest of the world in goods markets. There are two types of agents: traders (who have access to capital markets) and non-traders (who do *not* have access to capital markets). The fraction of traders is λ while that of non-traders is $1-\lambda$. There is no uncertainty in the model and agents are blessed with perfect foresight. The law of one price holds for the only good; hence, $P_t = E_t P_t^*$. Foreign inflation is assumed to be zero and, for simplicity, P_t^* is taken to be unity. Hence, $P_t = E_t$.

Both traders and non-traders are subject to a cash-in-advance constraint. For the case of traders, we follow Lucas' (1982) timing and assume that asset markets open first (say, in the morning) followed by goods markets (in the afternoon). By assumption, of course, non-traders do not have access to asset markets and hence only visit goods markets.[3]

There are two types of shocks: real and monetary. Both traders and non-traders face identical shocks. Real shocks are captured by fluctuations in the endowment of the only good, y. Following Alvarez, Lucas and Weber (2001), we capture monetary – or velocity – shocks by allowing both traders and non-traders to access a fraction v of current period sales ($v_t P_t y_t$) and letting v_t fluctuate over time.

To fix ideas, it proves useful to keep in mind the following scenario regarding the model's timing conventions. Households consist of two individuals: a shopper and a seller. As is standard, households do not consume their own endowment. As goods markets are about to open in the afternoon, the seller and the shopper part and, in the standard model, would not see each other until the end of the day. In other words, the seller stays in the store

[3] Asset market segmentation could be endogenized by assuming that there is a fixed cost of accessing asset markets. With idiosyncratic fluctuations in endowment, the number of agents that choose to gain access to asset markets would be endogenously determined.

selling the endowment to other households' shoppers and the shopper visits other stores to purchase goods. In the standard model, then, the shopper does *not* return to the store until after goods markets close and therefore has no access to the money balances accrued to the seller from the sale of the current-period endowment ($P_t y_t$). In the current model, we depart from the standard model by allowing the shopper to come back to the store once during the goods market session, empty the cash register, and go back to shopping. We assume that the amount of money in the cash register at the time the shopper comes back to the store is $v_t P_t y_t$, where $0 < v < 1$.

Finally, both traders (T) and non-traders (NT) have identical preferences, given by

$$U^i = \sum_{t=0}^{\infty} \beta^t u(c_t^i), \;\; i = T, NT. \tag{1}$$

where c^i denotes consumption of an agent of type i.

2.1 Non-traders

Non-traders do not have access to asset markets and, hence, hold only money. Their flow budget constraint is given by

$$M_{t+1}^{NT} = M_t^{NT} + E_t y_t - E_t c_t^{NT}, \tag{2}$$

where M_t^{NT} denotes end-of-period $t-1$ (and hence beginning of period t) nominal money balances in the hands of non-traders. The initial level of nominal money balances, M_0^{NT}, is given. Non-traders are subject to a cash-in-advance constraint of the form:

$$M_t^{NT} + v_t E_t y_t \geq E_t c_t^{NT}. \tag{3}$$

The nominal money balances that non-traders can use to purchase goods consist of the nominal money balances that they bring into period t, M_t^{NT}, and a fraction v_t of current-period sales (recall that, by assumption, $0 < v_t < 1$).

We will only consider equilibrium paths along which the cash-in-advance binds.[4] If the cash-in-advance constraint binds, then we can solve for c_t^{NT} from equation (3) to obtain:

[4] Appendix 5.1 derives sufficient conditions for the cash-in-advance constraint to bind. Contrary to what our intuition would first tell us – that the cash-in-advance constraint would rarely bind because non-traders would like to save some money for low endowment

$$c_t^{NT} = \frac{M_t^{NT} + v_t E_t y_t}{E_t}, \qquad t \geq 0. \tag{4}$$

To find out how much money balances non-traders will carry on to the next period, substitute (4) into (2) to obtain:

$$M_{t+1}^{NT} = (1 - v_t) E_t y_t. \tag{5}$$

When the cash-in-advance binds, the non-traders' problem becomes completely mechanical. In other words, their opportunity set consists of only one point in every period – given by (4) – and there is thus no need to carry out any maximization. Intuitively, non-traders begin their life with a given level of nominal money balances, M_0. They augment these cash balances with a fraction v_0 of period 0 sales, $v_0 E_0 y_0$. Since the cash-in-advance binds, they spend all of their money balances, $M_0^{NT} + v_0 E_0 y_0$, on consumption in period 0. Their end-of-period cash balances consist of the cash proceeds from selling their endowment, $E_0 y_0$, minus the amount of period 0 sales spent in period 0, $v_0 E_0 y_0$. They thus enter period 1 with $M_1 (= (1 - v_0) E_0 y_0)$ and the process begins anew.

