NBER WORKING PAPER SERIES

PAASCHE VS. LASPEYRES: THE ELASTICITY OF SUBSTITUTION AND BIAS IN MEASURES OF TFP GROWTH

Alwyn Young

Working Paper 6663
http://www.nber.org/papers/w6663

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
July 1998

Paasche vs. Laspeyres: The Elasticity of
Substitution and Bias in Measures of TFP Growth
Alwyn Young
NBER Working Paper No. 6663
July 1998

ABSTRACT

In recent papers, Nelson and Pack (1995) , Rodrik (1997), and Hsieh (1997a) argue that standard measures of total factor productivity growth in countries where the capital-labour ratio has risen rapidly, e.g. the East Asian NICS, will understate true productivity growth if the elasticity of substitution is less than one and there is labour augmenting technical change. This note shows that this argument increases a Paasche measure of productivity, at the expense of lowering a Laspeyres estimate. The conditions under which total factor productivity growth is consistently underestimated are clarified.

Alwyn Young
Graduate School of Business
University of Chicago
1101 East 58th Street
Chicago, IL 60637
and NBER
a-young2@uchicago.edu

In recent papers, Nelson and Pack (1995), Rodrik (1997) and Hsieh (1997a) argue that standard measures of total factor productivity growth in countries where the capital-labour ratio has risen rapidly will substantially understate true productivity growth if the elasticity of substitution is less than one and there is labour augmenting technical change. All four authors apply their argument to my (1995) estimates of productivity growth in the East Asian NICs, suggesting that, using standard techniques, I fail to measure the importance of technical change. This note discusses their proposition.

To understand the Nelson-Pack-Rodrik-Hsieh (NPRH) argument, consider a general production function of the form $Y = F(E_K K, E_L L)$, where the E_i's are factor augmenting technology parameters. In this case, the rate of growth of the share of capital is given by:

$$(1)\quad \hat{\theta}_K = (1-\theta_K)\left(\frac{\sigma-1}{\sigma}\right)\left[(\hat{K}-\hat{L})-(\hat{E}_L-\hat{E}_K)\right]$$

where σ is the elasticity of substitution. The standard approach weights the growth of the capital-labour ratio with the average of the *observed* initial and final shares of capital. NPRH argue that the elasticity of substitution may be small so that, absent technical change, the final share of capital would have been extremely low. Labour augmenting technical change could, however, sustain the capital share, in which case the standard approach overweights the contribution of capital accumulation and understates the role of technical change.

Image 1: Hsieh's (1997a) Figure I

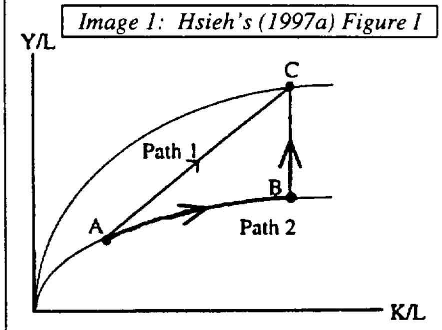


Figure 1

Hsieh (1997a), who provides the most extensive discussion and analysis, conceptualizes the NPRH argument in his Figure I (reproduced in Image I). In his view, the contribution of capital accumulation is the movement from A to B (taking into account the diminishing returns to capital), with the contribution of technical change being given, then, by the movement from B to C. As Hsieh explains:

> [The] difficulty of separately identifying the contribution of factor accumulation from that of technological change is related to the properties of the TFP index as a divisia (or chain) index. Since a divisia index is a line integral, we are essentially calculating a line integral when we estimate a TFP index. Since line integrals are generally path dependent, a TFP index is also path dependent. This means that for an economy that moved from point A to C in Figure 1, a TFP index calculated over path 1 will not be the same as an index computed over path 2. Since

we want the TFP index to measure the shift of the production function, which is the distance between B and C in Figure 1, the correct TFP index is obtained by integrating over path 2...