2.2 Traders

Traders have access to asset markets and thus behave like consumers in any standard model with perfect capital mobility. The only difference is that, like non-traders, they have access to a fraction v_t of current-period sales.

Let us first look at the flow constraint for the asset market. Traders enter the asset market with a certain amount of nominal money balances, M_t^T, and a certain amount of bonds, b_t. Once in the asset market, they receive/pay interest on the bonds they carried into the asset markets, $E_t r b_t$, receive transfers from the government, T, and buy/sell bonds in exchange for money.[5] Traders exit the asset market with a quantity $\hat{M}_t$ of nominal money

periods – the cash-in-advance may bind under very weak conditions because unspent money balances have an opportunity cost that is positively related to the state of the economy (i.e., the opportunity cost is higher in good times). In good times, therefore, non-traders would like to save for consumption smoothing motives but dissave for financial reasons.

[5] Given the open economy nature of the model, the private sector as whole must always be able to exchange money for foreign bonds (and viceversa) in the asset market (even under flexible rates), and bonds for goods (and viceversa) in the goods market. One can

balances and b_{t+1} of bonds. The flow constraint for the asset market is thus:

$$E_t b_{t+1} + \hat{M}_t^T = M_t^T + E_t(1+r)b_t + \frac{T_t}{\lambda}. \tag{6}$$

Traders are subject to a cash-in-advance constraint:

$$\hat{M}_t^T + v_t E_t y_t \geq E_t c_t^T. \tag{7}$$

What will traders' nominal money balances be at the end of period t? Traders will have the money brought from the asset market plus the proceeds from the sale of their endowment ($E_t y_t$) minus the money balances used to purchase goods ($E_t c_t$):

$$M_{t+1}^T = \hat{M}_t^T + E_t y_t - E_t c_t^T. \tag{8}$$

By substituting (8) into (6), we obtain the traders' flow constraint for period t as a whole:

$$E_t b_{t+1} + M_{t+1}^T = M_t^T + E_t(1+r)b_t + E_t y_t + \frac{T_t}{\lambda} - E_t c_t^T. \tag{9}$$

2.2.1 Utility maximization

For the purposes of the maximization – and by substituting (6) into (7) – we can rewrite the cash-in-advance constraint as:

$$M_t^T + E_t(1+r)b_t + \frac{T_t}{\lambda} - E_t b_{t+1} + v_t E_t y_t \geq E_t c_t^T. \tag{10}$$

Traders thus maximize lifetime utility subject to the flow budget constraint (9) and the cash-in-advance constraint (10), for given values of M_0^T and b_0. The Lagrangian is then given by:

imagine a trading agency that is in charge of such activities or, alternatively, that the household has a third member, a foreign trader, whose job is to put aside some of the household's money or bonds during the asset market and transact with foreigners during the goods market.

$$
\begin{aligned}
\underset{\{c^T, M^T_{t+1}, b_{t+1}, \eta_t, \Psi_t\}}{\text{Max}} \quad & \sum_{t=0}^{\infty} \beta^t u(c^T) \\
& + \sum_{t=0}^{\infty} \beta^t \eta_t \left[M_t^T + E_t(1+r)b_t + \frac{T_t}{\lambda} + E_t y_t - E_t c_t^T - E_t b_{t+1} - M_{t+1}^T \right] \\
& + \sum_{t=0}^{\infty} \beta^t \Psi_t \left[M_t^T + E_t(1+r)b_t + \frac{T_t}{\lambda} - E_t b_{t+1} + v_t E_t y_t - E_t c_t^T \right].
\end{aligned}
$$

The first-order conditions with respect to c^T, M_{t+1}^T, and b_{t+1} are given, respectively, by (assuming, as usual, that $\beta(1+r) = 1$):

$$
\begin{aligned}
u'(c_t^T) &= E_t(\eta_t + \Psi_t), && (11) \\
\eta_t &= \beta(\eta_{t+1} + \Psi_{t+1}), && (12) \\
E_{t+1}(\eta_{t+1} + \Psi_{t+1}) &= E_t(\eta_t + \Psi_t). && (13)
\end{aligned}
$$

The first-order condition with respect to η_t naturally recovers the flow constraint (9). Finally, the Kuhn-Tucker condition for Ψ_t recovers (10) and requires the complementary slackness condition:

$$
\left[M_t^T + E_t(1+r)b_t + \frac{T_t}{\lambda} - E_t b_{t+1} - E_t c_t \right] \Psi_t = 0. \tag{14}
$$

Combining first-order conditions (11) and (13) yields:

$$
u'(c_t^T) = u'(c_{t+1}^T).
$$

As in standard cash-in-advance models with Lucas' (1982) timing, traders will fully smooth consumption over time.

Combining first-order conditions (12) and (13) yields (using $\beta = \frac{1}{1+r}$):

$$
\eta_t \left[(1+r) \frac{E_{t+1}}{E_t} - 1 \right] = \Psi_t. \tag{15}
$$

Perfect capital mobility (for traders) implies that the interest parity condition holds:

$$
1 + i_t = (1+r) \frac{E_{t+1}}{E_t}, \tag{16}
$$

which enables us to rewrite condition (15) as

$$\eta_t i_t = \Psi_t. \tag{17}$$

Since, at an optimum, $\eta_t > 0$, equation (17) says that if $i_t > 0$, then $\Psi_t > 0$ which implies, from the complementary slackness condition (14) that the cash-in-advance constraint binds. Since we will only consider equilibria in which the nominal interest rate is positive, the cash-in-advance constraint will always bind and traders' end of period money balances can be obtained by combining (7) and (8):[6]

$$M_{t+1}^T = (1 - v_t) E_t y_t. \tag{18}$$

2.3 Government

The government's flow constraint is given by

$$E_t h_{t+1} = (1 + r) E_t h_t + M_{t+1} - M_t - T_t, \tag{19}$$

where h_t denotes net foreign bonds held by the government.

2.4 Equilibrium conditions

Money market equilibrium implies that:

$$M_t = \lambda M_t^T + (1 - \lambda) M_t^{NT}. \tag{20}$$

Equations (5) and (18) imply that $M_{t+1}^{NT} = M_{t+1}^T$. Together with the money market equilibrium condition (20), this implies that

$$M_t = M_t^{NT} = M_t^T.$$

Since there are no differences across agents in terms of the endowment, all agents hold the same amount of money (on a per-capita basis). Hence, (5) and (18) together with the money market equilibrium condition (20) yield a quantity theory equation:

$$M_{t+1} = (1 - v_t) E_t y_t, \qquad t \geq 0. \tag{21}$$

[6] Appendix 5.1 derives the restrictions needed to ensure a positive nominal interest rate.

To make it directly comparable to the quantity theory equation found in textbooks (typically written as $MV = Py$, where V denotes velocity), we can rewrite this last equation as

$$\frac{M_{t+1}}{1-v_t} = E_t y_t, \qquad t \geq 0.$$

Velocity is thus given by $1/(1-v_t)$. Hence, a higher v captures an increase in the velocity of circulation, which rationalizes our terminology of "velocity shocks" when referring to changes in v.

To obtain the economy's flow constraint, multiply the non-traders' flow constraint (equation (2)) by $1-\lambda$ and the traders' flow constraint (equation (9)) by λ and then add them up, taking into account the government's flow constraint (19)) and the money market equilibrium condition (20):

$$k_{t+1} - k_t = rk_t + y_t - [\lambda c_t^T + (1-\lambda)c_t^{NT}], \tag{22}$$

where $k \equiv h_t + \lambda b_t$ denotes the economy's per-capita net foreign assets.