...When is the change in the TFP index over path 1 equal to the change in the TFP index over path 2? In other words, when is the TFP index path independent?...This paper combines the main element of Hulten's proof (the fundamental theorem of calculus for line integrals) with Leontief's (1943) theorem on aggregation to show that the TFP index is path independent when technological change is Hicks-neutral...(1997a, pp. 3-4)

Following an "if and only if" mathematical proof on Hicks-neutrality and path independence, Hsieh uses a CES production function and the growth rates from my 1995 paper to show that if σ equals .3 the actual rate of total factor productivity growth in Singapore between 1966 and 1990 was 2.0% per annum, well above the 0.2% I found using standard techniques.

The easiest way to proceed is to produce the mathematics that underly Hsieh's estimate of 2.0% "true" productivity growth in Singapore. As a a preliminary, I note that Hsieh uses the CES production function:

$$(2)\ \ Y = \left(\gamma (E_L L)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)(E_K K)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

For this functional form, the share of capital is given by:

$$(3)\ \ \theta_K = \left(\frac{(1-\gamma)(E_K K)^{\frac{\sigma-1}{\sigma}}}{\gamma (E_L L)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)(E_K K)^{\frac{\sigma-1}{\sigma}}}\right) = \left(\frac{(1-\gamma)(E_K K)^{\frac{\sigma-1}{\sigma}}}{Y^{\frac{\sigma-1}{\sigma}}}\right)$$

Finally, to complete our preparation, we need the raw data for the NICs, which Hsieh (1997a, pp. 11-12) takes from my 1995 paper:[1]

Table I: Annual Growth Rates for the NICs

	Hong Kong (1966-1991)	Singapore (1966-1990)	South Korea (1966-1990)	Taiwan (1966-1990)
Output	7.3	8.7	10.3	8.9
Capital	8.0	11.5	13.7	12.3
Labour	3.2	5.7	6.4	4.9
Initial Capital Share	0.357	0.497	0.310	0.261

S: Hsieh 1997a, pp. 11-12, and Young 1995.

[1]For Taiwan, Hsieh reports the unadjusted growth of non-agricultural Taiwanese output (9.4%). I modify the Table to incorporate my adjustment (-0.5%) for the overstatement of public sector output growth in that economy. Otherwise, to stay close to his analysis, I follow Hsieh's rendition of my numbers. I should note that what Hsieh calls the initial capital share is actually the average share across the first period of my 1995 estimates.

To reproduce Hsieh's analysis, we first need to figure out what the share of capital at the end of the sample period would have been absent technical change. Let:

$$(4)\ \tau = \left(\frac{(1-\gamma)(E_K K)^{\frac{\sigma-1}{\sigma}}}{\gamma\,(E_L L)^{\frac{\sigma-1}{\sigma}}} \right) \text{ and } \theta_K = \left(\frac{\tau}{1+\tau} \right)$$

Assuming that σ equals .3, we know from the Singaporean data that, absent technical change:

$$(5)\ \frac{\tau_T}{\tau_0} = [\exp(.058 * 24)]^{-7/3} = f = .0389$$

while:

$$(6)\ \tilde{\theta}_K(T) = \left(\frac{f\tau_0}{1+f\tau_0} \right) = \left(\frac{f\theta_K(0)}{1-\theta_K(0)+f\theta_K(0)} \right) = .037$$

where the ~ reminds the reader that the share of capital in period T is the notional share, i.e. the share absent technical change. From equation (3), the reader can see that absent technical change the growth of output is given by:

$$(7)\ \ln(Y_T/Y_0) = \ln(K_T/K_0) + \frac{\sigma}{1-\sigma}\ln(\tilde{\theta}_K(T)/\theta_K(0)) = .069 \text{ (annual)}$$

Absent technical change, the growth of capital and labour in the Singaporean economy would have led to output growth of 6.9% per annum (the movement from A to B in Hsieh's figure). Since output actually grew 8.7% per annum, the contribution of technical change was 1.8% per annum (the movement from B to C in the figure).[2]