Iterating forward and imposing the transversality condition $\lim_{t\to\infty} \frac{k_{t+1}}{(1+r)^t} = 0$, we obtain the resource constraint:

$$(1+r)k_0 + \sum_{t=0}^{\infty} \beta^t y_t = \sum_{t=0}^{\infty} \beta^t [\lambda c_t^T + (1-\lambda)c_t^{NT}]. \tag{23}$$

In what follows, we will assume that $k_0 = 0$.[7]

2.5 Equilibrium consumption

We will now derive expressions for consumption of both traders and non-traders. To obtain non-traders' consumption, substitute the quantity theory equation (21) into (4) to obtain (recall that $M_t = M_t^{NT}$):

$$c_t^{NT} = \begin{cases} \frac{M_0}{E_0} + v_0 y_0, & t = 0 \\ \frac{(1-v_{t-1})E_{t-1}y_{t-1} + v_t E_t y_t}{E_t}, & t \geq 1. \end{cases} \tag{24}$$

This expression will prove useful when dealing with fixed exchange rates. When dealing with flexible exchange rates, however, it will prove convenient

[7] This assumption just ensures that the present discounted value of income is identical across traders and non-traders when the money supply or the exchange rate is fixed.

to use (21) to rewrite (24) as

$$c_t^{NT} = \; y_t - \tfrac{M_{t+1}-M_t}{E_t}, \quad t \geq 0. \tag{25}$$

To obtain traders' consumption, substitute (24) into (23) and solve for the constant level of c^T, denoted by $\overline{c^T}$, to obtain:

$$\overline{c^T} = y^p + \frac{1-\lambda}{\lambda}\left[y^p - \frac{r}{1+r}\left(\frac{M_0}{E_0} + v_0 y_0 + \sum_{t=1}^{\infty}\beta^t \frac{(1-v_{t-1})E_{t-1}y_{t-1} + v_t E_t y_t}{E_t}\right)\right], \tag{26}$$

where

$$y^p \equiv (1-\beta)\sum_{t=0}^{\infty}\beta^t y_t$$

denotes permanent income. Alternatively, substitute (25) into (23) and iterate to obtain:

$$\overline{c^T} = y^p + \frac{r}{1+r}\frac{1-\lambda}{\lambda}\sum_{t=0}^{\infty}\beta^t\left(\frac{M_{t+1}-M_t}{E_t}\right). \tag{27}$$

Equations (25) and (27) make clear the redistributive role that monetary policy plays in this model. If, say, money supply is constant, then non-traders consume their endowment ($c_t^{NT} = y_t$) and traders their permanent income ($\overline{c^T} = y^p$). An increase in the money supply (i.e., $M_{t+1} > M_t$) implies a transfer from non-traders to traders. The reverse is true in the case of a reduction in the money supply.

2.6 Flexible exchange rates

Consider a flexible exchange rate regime in which the monetary authority sets a constant path of the nominal money supply:[8]

$$M_t = \bar{M}, \qquad t \geq 0. \tag{28}$$

[8] We will consider only the extreme cases of a constant money supply (under flexible rates) and a fixed exchange rate (as opposed to time-varying paths of the exchange rate). For an extension of our main results to more general rules involving a fixed rate of growth of either the money supply or the exchange rate, see Lahiri, Singh, and Végh (2006b).

Substituting (28) into (25), we obtain non-traders' consumption:

$$c_t^{NT} = y_t, \qquad t \geq 0. \tag{29}$$

Two observations are worth making. First, consumption of non-traders will fluctuate one-to-one with fluctuations in the endowment. Flexible exchange rates provide no insulation whatsoever for non-traders from output fluctuations. Second, consumption of non-traders is not affected by velocity shocks.

Substituting (28) into (27), we obtain traders' consumption:

$$\overline{c^T} = y^p. \tag{30}$$

Let us now derive the path of the nominal exchange rate. From the quantity theory equation (21), we obtain:

$$E_t = \frac{\bar{M}}{(1 - v_t)y_t}, \qquad t \geq 0. \tag{31}$$

It follows that

$$\frac{E_{t+1}}{E_t} = \frac{(1 - v_t)}{(1 - v_{t+1})} \frac{y_t}{y_{t+1}}. \tag{32}$$

When output increases (i.e., $y_{t+1} > y_t$) – and for constant velocity – the nominal exchange rate will fall (i.e., the domestic currency appreciates). Intuitively, higher output increases real money demand and hence leads to a fall in the price level (i.e., in the nominal exchange rate). On the other hand, when there is an increase in velocity (i.e., $v_{t+1} > v_t$) – and for constant output – the nominal exchange rate will increase (i.e., the domestic currency depreciates). Intuitively, an increase in velocity implies that more money is available to purchase the same level of output, which will lead to a higher price level (i.e., a higher nominal exchange rate).

Finally, the path of the nominal interest rate follows from combining the interest parity condition (16) with (32):

$$1 + i_t = (1 + r)\frac{(1 - v_t)}{(1 - v_{t+1})} \frac{y_t}{y_{t+1}}. \tag{33}$$

2.7 Fixed exchange rates

Consider now a fixed exchange rate regime in which the monetary authority sets a constant value of the nominal exchange rate:

$$E_t = \bar{E}.$$

To ensure that initial conditions under fixed rates are consistent with those under flexible rates (in the sense that they generate the same initial level of real money balances as in the case of flexible exchange rates), we take initial nominal money holdings to be $M_0^N = M_0^T = M_0 = \bar{M}$. Further, we assume that the exchange rate is fixed at the level given by $\bar{E} = \bar{M}/(1 - v_0)\, y_0$. Under these assumptions, initial real money balances, $M_0/\bar{E}$ are given by $(1 - v_0)\, y_0$, as is the case under flexible rates (recall (31)).

Under a fixed exchange rate, we can use (24) to obtain consumption of non-traders:

$$c_t^{NT} = \begin{cases} \frac{M_0}{\bar{E}} + v_0 y_0, & t = 0, \\ (1 - v_{t-1}) y_{t-1} + v_t y_t, & t \geq 1. \end{cases} \tag{34}$$

Since $M_0/\bar{E} = (1 - v_0) y_0$, it follows that $c_0^{NT} = y_0$.

By the same token, using (26), consumption of traders is given by:

$$\overline{c^T} = y^p + \frac{1-\lambda}{\lambda} \left\{ y^p - \frac{r}{1+r} \left(y_0 + \sum_{t=1}^{\infty} \beta^t \left[(1 - v_{t-1}) y_{t-1} + v_t y_t \right] \right) \right\}. \tag{35}$$

Let us now derive the path of the nominal money supply, which is endogenous under fixed exchange rates. $M_0 = \bar{M}$, as remarked earlier. The path of M_t, for $t \geq 1$ then follows from the quantity theory equation (21):

$$M_{t+1} = (1 - v_t) \bar{E} y_t, \qquad t \geq 0.$$

3 Comparing flexible versus fixed exchange rates

We are now ready to ask our main question: which exchange regime is better?

3.1 Velocity shocks only

Suppose that there are only velocity shocks (i.e., set $y_t = y^p$). Then, under flexible rates, consumption of non-traders is completely flat and equal to y^p (as follows from equation (29)). Further, as equation (30) indicates, traders' consumption is also equal to permanent income. Clearly, this equilibrium corresponds to the first-best. Both traders and non-traders perfectly smooth consumption over time.

Under fixed rates, it follows from equation (34) that consumption of non-traders is given by

$$c_t^{NT} = \begin{cases} y^p, & t = 0, \\ y^p(1 + v_t - v_{t-1}), & t \geq 1. \end{cases} \tag{36}$$

In turn, consumption of traders is given by (from (21) and (27))

$$\overline{c^T} = y^p \left[1 - \frac{1-\lambda}{\lambda} \sum_{t=0}^{\infty} \beta^t (v_t - v_{t-1}) \right]. \tag{37}$$

In what follows, it will prove useful to define a "permanent" velocity shock, v^p, as

$$v^p \equiv (1-\beta) \sum_{t=0}^{\infty} \beta^t v_t.$$

Under the assumption that $v_0 = v^p$, it follows that (see Appendix 5.2)[9]

$$\sum_{t=1}^{\infty} \beta^t (v_t - v_{t-1}) = 0. \tag{38}$$

Substituting (38) into (37), we obtain traders' consumption:

$$\overline{c^T} = y^p.$$

Traders' consumption is therefore the same under flexible and fixed exchange rates and they are thus indifferent between the two regimes. As for non-traders, it follows from (36) and (38) that the present discounted value of non-traders' consumption under fixed rates is the same as under flexible

[9] In a stochastic version of the model, the equivalent assumption would be that velocity shocks are white noise.

exchange rates. As a result, non-traders are clearly better off under flexible exchange rates in which case they have a flat path of consumption. Since traders are indifferent, we conclude that flexible exchange rates dominate.