Allow me to suggest an alternative calculation. Instead of first calculating the contribution of factor accumulation, and then estimating the contribution of technical change, why not calculate the contribution of technical change first, and then estimate the contribution of factor accumulation. In other words, in terms of Hsieh's Figure I, take the contribution of technical change as a vertical movement up from point A, and the contribution of factor accumulation as the (subsequent) movement along the upper production function. Take the final

[2]As is the case in my (1998) discussion of Hsieh (1997b), I find it difficult to reproduce Hsieh's estimates, despite the fact that I use the data and functional forms specified by Hsieh himself. According to him, absent technical change the share of capital in Singapore would have fallen to .029 (p. 12). My number is .037. Using Hsieh's number, and equation (7), one sees that the contribution of technical change should then be 2.3%, not the 2.0% he reports. For Korea, Hsieh states the share of capital would have fallen to .005. I find that it would have fallen to .0075. Hsieh states that, with sigma equal to .3, technical change in South Korea would be 3.3% per annum. Using his number of .005 and equation (7) one finds it to be 4.0% per annum. My own computations indicate that it would be 3.3% per annum (see below), so this number of mine agrees with his, but only after ignoring the discrepancies in the intermediate steps. Similar problems appear with Hong Kong and Taiwan.

capital share in Singapore as .497.[3] Assuming an elasticity of substitution of .3, what would the initial capital share have been, absent technical change? Rearranging (6), one sees that:

$$(8)\quad \tilde{\theta}_K(0) = \frac{\theta_K(T)}{f(1-\theta_K(T)) + \theta_K(T))} = 0.962$$

Absent technical change, the contribution of factor accumulation (along the upper production function) is then given by:

$$(9)\quad \ln(Y_T/Y_0) = \ln(K_T/K_0) + \frac{\sigma}{1-\sigma}\ln(\theta_K(T)/\tilde{\theta}_K(0)) = .103 \text{ (annual)}$$

Given realized output growth of 8.7% per annum, it follows that the contribution of technical change was -1.6% (annualized).

What sorcery! How does one turn productivity growth of +1.8% per annum into -1.6% per annum? For the intuition, the reader need only examine equation (1) above. Nelson-Pack-Rodrik-Hsieh wish to argue that the elasticity of substitution between labour and capital is small, but must also deal with the fact that the capital share in the NICs has not collapsed. The solution is to argue that there is labour biased technical change which offsets the growth of the capital-labour ratio. Unfortunately, the growth of the capital-labour ratio in Singapore, South Korea and Taiwan is 6% or more per annum.[4] If $\hat{E}_K$ is to be positive, it is then necessary that $\hat{E}_L$ be greater than 6%. This, in turn, would imply enormous output growth which in my measures would have shown up as *high* productivity growth. Output, however, simply hasn't grown that fast. From equation (3) one sees that if the share of capital is roughly constant, then the growth of the capital augmentation factor, E_K, is equal to the growth of the output-capital ratio, -2.8% for Singapore (using my 1995 numbers), while the growth of the labour augmentation factor, E_L, is equal to the growth of the output-labour ratio (3.0%). In order for the Nelson-Pack-Rodrik-Hsieh framework to explain the facts of East Asian growth, it is not only necessary that factor augmenting technical change offset the growth of the capital-labour

[3]The capital shares in the NICs fluctuate, but do not have any statistically significant trend. In order to focus on the important issues, in what follows I assume the capital share is constant.

[4]Of course, with the reduction of investment in the 1980s and 1990s, the growth of the capital-labour ratio in Singapore has slowed (averaging only 4.6% across 1966-1995 in my 1998 estimates). Unfortunately, as this measure goes to zero, the motivation for the NPRH argument also vanishes.

ratio, it is also necessary (given the relatively slow growth of output relative to factor accumulation) that the production function *rotate*.