What is the underlying intuition? The key lies in the role of the exchange rate as a shock absorber in the presence of velocity shocks. If velocity increases, the nominal exchange rate also increases (a nominal depreciation of the domestic currency) thus offsetting the shock. Under fixed exchange rates, the natural adjustment mechanism (i.e., the agents' ability to recompose their nominal money balances through the central bank) is not fully operative because non-traders cannot access asset markets. Hence, fluctuations in velocity lead to fluctuations in consumption. Specifically, an increase in velocity (i.e., $v_t > v_{t-1}$) implies that more money balances are available for consumption; a decrease in velocity (i.e., $v_t < v_{t-1}$) implies that less money balances are available for consumption.

3.2 Output shocks only

Suppose that there are only output shocks (i.e., set $v_t = \bar{v} > 0$). Then under flexible rates, consumption of non-traders and traders continues to be given by (29) and (30). Non-traders absorb the full variability of the endowment path.

Under fixed exchange rates, consumption of non-traders follows from (34):

$$c_t^{NT} = \begin{cases} y_0, & t = 0, \\ y_t + (1 - \bar{v})(y_{t-1} - y_t), & t \geq 1. \end{cases} \tag{39}$$

Under the assumption that $y_0 = y^p$, it follows that (see Appendix 5.3)

$$\sum_{t=1}^{\infty} \beta^t (y_{t-1} - y_t) = 0. \tag{40}$$

From (39) and (40), it follows that the present discounted value of c_t^{NT} under fixed rates will be the same as under flexible rates.

Consumption of traders follows from (35) and (40):

$$\overline{c^T} = y^p. \tag{41}$$

As is the case under velocity shocks, traders' consumption is the same under flexible and fixed exchange rates. Traders are therefore indifferent between the two regimes.

For expositional clarity, it is useful to rewrite non-traders' consumption as:

$$c_t^{NT} = \begin{cases} y_0, & t = 0, \\ \bar{v}y_t + (1 - \bar{v})y_{t-1}, & t \geq 1, \end{cases} \tag{42}$$

which makes clear that from $t = 1$ onwards, non-traders' consumption is an average of this period's and last period's output. Clearly, consumption of non-traders will fluctuate under both flexible and fixed exchange rates but will fluctuate less under fixed rates. Since, as shown above, the present discounted value of c_t^{NT} is the same under both regimes, non-traders' welfare will be higher under fixed exchange rates.

Intuitively, (42) states that today's consumption is a weighted average of last period's and this period's real sales revenues. Fixed exchange rates allow purchasing power to be transferred across periods which, as equation (42) makes clear, results in some consumption smoothing over time. In contrast, under flexible rates, a constant money supply implies that the real value of last period's sales is equal to current output. As a result, current consumption depends solely on current output.

We conclude that since traders are indifferent between the two regimes and non-traders are better off under a fixed exchange rate, social welfare will be maximized if, in response to output shocks, a fixed exchange rate is adopted.

4 Concluding remarks

One of the most influential results in open economy macroeconomics – which follows from any standard Mundell-Fleming model – holds that the choice of the optimal exchange rate regime should depend on the type of shock hitting the economy. If shocks are predominantly real, a flexible exchange rate is optimal, whereas if shocks are predominantly monetary, a fixed exchange rate is optimal.

We have shown that this influential result critically depends on the assumption that while there are frictions in goods markets (i.e., sticky prices), asset markets are frictionless. If we reverse these assumptions – frictionless goods markets and segmented asset markets – we turn the famous Mundell-Fleming dictum on its head: flexible rates are called for in the presence of monetary shocks whereas fixed exchange rates are optimal in the presence of real shocks. We thus conclude that the optimal exchange rate depends not only on the type of shock (monetary versus real) but also on the type of friction (goods market versus asset market).

A more modern approach to exchange rate regimes would view fixed and flexible exchange rate regimes as two particular cases of a more general monetary policy rule, which could in turn incorporate a response to current (if observable) and past shocks. In Lahiri, Singh, and Végh (2006b), we follow this more general approach and show how an optimal monetary policy rule would actually involve responding to contemporaneous shocks. It is only in the absence of output shocks (i.e., a world with only velocity shocks) that a "pure" flexible exchange rate – as studied in this paper – would be optimal.

5 Appendices

5.1 Conditions for a binding cash-in-advance

This appendix derives the conditions needed for the cash-in-advance to bind for both non-traders and traders and then provides an example of the restrictions that need to be imposed on the output and velocity processes.

5.1.1 When does the cash-in-advance bind for non-traders?

Non-traders choose $\{c_t^N, M_{t+1}^N\}_{t=0}^{\infty}$ to maximize lifetime utility (1) subject to the sequence of flow constraints given by (2) and the sequence of cash-in-advance constraints given by (3) for a given M_0. In terms of the Lagrangian:

$$\underset{\{c_t^{NT}, M_{t+1}^N, \lambda_t, \Psi_t\}}{\text{Max}} \qquad L = \sum_{t=0}^{\infty} \beta^t u(c_t^N) + \sum_{t=0}^{\infty} \beta^t \lambda_t (M_t^N + E_t y_t - E_t c_t^N - M_{t+1}^N) + \sum_{t=0}^{\infty} \beta^t \Psi_t \left(M_t^N + v_t E_t y_t - E_t c^N \right).$$

The first-order conditions for c_t^N and $M_{t+1}^N$are given by:

$$u'(c_t^N) = E_t(\lambda_t + \Psi_t), \tag{43}$$

$$\beta(\lambda_{t+1} + \Psi_{t+1}) = \lambda_t. \tag{44}$$

The Kuhn-Tucker condition for Ψ_t is given by:

$$\begin{aligned} M_t^N + v_t E_t y_t &\geq E_t c^N, \quad \Psi_t \geq 0, \\ \left(M_t^N + v_t E_t y_t - E_t c^N\right) \Psi_t &= 0. \end{aligned}$$

Suppose that $\Psi_t > 0$; that is, the cash-in-advance constraint binds. Then, it follows from (43) and (44) that

$$\frac{u'(c_{t+1}^{NT})}{u'(c_t^{NT})} = \frac{1}{\beta} \frac{E_{t+1}}{E_t} \frac{\lambda_t}{\lambda_t + \Psi_t}.$$

Hence, for the cash-in-advance to bind, it must be the case that

$$u'(c_t^{NT}) > \beta \frac{E_t}{E_{t+1}} u'(c_{t+1}^{NT}). \tag{45}$$

If the cash-in-advance binds, it means that non-traders prefers not to carry over nominal money balances from one period to the next even though doing so would provide more consumption tomorrow. In other words, money balances are not used for saving purposes. In this case – and as condition (45) indicates – the consumer is unwilling to save and therefore today's marginal utility will be higher than tomorrow's adjusted by the discount factor and the return on money.

To fix ideas, consider the case of logarithmic preferences. Condition (45) then reduces to:

$$c_t^{NT} < c_{t+1}^{NT} \frac{1}{\beta} \frac{E_{t+1}}{E_t}. \tag{46}$$

Using the quantity theory (equation (21)), we can rewrite this equation as

$$\frac{c_t^{NT}}{c_{t+1}^{NT}} < \frac{1}{\beta}(1 + \mu_{t+1}) \left(\frac{1 - v_t}{1 - v_{t+1}}\right) \left(\frac{y_t}{y_{t+1}}\right). \tag{47}$$

Flexible exchange rates Consider the case of flexible exchange rates with a constant money supply. In this case, $c_t^{NT} = y_t$ and $\mu_{t+1} = 0$. Equation (47) then reduces to

$$\beta < \frac{1 - v_t}{1 - v_{t+1}}. \tag{48}$$

As long as this condition holds (which implies a restriction on the variability of the velocity shocks), the cash-in-advance constraint will bind. Clearly, since this conditions involves exogenous variables, one can always choose parameters such that it will hold.

Fixed exchange rates Consider the case of fixed exchange rates. In this case, $E_{t+1} = E_t = \bar{E}$. In this case, use condition (46), taking into account (34), to obtain:

$$\beta < \frac{(1 - v_t)y_t + v_{t+1}y_{t+1}}{(1 - v_{t-1})y_{t-1} + v_t y_t}. \tag{49}$$

Again, since this condition involves only exogenous variables, one can always choose β and output and velocity processes such that it holds.