In Figure I below I redraw Hsieh's figure I. Hsieh argues formally (as does Rodrik 1997) that the contribution of factor accumulation should be measured as the movement from A to B, with the contribution of technical change then given by the movement from B to C . This is, in effect, a Paasche-like measure of productivity growth, one where the productivity growth is conceptually deemed to have occurred at the end of the period of analysis. One could just as easily measure the contribution of technical change as the movement from A to D, and the contribution of factor accumulation as the movement from D to C, a Laspeyres-like measure in which the productivity growth is assumed to occur at the beginning of the period.[5] In Table II below I report the "Paasche", "Laspeyres" and standard measures of productivity growth derived using the data in Table I earlier.[6] As the reader can see, the NPRH Paasche procedure raises the TFP estimate above the standard measure, but only at the cost of producing a lower Laspeyres estimate. According to NPRH, labour augmenting technical change rescues the economy from the dismal growth that would have followed the rapid diminuition of the marginal product of capital. Using the very same framework, one can just as easily argue that rapid capital accumulation rescues the economy from the output collapse that would have followed capital augmenting technical regression.

Figure I: Revising Hsieh's Figure I

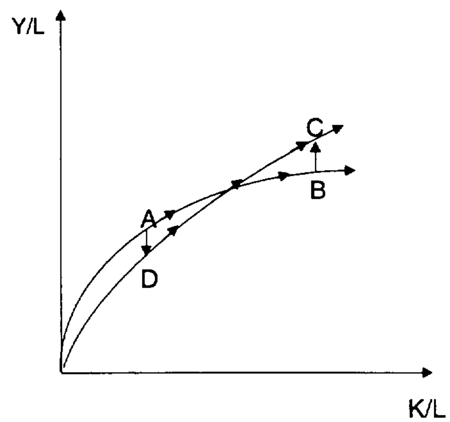


Table II: Raising and Lowering the Standard Estimates

	Hong Kong	Singapore	South Korea	Taiwan
"Paasche"	3.4	1.8	3.3	3.5
Standard	2.4	0.1	1.6	2.1
"Laspeyres"	0.9	-1.6	-1.4	-1.1

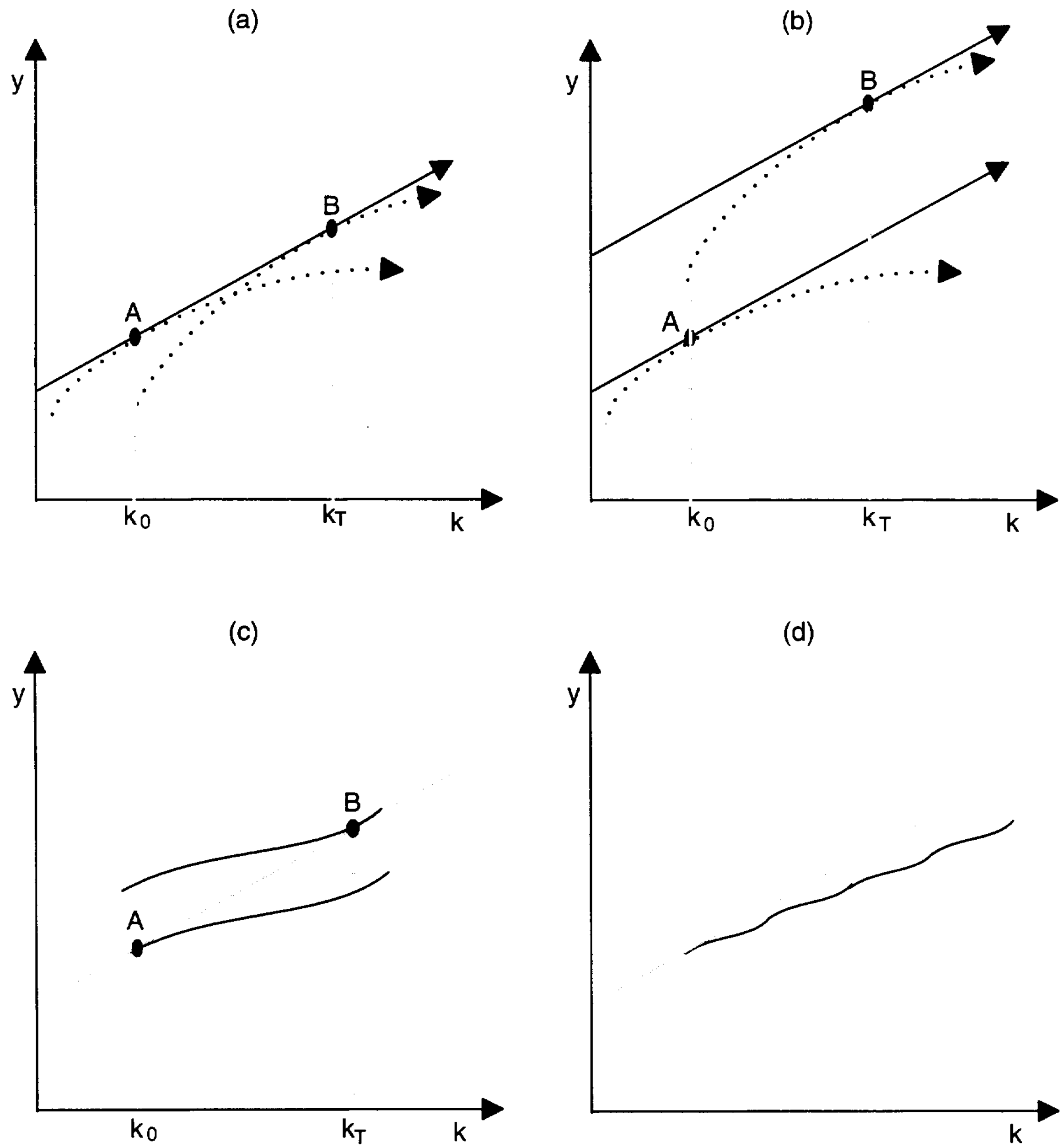
[5]These measures are not, of course, literally Paasche and Laspeyres as they do not make use of the observed initial and final factor shares but, rather, involve an argument concerning what the shares would have been absent (or following) technical change and capital accumulation.

[6]For the standard measure, I continue to follow the assumption (noted in an earlier footnote) that the share of capital is constant. Consequently, the standard measure I record in the Table does not agree exactly with those I reported in my 1995 paper (where I used the time varying shares).

An additional diagram provides further intuition, clarifying, in the process, a theoretical confusion introduced by Hsieh. Let lower case y and k denote the lns of output per worker and capital per worker. In Figure II I graph y against k. In this diagram, the slope of the production function represents the elasticity of output with respect to capital, i.e. θ_K. If the production function were Cobb-Douglas, it would be described by a straight-line such as the one that passes through A and B in panel (a). Say I observe the economy at time 0 and T, with k and y given by points A and B and with a constant θ_K equal to the slope of the ray going through A and B. Using standard growth accounting techniques, I would then conclude that there was zero productivity growth. NPRH wish to argue that the elasticity of output with respect to capital is rapidly declining in k, which I depict with the dashed curve running through point A. NPRH must also, however, match the fact that the share of capital at time T is the same as that observed at time 0. To do so, they have to turn their production function so that it is tangential to the ray at point B. Given that the capital elasticity is *declining* in k, this then implies that the function falls below the ray at points to the left of B. Consequently, while the Paasche measure yields positive estimates of productivity growth, the Laspeyres measure is negative. Were output to have grown fast enough, the Laspeyres measure could be positive, as depicted in panel (b). In this case, however, the standard measure will also be highly positive (equal to the distance between the two parallel rays). As long as the capital elasticity is monotonically declining in k, as is the case with the CES function emphasized by Rodrik and Hsieh, the Laspeyres measure will always be less than the standard measure of productivity growth.