Intuition To understand the intuition as to why the cash-in-advance may bind for non-traders, consider the case of flexible exchange rates and no velocity shocks (i.e., output shocks only). In that case, condition (48) will always hold because, by assumption, $\beta < 1$. Intuitively, suppose that $y_t > y_{t+1}$ and consider the non-trader's choice at time t. Based on the consumption smoothing motive, non-traders would want to save in order to consume more next period when output will be low. However, given that $\mu_t = 0$, periods of high output will coincide with periods in which the real return on nominal money balances is low. To see this, notice that using the cash-in-advance the gross real return on holding money is given by

$$\frac{E_t}{E_{t+1}} = \frac{y_{t+1}}{y_t}.$$

Since $y_t > y_{t+1}$, then $E_t/E_{t+1} < 1$ which means a negative real return on money. Hence, with logarithmic preferences, the non-trader's desire to dissave based on the negative real return on money more than offsets the desire to save based on consumption smoothing motives.

5.1.2 When does the cash-in-advance bind for traders?

For the CIA to bind for traders, we just need to ensure that the nominal interest rate is positive. The restrictions needed for this depend on the exchange rate regime.

Flexible exchange rates From the interest parity condition (16), a positive nominal interest rate requires that

$$\frac{E_{t+1}}{E_t} > \frac{1}{1+r}.$$

Using the quantity theory equation (21), it follows that

$$\frac{E_{t+1}}{E_t} = \frac{1-v_{t-1}}{1-v_t}\frac{y_{t-1}}{y_t}.$$

Combining the last two equations – and recalling that $\beta(1+r) = 1$ – it follows that if

$$\beta < \frac{1-v_{t-1}}{1-v_t}\frac{y_{t-1}}{y_t}, \tag{50}$$

then the nominal interest rate will always be positive and the CIA will always bind for traders as well.

Fixed exchange rates Under fixed exchange rates, the interest parity condition (16) indicates that the nominal interest rate will always be positive since $1+i = 1+r$.

5.1.3 An example

Let us illustrate the restrictions necessary to ensure a binding cash-in-advance constraint for the cases of only one shock at a time (the case studied in the text). Suppose $\beta = 0.96$.

Output shocks only Suppose that $v_t = \bar{v} = 0.2 > 0$ and that y_t alternates between 1.04 and 1. For non-traders, (48) holds since $\beta < 1$ and condition (49) becomes (assuming the most restrictive case which is $y_{t-1} = 1.04$, $y_t = 1$, and $y_{t+1} = 1.04$):

$$\beta < \frac{(1-\bar{v})y_t + \bar{v}y_{t+1}}{(1-\bar{v})y_{t-1} + \bar{v}y_t},$$

which reduces to $\beta < 0.977$ and hence holds. For traders, (50) is satisfied since $\beta < y_t/y_{t+1} = 0.962$ and hence the CIA binds under both flexible and fixed exchange rates.

Velocity shocks Suppose that $y_t = y_{t+1} = y^p$. The velocity variable alternates between two values: 0.20 and 0.22. Assume first that $v_{t-1} = 0.2$, $v_t = 0.22$, and $v_{t+1} = 0.2$. Then, for non-traders under flexible rates, it must be the case that

$$\beta < \frac{1 - v_t}{1 - v_{t+1}}, \tag{51}$$

which holds since $\beta < 0.975$. Under fixed rates, it must the case that

$$\beta < \frac{1 - v_t + v_{t+1}}{1 - v_{t-1} + v_t}, \tag{52}$$

which holds - since under the most restrictive case in which $v_{t-1} = 0.2$, $v_t = 0.22$, and $v_{t+1} = 0.2$, then $\beta < 0.961$.

For traders, the cash-in-advance always holds.

5.2 Proof that $\sum_{t=1}^{\infty} \beta^t (v_t - v_{t-1}) = 0$ if $v_0 = v^p$

Rewrite $\sum_{t=1}^{\infty} \beta^t (v_t - v_{t-1})$ as

$$\sum_{t=1}^{\infty} \beta^t (v_t - v_{t-1}) = -\beta v_0 + (1 - \beta) \sum_{t=1}^{\infty} \beta^t v_t.$$

But, by definition of v^p and given that $v_0 = v^p$, $\sum_{t=1}^{\infty} \beta^t v_t = \frac{v^p}{1-\beta} - v^p = \frac{\beta}{1-\beta} v^p$. Hence,

$$\sum_{t=1}^{\infty} \beta^t (v_t - v_{t-1}) = \beta(v^p - v_0) = 0.$$

as $v_0 = v^p$.

5.3 Proof that $\sum_{t=1}^{\infty} \beta^t (y_{t-1} - y_t) = 0$ if $y_0 = y^p$

Replace v by y in Section 5.2 above.

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