With the above in mind, it is easy to see how the NPRH approach can be modified so that both the Paasche and Laspeyres measures exceed the standard measure. In diagram (c) I draw a production function whose capital elasticity first falls, and then rises back to its original value. Between periods 0 and T this function shifts up uniformly. Consequently, I observe the two points along the dashed ray, both with the same θ_K and conclude, mistakenly, that technical progress is zero. The Paasche and Laspeyres measures, which are identical, are both positive. The "trick" here is that the capital elasticity is non-monotonic with respect to k. This example generates a "one shot" error. To make the error more persistent, I depict (in panel d) a production function whose capital elasticity follows a sinusoidal pattern, rising and falling periodically. If this production function is shifting up all the time, and I always observe the

Figure II: Paasche & Laspeyres Measures of Productivity Growth



capital elasticity at its maximum value, I will understate productivity growth consistently. However, as the reader can readily see, if I always observe the minimum value (lowest slope) I will consistently *overstate* the rate of productivity growth. Consequently, for the Paasche and Laspeyres measures to both exceed the standard measure, it is necessary that the capital elasticity be non-monotonic and, further, that in all my observations I always observe the capital share at its higher values.

In sum, while it possible to construct other examples, it is easy to see that they will all share the following features: (1) To maximize the error of the standard measure it is necessary that the capital elasticity vary substantially as one moves along a given production function. (2) Given that, empirically, the capital elasticities in the NICs have not moved that much, the theorist has to show that technical change constantly interacts with the production function, from period to period, to ensure that it is always tangent to, or very nearly so, a ray with a constant slope. And, (3) For both the Paasche and Laspeyres measures to be greater than the standard measure it is necessary that the constant slope of the ray exceed the average value of the capital elasticity of the production function (both before and after the technical change) measured across the capital output ratios that the economy has traversed (i.e. the average value between k_0 and k_T).

As a final point, I ask the reader to reexamine panel (c) in the Figure above. As the reader can see, the technical change depicted in this diagram is Hicks-Neutral, but the standard measure of productivity growth *is* biased. From this it follows that Hsieh's discussion and if and only if proofs of path independence, line integrals and Hicks Neutrality are somewhat unnecessary. The confusion here has to do with continuous versus discrete time. In continuous time, with Hicks-Neutral technical change a Divisia index will be path independent (integrate any of the curves in panel (c)). Consequently, in continuous time, to argue for bias, one has to specify non-neutral technical change. Further, to get the type of bias NPRH want, one has to argue that the capital elasticity is rapidly declining in k and, to match the facts, that the bias of technical change interacts with the rate of capital accumulation so as to keep the capital share virtually unchanged at all the *discrete* points of observation. In discrete time bias emerges as long as the capital elasticity, at the points it is measured, is higher than the average (either before or after technical change) between the points of observation. This is less stringent.

Bibliography

Hsieh, Chang-Tai. "Measuring Technological Change." Working Paper, University of California, Berkeley, March 1997(a).

Hsieh, Chang-Tai. "What Explains the Industrial Revolution in East Asia? Evidence from Factor Markets." Working Paper, University of California, Berkeley, November 1997(b).

Nelson, Richard and Pack, Howard. "The Asian Growth Miracle and Modern Growth Theory." Manuscript, 1995.

Rodrik, Dani. "TFPG Controversies, Institutions, and Economic Performance in East Asia." NBER Working Paper No. 5914, February 1997.

Young, Alwyn. "The Tyranny of Numbers: Confronting the Statistical Realities of the East Asian Growth Experience." Quarterly Journal of Economics (August 1995): 641-680.

Young, Alwyn. "Alternative Estimates of Productivity Growth in the NICs: A comment on the work of Chang-Tai Hsieh." Manuscript, 1998.